

Electrical Engineering

Quals Questions

2000



To: Diane Shankle <shankle@ee.stanford.edu>  
Subject: Re: Quals Question  
Date: Fri, 21 Jan 2000 15:15:26 -0800  
From: Mary Baker <mgbaker@plastique.Stanford.EDU>

1) Consider the following three file systems in terms of how they lay out file data on disk:

Extent-based file system  
Block-based file system (like most UNIX file systems)  
Log-structured file system

[I then described how each lays out its data and when it does its write operations.]

- a) What is the relative performance of these three systems if you attempt to write out a large file sequentially from beginning to end?
  - b) What is the relative performance of these three systems if you attempt to write out portions of a large file randomly skipping round in the file?
  - c) What is the relative performance of these three systems if you later attempt to reread that randomly-written file by reading it sequentially from beginning to end?
- 

2) Consider a system with dynamically-linked shared libraries. [I describe what that means.] In this system the libraries can be brought into a process's address space at any time and in any order, and they may be placed in different places in the different processes' virtual address spaces.

What are some of the issues you'd need to consider in the code for these different libraries? What kinds of support would you need for such libraries in the operating system or in the processes that use the libraries?

[Note: I graded based not just on correct answers but on how much pushing a student needed in order to get at those answers. Also, many students didn't get through as much of the questions as some other students, and that made a difference too.]

Quals - 2000

J. Cioffi

Given: linear discrete time system

$$y_k = x_k + a y_{k-1} \quad \text{where } x_k \text{ is zero mean Gaussian with variance } E[x_k^2] = E_x$$

$x_k$  has autocorrelation function

$$E[x_m x_{m-k}] = (1+b^2) \delta_k - b \delta_{k-1} - b \delta_{k+1}$$

$$\delta_k = \begin{cases} 1 & k=0 \\ 0 & k \neq 0 \end{cases}$$

Find:

- 1).  $E[y_k]$ , mean of  $y_k$  (1 pt)
- 2).  $\text{var}[y_k]$  (2 pts)
- 3). distribution of  $y_k$  (1 pt)
- 4). For what values of  $a$  is process  $y_k$  stable? (1 pt)
- 5). Find the power spectral density of  $x_k$ . (2 pts)
- 6). " " " " " " " "  $y_k$  (2 pts)
- 7). When is  $y_k$  a "white" process? (1 pt.)

From: Dawson Engler <engler@csl.Stanford.EDU>  
Subject: Re: Quals Question  
To: shankle@ee.stanford.edu (Diane Shankle)  
Date: Wed, 9 Feb 2000 16:42:24 -0800 (PST)

Hi Diane,

> I am still missing your Quals Question! Please try to get it in this week.!

here it is; good luck extracting one from the other people ;-)

Dawson

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Dr. Frankenstein (Jr.) has decided that people are too stupid to deal with concurrency and has started a company ("not-like-last-time.com") that plans to genetically modify the population to correct this problem.

As a stopgap measure while he works out a few details in the laboratory, he has hired you to eliminate the need for locks by building a system he has designed that works as follows:

1. On thread create, it creates a private copy of the current ("primary") address space. Every load or store done by the thread uses this private copy.

2. When a thread exits, the system attempts to merge the thread's private address space back with the primary copy. If the system detects there was a possible race condition, it discards the private address space and restarts the thread from scratch.

To do this step, the system allows you to suspend all threads, and to later resume them.

3. To allow you to detect races, the system tracks (1) the time the thread was created, (2) what pages have been read or written by the process and (3) every primary memory page has a time stamp of the last time it was read or written.

Explain how to use the system's provided information to detect when a possible race condition happened. Please be concretely precise about what a race condition will look like; and say each of the four possible combinations of read and write why it is ok or causes a race.

We rely on being able to reexecute threads. What assumptions are we making? (Hint, think about restarting instructions after a fault.) Give two concrete examples of things the thread cannot do.

*System status might not be same.*

*I/O or network packet may lost*

OFFICE MEMORANDUM ◊ STAR LABORATORY

January 21, 2000

To: Diane Shankle  
From: Tony Fraser-Smith *PECS*  
Subject: Ph.D. Quals Question, January 2000

**Probing the Surface of Europa**

The student is told about NASA's latest discoveries on Europa, the fourth largest moon of Jupiter: (1) images acquired during recent spaceprobe flybys show a surface covered with ice (discolored ice in some places) and the layer of ice appears to be quite thick, and (2) magnetic and gravity measurements suggest very strongly that there is a liquid ocean beneath the ice. Liquid oceans are very unusual in the solar system, and the existence of one on Europa suggests the possibility of life. NASA is therefore planning a mission specifically to Europa that will place a space probe in orbit around the moon (it will be called the Europa Orbiter) and which may land microprobes on the surface to learn more about the ice layer and – it is hoped – about the ocean underneath.

Question: Discuss what NASA might learn about the ice, and possibly ocean, by electromagnetic probing from a microprobe on the surface.

To get full marks for this question, the student was expected to draw attention, at some time during the discussion, to techniques other than electromagnetic probing that might be used. Obviously it would be impractical to drill through a thick layer of ice, given the logistics involved, but acoustic or seismic signals might be used to measure the thickness of the ice.

The student was then expected to consider what kind of frequencies would be best for electromagnetic probing. For this they would have their attention drawn to the discolorations in the ice and the likelihood that the ocean had salts dissolved in it – in other words, the ice is probably somewhat contaminated and "salty." With this information, they would first look to see when the ice could be considered to be a good or poor conductor. For this they could start with the quantity  $\sigma/\omega\epsilon$ , which is a measure of the relative magnitude of the conduction current to the displacement current in a medium ( $\sigma$  is the electrical conductivity,  $\omega$  is the angular frequency, and  $\epsilon$  is the permittivity), and discuss the "good conductor" approximation  $\sigma/\omega\epsilon \gg 1$ . Alternatively, they could start with the transition frequency  $\omega_c = \sigma/\epsilon$ , below which frequency the medium acts as a good conductor. ★

The European ice probably has a conductivity  $\sigma < 4 \text{ S/m}$ , where  $4 \text{ S/m}$  is typical for sea water on Earth, and a permittivity of around  $\epsilon = 81 \times 8.85 \times 10^{-12} \text{ F/m}$ . Out of this the students should conclude that they are likely to be dealing with the good conductor approximation for frequencies less than  $890 \text{ MHz}$ . Knowing that high frequency electromagnetic waves are usually rapidly attenuated in conducting materials, the student should conclude that NASA would have to use the lowest possible frequencies, and that they would be operating with ice that acted electromagnetically as a good conductor.

In the subsequent discussion the student is either asked about or derives the skin depth ( $\delta$ ) and inevitably he/she can write it down as  $\delta = \sqrt{2/(\omega\mu\sigma)}$ , where  $\mu$  is the permeability. We discuss the significance of each of the factors in the expression for  $\delta$  and its applicability.

Obviously the frequencies required for penetration through several km of slightly salty ice will be on the order of 1 Hz or less.

We end by discussing how we might send out a pulse of these low frequency radio waves through the ice and obtaining echoes back that could give us an indication of the depth of the ice. The important thing here is for the student to realize that there is dispersion, with the higher frequency components of the pulse travelling faster than the lower frequency components, and also frequency-dependent absorption, with the higher frequency components suffering greater absorption. These two effects change the shape of the radiated pulse and degrade the accuracy of the measurement of ice thickness.

Is this a general property of conductors?

X-Sender: hector@db.stanford.edu (Unverified)  
Date: Thu, 10 Feb 2000 14:09:54 -0800  
To: Diane Shankle <shankle@ee.stanford.edu>  
From: Hector Garcia-Molina <hector@cs.stanford.edu>  
Subject: Re: Quals Question

here is my quals question... sorry for the delay...  
hector

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EE Qual exam Question January 2000  
Hector Garcia-Molina

Consider a FIFO queue implemented with  
a doubly linked list.

- (a) Describe the data structure needed  
to implement this queue.
- (b) Using simple pseudo-code, write an Enqueue  
procedure that adds an element to the queue.
- (c) Write a Dequeue operation that returns an  
element from the queue.
- (d) Discuss what would happen in multiple processes  
concurrently used your enqueue and dequeue procedures.  
What can be done to prevent such conflicts?  
(Just discuss briefly; do not write code.)

Date: Wed, 19 Jan 2000 15:01:00 -0800  
From: "James F. Gibbons" <gibbons@cis.Stanford.EDU>  
Reply-To: gibbons@soe.stanford.edu  
Organization: Stanford University  
X-Accept-Language: en  
To: Diane Shankle <shankle@ee.Stanford.EDU>  
Subject: Re: Quals Question

Diane:

I had four questions that followed from each other. Scores were determined by how far through the list of questions an examinee got. The questions were:

1. What is an LED and how does it work?
2. How can you explain the difference between the apparent output power and the power supplied by the battery?
3. What is a semiconductor laser diode, and how is it different from an LED?
4. What is a solar cell and how does it work? What limits the maximum efficiency) of a solar cell?

Diane Shankle wrote:

> Please submit your Quals Question by email or hard copy.  
> Due date Friday, January 21st.  
> Thanks!  
> Diane  
>  
> \_\_\_\_\_  
> Tel: (650) 723-3194  
> FAX: (650) 723-1882  
> shankle@ee.stanford.edu  
> Stanford University  
> Dept of Electrical Engineering  
> Packard Building Rm. 165  
> Stanford, CA 94305-9505  
> <http://www-ee.stanford.edu>

Diane Joan Shankle

## 2000 PhD Quals

Jim Harris

1. Semiconductors, insulators and metals are all used in integrated circuits. What is the most distinguishing property which differentiates these materials?

$$10^2 - 10^{20} \text{ cm}^{-3}$$

2. Can you roughly sketch what the conductivity vs temperature ( $50^\circ$ - $500^\circ\text{K}$ ) and vs impurity incorporation ( $1\text{E}12$ - $1\text{E}20 \text{ cm}^{-3}$ ) looks like for each of these categories of materials? What is the order of magnitude of these changes and what is the change due to?

3. Most of the time, we design devices around a set of properties of a given semiconductor material. I have just become an alchemist—I haven't figured out how to change lead into gold, but I can give you any properties you would like in a semiconductor material. What would be the properties of your "ideal semiconductor" and why?

A. Bandgap =  $1.3$ - $1.5 \text{ eV}$  – Too small, large  $n_i$ , inability to control n & p doping, high  $\text{GaAs } 1.43 \text{ eV}$

leakage currents and poor temperature operating range. A bandgap in this range will yield low enough  $n_i$  to provide a semi-insulating substrate and ease isolation problems. Too large,

why ? { unable to ionize impurities, poor ohmic contacts, high thresholds on lasers, bipolar transistors, high power dissipation. Direct bandgap for optical devices.



B. Naturally forming oxide which is an insulator, has no surface states or fixed charge, passivates the surface, thermally stable, good high temperature diffusion and implantation barrier, easily etched with high etch selectivity compared to semiconductor.

C. A material which can be easily grown in high purity form and prepared as large wafers that are physically robust, chemically stable and compatible with metals and insulators, which can be doped n or p type over a large range.

D. A relatively low dielectric constant,  $<2$ . This provides a higher field in the semiconductor for a given gate voltage, hence much better control of the channel conductance in a MOSFET. Lower dielectric constants also reduce parasitic capacitance.

