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Statistical Methods and Data Analysis (EN.625.603)
Project 1

Project 1
Lead is toxic, particularly for young children, and for this reason government regulations severely restrict the amount of lead in our environment. But this was not always the case. In the early part of the 20th century, the underground water pipes in many US cities contained lead, and lead from these pipes leached into drinking water. In this exercise you will investigate the effect of these lead water pipes on infant mortality.

(a) Compute the average infant mortality rate (Inf) for cities with lead pipes and for cities with non-lead pipes. Is there a statistically significant difference in the averages

(b) The amount of lead leached from lead pipes depends on the chemistry of the water running through the pipes. The more acidic the water (that is, the lower the pH), the more lead is leached. Run a regression of Inf on $Lead, pH$, and the interaction term $Lead \times pH$.

i. The regression includes four coefficients (the intercept and the three coefficients multiplying the regressors). Explain what each coefficient measures.

ii. Plot the estimated regression function relating Inf to pH for $Lead = 0$ and $Lead = 1$. Describe the differences in the regression functions and relate these differences to the coefficients discussed in (i).

iii. Does *Lead* have a statistically significant effect on infant mortality? Explain.

iv. Does the effect of *Lead* on infant mortality depend on *pH*? Is this dependence statistically significant?

v. What is the average value of *pH* in the sample? At this *pH* level, what is the estimated effect of *Lead* on infant mortality? What is the standard deviation of *pH*? Suppose that the *pH* level is on standard deviation lower than the average level of *pH* in the sample; what is the estimated effect of *Lead* on infant mortality? What if *pH* is one standard deviation higher than the average value?

vi. Construct a 95% confidence interval for the effect of Lead on infant mortality when pH = 6.5.

(c) The analysis in (b) may suffer from omitted variable bias because it neglects factors that affect infant mortality and that might potentially be correlated with *Lead* and *pH*. Investigate this concern, using the other variables in the data set.

Solution

(a) Using Python with Pandas library, we have the following statistics with $n, \bar{x}, s_x$ for lead and $m, \bar{y}, s_y$ for non-lead.

$$n = 55 \quad \bar{x} = 0.3812 \quad s_x = 0.1478$$
$$m = 117 \quad \bar{y} = 0.4033 \quad s_y = 0.1531$$

Test hypothesis is as follows:

$$\begin{aligned} \mathrm{H}_0 &: \mu_x = \mu_y \\ \mathrm{H}_1 &: \mu_x < \mu_y \end{aligned}$$

The level of significance is $\alpha = 0.05$.
The degrees of freedom and critical value are as follows:

$$\begin{aligned} df &= n + m - 2 \\ &= 55 + 117 - 2 \\ &= 170 \\ t_{\alpha,df} &= t_{0.05,170} \\ &= 1.6539 \end{aligned}$$

The pooled standard deviation is as follows:

$$\begin{aligned} s_p &= \sqrt{\frac{(n-1)s_x^2 + (m-1)s_y^2}{n+m-2}} \\ &= \sqrt{\frac{(55-1)0.1478^2 + (117-1)0.1531^2}{170}} \\ &= 0.1514 \end{aligned}$$

The test statistic is as follows:

$$\begin{aligned} t &= \frac{\bar{x} - \bar{y}}{s_p\sqrt{\frac{1}{n} + \frac{1}{m}}} \\ &= \frac{0.3812 - 0.4033}{0.1514\sqrt{\frac{1}{55} + \frac{1}{117}}} \\ &= -0.8923 \end{aligned}$$

Because $t = -0.8923 > -t_{\alpha,df} = -1.6539$, we fail to reject the null hypothesis.

