University of Texas
MSBA
RM 294 - Optimization I

Project 1 Report

Team Members: Luke Hartfield, Pranit Yadav, Peyton Gibbs, Vyshnavi Maringanti

September 28th, 2025

Introduction:

In this project, we optimized a stock portfolio that minimizes the Conditional Value-at-Risk (CVaR), a risk metric that captures the average losses in the tail of the distribution. We used daily returns from 100 Nasdaq stocks in 2019 to construct the portfolio, then evaluated its performance on 2020 returns, an unusual year for the stock market due to COVID-19. This report also explores how confidence level beta and rebalancing frequency impact risk management.

Problem Formulation:

Goal: Construct a portfolio of stocks that minimizes CVaR while meeting a minimum expected return constraint.

Each decision variable represents how much of the portfolio is invested in a particular stock. The optimization problem determines the best combination of these weights to meet our return requirement while minimizing risk. To ensure the portfolio is properly structured, we apply a budget rule that requires all the weights to add up to 100%.

We also introduce a variable called alpha, which acts as a benchmark for defining the cutoff between normal losses and extreme losses. In addition, we use a set of auxiliary variables that measure how much the portfolio's losses in each scenario go beyond that cutoff. If the loss does not exceed the cutoff, the auxiliary variable is set to zero; if it does, the variable captures the excess amount.

By averaging these excess losses across all scenarios, the solver calculates the Conditional Value-at-Risk (CVaR). The optimization process then selects portfolio weights that minimize this average tail loss, giving us a portfolio that is designed to be more resilient under extreme market conditions.

Objective - Minimize CVaR:

```
stockMod.setObjective(
    alpha + cvar_coeff * gp.quicksum(u[k] for k in range(q)),
    GRB.MINIMIZE
)
```

Constraints:

```
# 1. Portfolio weights sum to 1
stockMod.addConstr(
    gp.quicksum(x[j] for j in range(n)) == 1.0,
    name="budget"
)
```

```
# 2. Expected return constraint
stockMod.addConstr(
    gp.quicksum(mean_returns[j] * x[j] for j in range(n)) >= R,
    name="min_return"
)

# 3. CVaR auxiliary constraints
for k in range(q):
    stockMod.addConstr(
        u[k] >= -gp.quicksum(Y[k, j] * x[j] for j in range(n)) - alpha,
        name=f"tail_{k}"
    )
```

Decision Variables:

```
# Decision variables
x = stockMod.addVars(n, lb=0, ub=1, name="x")  # Portfolio weights (0 to 1)
alpha = stockMod.addVar(lb=-GRB.INFINITY, name="alpha")  # VaR proxy
u = stockMod.addVars(q, lb=0, name="u")  # Auxiliary variables
```

CVaR Parameters:

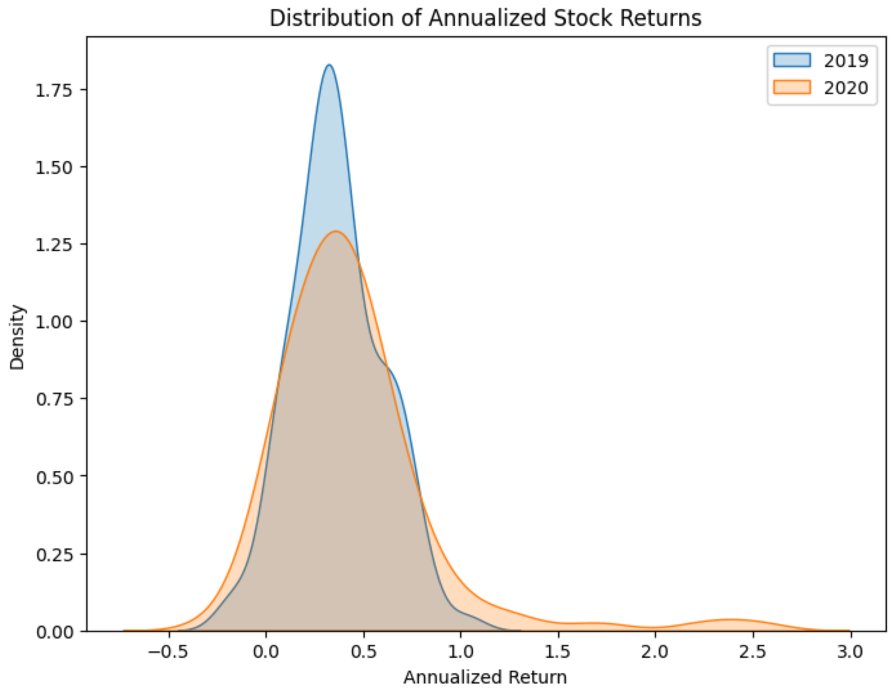
```
beta = 0.95
R = 0.0002  # 0.02% minimum daily return
cvar_coeff = 1.0 / ((1.0 - beta) * q)
```

Data and Preprocessing

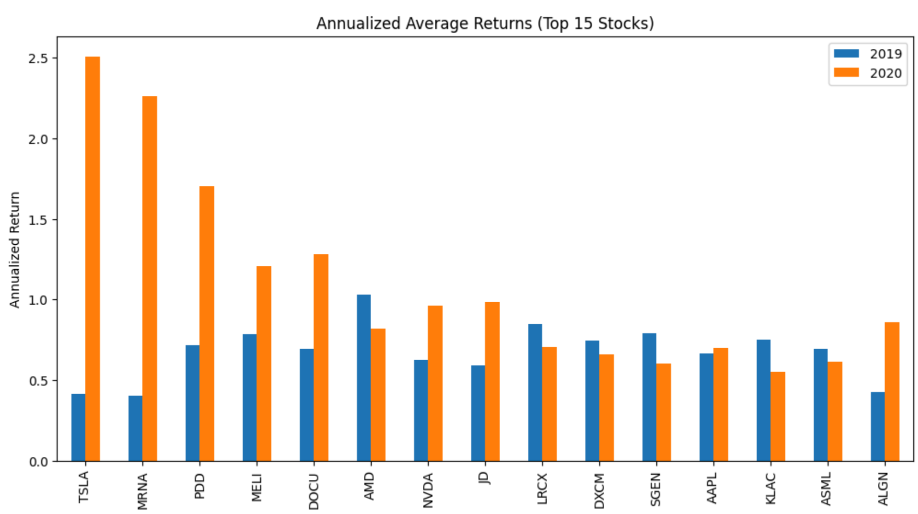
Dataset: Daily prices of 100 Nasdaq stocks 2019-2020

