Gearbox Design Report

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ME 330: Machine Design
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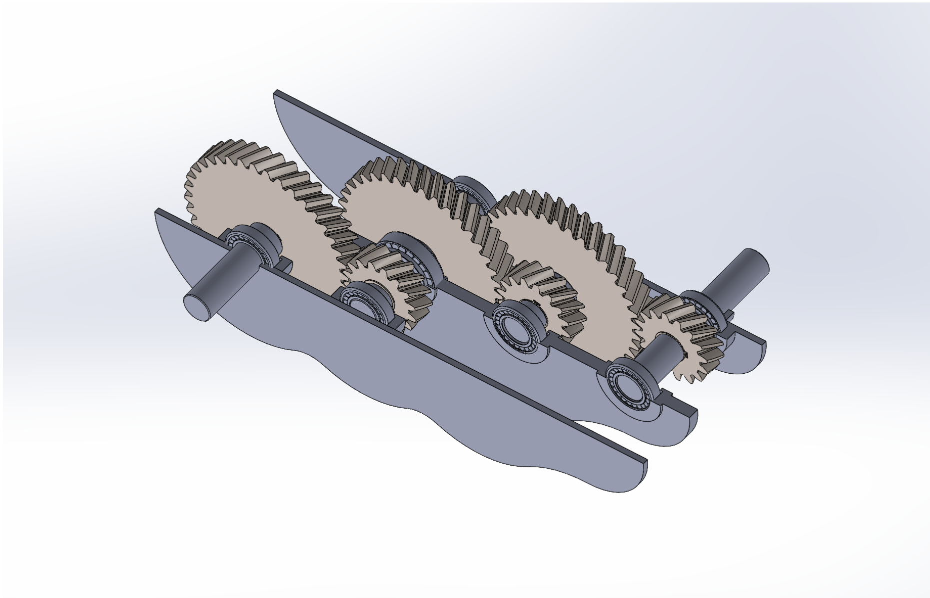
Contents



1 Introduction

1.1 Project Statement

Using machine design principles, comprehensively design and create a CAD model of a three-stage speed reduction gearbox.

1.2 Overall Requirements

- Reduce the shaft speed from an input of 1,750 rpm to an output of 130 ± 5 rpm.
- Tolerate an input power of 135 hp.
- Consist of four shafts, one input, two intermediate, and one output.
- Use helical gears only.
- Be lubricated and sealed.

1.3 Design Objectives

- Minimize the total size of the gearbox.
- Ensure the gearbox is easy to assemble and disassemble.
- Ensure manufacturability of all components.

1.4 Methods

- Design report will be prepared with LaTeX.
- CAD models, assemblies, drawing, simulations, and rendering will be prepared with Solidworks.

2 Gears Specifications

2.1 Requirements

- Overall gear train must convert 1,750 rpm to 130 ± 5 rpm.
- Gear train must handle an input power of 135 hp.
- Gear train must be as compact as possible.
- All gears must be helical.
- Gears must be designed for a service life of 2,000 hours per year and 6 years (12,000 hours total).
- Gears will be lubricated.
- AGMA (American Gear Manufacturers Association) design methodology will be used.

2.2 Design Decisions

- All gears will use a standard pressure angle of 20°. This is the most common pressure angle used for gearing and thus tooth cutters will be available for the greatest number of diametral pitches [1].
- All gears will have full-depth teeth.
- All gears will have a helix angle of 30°. This will allow for maximum advantage of strength and noise benefits of helical gears.

2.3 Gear Ratio

In order to achieve the smallest overall package size, we want our three stage ratios to be as close together as possible. Our requirements allow us to deviate from our target output speed of 130 rpm by as much as 5 rpm. If we assume the ideal case of all stage ratios being equal, we can calculate a target individual stage ratio:

$$\frac{N_1}{N_2} = \frac{N_3}{N_4} = \frac{N_5}{N_6} = \sqrt[3]{\frac{13}{175}} \approx \frac{1}{2.379}$$

Now we can consider possible ratios that get close enough to this value by limiting the denominator of this ideal ratio, to get the closest simple fraction approximations.

Table 1: Stage Gear Ratios Approximating a Ratio of 13:175

Stage Ratio $N_P : N_G$	Train Value e	Output Speed ω_{out} (rpm)	Output Error $\Delta\omega_{\text{out}}$ (rpm)
37 : 88	0.0743	130.1	0.075
29 : 69	0.0742	129.9	0.077
21 : 50	0.0741	129.7	0.35
8 : 19	0.0746	130.6	0.63
5 : 12	0.0723	126.6	3.4
3 : 7	0.0787	137.8	7.8
2 : 5	0.0640	112.0	18

See Appendix A for generation source code used to generate this data.

From Table 1, we see that we have five ratios to choose from that will produce a output speed within 5 rpm of our target output speed of 130 rpm. We will choose a stage ratio of 8 : 19 to get very close to the target speed, and allow for scaling the ratio if we need to change our diametral pitch. Starting by doubling the values to 16 : 38 and testing for meshing interference:

$$N_{\mathrm{G,\ max}} = \frac{N_P^2 \sin^2(\phi_t) - 4k^2\cos^2(\psi)}{4k\cos(\psi) - 2N_P\sin^2(\phi_t)} = \frac{16^2\sin^2(20°) - 4(1)\cos^2(30°)}{4\cos(30°) - 2(16)\sin^2(20°)} = -26.45 \qquad \text{(13-23) [1]}$$

This means that there is no gear size of this specification that would interfere with a pinion of 16 teeth, so we know there will not be any interference in the gear train. We can now obtain our values for torque and speed:

Table 2: Transmission Values by Shaft

	Input	Intermediate		Output
Shaft	A	B	C	D
Speed (rpm)	1750	736.8	310.2	130.6
Torque (lbf · ft)	405.2	962.3	2285	5428
Pinion Tooth Count	16	16	16	–
Gear Tooth Count	–	38	38	38

For now, we can now proceed with tooth counts:

$$\frac{N_P}{N_G} = \frac{N_1}{N_2} = \frac{N_3}{N_4} = \frac{N_5}{N_6} = \frac{16}{38}$$

2.4 Estimation of Diametrical Pitch

In order to reduce iteration, we will try to get a preliminary estimations of a diametrical pitch for our gears. To do so, we will analyze the likely critical gear in our gearbox, which is #5, (pinion of shaft C) as it is the pinion with the highest torque.

