Physics
UNIVERSITY OF TORONTO

The Millikan Oil-Drop Experiment
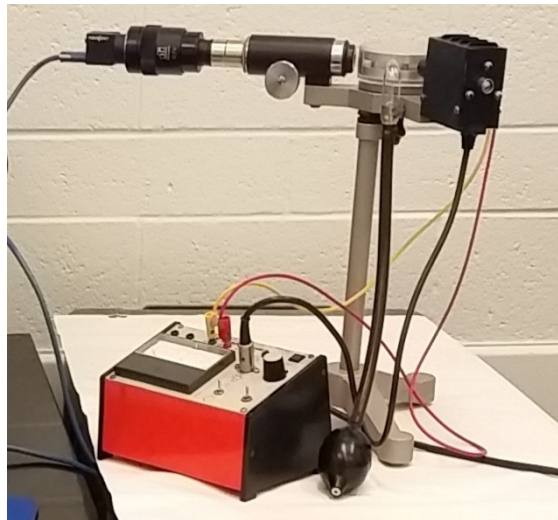
Revisions

2023	C. Lee
2018	N. Krasnopolskaia
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Introduction

The experiment is one of the most fundamental of the experiments in the undergraduate laboratory. The experimental apparatus is patterned after the original apparatus made and used by R.A. Millikan to show that electric charge exists as integral multiples of the elementary charge of a single electron.

This experiment first described by [Millikan, 1913] is based on the fact that different forces act on an electrically charged oil drop moving in the homogeneous electric field of a plate capacitor (Figure 1). Measuring the effect of these forces on an oil droplet makes it possible to measure the effect of excess electric charge on the droplet. With enough measurements, you will be able to measure the elementary electrical charge of a single electron.

Oil from the reservoir is 'atomized' by blowing air across the reservoir. As oil droplets are pushed into the chamber they acquire excess electric charges from friction (known as triboelectric charging). Once in the chamber the droplets will fall slowly under the influence of gravity while feeling the effect of the electrical charge between the plates, buoyancy effects from the surrounding air, and drag from falling through the air. The four relevant forces experienced by the droplet inside the chamber.

- A downward gravitational force $F_{\mathrm{grav}} = m_{oil}g$, where m_{oil} is the mass of the oil droplet, g is gravity. *note the sign of the force here.*

- An upward buoyancy force $F_{\mathrm{b}} = -m_{air}g$ where m_{air} is the mass of air displaced by the droplet.

- An electric force $F_E = qE$, where q is the excess charge on the droplet, and $E = \frac{V}{d}$ is the electic field inside the chamber using plates charged with a potential difference V separated by distance d.

- Resistance to the moving droplet following Stokes law $F_{drag} \approx 6\pi\, r\nu v$ for a particle of radius r falling at speed v through a fluid of viscosity ν.
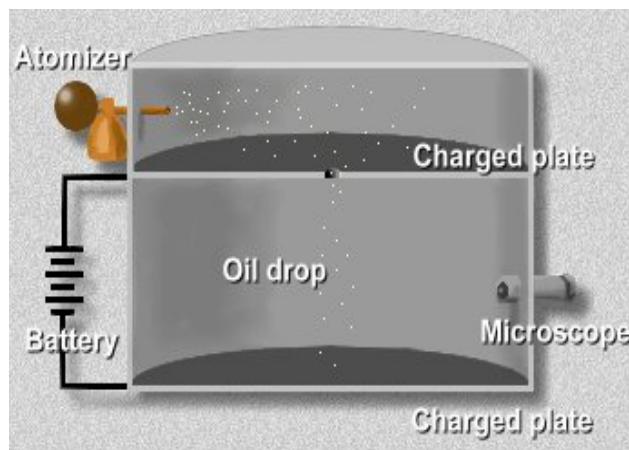

Figure 1: Sketch of a device to produce and observe motion of charged droplets. The voltage across the chamber can be adjusted to change the potential difference. In our equipment, the 'microscope' is attached to a digital camera.

