Optimization – Project 2
Marketing Campaign Allocations

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Executive Summary

We set out to determine the best optimal budget allocation for our company's marketing budget in order to maximize the return on investment. Given the budget of $10 million, we wanted to find the most profitable distribution of this budget across 10 different marketing platforms. We used linear programming and mixed integer programming to determine the best allocation and we compared two different consulting groups budgets to see what would work best. We also determine a monthly allocation that would allow us to reinvest in our monthly budget.

Optimal Budget Allocation – Linear Program

The ROI's calculated by company 1 allow for a linear program to be used to determine what marketing platforms should be used to achieve the maximum ROI. The first step was defining the decision variables and their upper and lower bounds. We added the decision variables one at a time, using a dictionary to define the separate marketing platforms and their tiers. The upper bound of each decision variable was defined by the variable's upper bound minus its lower bound. This subtraction step was needed to ensure the model was feasible, and it addressed the tiered structure the non-increasing function created.

We constructed this problem using 40 decision variables, 4 tiers of investment for each of the 10 available platforms. For a given platform, decision variables are as follows:

$$\begin{aligned} a_P &= \text{amount invested in tier 1 of platform } P \\ b_P &= \text{amount invested in tier 2 of platform } P \\ c_P &= \text{amount invested in tier 3 of platform } P \\ d_P &= \text{amount invested in tier 4 of platform } P \end{aligned}$$

$$X_P = a_P + b_P + c_P + d_P \quad \text{(total investment in platform P)}$$

For platforms with fewer than 4 tiers, the corresponding decision variable (i.e. d_p) was constrained to an upper bound of 0.

The linear programming model's objective was to maximize the ROI of all of the different marketing platforms. Formally,

$$\text{Maximize} \sum_{\text{platforms } P} \left(ROI_{P,1} \cdot a_P + ROI_{P,2} \cdot b_P + ROI_{P,3} \cdot c_P + ROI_{P,4} \cdot d_P \right)$$

We then added the four constraints needed:

1. The marketing budget of $10 million

$$\sum_{\text{all platforms } P} (a_P + b_P + c_P + d_P) \leq 10$$

2. The amount invested in Print and TV is less than or equal to the amount invested in Facebook and Email.

$$X_{\text{Print}} + X_{\text{TV}} \leq X_{\text{Facebook}} + X_{\text{Email}}$$

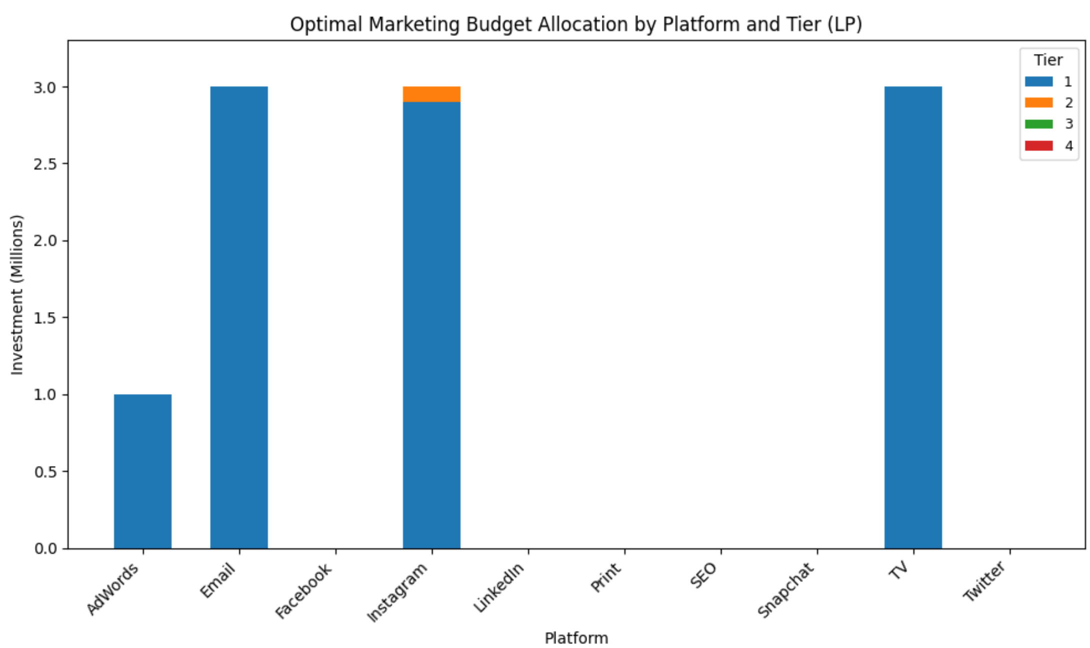
3. The amount invested in social media is greater than or equal to double the amount invested in SEO and AdWords

$$\sum_{P \in \text{Social Media}} X_P \geq 2 \cdot (X_{\text{SEO}} + X_{\text{AdWords}})$$

