

# Electrical Engineering

## Quals Questions

**1999**

To: Diane Shankle <shankle@ee.stanford.edu>  
Subject: Re: Quals Questions  
Date: Fri, 15 Jan 1999 16:58:26 -0800  
From: "Mary G. Baker" <mgbaker@gunpowder.stanford.edu>

Diane,

65

I already sent this to John Gill, but just in case I should send it to you as well, here it is.

Thanks,  
Mary

-----  
Question:

Assume a new type of memory has been invented. This memory is very cheap and usually extremely fast -- so fast that it's comparable to accessing a register on the CPU. Unfortunately, occasionally and entirely unpredictably, an access (read or write) to the memory is extremely slow -- as slow as a disk access. Overall, the average of the access times is faster than DRAM.

Assume you are designing a system and have replaced all the main memory in the computer with this new memory. What are some of the problems and issues that could come up with applications and the operating system as they run on this new system?

What are some of the techniques you might use to address the problem?

Answer:

The key issue is the unpredictability of the memory. Real-time applications would suffer since they could hit these slow accesses and fail to make their deadlines. The OS scheduling these real-time applications could also hang and fail to schedule them appropriately. Batch applications wouldn't suffer so, since overall access times are faster.

The OS itself could have many problems. For instance, getting such a slow access in a critical section, when you can't context switch would be a problem and result in inefficient use of the system as we stalled waiting for the access to complete. Any operations in the OS with timeouts would be a problem: responding to interrupts from peripheral devices such as disks or the network, etc. TCP would have problems if the network buffers used this sort of memory, since a receiver might stall while accessing data in the buffers and fail to send an acknowledgement in time. This would cause potentially unnecessary retransmissions of data.

There are also some pipelining issues that people got plus points for bringing up.

To fix the problem we might try in general to make it possible to context switch a process that stalled on one of these accesses. That assumes it is possible to determine one of these slow accesses and that it is possible to access the memory elsewhere while waiting for this other access to complete.

Or, if it is possible to preempt an access, we might try to determine in some out-of-band way when a slow access was occurring and then just re-poll that location hoping the next access would be fast.

Or, since the memory is cheap, we might have redundant banks of memory which we access in parallel. This would involve some extra complexity, since if we write to the two banks and one stalls, we need to know from which bank to perform a read on that data if the other write hasn't finished.

Overall, ~~I wouldn't want most of the OS itself running in this memory, since it could stall executing or accessing anything.~~

How I Graded the Exam:

In general, I looked for how many of these issues and topics the examinee brought up without my prompting. I gave more hints to those without an OS background and took their lack of background into account. I also explained the necessary features of what an OS does to these people at the beginning of the exam. The more hints and prompting, though, the less well an examinee did. Saying things that were backwards or otherwise wrong reduced the score.

candidates A, B, C, and D are ranked by 10 evaluators:

	1st	2nd	3rd	4th
eval. 1	A	C	B	D
eval. 2	B	A	C	D
:	:	:	:	:
eval. 10	A	B	D	C

To form a composite ranking, each candidate is given 1 point when ranked 1st, 2 points when ranked 2nd, etc. The candidate with the lowest total was taken as 1st in the composite ranking, the one with the second lowest total was taken as 2nd, etc. The resulting composite ranking is: A,B,C,D (say).

**Question:** What happens if we use different weights to determine the composite ranking? For example, we give 0.7 points when a candidate is ranked 1st, 1.3 points when ranked 2nd, etc.

It is possible to get the same composite ranking (ABCD) no matter what weights are used? How would you know if this occurs?

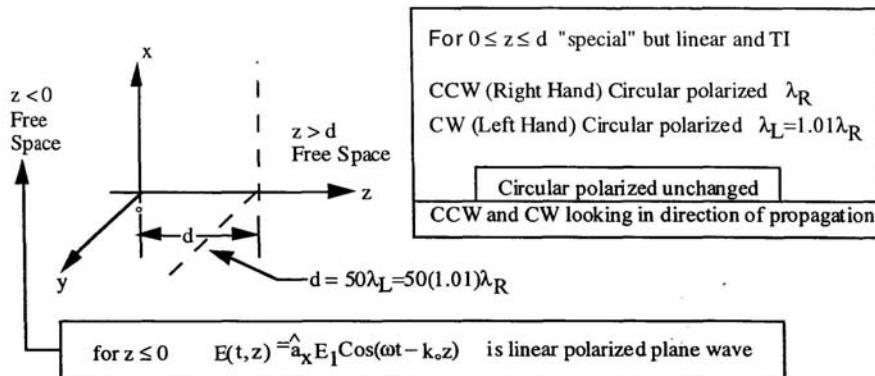
Discuss.

1999 Ph.D. Qualification Exam Question

Donald C. Cox  
January 1999

*Signal*

At the top of the whiteboard in the room was the following information:



I started by noting there was a problem on the board that I would describe and then the student could tell me something about the questions I will ask.

The description included:

- Pointing out the x, y, z coordinate system and noting it was a right hand coordinate system (coordinate system was in red color).
- Noting that for  $z < 0$  was free space and in the  $z < 0$  region was propagating a wave described by the equation (equation was in green). Noted that the wave was a linearly polarized plane wave with E field oriented in the x direction and was propagating as  $\cos(\omega t - k_0 z)$
- Noting that for  $z > d$  was again free space.
- Noting that for  $z$  between 0 and  $d$  was a "special" medium that was a little "strange" but was linear and time invariant. Noted that the propagation for  $0 \leq z \leq d$  was described in the box (which was in blue). Noted that circular polarizations propagated through the medium unchanged. Noted that counterclockwise (CCW) or right hand circular polarized waves propagated in the +z direction with a velocity that resulted in a wavelength  $\lambda_R$ . Noted that

clockwise (CW) or left hand circular polarized waves propagated in the +z direction with a different velocity that resulted in a wavelength  $\lambda_L = 1.01\lambda_R$

•Noting that CCW and CW polarizations were defined looking in the direction of propagation, that is in the +z direction.

•Noting that the other information needed was the distance d expressed in terms of the wavelengths and that  $d = 50\lambda_L = 50(1.01)\lambda_R$

- a) I stated that for the first question we would assume that the medium was lossless and we could assume other things if needed. The student was then asked what the polarization was of the wave that exited the medium at  $z = d$  propagated in the free space for  $z > d$ .

Solution:

•decompose the linearly polarized wave incident at  $z = 0$  into two orthogonal circular polarized waves (CCW and CW) in terms of x and y components (90° out of phase). ✓

•calculate the phase shift  $kd$  through the medium for CCW and CW polarizations.  
Note:  $d = 50\lambda_L$  was chosen to be no phase shift.  $d = 50(1.01)\lambda_R$  was chosen to be  $0.5\lambda_R$  or a  $\pi$  phase shift that reverses the sign of the CCW polarized wave components.

•adding the CCW and CW waves at  $z = d$  then results in a y directed linearly polarized wave.

•note: in the few cases where a student noted concern over reflections at  $z = 0$  or  $z = d$  I stated: assume no reflections at these boundaries (recall it is a "strange" medium).

About 40% of students completed this part of the exam. Another 10% nearly completed it, and 10% made good progress on it, i.e. decomposed the linearly polarized wave ok or described doing it; calculated the phase shift, etc.

About 20% made some progress; e.g. stated what needed to be done but couldn't do it; or wrote descriptions of the circular polarized waves but didn't see the decomposition, etc. The last 20% made no progress, even with several hints. Points were given for reasoning and deducing how to proceed either without or with hints.

- b) The problem was then changed to have the same total attenuation for the CCW and CW waves over the distance  $d$ . The student was asked what then was the polarization of the wave exiting the medium at  $z = d$  and what was the attenuation of the exiting wave in terms of the attenuation of the CCW and CW components. Answer: By inspection, same y directed linearly polarized wave attenuated by the same amount as each component.

About 20% of the students progressed this far.

- c) Again for the lossless case what was the polarization at  $z = \frac{d}{2}$  and describe the polarization change as the wave propagates from  $z = 0$  to  $z = d$ .

Answer: By inspection of the equations it can be seen that at  $z = \frac{d}{2}$  the wave is linearly polarized at  $45^\circ$  to the x and y axis and that as the wave propagates from  $z = 0$  to  $z = d$  the polarization remains linear and rotates smoothly from x direction to y direction.

If time was almost up this c) part was omitted. About 10% of students progressed this far and gave the correct answer.

- d) The student was then asked to discuss qualitatively the polarization of the wave at  $z = d$  if the CCW and CW components had different attenuations. Answer: Wave at  $z = d$  is elliptically polarized.