E. High and equal mobility for both electrons and holes (complementary circuits) and low resistivity for diodes, lasers, transistors, and high saturated drift velocity which is reached at low electric fields for high speed and low power dissipation

F. High thermal conductivity and small thermal expansion coefficient and one that matches oxide and metals.

G. Heterojunction possibilities for lasers, HBTs, QW devices, MODFETs, etc.

H. Properties that are independent of temperature.

modulation doped fet

I. High breakdown field strength for high power devices.

also called high electron mobility transistor

heterojunction bipolar transistor  
for high speed device.

J. Small and equal electron and hole effective masses for higher mobilities, higher tunneling currents for low contact resistance, minimize current for laser threshold.

K. Widely abundant, low cost and non-toxic.

4. If you could choose one property to be temperature independent, which would you choose?

A. Bandgap as temperature variation causes threshold variation in FETs, bipolar transistors, lasers and higher leakage currents in all devices.

X-Sender: horowitz@vlsi.stanford.edu  
Date: Mon, 24 Jan 2000 23:41:10 -0800  
To: Diane Shankle <shankle@ee.stanford.edu>  
From: Mark Horowitz <horowitz@stanford.edu>  
Subject: Re: Reminder: Quals Questions were due on Friday, 1/21/200

Here is my qual's question

We are looking at a machine that has variable length instructions. Assume that the top two bits indicate the instruction length. 0- indicates 2B instruction, 10 is 4B and 11 is 5bytes.

1. Show the logic for the length decoder. (it has 3 outputs)
2. Show the logic for getting the instruction data aligned for the next instruction decode
3. Assume you want to build a machine that can issue three instructions each cycle. Show how you would connect 3 instruction decoders to the register that contains the instruction bytes. Don't worry about delay, but show the critical path
4. Now assume that decoding the instruction length is complex, like in the x86 instruction set. What techniques could you use to speed up the implementation. You have as much hardware as you need.
5. Would making the long instructions 6 bytes make the decoding easier?

time constant for RC circuit is  $\tau = RC$

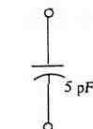
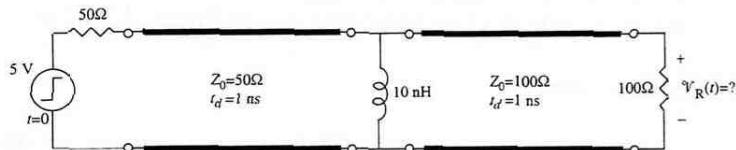
Professor Umran Inan  
Packard Bldg., Rm. 355  
inan@nova.stanford.edu  
650-7234994

time constant for RL circuit is  $\tau = \frac{L}{R}$

PhD Quals  
January 2000  
#1 of 2

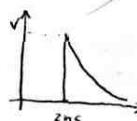
#### INDUCTIVELY COUPLED TRANSMISSION LINES

$$\tau_L = \frac{R_L - Z_0}{R_L + Z_0}$$



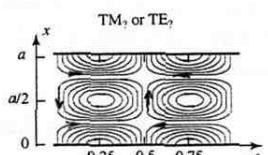
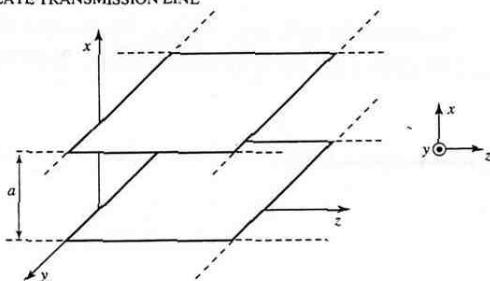
The key to this question was to realize that the inductor initially acts like an open circuit, so that the reflection coefficient at the junction between the two lines is  $\Gamma = 1/3$ , meaning that  $(1+\Gamma)(5/2) = 10/3$  V is transmitted down the 2nd line. This voltage reaches the  $100\Omega$  load at  $t=2$  ns, at which time  $V_R(t)$  jumps to  $10/3$  V. After this, the inductor begins to draw current, its current increasing exponentially with a time constant of  $\tau = L/R_{eq}$ , where  $R_{eq} = Z_0(Z_0/(Z_0) + Z_{00}) = 33.33\Omega$ . Since across its terminals, the inductor "sees" the two transmission line impedances in series. As the inductor current rises, it eventually becomes a short circuit, at which time  $V_R(t)$  reaches zero. The time constant of the exponential decay is  $\tau = L/R_{eq} = 0.3$  ns.

If we had a capacitor instead of the inductor, the result is the dual of that of the inductor; the voltage  $V_R(t)$  starts at being zero at  $t=2$  ns, rising exponentially to  $5(100)/(50+100) = 10/3$  V with a time constant of  $\tau = R_{eq} C$ .



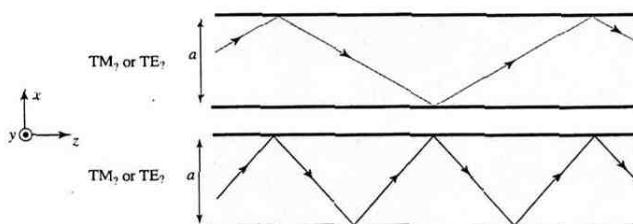
PARALLEL PLATE TRANSMISSION LINE

$$a = n\lambda$$



PhD Quals  
January 2000  
#2 of 2

Answer: ~~TM2~~  $\rightarrow$  TM2  
lines are normal to boundaries



Answer: TM2 or TE2

Answer: TM3 or TE3

The key to this one was to first realize that these could be both TE and TM. The students were additionally told that  $a = 2\lambda$ . They could then proceed to solve by measuring or estimating the angles. In fact, the value of the incidence angles were given to the students, if they asked for it.  
For details, see pp. 269-273 of Inan&Inan, Electromagnetic Waves, Prentice-Hall, 1999.

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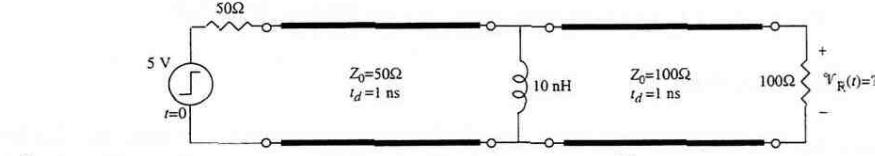
Professor Umran Inan  
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time constant for RC circuit is  $\tau = RC$   
time constant for RL circuit is  $\tau = \frac{L}{R}$

PhD Quals  
January 2000  
#1 of 2

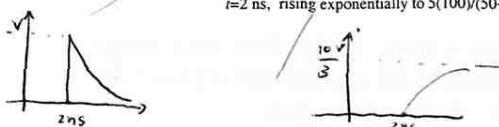
#### INDUCTIVELY COUPLED TRANSMISSION LINES

$$V_L = \frac{R_L - Z_0}{R_L + Z_0}$$

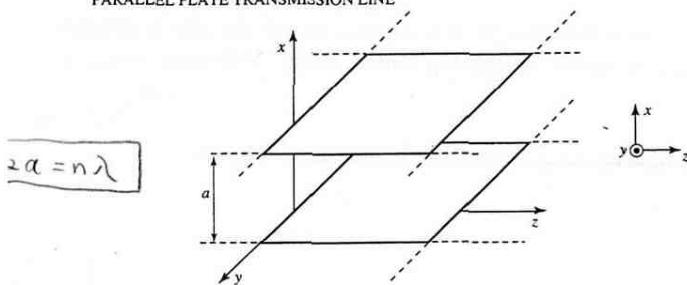


The key to this question was to realize that the inductor initially acts like an open circuit, so that the reflection coefficient at the junction between the two lines is  $\Gamma=1/3$ , meaning that  $(1+\Gamma)(5/2)=10/3$  V is transmitted down the 2nd line. This voltage reaches the  $100\Omega$  load at  $t=2$  ns, at which time  $V_R(t)$  jumps to  $10/3$  V. After this, the inductor begins to draw current, its current increasing exponentially with a time constant of  $\frac{Z_0}{R_{eq}}$ , where  $R_{eq}=Z_0(Z_0+Z_0)=33.33\Omega$ , since across its terminals, the inductor "sees" the two transmission line impedances in series. As the inductor current rises, it eventually becomes a short circuit, at which time  $V_R(t)$  reaches zero. The time constant of the exponential decay is  $\tau=\frac{Z_0}{R_{eq}}=0.3$  ns.

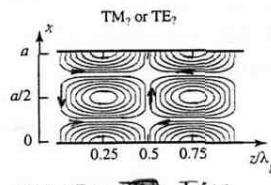
If we had a capacitor instead of the inductor, the result is the dual of that of the inductor: the voltage  $V_R(t)$  starts at being zero at  $t=2$  ns, rising exponentially to  $5(100)/(50+100)=10/3$  V with a time constant of  $\frac{Z_0}{R_{eq}}$ .



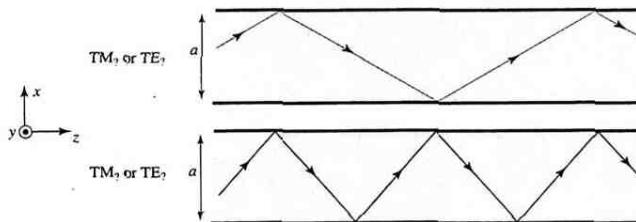
PARALLEL PLATE TRANSMISSION LINE



PhD Quals  
January 2000  
#2 of 2



Answer: ~~TM2~~ TM2  
lines are normal to boundaries



Answer: TM2 or TE2

Answer: TM3 or TE3

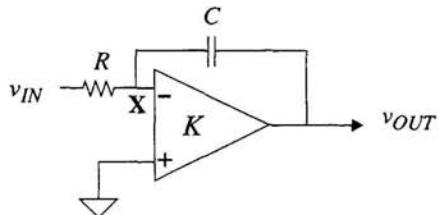
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For details, see pp. 269-273 of Inan&Inan, *Electromagnetic Waves*, Prentice-Hall, 1999.

Quals Question: Explain the operation of coupled electromechanical capacitors.  
Khuri-Yakub

**Question 1:** In the following circuit, assume that the op-amp is ideal in every respect (zero input current, zero output impedance, infinite bandwidth) *except* that the gain of the op-amp,  $K$ , is only 2. Sketch the unit step response. Label important features of your sketch.

FIGURE 1. Non-ideal integrator



**Answer:** This problem is solvable by brute force or near-inspection, since the circuit is only first order.

**Method 1** ("textbook"): KCL at the inverting input node (marked with an "X" above) says:

$$\frac{v_{IN}}{R} = \frac{\dot{v}_X - v_{OUT}}{(1/sC)} \quad (1)$$

A common error was to assume  $v_X = 0$ , an assumption which holds **only** for infinite op-amp gain. Recognize that  $K$  quantitatively relates the output voltage to the op-amp's input voltage:

$$v_{OUT} = -Kv_X \quad (2)$$

Clearly, a nonzero output requires a nonzero  $v_X$ . Now combine the two equations, eliminate  $v_X$ , and solve for the input-output transfer function:

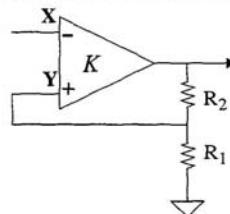
$$\frac{v_{OUT}}{v_{IN}} = -\frac{K}{(1+K)sRC + 1} = -\frac{2}{3sRC + 1} \quad (3)$$

The unit step response of this first-order transfer function is an exponential charging transient with a final value of  $-2V$  and a time constant of  $3RC$ .