Cleaning:

- Convert prices to percentage returns
- Drop NDX column that tracks the whole index itself
- Align dates correctly as indexes
- Build training set (2019) and testing set (2020)



This chart shows the distribution of annualized stock returns in 2019 and 2020. As you can see, 2019 returns are centered stronger around positive values while 2020 is shifted slightly to the left with wider tails. This shows how our in sample (2019) differs from out of sample (2020) returns.



This also shows the top 15 annualized average returns for 2019 and 2020. It is dominated by technology stocks with a few healthcare/innovative companies.

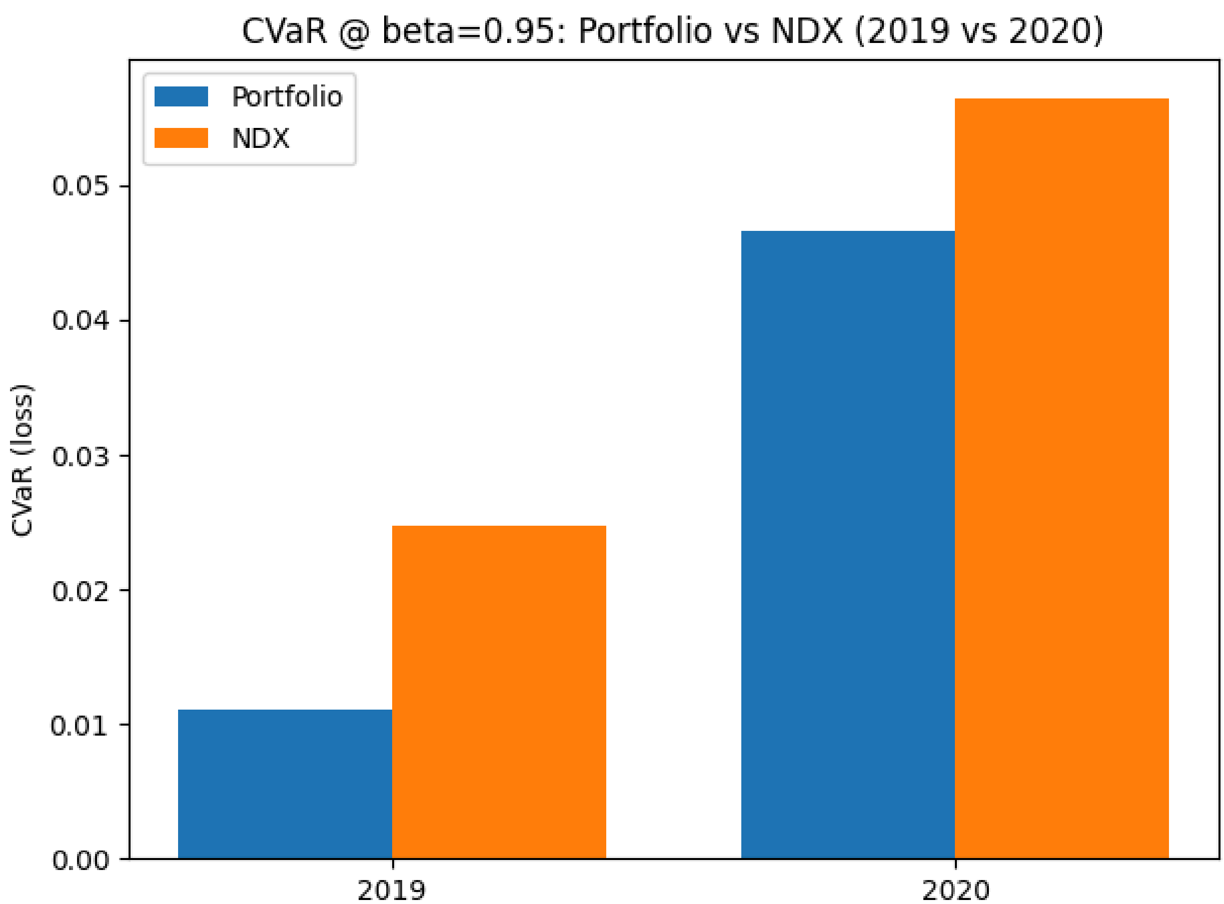
Results

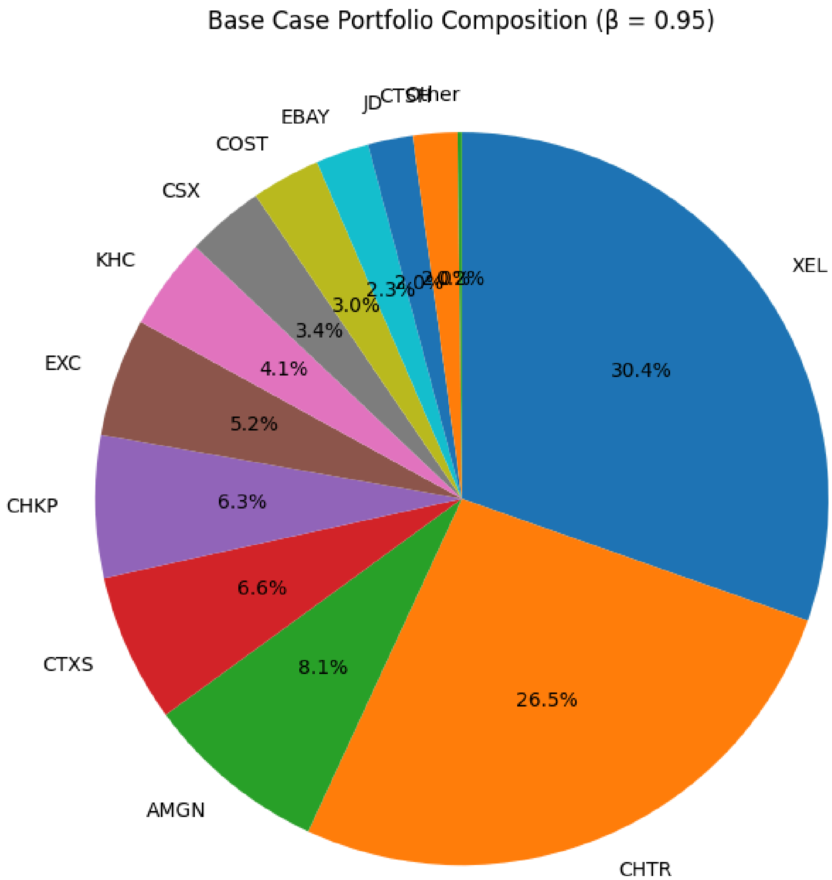
Base Case

To start, we optimized under a base case that has a beta (β) of .95 and a required return constraint (R) of 2%.

In the base case, the 2019 portfolio is in-sample and its performance in 2020 is out-of-sample. The portfolio achieved a low 2019 CVaR of 0.0111, but when applied to 2020 data, CVaR rose sharply to 0.0466, showing a substantial increase in tail risk. This illustrates the non-stationarity of markets, the concept that a portfolio that performs well in one year may not hold up under different conditions, making a static allocation across years not advisable for long-term risk management.