4. For each marketing platform, the amount invested is no more than \$3 million

$$a_P + b_P + c_P + d_P \leq 3$$

After completing these steps, we were able to optimize our model, and we achieved an optimal value of \$543,640. The optimal marketing budget allocation is shown below.



Optimal Budget Allocation – Mixed Integer Program

To further our analysis, we want to run the optimization on another ROI scenario where some later tiers have higher ROI than earlier ones. This makes the total return function non-concave, and if we used the LP, it would skip the earlier cheaper tiers and skip to a high ROI tier. To prevent this, we added binary variables. The added constraints that turn this into a mixed integer problem are shown below.

Subject to:

1. No spending unless tier is 'turned on' by the binary variable

$$x_{i,j} \leq \text{Width}_{i,j}\, z_{i,j}$$

2. No skipping tiers

$$z_{i,j-1} \geq z_{i,j} \quad \forall i, j = 2, 3, 4$$

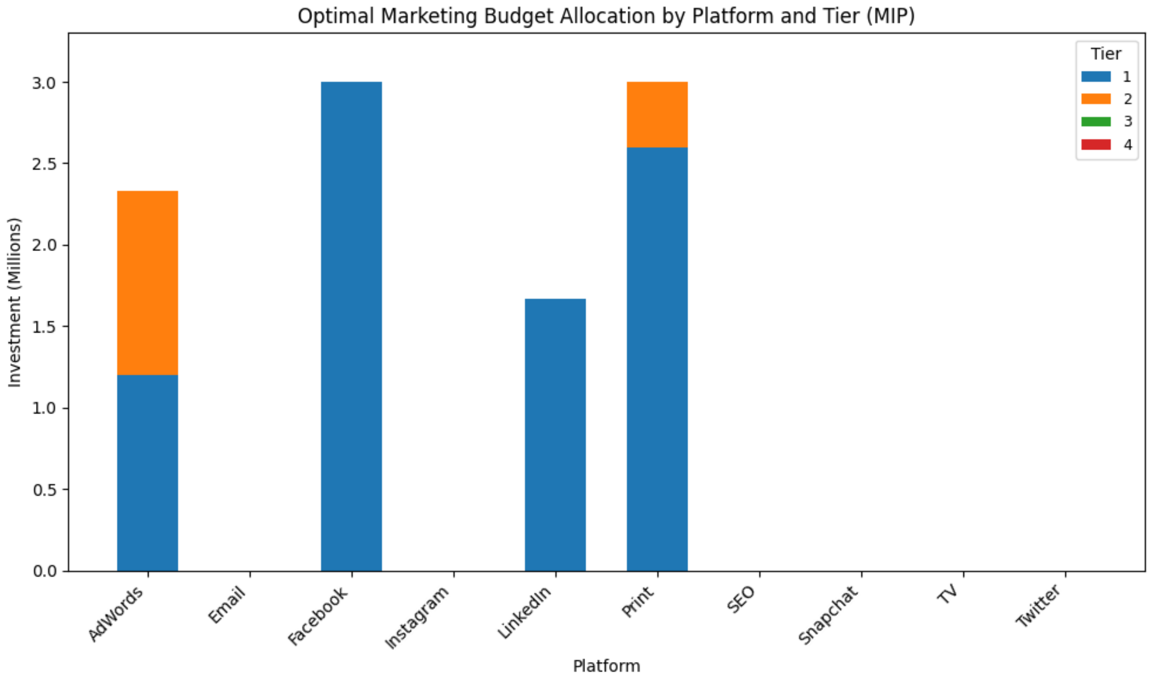
3. Fill lower tiers completely before moving to higher tiers

$$x_{i,j-1} \geq \text{Width}_{i,j-1}\, z_{i,j} \quad \forall i, j = 2, 3, 4$$

4. Binary variables

$$z_{i,j} \in \{0, 1\}$$

Building on the original constraints from the LP above, the mixed integer program (MIP) optimization using the 2$^{\text{nd}}$ ROI estimates yielded an ROI of $452,827 and an allocation shown below.



