Converting UTM to Latitude and Longitude (Or Vice Versa)

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Information on the UTM system
Spreadsheet For UTM Conversion
Help! My Data Doesn't Look Like A UTM Grid!

I get enough inquiries on this subject that I decided to create a page for it.

Caution! Unlike latitude and longitude, there is no physical frame of reference for UTM grids. Latitude is determined by the earth's polar axis. Longitude is determined by the earth's rotation. If you can see the stars and have a sextant and a good clock set to Greenwich time, you can find your latitude and longitude. But there is no way to determine your UTM coordinates except by calculation.

UTM grids, on the other hand, are created by laying a square grid on the earth. This means that different maps will have different grids depending on the datum used (model of the shape of the earth). I saw US military maps of Germany shift their UTM grids by about 300 meters when a more modern datum was used for the maps. Also, old World War II era maps of Europe apparently used a single grid for all of Europe and grids in some areas are wildly tilted with respect to latitude and longitude.

The two basic references for converting UTM and geographic coordinates are U.S. Geological Survey Professional Paper 1395 and U. S. Army Technical Manual TM 5-241-8 (complete citations below). Each has advantages and disadvantages.

For converting latitude and longitude to UTM, the Army publication is better. Its notation is more consistent and the formulas more clearly laid out, making code easier to debug. In defense of the USGS, their notation is constrained by space, and the need to be consistent with cartographic literature and descriptions of several dozen other map projections in the book.

For converting UTM to latitude and longitude, however, the Army publication has formulas that involve latitude (the quantity to be found) and which require reverse interpolation from tables. Here the USGS publication is the only game in town.

Some extremely tiny terms that will not seriously affect meter-scale accuracy have been omitted.

Converting Between Decimal Degrees, Degrees, Minutes and Seconds, and Radians

(dd + mm/60 +ss/3600) to Decimal degrees (dd.ff)

dd = whole degrees, mm = minutes, ss = seconds

dd.ff = dd + mm/60 + ss/3600

Example: 30 degrees 15 minutes 22 seconds = 30 + 15/60 + 22/3600 = 30.2561

Decimal degrees (dd.ff) to (dd + mm/60 +ss/3600)

For the reverse conversion, we want to convert dd.ff to dd mm ss. Here ff = the fractional part of a decimal degree.

mm = 60*ff

ss = 60*(fractional part of mm)

Use only the whole number part of mm in the final result.

30.2561 degrees = 30 degrees

.2561*60 = 15.366 minutes

.366 minutes = 22 seconds, so the final result is 30 degrees 15 minutes 22 seconds

Decimal degrees (dd.ff) to Radians

Radians = (dd.ff)*pi/180

Radians to Decimal degrees (dd.ff)

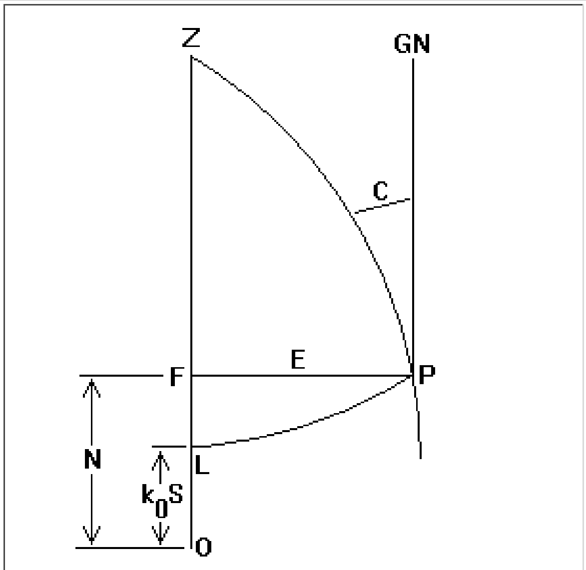
(dd.ff) = Radians*180/pi

Degrees, Minutes and Seconds to Distance

A degree of longitude at the equator is 111.2 kilometers. A minute is 1853 meters. A second is 30.9 meters. For other latitudes multiply by cos(lat). Distances for degrees, minutes and seconds in latitude are very similar and differ very slightly with latitude. (Before satellites, observing those differences was a principal method for determining the exact shape of the earth.)

Converting Latitude and Longitude to UTM

Okay, take a deep breath. This will get *very* complicated, but the math, although tedious, is only algebra and trigonometry. It would sure be nice if someone wrote a spreadsheet to do this.