**Method 2** ("intuit the answer"): By inspection, this is a first-order system (there's only one energy-storage element), so its step response must consist of a single exponential (whose time constant must be determined). The final value is readily found by recognizing that the capacitor is an open-circuit at DC. Thus, the final value is just  $-K$  times the input value. The time constant may be found any number of ways. One simple way is to find the impedance seen by the resistor (apply test voltage at  $v_X$ , measure response current flowing to the right; take ratio). The combination of the op-amp and the capacitor presents an effective capacitance of  $(1+K)C$  to the resistor (a result that can also be obtained from knowledge of the Miller effect), and therefore the time constant is  $3RC$ .

**Question 2:** Now suppose that the op-amp of the previous circuit is replaced by the following combination. Take careful note of the polarities:

FIGURE 2. Not-quite standard op-amp connection



Suppose that we select the resistors such that their voltage division factor is precisely equal to  $1/K$ :

$$\frac{R_1}{R_1 + R_2} = \frac{1}{K} \quad (4)$$

Now sketch the new unit step response when this combination is used to replace the op-amp of Question 1, again labeling all relevant features of your sketch.

**Answer:** This combination is to replace the gain-of- $K$  amplifier of the previous page, so simply substitute the effective gain of this combination for  $K$  in equation (3).

To find the gain apply a voltage,  $v_X$ , to the inverting terminal, and find the output voltage.

$$v_Y = v_{OUT} \left( \frac{R_1}{R_1 + R_2} \right) \quad (5)$$

$$v_{OUT} = K(v_Y - v_X) \quad (6)$$

Combine, eliminate  $v_Y$ , and solve for the gain:

$$\frac{v_{OUT}}{v_X} = -\frac{K}{1 - K \left( \frac{R_1}{R_1 + R_2} \right)} = -\infty \quad (7)$$

The positive feedback connection has boosted the gain to infinity, creating an ideal op-amp (*not* an oscillator or latch, as several students confidently, but erroneously, asserted). Using this improved amplifier in the previous circuit produces a perfect integrator, so the unit step response is a linear ramp of slope  $-1/VRC$ . Letting  $K$  go to (minus) infinity in equation (3) allows one to get this result even without remembering how ideal op-amp integrators work.

Bruce Lusignan

Quals Questions: for year 2000

- A. We will relay a data stream from one tower to another. As system engineer, what parameters of the system do you need to define?

1. Transmitter: Antenna gain or area; Transmit power. Data rate, modulation type, radio frequency.
2. Path: Distance, obstacles, rain loss, multi-path effects.
3. Receiver: Antenna gain or area; Effective Temperature, demodulation error correction.

- B. Can you relate these mathematically?

$$G = 4\pi \frac{A}{\lambda^2}; B \cdot \frac{C}{N_0} = R \cdot \frac{E_b}{N_0}$$
 for data  $\frac{C}{N_0}$  carrier to noise ratio

- C. To choose frequency, what are the tradeoffs?

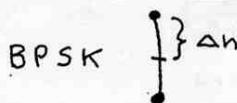
We can fix the antenna areas and use directional parabolas.  $\lambda \propto \frac{C}{N_0}$  use higher frequency. If choice is between 1GHz and 4GHz how much improvement?

The transmit gain  $G_T$  will be 16 times greater with  $\lambda$  being 4 times smaller at 4GHz.

$$P_R = \frac{W G_T G_R \lambda^2}{4\pi R^2}$$
  $G_R$  is also 16 times greater but "free space loss" is also 16 times greater to cancel out the improved  $G_R$ .  $P_R$  depends only on  $1/\lambda^2$ .  $\frac{W G_T G_R}{4\pi R^2} \left(\frac{\lambda}{4\pi R}\right)^2 = \frac{C}{N_0}$

- D. Choosing between Binary Phase Shift Key (BPSK) and Quadrature Phase shift Key (QPSK), what is the effect on power?

With two possible phases per symbol in BPSK you get 1 bit of data. With four phases in QUPSK you get 2 bits/symbol... The symbol rate for the same data rate is  $1/2$  in QPSK... The bandwidth, B, is about  $1/2$  and the noise allowed into the receiver is  $1/2$ .



But the amount of noise  $\Delta h$  to cause an error in QUPSK is  $1/\sqrt{2}$  as large as in BPSK, draw sketch... Since the phaser diagram represents voltage, in power the noise must be  $(1/\sqrt{2})^2$  or  $1/2$  as much. Therefore, with half the bandwidth and half the noise QPSK and BPSK have the same error rate, i.e. transmit power is the same.

- What about 16-ARY Amplitude Phase Shift?

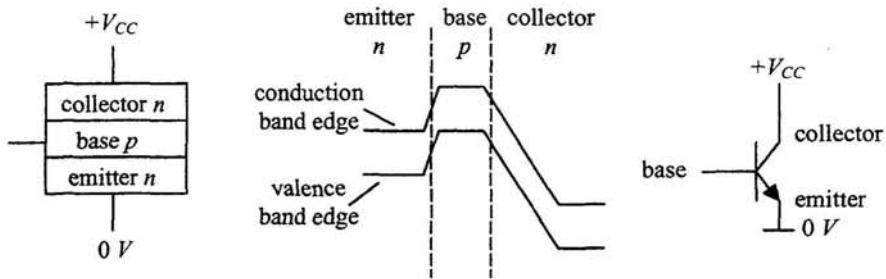
Noise in QPSK must be half of BPSK so power density will be twice of BPSK.

You would need  $1/4$  the bandwidth of BPSK since 16 symbol alternatives can represent 4 bits. But  $\Delta h$  is much smaller and noise must be much less than  $1/4$ , i.e. needs more power.

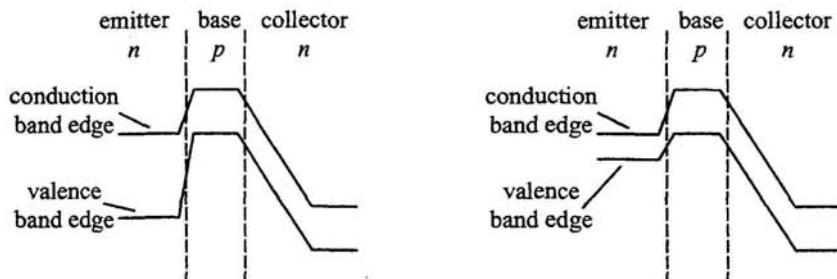


Runs into Shannon's Law!!!

Consider an *npn* bipolar transistor. This is shown below in a layer diagram, a simplified band diagram, and in its conventional circuit symbol with bias voltages. The device is biased in a typical manner, as shown in the layer and circuit diagrams.



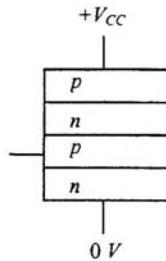
- Explain why, when the base voltage is increased, more current flows between the emitter and the collector.
- In practice in such a transistor, there is some base current that flows in the transistor when a positive voltage is applied to the base terminal. This current limits the current gain of the transistor. Give two possible reasons why there is a finite base current.
- It is proposed to improve the current gain of the transistor by changing the material used for the emitter. Supposing for simplicity that the materials and interfaces are of very high quality regardless of what specific materials are chosen, explain which of the following two band diagrams is preferable for improving the current gain.



Supplementary Question

d) Suppose now we make an *n**p**n**p* structure and bias it as shown to the right. Explain what will happen to the current flowing "vertically" through the whole *n**p**n**p* structure if apply positive voltage to the lower *p* layer. (You may wish to think of this device as two transistors connected together in a particular way.)

e) Can you think of a use for this *n**p**n**p* device?



Sketch of solution

p-n junction forward bias

a) The electrons in the emitter have thermal energy. Applying a positive voltage to the base lowers the potential barrier seen by the electrons in the emitter, and they ones with sufficient energy can travel through the base to be swept into the collector. (More formal descriptions involving diffusion were also acceptable as long as the students understood that the electrons had to have thermal energy to get over the barrier.)



b) Two main mechanisms are that (i) some of the electrons in the base recombine with holes in the base, and (ii) there is also a current of holes that goes from the base to the emitter.



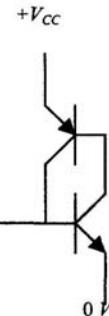
c) The diagram on the left is the better device because it has less current of holes from the base to the emitter as a result of the higher barrier for holes in the valence band. (This is a heterojunction bipolar transistor.)

Nearly all examinees got these first three parts, some with more prompting and correction than others.

d) The device can be drawn as an equivalent circuit with two transistors as shown below.

If base current is applied to the lower transistor, it starts to turn on, passing a larger collector current. This collector current is the base current for the upper transistor, which in turn causes a larger collector current to flow in the upper transistor, which in turn feeds more current into the base of the lower transistor, giving a regenerative mechanism that turns the device "on" into a highly conducting state.

Students were also asked orally, (i) what happens when you remove the external base current (answer: the device stays on), (ii) how would you turn the device off (in a really simple way (answer: remove the supply voltage!)), and (iii) whether the same regenerative mechanism would exist if the supply voltage polarity was reversed (answer: it would not).

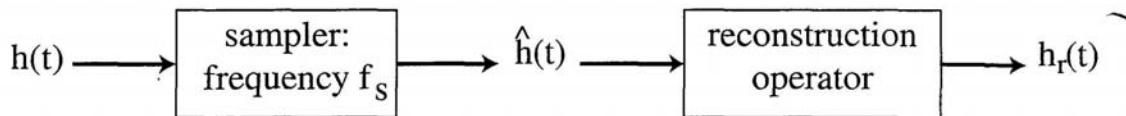


This device is known as a thyristor or silicon controlled rectifier (though it did not matter if the students did not know that).

- e) Almost no students knew the answer to this question when they started. The answer I was looking for is light dimmers, which are used in homes (and also university lecture theaters). The students were prompted to consider replacing  $V_{CC}$  with a large AC voltage, to add a series resistor, and try to devise a way of varying the average power in the resistor. The method used in practice is to vary what point in the positive AC cycle that the device is turned on (with the device then remaining on until the AC voltage goes through zero). Hence the average power in the load is controlled without substantial dissipation in the device (it either has high current and low voltage, or high voltage and no current). Creative answers were also accepted here (though I only got one!). After realizing how the device was used, some students realized it was used to make light dimmers. A surprising number of students had presumed that light dimmers were made simply using variable resistors (despite the large power dissipation that would result in the dimmer).

Students varied quite a lot in how far they got through parts (d) and (e), and these parts largely determined their final scores in the exam.

## Sample and Interpolate to Recover the Signal

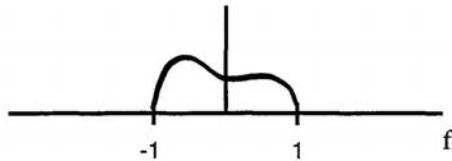


For the following cases:

State the *lowest* sampling frequency  $f_s$  necessary to recover  $h(t)$  from  $\hat{h}(t)$  (If possible)

Specify the required reconstruction operator.

1)  $h(t) = g(t)$ , with Fourier transform  $G(f)$



2)  $h(t) = \dot{g}(t)$

3)  $h(t) = g^3(t)$

4)  $h(t) = g(t) \exp(-i2\pi t)$

5)  $h(t) = g(t) \cos(6\pi t)$

+ assorted questions throughout; e.g.,

2) how to reconstruct  $g(t)$ ?

5) with your answer for  $f_s$ , is  $h(t)$  recoverable if sampler delayed by some  $\epsilon$ ?

