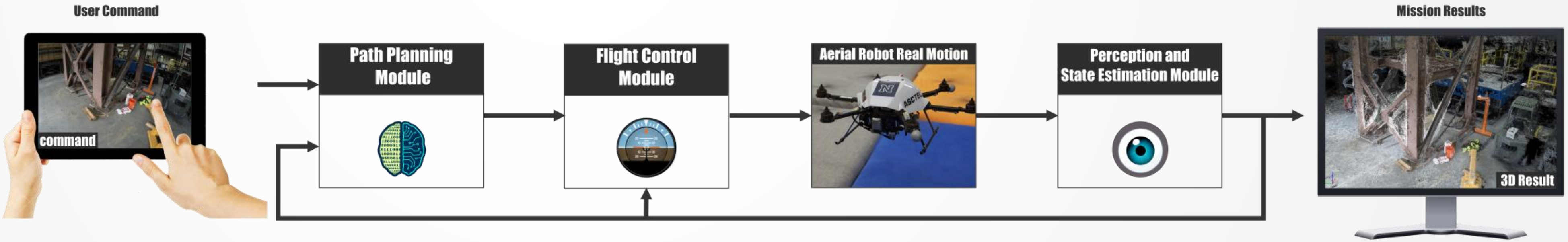
CS491/691: Introduction to Aerial Robotics

Topic: Midterm Preparation

Dr. Kostas Alexis (CSE)

Areas of Focus

- Coordinate system transformations (CST)
- MAV Dynamics (MAVD)
- Navigation Sensors (NS)
- State Estimation (SE)



CS491/691: Introduction to Aerial Robotics

Topic: Coordinate Systems Transformations

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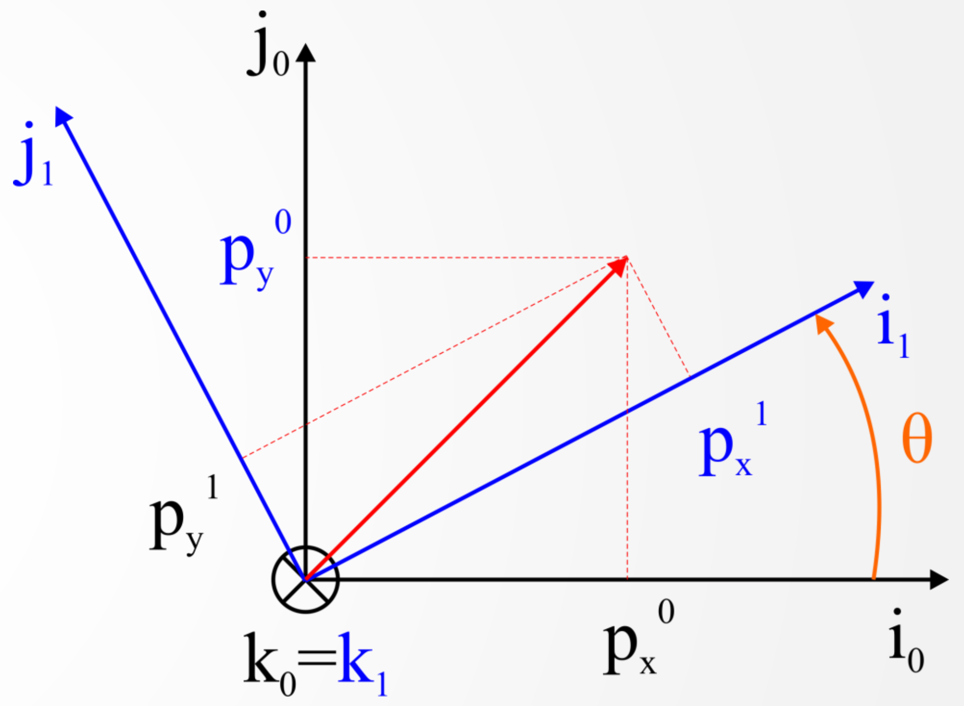
CST: Vector Rotation

- Rotation around the k-axis

$$\mathbf{p} = p_x^0 \mathbf{i}^0 + p_y^0 \mathbf{j}^0 + p_z^0 \mathbf{k}^0$$
$$\mathbf{p} = p_x^1 \mathbf{i}^1 + p_y^1 \mathbf{j}^1 + p_z^1 \mathbf{k}^1$$
$$\mathbf{p}^1 = \begin{pmatrix} p_x^1 \\ p_y^1 \\ p_z^1 \end{pmatrix} = \begin{pmatrix} \mathbf{i}^1\mathbf{i}^0 & \mathbf{i}^1\mathbf{j}^0 & \mathbf{i}^1\mathbf{k}^0 \\ \mathbf{j}^1\mathbf{i}^0 & \mathbf{j}^1\mathbf{j}^0 & \mathbf{j}^1\mathbf{k}^0 \\ \mathbf{k}^1\mathbf{i}^0 & \mathbf{k}^1\mathbf{j}^0 & \mathbf{k}^1\mathbf{k}^0 \end{pmatrix} \begin{pmatrix} p_x^0 \\ p_y^0 \\ p_z^0 \end{pmatrix}$$



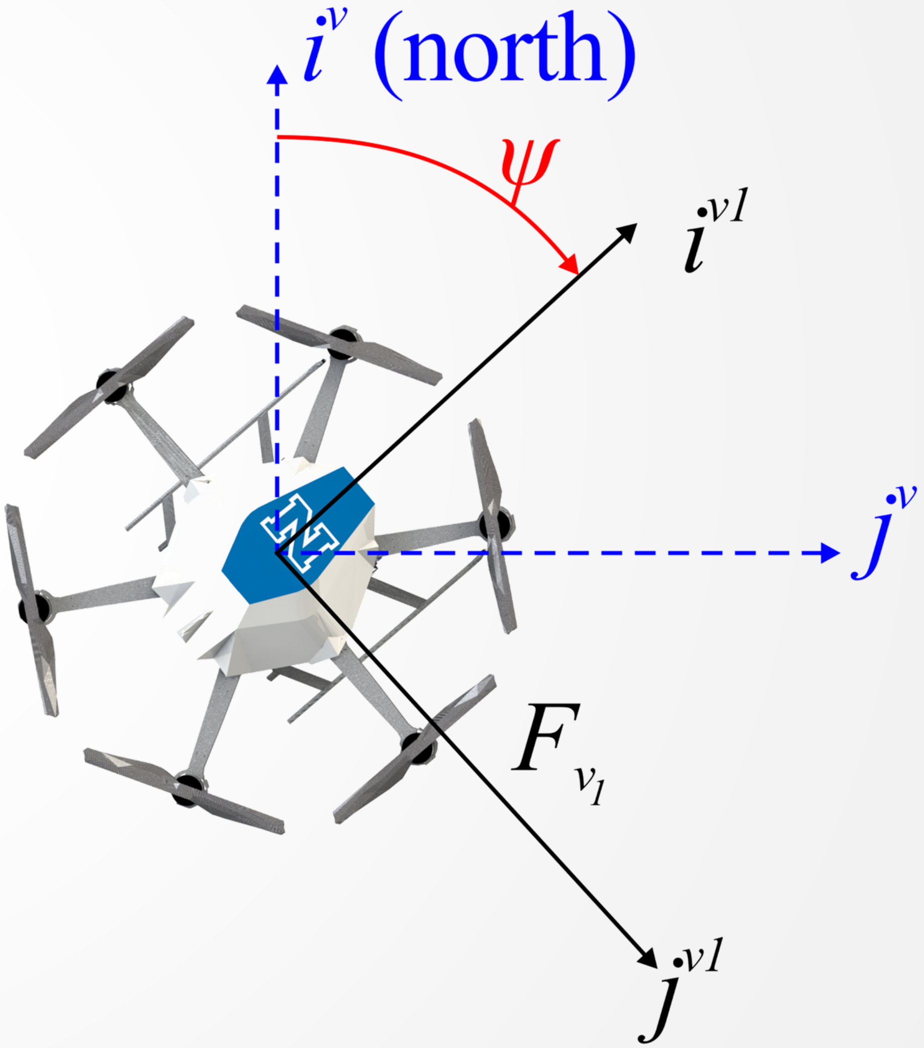
$$\mathbf{p}^1 = \mathcal{R}_0^1 \mathbf{p}^0, \ \mathcal{R}_0^1 = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

CST: Vehicle-1 Frame

$$\mathbf{p}^{v_1} = \mathcal{R}_v^{v_1}\mathbf{p}^v,$$

$$\mathcal{R}_v^{v_1} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- ψ represents the yaw angle