We will use equation 14-8 and rewrite it in terms of diametrical pitch P:

$$\sigma = \frac{K_v P W_t}{FY} \implies P = \frac{FY\sigma}{K_v W_t} \qquad \text{(14-8) [1]}$$

We can simplify the gear to a spur gear, and use the following assumptions [1]:

- F : The face width is recommended to be $3p$ to $5p$.
- Y : The Lewis form factor ranges from 0.24 to 0.485.
- σ : Endurance strength of steel gears range from 22 ksi to 75 ksi.
- K_v : The velocity factor is a function of pitch-line velocity V, which is dependent on P. The velocity factor ranges from $\frac{600+V}{600}$ to $\sqrt{\frac{78+\sqrt{V}}{78}}$.

We can now rewrite all variables as constants or in terms of P using $pP = \pi$, $r = \frac{N}{2P}$, $V = \omega r$, and $W_t = 135\,\mathrm{hp}/V$.

After solving numerically, we can obtain a very conservative value of P of 1.891 teeth/in, and a nonconservative value of 5.338 teeth/in. For now, we will choose from tooth sizes in general use [1], and select $P_t = 3$ teeth/in.

3 Shaft Layout

Our next step is specify the general layout of the shafts and axial locations of gears and bearings. In additon, we will address attaching components to shafts.

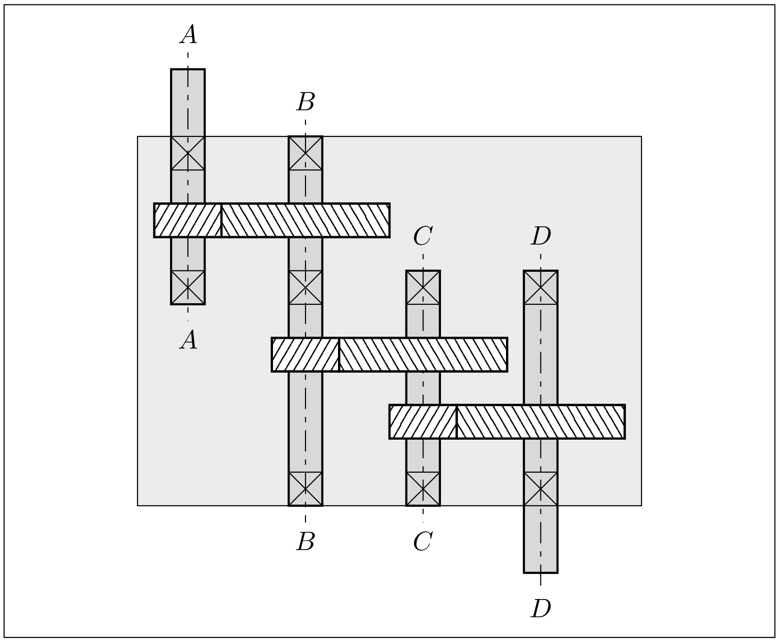


Figure 1: First Iteration of General Gearbox Shaft Layout

3.1 Requirements

- There must be 4 shafts: one input, two intermediate, and one output.
- Gearbox size must be minimized.
- Shafts should be as short as possible.
- Input and output shafts must extend outside of the gearbox housing by two times their diameter.
- Where possible, shaft diameters should be standard sizes.
- Shafts geometry should include means for rotating components to be affixed, such as keyways and retaining rings.

3.2 Decisions

- In order to transmit torque via shafts, keyways will be used.
- Because helical gears will have a great axial force on one side, and little to no force on the other, a mix of shoulders and retaining rings will be used.
- Shoulders will be used to hold shafts in place on bearings.

3.3 General Layout

Based on these requirements and design decisions I designed a first iteration of the shaft layout. My layout uses three planes of bearings and three planes of gears to eliminate interference between gears. In this layout, shaft B is the longest shaft, but is supported by three bearings, reducing the effective length. This layout has one pair of gears above the middle bearing plane, and two pairs below. I believe this is a flaw with this design: this causes shafts C and D to have to be longer to accommodate the additonal gears and clearance. As shaft C or D are likely critical due to the high torque on them, it is wise to adjust this design to reduce the length of these shafts.

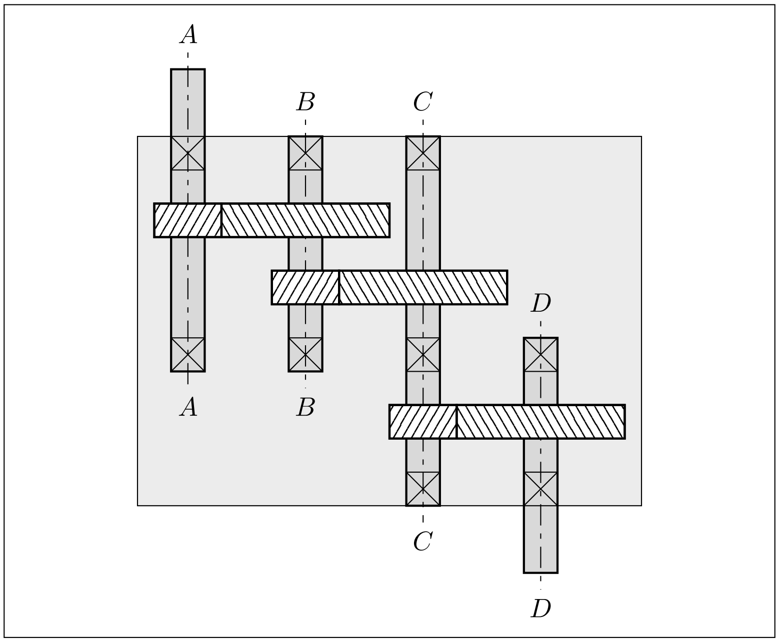


Figure 2: Final General Gearbox Shaft Layout

I modified the layout to move an additional pair of gears above the middle plane of bearings. This allowed me to reduce the length of shafts C and D by increasing the lengths of A and B. This is reasonable, because it reduces the stress of the most critical shafts, reducing the required diameter. Shaft C is now the longest shaft, but again, the middle bearing significantly decreases the effective length.