The allocations are completely different from the LP using the first ROI estimates. AdWords is the only platform in common, and in the MIP it allocates spending on AdWords at $2.333 million across tier 1 and 2. The MIP then allocates $3 Million to Facebook and Print, and the remaining $1.667 million to LinkedIn.

Assuming the first ROI data is correct, if you were to use the second allocation (the allocation that assumed the second ROI data was correct), the objective would be $128,067, which is $415,573 lower than the original optimal objective using the first ROI data of $543,640.

Assuming the second ROI data is correct, if you were to use the first allocation (the allocation that assumed the first ROI data was correct), the objective would be $76,150, which is $376,677 lower than the original optimal objective using the second ROI data of $452,827.

The $3M per-platform cap acts as a practical guardrail against dumping the entire budget into a single line item due to estimation noise. It improves robustness and mirrors real media buying constraints. With allocation from both solutions hitting the maximum of $3M per-platform, it's clear that this is needed to constrain the per-platform budgets.

Optimal Budget Allocation – Mixed Integer Program with Minimum Constraint

We added the minimum investment constraint into the mixed integer program and compared our results with the mixed integer program that did not include this constraint.

The minimum-investment rule is defined as:

$$\sum_{t^{x}_{(p,t,m)}} \geq MinInvestment_{p} y_{(p,m)}, \sum_{t^{x}_{(p,t,m)}} \leq 3.0 y_{(p,m)}, \; y_{(p,m)} \in \{0,1\}$$

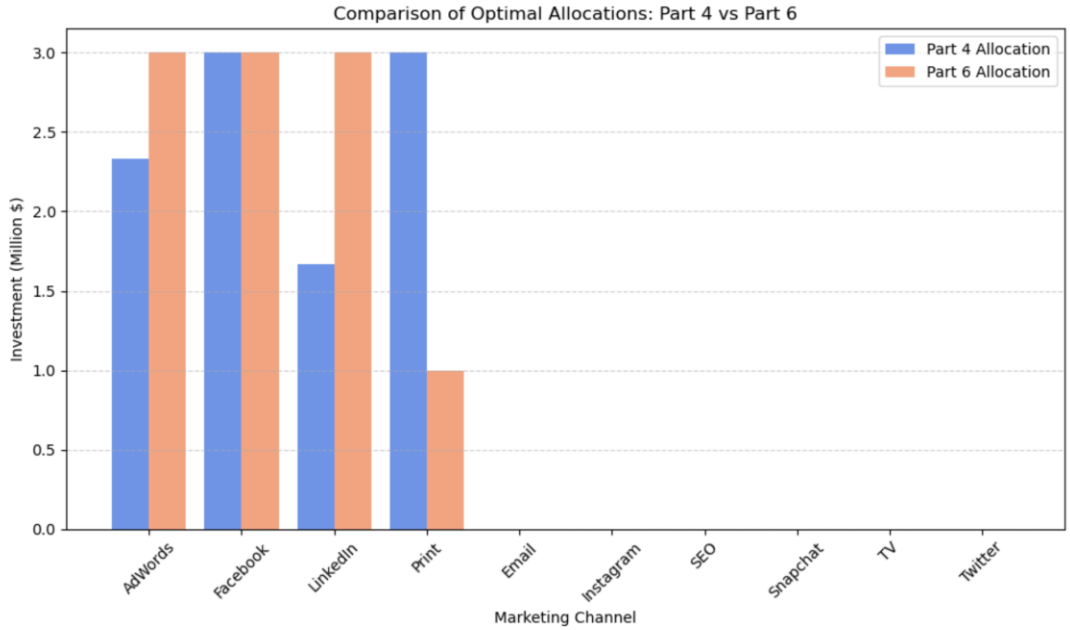
This guarantees that if a platform is active, its spending is at least the specified minimum; otherwise, it remains off.