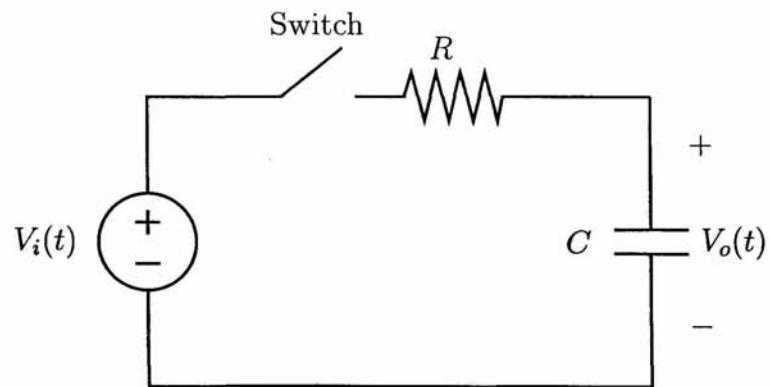
About 10% of students answered this question.

- e) The student was then asked what the polarization of the wave would be at  $z = d$  if one of the circular components was not attenuated and the orthogonal component was totally attenuated at  $z = d$ . Answer: Wave at  $z = d$  would be circularly polarized.

The question was then extended to ask what would be the power in the wave at  $z = d$  for this condition compared to the power at  $z = 0$ . (Could be clarified to be the Poynting vector at  $z = d$  compared to Poynting vector at  $z = 0$ ). Answer: Power in wave at  $z = d$  would be half that at  $z = 0$ .

Again, about 10% of students completed all parts of the question and reasoned well in providing answers.

2. Consider the RC circuit in the figure. The input voltage  $V_i(t)$  is a white noise process with power spectral density  $N$ . The switch closes at time  $t = 0$ . Find  $E(V_o^2(t))$ , for  $t \geq 0$ , assuming the capacitor was uncharged before  $t = 0$ .



$$\text{skin depth } \delta = \left( \frac{\omega}{\sigma \mu_0} \right)^{1/2}$$

$$\text{Good conductor approximation } \frac{\sigma}{\omega \epsilon} \gg 0$$

Electromagnetism

## OFFICE MEMORANDUM ♦ STAR LABORATORY

January 23, 1999

To: Diane Shankle

From: Tony Fraser-Smith

Subject: Ph.D. Quals Question, 1999

### Penetration of Low-Frequency Electromagnetic Fields

The student is presented with a thin sheet of plastic insulation and asked about its electric and magnetic properties. He/she comes up with, or is led to, an electrical conductivity  $\sigma = 0$ , electrical permittivity  $\epsilon = \epsilon_0$  (free space), and magnetic permeability  $\mu = \mu_0$  (free space).

$$\omega \rightarrow 0$$
$$\sigma \rightarrow \epsilon$$

A strong horseshoe magnet is produced; the steel keeper is removed, and placed on the desk. The piece of plastic is placed over the keeper and the student asked if the magnet will attract the keeper through the plastic sheet. In the subsequent discussion the student is asked about the skin depth ( $\delta$ ) and inevitably he/she can write it down as  $\delta = \sqrt{2/(\omega \mu \sigma)}$ , where  $\omega$  is the angular frequency,  $\mu$  is the permeability, and where  $\sigma$  is the electrical conductivity. We discuss the significance of each of the factors in the expression for  $\delta$  and its applicability. At this stage we will probably discuss the "good conductor" approximation  $\sigma/\omega \ll 0$  and how it applies at low frequencies (particularly very low frequencies). Needless to say, applicability of the good conductor approximation is a prerequisite for a material to have a skin depth.

The magnet is now brought up to the plastic sheet and it is seen that the keeper is strongly attracted through the plastic. This experimental result is discussed.

Next, a sheet of aluminum is produced and its electrical properties discussed (it is a good conductor, so  $\sigma$  is relatively large;  $\mu \approx \mu_0$ ;  $\epsilon \approx \epsilon_0$ ). Is the keeper attracted through the aluminum sheet? After the student arrives at an answer, a test is carried out and it is found that the keeper is attracted. A similar test is carried out with a sheet of copper, one of the best metallic conductors. Once again the keeper is attracted strongly through the metal. These results are then discussed in the context of the skin depth equation above. At this stage it is concluded that the low value of the frequency (in fact  $\omega \approx 0$ ) must be a crucial factor. For low frequency, skin depth is high

A thin sheet of steel is produced. The student is asked about its properties in the context of the skin depth equation. He/she is expected to come up with, or is led to, an electrical conductivity  $\sigma \approx \sigma_{\text{copper}}, \sigma_{\text{aluminum}}$ , electrical permittivity  $\epsilon \approx \epsilon_0$  (free space), and magnetic permeability  $\mu \gg \mu_0$  (free space). Will the magnet attract the steel keeper through the steel sheet? At this time the student is expected to fret over the fact that the high conductivity and permeability values will reduce the skin depth but the frequencies involved are still extremely small thus keeping the skin depth large. Given the previous results, it is hoped that the conclusion will be that the keeper is attracted. Experiment shows that the keeper is strongly attracted.

Finally, the student is asked if there was any means for preventing a low-frequency or DC magnetic field from penetrating through a material. High conductivity/high permeability materials might be briefly discussed, but it is hoped that reference will be made to the use of a superconducting material.

X-Sender: hector@db.stanford.edu  
Date: Tue, 27 Jul 1999 10:49:49 -0700  
To: shankle@ee.Stanford.EDU  
From: Hector Garcia-Molina <hector@CS.Stanford.EDU>  
Subject: my qual question  
Cc: siroker@DB.Stanford.EDU  
Mime-Version: 1.0

CS

EE QUALIFYING EXAM 1999  
Hector Garcia-Molina

Write (in any programming language, or in pseudo-code) a procedure for binary search. Assume you have an array of integers  $X[i]$  where "i" can range between 1 and N. The input to the procedure is an integer value "v" that you are searching for. The output is (1) a flag "found" which is true if "v" was one of the values in X, and (2) if flag is true, the value "k" such that  $X[k]=v$ .

Sort the table first

```
typedef struct
{
    int *x;
    int order;
} out;

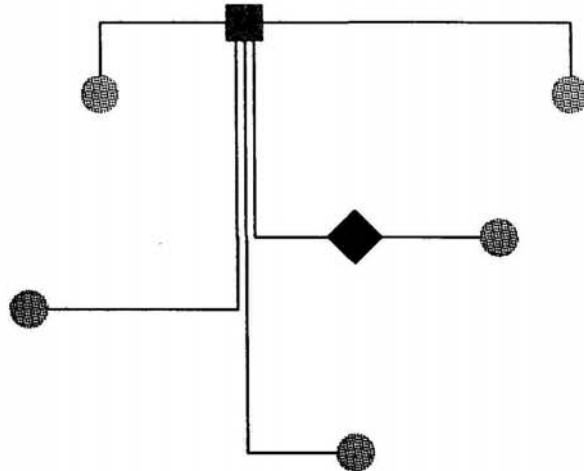
int SortSeq( int *pdwIn , OUT *po );
```

4:28

## 1998-1999 Qualifying Examination Question

JOHN GILL

A set of nodes (blue circles in the figure below) must be connected to a power node (red square) using separate wires that run horizontally or vertically.



Where in general should the power node be placed to minimize the *total* wire length?  
(The location shown in the figure above is *not* optimal.)

### Answer

Suppose that the  $i$ -th blue node is at location  $(x_i, y_i)$ . If  $(x, y)$  is the location of the red node, then the total wire length is

$$\sum_{i=1}^n (|x - x_i| + |y - y_i|) = \sum_{i=1}^n |x - x_i| + \sum_{i=1}^n |y - y_i|.$$

The two sums can be independently minimized; that is, the  $x$ -coordinate of the best location for the power node depends only on the  $x$ -coordinates of the blue nodes, and similarly for the  $y$ -coordinates. We can assume that  $x_1 \leq x_2 \leq \dots \leq x_n$ . If  $x_k \leq x \leq x_{k+1}$  then the total  $x$  cost is

$$\sum_{i=1}^k (x - x_i) + \sum_{i=k+1}^n (x_i - x).$$

The derivative of this piecewise-linear function is  $k - (n - k) = 2k - n$ . The derivative is negative if  $k < n/2$  and is positive if  $k > n/2$ . Thus the  $x$  cost is minimized by locating  $x$  at the *median* of the  $x$ -coordinates, where an equal number of  $x_i$  are less than  $x$  and greater than  $x$ . Similarly, the best  $y$  location is the median of the  $y$ -coordinates. The optimum power node location for the figure above is the green diamond. (If  $n$  is even then all values of  $x$  between  $x_{n/2}$  and  $x_{n/2+1}$  have the same cost.)

**QUESTION 1:**

The line and the capacitor are charged to 5 V.  
Switch is closed at  $t=0$ .  
Sketch  $V_L(t)$  for  $t>0$ .