Once the droplet is inside the chamber you can place it in one of three terminal states: Moving upwards at constant speed because of an excess charge in the chamber

$$qE = g(m_{\text{oil}} - m_{\text{air}}) + 6\pi\,r\nu\,v_u. \tag{1}$$

Moving downward at the constant terminal velocity v_d:

$$g(m_{\text{oil}} - m_{\text{air}}) = 6\pi\,r\nu\,v_d. \tag{2}$$

Not moving because the voltage across the plates and excess charge of the electron balances the forces of gravity.

$$g(m_{\text{oil}} - m_{\text{air}}) = q\frac{V_{stop}}{d}. \tag{3}$$

It is possible to see droplet the transition between these states, accelerating or decelerating under net forces, but these states are short-lived and not useful for your measurements.

Combining these equations gives an expression for the charge q:

$$q = \frac{(m_{\text{oil}} - m_{\text{air}})g(v_d + v_u)}{Ev_d}. \tag{4}$$

which can be simplified to give expressions relating the excess charge q to your measurements. For example, if you start with a stationary spherical droplet hovering in a potential field V_{stop}, and ignore buoyancy forces you can combine the equations to give

$$q = C_1\frac{v_d^{\frac{3}{2}}}{V_{stop}} \tag{5}$$

where v_d is the downward terminal velocity when there is no potential field.

> **i** You can reasonably ignore buoyancy forces since the density of the oil is 700 times higher than the air it displaces. You can also assume a spherical droplet in Stokes' law to relate the terminal velocity and radius. However, for the low fall speeds and small particles you have in this experiment you should modify Stokes' law with a correction to the viscosity [e.g. Leybold, 2023]. This correction was first derived by Cunningham [1997] and verified by Cunningham and Millikan [Millikan, 1917].

You will use the equations above and measurements of voltage and the droplet position with time to find the elementary charge e.

Method 1

The simplest method requires stopping droplet in the chamber using the plate voltage to get V_{stop}, then switching off the voltage and measuring the fall speed of the droplet v_d. These measurements can be used in equation 5 (or an equivalent equation with corrected viscosity) to calculate the excess charge q of each droplet.

Constant	Symbol	Numerical value
Density of oil	ρ_{oil}	875.3kgm^{-3}
Density of air (at the surface)	ρ_{air}	1.204kgm^{-3}
Acceleration due to gravity	g	9.80ms^{-2}
Dynamic viscosity of air at 1 atm	ν	$1.827 \times 10^{-5} \text{Ps s}$
The separation of the parallel plates	d	6.0mm

Table 1: Table of Physical constants for the Millikan experiment

However, since you will not know the number of excess charges in each droplet, you need to collect a large enough sample of droplets to make sure that some droplets have an excess charge of on electron. You should collect a large number ($N > 50$) of droplets and calculate the charge for each one. You will need to show that your charges are multiples a common divisor. The greatest common divisor will be the elementary charge of the electron.

Method 2

An alternative method uses the upward and downward motion in equation 4 along with the voltage on the plates (represented by the charge E) that **isn't** the stopping voltage. The analysis for the second method is the same as the first method but you no longer need a precise stopping voltage. Instead you need to find the motion in two directions using any voltage high enough to make the droplet rise.

The recommended procedure below takes both methods into account and suggests a procude that collects both datasets at the same time. First find V_{stop} (method 1), then choose a higher voltage to make the droplet rise (method 2), then let the droplet fall (method 1 and 2).

