MATH 456 — Mathematical Modeling

Assignment #9: Eigenface **Due Date:** April 29, 2022, 11:59 PM

In this assignment, you are expected to apply PCA to a large library of facial images to extract the most dominant correlations between images. The result of this decomposition is a set of eigenfaces that define a new coordinate system. Images may be represented in those principle coordinates. The goal of this assignment is to demonstrate such algorithm using the Extended Yale Face Database B, consisting of cropped and aligned images of 38 individuals (28 from the extended database, and 10 from the original database) under 9 poses and 64 lighting conditions. Each image is 192 pixels tall and 168 pixels wide, which can be reshaped into a large column vector with $192 \times 168 = 32,256$ elements. See the attached .mat file which contains a large matrix of human faces. Each column in the matrix corresponds to an image of human face. Please use the first 36 people in the database (there are 2410 in total) as our training data for eigenfaces example, and hold the 37th and 38th people as a test set.

Tasks:

1. Find the truncated SVD $X = U_r D_r V_r^T$ of the data matrix X with different values of r, e.g. r = 50,100,200,400,800,1600, etc. The approximate low-dimensional representation of a test image x_{test} can be found by the projection

$$\tilde{x}_{test} = U_r U_r^T x_{test}.$$

   Note that the projection is approximately an identity, i.e. $U_r U_r^T \approx I_d$.

2. It is interesting that the eigeface space of humans can be used to represent other objects, such as dogs. In the attached data, there is a .mat file containing images of dogs. Use the identified eigeface space to construct an approximation of a dog image.

You need to document your results in Word/PDF/Markdown/Jupyter Notebook. Upload the document along with the codes.