Traffic Modelling on a Single-Lane Motorway

MTH3024 Project

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Introduction, and Questions to Address:

Traffic modelling is an important part of transportation planning. This project's first goal is to implement the Intelligent Driver Model on a single-lane motorway, looping in a circle. The model will also take some macroscopic measurements such as the total traffic flow over time, and the density of this traffic. Once this is complete, we wish to implement some changes to the road system in order to find ways to improve the flow of traffic.

Ideally, we wish to find some model parameters that optimise the traffic system's efficiency while also remaining safe for all road users. We will also implement a "speed zone" in order to answer some more questions about model parameters. Does changing the speed limit affect the overall flow of traffic? Or are there other factors that are more important for the movement of cars through our road system?

Setting Up Our Model

We assume that the road is a single lane motorway, which means cars are unable to overtake each other. If the car in front is slower, they will also decelerate and potentially stop if they come too close to the leading car.

Defining Fundamental Values

The Intelligent Driver Model (IDM) requires us to calculate the velocity, acceleration, and position on the road as time passes. From Newtonian dynamics, we know we can define the position and velocity of each car

$$\begin{aligned} x_t &= x_0 + v_0 + \frac{1}{2}a_0 t^2 \\ v_t &= v_0 + a_0 t \end{aligned} \tag{1}$$

While acceleration is given by

$$\dot{v} = a\left[1 - \left(\frac{v}{v_0}\right)^{\delta} - \left(\frac{s^*(v, \Delta v)}{s}\right)^2\right] \tag{2}$$

Where a is the obstacle free acceleration at rest, v, v_0 are the current and desired velocity respectively (the desired velocity in this case will be the set speed limit of the motorway), δ is a parameter for the acceleration profile, [1] and s^* is the desired bumper-to-bumper distance to the preceding car, calculated through the current velocity and the approach velocity to this car (Δv). s is the current bumper-to-bumper distance to the preceding car, also known as the headway and the desired headway s^* is given by

$$s^* = s_0 + \max\left(0, vT + \frac{v\Delta v}{2\sqrt{ab}}\right). \tag{3}$$

This contains our minimum gap between bumpers, $s_0 + vT$, but also a dynamical term which accounts for our braking strategy, where b is a braking parameter for comfortable deceleration.

Using these terms, we can define the position of a car on the road in a continuous time scale. However, to run these in a programme, we will need to define them with respect to a discrete time step, Δt.

We can define the updating of positions and velocity using this time step:

$$\begin{aligned} x_{new} &= x + v\Delta t + \frac{1}{2}\dot{v}(\Delta t)^2 \\ v_{new} &= v + \dot{v}\Delta t. \end{aligned} \tag{4}$$

To stop our velocity becoming negative through this calculation, we can change these terms to instead become

$$\begin{aligned} x_{new} &= x + v(\Delta t)^* + \frac{1}{2}\dot{v}\big((\Delta t)^*\big)^2 \\ v_{new} &= v + \dot{v}(\Delta t)^*, \end{aligned} \tag{5}$$

where $(\Delta t)^*$ is the time required for the car to come to a halt, defined as $(\Delta t)^* = -\frac{v}{\dot{v}}$.

We can use these expressions to create a basic implementation of the Intelligent Driver Model.

Investigations

Quantification of Traffic Flow

The ultimate goal of the project is to model traffic flow according to the intelligent driver model, a way to measure this is by calculating global density ρ_g, global flow (Q_g), and their local counterparts Q_l, ρ_l. The definitions of the global parameters are

$$\begin{aligned} \rho_g &= \sum_{i=1}^{n} \frac{v_i}{L} \text{ and} \\ Q_g &= \sum_{i=1}^{n} \frac{v_i}{i} \times \rho_g, \end{aligned} \tag{6}$$

where n is the total number of cars, L the total length of the road and v_i is the speed of the i^{th} car on the road.