(b) i. The coefficients were calculated using Python and Pandas are as follows

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                infrate   R-squared:                       0.272
Model:                            OLS   Adj. R-squared:                  0.259
Method:                 Least Squares   F-statistic:                     20.91
Date:                Tue, 15 Aug 2023   Prob (F-statistic):           1.47e-11
Time:                        19:10:49   Log-Likelihood:                 108.52
No. Observations:                 172   AIC:                            -209.0
Df Residuals:                     168   BIC:                            -196.5
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.9189      0.174      5.267      0.000       0.574       1.263
lead           0.4618      0.221      2.087      0.038       0.025       0.899
ph            -0.0752      0.024     -3.098      0.002      -0.123      -0.027
lead-pH       -0.0569      0.030     -1.871      0.063      -0.117       0.003
==============================================================================
Omnibus:                        4.916   Durbin-Watson:                   1.946
Prob(Omnibus):                  0.086   Jarque-Bera (JB):                4.987
Skew:                           0.411   Prob(JB):                       0.0826
Kurtosis:                       2.861   Cond. No.                         252.
==============================================================================
```

The regression includes four coefficients (the intercept and the three coefficients multiplying the regressors).

$$\mathrm{Inf} = \beta_0 + \beta_1\mathrm{Lead} + \beta_2\mathrm{pH} + \beta_3\mathrm{Lead} \times \mathrm{pH}$$

$$\beta_0 = 0.9189$$
$$\beta_1 = 0.4618$$
$$\beta_2 = -0.0752$$
$$\beta_3 = -0.0569$$

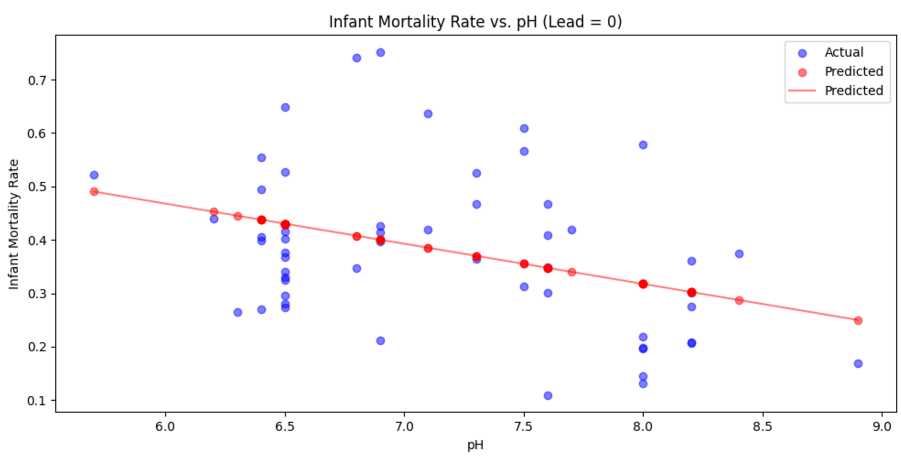
$\beta_0 = 0.9189$ is the average infant mortality when $Lead = 0$ and $pH = 0$.
$\beta_1 = 0.4618$ is the effect of $Lead$ on infant mortality when $pH = 0$.
$\beta_2 = -0.0752$ is the effect of pH on infant mortality when $Lead = 0$.
$\beta_3 = -0.0569$ is the effect of pH on infant mortality when $Lead = 1$.

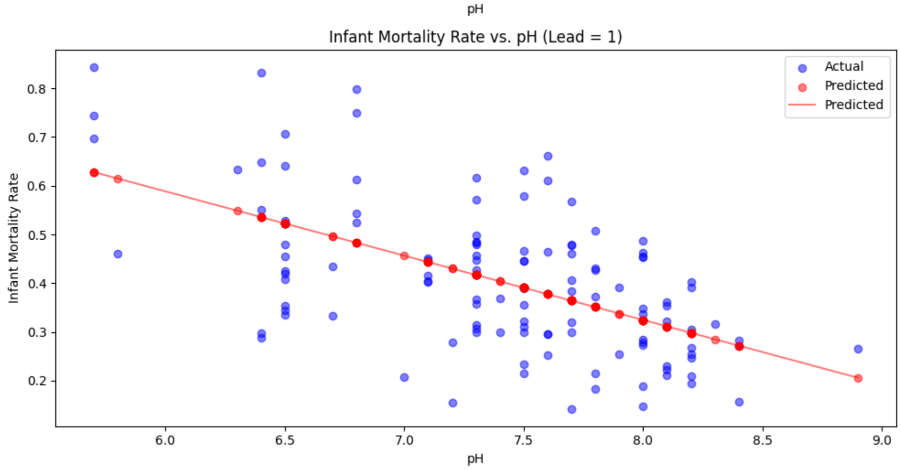
ii. The estimated regression function relating Inf to pH for $Lead = 0$ is as follows

$$\begin{aligned}\text{Inf} &= \beta_0 + \beta_2\text{pH} + \epsilon \\ &= 0.9189 - 0.0752\text{pH} + \epsilon\end{aligned}$$

The estimated regression function relating Inf to pH for $Lead = 1$ is as follows

$$\begin{aligned}\text{Inf} &= \beta_0 + \beta_1 + \beta_2\text{pH} + \beta_3\text{pH} + \epsilon \\ &= 0.9189 + 0.4618 - 0.0752\text{pH} - 0.0569\text{pH} + \epsilon \\ &= 1.3807 - 0.1321\text{pH} + \epsilon\end{aligned}$$





iii. To determine if Lead has a statistically significant effect on infant mortality, we use $F-$test from Python as follows

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                infrate   R-squared:                       0.234
Model:                            OLS   Adj. R-squared:                  0.229
Method:                 Least Squares   F-statistic:                     51.80
Date:                Tue, 15 Aug 2023   Prob (F-statistic):           1.88e-11
Time:                        23:58:28   Log-Likelihood:                 104.11
No. Observations:                 172   AIC:                            -204.2
Df Residuals:                     170   BIC:                            -197.9
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.1704      0.108     10.833      0.000       0.957       1.384
ph            -0.1057      0.015     -7.197      0.000      -0.135      -0.077
==============================================================================
Omnibus:                        5.592   Durbin-Watson:                   1.901
Prob(Omnibus):                  0.061   Jarque-Bera (JB):                5.729
Skew:                           0.427   Prob(JB):                       0.0570
Kurtosis:                       2.738   Cond. No.                         79.9
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
F-statistic: 4.419053361850366
p-value: 0.01347744945676196
```

