math 456 hw 4

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1 Question 3

This code is in R:
```
xbar=c()
for (n in 1:10000) {
xbar[n]=mean(rnorm(n,mean=0,sd=1))
}
n=1:10000
plot(xbar,n)
```
This code generates 10000 samples from a normal distribution

```
xbar=c()
for (n in 1:10000){
xbar[n]=mean(rcauchy(n))}
n=1:10000
plot(xbar,n)
```
This code generates 10000 samples from a cauchy distribution.
The reason the two plots look so different is because the shape of the normal distribution is bell shaped and the cauchy distribution is uniform distribution.

2 Question 4

PART A
This code is in R:
n $\leftarrow$ 10000
$nsums \leftarrow 1000$
$Xn \leftarrow array(dim = nsums)$
$Y \leftarrow array(dim = n)$
$for(iin1 : nsums)$
$\{$
$Y \leftarrow rbinom(n, 1, 0.5)$
$Y \leftarrow 2 * Y - 1$
$Xn[i] \leftarrow sum(Y)$

}
$mean(Xn)$
$var(Xn)$

The expected value of Xn is 0. We can prove this because $E(Xn) = \Sigma_{i=1}^{n} E(Yi) = 0$
The variance value of Xn is n. We can prove this because $Var(Xn) = \Sigma_{i=1}^{n} Var(Yi) = n$

PART B
The calculations in part a explain why the runs look very different even though they were generated the same way because different values of n were used. When n is a low number it will look very different from when n is 10,000.

3 Question 5

Here is a plot for fX written in R:
plot(dunif(x,0,1))
The expected value of Xn is 1/2 because E(X) = $\int_0^1 \frac{x}{1-0}\, dx = \frac{1+0}{2} = \frac{1}{2}$
The variance of Xn is $\frac{1}{12}$ since $V(Xn) = E[X^2] - (E(X))^2$
E[X^2] = $\int_0^1 \frac{x^2}{1-0} dx = \frac{1^3-0^3}{3(1-0)}$
and $(E(X))^2 = \frac{1}{4}$
so $E[X^2] - (E(X))^2 = \frac{1}{3}$ - $\frac{1}{4} = \frac{1}{12}$

After plotting Xn for n = 1, 5, 25, 100, I noticed that as Xn increases, the mean and variance get closer to the expected value and the expected varaince. When n is 1 and 5 the mean isn't always going to be close to 1/2 but when it's 25 and 100 it's far closer. Same goes for the variance. As n increases, the variance gets closer to $\frac{1}{12}$.