For comparison, the NDX index had CVaRs of 0.0248 (2019) and 0.0565 (2020). While our portfolio's risk worsened in 2020, it still outperformed the benchmark by maintaining a lower CVaR than the NDX. Overall, the portfolio reduced tail risk relative to the market, but its instability to 2020 emphasizes the need for rebalancing and adaptive strategies.





The base-case portfolio (β = 0.95) is heavily concentrated in two stocks: XEL (30.4%) and CHTR (26.5%), which together account for more than half of the allocation. The next largest holdings, AMGN (8.1%), CTXS (6.6%), CHKP (6.3%), and EXC (5.2%) round out the mid-sized positions. The remaining weights are spread thinly across smaller allocations, with no other stock above 5%.

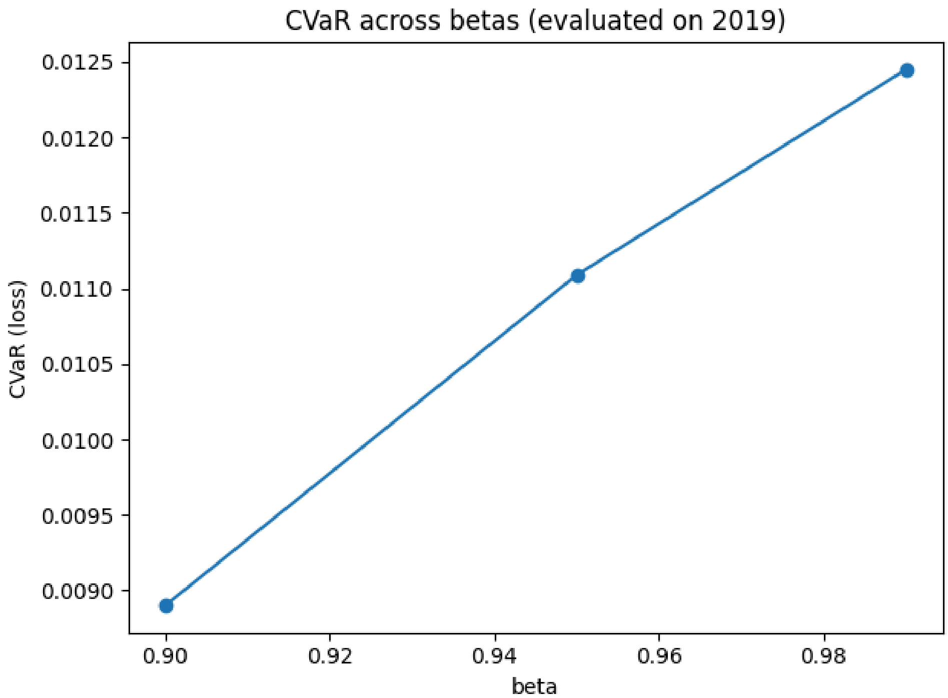
This allocation reflects the optimizer's preference for defensive and stable names, such as XEL (utilities) and CHTR (communications), which help reduce tail risk at the 95% confidence level. The portfolio therefore sacrifices diversification in favor of concentration in lower-volatility, recession-resilient sectors, consistent with the goal of minimizing downside risk.

