Chapter 1. Vector Spaces

Linear Algebra Done Right 4th ed., by Sheldon Axler

1A. $\mathbb{R}^n$ and $\mathbb{C}^n$

Problem 2

Show that

$$(\alpha + \beta) + \lambda = \alpha + (\beta + \lambda)$$

for all $\alpha, \beta, \lambda \in \mathbb{C}$.

Proof. Suppose we have

$$\alpha = a + bi \quad \beta = c + di \quad \lambda = e + fi$$

where $a, b, c, d, e, f \in \mathbb{R}$. Then by the addition of complex numbers we have

$$\begin{aligned}
(\alpha + \beta) + \lambda &= [(a + bi) + (c + di)] + (e + fi) \\
&= [(a + c) + (b + d)i] + (e + fi) \\
&= [(a + c) + e] + [(b + d) + f]i \\
&= [a + (c + e)] + [b + (d + f)]i \\
&= (a + bi) + [(c + e) + (d + f)i] \\
&= (a + bi) + [(c + di) + (e + fi)] \\
&= \alpha + (\beta + \lambda),
\end{aligned}$$

as desired. □

1B. Definition of Vector Space

1C. Subspaces