*Fabian*

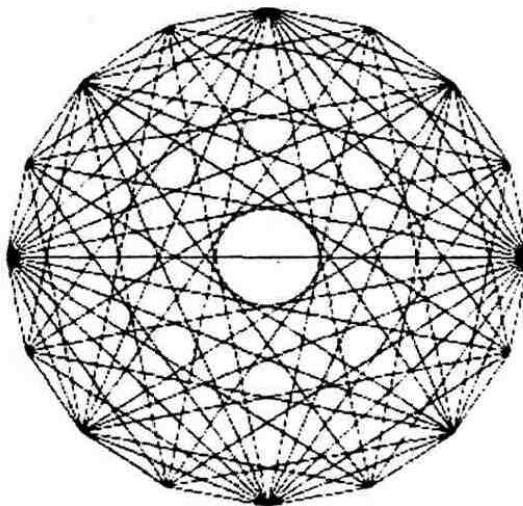
## Quals Question 2000, Pease

1. Why is the semiconductor industry spending \$B's on shrinking the features of IC's?
2. How do the speed and energy dissipated per clock cycle vary with linear dimensions (vertical and horizontal) when the speed is limited by the interconnect load (in a CMOS inverter ckt.)?
3. Why does the energy dissipated per clock cycle not depend upon the resistance when it is only in the resistance that energy is dissipated?
4. By driving the interconnect with a slow ramp instead of a step function can we lower the energy dissipated/clock cycle below  $CV^2$ ?
5. What fundamental factors (i.e. NOT lithography) limit how far we scale down dimensions of IC's?

Brad Osgood

EE Qualifying Exam  
Winter Quarter 2000

A friend of yours in computer graphics says: "I came across an intricate geometric design (shown below), but I can't figure out how to draw it." You say: "If you can do the actual drawing, I think I can find the points on the outer circle *and* how to join them, all with the help of the discrete Fourier transform. In fact, we can draw a whole series of these things. Oh, would it help if I first explained how it goes for four points . . ."



Date: Mon, 31 Jan 2000 01:54:49 -0700 (PDT)  
From: PIERO PIANETTA AT SSRL <pianetta@SLAC.Stanford.EDU>  
Subject: Re: Quals Question  
To: shankle@ee.stanford.edu

1. Describe a device that could be used to measure photons of visible light as well as the circuit in which it would be placed.

Two simple devices are a reverse biased PN junction or a silicon film between two ohmic contacts. In both cases the active area of the device must be thick enough to absorb all of the light and the light must be energetic enough to create electron hole pairs. The circuit would be a reverse bias for the PN junction or a simple bias for the resistive film with a current meter in series.

2. Assuming that  $1e10$  photons/sec are incident on the active area of the device, what is a first order estimate of the current that would be generated in the device.

Assume that all the photons are absorbed and that the device is sufficiently defect free that all the electrons generated by the photons can reach the contacts then the current is on the order of  $1e-9$  amps.

$$10^{10} \cdot 10^{-19} \approx 10^{-9}$$

3. In the case of the silicon film, determine the properties of the film that are required for it to be used to measure the above current.

The simplest answer involves realizing that  $1e-9$  amps is a very small current and that to have a dark current no more than 10% of the dark current, the resistance of the film must be on the order of  $1e9$  ohms for bias voltages on the order of 1 V.

$$1V / 1e9 = 10^{-9} \text{ amps}$$

Stanford University  
Department of Electrical Engineering

EE Quals. Jan 10-14, 2000.

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You are the manager of a large website which posts 10 million pages. When a new page arrives, it is timestamped and added to the site if there is room. If the site is full you would like to make room by evicting the oldest page from the site.

But there is a problem. Due to the sheer size of the website and the way in which pages are stored, it is very difficult to determine the oldest webpage.

So you use the following random procedure to approximate the eviction of the oldest page. You sample 10 pages at random from the website and evict the oldest of these.

**Question 1.** What is the probability that the evicted page belongs to the oldest 10<sup>th</sup> percentile of all pages in the website?

**Question 2.** Now suppose that you choose  $10 \times k$  samples, where  $k$  is a positive integer, and evict the oldest of these. What is the probability that the evicted page belongs to the oldest 10<sup>th</sup> percentile? How large should  $k$  be to make this probability bigger than 95%?

**Question 3.** Suppose you choose 30 samples. What is the probability that the oldest 2 pages in the sample belong to the oldest 10<sup>th</sup> percentile? Does it make sense to remember the second oldest page from the sample for the next eviction?

Balaji Prabhakar

X-Sender: quate@ee.stanford.edu (Unverified)  
Date: Thu, 20 Jan 2000 06:36:45 -0800  
To: Diane Shankle <shankle@ee.stanford.edu>  
From: Cal Quate <quate@ee.stanford.edu>  
Subject: Re: Quals Question

Diane

In the Quals I asked about Coulombs law and following this by asking the students to design an experiment that would show that the force varied as the inverse of the square of the distance between charges.

I, also, asked them to discuss the force between two parallel wires carrying equal currents.

cal quate

magnetic force is opposite to static electrical  
force

At 10:13 AM 1/19/00 -0800, you wrote:

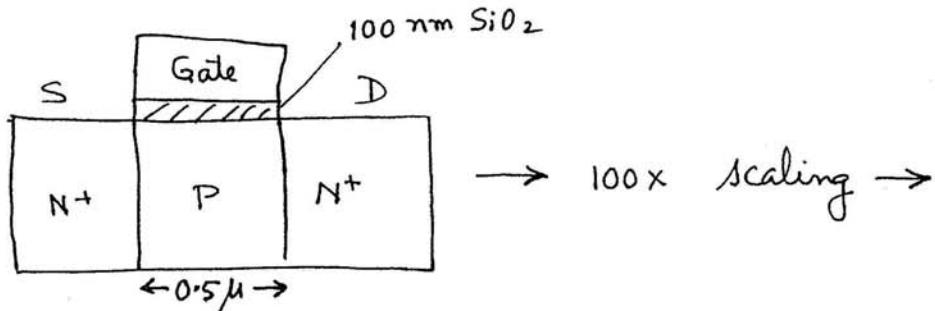
Please submit your Quals Question by email or hard copy.  
Due date Friday, January 21st.  
Thanks!  
Diane

Diane Joan Shankle

---

Tel: (650) 723-3194  
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Stanford, CA 94305-9505  
<http://www-ee.stanford.edu>

## Krishna Saraswat 2000 PhD Quals Question



My question was that if the device shown in the figure is scaled by a factor of 100 what would you expect?

I steered them on performance and reliability issues relating to an ultra short channel length of 5 nm and ultrathin gate oxide of 1 nm.

$$100 \text{ nm} \rightarrow 100 \rightarrow 1 \text{ nm} \approx 10 \text{ Å}$$

Radius of silicon is  $5 \text{ Å}$ , one layer of atoms

$$0.5 \mu\text{m} \rightarrow 100 \rightarrow 5 \text{ nm} \approx 50 \text{ Å}, 5 \text{ layers atoms}$$

very thin dielectric cause "quantum noise"

Electrical Engineering Quals Questions, Olav Solgaard, Jan. 2000

1. What is this? (Giving the students a diffractive lens)

*It is a diffractive lens.*

2. Consider a plano-convex lens with an incident plane wave (drawing on the board).

- a. What happens after the light passes through the lens?

*The light is focussed to a point one focal length behind the lens.*

- b. What are the phase relationships at the focus?

*All the different parts of the plane wave arrive at the focus in phase.*

- c. How is the focus altered if the different parts of the beam are out of phase by multiples of  $2\pi$  at the focus? (Monochromatic light)

*The focus is the unaltered.*

3. How can you use this information (answer to 2c) to explain the "flat" lens I just showed you?  
*You start with an ordinary refractive lens, remove cylinders corresponding to a phase difference of  $2\pi$ , and collapse the structure to a surface with phase steps of  $2\pi$ .*

4. What is the physical step size, d, on the surface of an ideal refractive lens? (Assume a vacuum wavelength,  $\lambda_0$ , of 500 nm, and an index, n, of 1.5 for the lens material.)

*By considering the phase of two parts of the beam that pass the lens on either side of the step, you find the following equation for the step height:*

$$d = \frac{\lambda_0}{(n-1)} = 1 \mu m$$

$$\frac{z\phi l}{\lambda_0} - \frac{z\phi l}{(\frac{\lambda_0}{n})} = 1 \\ ? \quad d = \frac{\lambda_0}{2(n-1)} = 0.5 \mu m$$



5. How do you explain the fact that the steps on the lens have an approximately constant spacing?

*The steps are increased from  $2\pi$  to higher integers of  $2\pi$  as we are going from the center towards the edge of the lens.*

6. What part of the lens is more efficient in spatially separating the spectral components of an incident wave?

*The edge. The larger phase steps are more dispersive.*

7. Consider an ideal diffractive lens with an incident plane wave. Assume a wavelength of 500 nm, a lens-material index of 1.5, and a focal length of 10 cm. What happens if we make the front surface of the lens reflecting (e.g. by applying a thin layer of metal that doesn't change the shape of the lens)?

*The light is reflected and spread as if originating from a point  $2.5 \text{ cm}$  behind the lens.*

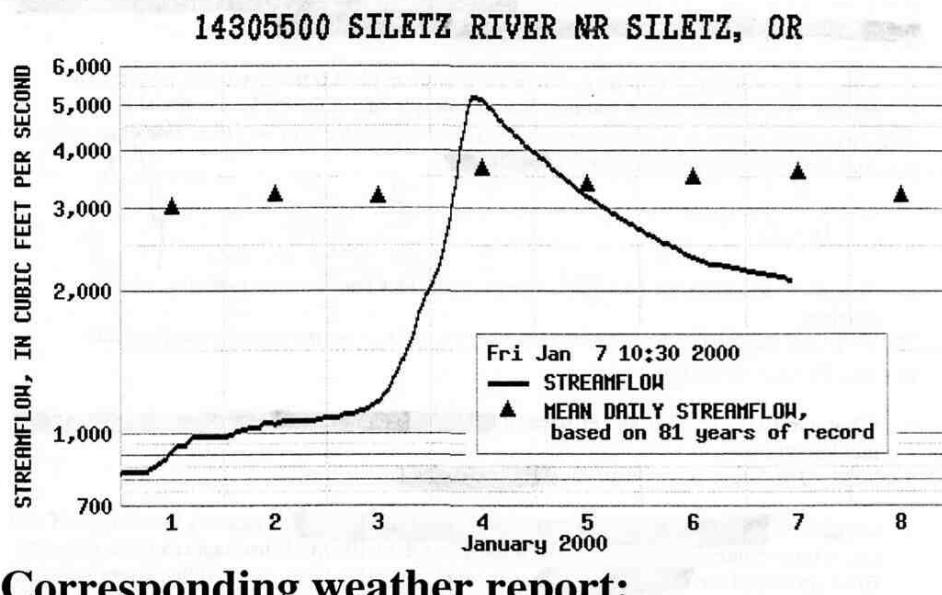


*how does 2.5cm come from?*

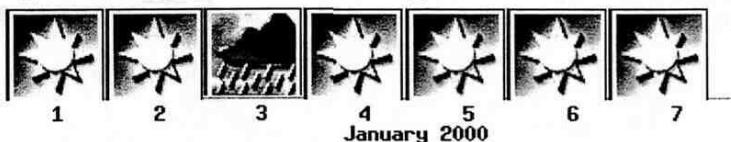
You've recently graduated and taken a new job in charge of the river monitoring station on the Siletz river. Your job is to provide continuous updates of the water flow to the National Weather Service (for flood warnings, etc.).