**ANSWER:**

$$V_1^-(z,0) = V_L(0^+) - V_L(0^-) = V_L(0^+) - 5$$

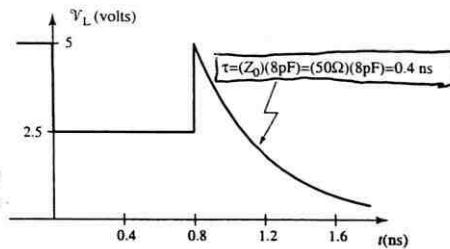
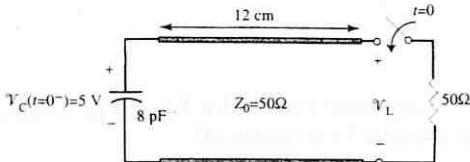
$$I_1^-(z,0) = I_L(0^+) - I_L(0^-) = I_L(0^+)$$

$$V_L(0^+) = (50\Omega)I_1(0^+) = (50\Omega)I_1^-(z,0) = -(50\Omega)V_1^-(z,0)/Z_0 = -V_1^-(z,0)$$

$$V_1^-(z,0) = -V_1^-(z,0) - 5$$

$$\boxed{V_1^-(z,0) = -2.5 \text{ V}}$$

Since the disturbance propagates in the  $-z$  direction



Although the formal solution is as given above, one can solve this problem simply by considering the fact that when the switch is closed, the 5 V is divided between the  $50\Omega$  load and  $Z_0$ , resulting in the launching of  $-2.5\text{V}$ , which then travels to the left and reflects (with  $\Gamma=-1$ ) from the capacitor, the voltage on which cannot change instantaneously.

One can also use superposition and think in terms of two voltage sources, one  $-5\text{V}$  and another  $+5\text{V}$ , initially in series with the  $50\Omega$  resistor. When the switch is closed the  $+5\text{V}$  is matched by the  $+5\text{V}$  voltage of the already charged line, while the  $-5\text{V}$  is divided between the  $50\Omega$  load and  $Z_0$ .

WEY!

**QUESTION 2:**

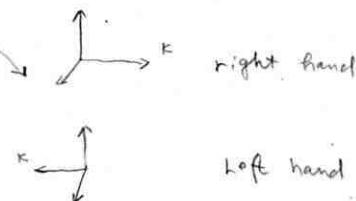
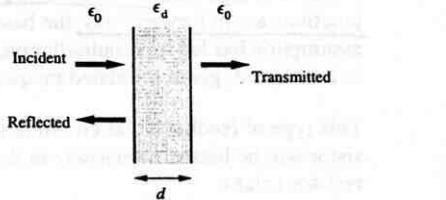
The incident wave is right-hand circularly polarized.  
What is the polarization of the reflected wave?

Transmitted wave?

**ANSWER:**

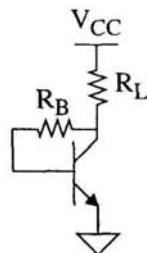
The polarization of the reflected wave is left-hand circular, regardless of the values of  $\epsilon_d$  or  $d$ . Under normal incidence, both the perpendicular and parallel components of the electric field are reflected in the same manner -- the sense of polarization changes simply because of the change in propagation direction.

The transmitted wave is right-hand circular, exactly like the incident wave.



- 1) In the following circuit, assume that  $V_{CC} > V_{BE} > V_{CEsat}$ . Under what conditions, if any, will the transistor be in saturation?

FIGURE 1. Circuit for Problem 1

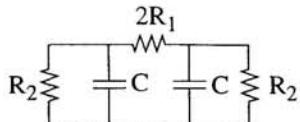


**Solution:** This problem can be solved any number of ways. Perhaps the most expedient is a *reductio ad absurdum* argument. First assume that the transistor is in saturation. We are given that  $V_{BE} > V_{CEsat}$ , so the base voltage would exceed the collector voltage with our assumption of saturation. Now, the current through  $R_B$  is the base current, and would be negative in sign here. Because saturation for bipolars occurs when both the B-C and B-E junctions are in forward bias, the base current must be positive in saturation. Because our assumption has led to a contradiction, it is wrong. Therefore, the transistor *never saturates* in this circuit, given the stated inequalities.

This type of feedback bias circuit is versatile precisely because it *guarantees* that the transistor will be biased *somewhere* in the forward active region of operation, independent of resistor values.

2) Provide expressions for the pole(s) of the following symmetric RC network:

FIGURE 2. *RC* network for Problem 2



**Solution:** As in the previous question, this one may be solved a number of ways. By far the fastest is to remember that a pole is the (generalized complex) frequency of a signal that can exist in a network **in the absence of any input**. That is why no input or output is shown: the poles of a system do not depend on how we define our inputs and outputs (but zeros do); the denominator polynomial and its roots are always the same.

Given that understanding, observe that we have two independent energy storage elements. Therefore, there will be two poles. To find them, let's figure out a way to deduce the pole frequencies by giving the capacitors some initial charge and watching how the network relaxes. By definition, the frequencies with which the system's state evolves are the pole frequencies, because there is no input. If we're clever about the choice of initial conditions, we might be able find the poles with very little work. That's where the symmetry comes into play.

Suppose we give both capacitors the same initial charge. Then, by symmetry, however the network relaxes, the voltage across  $2R_1$  must remain zero. We may therefore remove (open- *or* short-circuit) that resistor. Clearly, one pole's time constant is then simply  $R_2C$ .

Since a common-mode initial condition was helpful in discovering one pole frequency, perhaps a differential-mode one will give us the other. Specifically, let one capacitor have an initial voltage  $V_0$  and the other have  $-V_0$ . Again by symmetry, however the network relaxes, it must do so in such a fashion as to leave the midpoint of  $2R_1$  at the same potential as the bottom node of the network (you may call it ground if you wish). We may thus again decouple the left from the right. In this case, each of the resulting decoupled subcircuits has  $R_1$  in parallel with  $R_2$ . The other pole's time constant is therefore  $(R_1 \parallel R_2)C$ .

If all of this "common-mode" and "differential-mode" stuff seems foreign, the problem is still readily solved by, say, squirting in a current into the left of  $2R_1$ , and measuring the voltage across the right half circuit (this is only one of several suitable choices). Find the roots of the denominator polynomial for that transfer function, and you're done. It's only second-order, so it's do-able in the given time.

A common error was to apply a *voltage* source to the left half circuit when trying the transfer-function approach. This choice makes irrelevant the left half circuit (the Thévenin equivalent resistance of a voltage source is zero ohms!), leading to a single-pole network whose pole frequency does not correspond to a pole of the original network.

A space craft is talking to an astronaut. They are separated by 100 miles and 1 watt is just enough to get a good voice signal from Astronaut to Station. The astronaut uses an omni directional antenna the Space station uses a 10 foot parabolic dish. The radio frequency is 1GHz. How much power is required if:

- a. The space station uses a 3 foot dish instead of the 10 foot dish?
- b. The space station moves from 100 miles to 1,000 miles from the astronaut?
- c. The frequency is changed from 1GHz to 10GHz?
- d. The voice is transmitted digitally rather than analog? The digital requires 10kb/sec and 5dB Eb/N<sub>0</sub> and the analog required 30Khz bandwidth and 10dB C/N.

The explanations of a, b, and c illustrated understanding of the link equations; d. illustrated modulation performance.

Bruce Lusignan

Electromagnetics

$$\frac{P_R}{P_T} = \frac{\eta G_T G_E \lambda^2}{4\pi R^2} \quad G_T = 4\pi \frac{A_T}{\lambda^2} \quad \Rightarrow \frac{P_R}{P_T} \propto \frac{1}{\lambda^2}$$

only if both side use directional antenna, because if one side use omni-directional

> Analog:  $30 \text{ kHz} * 10 \text{ dB C/N} = (10 \text{ KB}) + 2 \text{ dB} + 10 \text{ dB}$   
 digital:  $10 \text{ kb/sec} * 5 \text{ dB E}_b/N_0 :$

Convert both to dB. analog will require 7 more dB.

Date: Mon, 18 Jan 1999 13:13:36 -0700 (PDT)  
From: PIERO PIANETTA AT SSRL <pianetta@SLAC.Stanford.EDU>  
Subject: Re: Quals Questions  
To: shankle@ee.stanford.edu  
MIME-version: 1.0

## Electronic Devices

Diane,

Here is my quals question:

Part A. How do the properties of an MOS device change if the band gap of the oxide is varied. What would you need to know if you were asked to design a MOS device with an insulator whose band gap was to be kept at a minimum value.

For very large gaps, tunnelling is reduced. For small gaps, there is increased gate leakage. Expected the discussion to use the band diagram and show how it was affected by a change in the gap. Would at least need to know operating voltage and temperature.  $\star$  Do not forget temperature.

Part B. Explain why the oxide in a MOS device is thinner when the lateral dimensions of the device are made smaller. What is a major problem with thin oxides? What are some theoretical ways of solving this problem if you are allowed to vary any property of the gate dielectric?

Expected a discussion of basic scaling such as wanting to keep the electric field constant and then showing how capacitance and threshold voltage scaled with oxide thickness. Problem with thinner oxides is tunneling. Can solve this by either going to infinite bandgap or by increasing the thickness of the dielectric and then solving the resulting problems with lowered capacitance and increased threshold voltage by increasing the dielectric constant of the oxide.  $\star$  ?

Basic scaling: to keep electric field constant.