We have the following statistics

$$F = 4.4191$$
$$p - value = 0.0134$$

Thus, we can conclude that Lead has a statistically significant effect on infant mortality.

iv. $p - value$ of the interaction $lead \times pH$ is 0.0631 which is greater than $\alpha = 0.05$. Thus, we can conclude that the effect of $Lead$ on infant mortality does not depend on pH.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                infrate   R-squared:                       0.272
Model:                            OLS   Adj. R-squared:                  0.259
Method:                 Least Squares   F-statistic:                     20.91
Date:                Wed, 16 Aug 2023   Prob (F-statistic):           1.47e-11
Time:                        01:04:58   Log-Likelihood:                 108.52
No. Observations:                 172   AIC:                            -209.0
Df Residuals:                     168   BIC:                            -196.5
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.9189      0.174      5.267      0.000       0.574       1.263
lead           0.4618      0.221      2.087      0.038       0.025       0.899
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==============================================================================
Omnibus:                        4.916   Durbin-Watson:                   1.946
Prob(Omnibus):                  0.086   Jarque-Bera (JB):                4.987
Skew:                           0.411   Prob(JB):                       0.0826
Kurtosis:                       2.861   Cond. No.                         252.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
p-value for pH X lead: 0.0631
```

v. We have mean and standard deviation of pH as follows

$$\bar{pH} = 7.3227$$
$$s_{pH} = 0.6917$$

