OLS With L2 Regularization

June 8, 2016

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        %matplotlib inline
```

0.1 The function

L2 regularization, a.k.a. ridge regression has a closed-form solution for w: $w = (X^T X + \alpha I)^{-1} X^T Y$
We can then calculated predicted values of Y using: $\hat{Y} = Xw$
In practice, we can use cross-validation to determine an optimal value for alpha.

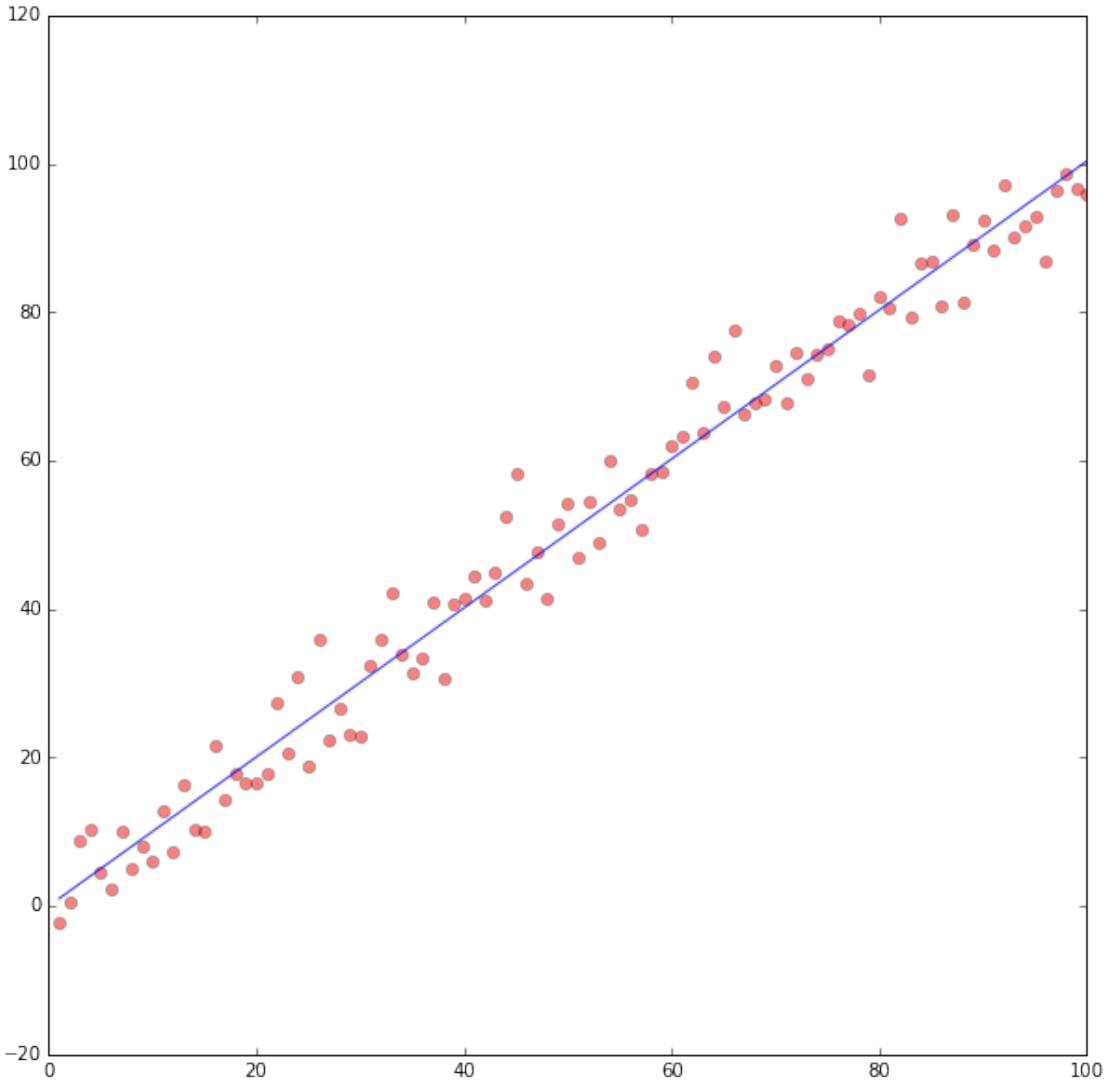
```
In [2]: def weight_vector(Xmat, Y, alpha):
            # catch cases where there is only 1 feature
            if type(Xmat) != np.ndarray:
                Xmat = np.array(Xmat)
                Xmat = Xmat.reshape((len(Xmat),1))
            num_features = Xmat.shape[1]
            numerator = np.dot(Xmat.T, Xmat) + alpha*np.identity(num_features)
            numerator = np.linalg.inv(numerator)
            denominator = np.dot(Xmat.T, Y)
            w = np.dot(numerator, denominator)
            y_hat = np.dot(Xmat, w)
            return w, y_hat
```

0.1.1 1 Dimensional Case