## What question

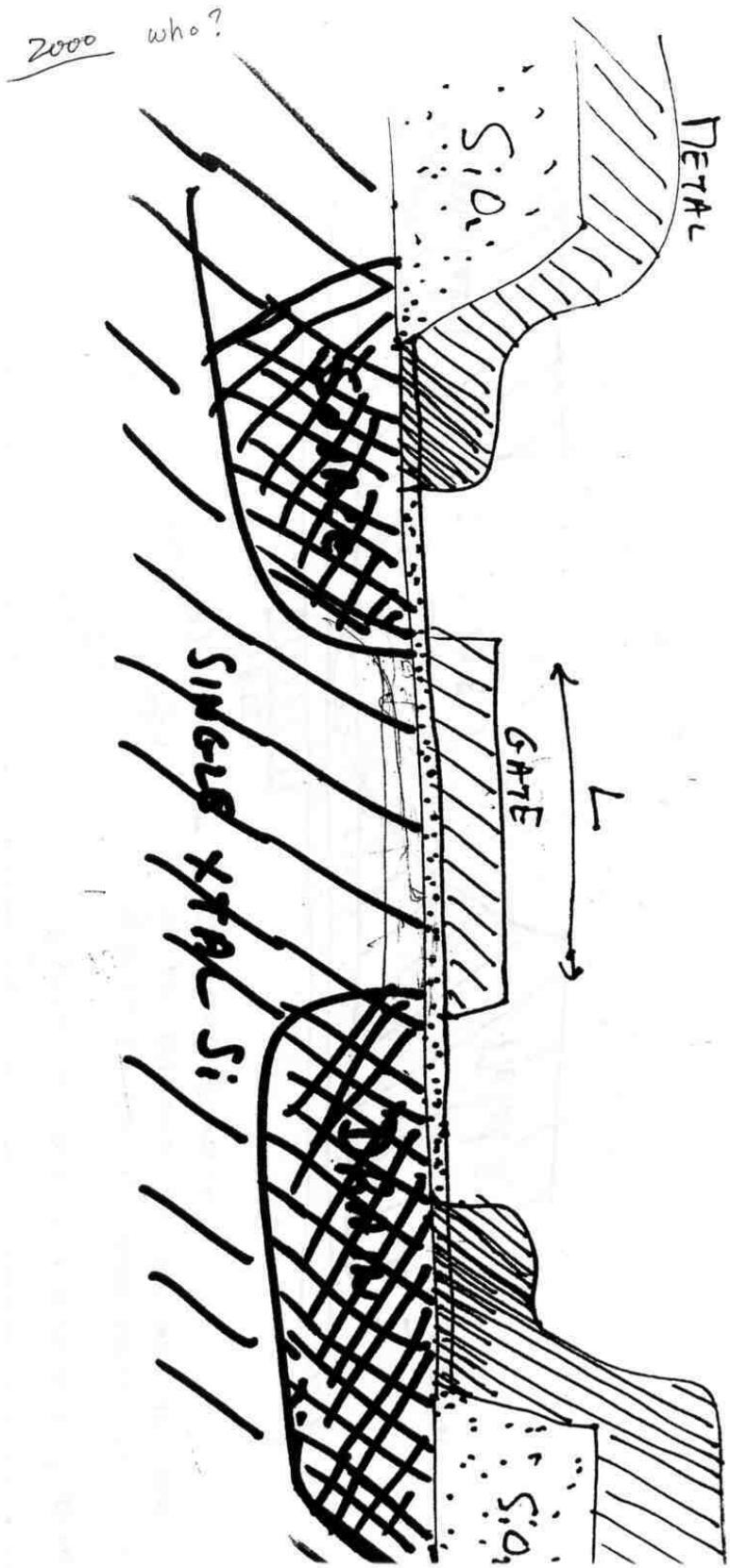
*Computer Architecture*

Which type of processor will benefit most from each feature?  
Explain your reasoning.

	Embedded/ DSP	Desktop	Super- computer
Large # of FUs			
Short displacements and few GF registers			
High data memory bandwidth			
High DRAM fill frequency			
Small die size			
On chip data cache			
High clock frequency			
Dynamic register renaming			
Integer vector support			
Precise interrupts			
VLIW ISA			
Programmer managed data buffers			
Multiple threads of control			
Global branch correlation			

WHAT DEVICE IS THIS?  
WHY IS THE STARTING MATERIAL : THE SEMICONDUCTOR,  
SINGLE CRYSTAL?  
SILICON?

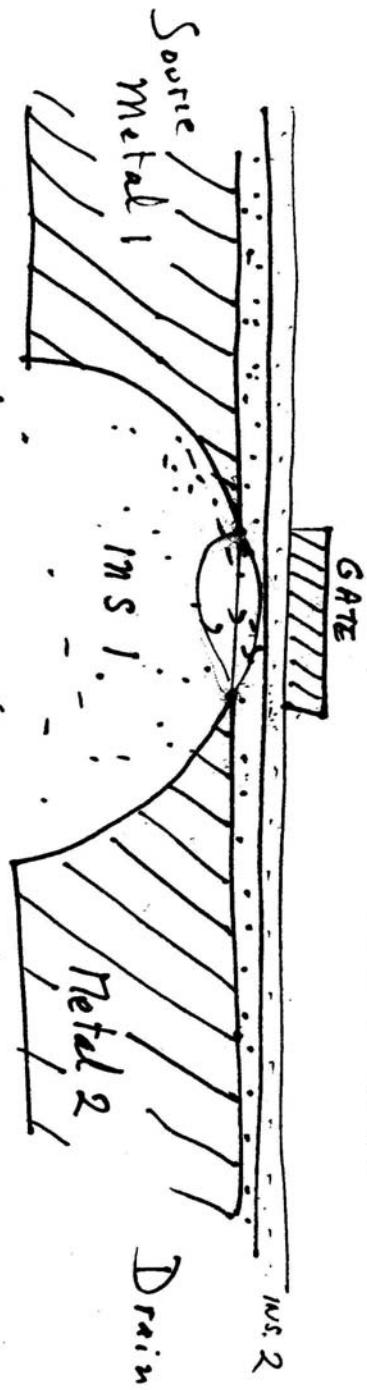
Fig 1



BE HIR OF THE SINGLE X-TAL STATE POINT. THE DEVICE BELOW (NO SEMI CONDUCTOR, A NO SINGLE X-TAL) HAS BEEN PROPOSED AS AN ALTERNATIVE THAT CAN BE GROWN IN A 3-D STRC

1) WOULD YOU FUND SUCH A PROPOSAL? IF NOT,

2) IF NEW DIA FUNDS IT WHICH ISSUE WOULD  
TO BE RESOLVED AT THE END OF 17EOK TO  
CONTINUED FUNDING?

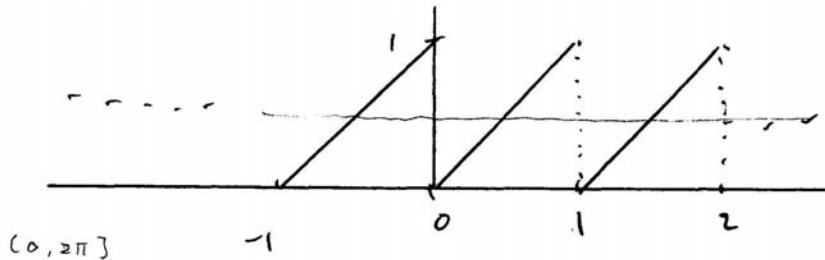


\* FROM THE POINT OF VIEW OF A FUNDING AGENCY.

EE Qualifying Exam  
Winter Quarter 1999

- Does the 'saw tooth' signal drawn below have a Fourier transform? Does it have a Fourier series? Please explain.
- Use one or the other to show that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$



$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx dx$$

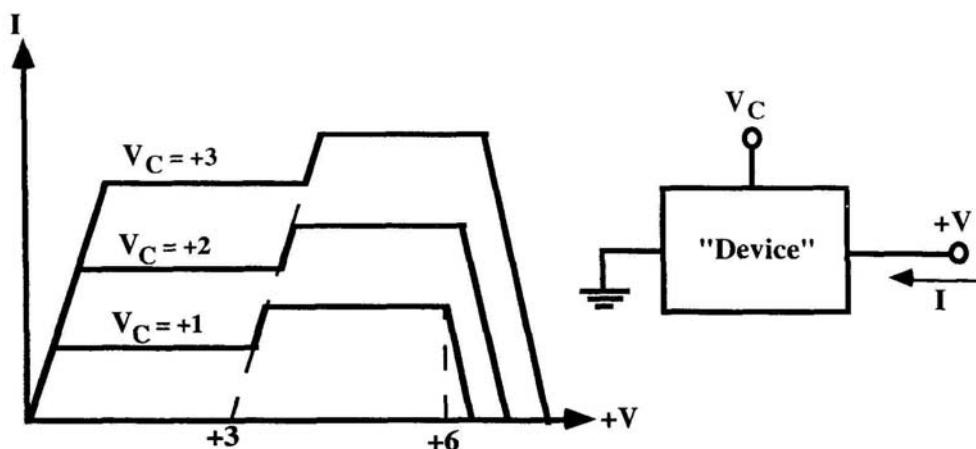
$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx dx$$

$\left[\frac{1}{2}, \frac{1}{2}\right]$

$$a_n = \omega \int_{-\frac{1}{2}}^{\frac{1}{2}} f(x) \sin 2\pi n x dx$$

# 1998-99 Ph.D. Quals

J. D. Plummer



The problem posed was as follows. A 3 terminal "black box" is to be designed that will produce the I-V characteristics shown on the left. There is no unique solution. The student was told that he/she could design a circuit, a device, a structure or any combination of these to realize the I-V characteristics.

