

# Electrical Engineering

## Quals Questions

2001

Anonymous :  
Instructors

Stanford University  
Department of Electrical Engineering

EE Quals. Jan 22-26, 2001.

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There are 25 old ladies, and there are 25 rooms each with a light bulb. All light bulbs are initially off.

1. Suppose that there is a fair coin in each room. Each old lady goes to *each room* and flips the coin. If the coin comes up heads, she changes the state of the bulb. Else, she moves to the next room and repeats. What is the average number of bulbs that are on when all 25 ladies are done?
2. What is the average number of bulbs that are on, given that at least 5 are on?
3. Now suppose that all the coins are removed (no probabilities), and all bulbs are initially off. Suppose that old lady no. 1 goes through all the rooms and turns on all the bulbs. Old lady no. 2 goes through even numbered rooms (2, 4, 6, ...) and turns the bulbs off. Old lady number  $I$  goes through rooms numbered  $kI$ ,  $k = 1, 2, \dots$  and changes the state of the bulbs. After all 25 ladies are done, which are the bulbs that are on?

To: Diane Shankle <shankle@ee.stanford.edu>  
Subject: Re: Quals Meeting Today!  
Cc: mgbaker@ee.Stanford.EDU  
Date: Mon, 29 Jan 2001 13:48:59 -0800  
From: Mary Baker <mgbaker@plastique.Stanford.EDU>

Question:

1. You're working at a desktop computer and it crashes. You would like it to boot up into the same state it was in before it crashed, so you can keep on working. What would the system need to do to support this?

If students didn't get to these topics on their own, I asked the further subquestions:

- a. What kinds of state would need to be saved/restored?
- b. What special issues are there with peripheral devices? (especially the network?)
- c. What device will not come up in the same state (and we don't want it to), and software issues may be associated with that?
- d. Let's assume we're successful and the machine is restored to the identical state it was in right before it crashed. What further issues might we need to worry about?

Answer:

i.a. We'd need to save and restore processor state, virtual memory mappings and the contents of process address spaces, open file descriptor tables, etc. There's also OS state of various kinds that some students got extra points for.

b. Some peripherals might have been active at the time of the crash. For instance, if we were sending a document to a printer, do we know if the whole transfer took place? Do we need to communicate with the printer to find out if we need to reprint the file? Devices with buffers may be an issue if we don't know if the whole buffer has been processed or not. With a disk it may be safe to reissue the command, but with a tape, for instance, we will need to seek back to the right location before reissuing any commands.

The network presents special problems: some protocols may not be restartable, and remote machines may have state built up on them that cannot easily be recreated.

c. The system clock should probably not come back in the same state it was in when the crash occurred, and it is possible that some software may break due to this. If events are scheduled to occur at a particular time (that is missed due to the crash) and if they are cumulative events and we don't check whether the last even occurred, this could be a problem. If we were timing a process or benchmarking it, this could also be a problem, and of course real-time processes present problems when there's a crash.

d. If the system is in the identical state it was in before it crashed, and if the cause of the crash is software-based, then we are likely to crash the same way again.

NOTE: Most students managed to get many of these answers, but they required very different amounts of prodding and hints. I graded based on how independently they were able to reason about the problem.

## **2001 Quals**

**J. Cioffi**

TOTAL - 10 pts

### **Random Processes and Discrete-Time DSP - 10 pts**

You have just started to invest in the stock market and have invested  $y_0 = \$10$  in Company X. Assume there are 250 trading days in one year and that your investment grows by a factor of  $(1 + \alpha)$  each day, where numerical answers in this problem can (eventually) substitute  $\alpha = .001$ . However, there is an additive random Gaussian disturbance to this stock growth that has zero mean, but variance  $\sigma^2 = .0625$  (dollars squared). The Gaussian noise is white, that is, the disturbance on the daily growth of your money on any particular day is independent of all other days.