3.4 Dimensions

From the general layout, axial dimensions now have to be specified. Each shaft's length is a combination of the following lengths:

- F Face Width for helcial gears is recommended to be at least two times the axial pitch. Using formula (13-17)[1], $p_x = p_t/2 \cdot \tan(\psi) = 2 \cdot \pi/3/\tan(30°) = 1.81$. Increase to 2 in and revisit if needed.
- S : The axial length of the shoulder around the gear is determed by proportionality to the shaft minor diameter and shoulder diameter. This length will also accommodate the retaining hardware on the side of each gear opposite the axial force and shoulder. To meet both requirements, choose 1 in.
- B : The axial length of the bearing is determined by the shortest bearing able to support the required load of each shaft. Estimate to be 1 in.
- E : The external length of the input and output shafts from the gearbox housing. Per design requirements this must be double the diameter of the external shaft. As diameters are currently unknown, estimate to be 2.5 in. This dimension will later accommodate shaft seals and likely need to be increased. but this portion of the shafts will just be under torsion and will not be critical, and therefore changes in E will likely be inconsequential for stress life.

3.5 Force Analysis

Now that gear diameters and axial dimension are known we can produce free-body diagrams, shear-moment diagrams, and obtain maximum bending moments for each shaft.

4 Shaft Design

4.1 Decisions

- Since all shafts must be designed for an infinite life, all shafts will be made of steel.
- To reduce costs and manufacturing difficulty, all shafts will be made of a single material.

4.2 Force Analysis

Due to the speed of the gearbox decreasing and the torque increasing, shaft C or D are the critical shaft. I will conduct a force analysis on both to find the most critical point.

Shaft C:

$$
\begin{aligned}
V_{34} &= \frac{\omega_C N_4}{P} = 1{,}029\,\text{ft/min} & V_{56} &= \frac{\omega_C N_5}{P} = 433\,\text{ft/min} \\
W_{34}^t &= P/V_{34} = 4{,}330\,\text{lbf} & W_{56}^t &= P/V_{56} = 10{,}284\,\text{lbf} \\
W_{34}^r &= W_{34}^t \tan(\phi_t) = 1{,}820\,\text{lbf} & W_{56}^r &= W_{56}^t \tan(\phi_t) = -4{,}322\,\text{lbf} \\
W_{34}^x &= W_{34}^t \tan(\psi) = 2{,}500\,\text{lbf} & W_{56}^x &= W_{56}^t \tan(\psi) = -5{,}937\,\text{lbf}
\end{aligned}
$$

Shaft C has three bearings, two gears. Split shaft into two portions, left and right and evaluate independently. The middle bearing will support the loads on both sides.

$$
\begin{aligned}
R_{ay} &= -W_{34}^r \frac{B/2 + S + F/2}{l_{\text{left}}} = -569\,\text{lbf} & R_{az} &= -W_{34}^t \frac{B/2 + S + F/2}{l_{\text{left}}} = -1{,}351\,\text{lbf} \\
R_{by} &= -W_{34}^r \frac{B/2 + 2S + \frac{3}{2}F}{l_{\text{left}}} - \frac{1}{2} W_{56}^r = 910\,\text{lbf} & R_{bz} &= -W_{34}^t \frac{B/2 + 2S + \frac{3}{2}F}{l_{\text{left}}} - \frac{1}{2} W_{56}^t = -8{,}119\,\text{lbf} \\
R_{cy} &= -\frac{1}{2} W_{56}^r = 2{,}161\,\text{lbf} & R_{cz} &= \frac{1}{2} W_{56}^t = -5{,}142\,\text{lbf}
\end{aligned}
$$

$$
R_{bx} = -3{,}438\,\text{lbf} \quad T = \frac{W_{56}^t N_5}{2P} = 27{,}424\,\text{lbf} \cdot \text{in}
$$

Shaft D:

$$
\begin{aligned}
W_{56}^t &= P/V_{56} = -9{,}331\,\text{lbf} & R_y &= -\frac{1}{2} W_{56}^r = 1{,}960\,\text{lbf} \\
W_{56}^r &= W_{56}^t \tan(\phi_t) = -3{,}921\,\text{lbf} & & \\
W_{56}^x &= W_{56}^t \tan(\psi) = -5{,}386\,\text{lbf} & R_z &= -\frac{1}{2} W_{56}^t = 4{,}665\,\text{lbf}
\end{aligned}
$$

$$
T = \frac{W_{56}^t N_6}{2P} = 65{,}133\,\text{lbf} \cdot \text{in}
$$

4.3 Stress Analysis

Shafts C and D are both critical at the point of the meshing pair gears 5 and 6. Both shafts have the same bending moment at that point, but shaft D has a far higher torque. Therefore the axial location of gear 6 on shaft D is critical.