Most students saw that the problem could be divided up into 3 regions on the left, the center and on the right of the I-V plot. The left and center regions could be treated as two parallel devices, with the center device starting to conduct when the applied voltage reaches 3 volts. Many students tried to use 2 parallel MOSFETs. The center region 3 volt offset can be achieved, for example, by putting several diodes in the drain of the MOSFET so that it does not turn on until the applied  $+V$  reaches 3 volts. The right hand region can be achieved in a number of ways. Common solutions included putting a third MOS device in series with the other two, hooked up to turn off when the applied voltage reaches 6 volts. Another solution is to put a 3<sup>rd</sup> MOSFET in the gate circuit of the other 2 MOSFETs such that the gate is grounded when the supply voltage reaches 6 volts.

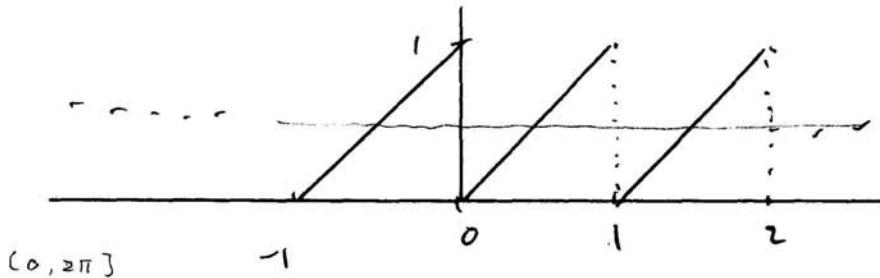
Subtleties include the fact that if MOSFETs are used for the left and center regions, they do not normally produce equally spaced curves and the saturation region currents do not all originate from the same linear region characteristic. What is shown is more typical of BJT devices and some students realized this. Alternatively, resistors could be used in the MOSFET source regions to make the MOSFETs more closely approach what is shown. Also, the turn-off region on the right does not strictly start at 6 volts for all 3 curves. As shown, it starts at higher voltages for higher  $V_C$  values. Some schemes for accomplish the turn-off work this way and others don't.

I was mainly looking for how students approached the problem and how they reasoned through a possible solution. Grading depended on how far they got and on the quality of their reasoning about the problem.

EE Qualifying Exam  
Winter Quarter 1999

- Does the 'saw tooth' signal drawn below have a Fourier transform? Does it have a Fourier series? Please explain.
- Use one or the other to show that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$



$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx dx$$

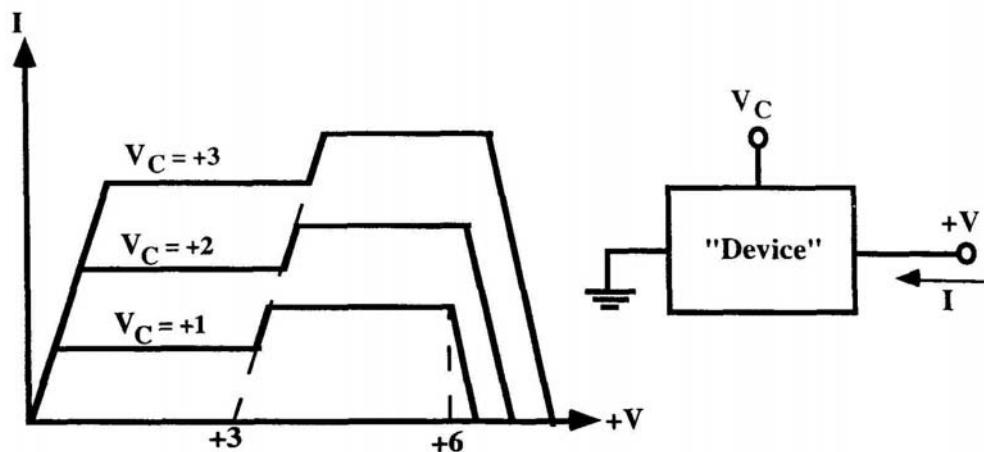
$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx dx$$

$$[\frac{1}{2}, \frac{1}{2}]$$

$$a_n = \frac{1}{\pi} \int_{-\frac{1}{2}}^{\frac{1}{2}} f(x) \cos nx dx$$

# 1998-99 Ph.D. Quals

J. D. Plummer



The problem posed was as follows. A 3 terminal "black box" is to be designed that will produce the I-V characteristics shown on the left. There is no unique solution. The student was told that he/she could design a circuit, a device, a structure or any combination of these to realize the I-V characteristics.

Most students saw that the problem could be divided up into 3 regions on the left, the center and on the right of the I-V plot. The left and center regions could be treated as two parallel devices, with the center device starting to conduct when the applied voltage reaches 3 volts. Many students tried to use 2 parallel MOSFETs. The center region 3 volt offset can be achieved, for example, by putting several diodes in the drain of the MOSFET so that it does not turn on until the applied  $+V$  reaches 3 volts. The right hand region can be achieved in a number of ways. Common solutions included putting a third MOS device in series with the other two, hooked up to turn off when the applied voltage reaches 6 volts. Another solution is to put a 3<sup>rd</sup> MOSFET in the gate circuit of the other 2 MOSFETs such that the gate is grounded when the supply voltage reaches 6 volts.

Subtleties include the fact that if MOSFETs are used for the left and center regions, they do not normally produce equally spaced curves and the saturation region currents do not all originate from the same linear region characteristic. What is shown is more typical of BJT devices and some students realized this. Alternatively, resistors could be used in the MOSFET source regions to make the MOSFETs more closely approach what is shown. Also, the turn-off region on the right does not strictly start at 6 volts for all 3 curves. As shown, it starts at higher voltages for higher  $V_c$  values. Some schemes for accomplish the turn-off work this way and others don't.

I was mainly looking for how students approached the problem and how they reasoned through a possible solution. Grading depended on how far they got and on the quality of their reasoning about the problem.

where  $g$  is any positive constant. Now all they had to do was calculate the constant via

$$j = g \frac{j - 1}{j + 1} = g \frac{-(j - 1)^2}{|j + 1|^2} = gj \Rightarrow g = 1.$$

Everyone could do this, but their speeds varied greatly. Sign errors were common, and I pointed them out before further results were contaminated.

4. Apply this bilinear transformation to  $H(s)$  to obtain  $H_d(z)$ .

$$\begin{aligned} H_d(z) &= H(s) |_{s=\frac{z-1}{z+1}} = \frac{1}{\frac{z-1}{z+1} + 1} = \frac{z+1}{z-1 + (z+1)} = \frac{z+1}{2z} = \frac{1}{2} + \frac{1}{2}z^{-1} \\ \leftrightarrow h_d &= \left[ \frac{1}{2}, \frac{1}{2}, 0, \dots \right] \quad (\text{for the unilateral } z \text{ transform}) \end{aligned}$$

5. What is the corresponding discrete-time impulse response  $h_d(n)$ ?

$$h_d(n) = \mathcal{Z}^{-1}\{H_d(z)\} = \frac{1}{2}\delta(n) + \frac{1}{2}\delta(n-1) = \begin{cases} \frac{1}{2}, & n = 0, 1 \\ 0, & \text{otherwise} \end{cases}$$

Ideally, the student was somewhat surprised by this result. Usually this was expressed by a bit of a pause.

6. Explain how the continuous-time and discrete-time filters correspond.

I let the student formulate an initial answer here, to see how they preferred to plunge in, but within a minute or two, I would guide the discussion with questions such as "What happened to our exponential?", "How did we manage to convert a one-pole continuous-time filter into a one-zero digital filter?", "What happened to the pole and its associated exponential decay?", "Where did the new zero come from?", (etc.). If the student remained at a loss, (or more commonly giving wrong answers such as "sampling entails a loss of information" or opaque answers such as "it's the bilinear transform's fault"), I suggested drawing a pole-zero diagram in the  $z$  plane and thinking about the bilinear transform mapping, reminding them that it is a one-to-one, that the LHP maps to the unit circle, etc. I also would ask for a quick sketch of the amplitude response from the pole-zero diagrams in the  $s$  and  $z$ -plane cases and get them to say that they looked about right.

Sooner or later, most students recognized that the pole was mapped to  $z = 0$ . In most cases I asked for the time constant associated with a pole at the origin (looking for the simple answer "zero"). A lot of students had trouble with this. Most knew  $z^{-n}$  corresponds to a delay by  $n$  samples, and so the concept of time-constant didn't seem to apply. Surprisingly few students could get this part without a lot of coaching (e.g., "Try thinking about a limit as the pole goes from  $z = 1$  to  $z = 0$ "). But then they were also normally short on time at this point.