As a supervisor, you hire a graduate student to collect the data. As has been done for many years, you have the student measure the flow every 15 min, 24 hrs/day, 7 days/wk.

### Last week's data:



### Corresponding weather report:



Daniel Spielman

Date: Fri, 21 Jan 2000 11:45:07 -0800 (PST)  
From: "Fouad A. Tobagi" <tobagi@stanford.edu>  
To: Diane Shankle <shankle@ee.stanford.edu>  
Cc: Fouad Tobagi <tobagi@stanford.edu>  
Subject: Re: Quals Question

Quals Question - January 2000:

Consider the design of a Video-On-Demand Server where digitized and compressed video is stored and from which it is streamed to users over a network. It is important that the design be scalable and flexible. What are the principal components of the video server and their underlying design parameters? What are the design issues and tradeoffs? The focus is on the [storage aspects and data flow aspects.]

Fouad A. Tobagi Tel: (650) 723-1708  
Professor of Electrical Engineering Fax: (650) 725-6221  
and by courtesy, Computer Science Email: Tobagi@stanford.edu

On Wed, 19 Jan 2000, Diane Shankle wrote:

> Please submit your Quals Question by email or hard copy.  
> Due date Friday, January 21st.  
> Thanks!  
> Diane  
>  
> Tel: (650) 723-3194  
> FAX: (650) 723-1882  
> shankle@ee.stanford.edu  
> Stanford University  
> Dept of Electrical Engineering  
> Packard Building Rm. 165  
> Stanford, CA 94305-9505  
> <http://www-ee.stanford.edu>  
>

Diane Joan Shankle

Cpu. memory

may need dedicated video decoder chip if we have to decompress video

latency, jitter

To: Diane Shankle <shankle@ee.stanford.edu>  
Subject: Re: Quals Question  
Date: Wed, 19 Jan 2000 12:00:36 -0800  
From: Jennifer Widom <widom@DB.Stanford.EDU>

Jennifer Widom's EE Quals Question, 2000  
=====

Consider a database that stores a set of data items that are numeric values. However, instead of storing the values themselves, the database stores a range for each value that the actual value is known to lie in. For example, the database might contain:

Item 1: [ 4, 9]  
Item 2: [-8, 7]  
Item 3: [ 6, 6]  
Item 4: [-3, -1]  
Item 5: [ 0, 5]  
Item 6: [ 2, 10]  
Item 7: [-3, 0]  
Item 8: [ 2, 4]

In the general case, the database contains a set of n items specified as low and high values: { [L<sub>1</sub>, H<sub>1</sub>], [L<sub>2</sub>, H<sub>2</sub>], ..., [L<sub>n</sub>, H<sub>n</sub>] }

Question 1

Suppose we are interested in finding the maximum value in our data set, and computing the sum of the values. Since we don't know the actual values, the computed maximum and sum must also be ranges.

Give a general specification for the maximum (range) [L<sub>m</sub>, H<sub>m</sub>] and the sum (range) [L<sub>s</sub>, H<sub>s</sub>] as functions over the items in the set: { [L<sub>1</sub>, H<sub>1</sub>], [L<sub>2</sub>, H<sub>2</sub>], ..., [L<sub>n</sub>, H<sub>n</sub>] }. Also illustrate the maximum and sum results over the sample data.

Answer 1

MAXIMUM: [L<sub>m</sub>, H<sub>m</sub>] = [max(L<sub>i</sub>, i=1..n), max(H<sub>i</sub>, i=1..n)]  
on sample data: [6, 10]

SUM: [L<sub>s</sub>, H<sub>s</sub>] = [L<sub>1</sub>+L<sub>2</sub>+...+L<sub>n</sub>, H<sub>1</sub>+H<sub>2</sub>+...+H<sub>n</sub>]  
on sample data: [0, 40]

Question 2

Now suppose we have a way of finding the precise value for each data item, but it is expensive to do so. Further suppose that we are interested in computing our sum within a certain precision, i.e., we want to guarantee that the result range is no wider than some specified number W. We would like to find the smallest subset of our data items such that if we get the precise values for those items, no matter what the values turn out to be (within the known bounds), our sum result is guaranteed to have the desired precision.

For example, the sum over the sample data is [0, 40]. Suppose that we want a result with a width of no more than 10, and the precise values for our sample data items are:

```
Item 1: [ 4,  9] (value is 6)
Item 2: [-8,  7] (value is -4)
Item 3: [ 6,  6] (value is 6)
Item 4: [-3, -1] (value is -1)
Item 5: [ 0,  5] (value is 0)
Item 6: [ 2, 10] (value is 9)
Item 7: [-3,  0] (value is -1)
Item 8: [ 2,  4] (value is 2)
```

If we get the precise values for items 1, 2, 5, and 6, then our new data items are:

```
Item 1: [ 6,  6] (value is 6)
Item 2: [-4, -4] (value is -4)
Item 3: [ 6,  6] (value is 6)
Item 4: [-3, -1] (value is -1)
Item 5: [ 0,  0] (value is 0)
Item 6: [ 9,  9] (value is 9)
Item 7: [-3,  0] (value is -1)
Item 8: [ 2,  4] (value is 2)
```

and our new sum = [13, 20] satisfies our precision constraint. In fact, no matter what values items 1, 2, 5, and 6 take within their known ranges, our precision constraint will be met.

Suggest a general rule for picking the fewest items for which, when we can get their precise values, the sum is guaranteed to have a width no greater than W.

Answer 2

---

Let  $[L_s, H_s]$  be the result of the sum before getting any precise values. Suppose that we need to "shrink" this range (i.e., reduce  $H_s - L_s$ ) by  $S$  in order to meet our width constraint  $W$ . If we get the precise value for an item  $[L_i, H_i]$  then the width of the sum result shrinks by  $(H_i - L_i)$ . Therefore we should select the fewest number of data items such that the sum of their widths is equal to or greater than  $S$ . We can do so by picking the items in order by decreasing width.

Question 3

---

Consider the same problem as for Question 2 but for maximum instead of sum. For example, the maximum over the sample data is [6, 10]. Suppose that we want a result with a width of no more than 2, and the precise values for our sample data items are:

```
Item 1: [ 4,  9] (value is 6)
Item 2: [-8,  7] (value is -4)
Item 3: [ 6,  6] (value is 6)
Item 4: [-3, -1] (value is -1)
Item 5: [ 0,  5] (value is 0)
Item 6: [ 2, 10] (value is 9)
Item 7: [-3,  0] (value is -1)
Item 8: [ 2,  4] (value is 2)
```

If we get the precise values for items 1 and 6, then our new data items are:

Item 1: [ 6, 6] (value is 6)  
Item 2: [-8, 7] (value is -4)  
Item 3: [ 6, 6] (value is 6)  
Item 4: [-3, -1] (value is -1)  
Item 5: [ 0, 5] (value is 0)  
Item 6: [ 9, 9] (value is 9)  
Item 7: [-3, 0] (value is -1)  
Item 8: [ 2, 4] (value is 2)

and our new maximum = [9, 9] satisfies our precision constraint. In fact, no matter what values items 1 and 6 take within their known ranges, our precision constraint will be met.

Suggest a general rule for picking the fewest items for which, when we can get their precise values, the maximum is guaranteed to have a width no greater than W.

Answer 3

-----

Let  $[L_m, H_m]$  be the result of the maximum before getting any precise values. We will get the precise value for each data item  $[L_i, H_i]$  such that  $H_i > (L_m + W)$ . The reasoning is as follows. Consider an item  $[L_i, H_i]$  such that  $H_i > (L_m + W)$ . Suppose we don't get the precise value for this item, and for all items that we do get, they have their lowest possible values. Then the result will be  $[L_m, H_j]$  with  $H_j \geq H_i$ , and our precision constraint will not be met. If we get the precise values for all such items  $[L_i, H_i]$ , then one of two things happens: (1) Some precise value turns out to be the actual maximum (as in our example), so the result has width 0; or (2) All precise values are smaller than some other  $H_j$ , in which case our precision constraint is met since  $H_j \leq (L_m + W)$ .



Quals Question  
January 2000  
Tom Cover

**Question:** Can a mouse smell the direction of a piece of cheese? A drawing of a piece of cheese and a fixed mouse with two nostrils was provided.

**Purpose of question:**

This is really a question about stochastic processes, Brownian motion in particular, and was designed to cover the sort of material given in a course like EE278, Statistical Signal Processing, or an equivalent course given at another university. The entire 10 minutes of the exam was devoted to this question.

Some interesting wrong approaches could be given. For example, many people leapt to the conclusion that the intensity of the cheese smell fell off like  $\frac{1}{r^2}$ . When questioned about this, they came dangerously close to suggesting that there were such things as cheese waves and a wave equation for cheese.

Most people correctly identified, after a minute or two, the process as a random walk or Brownian motion. This led naturally to questions of gamblers ruin, waiting time to absorption, multivariate Gaussian distributions, etc.

I made it a point to ask one specific question along the way. Change to 1 dimension. If the cheese is located at a point somewhere in the unit interval, what is the relative probability of absorption at each of the end points? Since the random walk has no drift, this becomes a problem with two equations and two unknowns. And the probability of being absorbed at 1 rather than at 0, given a starting point  $\alpha$  is equal to  $\alpha$ .

How does the information about the direction of the cheese (given two nostrils) change with time? It turns out that the first whiff of the cheese gives the most directional information. As time goes on, the gradient in cheese smell tends to 0.

Here I might mention a great answer. I asked the question, is the direction of the cheese known perfectly after a sufficient time so that all the cheese molecules have either been absorbed by the left nostril or the right nostril? The answer came back: No, but after all the cheese molecules have been absorbed, the mouse is no longer interested in the direction of the cheese.

---

# Qualifying Examination 2000

## Logic Design

Giovanni De Micheli

January 2000

You have to implement several combinational logic circuits in a new technology. This technology uses a 2-input multiplexer as primitive. Input variables should connect to the select input of the multiplexers only.

- Explain how you would implement an arbitrary single-output logic function using the primitive.

*The easiest way is to show how ANDs, ORs and INVs can be realized by muxes. The desired answer was to show how the recursive Shannon expansion of a function can be implemented by multiplexers*

- Give an example using the function  $f = ac + bc$

Use recursive Shannon expansion.  $f = ac + bc = a(c) + a'(bc)$ . We use a mux controlled by  $a$  with inputs  $c$  and  $bc$ . Next:  $bc = b(c) + b'(0)$ . Thus  $bc$  can be implemented by a mux controlled by  $b$ . Last, a multiplexer driven by  $c$  can be used to generate signal  $c$  to be fed to both the first and second multiplexers.

- Is the implementation unique? If not, how can I find the implementation with a minimum number of muxes?