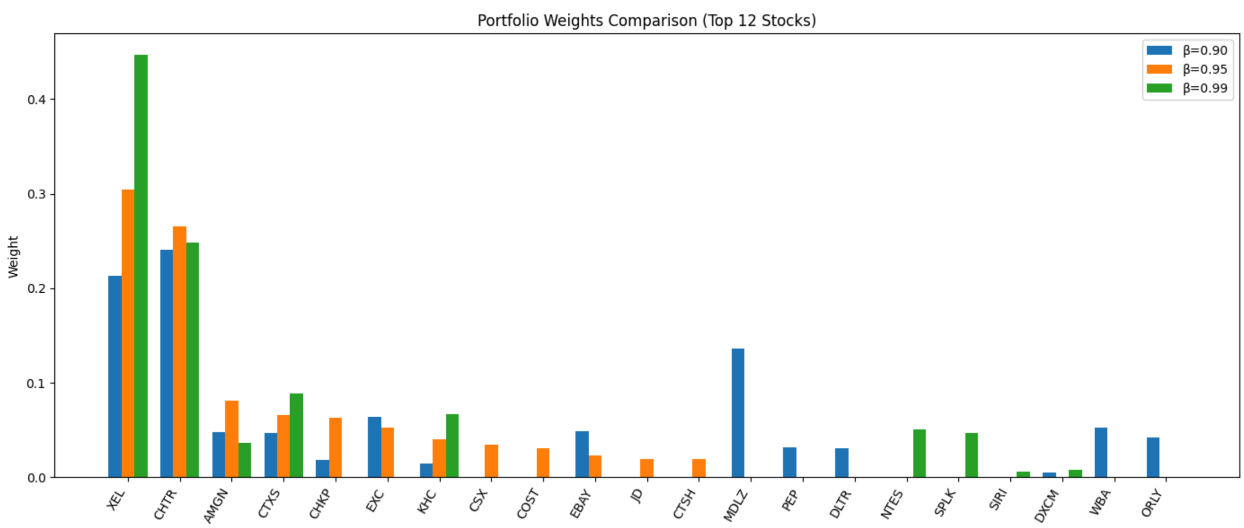
Sensitivity Analysis on Beta

We compared β = 0.90, 0.95, and 0.99, which correspond to examining the worst 10%, 5%, and 1% of days, respectively. As expected, higher β values produced stricter risk measures, with the 2019 CVaR rising from 0.0089 → 0.0111 → 0.0125 as β increased. This shows that focusing on more extreme tail events raises the assessed downside risk.

Portfolio allocations also shifted with β. At β = 0.90, the portfolio was more diversified, while at β = 0.99, it became highly concentrated in defensive stocks. XEL rose to 45% of the allocation under the highest beta. Higher β values protect more aggressively against losses, driving the optimizer to overweight utilities and consumer staples, sectors that tend to hold up even when the broader economy struggles.

The graph below shows how the CVaR varies across our sensitivity analysis.



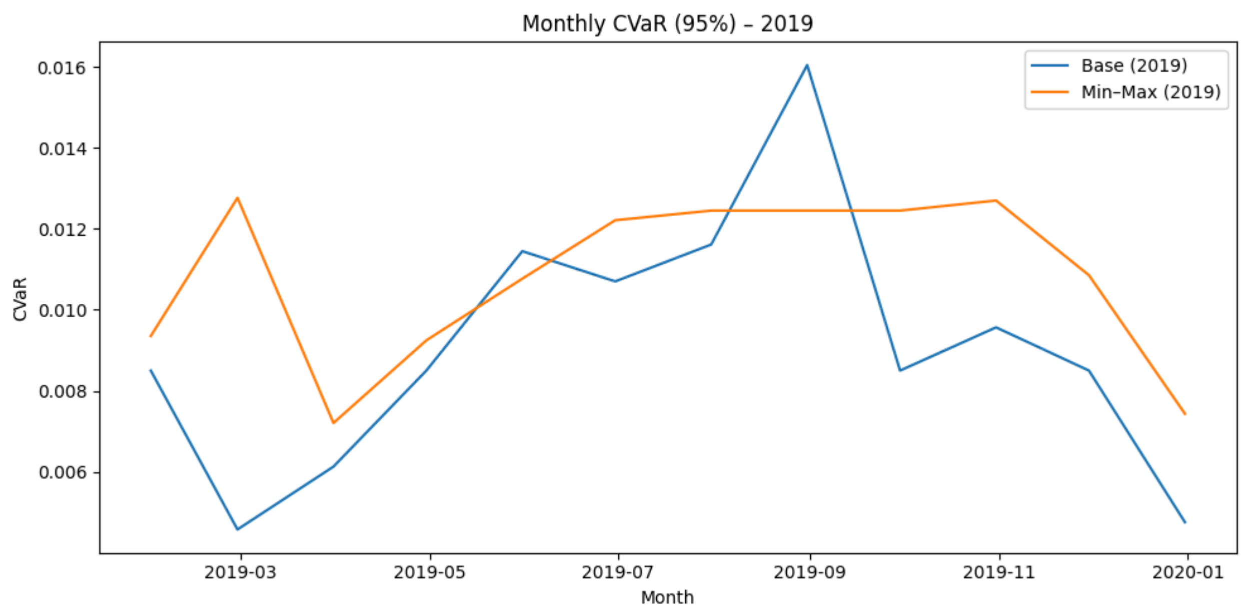


This chart compares portfolio weights under β = 0.90, 0.95, and 0.99, showing how allocations shift as the confidence level tightens. At β = 0.90 (blue), the portfolio is more diversified, with moderate weights spread across multiple stocks. As β increases, the model becomes stricter, concentrating more heavily in defensive stocks.