```
Average pH: 7.3227
Standard Deviation of pH: 0.6917
pH 1 Standard Deviation Above Average: 8.0144
pH 1 Standard Deviation Below Average: 6.6309
```

The estimated regression function relating Inf to pH for $Lead = 0$ is as follows

$$\begin{aligned}\text{Inf} &= \beta_0 + \beta_2\text{pH}\\ &= 0.9189 - 0.0752(7.3227)\\ &= 0.3682\end{aligned}$$

The estimated regression function relating Inf to pH for $Lead = 1$ is as follows

$$\begin{aligned}\text{Inf} &= \beta_0 + \beta_1 + \beta_2\text{pH} + \beta_3\text{pH}\\ &= 0.9189 + 0.4618 - 0.0752(7.3227) - 0.0569(7.3227)\\ &= 0.4134\end{aligned}$$

Thus, the difference in the estimated infant mortality rates for $Lead = 0$ and $Lead = 1$ is

$$\begin{aligned}\text{Inf}_{Lead=0} - \text{Inf}_{Lead=1} &= 0.3682 - 0.4134\\ &= -0.0452\end{aligned}$$

pH one standard deviation above its mean is $7.3227 + 0.6917 = 8.0144$. The estimated regression function relating Inf to pH for $Lead = 0$ is as follows

$$\begin{aligned}\text{Inf} &= \beta_0 + \beta_2\text{pH}\\ &= 0.9189 - 0.0752(8.0144)\\ &= 0.3162\end{aligned}$$

The estimated regression function relating Inf to pH for $Lead = 1$ is as follows

$$\begin{aligned}\text{Inf} &= \beta_0 + \beta_1 + \beta_2\text{pH} + \beta_3\text{pH}\\ &= 0.9189 + 0.4618 - 0.0752(8.0144) - 0.0569(8.0144)\\ &= 0.3220\end{aligned}$$

Thus, the difference in the estimated infant mortality rates for $Lead = 0$ and $Lead = 1$ is

$$\begin{aligned}\text{Inf}_{Lead=0} - \text{Inf}_{Lead=1} &= 0.3162 - 0.3220\\ &= -0.0058\end{aligned}$$

pH one standard deviation below its mean is $7.3227 - 0.6917 = 6.6310$. The estimated regression function relating Inf to pH for $Lead = 0$ is as follows

$$\begin{aligned}\text{Inf} &= \beta_0 + \beta_2\text{pH}\\ &= 0.9189 - 0.0752(6.6310)\\ &= 0.4202\end{aligned}$$

The estimated regression function relating Inf to pH for $Lead = 1$ is as follows

$$\begin{aligned}\text{Inf} &= \beta_0 + \beta_1 + \beta_2\text{pH} + \beta_3\text{pH}\\ &= 0.9189 + 0.4618 - 0.0752(6.6310) - 0.0569(6.6310)\\ &= 0.5047\end{aligned}$$

Thus, the difference in the estimated infant mortality rates for *Lead* = 0 and *Lead* = 1 is

$$\begin{aligned}\text{Inf}_{Lead=0} - \text{Inf}_{Lead=1} &= 0.4202 - 0.5047 \\ &= -0.0844\end{aligned}$$

vi. The standard error of the estimated infant mortality rate is

$$s_{\text{Inf}} = 0.1513$$

The estimated mortality rate for *Lead* = 0 and *pH* = 6.5 is

$$\begin{aligned}\text{Inf} &= \beta_0 + \beta_2\text{pH} \\ &= 0.9189 - 0.0752(6.5) \\ &= 0.4301\end{aligned}$$

The estimated mortality rate for *Lead* = 1 and *pH* = 6.5 is

$$\begin{aligned}\text{Inf} &= \beta_0 + \beta_1 + \beta_2\text{pH} + \beta_3\text{pH} \\ &= 0.9189 + 0.4618 - 0.0752(6.5) - 0.0569(6.5) \\ &= 0.5221\end{aligned}$$

Degrees of freedom and t-critical value for $\alpha = 0.05$ are as follows

$$\begin{aligned}n - p - 1 &= 172 - 3 - 1 \\ &= 168 \\ t_{\alpha/2,df} &= t_{0.025,168} \\ &= 2.262\end{aligned}$$

The 95% confidence interval for the difference in the estimated infant mortality rates for *Lead* = 0 and *Lead* = 1 is

$$\begin{aligned}&= \text{Inf}_{Lead=0} - \text{Inf}_{Lead=1} \pm t_{\alpha/2,df}\, s_{\text{Inf}}\sqrt{\frac{1}{n_0} + \frac{1}{n_1}} \\ &= -0.0920 \pm 2.262(0.1513)\sqrt{\frac{1}{55} + \frac{1}{117}} \\ &= (-0.1480, -0.0360)\end{aligned}$$

(c) Of the 15 columns in the dataset, the analysis obmitted the majority of other variables which might have an effect on infant mortality and correlated with *Lead* and *pH*.

i. We can investigate water hardness index.
After adding *Hardness* to the model, and we can see that its *p*-value is 0.924, which is greater than 0.05. Thus, we can conclude that *Hardness* is not statistically significant in the model.

ii. We can investigate mom age index.
After adding *Age* to the model, and we can see that its *p*-value is 0.416, which is greater than 0.05. Thus, we can conclude that *Age* is not statistically significant in the model.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                infrate   R-squared:                       0.295
Model:                            OLS   Adj. R-squared:                  0.269
Method:                 Least Squares   F-statistic:                     11.51
Date:                Wed, 16 Aug 2023   Prob (F-statistic):           9.68e-11
Time:                        02:30:00   Log-Likelihood:                 111.30
No. Observations:                 172   AIC:                            -208.6
Df Residuals:                     165   BIC:                            -186.6
Df Model:                           6
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
const             0.9926      0.234      4.235      0.000       0.530       1.455
lead              0.2878      0.269      1.068      0.287      -0.244       0.820