After dispensing with the pole, I focused them on the new zero. With variable amounts of coaching, most students came up with the result that it was the image under the mapping of the original zero at infinity. So, the zero at infinity was mapped to  $z = -1$ , giving rise to the term  $(z + 1)$  in the numerator of the discrete-time transfer function, and hence an impulse-response component proportional to  $[1, 1, 0, \dots]$ . A fine point that nobody mentioned is that the pole at  $z = 0$  can be interpreted as making the one-zero response causal.

Mime-Version: 1.0  
X-Sender: sxwang@ee.stanford.edu  
Date: Tue, 19 Jan 1999 09:35:01 -0800  
To: Diane Shankle <shankle@ee.stanford.edu>  
From: "Shan X. Wang" <sxwang@ee.stanford.edu>  
Subject: Re: Quals Questions physics

Diane,

Here it is. Regards, Shan Wang

### Shan Wang's Quals question for 1999:

- 1) How do you express (or describe) the momentum of an electron in a crystalline solid?
  - 2) Is it true that the property of such an electron will not change if its wavevector is increased by a reciprocal lattice vector? If yes, what is the underlying physical law or theorem?
  - 3) Show that the real (or average) momentum of the electron indeed does not change if its wavevector is increased by a reciprocal lattice vector.

X-Authentication-Warning: Manta.Stanford.EDU: widom owned process doing -bs  
 To: Diane Shankle <shankle@ee.stanford.edu>  
 Subject: Re: Quals Questions  
 Date: Fri, 15 Jan 1999 17:04:56 -0800  
 From: Jennifer Widom <widom@DB.Stanford.EDU>

CS

Jennifer Widom  
 1999 EE Quals Question

Question for students with no database implementation background

---

Consider two tables of information stored in a computer, for example:

Employee table:	ID	name	deptNum
	123	Joe	55
	456	Mary	22
	789	Fred	55
	135	Susan	13
	246	John	22
	...	...	...

Department table:	num	name
	10	research
	55	support
	22	sales
	18	HR
	13	develop.
	...	...

Your goal is to write an algorithm that computes the "join" of these two tables based on Employee.deptNum = Department.num:

ID	emp-name	deptNum/num	dept-name
123	Joe	55	support
456	Mary	22	sales
789	Fred	55	support
135	Susan	13	develop.
246	John	22	sales
...	...	...	...

Suggest up to three different algorithms for computing the join of two tables T1 and T2. Contrast the algorithms in terms of their time complexity and storage requirements.

POSSIBLE ANSWERS:

Algorithm 1: simple nested-loop join

```
for each row R1 in table T1:  
    for each row R2 in table T2:  
        if R1 and R2 satisfy the joining condition then combine  
            R1 and R2 and append to the result table
```

Time complexity:  $O(|T_1| * |T_2|)$

Storage requirement: essentially none

Algorithm 2: single-sort join

```
sort table T1 on the joining value;  
for each row R2 in T2:  
    use binary search on sorted T1 to find all matching values and  
    add to the result table
```

Time complexity:  $O(|T_1| * \log(|T_1|))$  to sort T1  
 $O(|T_2| * \log(|T_1|))$  for second step

Algorithm 3: sort-merge join Sort both

```
sort table T1 on the joining value;  
sort table T2 on the joining value;  
traverse the two tables linearly (with some "backtracking" for  
duplicate values), matching join values and adding joining tuples  
to the result table
```

Time complexity:  $O(|T_1| * \log(|T_1|))$  to sort T1  
 $O(|T_2| * \log(|T_2|))$  to sort T2  
 $O(\max(|T_1|, |T_2|))$  for "merge" phase assuming not too  
many duplicate joining values

Storage requirement: not much, depends on sorting algorithm used

Algorithm : hash join

```
set up hash table;  
for each row R1 in T1:  
    hash R1's join value and put R1 in the appropriate hash bucket  
for each row R2 in T2:  
    hash R2's join value;  
    find all matching tuples in the hash bucket and add to the result table
```

? Time complexity:  $|T_1| + |T_2|$  assuming well-distributed hash table  
 Storage requirement:  $O(|T_1|)$  for hash table

Question for students with database implementation background

Consider the standard [tuple]-based nested-loop join algorithm for computing T1 JOIN T2:

```
for each row R1 in T1:  
    for each row R2 in T2:  
        if R1,R2 satisfy the join condition then add R1/R2 to result
```

Suggest three separate possible improvements to this algorithm.  
Assume a standard DBMS storage system and query processing context.  
Improvements could depend on additional assumptions or scenarios.

ANSWER:

1. Nested-block join

Process T1 and T2 block-at-a-time instead of row-at-a-time.  
Considerably reduces the number of times T2 is scanned, without  incurring extra I/O's for T1.

"block at a time"

2. "Rocking"

For T2, scan it forwards the first time, backwards the second time,  
forwards the third time, etc. Takes advantage of LRU page replacement  
policy typically used by database buffer managers.

last recent use

forward-backward

3. Use of keys

If the join condition is T1.A = T2.B and B is a key for T2, then once  
a match is found the algorithm can break out of the inner loop.

4. Use of index

If the join condition is T1.A = T2.B and there is an index on T2.B  
then the inner loop can find matching T2 rows using the index instead  
of by scanning the whole relation. (Also works for inequality join  
conditions if the index is a B-tree.)

Simon Wong, 1/15/99 3:24 PM -0800, Re: Quals Questions

---

1

Mime-Version: 1.0  
Date: Fri, 15 Jan 1999 15:24:10 -0800  
To: Diane Shankle <shankle@ee.stanford.edu>  
From: Simon Wong <swong@snf.stanford.edu>  
Subject: Re: Quals Questions

99 Qualification Exam Question  
Simon Wong

*circuits*

Sketch a typical output buffer of a digital chip. (Most students sketch a CMOS inverter.)

Sketch a tri-state output buffer. (Most students sketch a CMOS inverter in series with a pair of control MOSFETs, which takes up about 4 times the area of a simple inverter.)

Discuss other more area efficient implementations of the tri-state output buffer.

EE Qualifying Examination

January, 1999

Examiner: Yoshihisa Yamamoto

1. Describe the three essential elements of a negative conductance oscillator (such as laser and transistor oscillators).
  
2. What is the fundamental difference between an oscillator and an amplifier?

**Hint #1:** form a  $4 \times 4$  matrix  $R$ , with  $R_{11}$  the number of times  $A$  is ranked 1st,  $R_{12}$  the number of times  $A$  is ranked 2nd,  $R_{21}$  the number of times  $B$  is ranked 1st, etc.

the composite scores are then

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ R_{21} & R_{22} & R_{23} & R_{24} \\ R_{31} & R_{32} & R_{33} & R_{34} \\ R_{41} & R_{42} & R_{43} & R_{44} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}$$

where  $0 < w_1 < w_2 < w_3 < w_4$  are the weights used.

**Hint #2:** suppose  $y = Tx$ , where  $T$  is  $4 \times 4$ . When does  $x_1 > 0, x_2 > 0, x_3 > 0, x_4 > 0$  imply  $y_1 > 0, y_2 > 0, y_3 > 0, y_4 > 0$ ?

(In other words, when does a matrix map positive vectors into positive vectors?)

## **Matrices that preserve vector order**

we say  $x \preceq y$  if  $x_1 \leq y_1, \dots, x_n \leq y_n$

**Part 1.** what matrices  $A$  preserve order, i.e., satisfy

$$u \preceq v \implies Au \preceq Av?$$

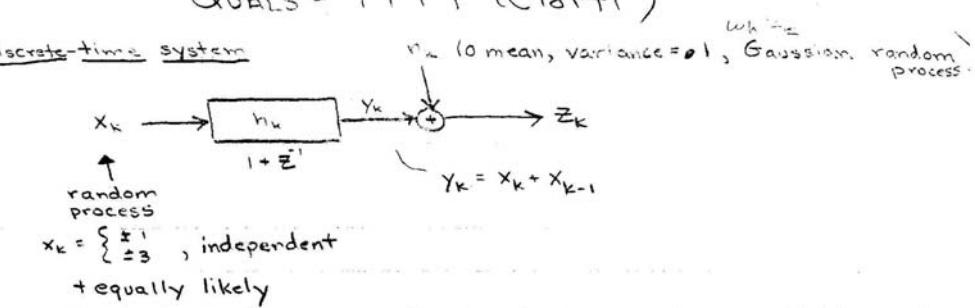
**Part 2.** what matrices  $A$  satisfy the stronger condition

$$u \preceq v \iff Au \preceq Av?$$

Signal

## Quals - 1999 (Cioffi)

Discrete-time system



- (1) a). FIND mean + variance of  $x_k$ .
- (1) b). FIND possible values of  $y_k$  and their probabilities.
- (1) c). FIND mean + variance of  $y_k$ .
- (2) d). FIND  $R_{zz,k} = E[z_m z_{m-k}]$ , the autocorrelation function.
- (2) e). FIND  $S_{zz}(\omega)$ , the power spectral density.
- (2) f). Describe a receiver that at each time  $k$  best detects  $\hat{x}_k$  (you may assume last decisional  $k-1$  is correct)
- (1) g). Find probability of error for your receiver.