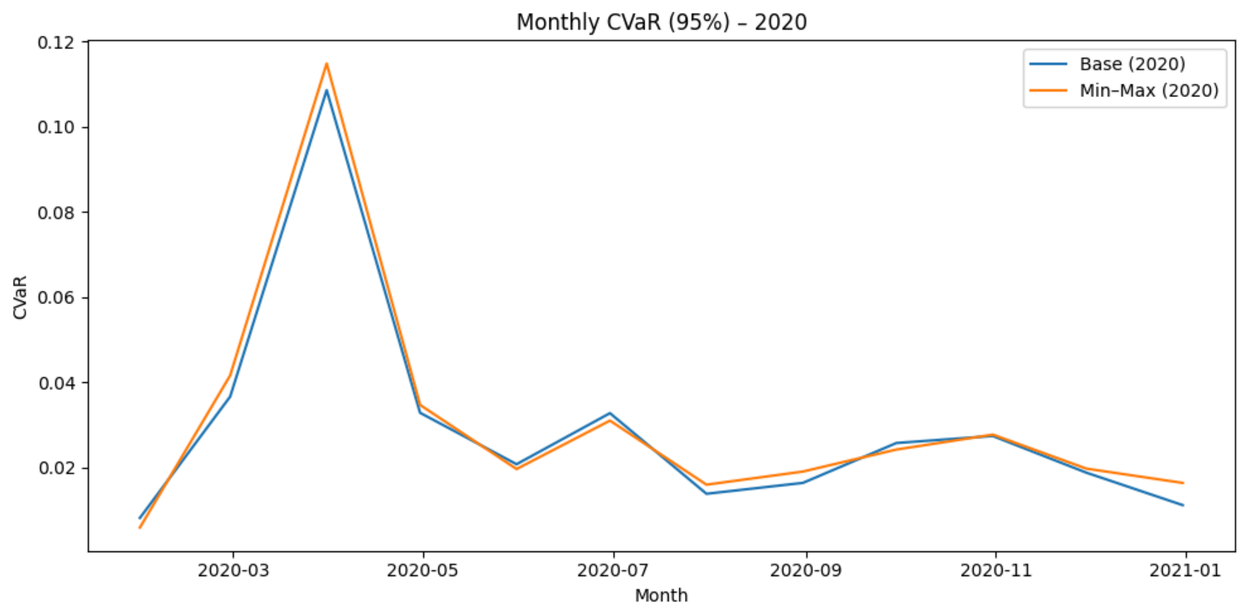
Alternative Risk Objective

The min-max CVaR approach, which minimizes the worst monthly tail loss rather than the overall average, produced a more conservative portfolio compared to the base case. While the original base case 2019 portfolio achieved a lower in-sample CVaR of 0.0111, the min-max version posted a slightly higher maximum monthly CVaR of 0.0125, reflecting its focus on controlling the single worst month rather than minimizing the average risk.

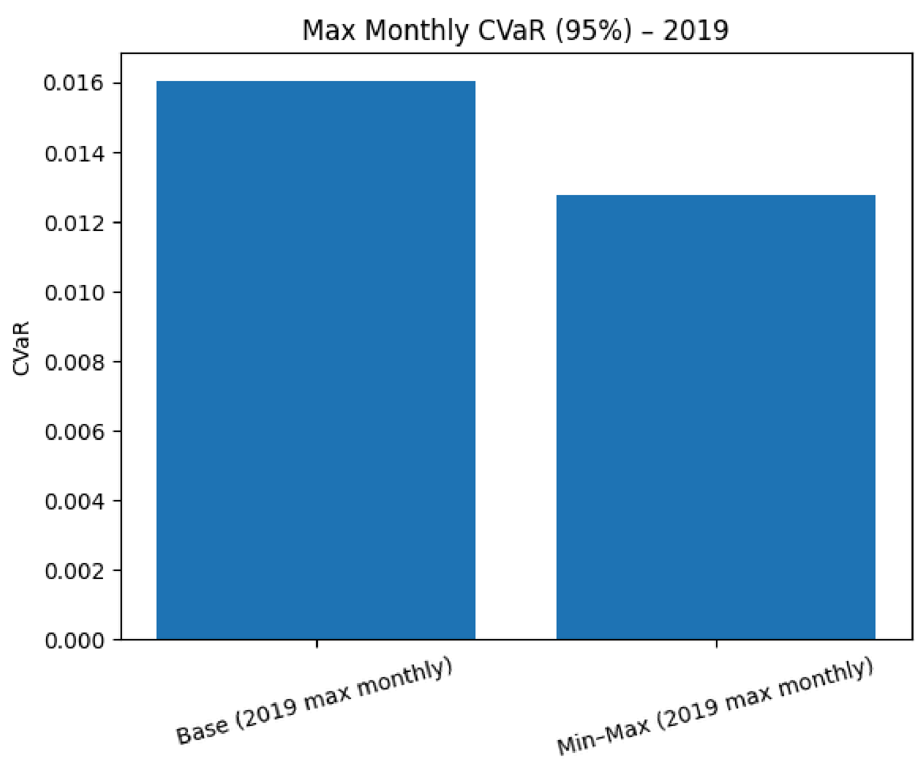
This conservative stance paid off in stress conditions. In 2020, the min-max portfolio delivered an out-of-sample CVaR of 0.0466, nearly the same as the base case but slightly better under extreme conditions. Importantly, both approaches outperformed the NDX benchmark CVaR of 0.0565, but the min-max method demonstrated the value of prioritizing downside protection. It highlights why managing the worst tail events is critical when market regimes shift as dramatically as they did between 2019 and 2020.