ph               -0.0870      0.035     -2.502      0.013      -0.156      -0.018
hardness         -0.0002      0.002     -0.096      0.924      -0.004       0.004
lead-pH          -0.0264      0.040     -0.666      0.506      -0.105       0.052
lead-hardness    -0.0004      0.000     -1.206      0.230      -0.001       0.000
ph-hardness    4.252e-05      0.000      0.171      0.865      -0.000       0.001
==============================================================================
Omnibus:                        4.830   Durbin-Watson:                   1.921
Prob(Omnibus):                  0.089   Jarque-Bera (JB):                4.851
Skew:                           0.409   Prob(JB):                       0.0884
Kurtosis:                       2.903   Cond. No.                     3.97e+04
==============================================================================
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                infrate   R-squared:                       0.278
Model:                            OLS   Adj. R-squared:                  0.252
Method:                 Least Squares   F-statistic:                     10.60
Date:                Wed, 16 Aug 2023   Prob (F-statistic):           5.99e-10
Time:                        02:35:39   Log-Likelihood:                 109.29
No. Observations:                 172   AIC:                            -204.6
Df Residuals:                     165   BIC:                            -182.5
Df Model:                           6
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
const             1.5062      0.742      2.030      0.044       0.041       2.971
lead              0.4023      0.265      1.521      0.130      -0.120       0.925
ph               -0.1597      0.106     -1.513      0.132      -0.368       0.049
mom_rate         -2.8985      3.555     -0.815      0.416      -9.918       4.121
lead-pH          -0.0551      0.031     -1.803      0.073      -0.116       0.005
lead-mom_rate     0.2426      0.677      0.358      0.721      -1.095       1.580
ph-mom_rate       0.4176      0.507      0.823      0.412      -0.584       1.419
==============================================================================
Omnibus:                        4.682   Durbin-Watson:                   1.975
Prob(Omnibus):                  0.096   Jarque-Bera (JB):                4.765
Skew:                           0.398   Prob(JB):                       0.0923
Kurtosis:                       2.827   Cond. No.                     3.44e+03
==============================================================================
```

```python
# All code used to generate the results are shown below
import pandas as pd
import numpy as np
import statsmodels.api as sm
from scipy import stats
import scipy.stats
import matplotlib.pyplot as plt


class LeadMortalityDataframe:

    df = None

    column_name_year = None
    column_name_city = None
    column_name_state = None
    column_name_age = None
    column_name_hardness = None
    column_name_ph = None
    column_name_infrate = None
    column_name_typhoid_rate = None
    column_name_np_tub_rate = None
    column_name_mom_rate = None
    column_name_population = None
    column_name_precipitation = None
    column_name_temperature = None
    column_name_lead = None
    column_name_foreign_share = None

    def __init__(self):
        file_path = 'lead_mortality.xlsx'
        file_sheet_name = 'Data'

        self.df = pd.read_excel(file_path, sheet_name=file_sheet_name)
        self.column_name_year = self.df.columns[0]
        self.column_name_city = self.df.columns[1]
        self.column_name_state = self.df.columns[2]
        self.column_name_age = self.df.columns[3]
        self.column_name_hardness = self.df.columns[4]
        self.column_name_ph = self.df.columns[5]
        self.column_name_infrate = self.df.columns[6]
        self.column_name_typhoid_rate = self.df.columns[7]
        self.column_name_np_tub_rate = self.df.columns[8]
        self.column_name_mom_rate = self.df.columns[9]
        self.column_name_population = self.df.columns[10]
        self.column_name_precipitation = self.df.columns[11]
        self.column_name_temperature = self.df.columns[12]
        self.column_name_lead = self.df.columns[13]
        self.column_name_foreign_share = self.df.columns[14]

    def log_dataframe_info(self):
        print(f'Number of rows: {self.df.shape[0]}')
        print(f'Number of columns: {self.df.shape[1]}')
        print('Column names:')
        print(self.df.columns.tolist())
        print('Data summary:')
        print(self.df.describe(include='all'))

    def get_lead_by_condition(self, condition):
        return self.df[self.df[self.column_name_lead] == condition]

