HW5

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6.5

An engineer is interested in the effects of cutting speed (A), tool geometry (B), and cutting angle (C) on the life (in hours) of a machine tool. Two levels of each factor are chosen, and three replicates of a 23 factorial design are run. The results are as follows:

(a) Estimate the factor effects. Which effects appear to be large?

(b) Use the analysis of variance to confirm your conclusions for part (a).

(c) Write down a regression model for predicting tool life (in hours) based on the results of this experiment.

(d) Analyze the residuals. Are there any obvious problems?

(e) On the basis of an analysis of main effect and interaction plots, what coded factor levels of A, B, and C would you recommend using?

6.6 (OPTIONAL)

Reconsider part (c) of Problem 6.5. Use the regression model to generate response surface and contour plots of the tool life respnse. Interpret these plots. Do they provide insight regarding the desirable operating conditions for this process?

6.7

Find the standard error of the factor effects and approximate 95 percent confidence limits for the factor effects in Problem 6.5. Do the results of this analysis agree with the conclusion from the analysis of variance?

6.10

Reconsider the experiment described in Problem 6.5. Suppose that the experimenter only performed the eight trials from replicate I. In addition, he ran four center points and obtained the following response values: 36, 40, 43, 45.

(a) Estimate the factor effects. Which effects are large?

(b) Perform an analysis of variance, including a check for pure quadratic curvature. What are your conclusions?

(c) Write down an appropriate model for predicting tool life, based on the results of this experiment. Does this model differ in any substantial way from the model in Problem 6.5, part (c)?

(d) Analyze the residuals.

(e) What conclusions would you draw about the appropriate operating conditions for this process?

6.11

An experiment was performed to improve the yield of a chemical process. Four factors were selected, and two replicates of a completely randomized experiment were run. The results are shown in the following table:

(a) Estimate the factor effects.

(b) Prepare an analysis of variance table and determine which factors are important in explaining yield.

(c) Write down a regression model for predicting yield, assuming that all four factors were varied over the range from –1 to + 1 (in coded units).

(d) Plot the residuals versus the predicted yield and on a normal probability scale. Does the residual analysis appear satisfactory?

(e) Two three-factor interactions, ABC and ABD, apparently have large effects. Draw a cube plot in the factors A, B, and C with the average yields shown at each corner. Repeat using the factors A, B, and D. Do these two plots aid in data interpretation? Where would you recommend that the process be run with respect to the four variables?

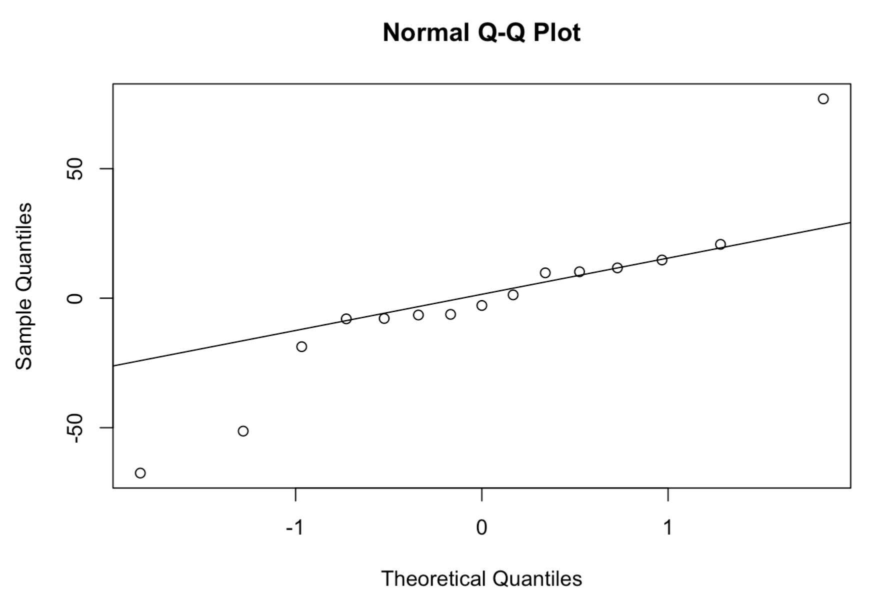
6.21

An experimenter has run a single replicate of a 2^4 design. The following effect estimates have been calculated:

```
effects <- c(76.95, -67.52, -7.84, -18.73,
             -51.32, 11.69, 9.78, 20.78, 14.74, 1.27,
             -2.82, -6.50, 10.20, -7.98, -6.25)
names(effects) <- c("A", "B", "C", "D",
            "AB", "AC", "AD", "BC", "BD", "CD",
            "ABC", "ABD", "ACD", "BCD", "ABCD")
```