```
In [3]: x_lin = range(1,101)
        y_lin = range(1,101)
        for i in range(len(y_lin)):
            if np.random.random(1) > 0.5:
                y_lin[i] += np.random.normal(loc=0, scale=5, size=1)[0]
            else:
                y_lin[i] -= np.random.normal(loc=0, scale=5, size=1)[0]

        w, y_hat = weight_vector(x_lin, y_lin, alpha = 0.5)

        plt.figure(figsize=(10,10))
        plt.plot(x_lin, y_lin, 'ro', alpha = 0.5, color='r')
        plt.plot(x_lin, y_hat, alpha = 0.8, color='b')
        plt.show()
```



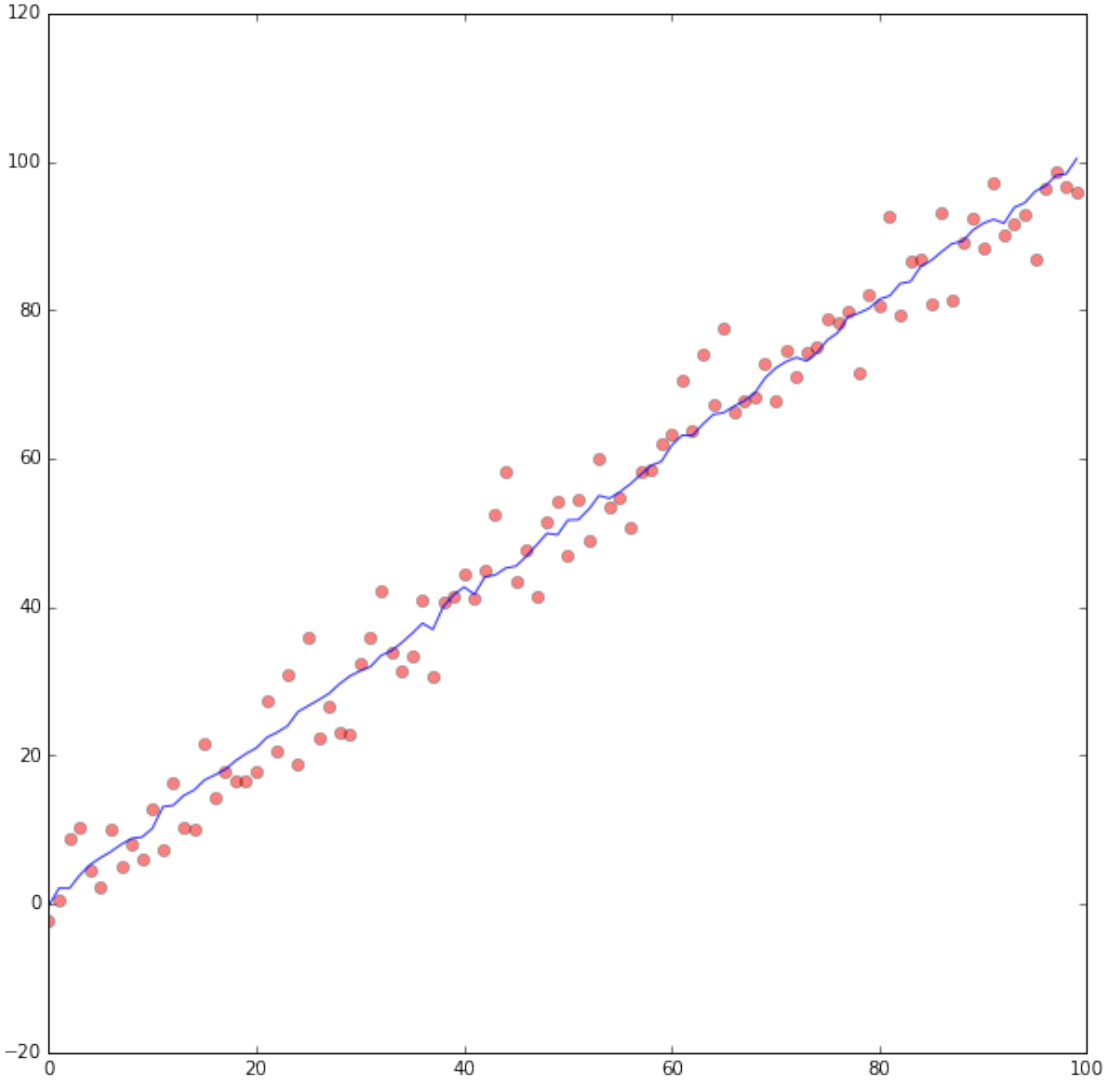
0.1.2 Multidimensional case

In [4]:
```
# generate random data - note there will be multicollinearity between X1 and X2 due to the way
X1 = x_lin
X2 = np.random.normal(loc=X1, scale=5, size=100)
Y = y_lin

Xmat = np.vstack((X1,X2)).T
```

In [5]:
```
w, y_hat = weight_vector(Xmat=Xmat, Y=Y, alpha=0.7)
```

In [6]:
```
plt.figure(figsize=(10,10))
plt.plot(Y, 'ro', alpha = 0.5, color='r')
plt.plot(y_hat, alpha = 0.8, color='b')
plt.show()
```


120
100
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