Gale-Shapley Algorithm

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Gale-Shapley Introduction

In the early 1960's David Gale and Lloyd Shapley proved that, for any equal number of participants of each type, it is always possible to find a matching in which every pair is stable, and presented an algorithm that would do so. This algorithm is used to solve what is known as a stable matching problem or SMP. An SMP is the problem of finding stable matching between two equally sized sets of elements given an ordering of preferences for each element.

Gale-Shapley Introduction

The Gale-Shapley algorithm would first be put to practical use in the 1950’s in the National Resident matching program, which was a government program created to place medical students into residency training programs within the US. Since then this algorithm has had many uses for pairing things like: employees and employers, schools and applicants, organ donors and those that need them, college admissions and even online dating

Gale-Shapley Algorithm Intuition

This algorithm models a scenario where elements of each group have a preference to whom they’re matched with(eg organ donors). And the goal of this algorithm is to find a stable matching where no two individuals would prefer to be matched with each other over their current partners. This is achieved by aribairtaily pairing individuals and then iteratively improving them based on the preferences of each individual. Also the matching elements from each set are bijections.

Gale-Shapley Algorithm Pseudocode

in VScode

Gale-Shapley Algorithm Run-Time

The worst case scenario for this algorithm is if each donor has to be proposed to each patient resulting in a run time of O(n^2) . However due to the algorithm terminating when a stable match is found the worst case scenario is often not the average runtime, and with the addition of ordering the preference lists we can cut down the average runtime to O(nlogn) which still isn’t great but much more practical than O(n^2).

References

https://en.wikipedia.org/wiki/Gale%E2%80%93Shapley_algorithm