First choose inexpensive 1020 CD Steel. Calculate endurance limit based off of 90% reliability and a estimated 2 in diameter:

$$
S_e = k_a k_b k_e S_e' = 0.80 \cdot 0.82 \cdot 0.90 \cdot 0.5 \cdot 68\,\text{ksi} = 19.9\,\text{ksi} \qquad \text{(6-17) [1]}
$$

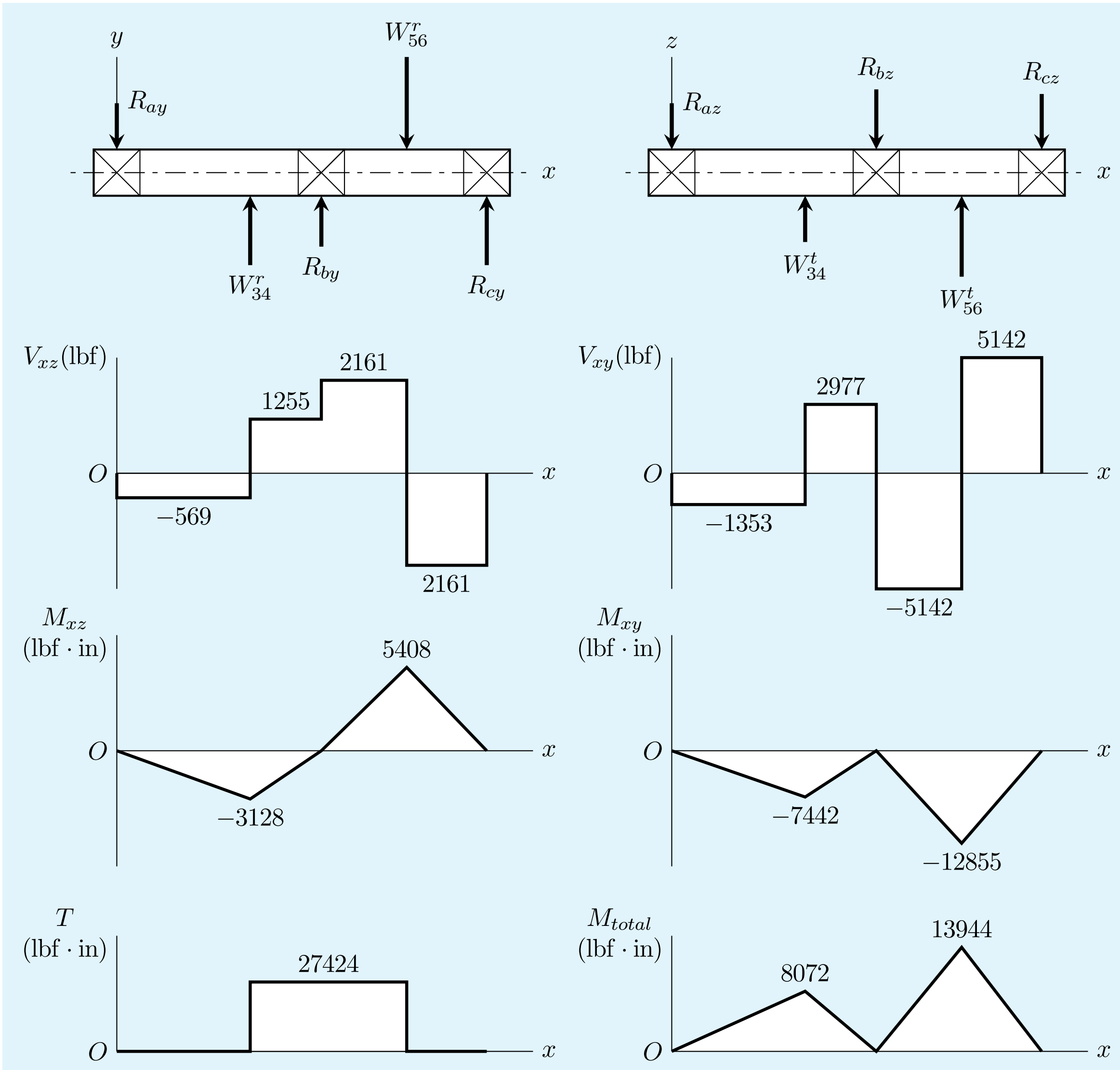


Figure 3: Force Analysis of Shaft C

Using the DE-Goodman criteria, find the critical diameter for shaft D. This location will have an end-milled keyway to transmit motion between gears, so calculate fatigue notch factors [1]:

$$r = 0.02 \cdot 2\,\text{in} = 0.04\,\text{in} \implies q = 0.65, q_s = 0.7$$
$$K_f = 1 + q\,(K_t - 1) = 1 + 0.65(2.14 - 1) = 1.74$$
$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.7(3.0 - 1) = 2.4$$

For a constantly rotating shaft, $T_a = M_m = 0$ and use values from force analysis for T_m and M_a. Also use $n = 1.25$ for a factor of safety.