The addition of minimum-investment constraints did **not change the optimal allocation** relative to the original mixed integer program, because the active channels (AdWords, Facebook, LinkedIn, Print) were already spending above their minimums. Channels with low ROI and positive minimum thresholds (e.g., Email, SEO, Instagram) remain inactive because turning them "on" would require removing funds from higher-yield channels. The total expected return increased slightly to **$0.495 M**, indicating a more efficient allocation once infeasible fractional investments were ruled out.

Platform	Invested (M$)	Minimum (M$)	Active
AdWords	3.000	0.800	✓
Facebook	3.000	0.400	✓
LinkedIn	3.000	0.200	✓
Print	1.000	0.300	✓
Email	0.000	0.500	✗
Instagram	0.000	0.700	✗
SEO	0.000	0.600	✗
Snapchat	0.000	0.800	✗
TV	0.000	0.300	✗
Twitter	0.000	0.300	✗

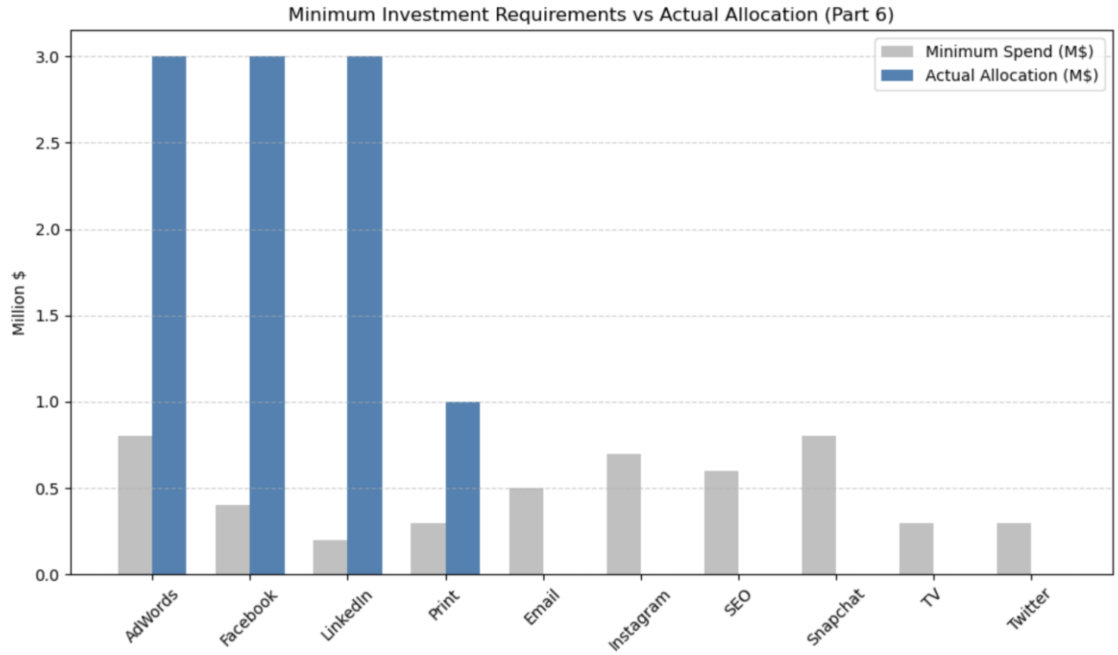
All four chosen media exceed their minimum required investments, and the full $10 M budget is utilized. The per-platform cap and the "no-skip tier" rules remain active. None of the new minimum constraints became binding because the selected platforms already met or exceeded the required thresholds.

Overall, the model successfully maximized return on investment while adhering to business rules and maintaining logical growth across months.



As shown in the graph above, the allocation pattern is unchanged when minimum-spend thresholds are introduced. The model continues to assign \$3M each to AdWords, Facebook, LinkedIn, and \$1M to Print. These channels already exceeded their respective minimum thresholds, so the additional constraint does not alter the optimal investment mix.

This finding demonstrates that the selected media were already at or above the "impactful investment" levels, while lower-performing channels (e.g., Email, Instagram, SEO) remain inactive due to lower expected returns per dollar spent.



The figure above highlights that the optimal solution fully satisfies all minimum investment requirements while strategically avoiding activation of channels that would force inefficient use of the budget. AdWords, Facebook, and LinkedIn exceed their minimum thresholds substantially, confirming their strong marginal ROI. Print remains active at exactly $1M due to the Print/TV and Social vs. Search balance constraints.