Apparatus and Measurements

Familiarize yourself with the apparatus before taking actual measurements. The Leybold-Heraeus apparatus in figure 2 allows you to determine the elementary electric charge using the digital camera image and `LabVIEW` software that returns the position in pixels of the selected droplet frame by frame with the frame rate of 10Hz. We calibrated the camera to convert the outcome in pixels into the distance in millimeter, and found the calibration factor being 540 ± 1px/mm for the left device, and 520 ± 1px/mm for the right device.
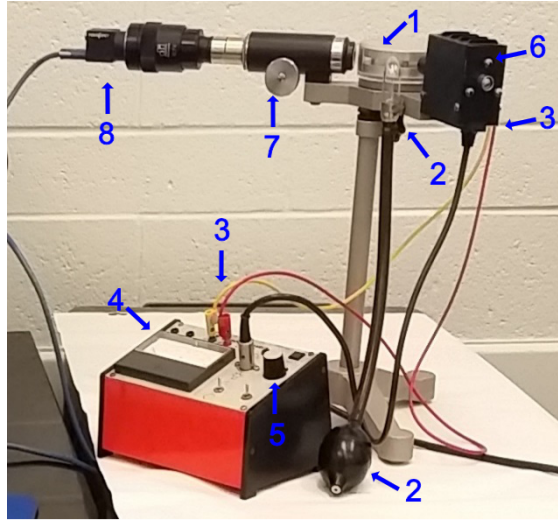
Figure 2: Leybold-Heraeus apparatus: **1**: Chamber (plate capactior), **2**: oil atomizer with a rubber bulb. **3**: socket pair for charging the plate capacitor connected to the DC power supply. **4**: DC power supply with adjusting knob (**5**). **6**: light source, **7**: microscope connected to a CCD camera. **8**: CCD camera.

Make sure that the spray nozzle of the oil atomizer is positioned before the small holes in the acrylic glass cover of the Millikan chamber.

Preparation of software

The Millikan experiment software will track droplets and record the position. You can also save the position information to an Excel file. You will also need to separately record any voltages and filenames for the each droplet.

Launch the Millikan experiment application on the desktop and open the software. Click "OK" when required to initialize the camera. Figure 3 shows the application with a droplet being measured.
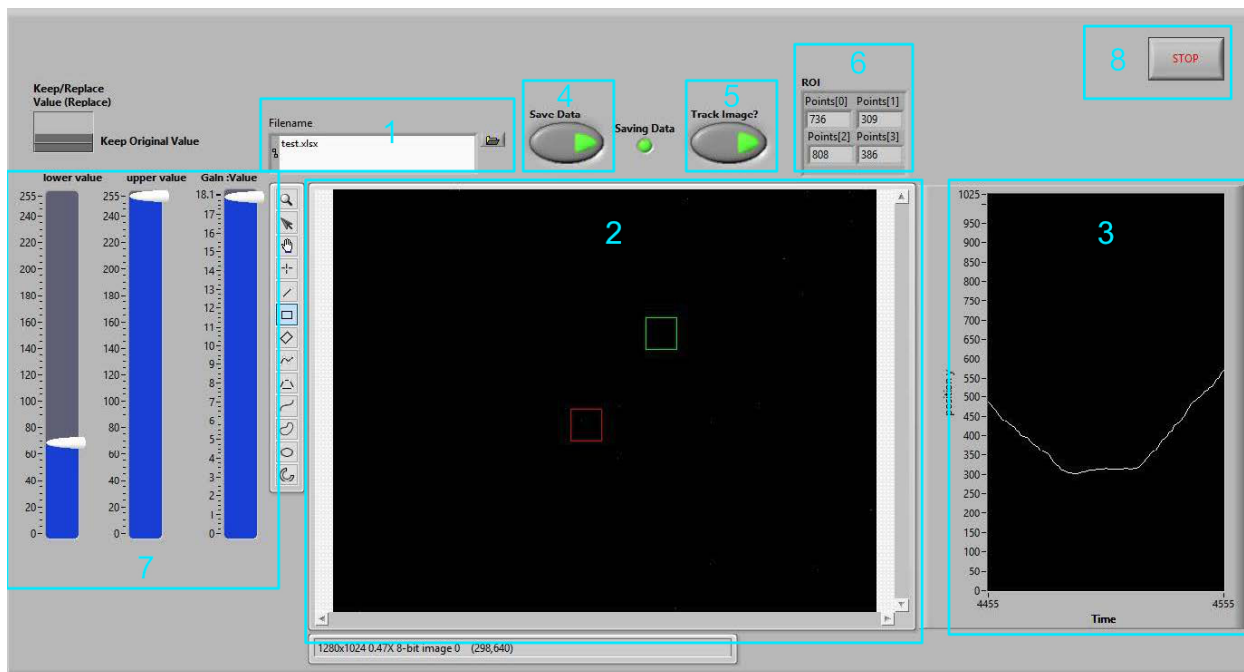
Figure 3: Software display image. **1**: A text window with a file name to save the data. **2**: large central window to observe the chamber; **3**: a window showing the position of the droplet versus frame number; **4**: `Save Data` button to start recording the data in the open file; **5**: `Track Image` button to track the chosen droplet; **6**: four small `ROI` windows with coordinates of the corners of the frame you used to trap the droplet; **7**: Three sliders to adjust brightness and contrast of the field of view; **8**: `STOP` button at the upper right corner to reload the software for new series of data acquisition.