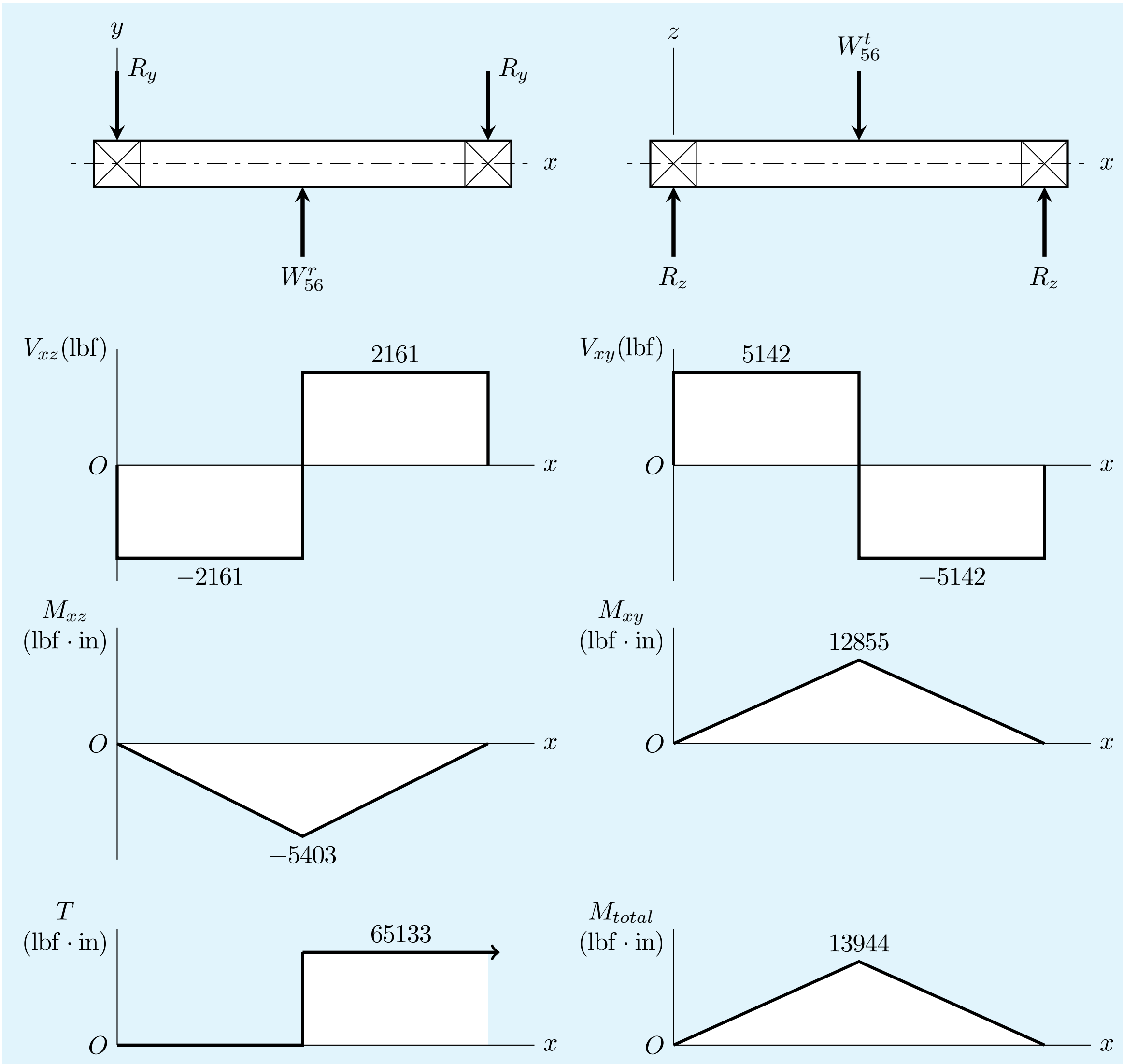


Figure 4: Force Analysis of Shaft D

$$A = \sqrt{4 \cdot (K_f M_a)^2} = \sqrt{4 \cdot (1.74 \cdot 13944)^2}$$

$$B = \sqrt{3 \cdot (K_{fs} T_m)^2} = \sqrt{3 \cdot (2.4 \cdot 65133)^2}$$

$$d = \left[\frac{16}{n}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)\right]^{\frac{1}{3}} = \left[\frac{16}{1.25}\left(\frac{A}{19.9} + \frac{B}{68}\right)\right]^{\frac{1}{3}} = 3.44\,\text{in} \quad \text{(7-7) [1]}$$

This is higher than I wanted the diameter to be due to having to increase it further for shoulders and bearing, and in order to make the gearbox profile size smaller, I will upgrade the material to Ground 4140 Q&T Steel. Now using $S_{ut} = 225\,\text{ksi}$, $k_a = 1.21 \cdot 225^{-0.067} = 0.84$, and $S_e = 61.6\,\text{ksi}$.

$$d = \left[\frac{16}{1.25}\left(\frac{A}{61.6} + \frac{B}{225}\right)\right]^{\frac{1}{3}} = 2.33\,\text{in}$$

Increase this to 2.5 in. This diameter will be the minimum diameter of all shafts in the gearbox. Next check first cycle yielding by checking critical point of shaft D:

$$\begin{aligned}
\sigma'_{\max} &= \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}T_m}{\pi d^3}\right)\right]^{\frac{1}{2}} \\
&= \left[\left(\frac{32 \cdot 2.4 \cdot 13944}{\pi \cdot 2.5^3}\right)^2 + 3\left(\frac{16 \cdot 2.4 \cdot 65133}{\pi \cdot 2.5^3}\right)\right]^{\frac{1}{2}} \\
&= 112.3\,\text{ksi} \\
n_y &= \frac{S_y}{\sigma'_{\max}} = \frac{208}{90.9} = 2.29
\end{aligned}$$

Next check sensitive points for deflection and slope. One option would be to use beam deflection equations, but I chose to use the derivative relationship between bending moment, slope, and deflection:

$$\begin{aligned}
\theta(x) &= \frac{1}{EI}\int M dx \\
y(x) = \int \theta(x) &= \frac{1}{EI}\int\int M dx dx \\
E = 30{,}000\,\text{ksi} \quad I &= \frac{\pi(2.5\,\text{in})^4}{64}
\end{aligned}$$

Table 3: Slope and Deflection Values along Shaft C

Point of Interest	Position x (in)	Slope θ rad	Deflection y (in)
Left Bearing	0.5	0.0000032	0.00000053
Gear 4	5.5	0.00039	0.00071
Middle Bearing	8	0.00056	0.0020
Gear 5	10.5	0.00086	0.0036
Right Bearing	13	0.0012	0.0063

All bearings and gears are within the required limits, although the right bearing slope is just barley within limits[1]. Consider during bearing selection.

5 Gears

Continuing with perviously used specification for gears and loading, I now need to fully specify gears. Since gear 5 is the smallest gear transmitting the largest load, it will be critical.

Gear 5 Wear:

$$I = \frac{\cos\phi_t \sin\phi_t m_G}{2m_N(m_G - 1)} = \frac{\cos(22.80°)\sin(22.80°)\frac{38}{16}}{2 \cdot 1(\frac{38}{16} - 1)} = 0.31$$

$$Q_v = 7, B = 0.731, A = 65.1$$

$$K_v = \left(\frac{A + \sqrt{V_5}}{A}\right)^B = 1.22$$

$$K_m = 1.21$$

$$C_p = 2300$$

$$K_o = K_s = C_f = 1$$

$$\sigma_c = C_p\sqrt{W_t K_o K_v K_s \frac{K_m C_f}{d_p F I}} = 148{,}446\,\text{psi}$$

For life factor Z_N get number of cycles for specified life of 12,000 hours.

$$L_4 = 12{,}000\,\text{hours} \cdot 310\,\text{rpm} = 2.23 \times 10^8\text{rev}$$

$$Z_N = 0.95, K_R = K_T = C_H = 1, S_H = 1.1$$

$$S_c = \frac{S_H\sigma_c}{Z_N} = \frac{1.1 \cdot 148446}{0.95} = 171{,}885\,\text{psi}$$

Select grade 2 carburized and hardened steel with $S_c = 225{,}000$ psi. $n_c = 225000 \cdot 0.9/197928 = 1.18$.