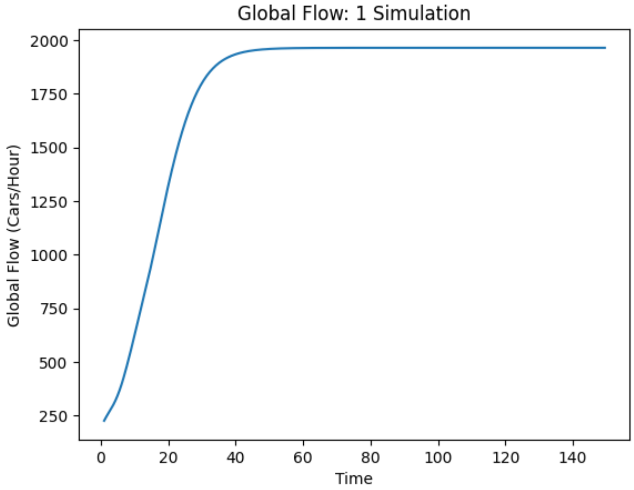

Figure 1: *The Global Flow of Traffic over the course of one simulation of our initial model for 20 cars, where the traffic reaches its desired velocity and stays there until the simulation ends.*

This data is taken from a simulation of 20 cars, which was chosen as it's a sufficient number to show interactions between the cars, initialised onto our simulated road. In this first version of our model, cars are initialised, then reach their desired velocity until our simulation ends. The graph reflects this, with the flow rising at first, then flattening out once the cars all reach the speed limit. There's little to no interaction between cars once this happens.

Iterating Upon Our Model

To create some more realistic interactions between cars in our model for analysis, we implemented a different speed zone, located halfway along our road. We also implemented various "weather conditions" which would affect certain parameters from our model. For setting up the cars on the road, we introduced an element of randomness to the distance between the starting positions of cars. Once these changes were made, we swept through the number of cars initialised in a simulation, to see how it affected traffic flow and density. We also included a detection function which would trigger upon a car passing its location, and append the velocity of each car in order to calculate the local flow and density at this point in the road.

Analysing Flow and Density Trends

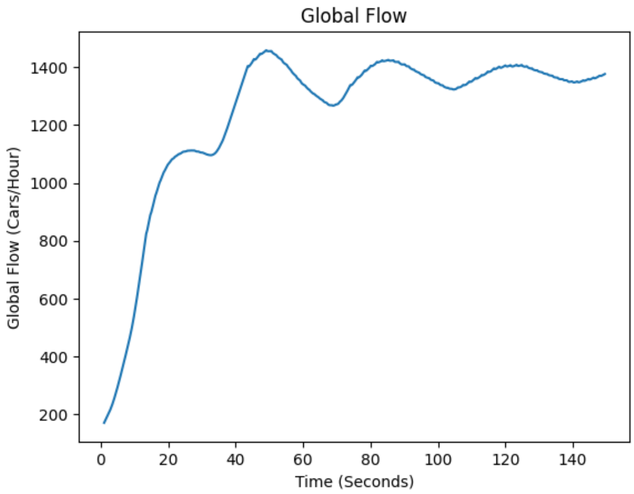

Figure 2: *The flow of traffic over time in one Simulation after the introduction of a speed zone*

Since the flow is calculated based on the average velocity of all the cars in the system, the shape of this graph makes sense as the average velocity would fluctuate much more with the addition of the slower speed limit zone. Now, we can sweep over the number of cars on the road, taking values of global and local density and flow at our detection point.

