Continuous and Discrete Random Variables

Rebecca C. Steorts

Agenda

- Continuous Random Variables
- Discrete Random Variables

Continuous Random Variables

A continuous random variable (r.v.) can take on an uncountably infinite number of values.

Given a probability density function (pdf), f(y), allows us to compute

$$P(a \leq Y \leq b) = \int_a^b f(y)\, dy.$$

Properties of continuous random r.v.'s:

- $\int f(y)\, dy = 1$.
- For any value y,

$$P(Y = y) = \int_y^y f(y)\, dy = 0 \implies$$

$$P(y < Y) = P(y \leq Y).$$

Discrete Random Variables

A discrete random variable has a countable number of possible values, where the associated probabilities are calculated for each possible value using a probabilitiy mass function (pmf).

A pmf is a function that calculates $P(Y = y)$, given each variable's parameters.

Common Discrete distributions

- Bernoulli
- Geometric
- Negative Binomial
- Hypergeometric

Common Continuous distributions

- Exponenial
- Gamma
- Normal (Gaussian)
- Beta

Beta distribution

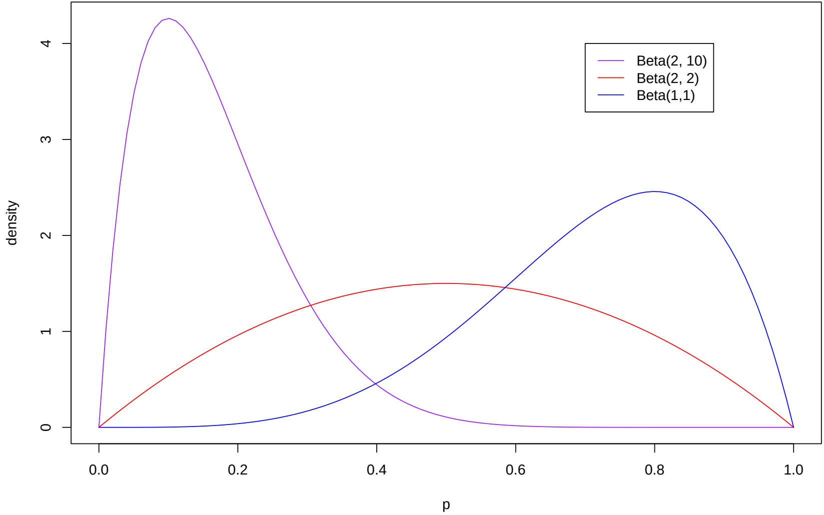
Given $a, b > 0$, we write $\theta \sim \text{Beta}(a, b)$ to mean that θ has pdf

$$p(\theta) = \text{Beta}(\theta | a, b) = \frac{1}{B(a, b)} \theta^{a-1} (1-\theta)^{b-1} \mathbb{1}(0 < \theta < 1),$$

i.e., $p(\theta) \propto \theta^{a-1}(1-\theta)^{b-1}$ on the interval from 0 to 1.

- Here,
$$B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$
.
- Parameters a, b control the shape of the distribution.
- This distribution models random behavior of percentages/proportions.

Beta distribution



Beta distribution example

Suppose that a college models probabilities of student accepting admission via the Beta(a, b) distribution, where a,b $>$ 0 are fixed and known.

What is the probability that a randomly selected student has prob of accepting an offer larger than 80 percent, where a=4/3 and b=2.

```
pbeta(0.8, shape1 = 4/3, shape2 = 2, lower.tail = FALSE)
```

```
## [1] 0.05930466
```