Gear 5 Bending:

$$J = 0.40, K_B = 1$$

$$\sigma = W_t K_v \frac{P_d K_m}{FJ} = 51{,}654\,\text{psi}$$

$$Y_N = 0.95$$

$$\sigma_{\text{allow}} = S_t Y_N = 58{,}500\,\text{psi}$$

$$n = \frac{58500}{51654} = 1.13$$

6 Other Components

6.1 Bearings

Tapered roller bearings must be used to allow for axial loads due to the helical gears. These bearings should be able to be housed by the gearbox casting. The greatest axial force is from gear 6 pushing onto the outer bearing of shaft D at 5,938 lbf. The greatest radial force is on both bearings of shaft D with a value of 2,161 lbf. This is a combined load of 6,318 lbf.

Using the supplier SXF's catalog to find bearings with a bore diameter of 2.5 in, and with load limits above those required, I selected part number 39585/39520 [2]. This bearing is also compatible with the required speeds, cycles, and mounting requirements. Additionally, bearing 663/653 will be used for the center of shaft C in order to accommodate a larger shaft diameter.

6.2 Lubrication

For helical gears with continuous duty and pitch line velocity over 1,000 ft/min, select AGMA 6 EP.

6.3 Keys

Standard 0.5 in square keys will be used. Using same 4140 Q&T Steel, check for failure by crushing:

$$F = \frac{T}{r} = \frac{65133}{2.5} = 26{,}053\,\mathrm{lbf}$$
$$l = \frac{2Fn}{tS_y} = \frac{2 \cdot 26053 \cdot n}{0.5 \cdot 225000} = 0.93\,\mathrm{in}1\,\mathrm{in}$$

6.4 Other Considerations

- Gearbox housing
- Gearbox mounting
- Lubrication
- Sealing
- Maintenance/Inspection Schedule
- FMEA

7 Appendices

Appendix A Source Code

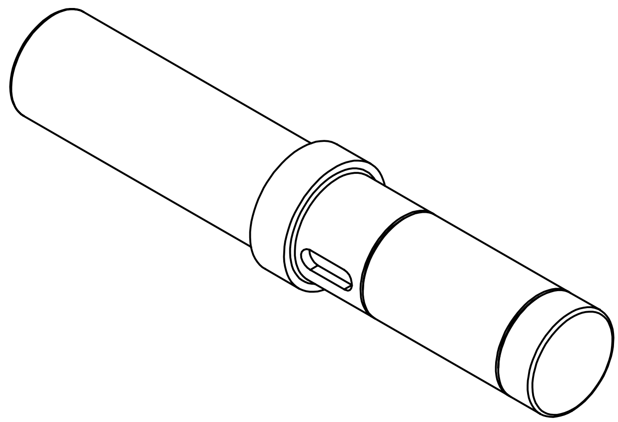
Script Used to Generate Possible Gear Ratios (approximate_gear_ratio.py)

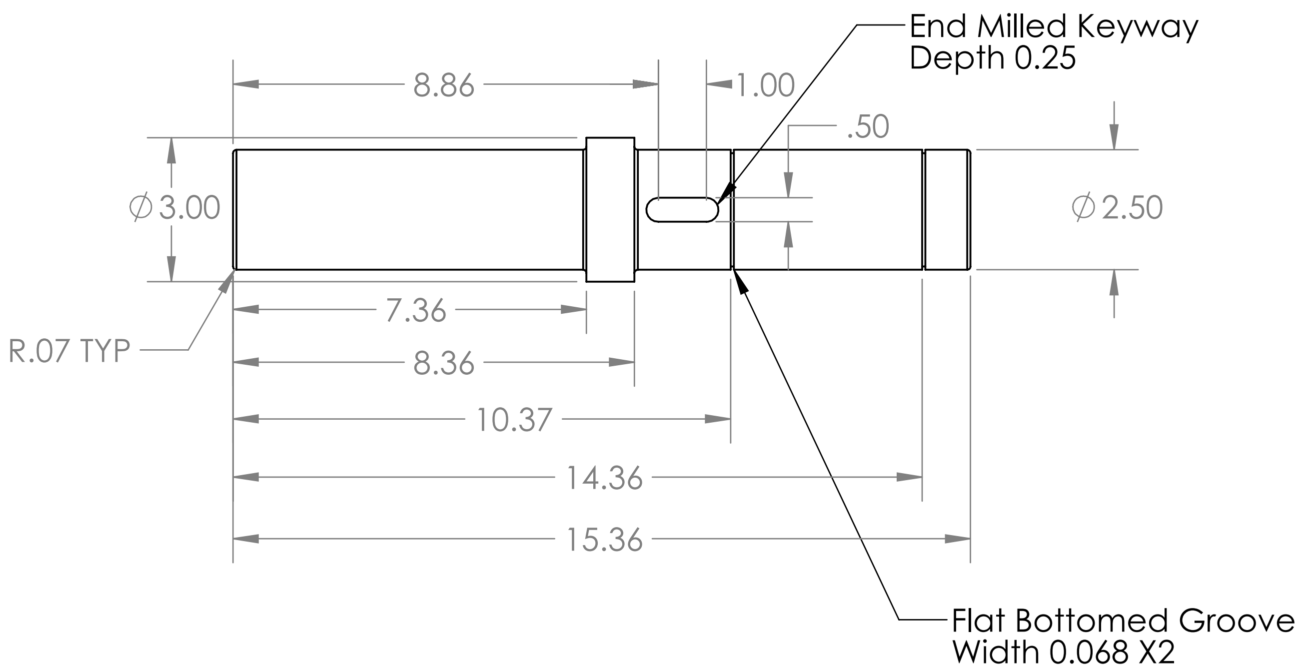
```python
"""Generates possible intermediate gear ratios.

Ratios are based on an specified overall ratio, a maximum tooth count and a
number of stages. Generates stage ratios where all stages are identical to
minimize total package size.
"""

from fractions import Fraction
import pandas as pd

input_speed = 1750 # rpm
target_output_speed = 130 # rpm
number_of_stages = 3
maximum_tooth_count = 150

target_ratio = Fraction(target_output_speed, input_speed)
target_stage_ratio = Fraction(target_ratio ** (1/number_of_stages))

ratios = []
max_denominator = maximum_tooth_count

# Iteratively reduce denominator until all close ratios are found.
while max_denominator > 1:
    stage_ratio = target_stage_ratio.limit_denominator(max_denominator)
    ratios.append(stage_ratio)
    max_denominator = stage_ratio.denominator - 1

# Store data in Dataframe.
df = pd.DataFrame({"Stage Ratio": ratios})
df["Overall Train Value"] = df["Stage Ratio"].astype(float) ** 3
df["Output Speed (rpm)"] = df["Overall Train Value"] * input_speed
df["Output Speed Error (rpm)"] = (df["Output Speed (rpm)"] - target_output_speed).abs()
```