Quals Solutions - 1999 (Cioffi)

a).  $m_x = 0 \quad m_y = \frac{1}{2}(1+9) = 5$

b).  $y_k = \begin{cases} +6 & \left(\frac{1}{16}\right) \\ +4 & \left(\frac{3}{16}\right) \\ +2 & \left(\frac{5}{16}\right) \\ 0 & \left(\frac{7}{16}\right) \\ -2 & \left(\frac{9}{16}\right) \\ -4 & \left(\frac{11}{16}\right) \\ -6 & \left(\frac{13}{16}\right) \end{cases}$

Add z r.v.s, convolve prob. of z

c).  $m_y = 0 \quad E_y = 2 \cdot 5 = 10$

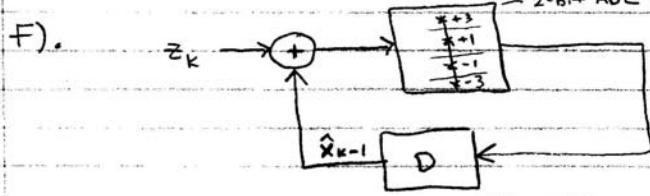
d).  $r_{zz}(z') = E_x \cdot (1+z')(1+z) + .1 = 5(z+z+z') + .1$

$$= 5z + 10.01 + 5z' \downarrow \quad \downarrow r_{zz,0} \quad \downarrow r_{zz,1}$$

all others  $r_{zz,k} = 0$

e).  $S_{zz}(\omega) = 5e^{j\omega} + 10.01 + 5e^{-j\omega}$

$$= 10.01 + 10 \cos \omega \quad |\omega| \leq \pi$$



g).  $P_e = 1.5 Q\left(\frac{1}{\sqrt{6}}\right) = 1.5 \times 10^{-3}$

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$$

Tom Cover  
'99 Quals      *signals*  
=====

Lottery Tickets.

The examinee is led to a paradoxical result. I am most interested in how the examinee interprets and explains the contradictions.

Question:

500 lottery tickets have been sold for a prize worth \$1000. Tickets cost \$1 each. Suppose no more tickets will be sold, except to you. A drawing will be held, and the winning ticket will receive the prize.

1. Should you buy a ticket? What is the expected profit on your ticket?

2. Suppose now that 999 tickets have been sold. Should you buy a ticket?

The above suggests that tickets should be bought until 1000 have been sold.

3. Suppose again that 500 tickets have been sold and no more tickets will be sold other than the tickets you purchase. How many tickets should you buy?

4. Why not buy more, since each ticket (past 707) still has positive expected profit?

Answers:

1. Expected profit:  $(1000/501)-1$ . Therefore, buy.

2. Yes, This is the break even point.

3. Buy  $(\sqrt{2}-1)500 = \text{approx } 207$  tickets.

4. Because a new purchase, while good by itself, diminishes the probability of winning for your previous holdings.

Subject: Dutton's Quals Question ('99)  
Date: Fri, 15 Jan 1999 16:59:37 -0800  
From: "Robert W. Dutton" <dutton@gloworm.Stanford.EDU>

*Circuits*

Part 1

Given three transmitters as follows:

1 uV @ 1.400 GHz

1 mV @ 1.405 GHz

1 mV @ 1.410 GHz

(these are all considered to be signals on the receiving antenna)

What are all the considerations of the receiver in order to capture/process the 1.40 GHz signal (i.e. that's the one you want to "talk to")

Part 2

Given three possible transistors with which you will design the receiver:

BJT where  $I_{out} \sim \exp(V_{in})$

MOS where  $I_{out} \sim (V_{in})^2$

MOS where  $I_{out} \sim V_{in}$

Which one would you use and what are all the considerations that need to be taken into account

Part 3

Consider two blocks of gain (and filtering) and a multiplier (with 1.39 GHz local oscillator to be "mixed" for down-conversion). What are the trade-offs and design choices between having:  
two gain stages followed by mixer  
vs  
one gain stage, mixer and then another gain stage.

Abbas  
S. Jia  
Elgammal

## Quals 99

1. Let  $X \sim U[0, 1]$  be a random variable uniformly distributed between 0 and 1. The outcome of  $X$  divides the  $[0, 1]$  interval into two subintervals  $[0, X)$  and  $[X, 1]$ . We pick one of the subintervals at random as follows: Let  $Y \sim U[0, 1]$  be independent of  $X$ . We pick the subinterval that the outcome of  $Y$  lies in. Let  $L$  be the length of the chosen subinterval. Find the  $E(L)$ .

January 1999

system

R.M. Gray's quals problem and solution

1.  $\dots, x_{-2}, x_{-1}, x_0, x_1, x_2, \dots$  is a real-valued discrete time signal, a sequence of real numbers

The sequence is put into a system, the output of which is a sequence of functions

$\dots, y_{-2}(f), y_{-1}(f), y_0(f), y_1(f), y_2(f), \dots$  where each  $y_n(f)$  is defined for all real  $f$  by

$$y_n(f) = \begin{cases} 0 & \text{if } n \leq 0 \\ \sum_{l=0}^{n-1} x_l e^{-i2\pi f l} & n = 1, 2, \dots \end{cases}$$

(a)  $\Rightarrow$  Is this system linear? time-invariant?

(b)  $\Rightarrow$  Suppose that  $n = N$  is a fixed positive integer and you are told only  $y_N(f)$  (for all real  $f$ ). What is the energy of the first  $N$  samples,

$$\mathcal{E}_N(x) = \sum_{n=0}^{N-1} |x_n|^2 ?$$

This problem was meant to get students with a solid Fourier background started, and provide students with a minimal background some basics to work on. Usually this question counted for about 4 points out of 10.

The system is easily seen to be linear since linear combinations of inputs produce the corresponding linear combination of outputs. The only novelty here is that the outputs are functions of  $f$  instead of real numbers, but linearity still holds. The system is not time invariant, e.g., since shifting the input to the left does not simply shift the output to the left since the output is always 0 for negative time. (A Kronecker delta function at the origin as an input provides a specific counterexample.)

The output to the system at time  $N$  should be recognized as being the discrete Fourier transform (DFT) of the first  $N$  input samples, but since it is the DFT only the frequencies  $k/N$  for  $k = 0, 1, \dots, N-1$  are needed to recover  $x_0, \dots, x_{N-1}$  or to compute the energy. This means Parseval's theorem holds:

$$\sum_{n=0}^{N-1} x_n x_n^* = \frac{1}{N} \sum_{k=0}^{N-1} |y_N(\frac{k}{N})|^2.$$

The result should either come from memory or from a quick derivation, e.g., since

$$|y_N(\frac{k}{N})|^2 = \left| \sum_{l=0}^{n-1} x_l e^{-i2\pi \frac{k}{N} l} \right|^2 = \sum_{l=0}^{n-1} \sum_{m=0}^{n-1} x_l e^{-i2\pi \frac{k}{N} l} x_m e^{+i2\pi \frac{k}{N} m}$$

from the orthogonality of complex exponentials summing over  $k$  yields

$$\begin{aligned}\sum_{k=0}^{N-1} |y_N(\frac{k}{N})|^2 &= \sum_{l=0}^{n-1} \sum_{m=0}^{n-1} x_l x_m \sum_{k=0}^{N-1} e^{-i2\pi \frac{k}{N}(l-m)} \\ &= \sum_{l=0}^{n-1} \sum_{m=0}^{n-1} x_l x_m N \delta_{l-m} \\ &= N \sum_{l=0}^{n-1} |x_l|^2\end{aligned}$$

If a student wrote from memory an integral formula, they were directed towards the sum appropriate for a DFT (and more amenable to digital computer implementation, e.g., using an FFT). Even though this is a DFT and not the DTFT of an infinite sequence, an integral can be used, e.g., a few students observed that integrating  $|y_N(f)|^2$  over  $[0, 1)$  would give the energy because of the orthogonality of exponentials. This provided a formula for the energy, but not the simpler finite sum. Integrating over infinite limits, however, does not work.

2.  $\dots, x_{-2}, x_{-1}, x_0, x_1, x_2, \dots$  is a real-valued discrete time signal, a sequence of real numbers

Suppose that the previous system is replaced by one which produces an output

$$w_n(f) = \begin{cases} 0 & \text{if } n < 0 \\ \sum_{k=0}^{n-1} e^{i2\pi f x_k} & n = 0, 1, 2, \dots \end{cases}$$

- (a)  $\Rightarrow$  Is this system linear? time-invariant?  
 (b)  $\Rightarrow$  Suppose that  $n = N$  is a fixed positive integer and you are told only  $w_N(f)$  (for all real  $f$ ). What is the energy of the first  $N$  inputs,

$$\mathcal{E}_N(x) = \sum_{n=0}^{N-1} |x_n|^2 ?$$

This system is not linear because the input appears in the exponential. Time invariance is violated as before.