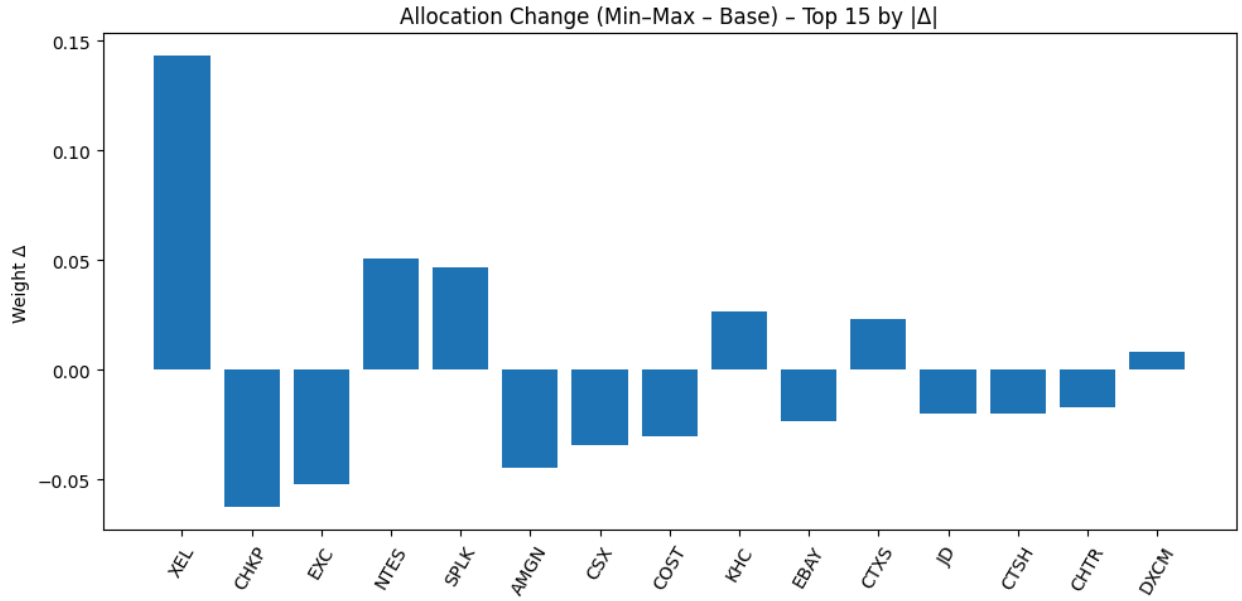
The base-case portfolio experienced larger swings in monthly tail risk, peaking near 0.016 in September 2019. The min-max portfolio smoothed these extremes, holding CVaR more consistently around 0.012-0.013. This illustrates how the min–max objective sacrifices some average performance to better control the single worst month's losses.



Both portfolios spiked sharply during March-April 2020, when the COVID-19 shock hit, with CVaRs above 0.10. After the crisis, tail risk declined and tracked closely for both approaches. This shows that in systemic downturns, even conservative strategies cannot avoid sharp increases in downside risk, but they still provide structure for recovery.



The worst-case monthly CVaR in 2019 was 0.016 for the base portfolio compared to 0.013 for the min-max portfolio. This confirms that the min–max objective achieved its design: it lowered the single worst month's tail losses at the cost of a slightly higher average CVaR across the year.



The min-max portfolio reallocated heavily toward defensive names, most notably raising XEL by ~15 percentage points. It reduced exposure to more volatile stocks like CHKP and AMGN. This shift demonstrates how the min-max approach concentrates in stable, low-volatility sectors to better protect against extreme tail losses.

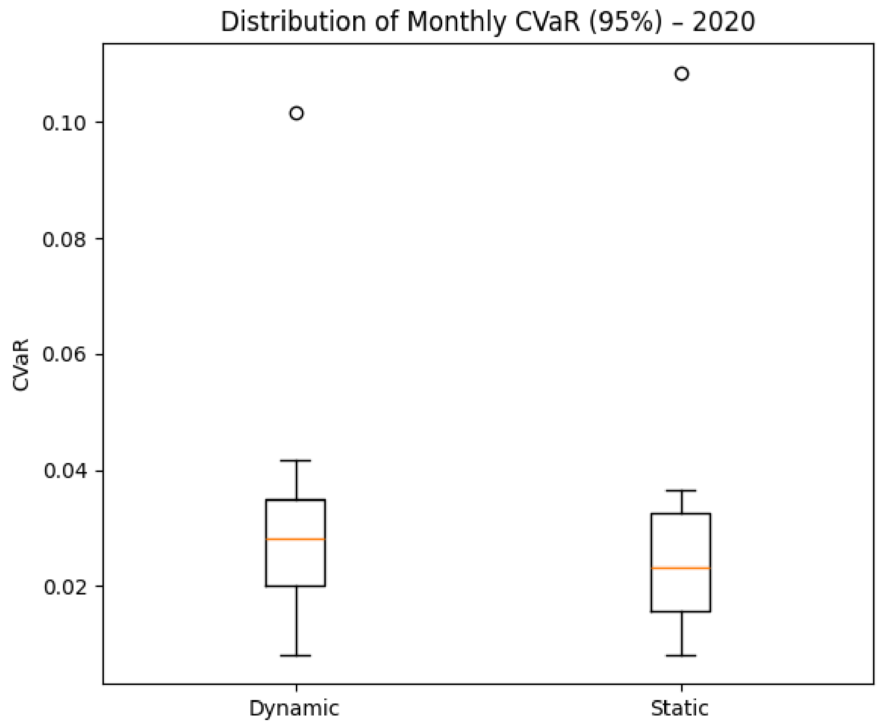
Dynamic Rebalancing

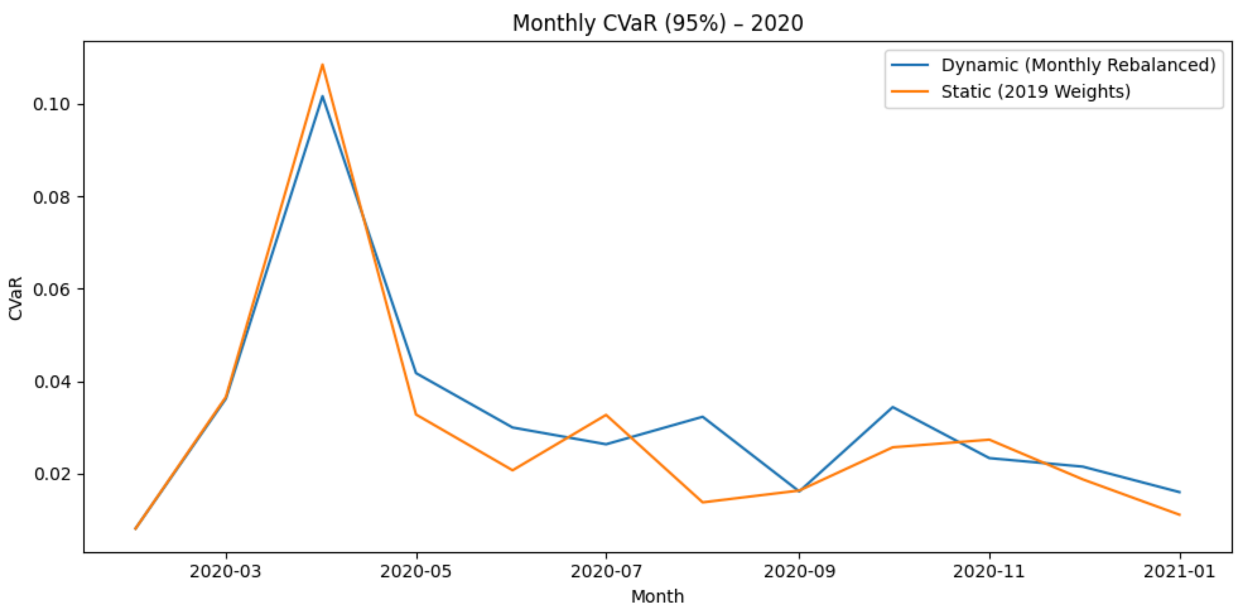
In the static allocation approach, a single set of portfolio weights is derived from the 2019 training data and then held fixed throughout 2020. While this provides a clean in-sample vs out-of-sample comparison, it ignores the fact that market conditions can shift drastically within a year.