(a) Construct a normal probability plot of these effects.

```
qqnorm(effects)
qqline(effects)
```




```
print(sort(effects)[c(1,2,15)])
```

```
##       B     AB      A
## -67.52 -51.32  76.95
```

(b) Identify a tentative model, based on the plots of the effects in part (a).

```
mean(effects)
```

```
## [1] -1.57
```

```
76.95/2
```

```
## [1] 38.475
```

```
67.52/2
```

```
## [1] 33.76
```

```
51.32/2
```

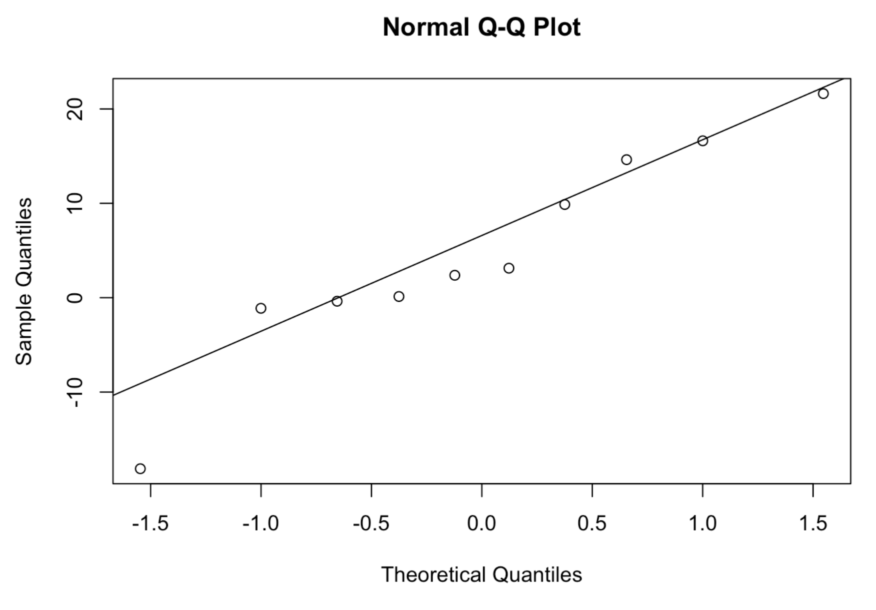
```
## [1] 25.66
```

y = -1.57 + 38.475 * A - 33.76 * B - 25.66 * AB

6.29

Consider the single replicate of the 24 design in Example 6.2. Suppose that we had arbitrarily decided to analyze the data assuming that all three- and four-factor interactions were negligible. Conduct this analysis and compare your results with those obtained in the example. Do you think that it is a good idea to arbitrarily assume interactions to be negligible even if they are relatively high-order ones?

```
effects <- c(21.625, 3.125, 9.875, 14.625, 0.125, -18.125, 16.625, 2.375, -0.375, -1.125)
names(effects) <- c("A", "B", "C", "D", "AB", "AC", "AD", "BC", "BD", "CD")
qqnorm(effects)
qqline(effects)
```




```
sort(effects)
```

```
##      AC      CD      BD      AB      BC       B       C       D      AD       A
## -18.125  -1.125  -0.375   0.125   2.375   3.125   9.875  14.625  16.625  21.625
```

From the normal probability plot, it seems only AC takes a significant deviation from its expected value. However, when looking at the total contribution of the effects, **we arrive at the same conclusion as we did in example 6.2**, that A, C, D, AC, and AD, all have significant effects for this model. Removing the three- and four-factor interactions changed little as their contribution to the model was negligible. However, **I would not advise to arbitrarily assume interactions to be negligible if higher-order**. I would use the normal probability plot of effect estimates as a guide to choose which effects to include.