```

```
    def get_infrate_by_lead_condition(self, lead_condition):
        df = self.get_lead_by_condition(lead_condition)
        return df[self.column_name_infrate]


def part_a_solution():
    lead_mortality = LeadMortalityDataframe()

    infrate_lead_0 = lead_mortality.get_infrate_by_lead_condition(0)
    infrate_lead_1 = lead_mortality.get_infrate_by_lead_condition(1)

    n = infrate_lead_0.shape[0]
    m = infrate_lead_1.shape[0]
    avg_x = infrate_lead_0.mean()
    avg_y = infrate_lead_1.mean()
    std_x = infrate_lead_0.std()
    std_y = infrate_lead_1.std()
    level_of_significance = 0.05
    d_freedom = n + m - 2
    t_a_df = -scipy.stats.t.ppf(level_of_significance, d_freedom)
    std_pooled = np.sqrt(
        ((n - 1) * pow(std_x, 2) + (m - 1) * pow(std_y, 2)) / d_freedom)
    t_statistics = (avg_x - avg_y) / (std_pooled * np.sqrt(1/n + 1/m))
    print(f'n: {n}')
    print(f'm: {m}')
    print(f'avg_x: {avg_x:.4f}')
    print(f'avg_y: {avg_y:.4f}')
    print(f'std_x: {std_x:.4f}')
    print(f'std_y: {std_y:.4f}')
    print(f'std_pooled: {std_pooled:.4f}')
    print(f't_a_df: {t_a_df:.4f}')
    print(f't_statistics: {t_statistics:.4f}')


def part_b_i_solution():
    lead_mortality = LeadMortalityDataframe()
    lead_mortality.df["lead-pH"] = lead_mortality.df[lead_mortality.column_name_lead] * \
        lead_mortality.df[lead_mortality.column_name_ph]

    x = sm.add_constant(lead_mortality.df[[lead_mortality.column_name_lead,
                                           lead_mortality.column_name_ph, "lead-pH"]])
    y = lead_mortality.df[lead_mortality.column_name_infrate]
    model = sm.OLS(y, x)
    results = model.fit()
    print(results.summary())


def part_b_ii_solution():
    lead_mortality = LeadMortalityDataframe()
    lead_mortality.df["lead-pH"] = lead_mortality.df[lead_mortality.column_name_lead] * \
        lead_mortality.df[lead_mortality.column_name_ph]

    x = sm.add_constant(lead_mortality.df[[lead_mortality.column_name_lead,
                                           lead_mortality.column_name_ph, "lead-pH"]])
    y = lead_mortality.df[lead_mortality.column_name_infrate]
    model = sm.OLS(y, x)
    results = model.fit()
    fig, ax = plt.subplots(2, 1, figsize=(10, 10))