*The implementation is not unique. The size of the implementation depends on both the function and on the procedure. As far as the function is concerned, the number of support variables affects the size. If the function is fixed, the number of muxes depends on the procedure in two ways. First, it depends on the order of the variables chosen for the Shannon expansion. Second, it depends on any redundancy that may appear in the circuit. For any given variable order, redundancy should be eliminated by sharing signals representing common sub-functions and eliminating multiplexers with identical inputs.*

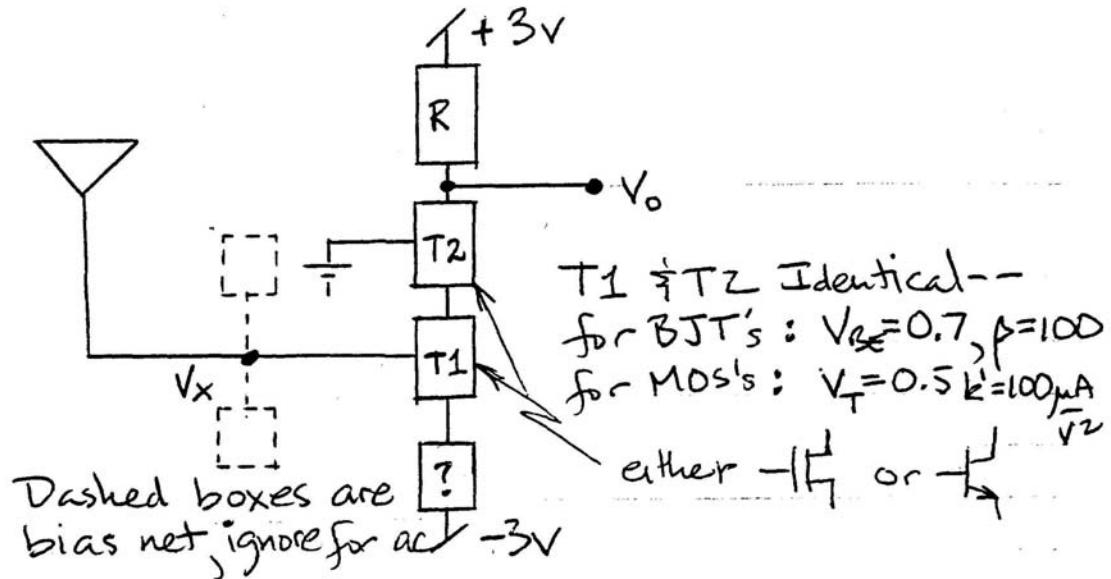
Ben Van Roy

There are two neighboring cities, Boston and Cambridge. Each year, the population each city multiplies by  $5/4$ . There is also at the end of each year a migration between the cities proportional to the difference between prevailing populations. A migration rate  $\alpha$  specifies the constant of proportionality here. To simplify things, assume that populations need not be integer valued.

1. Write a discrete time linear equation describing how populations of the two cities change each year.
2. Define the balance  $b_t$  to be the ratio of populations (Boston/Cambridge) on year  $t$ . For what values of  $\alpha$  will the balance converge to 1?
3. For what values of  $\alpha$  would the direction of migration oscillate?

$$X_{n+1} = \frac{5}{4}X_n + \alpha(Y_n - X_n)$$

$$Y_{n+1} = -\frac{5}{4}Y_n - \alpha(Y_n - X_n)$$



Given circuit topology above, choose one: either BJT's or MOS's for T<sub>1</sub> and T<sub>2</sub> and do the following:

1. Explain strategy in choosing bias  $V_x$  and select  $V_x$  and components to get small signal gain from antenna to  $V_o$  (the box with ? you can choose any component or short it)
2. What is small signal gain? How could you change circuit to improve it?
3. What is impedance seen by antenna looking into transistor? What can you do to make it a good match to antenna?
4. What are dominant noise sources in amplifier?

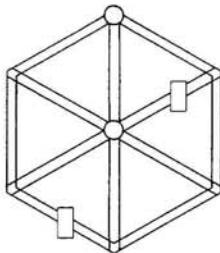
#### Note:

As a principle, I do not give an "answer", the exam is about a process of your thinking, explaining and interacting. Certainly details on this question can be found in notes for EE113 and EE133 as well as standard texts such as Gray and Meyer, Analysis and Design of Analog Integrated Circuits.

## 1999-2000 Qualifying Examination Questions

JOHN GILL

The Hexagon has two sets of corridors as shown in the figure below. Each corridor segment is 1 km long. These corridors contain a very large number of light bulbs, which burn out at random independent times. Two battery-powered robots (represented by rectangles) are available to replace the light bulbs. There are two recharging stations (circles), one at the top of the Hexagon and one in the center.



One robot is dedicated to the outside (circumferential) corridors and the other robot serves the inside (radial) corridors.

### Primary questions

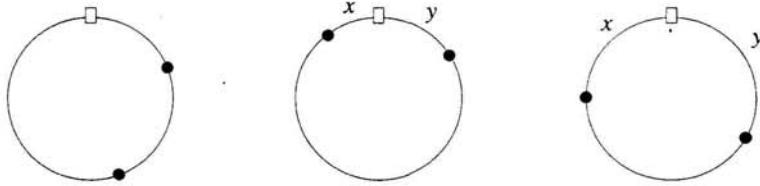
1. Suppose that the outside robot returns to the charging station at the top of the Hexagon after replacing a light bulb. On average how far does it travel to change the light bulb and return to the charging station?
2. Suppose that the outside robot waits until two light bulbs have burned out. It then travels by the most efficient overall path to replace the two bulbs, finally returning to the charging station. On average how far does it travel?

### Secondary and bonus questions

3. What is the average distance traveled by the outside robot to change a single light bulb if it remains at the location of the last replaced light bulb?
4. What is the average distance traveled by the inside robot to change a single light bulb and return to the charging station at the center?
5. What is the average distance traveled by the inside robot to change a single light bulb if it remains at the location of the last replaced light bulb?

### Answers to primary questions

1. The maximum round trip is 6 km, since the robot travels by the shortest path. The distance is uniform from 0 to 6, so the average round trip is 3 km.
2. With probability  $\frac{2}{4} = \frac{1}{2}$ , the two light bulbs are on the same side of the Hexagon. In this case, the robot travels to the closer light bulb, then to the more distant one, then returns by the reverse path. The distance to the farther light bulb is a random variable that is the maximum of two independent random variables that are uniformly distributed from 0 to 3. The average value of the distance is  $\frac{2}{3}$  of the maximum, so the average round trip in this case is 4 km.



When the light bulbs are on opposite sides of the Hexagon, the robot has two possible routes. If the bulbs are close together, the robot travels first to one, then returns to the charging station, then travels to the other and returns. The total distance is  $2x + 2y$ . If the light bulbs are far apart, it is more efficient to travel to one, then continue in the same direction to the second, then continue in the same direction to return to the charging station, for a total distance of 6. The robot chooses the shorter of these two paths, depending on whether  $x + y < 3$ . Since  $x$  and  $y$  are independent and uniformly distributed from 0 to 3, their sum  $x + y$  ranges from 0 to 6 and  $\Pr(x + y < 3) = \frac{1}{2}$ . The conditional probability density for  $z = x + y$  given  $x + y < 3$  is the triangle  $f(z) = 2z/9$ , so the conditional expectation is  $x + y$  is 2 and the average round trip in this case is 4. Obviously, when  $x + y > 3$  the average distance is 6. The overall average round trip distance is

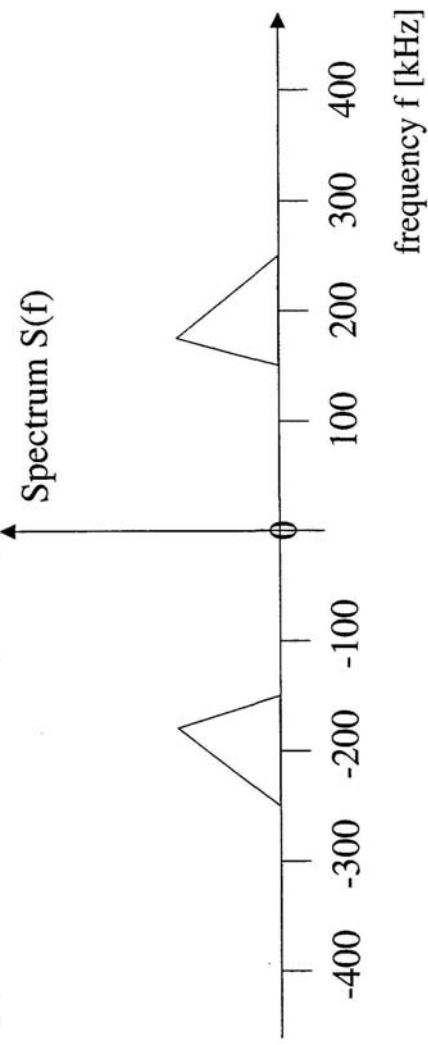
$$\frac{1}{2} \cdot 4 + \frac{1}{4} \cdot 4 + \frac{1}{4} \cdot 6 = 4\frac{1}{2},$$

which is obtained by combining the conditional expectations for the three cases.

### Answers to secondary and bonus questions

3. The locations of successive light bulbs are independent, so distance from one to the next by the shortest path is a maximum of 3 km and an average of 1.5 km.
4. The maximum round trip for the inside robot is 2 km, so the average is 1 km.
5. With probability  $\frac{5}{6}$  the next light bulb is in another radial corridor, in which case the average distance is  $\frac{1}{2} \cdot 2 = 1$ . With probability  $\frac{1}{6}$ , the next light bulb is in the same corridor, in which case the average distance is  $\frac{1}{3}$ . The overall average round trip for the inside robot is  $\frac{5}{6} + \frac{1}{18} = \frac{8}{9}$ .

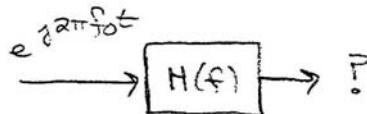
A time-continuous, real-valued bandpass signal  $s(t)$  with spectrum  $S(f)$  shall be represented by samples, such that a perfect (i.e. aliasing-free) interpolation from these samples is possible.



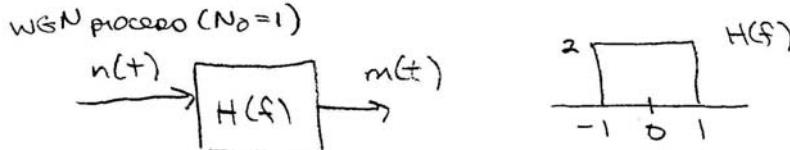
- What do we mean by „aliasing“?
- What is the lowest sampling rate possible?
- Sketch the spectrum after sampling
- Sketch the impulse response of the interpolation filter

Quals Question - Winter 2000 - Goldsmith

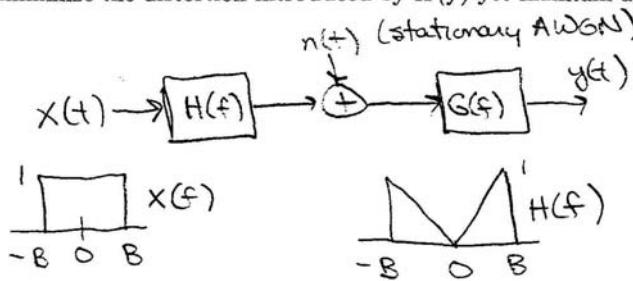
1. What is a linear time-invariant filter?
2. If you input a complex exponential into a linear time-invariant filter with frequency response  $H(f)$ , as shown below, what is the output signal?



3. If you are given a black box with a linear time-invariant filter inside (with an input wire and an output wire), how would you obtain the frequency response of the filter through measurements in the lab?
4. Can you have a signal that is both time-limited and band-limited? Give an example or prove that it is not possible.
5. What is a random process?
  - How do we usually characterize random processes?
  - Do all random processes have a power spectral density?
  - What is a white Gaussian random process?
  - If a white Gaussian process is sampled in time, what property do these samples have?
  - Why are uncorrelated Gaussian random variables also independent?
6. For the filter shown below with a white Gaussian random process as input, what is  $E[m(.5)m(1)]$ .