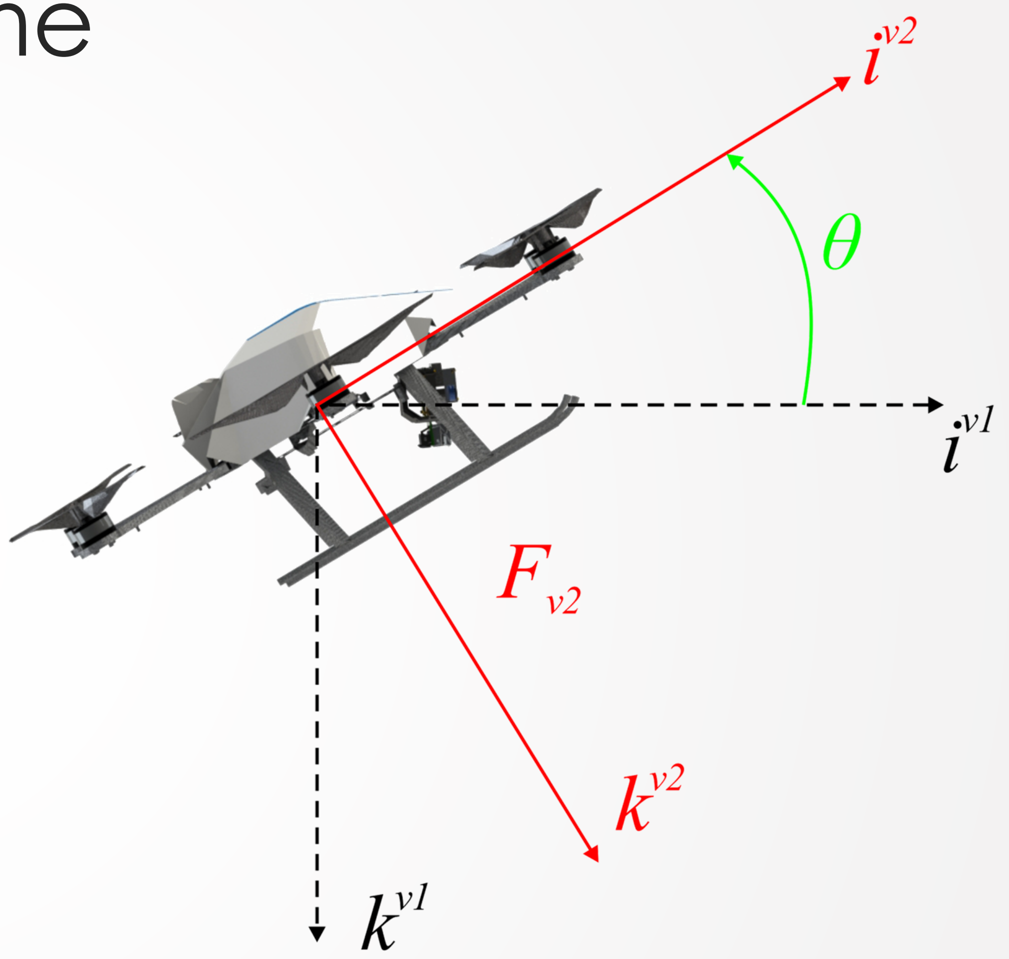


CST: Vehicle-2 Frame

$$\mathbf{p}^{v_2} = \mathcal{R}_{v_1}^{v_2}\mathbf{p}^{v_1},$$

$$\mathcal{R}_{v_1}^{v_2} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

- θ represents the pitch angle

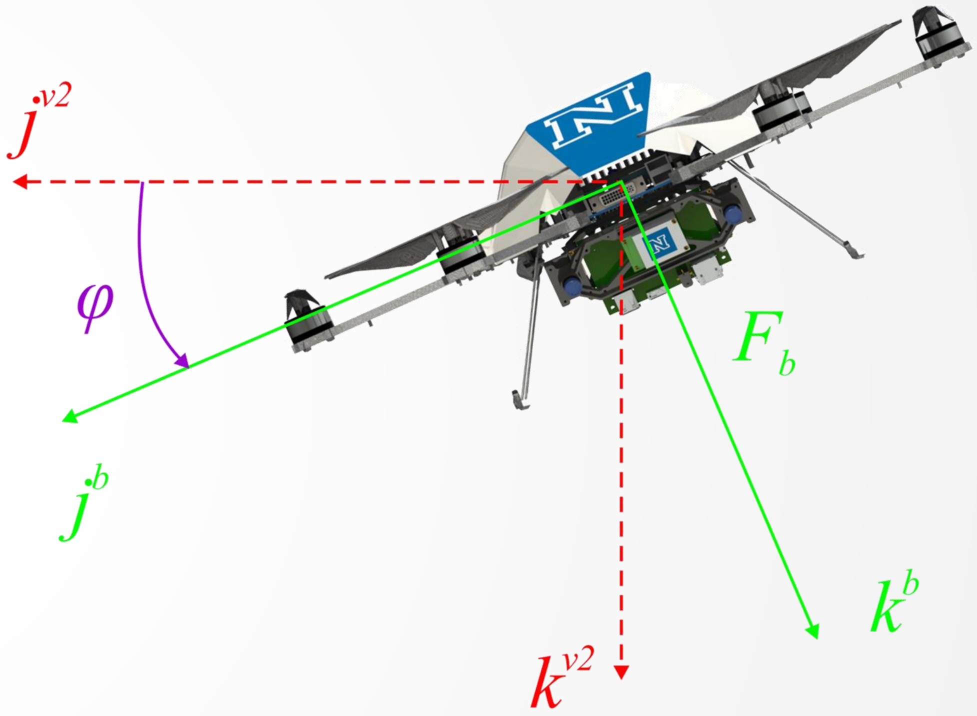


CST: Body Frame

$$\mathbf{p}^b = \mathcal{R}_{v_2}^b \mathbf{p}^{v_2},$$

$$\mathcal{R}_{v_2}^b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$$

- φ represents the roll angle



CST: Inertial Frame to Body Frame

- Let:

$$
\begin{aligned}
\mathcal{R}_v^b(\phi,\theta,\psi) &= \mathcal{R}_{v_2}(\phi)\mathcal{R}_{v_1}^{v_2}(\theta)\mathcal{R}_v^{v_1}(\psi) \\
&= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi c_\theta \\ c_\phi s_\theta c_\psi + s_\phi s_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi c_\theta \end{bmatrix}
\end{aligned}
$$

- Then:

$$\mathbf{p}^b = \mathcal{R}_v^b \mathbf{p}^v$$

CST: Rotation of Reference Frame

- Rotation around the i-axis

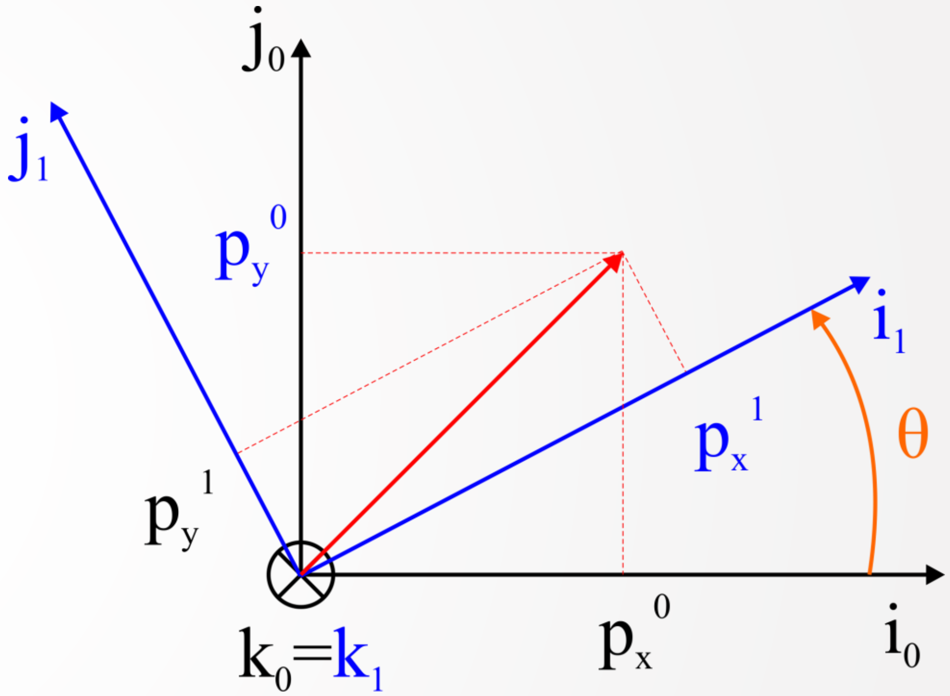
$$\mathcal{R}_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

- Rotation around the j-axis

$$\mathcal{R}_0^1 = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

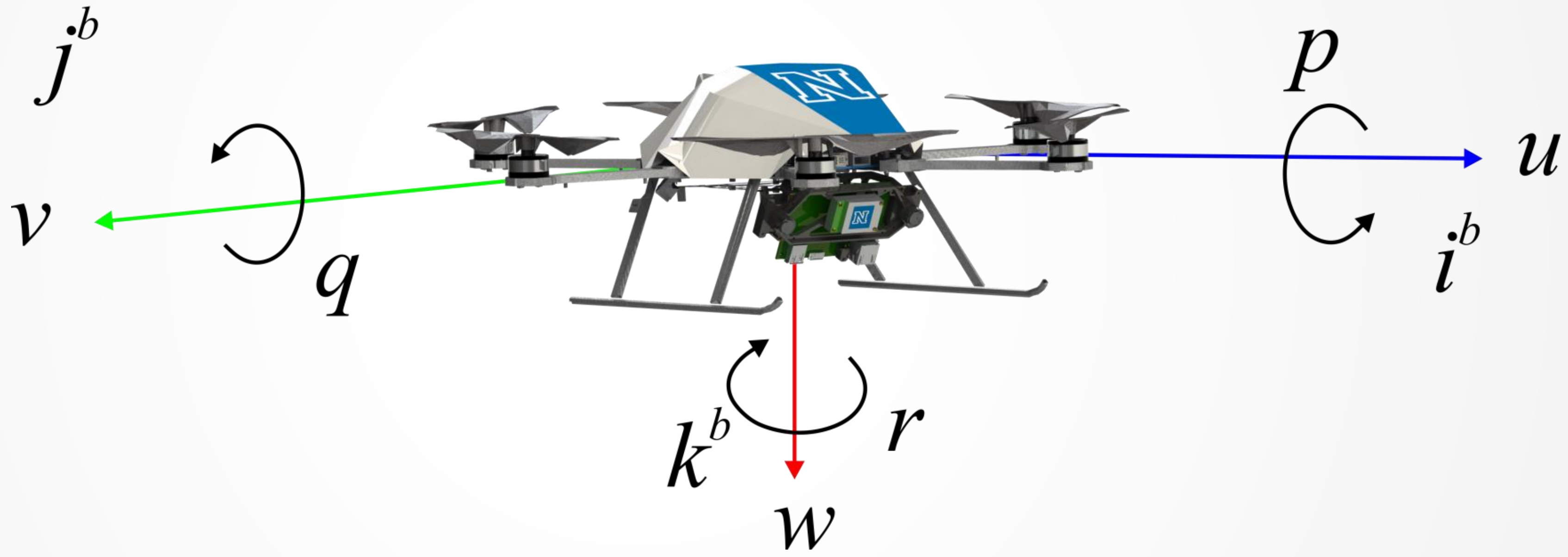
- Rotation around the k-axis

$$\mathcal{R}_0^1 = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



- Orthonormal matrix properties
  - $(\mathcal{R}_a^b)^-1 = (\mathcal{R}_a^b)^T = \mathcal{R}_b^a$
  - $\mathcal{R}_b^c\mathcal{R}a^b = \mathcal{R}_a^c$
  - $\det(\mathcal{R}_a^b) = 1$

CST: Application to Robot Kinematics



- *[p,q,r]* : body angular rates
- *[u,v,w]* : body linear velocities

CST: Relate Translational Velocity-Position

- Let *[u,v,w]* represent the body linear velocities

$$\frac{d}{dt}\begin{bmatrix} p_n \\ p_e \\ p_d \end{bmatrix} = \mathcal{R}_b^v \begin{bmatrix} u \\ v \\ w \end{bmatrix} = (\mathcal{R}_v^b)^T \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

- Which gives:

$$\begin{bmatrix} \dot{p}_n \\ \dot{p}_e \\ \dot{p}_d \end{bmatrix} = \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\phi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_{c\psi} \\ -s_\theta & s_\phi c_\theta & c_{c\theta} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

CST: Body Rates – Euler Rates

- Let *[p,q,r]* denote the body angular rates

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + \mathcal{R}_{v_2}^{b}(\phi) \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \mathcal{R}_{v_2}^{b}(\phi)\mathcal{R}_{v_1}^{v_2}(\theta) \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} =$$

$$\begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

- Inverting this expression:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

CST: Indicative questions

- What is the rotation matrix for a roll motion of $\pi/6$ and a pitch motion of $\pi/4$?
- Correlate the body rates and the Euler angles derivatives for a roll motion of $\pi/6$ and a pitch motion of $\pi/4$
- Where do the Euler angles present singularity?
- Your vehicle is performing a yaw rotation of $\pi/2$. Draw and provide the rotation matrix
- Explain the chain of operations required to conduct a roll, pitch and yaw rotation

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Topic: MAV Dynamics

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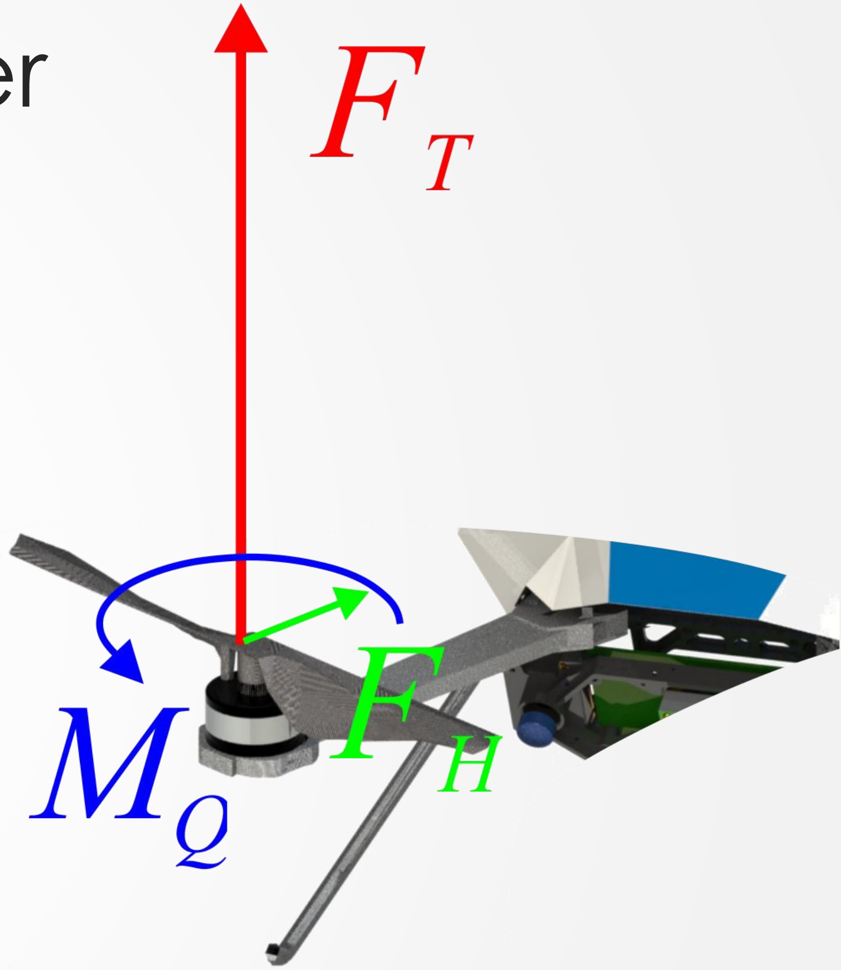
MAVD: The MAV Propeller

- Simplified model forces and moments:
  - **Thrust Force:** the resultant of the vertical forces acting on all the blade elements.