The inclusion of minimum-spend constraints reinforces the robustness of the previously identified allocation. The model confirms that investment levels in key digital platforms were already above the required impact thresholds, and additional constraints did not alter the optimal solution. This suggests that the firm's optimal allocation strategy is both **efficient** and **stable** under realistic spending rules.

Multi-Month Optimization with Minimum-Investment and Reinvestment Constraints

The goal of this stage was to extend our single-month marketing optimization into a multi-month dynamic allocation model that automatically reinvests a portion of each month's return into the next month's budget.

The model begins with a $10 M starting budget and reinvests 50% of the monthly return for each period, allowing the total budget to grow over time. Investment decisions were made across multiple digital and traditional marketing platforms, each with tiered ROI structures and spending caps.

To ensure practical business decisions, the following constraints were enforced:

- Per-platform investment caps: ≤ $3 M per platform per month.
- Minimum-investment rule (from Part 6):

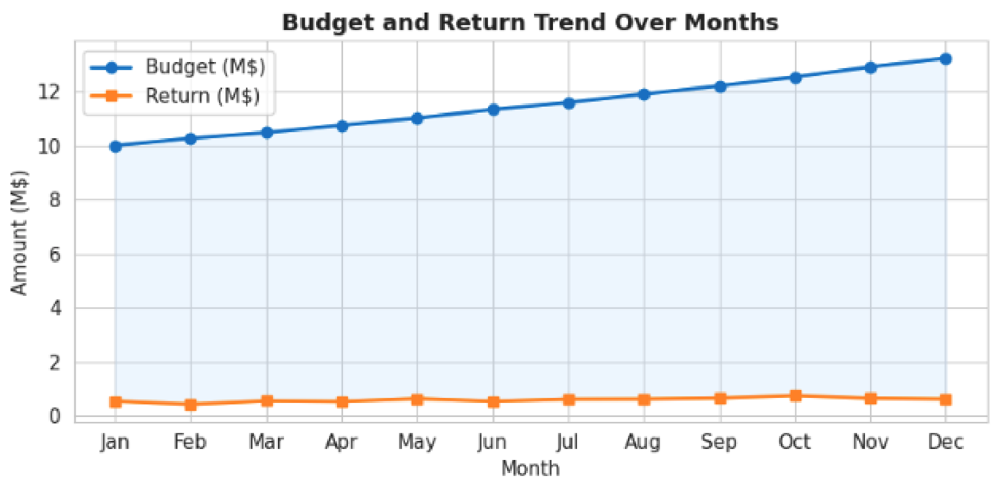
The minimum-investment rule is defined as:

$$\sum_{t^{x}_{(p,t,m)}} \geq MinInvestment_p y_{(p,m)}, \sum_{t^{x}_{(p,t,m)}} \leq 3.0 y_{(p,m)}, \; y_{(p,m)} \in \{0,1\}$$

This guarantees that if a platform is active, its spending is at least the specified minimum; otherwise, it remains off.

- Managerial rules:
  - *Print + TV ≤ Facebook + Email*
  - *Social ≥ 2 × (SEO + AdWords)*

The optimization was implemented using Gurobi, solving a series of linked monthly models. The results show that the total budget gradually increases from $10 M (January) to $13.23 M (December), while monthly returns stay consistent, reflecting sustainable reinvestment growth.



