HW2

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2.25

```
cov_X <- matrix(c(25, -2, 4, -2, 4, 1, 4, 1, 9),
         nrow=3, ncol=3, byrow=T)
V <- diag(sqrt(c(25,4,9)))
```

a. Determine rho and V^(1/2)

```
V
```

```
##      [,1] [,2] [,3]
## [1,]    5    0    0
## [2,]    0    2    0
## [3,]    0    0    3
```

```
cor_X <- solve(V) %*% cov_X %*% solve(V)
cor_X
```

```
##            [,1]       [,2]      [,3]
## [1,]  1.0000000 -0.2000000 0.2666667
## [2,] -0.2000000  1.0000000 0.1666667
## [3,]  0.2666667  0.1666667 1.0000000
```

b. Multiply your matrices to check V^(1/2) * rho * V^(1/2) = cov_X

```
V %*% cor_X %*% V
```

```
##      [,1] [,2] [,3]
## [1,]   25   -2    4
## [2,]   -2    4    1
## [3,]    4    1    9
```

```
cov_X == V %*% cor_X %*% V
```

```
##      [,1] [,2] [,3]
## [1,] TRUE TRUE TRUE
## [2,] TRUE TRUE TRUE
## [3,] TRUE TRUE TRUE
```

This checks out!

2.26

a. Find rho (1,3)

```
cor_X[1,3]
```

```
## [1] 0.2666667
```

b. Find the correlation between X1 and 1/2(X2) + 1/2(X3)

```
(cor_X[2,1] + cor_X[3,1])/2
```

```
## [1] 0.03333333
```

2.32

```
cov_X <- matrix(c(4,-1,1/2,-1/2,0,
                  -1,3,1,-1,0,
                  1/2,1,6,1,-1,
                  -1/2,-1,1,4,0,
                  0,0,-1,0,2),
                nrow = 5,ncol = 5, byrow = T)
mu_X <- c(2,4,-1,3,0)
A <- matrix(c(1,-1,1,1),
            nrow = 2,ncol = 2,byrow = T)
B <- matrix(c(1,1,1,1,1,-2),
            nrow = 2,ncol = 3, byrow = T)
```

a. E[X1]

```
mu_X[1:2]
```

```
## [1] 2 4
```

b. E[A * X1]

```
A %*% mu_X[1:2]
```

```
##      [,1]
## [1,]   -2
## [2,]    6
```

c. Cov(X1)

```
cov_X[1:2,1:2]
```

```
##      [,1] [,2]
## [1,]    4   -1
## [2,]   -1    3
```

d. Cov(A * X1)

```
A %*% cov_X[1:2,1:2] %*% t(A)
```

```
##      [,1] [,2]
## [1,]    9    1
## [2,]    1    5
```

e. E[X2]

```
mu_X[3:5]
```

```
## [1] -1  3  0
```

f. E[B * X2]

```
B %*% mu_X[3:5]
```

```
##      [,1]
## [1,]    2
## [2,]    2
```

g. Cov(X2)

```
cov_X[3:5,3:5]
```

```
##      [,1] [,2] [,3]
## [1,]    6    1   -1
## [2,]    1    4    0
## [3,]   -1    0    2
```

h. Cov(B * X2)

```
B %*% cov_X[3:5,3:5] %*% t(B)
```

```
##      [,1] [,2]
## [1,]   12    9
## [2,]    9   24
```

i. Cov(X1, X2)

```
cov_X[1:2,3:5]
```

```
##      [,1] [,2] [,3]
## [1,]  0.5 -0.5    0
## [2,]  1.0 -1.0    0
```

j. Cov(A * X1, B * X2)

```
A %*% cov_X[1:2,3:5] %*% t(B)
```

```
##      [,1] [,2]
## [1,]    0    0
## [2,]    0    0
```

2.40

Verify E[X + Y] = E[X] + E[Y] and E[A * X * B] = A * E[X] * B

2.40 Verify $\mathbb{E}[\underline{X}+\underline{Y}] = \mathbb{E}[\underline{X}] + \mathbb{E}[\underline{Y}]$

$$\mathbb{E}[X+Y] = \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{(x_{ij}+y_{ij})}{n} = \frac{1}{n}\sum_{i=1}^{m}\sum_{j=1}^{n}(x_{ij}+y_{ij}) = \frac{1}{n}\sum_{i=1}^{m}\sum_{j=1}^{n}x_{ij} + \frac{1}{n}\sum_{i=1}^{m}\sum_{j=1}^{n}y_{ij}$$

$$= \mathbb{E}[\underline{X}] + \mathbb{E}[\underline{Y}]$$

Verify $\mathbb{E}[AXB] = A\,\mathbb{E}[X]\,B$

$$\mathbb{E}[AXB] = \mathbb{E}\left[\sum_{k=1}^{n}\sum_{l=1}^{n} a_{ik}x_{kl}b_{lj}\right] = \sum_{k=1}^{n}\sum_{l=1}^{n} a_{ik}\,\mathbb{E}[X]\,b_{lj} = A\,\mathbb{E}[X]\,B$$

2.41

```
cov_X <- diag(c(3,3,3,3))
mu_X <- c(3,2,-2,0)
A <- matrix(c(1,-1,0,0,
              1,1,-2,0,
              1,1,1,-1),
           nrow = 3,ncol = 4,byrow = T)
```

a. E[A * X]

```
A %*% mu_X
```

```
##      [,1]
## [1,]    1
## [2,]    9
## [3,]    3
```

b. Cov(A * X)

```
A %*% cov_X %*% t(A)
```

```
##      [,1] [,2] [,3]
## [1,]    6    0    0
## [2,]    0   18    0
## [3,]    0    0   12
```

c. Which pairs of linear combinations have zero covariances? The linear combinations which have zero covariances are the off-diagonal entries