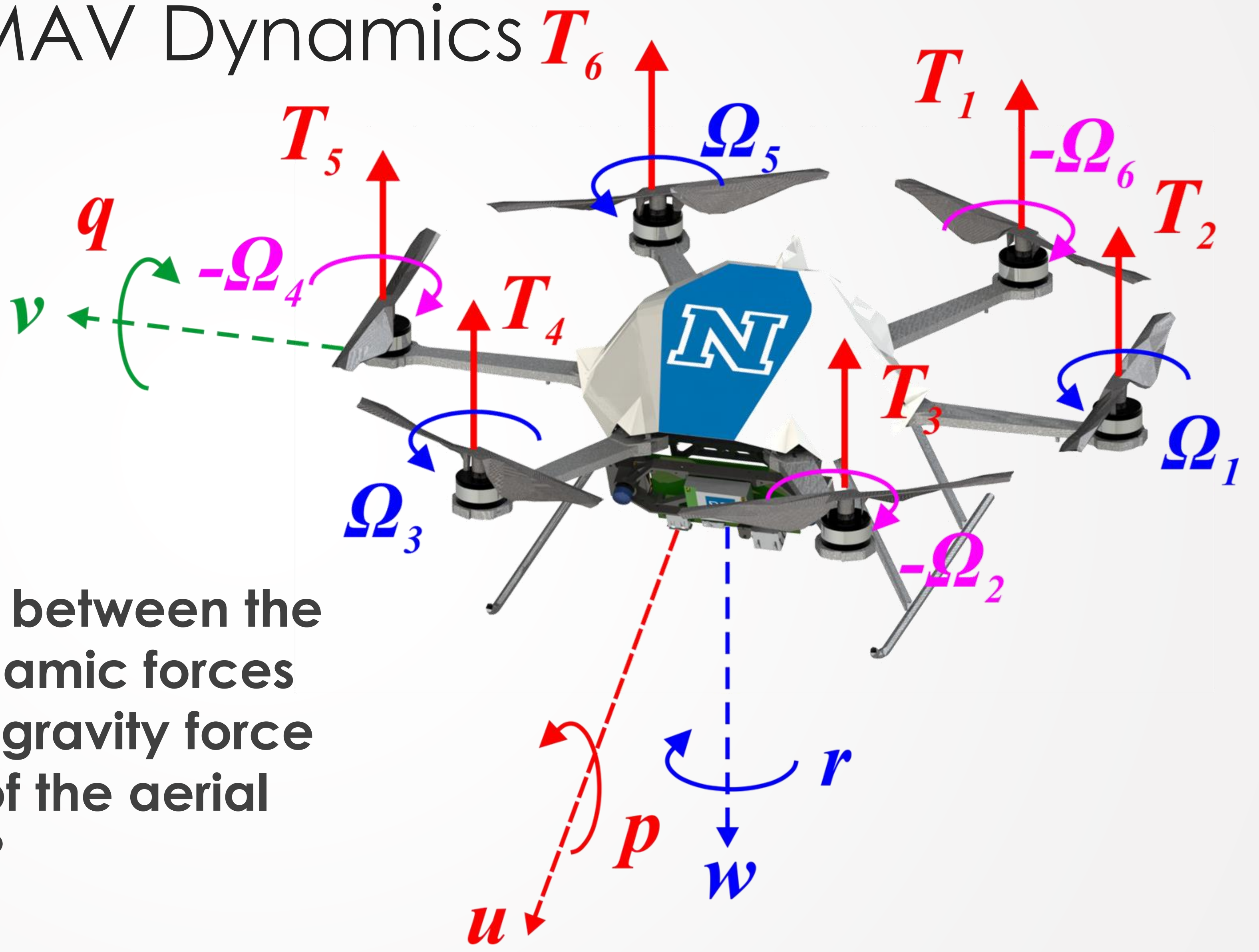
$$F_T = T = C_T \rho A (\Omega R)^2$$

  - **Drag Moment:** This moment about the rotor shaft is caused by the aerodynamic forces acting on the blade elements. The horizontal forces acting on the rotor are multiplied by the moment arm and integrated over the rotor. Drag moment determines the power required to spin the rotor.

$$M_Q = Q = C_Q \rho A (\Omega R)^2 R$$



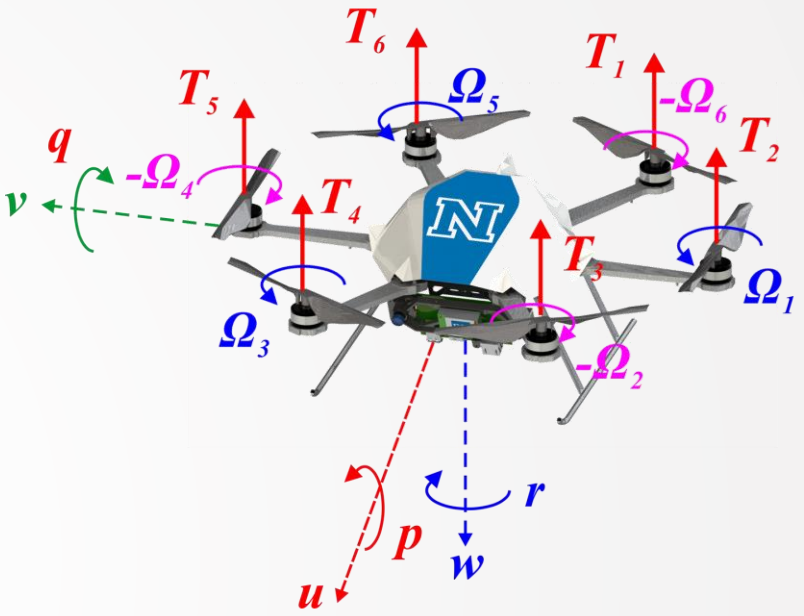
MAVD: MAV Dynamics



What is the relation between the propeller aerodynamic forces and moments, the gravity force and the motion of the aerial robot?

MAVD: MAV Dynamics

- Assumption 1: the Micro Aerial Vehicle is flying as a rigid body with negligible aerodynamic effects on it – for the employed airspeeds.
- The propeller is considered as a simple propeller disc that generates thrust and a moment around its shaft.



- Recall:

$$F_T = T = C_T \rho A (\Omega R)^2$$
$$M_Q = Q = C_Q \rho A (\Omega R)^2 R$$

- And let us write:

$$T_i = k_n \Omega_i^2$$
$$M_i = (-1)^{i-1} k_m T_i$$

MAVD: MAV Dynamics

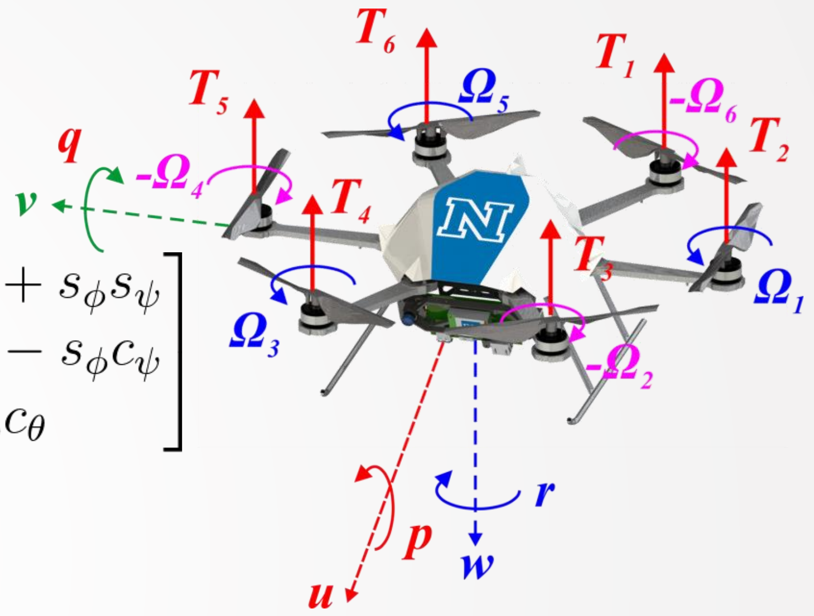
- To append the forces and moments we need to combine their formulation with

$$\begin{bmatrix}\dot{p}_n\\ \dot{p}_e\\ \dot{p}_d\end{bmatrix} = \mathcal{R}_b^v \begin{bmatrix}u\\ v\\ w\end{bmatrix}, \quad \mathcal{R}_b^v = \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix}$$

$$\begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{bmatrix} = \begin{bmatrix} rv - qw \\ pw - ru \\ qu - pv \end{bmatrix} + \frac{1}{m}\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

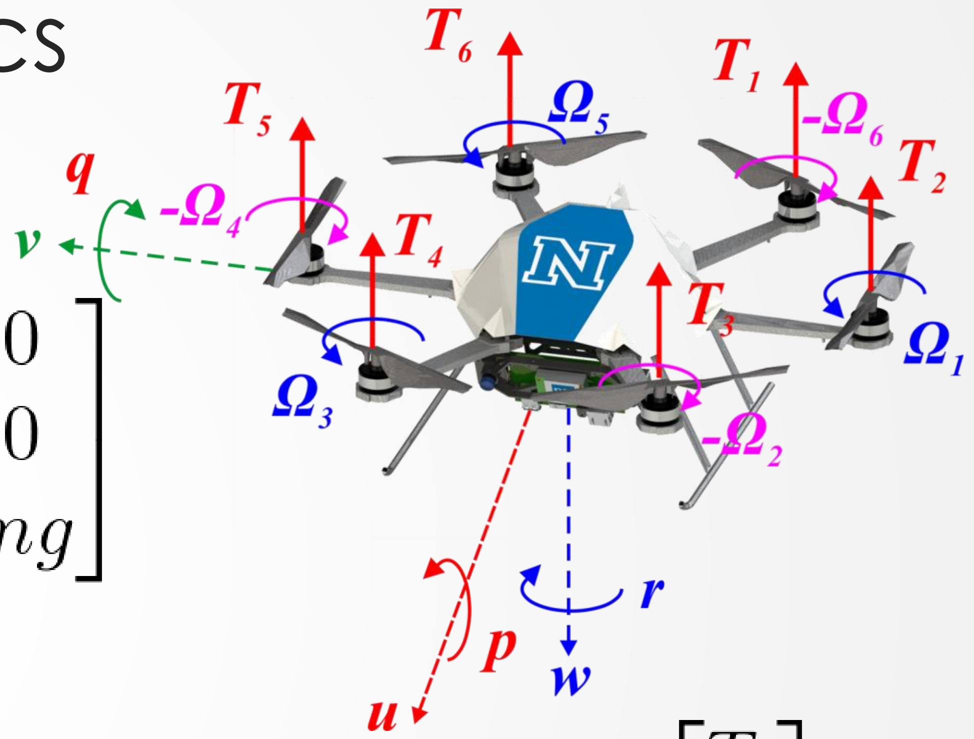
$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$\begin{bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \begin{bmatrix} \frac{J_y - J_z}{J_x} qr \\ \frac{J_z - J_x}{J_y} pr \\ \frac{J_x - J_y}{J_z} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \begin{bmatrix} \frac{1}{J_x} M_x \\ \frac{1}{J_y} M_y \\ \frac{1}{J_z} M_z \end{bmatrix}$$