6.54 (T/F)

A half-normal plot of factor effects plots the expected normal percentile versus the effect estimate.

False, it plots the absolute value of the effect.

6.55 (T/F)

In an unreplicated design, the degrees of freedom associated with the "pure error" component of error are zero.

True

6.56 (T/F)

In a replicated 2^3 design (16 runs), the estimate of the model intercept is equal to one-half of the total of all 16 runs.

False, it is equal to the average of all 16 runs.

6.58 (T/F)

The mean square for pure error in a replicated factorial design can get smaller if non-significant terms are added to a model.

False, if one term is added the denominator (sample size - number of variables) for MSPE decreases resulting in MSPE increasing.

6.62

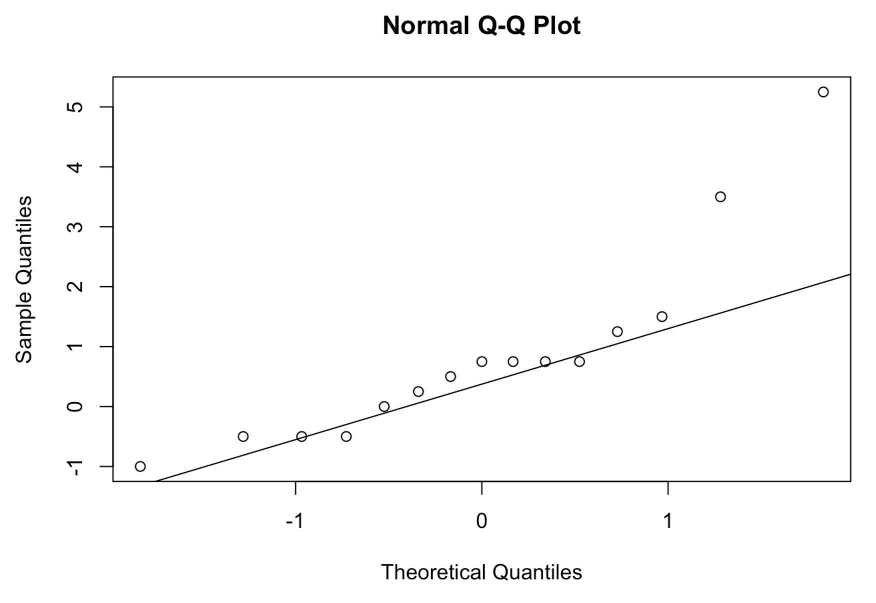
The display below summarizes the results of analyzing a 2^4 factorial design.

(a) Fill in the missing information in this table.

Term Intercept	Effect Estimate	Sum of Squares	% Contribution
A	1.25	6.25	3.25945
B	5.25	110.25	57.4967
C	3.5	49	25.5541
D	0.75	2.25	1.1734
AB	0.75	2.25	1.1734
AC	–0.5	1	0.521512
AD	0.75	2.25	1.1734
BC	1.5	9	4.693608
BD	0.25	0.25	0.130378
CD	0.5	1	0.521512
ABC	–1	4	2.08605
ABD	0.75	2.25	1.1734
ACD	–0.5	1	0.521512
BCD	0	0	0
$ABCD$	–0.5	1	0.521512

(b) Construct a normal probability plot of the effects. Which factors seem to be active?

```
effects <- c(1.25, 5.25, 3.5, 0.75, 0.75, -0.5, 0.75, 1.5, 0.25, 0.5, -1, 0.75, -0.5, 0, -0.5)
names(effects) <- c("A", "B", "C", "D", "AB", "AC", "AD", "BC", "BD", "CD", "ABC", "ABD", "ACD", "BCD", "ABCD")
qqnorm(effects)
qqline(effects)
```




```
sort(effects)
```

```
##   ABC    AC   ACD  ABCD   BCD    BD    CD     D    AB    AD   ABD     A    BC
## -1.00 -0.50 -0.50 -0.50  0.00  0.25  0.50  0.75  0.75  0.75  0.75  1.25  1.50
##     C     B
##  3.50  5.25
```

It looks like factors **C** and **B** are active.