14) $X \sim N_p(\mu, \Sigma)$ with $|\Sigma| \neq 0$

Show joint density can be written as product of marginal densities for X_1 and X_2 if $\Sigma_{12} = 0$

$$\Sigma = \begin{pmatrix} \Sigma_{11} & 0 \\ 0 & \Sigma_{22} \end{pmatrix} \quad \text{cov matrix} \quad \Sigma^{-1} = \begin{pmatrix} \Sigma_{11}^{-1} & 0 \\ 0 & \Sigma_{22}^{-1} \end{pmatrix}$$

$$f(x_1)f(x_2) = N_q(\mu_1, \Sigma_{11})\, N_{p-q}(\mu_2, \Sigma_{22})$$
$$= (2\pi)^{-p/2} |\Sigma_{11}|^{-1} |\Sigma_{22}|^{-1} \exp\{(x_1-\mu_1)'\Sigma_{11}^{-1}(x_1-\mu_1) + (x_2-\mu_2)'\Sigma_{22}^{-1}(x_2-\mu_2)\}$$
$$= (2\pi)^{-p/2} |\Sigma|^{-1} \exp\{-(x-\mu)'\Sigma^{-1}(x-\mu)/2\}$$

15) Show that $\sum_{j=1}^{n} (x_j - \bar{x})(\bar{x} - \mu)'$ and $\sum_{j=1}^{n} (\bar{x} - \mu)(x_j - \bar{x})'$ are both $p \times p$ matrices of zeroes

$$\Sigma\left(\begin{pmatrix} x_1 \\ \vdots \\ x_j \end{pmatrix} - \begin{pmatrix} \bar{x} \\ \vdots \\ \bar{x} \end{pmatrix}\right) = \Sigma \begin{pmatrix} x_1 - \bar{x} \\ \vdots \\ x_j - \bar{x} \end{pmatrix} \underset{1 \times j}{(\bar{x} - \mu)'} \quad j = 1$$

$$(\bar{x} - \mu) \sum_{j=1}^{n} (x_j - \bar{x})'$$
$$= (\bar{x} - \mu)(n\bar{x} - n\bar{x})'$$
$$= (\bar{x} - \mu)(0) = 0$$

$\Sigma(x_j\bar{x} - \bar{x}^2 + \bar{x}\mu - x_j\mu) \quad j=j$

$\Sigma(x_j\bar{x} - x_j\mu - \bar{x}^2 + \bar{x}\mu)$

$\Sigma(x_j(\bar{x} - \mu) - \bar{x}(\bar{x} - \mu)) = \Sigma(x_j - \bar{x})(\bar{x} - \mu)$ for each entry

$$\cancel{(n\bar{x} - n\bar{x})}(\bar{x} - \mu)$$
$$(0)(\bar{x} - \mu) = 0$$

16) Let $X_1\, X_2\, X_3\, X_4$ be ind $N_p(\mu, \Sigma)$ random vectors

a) Find
$$V_1 = \tfrac{1}{4}X_1 - \tfrac{1}{4}X_2 + \tfrac{1}{4}X_3 - \tfrac{1}{4}X_4$$
$$V_2 = \tfrac{1}{4}X_1 + \tfrac{1}{4}X_2 - \tfrac{1}{4}X_3 - \tfrac{1}{4}X_4$$

By 4.8 in txtbook $N_p\left(\sum^{n} c_i\mu, \left(\sum^{n} c_i^2\right)\Sigma\right)$

$V_1 \sim N_p(0, \tfrac{1}{4}\Sigma) \qquad V_2 \sim N_p(0, \tfrac{1}{4}\Sigma)$

b) Find joint density random vectors V_1, V_2 defined in (a)

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \sim N_{2p}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tfrac{1}{4}\Sigma & 0 \\ 0 & \tfrac{1}{4}\Sigma \end{pmatrix}\right)$$

$\leftarrow$ Od diagonals ar

$(b^T c)\Sigma$

$$0 = \begin{pmatrix} \tfrac{1}{4} \\ \tfrac{-1}{4} \\ \vdots \end{pmatrix} \left(\tfrac{1}{4}\ \tfrac{1}{4}\ -\tfrac{1}{4}\ -\tfrac{1}{4}\right)$$