- ***Next step: append the MAV forces and moments***

MAVD: MAV Dynamics



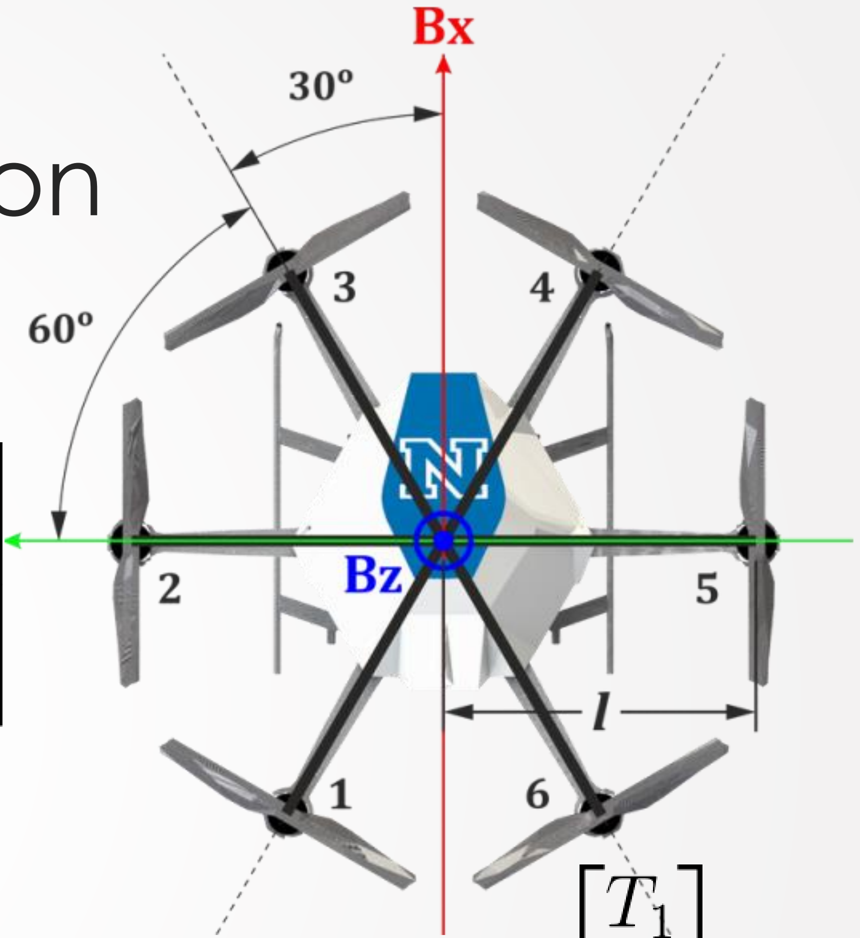
- MAV forces in the body frame:

$$\mathbf{f}_b = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^{6} T_i \end{bmatrix} - \mathcal{R}_v^b \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}$$

- Moments in the body frame:

$$\mathbf{m}_b = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} ls_{30} & l & ls_{30} & -ls_{30} & -l & -ls_{30} \\ -lc_{60} & 0 & lc_{60} & lc_{60} & 0 & -lc_{60} \\ -k_m & k_m & -k_m & k_m & -k_m & k_m \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix}$$

MAVD: Control Allocation



- MAV forces in the body frame:

$$\mathbf{f}_b = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^{6} T_i \end{bmatrix} - \mathcal{R}_v^b \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}$$

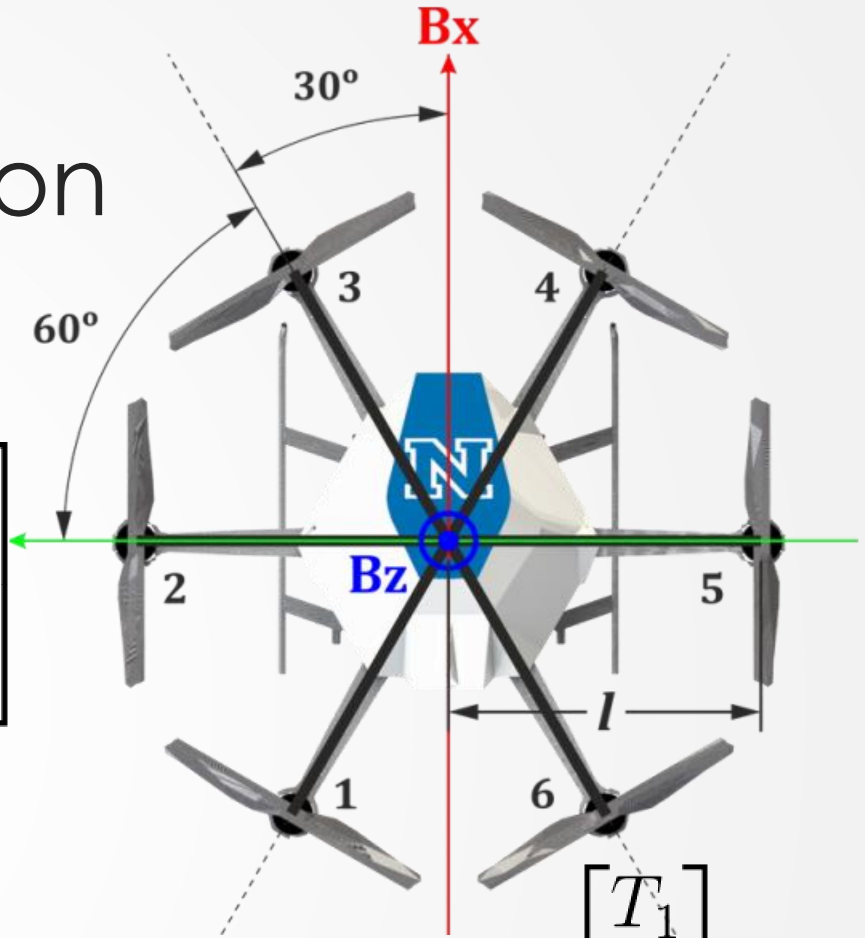
- Moments in the body frame:

$$\mathbf{m}_b = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} ls_{30} & l & ls_{30} & -ls_{30} & -l & -ls_{30} \\ -lc_{60} & 0 & lc_{60} & lc_{60} & 0 & -lc_{60} \\ -k_m & k_m & -k_m & k_m & -k_m & k_m \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix}$$

MAVD: Control Allocation

- MAV forces in the body frame:

$$\mathbf{f}_b = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^{6} T_i \end{bmatrix} - \mathcal{R}_v^b \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}$$



- Moments in the body frame:

$$\mathbf{m}_b = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} ls_{30} & l & ls_{30} & -ls_{30} & -l & -ls_{30} \\ -lc_{60} & 0 & lc_{60} & lc_{60} & 0 & -lc_{60} \\ -k_m & k_m & -k_m & k_m & -k_m & k_m \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix}$$

MAVD: Indicative questions

- What is the control allocation matrix for an octarotor with arm length equal to 0.3m? Describe how the control methodology of this control allocation actuates the roll, pitch and yaw rotations as well as how the aerial robot moves in the x,y,z axis.
- What is the thrust force necessary to compensate for the weight given any possibly orientation of the MAV that can be continuously defined using Euler angles?
- What parameters can you change to improve the capability of a motor-propeller system to provide more thrust? Correlate your answer with the terms of the thrust force equation.

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Topic: Navigation Sensors

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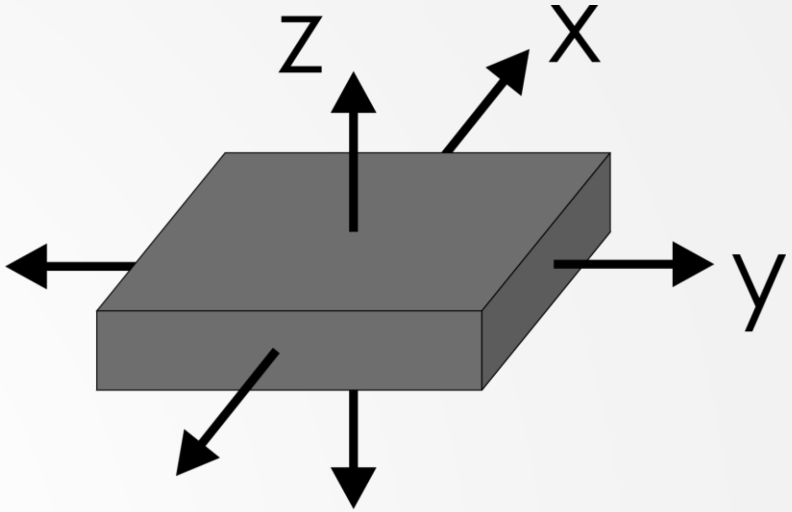
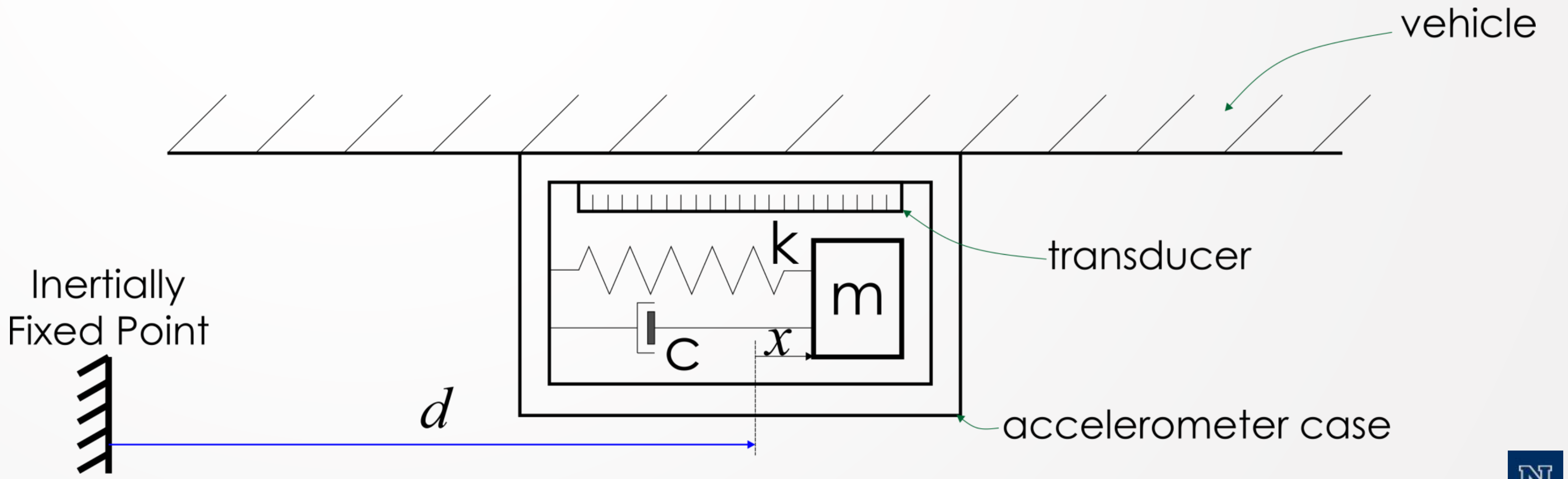
NS: Accelerometer



- Accelerometers are devices that **measure proper acceleration** ("g-force"). Proper acceleration is not the same as coordinate acceleration (rate of change of velocity). For example, an accelerometer at rest on the surface of the Earth will measure an acceleration g= 9.81 m/s^2 straight upwards.
- Accelerometers are electromechanical devices that are able of measuring static and/or dynamic forces of acceleration. Static forces include gravity, while dynamic forces can include vibrations and movement. Accelerometers can measure acceleration on 1, 2 or 3 axes.

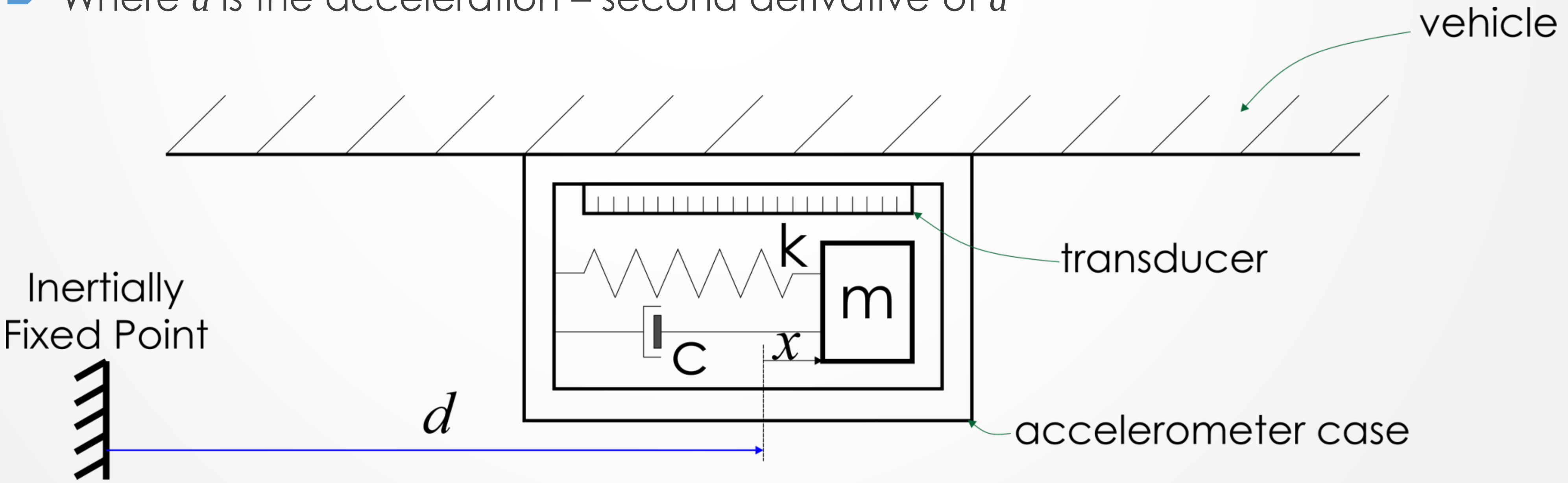


NS: Accelerometer

- **Simplified Accelerometer Model:**

$$m(\frac{d^2}{dt}(d+x)) = F_x \Rightarrow m(\frac{d^2}{dt}(d+x)) = -c\frac{dx}{dt} - kx \Rightarrow$$
$$m(\ddot{d}+\ddot{x}) + c\dot{x} + kx = 0 \Rightarrow m\ddot{x} + c\dot{x} + kx = -ma$$