P = point under consideration
F = foot of perpendicular from P to the central meridian. The latitude of F is called the *footprint latitude*.
O = origin (on equator)
OZ = central meridian
LP = parallel of latitude of P
ZP = meridian of P
OL = k_0S = meridional arc from equator
LF = ordinate of curvature
OF = N = grid northing
FP = E = grid distance from central meridian
GN = grid north
C = convergence of meridians = angle between true and grid north

Another thing you need to know is the datum being used:

Datum	Equatorial Radius, meters (a)	Polar Radius, meters (b)	Flattening (a-b)/a	Use
NAD83/WGS84	6,378,137	6,356,752.3142	1/298.257223563	Global
GRS 80	6,378,137	6,356,752.3141	1/298.257222101	US
WGS72	6,378,135	6,356,750.5	1/298.26	NASA, DOD
Australian 1965	6,378,160	6,356,774.7	1/298.25	Australia
Krasovsky 1940	6,378,245	6,356,863.0	1/298.3	Soviet Union
International (1924) -Hayford (1909)	6,378,388	6,356,911.9	1/297	Global except as listed
Clake 1880	6,378,249.1	6,356,514.9	1/293.46	France, Africa
Clarke 1866	6,378,206.4	6,356,583.8	1/294.98	North America
Airy 1830	6,377,563.4	6,356,256.9	1/299.32	Great Britain
Bessel 1841	6,377,397.2	6,356,079.0	1/299.15	Central Europe, Chile, Indonesia
Everest 1830	6,377,276.3	6,356,075.4	1/300.80	South Asia

Don't interpret the chart to mean there is radical disagreement about the shape of the earth. The earth isn't perfectly round, it isn't even a perfect ellipsoid, and slightly different shapes work better for some regions than for the earth as a whole. The top three are based on worldwide data and are truly global. Also, you are very unlikely to find UTM grids based on any of the earlier projections.

The most modern datums (jars my Latinist sensibilities since the plural of *datum* in Latin is *data*, but that has a different meaning to us) are NAD83 and WGS84. These are based in turn on GRS80. Differences between the three systems derive mostly from redetermination of station locations rather than differences in the datum. Unless you are locating a first-order station to sub-millimeter accuracy (in which case you are way beyond the scope of this page) you can probably regard them as essentially identical.

I have no information on the NAD83 and WGS84 datums, nor can my spreadsheet calculate differences between those datums and WGS84.

Formulas For Converting Latitude and Longitude to UTM

These formulas are slightly modified from Army (1973). They are accurate to within less than a meter within a given grid zone. The original formulas include a now obsolete term that can be handled more simply - it merely converts radians to seconds of arc. That term is omitted here but discussed below.

Symbols

- lat = latitude of point
- long = longitude of point
- $long_0$ = central meridian of zone
- k_0 = scale along $long_0$ = 0.9996. Even though it's a constant, we retain it as a separate symbol to keep the numerical coefficients simpler, also to allow for systems that might use a different Mercator projection.
- e = $SQRT(1-b^2/a^2)$ = .08 approximately. This is the eccentricity of the earth's elliptical cross-section.
- $e'^2 = (ea/b)^2 = e^2/(1-e^2)$ = .007 approximately. The quantity e' only occurs in even powers so it need only

be calculated as e'^2.

- $n = (a-b)/(a+b)$
- $rho = a(1-e^2)/(1-e^2\sin^2(lat))^{3/2}$. This is the radius of curvature of the earth in the meridian plane.
- $nu = a/(1-e^2\sin^2(lat))^{1/2}$. This is the radius of curvature of the earth perpendicular to the meridian plane. It is also the distance from the point in question to the polar axis, measured perpendicular to the earth's surface.
- $p = (long-long_0)$ **in radians** (This differs from the treatment in the Army reference)

Calculate the Meridional Arc

S is the meridional arc through the point in question (the distance along the earth's surface from the equator). All angles are in radians.

- $S = A'lat - B'\sin(2lat) + C'\sin(4lat) - D'\sin(6lat) + E'\sin(8lat)$, where lat is in radians and
- $A' = a[1 - n + (5/4)(n^2 - n^3) + (81/64)(n^4 - n^5) ...]$
- $B' = (3 \tan/2)[1 - n + (7/8)(n^2 - n^3) + (55/64)(n^4 - n^5) ...]$
- $C' = (15 \tan^2/16)[1 - n + (3/4)(n^2 - n^3) ...]$
- $D' = (35 \tan^3/48)[1 - n + (11/16)(n^2 - n^3) ...]$
- $E' = (315 \tan^4/512)[1 - n ...]$

The USGS gives this form, which may be more appealing to some. (They use M where the Army uses S)

- $M = a[(1 - e^2/4 - 3e^4/64 - 5e^6/256 ....)lat$
  $- (3e^2/8 + 3e^4/32 + 45e^6/1024...)\sin(2lat)$
  $+ (15e^4/256 + 45e^6/1024 + ...)\sin(4lat)$
  $- (35e^6/3072 + ...) \sin(6lat) + ...)]$ where lat is in radians

This is the hard part. Calculating the arc length of an ellipse involves functions called *elliptic integrals*, which don't reduce to neat closed formulas. So they have to be represented as series.