Appendix B Drawings

2 1

B B





A A

		NAME	DATE
DRAWN		Luke McGinnis	
CHECKED			
ENG APPR.			
MFG APPR.			
Q.A.			

DIMENSIONS ARE IN INCHES
TOLERANCES:
FRACTIONAL ±
ANGULAR: MACH ± BEND ±
TWO PLACE DECIMAL ±
THREE PLACE DECIMAL ±

MATERIAL

FINISH

DO NOT SCALE DRAWING

NEXT ASSY | USED ON

APPLICATION

COMMENTS:

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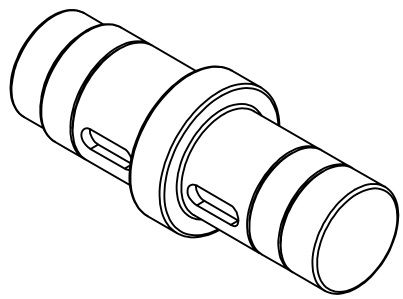

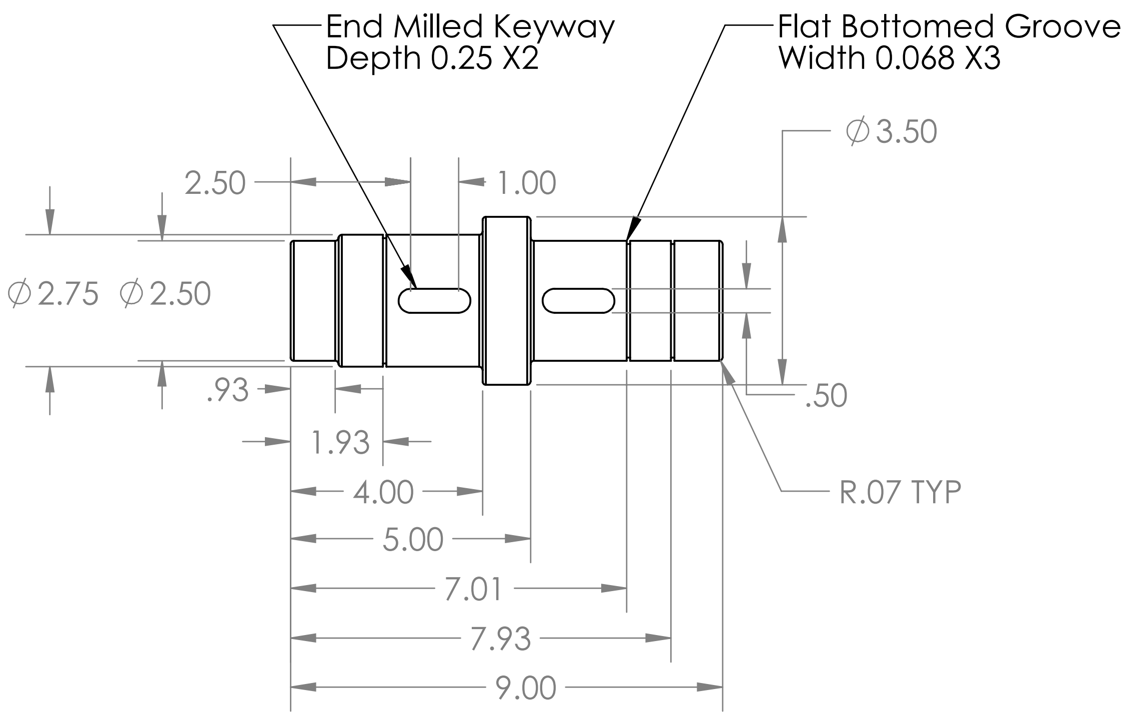

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	NAME	DATE
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MFG APPR.		
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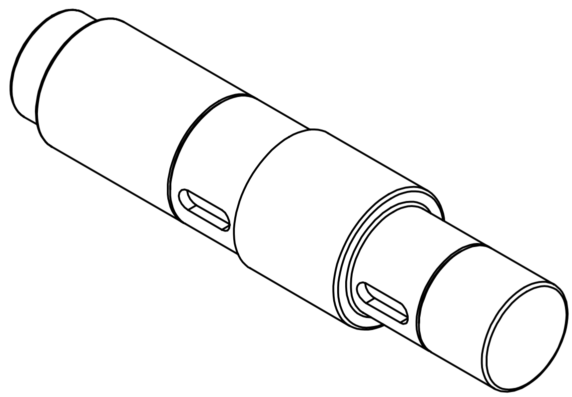