- Where a is the acceleration – second derivative of d

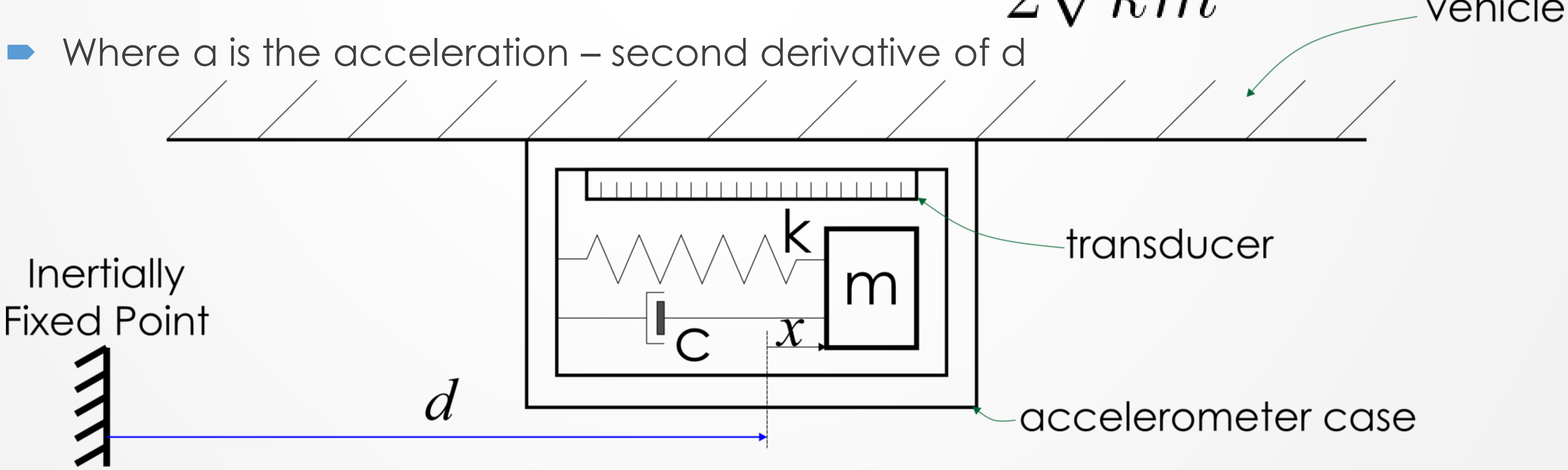


NS: Accelerometer

- For the cases within which, the vehicle acceleration is constant, then the steady state output of the accelerometer is also constant, therefore indicating the existence and value of the acceleration.
- The **undamped natural frequency** and the **damping ratio** of the accelerometer are:

$$\omega_n = \sqrt{k/m}, \quad \zeta = \frac{c}{2\sqrt{km}}$$

- Where a is the acceleration – second derivative of d

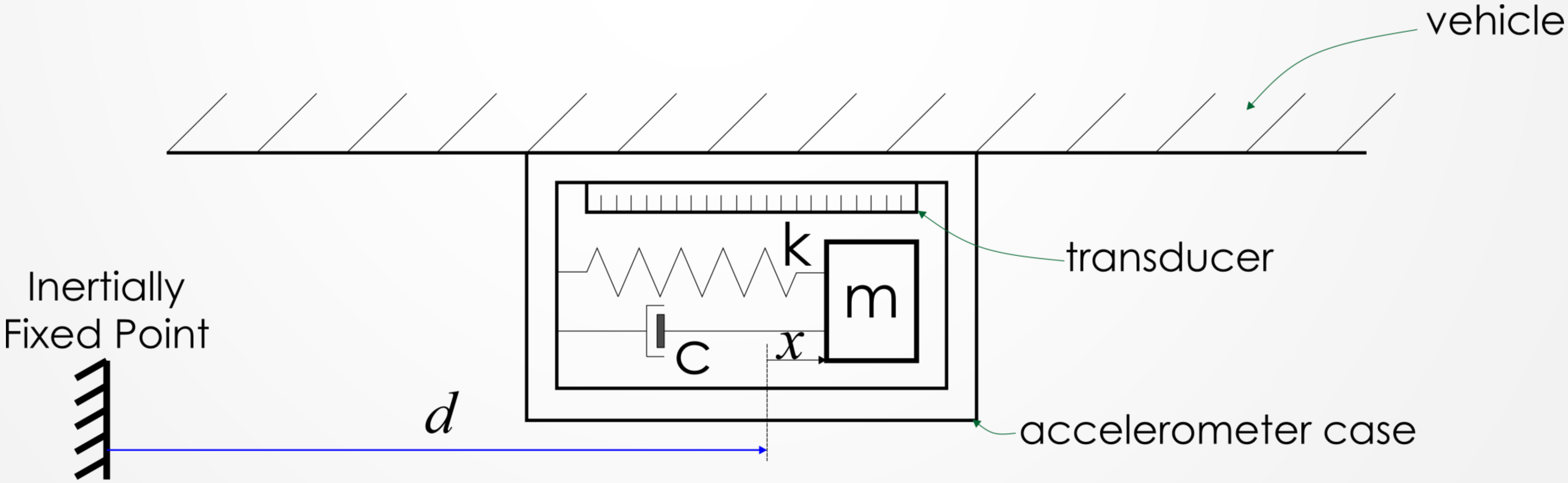


NS: Accelerometer

- **Bias effects on accelerometers:** accelerometer measurements are degraded by scale errors and bias effects. A typical error model takes the form:

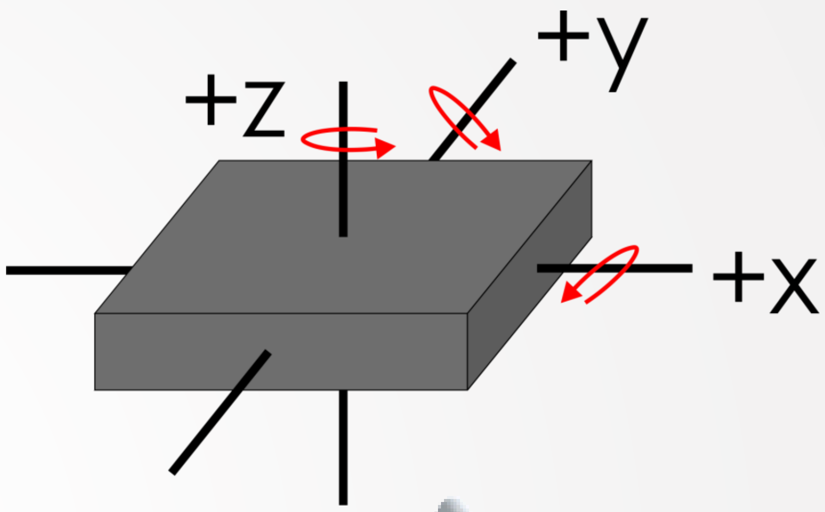
$$\mathbf{a}_{3D} = \mathbf{M}_{acc}\mathbf{a}_{3D}^{m} - \mathbf{a}_{bias} + \mathbf{a}_{n}$$

- Where a_{3D} stands for the 3-axes acceleration, M_{acc} for combined scale factor and misalignment compensation, $a_{3D}{}^{m}$ for the measurement, a_{bias} for bias signal and a^{n} for zero mean noise.



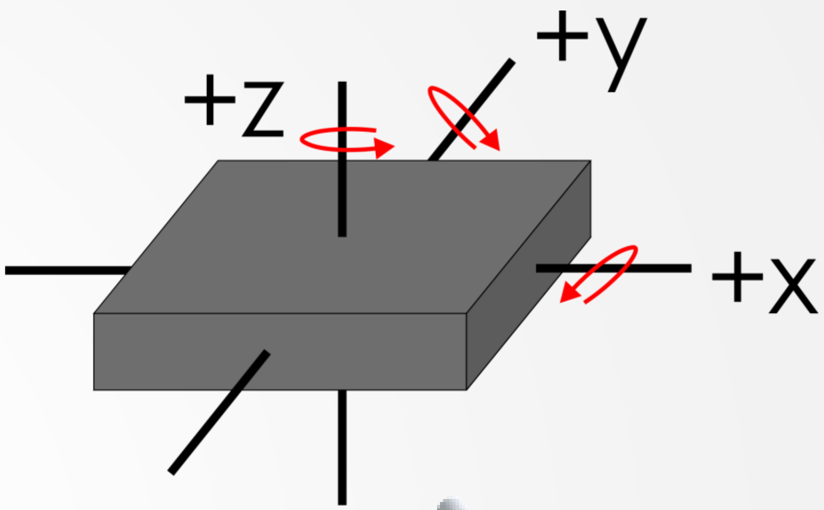
NS: Gyroscope

- **A gyroscope is - conceptually - a spinning wheel in which the axis of rotation is free to assume any possible orientation.** When rotating, the orientation of this axis remains unaffected by tilting or rotation of the mounting, according to the conservation of angular momentum. Due to this principle, a gyroscope can lead to the measurement of orientation and its rate of change. The word comes from the Greek "γύρος" and σκοπεύω which mean "circle" and "to look" correspondingly.
- Nowadays, we are mostly using gyroscopes that are based on different operating principles. In aviation we especially focus on MEMS gyroscopes or solid-state ring lasers, and fibre optic gyroscopes. In small-scale aerial robotics, we mostly care for MEMS gyroscopes.



NS: Gyroscope

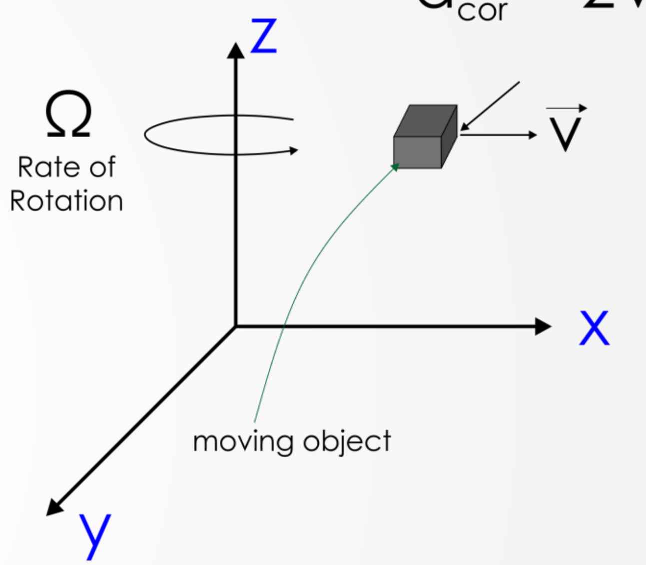
- A classical rotary gyroscope relies on the **law of conversation of angular momentum**.
  - ***The total angular momentum of a system remains constant in both magnitude and direction of the resultant external torque acting upon the system is zero.***
- Gyroscopes exploiting this principle, typically consist of a spinning disk or mass on an axle, which is then mounted on a series of gimbals. Each of these gimbals provides the spinning disk an additional degree of freedom.
- Therefore, as long as the gyroscope is spinning, it will maintain a constant orientation. In the case that external torques or rotations about a given axis are present in these devices, orientation can be maintained, and measurement of angular velocity can take place - due to the phenomenon of precession.
  - **The phenomenon of precession takes place when an object that is spinning about some axis (its "spin axis") has an external torque applied in a direction perpendicular to the spin axis (the input axis).** In a rotational system, when net external torques are present, the angular momentum vector (along the spin axis) will move in the direction of the applied external torque vector. Consequently, the spin axis rotates about an axis that is perpendicular to both the input axis and the spin axis (this is now the output axis).
- This rotation about the output axis is then sensed and fed back to the input axis where a motor-like device applies torque in the opposite direction therefore canceling the precession of the gyroscope and maintaining its orientation.
  - To measure rotation rate, counteracting torque is pulsed at periodic time intervals. Each pulse represents a fixed step of angular rotataion, and the pulse count in a fixed time interval will be proportional to the angle change θ over that time period.