To address this, we tested a dynamic rolling rebalancing strategy. For each month in 2020, the portfolio was re-optimized using the previous 12 months of returns as the training window. For example, the January 2020 portfolio was trained on returns from January-December 2019, while the February 2020 portfolio used February 2019-January 2020, and so on. This rolling approach produced a sequence of 12 monthly portfolios for 2020, each tailored to the most recent market information.

This setup allows us to:

- Capture evolving market conditions (earnings announcements, macro events, COVID crash).
- Evaluate monthly CVaR values instead of a single annual measure.
- Measure both average tail risk and its variability across months.



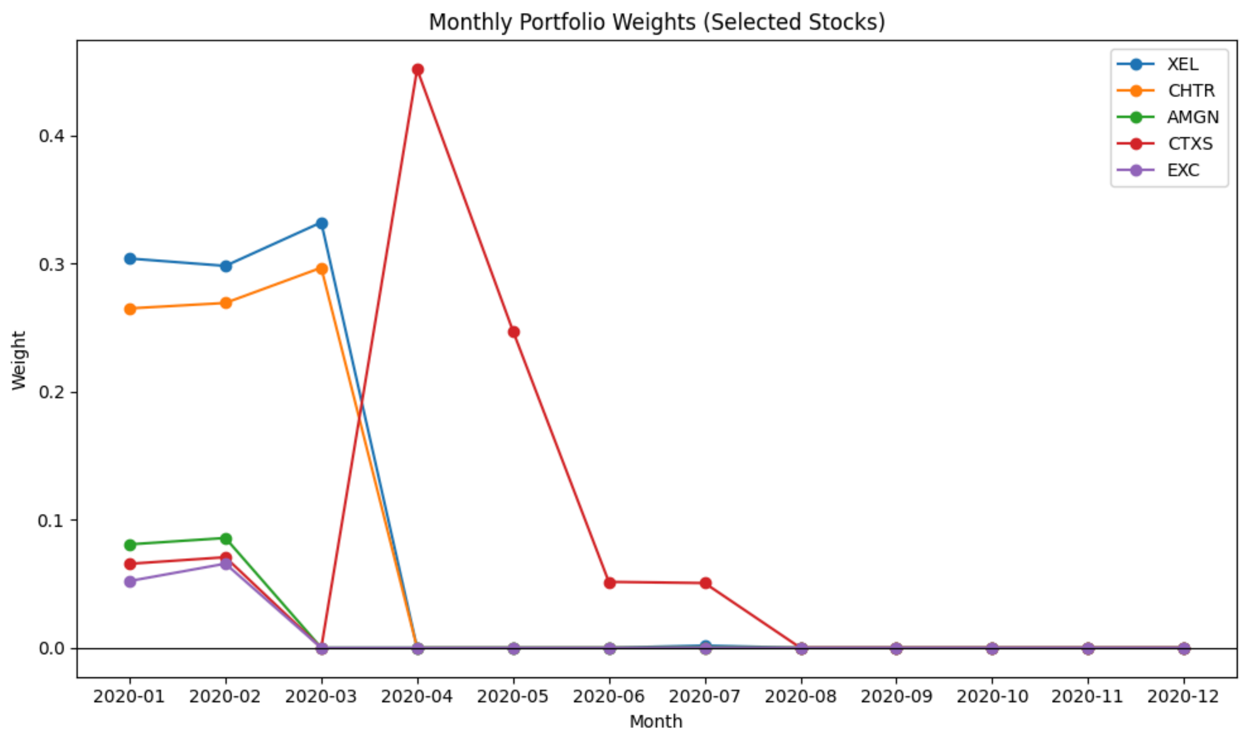


The results show that monthly rebalancing provided clear benefits relative to a static 2019 allocation. The average CVaR in 2020 was lower than the out-of-sample CVaR of the static portfolio, suggesting that the rolling strategy reduced exposure to extreme downside events. By examining CVaR month by month on the graph above, we gained insight into the volatility of tail risk over time. For instance, January 2020 appeared relatively safe with very low tail losses, while March 2020 experienced a dramatic spike in CVaR above 0.10, coinciding with the COVID-driven market crash.

The distribution of monthly CVaRs in the boxplots above further illustrates this volatility: the standard deviation of monthly CVaR was 0.024, showing meaningful variation across 2020. While the dynamic strategy generally maintained lower median risk than the static allocation, both approaches were still vulnerable to systemic shocks. Overall, these results confirm that re-optimizing monthly is worthwhile. It not only lowered the average tail loss but also revealed the instability of market risk from month to month. While frequent rebalancing introduces transaction costs in practice, from a risk management perspective it delivers a more responsive and resilient portfolio than a static allocation.