7. For the communication system shown below, if you neglect the noise ( $n(t) = 0$ ), what  $G(f)$  gives  $x(t) = y(t)$ . Now using this  $G(f)$ , what is the SNR at the output for  $n(t)$  and white noise process with power spectral density  $N_0$ . How would you alter the design of  $G(f)$  to minimize the distortion introduced by  $H(f)$  yet maintain a high SNR.



Gray's January 2000 EE Quals Question

A system  $\mathcal{S}$  has as input a real-valued discrete-time (DT) signal  $\dots, x_{-2}, x_{-1}, x_0, x_1, x_2, \dots$ . The output of the system is a continuous-time (CT) signal

$$v(t) = \sum_{n=-\infty}^{\infty} x_n \text{sinc}(t - n), \quad t \in (-\infty, \infty),$$

where  $\text{sinc}(t) = (\pi t)^{-1} \sin(\pi t)$ .

---

1. Is the system  $\mathcal{S}$  linear? causal?
2. Find a simple expression for  $\int_{-\infty}^{\infty} v(\tau) \text{sinc}(t - \tau) d\tau$ .
3. Find the energy  $\mathcal{E}(v) = \int_{-\infty}^{\infty} |v(t)|^2 dt$ .
4. If the input signal has a DT Fourier transform

$$X(f) = \sum_{n=-\infty}^{\infty} x_n e^{-j2\pi f n},$$

find the output CT Fourier transform

$$V(f) = \int_{-\infty}^{\infty} v(t) e^{-j2\pi f t} dt.$$


---

### *Solutions*

Ideally students would recognize the similarity of the formula defining  $v$  to the sampling theorem, which provides some shortcuts. In particular, since  $\text{sinc}(k)$  is 1 for  $k = 0$  and 0 for all other integer  $k$ ,  $v(n) = x_n$  for all  $n$  so that the inputs  $x_n$  can be considered as the samples of the output CT signal  $v$ . Another useful observation is that  $v(t)$  is formed by interpolating the  $x_n$  in the same way that the sampling theorem reconstruction interpolates using sincs.

1. Is the system  $\mathcal{S}$  linear? causal?

**Solution:** The system is linear because a linearly weighted combination of input signals will yield the corresponding linearly weighted combination of output signals. It is not causal since the output at any time depends on the infinite future of the input as well as on the past.

2. Find a simple expression for  $\int_{-\infty}^{\infty} v(\tau) \text{sinc}(t - \tau) d\tau$ .

**Solution:** The simplest way to solve this is to observe that the integral is a convolution of  $v$  with a sinc, which in the frequency domain corresponds to multiplying the Fourier transform of  $v$ , say  $V$ , by a rect or box function, i.e., it is a low pass filtered over  $[-1/2, 1/2]$ . But from its definition,  $v(t)$  is simply a linear combination of shifted sinc functions, all of which have 0 frequency content outside of  $[-1/2, 1/2]$  (since their Fourier transforms are just rects multiplied by complex exponentials), hence  $V$  is unchanged by multiplication with a rect, hence the integral evaluates to  $v$ .

This result can also be obtained mathematically with more care, an o.k. but longer approach. Two possible ways are the following:

**Method I:** As above we know the Fourier transform of the integral is  $V(f) \Pi(f)$ , where  $\Pi(f)$  is 1 for  $|f| \leq 1/2$  and 0 otherwise. This can be evaluated by evaluating  $V(f)$ , the CT Fourier transform of  $v$  which is

$$\begin{aligned}
V(f) &= \int_{-\infty}^{\infty} v(t) e^{-j2\pi ft} dt \\
&= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x_n \text{sinc}(t-n) e^{-j2\pi ft} dt \\
&= \sum_{n=-\infty}^{\infty} x_n \int_{-\infty}^{\infty} \text{sinc}(t-n) e^{-j2\pi ft} dt \\
&= \sum_{n=-\infty}^{\infty} x_n \Pi(f) e^{-j2\pi fn} \\
&= \Pi(f) \sum_{n=-\infty}^{\infty} x_n e^{-j2\pi fn}
\end{aligned} \tag{1}$$

using the fact that the Fourier transform of a shifted sinc is simply a rect weighted by a complex exponential. Obviously  $V(f)\Pi(f) = V(f)$  from this formula, yielding the conclusion as above.

### Method II

$$\begin{aligned}
\int_{-\infty}^{\infty} v(\tau) \text{sinc}(t-\tau) d\tau &= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x_n \text{sinc}(\tau-n) \text{sinc}(t-\tau) d\tau \\
&= \sum_{n=-\infty}^{\infty} x_n \int_{-\infty}^{\infty} \text{sinc}(\tau-n) \text{sinc}(t-\tau) d\tau
\end{aligned}$$

assuming the integral and sum can be exchanged. For each  $n$  the integral is simply  $\text{sinc}(t-n)$  since the convolution of two sinc functions is another sinc function (since in the Fourier domain the product of two rects is another rect). This can be shown in more detail using Parceval's theorem. Thus

$$\int_{-\infty}^{\infty} v(\tau) \text{sinc}(t-\tau) d\tau = \sum_{n=-\infty}^{\infty} x_n \text{sinc}(t-n) = v(t).$$

- Find the energy  $\mathcal{E}(v) = \int_{-\infty}^{\infty} |v(t)|^2 dt$ .

Solution: The quick way here was to realize that  $x_n = v(n)$  and hence the result follows from the sampling theorem. Another way was to observe that the sinc functions are orthonormal in the sense that

$$\int_{-\infty}^{\infty} \text{sinc}(t-n) \text{sinc}(t-k) dt = \delta_{n-k},$$

where  $\delta_l$  is the Kroncker delta, 1 if  $l = 0$  and 0 otherwise. Thus Parceval's theorem using sincs as basis functions immediately gives this result.

A third way is to essentially prove the previous argument by plugging in and massaging:

$$\begin{aligned}
\int_{-\infty}^{\infty} |v(t)|^2 dt &= \int_{-\infty}^{\infty} \left| \sum_{n=-\infty}^{\infty} x_n \text{sinc}(t-n) \right|^2 dt \\
&= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x_n \text{sinc}(t-n) x_k \text{sinc}(t-k) dt \\
&= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x_n x_k \int_{-\infty}^{\infty} \text{sinc}(t-n) \text{sinc}(t-k) dt
\end{aligned}$$

$$\begin{aligned}
&= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x_n x_k \delta_{n-k} \\
&= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} |x_n|^2
\end{aligned}$$

A fourth way is to use the ordinary Parceval's theorem and Equation (4), but this way gets messy.

A fifth way (the shortest mathematical way) is to write

$$\begin{aligned}
\int_{-\infty}^{\infty} |v(t)|^2 dt &= \int_{-\infty}^{\infty} v(t) \sum_{n=-\infty}^{\infty} x_n \text{sinc}(t-n) dt \\
&= \sum_{n=-\infty}^{\infty} x_n \int_{-\infty}^{\infty} v(t) \text{sinc}(t-n) dt \\
&= \sum_{n=-\infty}^{\infty} |x_n|^2
\end{aligned}$$

since  $\int_{-\infty}^{\infty} v(t) \text{sinc}(t-n) dt = v(n) = x_n$ .

4. If the input signal has a DT Fourier transform

$$X(f) = \sum_{n=-\infty}^{\infty} x_n e^{-j2\pi f n},$$

find the output CT Fourier transform

$$V(f) = \int_{-\infty}^{\infty} v(t) e^{-j2\pi f t} dt.$$

Solution: Just plug into  $V$  the definition of  $v$  to find  $V(f)$  (see equation ( above) and compare to the definition of  $X(f)$  to find  $V(f) = X(f) \square(f)$ ).

5. Suppose that  $\{x_n\}$  has a DT Fourier series

$$x_n = \sum_{k=0}^{N-1} a_k e^{j2\pi \frac{k}{N} n}$$

where  $N$  is a fixed positive integer. Find a CT Fourier series representation for  $v(t)$ .

Solution:

Here I was mainly after comments on the form of the CT Fourier series and what was needed to find it. In particular, it had to be argued or shown that  $v(t)$  was periodic and the period had to be found. General comments on the Fourier coefficients are good guesses with a suggestion of how to back them up sufficed.

A CT Fourier series has the similar form

$$v(t) = \sum_k b_k e^{j2\pi \frac{k}{T} t}$$

the trick is to determine  $T$  and  $b_k$ . Since  $x_n = v(n)$  for all integer  $n$ , a good guess is that the period  $T$  is just  $N$  and that the formula for  $v(t)$  should look like the interpolation of  $x_n$ :

$$v(t) = \sum_{k=0}^{N-1} a_k e^{j2\pi \frac{k}{N} t},$$

i.e., use the same coefficients and just replace the integer  $n$  by  $t$ . To check the period, it needs to be shown that  $v(t) = v(t + N)$  for all  $t$ . From the definition of  $v$

$$v(t + N) = \sum_{n=-\infty}^{\infty} x_n \text{sinc}(t - n + N).$$

Make the summation dummy variable  $k = n - N$ :

$$v(t + N) = \sum_{k=-\infty}^{\infty} x_{k+N} \text{sinc}(t - k) = \sum_{k=-\infty}^{\infty} x_{k+N} \text{sinc}(t - k) = v(t)$$

since  $x_{n+N} = x_n$  for all  $n$ . Thus  $v$  indeed has period  $N$  and must have a Fourier series. Next note that

$$\begin{aligned} v(t) &= \sum_{n=-\infty}^{\infty} x_n \text{sinc}(t - n) \\ &= \sum_{n=-\infty}^{\infty} \sum_{k=0}^{N-1} a_k e^{j2\pi \frac{k}{N} n} \text{sinc}(t - n) \\ &= \sum_{k=0}^{N-1} a_k \sum_{n=-\infty}^{\infty} e^{j2\pi \frac{k}{N} n} \text{sinc}(t - n) \end{aligned}$$

This will indeed have the form of a CT Fourier series with period  $N$  and coefficients  $b_k = a_k$  for  $k = 0, 1, \dots, N - 1$  and zero otherwise if it is true that

$$e^{j2\pi t \frac{k}{N}} = \sum_{n=-\infty}^{\infty} e^{j2\pi \frac{k}{N} n} \text{sinc}(t - n)$$

for all real  $t$ , but this is simply the sampling theorem applied to a complex exponential of frequency  $k/N$ . This is not rigorous, but any suggestions along these lines were sufficient.

PH.D. QUALS (Jan 2000).

T. KAILATH

1. Let  $X(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi k n}{N}}$ , the DFT of  $\{x(n)\}$

What is the result of applying the DFT  $L$  times in succession to  $\{x(n)\}$ .

Solution : Recall  $x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{+j \frac{2\pi k n}{N}}$

So if  $L = 2$ , we get  
 $x(n) \xrightarrow{X(k)} y(n) = x(-n) \triangleq x(N-n)$

For  $L = 3$ , we get  $X(-k)$  and so on.

2. Consider a single-input, single-output system

$$x(k+1) = Ax(k) + bu(k), \quad y = c^T x(k) + du(k)$$

or 
$$\begin{bmatrix} x(k+1) \\ y(k) \end{bmatrix} = \begin{bmatrix} A & b \\ c & d \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix}$$

a) The system is i) "lossless", i.e.,  $\forall k$   
 $x^*(k+1)x(k+1) + y^*(k)y(k) = x^*(k)x(k) + u^*(k)u(k)$

b) The pair  $\{A, b\}$  is controllable.

SHOW that  $A$  is a stable matrix, i.e., that

$|\lambda_i(A)| < 1$ , where  $\{\lambda_i(A)\}$  are the eigenvalues of  $A$ .