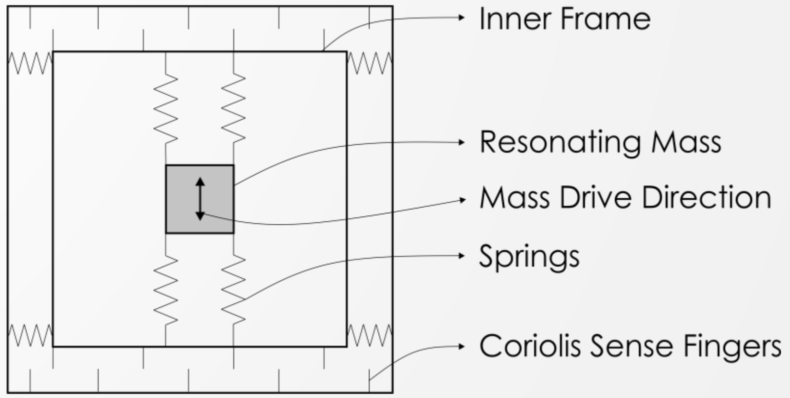


NS: Gyroscope

- MEMS gyroscopes are micro vibrating structures that base their operation the phenomenon of **Coriolis force**.
- In a rotating system, every point rotates with the same rotational speed. As one approaches the axis of rotation of this system, the rotational speed remains the same, but the speed in the direction perpendicular to the rotation axis decreases.
  - In order to travel along a straight line towards or away from the axis of rotation, lateral speed must be adjusted in order to maintain the same relative angular position on the body.
  - The Coriolis force corresponds to the product of the object mass (whose longitude is to be maintained) times the acceleration that leads to the required slowing down or speeding up.
  - **The Coriolis force is proportional to both the angular velocity of the rotating object, as well as to the velocity of the object moving towards or away from the axis of rotation**.

$$\vec{a}_{cor} = 2\vec{V} \times \vec{\Omega}$$

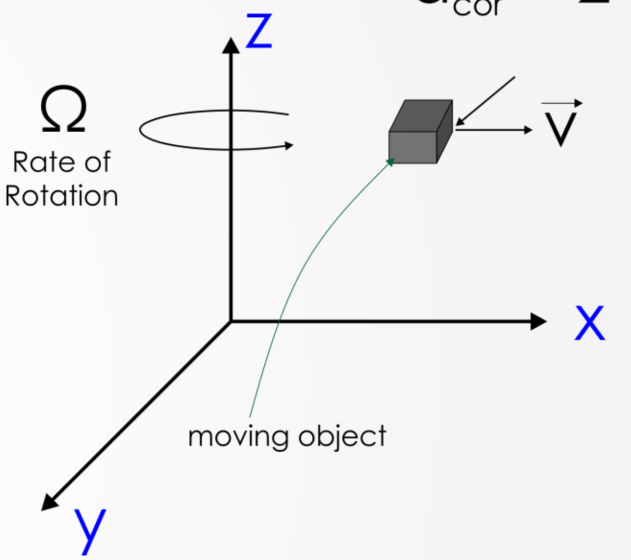


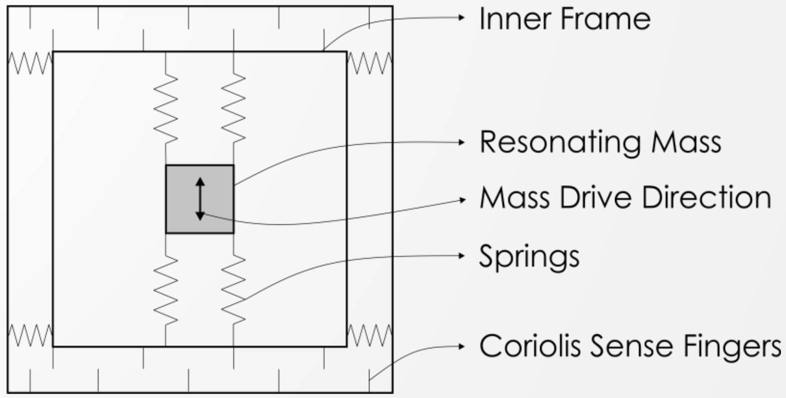


NS: Gyroscope

- **Fabrication:** a micro-machined mass which is connected to an outer housing by a pair of springs. This outer housing is then connected to the fixed circuit board using a second set of orthogonal springs.
  - The test mass is continuously driven sinusoidally along the first set of springs. As any rotation of the system will induce Coriolis acceleration in the mass, it will subsequently push it in the direction of the second set of springs.
  - As the mass is driven away from the axis of rotation, the mass will be pushed perpendicularly in one direction, and as it is driven back toward the axis of rotation, it will be pushed in the opposite direction, due to the Coriolis force acting on the mass.
- **Coriolis force sensing:** Coriolis force is sensed and detected by capacitive sense fingers that are integrated along the test mass housing and the rigid structure.
  - **As the test mass is pushed by the Coriolis force, a differential capacitance will develop and will be detected as the sensing fingers are brought closer together.** When the mass is pushed in the opposite direction, different sets of sense fingers are brought closer together.
  - The sensor can detect both the magnitude as well as the direction of the angular velocity of the system.

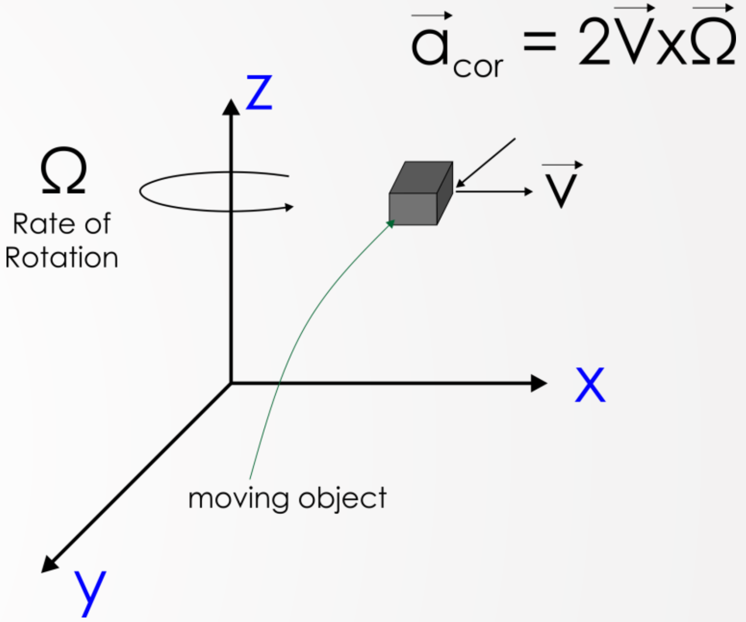
$$\vec{a}_{cor} = 2\vec{V} \times \vec{\Omega}$$

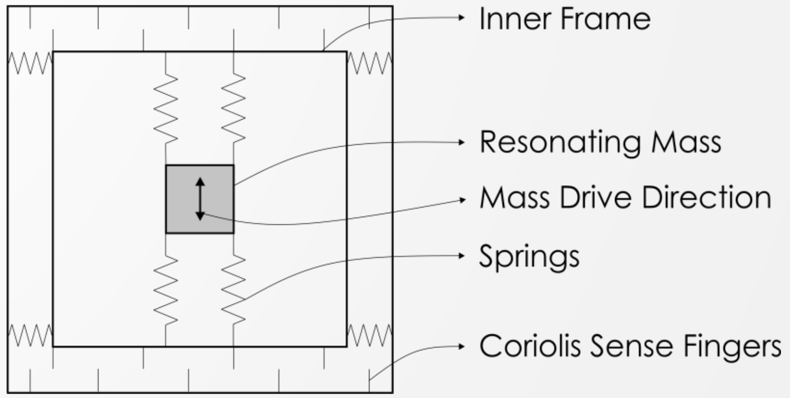




NS: Gyroscope

- **Bias effects on Gyros:** The biggest problem with gyros (and what essentially constraints us from simply performing integrating actions on their measurements), is the existence of bias effects. Bias are mostly caused by:
  - Drive excitation feedthrough
  - Output electronics offsets
  - Bearing torques
- **Biases are present in three forms** - as far as their expression and time evolution is concerned - namely:
  - Fixed bias ("const")
  - Bias variation from one turn-on to another (thermal), called bias stability ("BS")
  - Bias drift, usually modeled as a random walk ("BD")





NS: Gyroscope

- **As the bias effect are additive, we may write:**

$$\delta\omega_{bias} = \delta\omega_{const} + \delta\omega_{BS} + \delta\omega_{BD}$$
$$\frac{d}{dt}\omega_{BD} = \omega(t); \omega \sim N(O, Q)$$

- Where Q is known
- **Error model a single-axis gyroscope:**

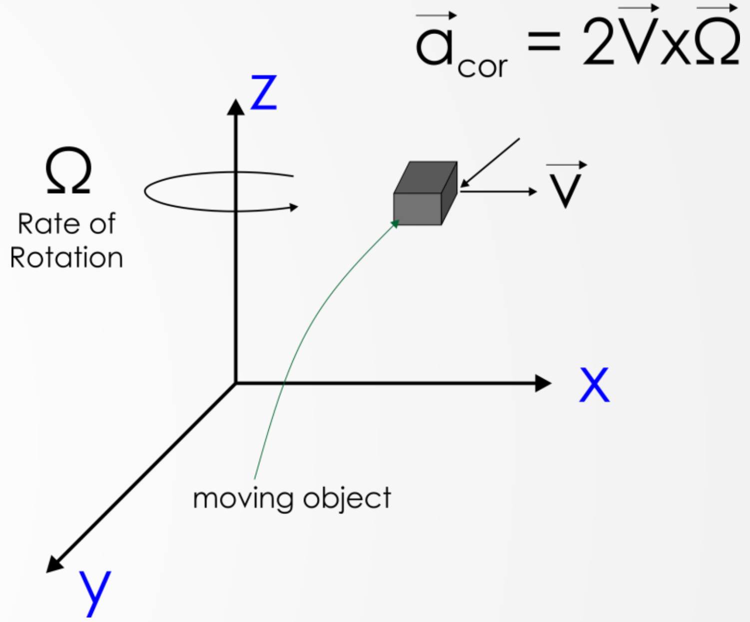
$$\omega_{1D} = k_g \omega_{1D}^m - \omega_{bias} + \omega_n$$

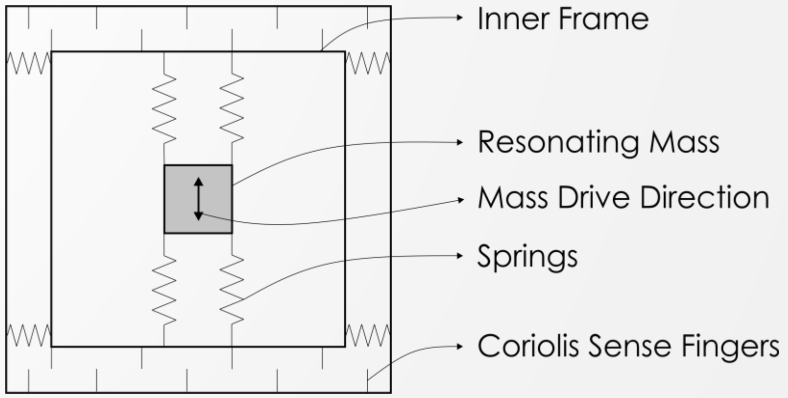
  - ω_{bias} : bias model

    $\dot{\omega}_{bias} = n_\omega, E[n_\omega] = 0, E[n_\omega(t)n_\omega^T(t)] = n(t-t'), E[n_\omega(t)n_\omega^T](t')] = 0$