Procedure

- This experiment is best performed in a dark room. The chamber has a built in light source that lights the droplets from the side. Check that the setup if connected as in figure 2. Turn on the power supply (figure 2, item **4**)).

- Set the `Upper Gain` slider to its maximum. Adjust display illumination by moving the `Lower Gain` slowly up to increase brightness to dark grey. The slides will need to be adjusted for each droplet to optimize the image.

- Set the voltage to a high initial value to force almost all droplets to rise. This will simplify selecting a droplet.

- Squeeze the rubber bulb two or three times to send droplets into the chamber. The main display should show falling droplets of different sizes and brightness. Choose a droplet and adjust the voltage to make it rise slowly. This will let you adjust the slider settings, adjust the microscope focus, and set the tracking box in the main window (**2**).

- Click on the button `Track Image` to start tracking the image. A red box will appear and follow the droplet as it moves. The location of this box is the data collected by the application. Click the `Save Data` button to start recording the droplet coordinates to the Excel file.

- When you are successfully tracking a droplet, record the voltage needed to stop the droplet rising (method 1).

- Increase the voltage to cause the droplet rise. Record this voltage (method 2).

- Switch off the voltage to cause the droplet to fall (method 1 *and* method 2).

- While the droplet is falling, the right window (3) shows the position in pixels (higher indexed pixels are lower in the chamber). If the droplets are moving at their terminal speed, you should see a straight line.

- Click `Save Button` to stop recording data. Click `Track Image` to stop the tracking.

- Open the Excel file you used for recording the data. The data is obtained as one column with the values of position of the droplet in each frame. You can add a column for frame number or time here, or add the information later with Python.

- Save this file on your USB drive.

Using the collected voltages and droplet positions, calculate the velocities required for each method, and calculate the charges q using the relevant equation. You will need to read the file into Python and split the data into the sections that are relevant for each method. Fit each section of the data to find the constant speed of rising or falling motion. Use these velocities to find the charges q.

> ⚠ Each droplet is only a few microns across and the pixel scale of the display is approximately 2 microns/pixel. It will take many attempts to successfully find and track the first droplets using the microscope and tracking application. Once you have found a working procedure it will also take time to collect voltages and position data for the > 50 droplets needed for the statistical analysis below.

Quantification of the elementary charge

Using the table of charges q derived from the particle tracking you can try two different methods to find the elementary charge.

In the first method you need to find the greatest common divisor (GCD) for the set of q values, which should correspond to the elementary charge if you had droplets with a single excess electron. You will likely need to divide by a candidate GCD value and determine if the divisor fits your data, and repeat this to find the best divisor.

> ℹ️ Once you have found a divisor, you should be able to group values into multiples of this GCD. If your GCD represents 1 electron charge you should find integer multiples separated by 1 charge. If your GCD represents 2 or more electron charges you will have more obvious non-integer multipler (e.g. a GCD of 2e might give multiples near 1, 1.5, 2, 2.5…).

Another method is to plot the q values in a histogram. If the histogram bin size is many times smaller than the smallest value in your dataset you should see data points grouped around multiples of the greatest common divisor (as above) and a lot of bins with no counts.

You can use either fitting procedure with either data experimental method. You will need to determine the uncertainty in your elementary charge and explain how you calculate it. Discuss the accuracy of your result and use it to compare your calculated charge with the expected value.

Acknowledgements

References

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