Portfolio Stability

The sequence of monthly portfolios we generated in 2020 was not stable. Several stocks exhibited changes in allocation greater than 5 percentage points (0.05) from one month to the next, which means the portfolio composition was shifting more aggressively than what might be practical in a real-world setting. Frequent large reallocations could drive up transaction costs and increase turnover, making the strategy less feasible for implementation

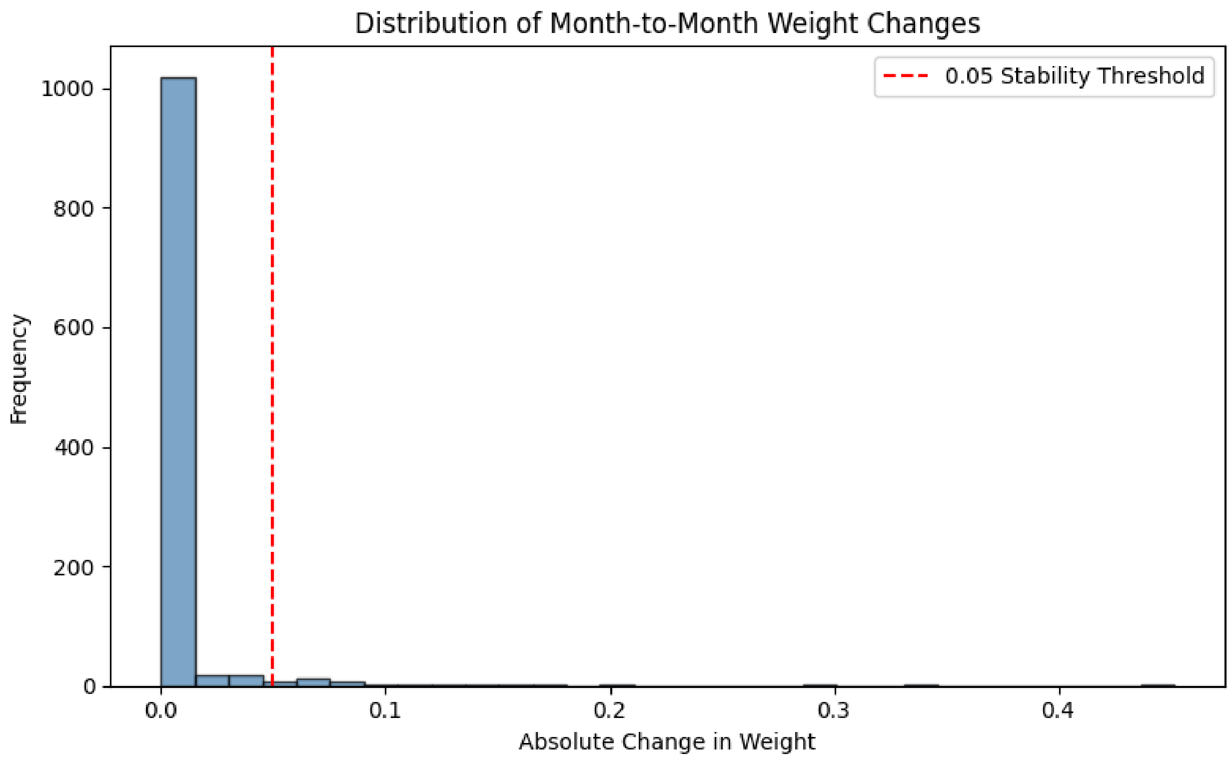


The weight paths of key holdings (XEL, CHTR, AMGN, CTXS, EXC) show that allocations were highly unstable from month to month. These swings are far larger than the 0.05 stability threshold, confirming that the unconstrained rebalancing process produced volatile and impractical allocation changes.

To address this, we introduced a stability constraint into the optimization model. The approach was straightforward: for each month's optimization, we carried forward the weights from the previous month's portfolio and enforced an additional restriction in the Gurobi model.

Specifically, each stock's weight in the current month could not deviate more than ±0.05 from its allocation in the prior month.

This formulation ensures that month-to-month portfolio changes remain gradual, producing a more stable allocation path. While the stability constraint may slightly reduce the optimizer's ability to minimize CVaR in the short term, it better balances risk management with portfolio turnover, making the strategy more realistic for practical asset management.



The histogram of absolute weight differences further illustrates the instability. While most weight changes clustered near zero, there is a noticeable tail of changes well beyond the 0.05 threshold (shown by the red dashed line), with some reallocations exceeding 0.3–0.4. This demonstrates that several positions experienced abrupt, outsized adjustments, which would create excessive turnover in practice.

Conclusion

The base-case portfolio (β = 0.95) demonstrated strong risk reduction relative to the benchmark NDX, but it was also highly concentrated in a few defensive names and performed inconsistently when exposed to new conditions in 2020. This highlighted the non-stationarity of financial markets and the dangers of relying on a static allocation across years

Sensitivity testing showed that higher confidence levels ($\beta = 0.99$) increase protection against extreme tail events, though at the cost of diversification. An alternative min–max objective further strengthened resilience by controlling the single worst month's CVaR, underscoring the importance of preparing for extreme downside scenarios

Dynamic monthly rebalancing proved especially valuable. By retraining on rolling 12-month windows, the strategy lowered the average tail loss in 2020 and captured the volatility of tail risk from month to month. Although this approach introduces higher turnover, it provides a more adaptive and resilient portfolio that adjusts as conditions evolve. Adding stability constraints, which limit weight changes to ±5% month over month, strikes a balance between responsiveness and practicality, ensuring smoother transitions without excessive reallocations.

Final Recommendation: Implement a CVaR-minimization strategy at $\beta = 0.95$–0.99, with regular rebalancing (monthly or quarterly) and stability constraints on position changes. This approach reduces tail risk, adapts to shifting market regimes, and ensures operational feasibility by keeping turnover manageable. Overall, this framework provides the best balance between downside protection, adaptability, and portfolio stability, making it a practical strategy for constructing low-risk portfolios going forward.