	Month	Budget(M$)	Return(M$)	AdWords	Email	Facebook	Instagram	LinkedIn	Print	SEO	Snapchat	TV	Twitter
0	Jan	10.0000	0.5394	0.0000	0.0000	3.0000	0.0000	3.0	3.0000	0.0	0.0000	0.0	1.0000
1	Feb	10.2697	0.4209	2.6566	0.0000	3.0000	0.0000	2.1	2.3000	0.0	0.2131	0.0	0.0000
2	Mar	10.4802	0.5488	2.6960	0.0000	2.3921	0.0000	3.0	2.3921	0.0	0.0000	0.0	0.0000
3	Apr	10.7545	0.5237	1.7545	0.0000	3.0000	0.0000	3.0	3.0000	0.0	0.0000	0.0	0.0000
4	May	11.0164	0.6315	0.0000	3.0000	0.0000	2.0164	3.0	3.0000	0.0	0.0000	0.0	0.0000
5	Jun	11.3322	0.5305	0.0000	0.0000	3.0000	0.0000	3.0	3.0000	0.0	0.0000	0.0	2.3322
6	Jul	11.5974	0.6114	0.0000	0.0000	3.0000	0.0000	3.0	2.5974	3.0	0.0000	0.0	0.0000
7	Aug	11.9031	0.6167	3.0000	0.0000	0.6000	0.0000	3.0	0.6000	0.0	1.7031	0.0	3.0000
8	Sep	12.2115	0.6519	2.3705	2.1000	0.9000	0.8410	3.0	3.0000	0.0	0.0000	0.0	0.0000
9	Oct	12.5374	0.7483	0.0000	0.5374	3.0000	0.0000	3.0	3.0000	0.0	0.0000	0.0	3.0000
10	Nov	12.9116	0.6440	0.0000	0.9116	3.0000	0.0000	0.0	3.0000	0.0	3.0000	0.0	3.0000
11	Dec	13.2336	0.6165	1.9336	3.0000	0.0000	0.0000	3.0	3.0000	0.0	0.0000	0.0	2.3000

Stability Analysis and Modeling Discussion:

A stable budget is defined as a monthly allocation such that, for each platform,

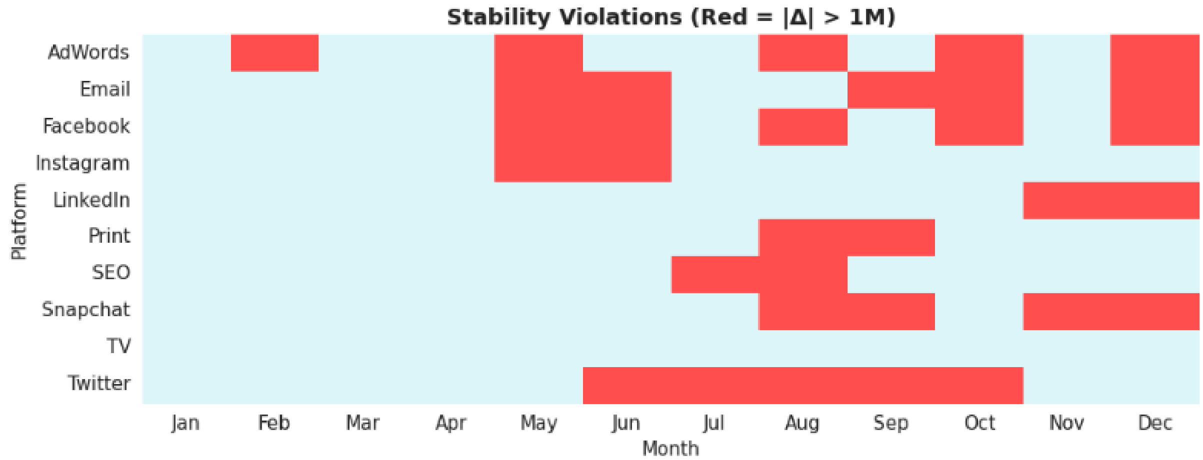
the month-to-month change in spend does not exceed ± $1 million.

After computing the absolute changes in spend between consecutive months, several violations were identified. Platforms such as AdWords, Facebook, SEO, and Twitter exhibited fluctuations greater than $1 M in multiple periods.

For instance:

- AdWords increased by ≈ $2.55 M (Jan → Feb)
- Facebook jumped by ≈ $3 M (May → Jun)
- SEO spiked by ≈ $3 M (Aug → Sep)
- Twitter saw repeated 2–3 M changes throughout the year

The heatmap below (red = |Δ| > 1 M) clearly highlights these instability points.



The allocation is not stable. The model aggressively reallocates budgets month-to-month to chase short-term ROI improvements, causing large swings that are impractical in real marketing operations. In practice, such shifts disrupt campaign consistency, vendor contracts, and audience reach continuity. Stability could be enforced mathematically by adding the following constraint for each platform (p) and month (m):

$$|X_{p,m} - X_{p,m-1}| \leq 1$$

This limits any platform's month-to-month change to within ± $1 M, producing smoother and more realistic allocations without drastically changing the optimization framework. To implement this, we would extend the multi-month model into a joint mixed-integer program that includes these cross-month "continuity" constraints, ensuring that performance growth is balanced with operational stability.