Here more thought is required to find the energy. The idea was to consider possible operations on  $w_N(f)$  and to find something that would produce the desired sum. A short cut is to see the resemblance to the moment generating property of the Fourier transform. Either way, differentiating  $w_N(f)$  with respect to  $f$  results in bringing the  $x_n$  out of the exponential:

$$w'_N(f) = \frac{d}{df} w_N(f) = \sum_{k=0}^{n-1} (i2\pi x_k) e^{i2\pi f x_k}$$

so that setting  $f = 0$  yields the first moment (mean) of the first  $N$  values of the input. The second moment, the energy, can be then found by differentiating again with respect to  $f$ :

$$w_N''(f) = \frac{d^2}{df^2} w_N(f) = \sum_{k=0}^{n-1} (i2\pi x_k)^2 e^{i2\pi f x_k}.$$

Evaluating this at 0 yields

$$\mathcal{E}_N(w) = -\frac{w_N''(0)}{(2\pi)^2}$$

For those who reached this point a final question was asked: Can the individual  $x_n$  be recovered from knowledge of  $w_N(f)$  for all  $f$ ? Unlike the DFT of the first question, the answer here is no. If you simply rearrange the  $x_n$  you get the same  $w_N(f)$ , e.g., if  $N = 2$  then  $x_0 = 0$  and  $x_1 = 1$  yields the same  $w_N(f)$  as  $x_0 = 1$  and  $x_1 = 0$ , so you cannot recover the individual inputs, but you can recover any moment.

X-Sender: horowitz@vlsi.stanford.edu  
Date: Sat, 16 Jan 1999 01:07:52 -0800  
To: shankle@ee.stanford.edu  
From: Mark Horowitz <horowitz@stanford.edu>  
Subject: Quals question:  
Cc: horowitz@vlsi.Stanford.EDU  
Mime-Version: 1.0

Computer Architecture and  
VLSI design

We will be looking at the question of wire delay in an IC. To solve this problem we will model delay using simple RC models.

I present the student with an example wire 1u wide, and 1mm long. It has a resistance of 100ohms and a capacitance of .13pF

1. How does the resistance and capacitance of the wire vary with length  
(both are linear)
2. How does the delay vary with length  
(quadratic)
3. What is the delay of a 10mm wire.  
$$1/2 \text{RC} = 650\text{ps}$$
4. If that is too large how could you reduce it?  
wire wider, repeaters
5. Shown on the board is how the capacitance of the wire varies with width.  
At 1u = .13pF, 2u=.16pF, 3u=.19pF. Does this data make sense? What does the extrapolated value of 0.1pF at zero width represent?  
Makes sense. 0.1pF is the fringe capacitance
6. Roughly how much faster can you make the wire by making it wider?  
4 times
7. Why does adding buffers speed up the wire?  
breaks the increasing resistance with length
8. What parameters should you optimize in a wire with repeaters to minimize delay.  
distance between inverters and the inverter size
9. Write the delay equation for one segment of a buffered wire

RC delay model, repeater, fringe capacitance, buffer

Balaaji Prabhakar

signal

## Balls and Urns

There are  $B$  balls and  $U$  urns.  $B$  is random and has a Poisson distribution with mean  $U$ ; that is,

$$P(B = b) = e^{-U} \frac{U^b}{b!}.$$

Each ball is placed in an urn chosen at random uniformly across all the urns. The placement of a ball is independent of the placement of all other balls.

- 1) What is the probability that urn number 1 has 3 balls?
- 2) What is the probability that Urn 1 has 3 balls and Urn 2 has 4 balls?
- 3) What is the expected number of empty urns?

Now suppose that  $U$  is also random, being uniformly distributed between 5 and 10.

- 4) What is the expected number of empty urns?

EE Quals Problem Solution

January 11-15, 1999

Julius Smith

Consider the following pole-zero diagram for a continuous-time filter:

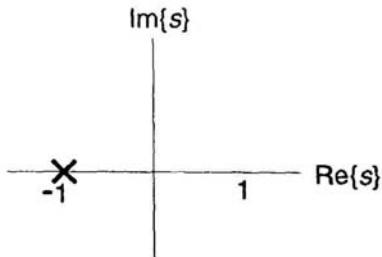


Figure 1: Pole-zero diagram ( $s$ -plane).

1. What is the transfer function  $H(s)$ ?

$$H(s) = \frac{g}{s+1}, \quad g \text{ any constant (henceforth } g=1)$$

2. What is the impulse response  $h(t)$ ?

$$h(t) = u(t)e^{-t}$$

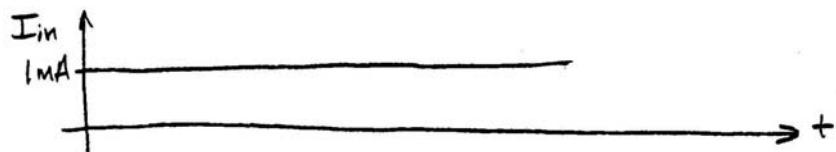
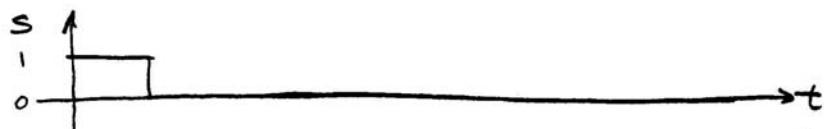
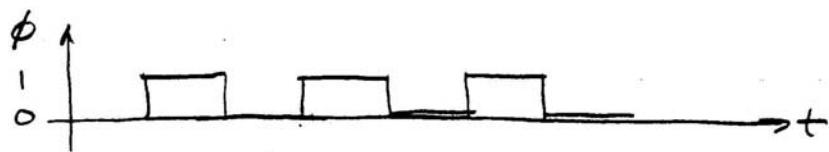
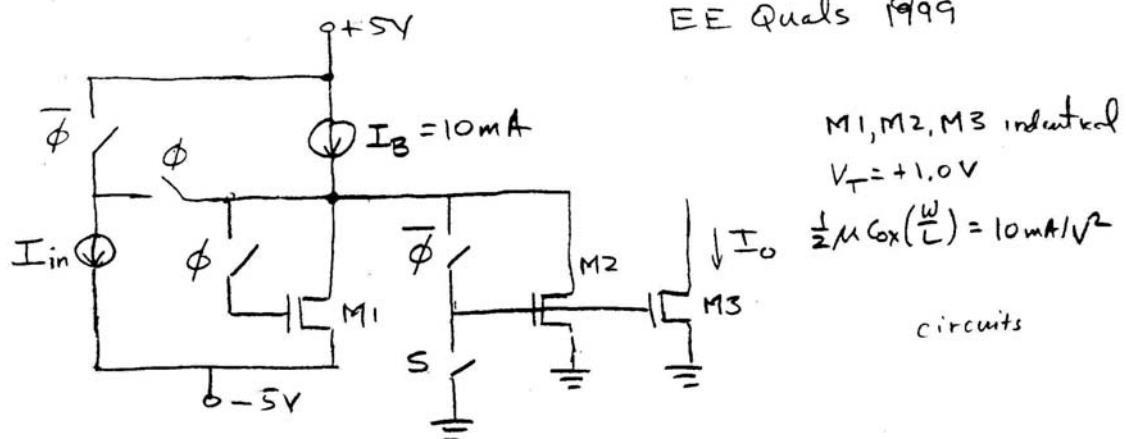
Some students asked if they could assume causality, which was nice. For the very few students who finished all problem parts with time remaining, I returned to these first two parts and asked about the region of convergence, to write down the inversion integral, repeat for a pole in the right-half plane, etc. About ten percent of the examinees could correctly answer any such additional question immediately, demonstrating a significantly superior mastery of  $s$  and  $z$  plane concepts. One student, in contrast, tried to recast the problem in the form of differential equations, noting that he generally preferred to avoid the Laplace transform.

3. What is the bilinear transformation which maps  $\omega = 1$  ( $s = j$ ) in the  $s$ -plane to  $\omega_d = \pi/2$  ( $z = j$ ) in the  $z$ -plane?

If the student didn't know the bilinear transform or barely remembered anything about it, I asked for a quick general discussion of what they did remember. Most knew that it is used to convert certain analog filters (e.g., lowpass) from the  $s$  plane to the  $z$  plane, without aliasing, with frequency warping, and they either remembered the general form of the transform (modulo signs) or else only the tangent expression for the frequency warping. After a short while, if they didn't figure it out from first principles, I wrote down for them the general, stability-preserving bilinear transform:

$$s = g \frac{z-1}{z+1}$$

Bruce Woolley  
EE Quals 1999



Sketch  $I_o$

**EE Qualifying Examination**

**January, 1999**

**Examiner: Yoshihisa Yamamoto**

*physics*

1. Describe the three essential elements of a negative conductance oscillator (such as laser and transistor oscillators).
  
2. What is the fundamental difference between an oscillator and an amplifier?