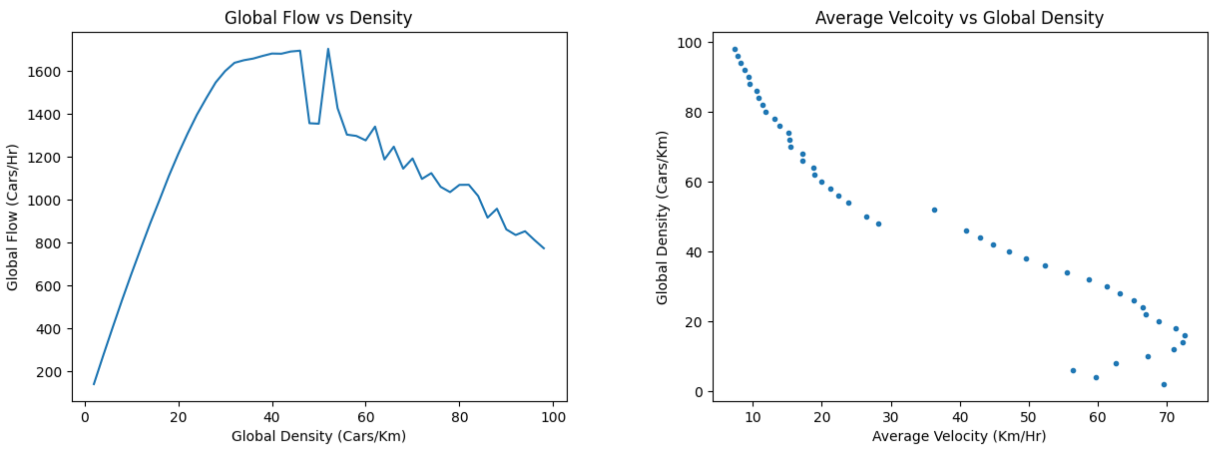

Figure 3: *Graphs of the relationships between global flow against global density and global density against velocity as more cars are introduced into the system.*

Firstly, the flow of traffic overall through the system was increased with the first additions of cars into the system, then sharply decreased as more were added, and it became too difficult for the cars to drive at the desired velocity, while also maintaining the ideal headway s^* between each vehicle. Clearly, once there are roughly 50 cars or more on the road, the flow generally decreases from its highest values. We would expect this since there are too many cars on the road for traffic to flow as freely as when there were less cars on the road.

The density is equal to the number of cars in the simulation, since the road length is only 1 kilometer. Clearly, the density of traffic negatively impacts the average velocity of vehicles on the road, which we would expect to be true already. This makes us more certain our model accurately simulates the flow of traffic through our motorway.

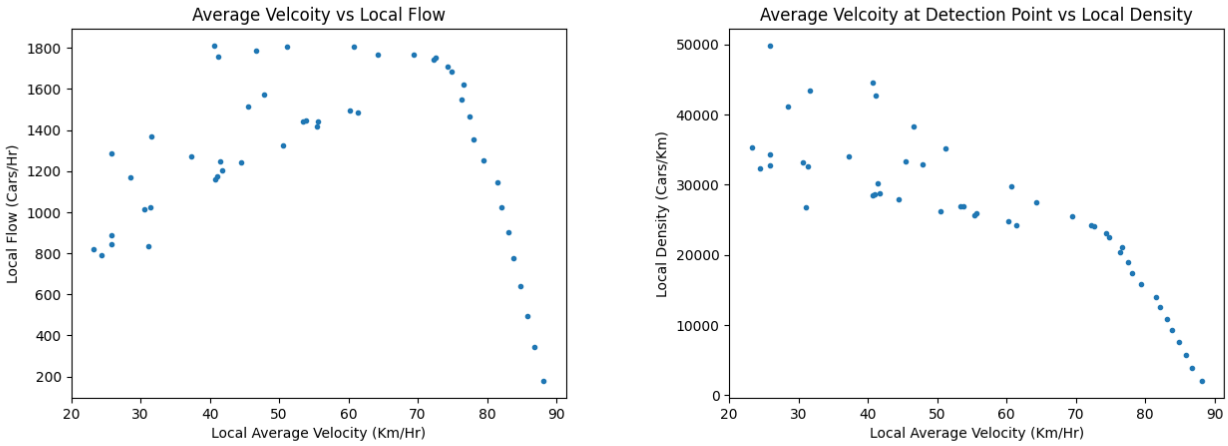

Figure 4: *Scatter plots of the relationships between the local flow and local density, both against the average velocity of cars passing the detection point.*

Our local flow against velocity averaged at the detection point is what we would expect, where the faster a car is going, the less cars are at the detection point at a time. This inference is also reinforced by a similar trend in local density, where there are less cars through the detection point when the average velocity detected is higher, since the car passing this point does not have to slow down to maintain a safe distance from the car in front of it.