Converting Latitude and Longitude to UTM

All angles are in radians.

$y = northing = K1 + K2p^2 + K3p^4$, where

- $K1 = Sk_0$,
- $K2 = k_0\ nu \sin(lat)\cos(lat)/2 = k_0\ nu \sin(2\ lat)/4$
- $K3 = [k_0\ nu \sin(lat)\cos^3(lat)/24][(5 - \tan^2(lat) + 9e'^2\cos^2(lat) + 4e'^4\cos^4(lat)]$

$x = easting = K4p + K5p^3$, where

- $K4 = k_0\ nu \cos(lat)$
- $K5 = (k_0\ nu \cos^3(lat)/6)[1 - \tan^2(lat) + e'^2\cos^2(lat)]$

Easting x is relative to the central meridian. For conventional UTM easting add 500,000 meters to x.

What the Formulas Mean

The hard part, allowing for the oblateness of the Earth, is taken care of in calculating S (or M). So K1 is simply the arc length along the central meridian of the zone corrected by the scale factor. Remember, the scale is a hair less than 1 in the middle of the zone, and a hair more on the outside.

All the higher K terms involve nu, the local radius of curvature (roughly equal to the radius of the earth or roughly 6,400,000 m), trig functions, and powers of e'^2 (= .007). So basically they are never much larger than nu. Actually the maximum value of K2 is about nu/4 (1,600,000), K3 is about nu/24 (267,000) and K5 is about nu/6 (1,070,000). Expanding the expressions will show that the tangent terms don't affect anything.

If we were just to stop with the K2 term in the northing, we'd have a quadratic in p. In other words, we'd approximate the parallel of latitude as a parabola. The real curve is more complex. It will be more like a hyperbola equatorward of about 45 degrees and an ellipse poleward, at least within the narrow confines of a UTM zone. (At any given latitude we're cutting the cone of latitude vectors with an inclined plane, so the resulting intersection will be a conic section. Since the projection cylinder has a curvature, the exact curve is not a conic but the difference across a six-degree UTM zone is pretty small.) Hence the need for higher order terms. Now p will never be more than +/-3 degrees = .05 radians, so p^2 is always less than .0025 (1/400) and p^4 is always less than .00000625 (1/160000). Using a spreadsheet, it's easy to see how the individual terms vary with latitude. $K2p^2$ never exceeds 4400 and $K3p^4$ is at most a bit over 3. That is, the curvature of a parallel of latitude across a UTM zone is at most a little less than 4.5 km and the maximum departure from a parabola is at most a few meters.

K4 is what we'd calculate for easting in a simple-minded way, just by calculating arc distance along the parallel of latutude. But, as we get farther from the central meridian, the meridians curve inward, so our actual easting will be less than K4. That's what K5 does. Since p is never more than +/-3 degrees = .05 radians, p^3 is always less than .000125 (1/8000). The maximum value of $K5p^3$ is about 150 meters.

That Weird Sin 1" Term in the Original Army Reference

The Army reference defines p in seconds of arc and includes a sin 1" term in the K formulas. The Sin 1" term is a holdover from the days when this all had to be done on mechanical desk calculators (pre-computer) and terms had to be kept in a range that would retain sufficient precision at intermediate steps. For that small an angle the difference between sin 1" and 1" in radians is negligible. If p is in seconds of arc, then (psin 1") merely converts it to radians.

The sin 1" term actually included an extra factor of 10,000, which was then corrected by multiplying by large powers of ten afterward.

The logic is a bit baffling. If I were doing this on a desk calculator, I'd factor out as many terms as possible rather than recalculate them for each term. But perhaps in practice the algebraically obvious way created overflows or underflows, since calculators could only handle limited ranges.

In any case, the sin1" term is not needed any more. Calculate p in radians and omit the sin1" terms and the large power of ten multipliers.

Converting UTM to Latitude and Longitude

y = northing, x = easting (relative to central meridian; subtract 500,000 from conventional UTM coordinate).

