

Homework 4

Due Date: 11:59 PM, 11/19/2025

You should create a single GitHub repository for this class and share it with the instructor (khalidjm@seas.ucla.edu). All the homeworks, reports, presentations, and proposal should be uploaded to this repository. Do not create a separate repository for each assignment. Within your repository, create a separate folder for each assignment (e.g., `homework_1`, `homework_2`, `homework_3`, `homework_4`, `homework_5`, `proposal`, `midterm_report`, and `final_report`).

Submission Instructions: Your submission on BruinLearn should only contain the URL to your GitHub repository. Your GitHub repository should include the following items:

1. **Report in PDF format:** Submit a report in `.pdf` format (file name should be `Homework4_LASTNAME.pdf`, replacing `LASTNAME` with your last name) addressing the questions asked in the deliverables. Include all the plots and figures requested in the assignment and discuss them in the report. See the syllabus for formatting requirements. As stated in the syllabus, you must use one of the provided templates.
 2. **Source code:** The submission should include one main file named exactly as `HomeworkX.[ext]` (where `X` is the homework number) along with any additional files (e.g., functions or text files) as necessary. The main file should require no more than a single command to run or one click for execution.
 3. **README file:** Add a `README.md` file on GitHub to provide clear instructions on how to run your code and describe the purpose of each file included in your repository.
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Axial Response of a Helical Spring in Discrete Elastic Rod

In this homework, you will model a helical coil as a 3D Discrete Elastic Rod (DER) and quantify its axial stiffness from dynamic relaxations to steady states.

Geometry & Material.

Helix parameters:

wire diameter $d = 0.002$ m,

mean coil diameter $D = 0.04$ m (helix radius $R = D/2$),

pitch per turn $p = d$,

number of turns $N = 5$.

The helix axis is the global z -axis. Axial (end-to-end) length $L_{\text{axial}} = Np$.

Arc length per turn $L_{\text{turn}} = \sqrt{(2\pi R)^2 + p^2}$.

Total contour length $L = N L_{\text{turn}}$.

Material parameters:

Young's modulus $E = 10$ MPa,

Poisson ratio $\nu = 0.5$,

shear modulus $G = \frac{E}{2(1+\nu)}$.

Section properties (circular wire):

area $A = \frac{\pi d^2}{4}$,

second moment $I = \frac{\pi d^4}{64}$,

polar moment $J = \frac{\pi d^4}{32}$.

Discretization. Construct the rest configuration of the helix using at least 10 nodes per turn (i.e., $\geq 10N$ nodes).

Boundary Conditions. Clamp the first two nodes and the first edge material angle (first seven DOFs of DER). These DOFs remain fixed for all time steps.

Loading & Characteristic Force. Apply a constant tensile force F at the *last* node, aligned with the \hat{z} , such that the helix extends axially. As described in Parts (1–3), the magnitude of this load is related to the characteristic bending force magnitude

$$F_{\text{char}} = \frac{EI}{L_{\text{axial}}^2}.$$

Tip Displacement. Let the end node position be $(x_{\text{end}}(t), y_{\text{end}}(t), z_{\text{end}}(t))$. Define the axial tip displacement

$$\delta_z(t) = z_{\text{end}}(t) - z_{\text{end}}(0),$$

so that $\delta_z(0) = 0$. Integrate in time long enough for transients to decay and $\delta_z(t)$ to reach a steady value.

Part (1): Single Load Level. Use $F = F_{\text{char}}$. Run the simulation to steady state and record δ_z^* . In your report, include at least five snapshots of the helix centerline at different simulation times. Use **axis equal** (or its equivalent) and label the x , y , and z axes with units. On each snapshot, clearly indicate the simulation time by placing a text label directly on the figure. For example, if the snapshot corresponds to $t = 5$ s, then include a visible annotation such as “**t = 5 s**” on that snapshot.

Plot time t vs. displacement $\delta_z(t)$ starting from $(0,0)$. State clearly how you decide that the system has reached steady state. For example, you may choose a simple rule such as: the value of $\delta_z(t)$ changes by less than one percent over a one-second interval. Once you determine that the motion has settled according to your chosen rule, mark the corresponding steady value δ_z^* on your time–displacement plot.

Part (2): Force Sweep and Linear Fit. Vary the axial force F using **logspace** (for example, in NumPy) so that the sampled force values range from $0.01 F_{\text{char}}$ to $10 F_{\text{char}}$ on a logarithmic scale. For each force value, run the simulation to steady state and record the corresponding displacement δ_z^* . Plot the applied force F versus the steady displacement δ_z^* . To extract the linear stiffness k , fit the data in the small-displacement region using the model

$$F = k \delta_z^*.$$

Do not use a polynomial fit of the form $y = mx + b$; the intercept must be fixed at zero.

Part (3): Diameter Sweep vs. Textbook Trend. Textbooks often approximate the axial stiffness of a close-coiled helical spring using the formula

$$k_{\text{text}} = \frac{G d^4}{8 N D^3}, \quad (1)$$

which is valid only in the linear, small-deflection regime where the spring behaves like a torsion-dominated structure. In this part, you will examine how your DER-based numerical stiffness compares to this classical prediction.

Vary the helix diameter D between 0.01 m and 0.05 m using at least ten values, and keep all other parameters (d, p, N, E, ν) fixed. For each chosen D , run your DER simulation as in Part (B) to obtain the linear stiffness k as a function of D .

Make a plot with horizontal axis $G d^4 / (8 N D^3)$ and vertical axis k . On the same plot, draw the textbook relation in Eq. 1 (i.e., a straight line of slope 1 through the origin) for reference.

Discuss the trends you observe in your results. The goal of this part is not to obtain exact numerical agreement with the textbook formula. You should not expect quantitative agreement between your DER simulations and the classical expression. Instead, focus on whether the qualitative behavior matches the expected trend. For example, as the helix diameter D increases, the spring becomes more flexible and the axial stiffness k decreases. Your discussion should compare this qualitative trend with the textbook prediction and comment on any deviations.

Deliverables. You are encouraged to include as many plots, figures, and snapshots as needed to make your report clear; however, at a minimum, the following items must be included:

- Summary of the plots: (1) t vs. $\delta_z(t)$ with steady-state identification (accompanied by a description of steady-state criterion used), and helix snapshots (≥ 5) with labeled axes/units/time; (2) F vs. δ_z^* with zero-intercept best-fit line; (3) k vs. $G d^4 / (8 N D^3)$ with the slope-1 reference.
- Numerical values: reported k from the linear fit(s) and the steady δ_z^* at $F = F_{\text{char}}$.

Figure Requirements. All plot must have axis labels with quantity and *units*, legends (if appropriate). Each figure in your report must have captions with figure numbers (e.g., “Figure X: Five snapshots of the helix at (a) $t = 0$ s, (b) $t = 1$ s, (c) $t = 5$ s, (d) $t = 10$ s, (e) $t = 50$ s”). Do not use screenshots; export high-resolution vector graphics (PDF) and include them in the report.