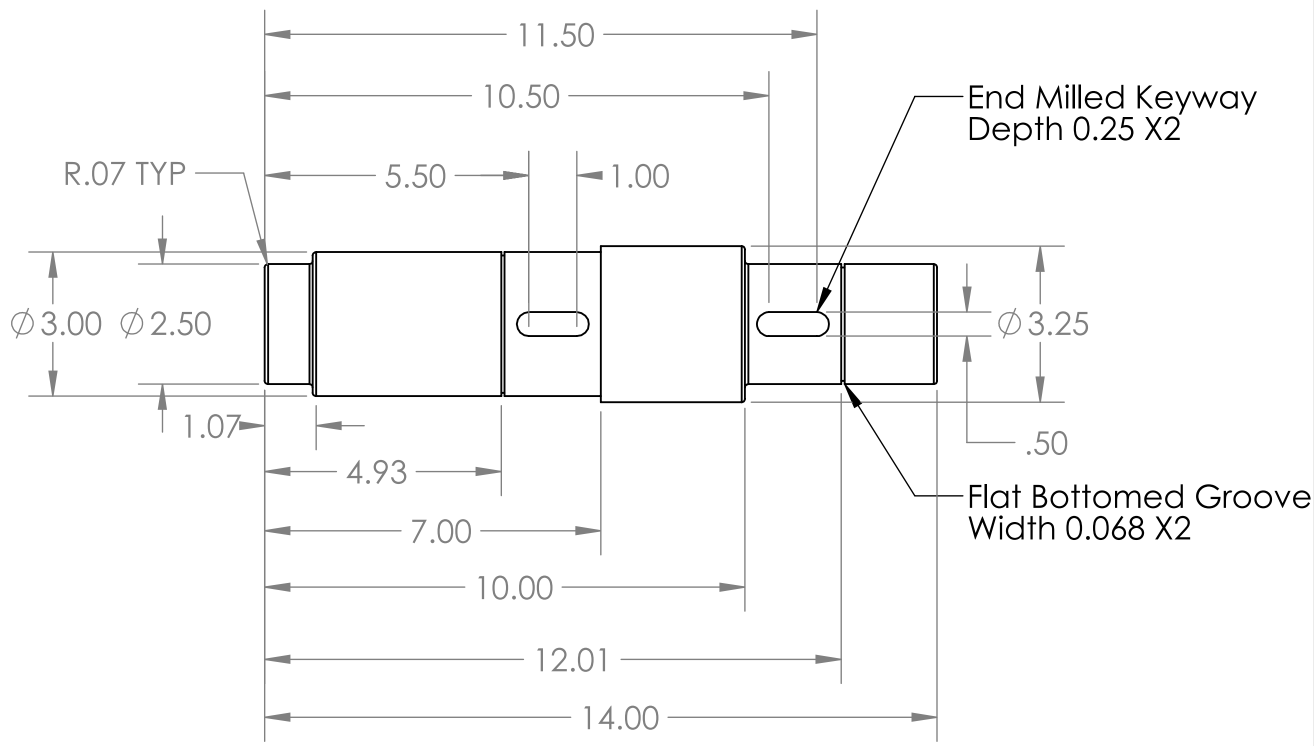

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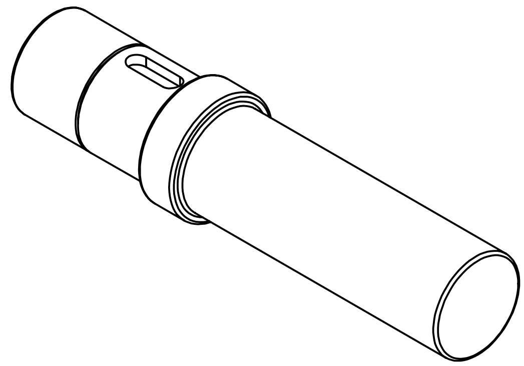

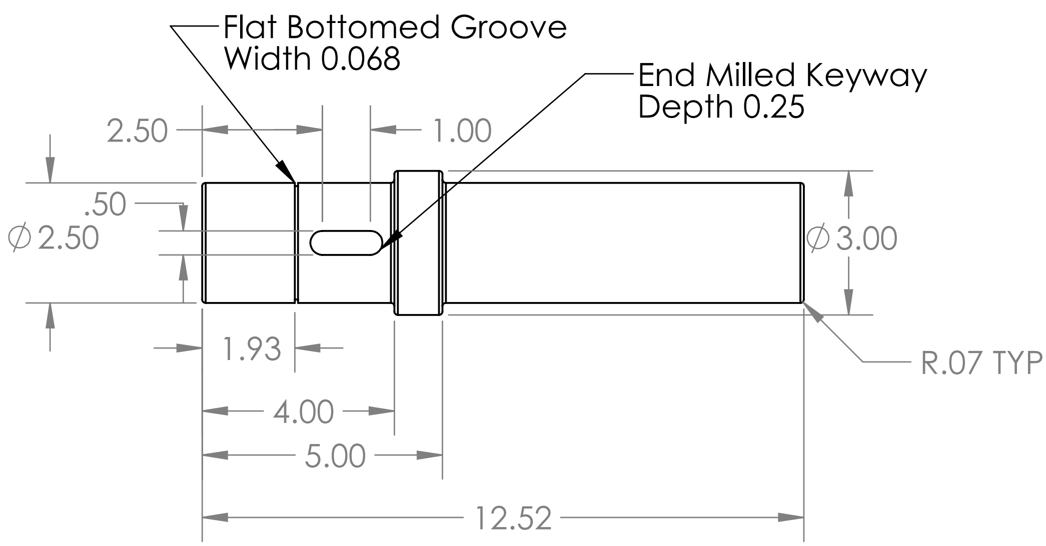

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	NAME	DATE
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MFG APPR.		
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COMMENTS:		

MATERIAL

FINISH

NEXT ASSY	USED ON
APPLICATION	

DO NOT SCALE DRAWING

SIZE A | DWG. NO. ShaftD | REV.

SCALE:1:4 | WEIGHT: | SHEET 1 OF 1

References

[1] Richard G. Budynas and J. Keith Nisbett. *Shigley's Mechanical Engineering Design*. 2024 Release. McGraw-Hill, 2024.

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