  - ω_n : noise model

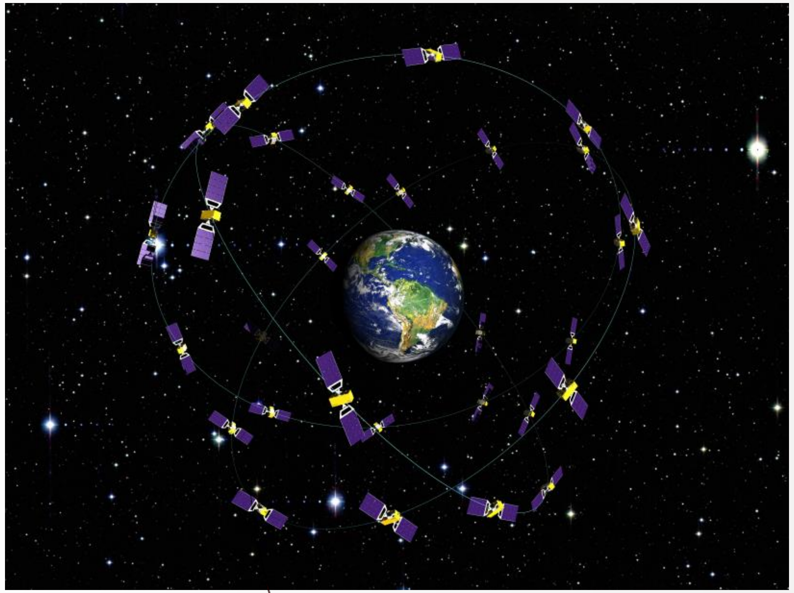
    $\omega_n : \quad E[\omega_n] = 0, E[\omega_n(t)\omega'(t')] = N_r\delta(t-t')$





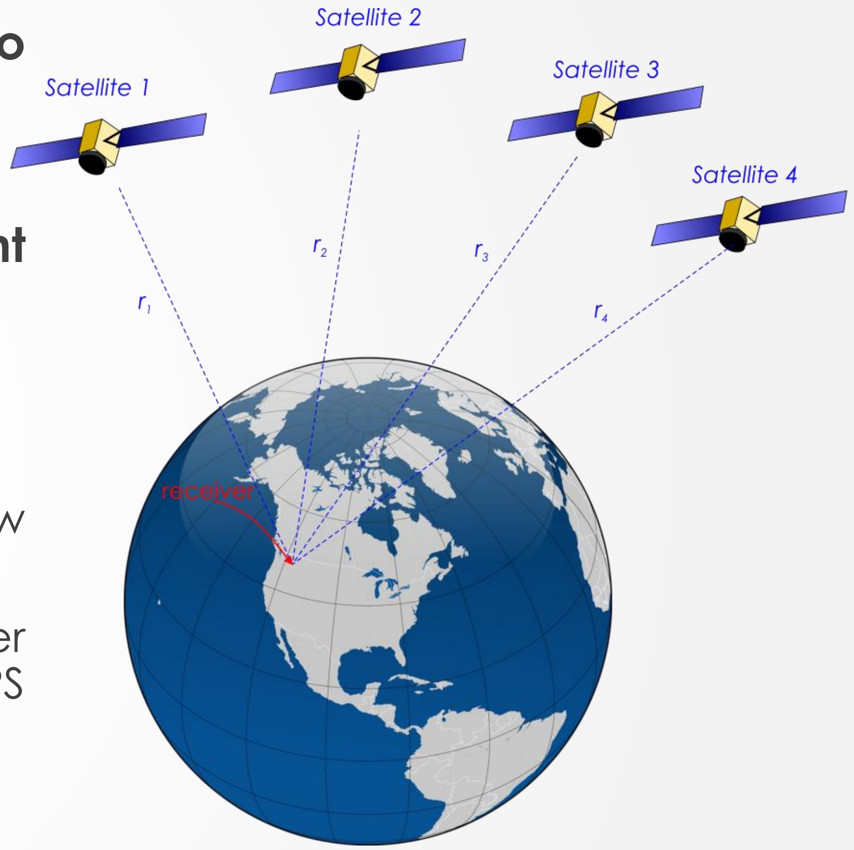
NS: Global Positioning System

- 24 Satellites orbiting the Earth (*and some back-ups*).
- Altitude set at 20,180km
- Any point on Earth's surface can be seen by at least 4 satellites at all times.
- Time-of-Flight of radio signal from 4 satellites to receiver in 3 dimensions.
- 4 range measurements needed to account for clock offset error.
- 4 nonlinear equations in 4 unknown results:
  - Latitutde
  - Longitutde
  - Altitude
  - Receiver clock time offset

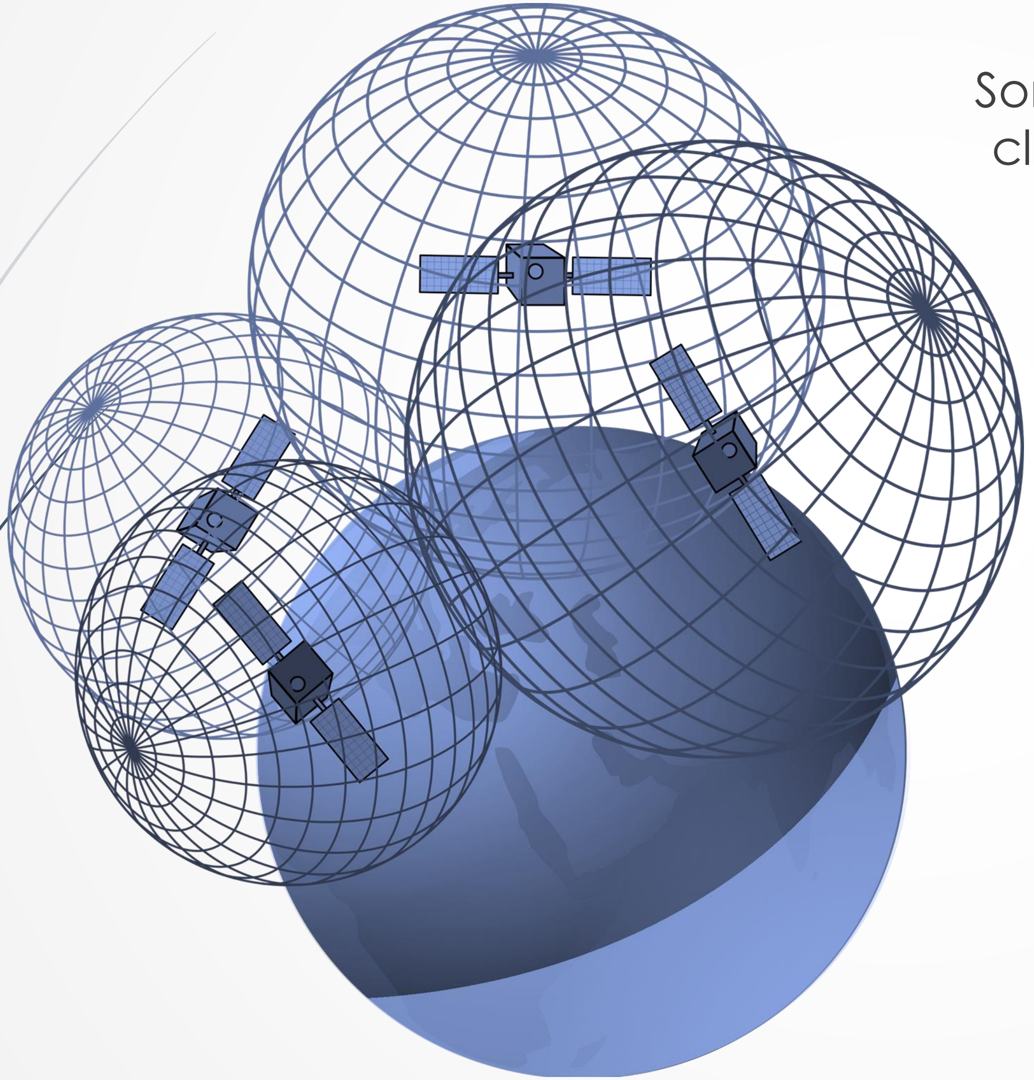


NS: Global Positioning System

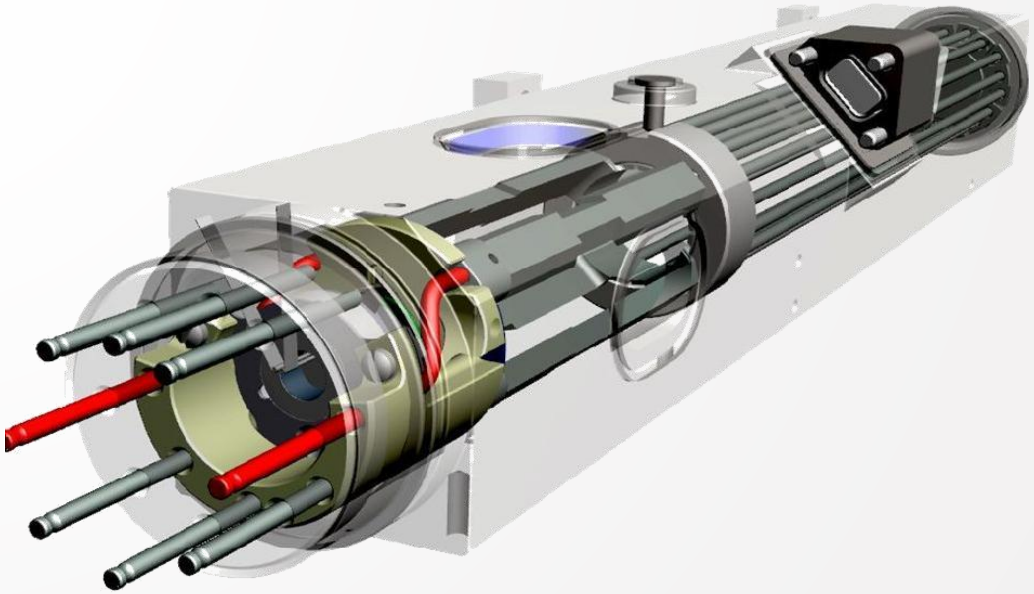
- **Time-of-Flight of the radio signal from satellite to receiver used to calculate pseudorange.**
  - Called pseudorage to distinguish it from true range.
- **Numerous sources of error in time-of-flight measurement:**
  - Ephemeris Data – errors in satellite location
  - Satellite clock – due to clock drift.
  - Ionosphere – upper atmosphere, free electrons slow transmission of the GPS signal.
  - Troposphere – lower atmosphere, weather (temperature and density) affect speed of light, GPS signal transmission.
  - Multipath Reception – signals not following direct path
  - Receiver measurement – limitations in accuracy of the receiver timing.
- **Small timing errors can result in large position deviations:**
  - 10ns timing error leads to 3m pseudorange error.