Weather Conditions

The "weather conditions" change our modelled parameters in the following ways:

- **Clear:** No parameter changes, the "default" simulation
- **Rain:** The braking parameter is halved, simulating increased stopping distance as water spray restricts vision.

- **Snow:** The braking parameter was again halved, and the maximum speed v_0 was lowered by 25 mph, simulating general behaviour in these conditions.

- **Wind**: The maximum speed was again reduced but only by 10 mph, to simulate behaviour in windy conditions.

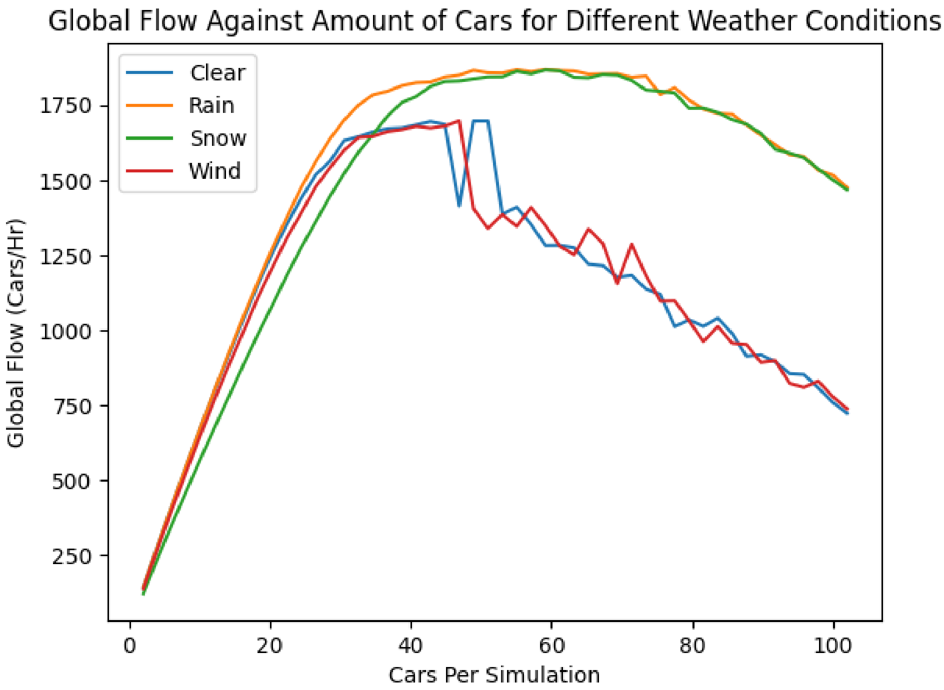

Figure 5: *Global Flow of each "Weather Condition" (blue = clear, orange = rain, green = snow and red = wind) against the number of cars in the simulation (equivalent to the global density)*

We can see from our graph that changing the braking parameter yielded the largest difference in the flow. This braking parameter outlines the maximum comfortable deceleration, so reducing it would keep the velocity higher on average, resulting in a higher global flow value. However, changing the speed limit alone seemed to have little effect on the overall flow of traffic as more cars were added. This tells us that changing the speed limit of the total road likely would not improve the traffic efficiency on the road.

Adjusting Road Length

In order to get a better idea of the relationship between road length and the flow and density values, we ran a sweep over road lengths with the same number of cars (20). We know that density will decrease with road length (due to how we defined global density) and the flow will also decrease for the same reason. We would expect the shorter length of road to have higher values of both flow and density, decreasing dramatically as the road length increases, especially past the one kilometer threshold.