Solution:

a) Losslessness implies that  $M \triangleq \begin{bmatrix} A & b \\ c & d \end{bmatrix}$

is unitary.

$$M^* M = I = MM^* = \begin{bmatrix} AA^* + bb^* & - \\ - & - \end{bmatrix}$$

so that  $AA^* + bb^* = I$

b) The PBH criterion for controllability relates controllability to the eigenvalues of  $A$ . Specifically it states that if  $v$  is a left eigenvector of  $A$  i.e.,  $vA = \lambda v$ , then  $vb \neq 0$  iff  $\{A, b\}$  is control

So from

$$v(AA^* + bb^*)v^* = vIv^*$$

$$\text{or } \lambda vAA^*v^* + (vb)(v^*b^*) = vv^*, \text{ since } Av^* = \lambda v^*$$

$$\text{or } |\lambda|^2 vv^* + |vb|^2 = vv^*$$

$$\text{or } 0 < |vb|^2 = (\cancel{|\lambda|^2}) (1 - |\lambda|^2) \underbrace{vv^*}_{>0}$$

Hence  $1 - |\lambda|^2 > 0 \quad \text{or} \quad |\lambda| < 1$ .

1. What is the Laplace transform? What is the Fourier transform? What is the relationship between the Laplace transform and the Fourier transform of a continuous-time signal?

2. What is the Z transform? What is the discrete-time Fourier transform (DTFT)? What is the relationship between the Z transform and the DTFT of a discrete-time signal?

Teresa Meng

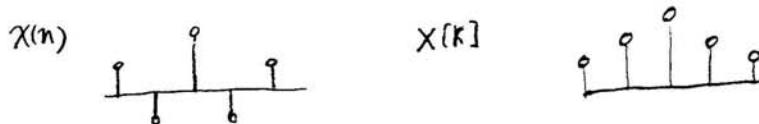
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January 11, 2000

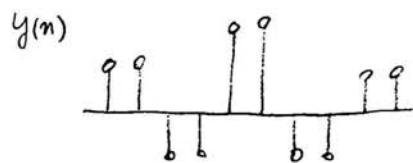
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3. What is the relationship between the discrete Fourier transform (DFT) and DTFT of a finite-length discrete-time signal?

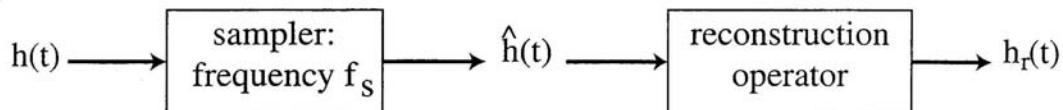
4. Assume that  $x(n)$  and  $X[k]$  are a DFT pair:



How would you predict the DFT of  $y(n)$  where  $y(n)$  is the sequence generated by repeating every sample of  $x(n)$ ?



**ANSWERS TO QUESTIONS**  
**Sample and Interpolate to Recover the Signal**

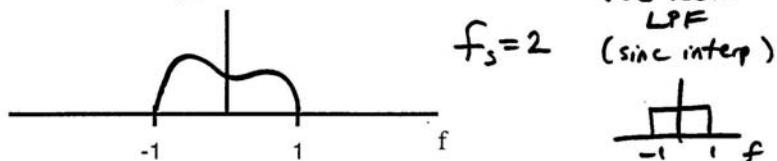


For the following cases:

State the *lowest* sampling frequency  $f_s$  necessary to recover  $h(t)$  from  $\hat{h}(t)$  (If possible)

Specify the required reconstruction operator.

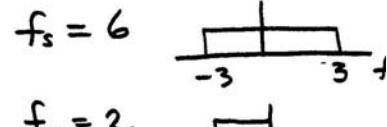
1)  $h(t) = g(t)$ , with Fourier transform  $G(f)$



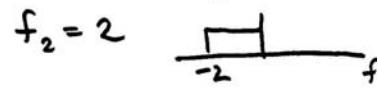
2)  $h(t) = \dot{g}(t)$

same answer as 1)

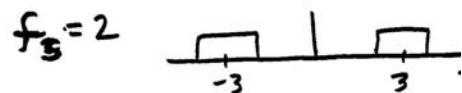
3)  $h(t) = g^3(t)$



4)  $h(t) = g(t) \exp(-i2\pi t)$



5)  $h(t) = g(t) \cos(6\pi t)$



+ assorted questions throughout; e.g.,

2) how to reconstruct  $g(t)$ ?

$$\frac{\pi(f)}{i2\pi f}$$

5) with your answer for  $f_s$ , is  $h(t)$  recoverable if sampler delayed by some  $\epsilon$ ?

No

### Quals Question: Professor Olukotun

1. Why is dynamic branch prediction important in modern processors?
2. Given then general classes of scientific and transaction processing applications. In which class are branches harder to predict? Why?
3. What is the simplest dynamic predictor you could use?
4. Given an unlimited number of single bit predictors without using any other kinds of history for any particular application at what point will the branch prediction accuracy saturate?
5. Suppose we want to improve the branch prediction accuracy beyond this point by using more branch history. There are two types of history, what are their names?
6. Define local and global history, explain how they can be used to improve branch prediction accuracy. Why are more predictors required? Why does this work?
7. Assume branch B1, B2 and B3 are executed repeatedly in a loop and two bits of history are kept for each branch. For each history bit determine which history information (local or global) will provide the best branch accuracy for branches B2 and B3. Indicate your answer by placing a G or an L in the appropriate box.

Assume any initial condition of the predictors that you like.

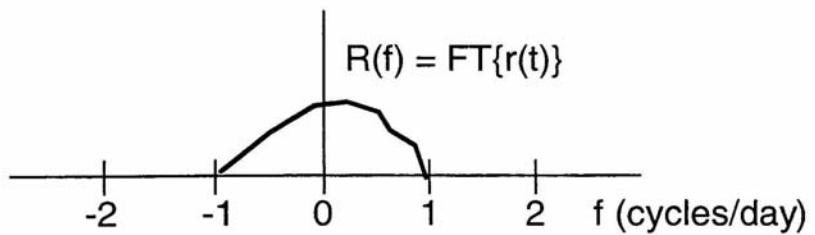
b1:	T	N	T	T	T	N	T	N	G/L	G/L				
b2:	N	N	T	N	T	T	N	N	<table border="1" style="margin-left: auto; margin-right: auto;"><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table>					
b3:	T	T	N	T	N	T	T	T						

8. How can you decide dynamically which history to use?

1. The student claims, that based on last week's data, measurements only need to be made twice a day (once in the morning and once at night).

Do you agree?

2. From the past 10 yrs worth of data, you determine that the streamflow,  $r(t)$  (as measured in cubic feet/s), is a bandlimited function such that:



How often do you need to sample?

3. The student hates getting up in the morning, so proposes to only measure the river level once/day and report:

$$r(t) \text{ and } r'(t) = dr(t)/dt$$

Is this acceptable to you?

4. Given  $r(n)$  and  $r'(n)$ ,  $n = \dots -2, -1, 0, 1, 2 \dots$ ,  
how would you compute  $r(t)$ ?

5. Over time, the student gets really lazy and  
proposes to just measure:

$$r(t), r'(t), r''(t), \dots, r^{(k)}(t), k=14$$

once a week.

What's wrong with this proposal?

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# Partial Answers and Hints

## Question 1.

Is the function bandlimited?

Estimate the bandwidth, based on the approximate impulse response. Is this really an LTI system?

## Question 2.

Nyquist theorem.

## Question 3.

What are the properties of  $r(t)$  such that ordinate and slope sampling work?

## Question 4.

Describe the process of interpolation.

$r(t) = a(t) * r(n) + b(t) * r'(n)$ , what are the desireable properties of  $a(t)$  and  $b(t)$ ?

## Question 5.

When is the Taylor's series expansion of a function valid?

What about measurement noise?

Is  $r(t)$  really bandlimited? What happens if it is not?

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**Simon Wong, 2/3/00 9:05 AM -0800, qual question**

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**1**

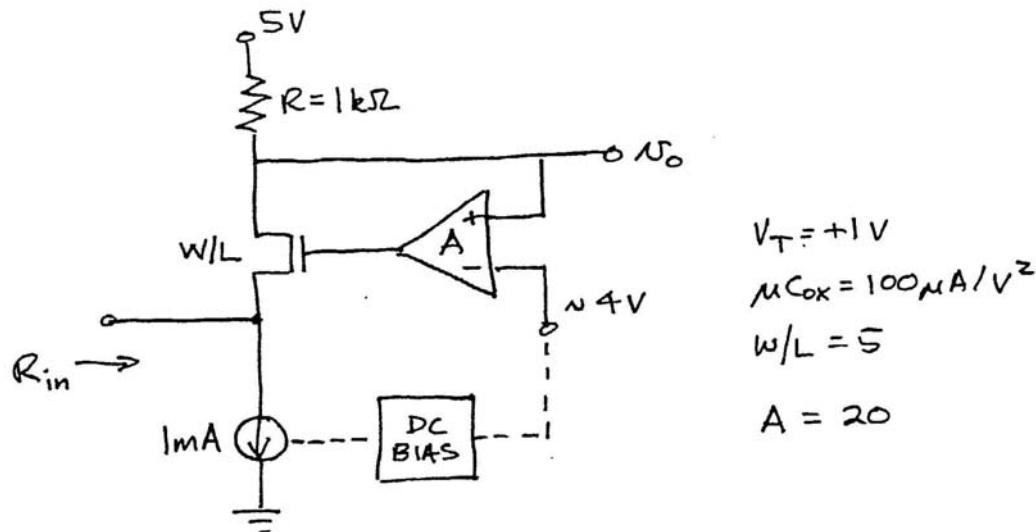
X-Sender: [swong@holst.stanford.edu](mailto:swong@holst.stanford.edu)  
Date: Thu, 03 Feb 2000 09:05:05 -0800  
To: [shankle@ee.stanford.edu](mailto:shankle@ee.stanford.edu)  
From: Simon Wong <[wong@ee.stanford.edu](mailto:wong@ee.stanford.edu)>  
Subject: qual question  
Cc: [wong@ee.stanford.edu](mailto:wong@ee.stanford.edu)

Simon Wong  
Area : Electronic Circuits

General characteristics of differential pair : voltage gain, common mode  
input range, active load, frequency limitations

QUALS QUESTION 99-00

Bruce Wooley



$$\begin{aligned}V_T &= +1\text{ V} \\ \mu C_{ox} &= 100\text{nA/V}^2 \\ W/L &= 5 \\ A &= 20\end{aligned}$$

What is the small-signal input resistance,  $R_{in}$ ?

**Howard Zebker, 2/2/00 3:32 PM -0800, Re: Quals Question**

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1

Date: Wed, 2 Feb 2000 15:32:05 -0800 (PST)  
From: Howard Zebker <zebker@jakey.stanford.edu>  
To: shankle@ee.stanford.edu  
Subject: Re: Quals Question

Hi Diane,

My qual's question is attached, but it's a bit hard to describe.

Howard

Quals '00 question:

The student is presented with a small tester that helps elicit wiring problems in AC wall sockets. The student is asked to reverse engineer the box.

After this, the student is presented with a set of four wall sockets that have been incorrectly wired. The task is to determine (non-destructively) how the wires in the connection box are configured. The tester from the first part of the problem is available, as is a light bulb with two wires attached. Logical reasoning plus a bit of common sense is needed to solve the problem.

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SS      T1  
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