NS: GPS Trilateration

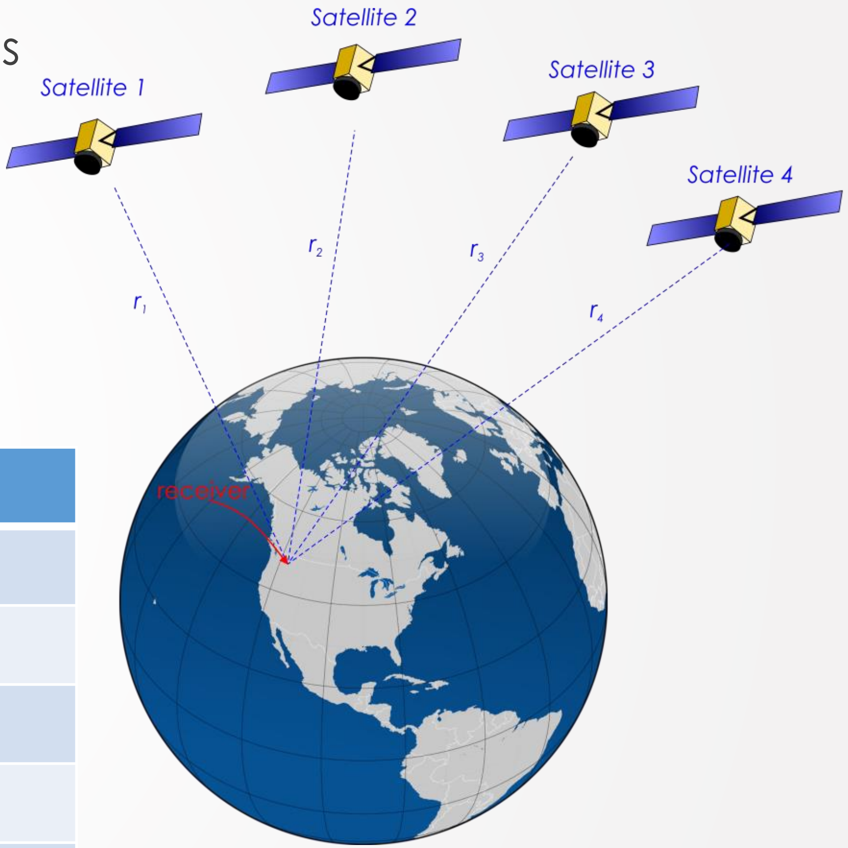


Some math and an atomic clock-based “stopwatch”



NS: GPS Error Characterization

- Cumulative effect of GPS pseudorange errors is described by the **User-Equivalent Range Error (UERE)**.
- **UERE has two components:**
  - Bias
  - Random

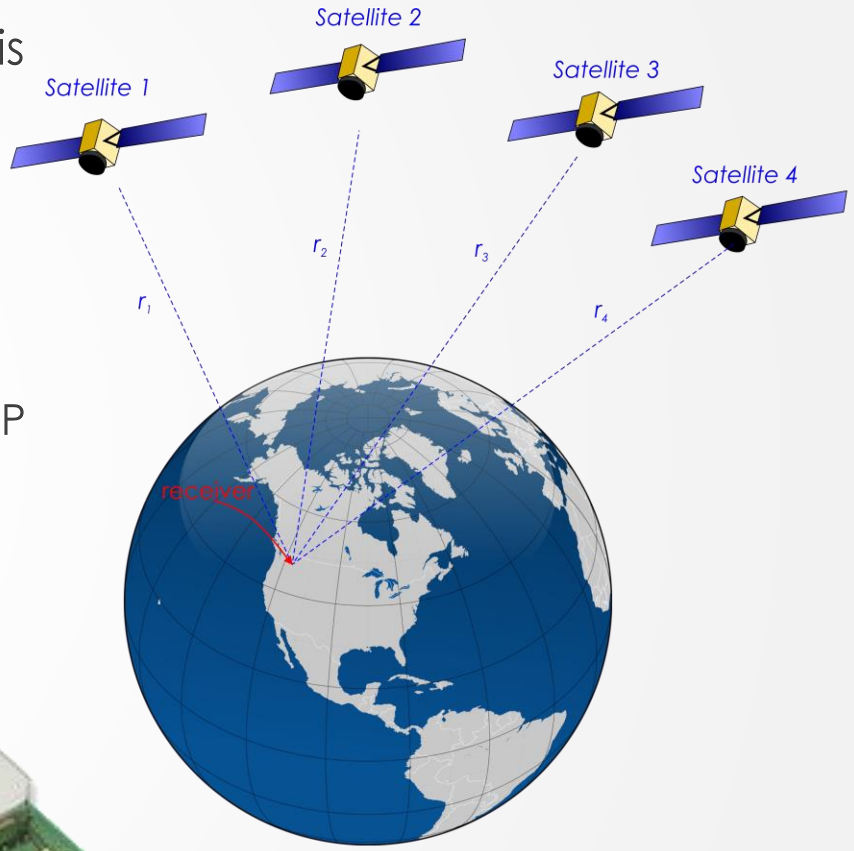


1σ, in m

Error source	Bias	Random	Total
Ephemeris data	2.1	0.0	2.1
Satellite clock	2.0	0.7	2.1
Ionosphere	4.0	0.5	4.0
Troposphere monitoring	0.5	0.5	0.7
Multipath	1.0	1.0	1.4
Receiver measurement	0.5	0.2	0.5
UERE, rms	5.1	1.4	5.3
Filtered UERE, rms	5,1	0.4	5.1

NS: GPS Error Characterization

- Effect of satellite geometry on position calculation is expressed by dilution of precision (DOP).
  - Satellites close together leads to high DOP.
  - Satellites far apart leads to low **DOP**.
  - DOP varies with time.
  - Horizontal DOP (**HDOP**) is smaller than Vertical DOP (**VDOP**):
    - Nominal HDOP = 1.3
    - Nominal VDOP = 1.8



NS: Total GPS Error

- **Standard deviation of RMS error in the north-east plane:**

$$E_{n-e,rms} = \mathrm{HDOP} \times \mathrm{UERE}_{rms} \Rightarrow$$
$$E_{n-e,rms} = (1.3)(5.1) = 6.6\mathrm{m}$$

- **Standard deviation of RMS altitude error:**

$$E_{h,rms} = \mathrm{VDOP} \times \mathrm{UERE}_{rms} \Rightarrow$$
$$E_{h,rms} = (1.8)(5.1) = 9.2\mathrm{m}$$

- As expected: an **ellipsoidal error model**.

NS: Indicative questions

- Provide the measurement model for the accelerometer and describe the types of errors as well as their nature.
- Provide the measurement model for the accelerometer and describe the types of errors as well as their nature.
- Assuming an ideal 2-axis accelerometer, how would you use it to derive an estimate of the pitch orientation of a vehicle performing only this motion?
- Define the sources of error for GPS systems
- What is the minimum amount of satellites required for a complete GPS measurement?

CS491/691: Introduction to Aerial Robotics

Topic: State Estimation – Reasoning with Bayes Law

Dr. Kostas Alexis (CSE)

SE: The State Estimation problem

- We want to estimate the world state x from:
  - Sensor measurements z and
  - Controls u

- We need to model the relationship between these random variables, i.e:

$$p(\mathbf{x}|\mathbf{z}) \qquad\qquad p(\mathbf{x}'|\mathbf{x},\mathbf{u})$$

SE: Causal vs. Diagnostic Reasoning

$P(\mathbf{x}|\mathbf{z})$ Is diagnostic

$P(\mathbf{z}|\mathbf{x})$ Is causal

- Diagnostic reasoning is typically what we need.
- Often causal knowledge is easier to obtain.
- Bayes rule allows us to use causal knowledge in diagnostic reasoning.

SE: Bayes rule

- Definition of **conditional probability**:

$$P(x, z) = P(x|z)P(z) = P(z|x)P(x)$$

- **Bayes rule:**

Observation likelihood

Prior on world state

$$P(x|z) = \frac{P(z|x)P(x)}{P(z)}$$

Prior on sensor observations

SE: Normalization

- Direct computation of $P(\mathbf{z})$ can be difficult.
- Idea: compute improper distribution, normalize afterwards.

- **STEP 1:** $$L(x|z) = P(z|x)P(x)$$
- **STEP 2:** $$P(z) = \sum_x P(z,x) = \sum_x P(z|x)P(x) = \sum_x L(x|z)$$
- **STEP 3:** $$P(x|z) = L(x|z)/P(z)$$

SE: Normalization

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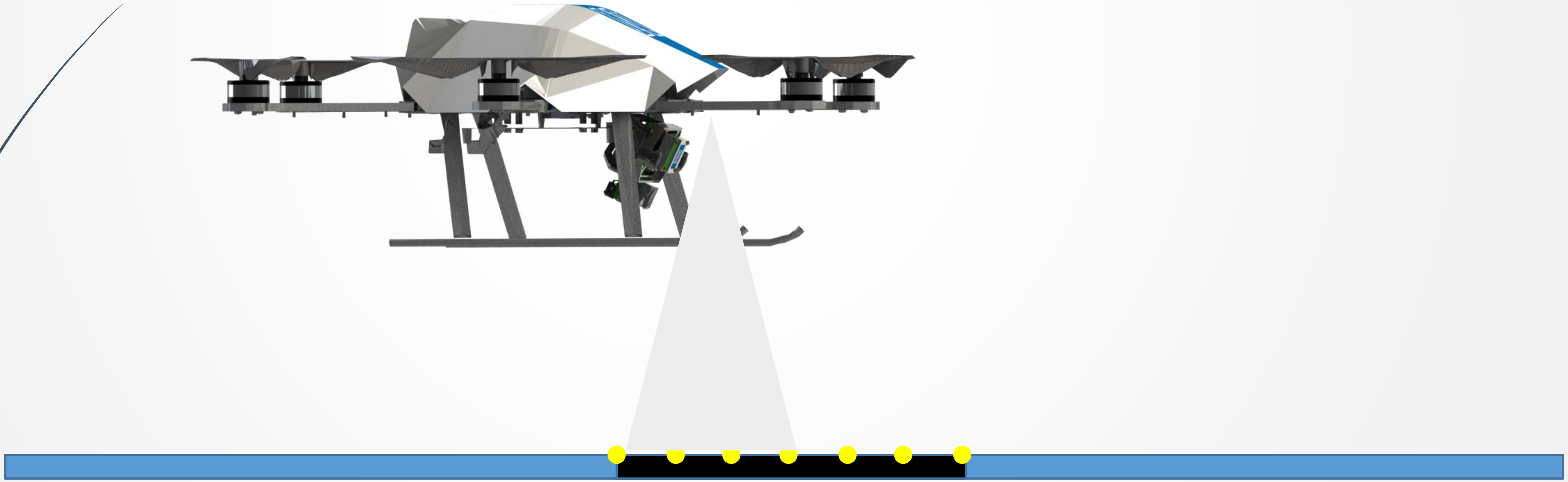
- STEP 3: $$P(x|z) = L(x|z)/P(z)$$

SE: Example: Sensor Measurement

- Quadrotor seeks the Landing Zone
- The landing zone is marked with many bright lamps
- The quadrotor has a light sensor.



SE: Example: Sensor Measurement

- Binary sensor $Z \in \{bright, \bar{bright}\}$
- Binary world state $X \in \{home, \bar{home}\}$
- Sensor model $$P(Z = bright | X = home) = 0.6$$ $$P(Z = bright | X = \bar{home}) = 0.3$$
- Prior on world state $P(X = home) = 0.5$
- Assume: robot observes light, i.e. $Z = bright$
- What is the probability $P(X = home | Z = bright)$ that the robot is above the landing zone.

SE: Example: Sensor Measurement

- Sensor model: $$P(Z = bright | X = home) = 0.6$$ $$P(Z = bright | X = \bar{home}) = 0.3$$
- Prior on world state: $P(X = home) = 0.5$
- Probability after observation (using Bayes):

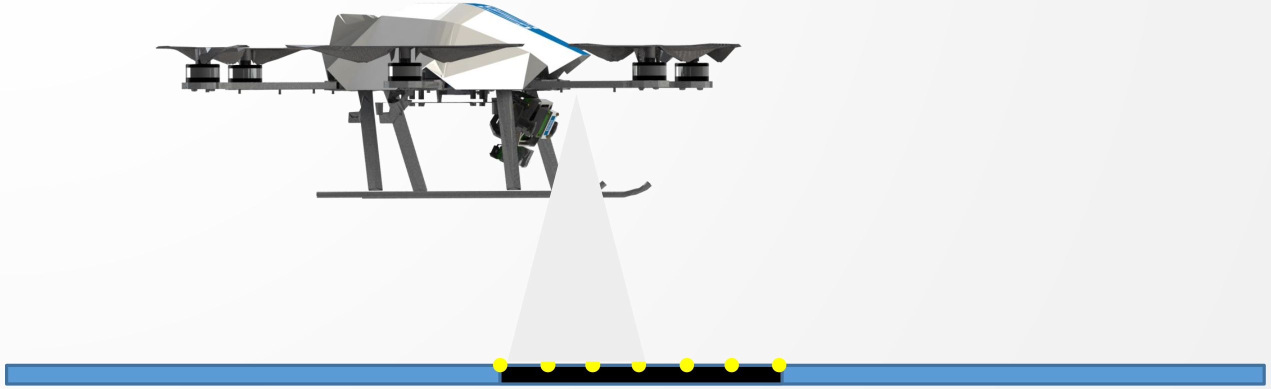
$$P(X = home | Z = bright) = \frac{P(bright|home)P(home)}{P(bright|home)P(home) + P(bright|\bar{home})P(\bar{home})} = \frac{0.6 \cdot 0.5}{0.6 \cdot 0.5 + 0.3 \cdot 0.5} = 0.67$$

SE: Indicative questions

- Consider an aerial robot searching for its landing spot using a brightness sensor. Given the sensor model:

$$P(Z = bright | X = home) = 0.6$$
$$P(Z = bright | X = \bar{home}) = 0.3$$

- And a known prior probabilistic prior, compute the probabilities after two consecutive observations.



Thank you!

Please ask your question!