```

```python
    lead_mortality.df['pred_infrate'] = results.predict(x)

    df_lead0 = lead_mortality.df[lead_mortality.df['lead'] == 0]
    ax[0].scatter(df_lead0['ph'], df_lead0['infrate'],
                  color='blue', alpha=0.5, label='Actual')
    sorted_df_lead0 = df_lead0.sort_values(by='ph')
    ax[0].plot(sorted_df_lead0['ph'], sorted_df_lead0['pred_infrate'],
               color='red', alpha=0.5, label='Predicted')
    ax[0].set_title('Infant Mortality Rate vs. pH (Lead = 0)')
    ax[0].set_xlabel('pH')
    ax[0].set_ylabel('Infant Mortality Rate')
    ax[0].legend()

    df_lead1 = lead_mortality.df[lead_mortality.df['lead'] == 1]
    ax[1].scatter(df_lead1['ph'], df_lead1['infrate'],
                  color='blue', alpha=0.5, label='Actual')
    sorted_df_lead1 = df_lead1.sort_values(by='ph')
    ax[1].plot(sorted_df_lead1['ph'], sorted_df_lead1['pred_infrate'],
               color='red', alpha=0.5, label='Predicted')
    ax[1].set_title('Infant Mortality Rate vs. pH (Lead = 1)')
    ax[1].set_xlabel('pH')
    ax[1].set_ylabel('Infant Mortality Rate')
    ax[1].legend()

    plt.tight_layout()
    plt.show()


def part_b_iii_solution():
    lead_mortality = LeadMortalityDataframe()
    lead_mortality.df["lead-pH"] = lead_mortality.df[lead_mortality.column_name_lead] * \
        lead_mortality.df[lead_mortality.column_name_ph]
    x = sm.add_constant(lead_mortality.df[[lead_mortality.column_name_lead,
                                           lead_mortality.column_name_ph, "lead-pH"]])
    y = lead_mortality.df[lead_mortality.column_name_infrate]
    results = sm.OLS(y, x).fit()

    x_ph = sm.add_constant(lead_mortality.df[[lead_mortality.column_name_ph]])
    results_ph = sm.OLS(y, x_ph).fit()

    f_test = results.compare_f_test(results_ph)
    print(results_ph.summary())
    print('F-statistic:', f_test[0])
    print('p-value:', f_test[1])


def part_b_iv_solution():
    lead_mortality = LeadMortalityDataframe()
    lead_mortality.df["lead-pH"] = lead_mortality.df[lead_mortality.column_name_lead] * \
        lead_mortality.df[lead_mortality.column_name_ph]
    x = sm.add_constant(lead_mortality.df[[lead_mortality.column_name_lead,
                                           lead_mortality.column_name_ph, "lead-pH"]])
    y = lead_mortality.df[lead_mortality.column_name_infrate]
    results = sm.OLS(y, x).fit()
    model = sm.OLS(y, x)
    results = model.fit()
    print(results.summary())
    p_values_pHxLead = results.pvalues["lead-pH"]
    print(f'p-value for pH X lead: {p_values_pHxLead:.4f}')
```

```python


def part_b_v_solution():
    lead_mortality = LeadMortalityDataframe()
    avg_ph = lead_mortality.df[lead_mortality.column_name_ph].mean()
    std_ph = lead_mortality.df[lead_mortality.column_name_ph].std()
    ph_1_std_above = avg_ph + std_ph
    ph_1_std_below = avg_ph - std_ph
    print(f'Average pH: {avg_ph:.4f}')
    print(f'Standard Deviation of pH: {std_ph:.4f}')
    print(f'pH 1 Standard Deviation Above Average: {ph_1_std_above:.4f}')
    print(f'pH 1 Standard Deviation Below Average: {ph_1_std_below:.4f}')


def part_b_vi_solution():
    lead_mortality = LeadMortalityDataframe()
    std_infrate = lead_mortality.df[lead_mortality.column_name_infrate].std()

    print(f'Standard Deviation of Infant Mortality Rate: {std_infrate:.4f}')


def part_c_i_solution():
    lead_mortality = LeadMortalityDataframe()
    lead_mortality.df["lead-pH"] = lead_mortality.df[lead_mortality.column_name_lead] * \
        lead_mortality.df[lead_mortality.column_name_ph]
    lead_mortality.df["lead-hardness"] = lead_mortality.df[lead_mortality.column_name_lead] * \
        lead_mortality.df[lead_mortality.column_name_hardness]
    lead_mortality.df["ph-hardness"] = lead_mortality.df[lead_mortality.column_name_ph] * \
        lead_mortality.df[lead_mortality.column_name_hardness]

    x = sm.add_constant(lead_mortality.df[[lead_mortality.column_name_lead,
                                           lead_mortality.column_name_ph,
                                           lead_mortality.column_name_hardness,
                                           "lead-pH",
                                           "lead-hardness",
                                           "ph-hardness"]])
    y = lead_mortality.df[lead_mortality.column_name_infrate]
    model = sm.OLS(y, x)
    results = model.fit()
    print(results.summary())


def part_c_ii_solution():
    lead_mortality = LeadMortalityDataframe()
    lead_mortality.df["lead-pH"] = lead_mortality.df[lead_mortality.column_name_lead] * \
        lead_mortality.df[lead_mortality.column_name_ph]
    lead_mortality.df["lead-mom_rate"] = lead_mortality.df[lead_mortality.column_name_lead] * \
        lead_mortality.df[lead_mortality.column_name_mom_rate]
    lead_mortality.df["ph-mom_rate"] = lead_mortality.df[lead_mortality.column_name_ph] * \
        lead_mortality.df[lead_mortality.column_name_mom_rate]

    x = sm.add_constant(lead_mortality.df[[lead_mortality.column_name_lead,
                                           lead_mortality.column_name_ph,
                                           lead_mortality.column_name_mom_rate,
                                           "lead-pH",
```

```
                                                  "lead-mom_rate",
                                                  "ph-mom_rate"]])
    y = lead_mortality.df[lead_mortality.column_name_infrate]
    model = sm.OLS(y, x)
    results = model.fit()
    print(results.summary())


if __name__ == '__main__':
    part_c_ii_solution()
```