Calculate the Meridional Arc

This is easy: $M = y/k_0$.

Calculate Footprint Latitude

- $mu = M/[a(1 - e^2/4 - 3e^4/64 - 5e^6/256...)$
- $e_1 = [1 - (1 - e^2)^{1/2}]/[1 + (1 - e^2)^{1/2}]$

footprint latitude fp = mu + J1sin(2mu) + J2sin(4mu) + J3sin(6mu) + J4sin(8mu), where:

- $J1 = (3e_1/2 - 27e_1^3/32 ..)$
- $J2 = (21e_1^2/16 - 55e_1^4/32 ..)$
- $J3 = (151e_1^3/96 ..)$
- $J4 = (1097e_1^4/512 ..)$

Calculate Latitude and Longitude

- $e'^2 = (ea/b)^2 = e^2/(1-e^2)$
- $C1 = e'^2\cos^2(fp)$
- $T1 = \tan^2(fp)$
- $R1 = a(1-e^2)/(1-e^2\sin^2(fp))^{3/2}$. This is the same as rho in the forward conversion formulas above, but calculated for fp instead of lat.
- $N1 = a/(1-e^2\sin^2(fp))^{1/2}$. This is the same as nu in the forward conversion formulas above, but calculated for fp instead of lat.
- $D = x/(N1k_0)$

lat = fp - Q1(Q2 - Q3 + Q4), where:

- $Q1 = N1 \tan(fp)/R1$
- $Q2 = (D^2/2)$
- $Q3 = (5 + 3T1 + 10C1 - 4C1^2 -9e'^2)D^4/24$
- $Q4 = (61 + 90T1 + 298C1 +45T1^2 - 3C1^2 -252e'^2)D^6/720$

long = long0 + (Q5 - Q6 + Q7)/cos(fp), where:

- $Q5 = D$
- $Q6 = (1 + 2T1 + C1)D^3/6$
- $Q7 = (5 - 2C1 + 28T1 - 3C1^2 + 8e'^2 + 24T1^2)D^5/120$

What Do The Formulas Mean?

As the sketch above shows, because of the poleward curve of parallels, the footprint latitude is always greater than the true latitude. Q1 is just a scaling coefficient and is constant for any given fp. The tangent term basically means the closer to the pole you are, the faster the parallels curve. Q2 is a quadratic term in x. Again, as with converting from geographic coordinates to UTM, we approximate the parallel as a parabola and add higher order corrections.

To determine longitude, we make a simple minded approximation that longitude is proportional to easting, but then, since fp is too large, the true longitude is smaller, since it lies on a parallel closer to the the equator. The divisor cos(fp) corrects for the varying length of degrees of longitude as latitude varies.

A Spreadsheet Program

Before linking to the program, note:

- It is an Excel spreadsheet. If you have Excel on your computer, it will (or should) open when you click the link. Most major spreadsheet programs can read spreadsheets in other formats.
- A spreadsheet is not an applet or program. In particular, you can't manually enter data into a cell and preserve any formulas that are there. That's why some aspects of data entry are clunkier than they might otherwise be with, say, a Visual Basic program.
- There are three computation pages, one for single conversions, the other two for batch conversions. Other pages contain information on datums and the specific conversion formulas. To use the batch conversions you need to be somewhat proficient in spreadsheets as you will have to copy data and cell formulas.
- For our mutual peace of mind, run a virus check.
- You may copy the program for your own non-commercial use and for non-commercial distribution to others, but not for commercial use. Please give appropriate credit when citing your calculations. You may also modify it as needed for your personal use. In Internet Explorer, right click on the link and select Save Target As... to save the spreadsheet to your own disk.

Spreadsheet For UTM Conversion

References

Snyder, J. P., 1987; Map Projections - A Working Manual. U.S. Geological Survey Professional Paper 1395, 383 p. **If you are at all serious about maps you need this book.**

Army, Department of, 1973; Universal Transverse Mercator Grid, U. S. Army Technical Manual TM 5-241-8, 64 p.

NIMA Technical Report 8350.2, "Department of Defense World Geodetic System 1984, Its Definition and Relationships with Local Geodetic Systems," Second Edition, 1 September 1991 and its supplements. The report is available from the NIMA Combat Support Center and its stock number is DMATR83502WGS84. Non-DoD requesters may obtain the report as a public sale item from the U.S. Geological Survey, Box 25286, Denver Federal Center, Denver, Colorado 80225 or by phone at 1-800-USA-MAPS.

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