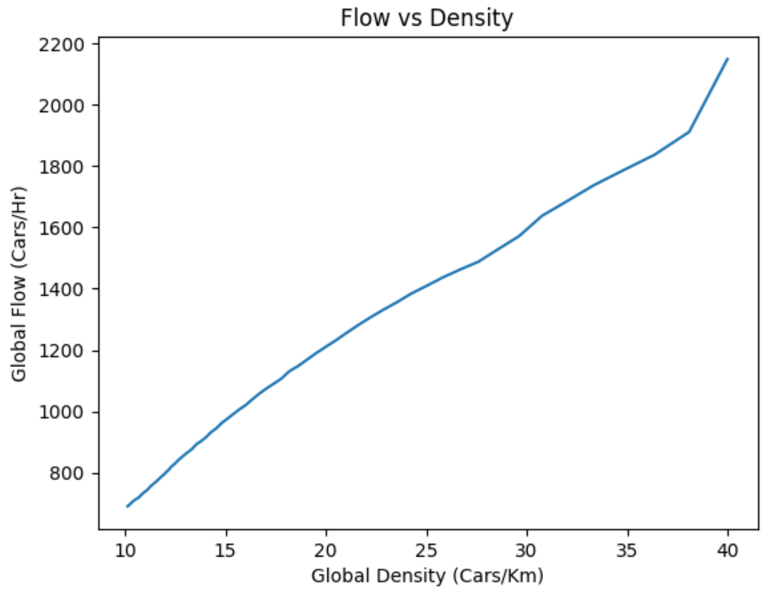

Figure 6: *A linear relationship of Global flow against global density as the length of the simulated road increases.*

The values where the flow and density are lowest are where the road is at its longest. However, the length of the road doesn't seem to have a huge effect on the traffic efficiency. The number of cars on the road and the parameters surrounding their behaviour are larger components of the efficiency of the road system.

Conclusions

In summary, after investigating the various factors involved in designing a model of a simple traffic system. We found that the way to maximise traffic flow through the system is to find ways for the cars to remain at their target velocity (the speed limit) for as long as possible. While the speed limit itself does not seem to be a major factor in the flow, having the cars stay at that limit for the maximum amount of time ensures all the cars are moving as efficiently as possible. We saw by changing the braking parameter the flow was higher for longer because the cars take longer to decelerate and the cars stay closer to that desired velocity for longer. Despite making multiple changes to the model, the time spent near our desired velocity v_0 was shown to be the single largest factor for traffic flow efficiency on this single-lane motorway scenario. While this road system is much simpler, an analysis of the relationship between speed and traffic flow on German Autobahns revealed that the factors influencing these values were much more based on human behaviour and time of day than on the speed limit. [2] The model in its current form models a driver's basic behaviour, but has no possibility of road accidents, drivers not accelerating to the full speed limit, and many other factors that are more based on randomness or human behaviour than on factors of the Intelligent Driver Model we have been implementing.

Further Work

Valid extensions to improve the realism of the model could include factoring in a driver's reaction time to changes in the car's behaviour. Implementing this in the model's current form would require a uniform reaction time over all drivers, and it would have to be a multiple of our time step Δt. As stated in our conclusions, the human factors of the road system are much more important to a full model than just the movement of cars in the system.

References

[1] M. Treiber and A. Kesting, *Car-Following Models Based on Driving Strategies*. Berlin, Heidelberg: Springer Berlin Heidelberg, 2013, pp. 181–204. [Online]. Available: https://doi.org/10.1007/978-3-642-32460-4_11

[2] W. Brilon and M. Ponzlet, "Variability of speed-flow relationships on german autobahns," *Transportation Research Record*, vol. 1555, no. 1, pp. 91–98, 1996. [Online]. Available: https://doi.org/10.1177/0361198196155500112