- a). Call the Gaussian disturbance,  $n_k$ , and write a simple difference equation to describe the growth of your investment in one share of this company per day, where the stock value is  $y_k$ . What is the distribution type for  $y_k$ ? (1 pts)
- b). Determine the daily interest rate and the compounded annual interest rate if the noise were somehow zeroed ( $\sigma = 0$ ). (1 pts)
- c). Find the expected value for the stock on any day, and determine its value specifically in dollars on day 250 when the noise variance is not zero. (2 pt)
- d). Find an expression for the variance of the stock on any day, and compute the specific value for the standard deviation on day 250. (2 pts)
- e). Find the probability that your investment has lost money after 1 day, 10 days, and 1 year. (2 pts)
- f). For what alternative value of  $\alpha$  does the risk in investing in this stock diminish if the stock is split each year (by factor of 2)? (2 pts)

$$y_k = (1+\alpha)y_0 + n_k$$

$$y_0 = \$10$$

Solution:

$$a). \quad y_k = (1+\alpha)^k y_0 + \sum_{i=1}^k n_i (1+\alpha)^{k-i} \quad \sigma = .25 \\ \sigma^2 = .0625$$

b). interest

$$\text{daily } \frac{\alpha}{100} = .1\% \text{ if } \alpha = .001$$

$$\text{annually } (1+\alpha)^{250} = 1.2839 \Rightarrow 28.39\%$$

$$c). \quad E[y_k] = (1+\alpha)^k y_0 = 10(1.2839) = \$12.84$$

Gaussian

$$d). \quad E[y_k^2] = (1+\alpha)^{2k} y_0^2 + \underbrace{\sum_{i=0}^{k-1} \sigma^2 (1+\alpha)^{2[k-i-1]}}_{\text{variance}}$$

$$\text{var}(y_k) = \sigma^2 \left[ (1+\alpha)^{2k} - \frac{1}{(1+\alpha)^2 - 1} \right]$$

$$\sigma_{y_k}^2 = \sigma^2 (1+\alpha)^{2k} + \sigma^2$$

$$\sum_{i=0}^{k-1} (1+\alpha)^{2i} \sigma^2 = \text{var}(y_{250}) = \sigma^2 \cdot \frac{(1.001)^{500} - 1}{(1.001)^2 - 1} = 3245 \sigma^2$$

$$\text{std} = 18.5 \sim 4.50$$

$$e). \quad \Pr\{y_k < y_0\} = \Pr\{n_0 > .01\} = Q\left[\frac{.01}{.0625}\right] = Q[.0016] \approx \boxed{.5}$$

$$\Pr\{y_{250} < y_0\} \Rightarrow E[y_{250}] = \$11.00 \quad \text{For } 250$$

$$= \Pr\{\text{noise} < -\$1.00\} = Q\left[\frac{1}{\sigma_{10} = 3.176}\right] \approx \boxed{.5}$$

$$\text{var}(y_{10}) = \sigma^2 \left[ 10.1107 \right] \quad 3.1791 \sigma \quad -10 \text{dB}$$

$$\Pr\{y_{250} < y_0\} = \Pr\{\text{noise} < -\$2.38\} \quad Q\left[\frac{2.38}{4.50}\right] \approx \boxed{.2} \quad -2.7 \text{dB}$$

$$\Pr\{y_{250} > y_0\} = \Pr\left\{ \left(\frac{2.84}{3.1791}\right) = y_2 \right\} \quad \begin{aligned} e). \quad (1+\alpha)^{250} &= 2 \\ \alpha &= .0028 \end{aligned}$$

Quals Question  
January 2001  
Tom Cover

**Question 1:**

Does every (convex) polygon have a stable edge? (An edge is stable if the polygon won't tip over when placed on that edge.)

**Answer to question 1:**

Yes. Otherwise one would have perpetual motion. This answer identifies the nearest face to the center of gravity as a stable face. It should also be noted that the farthest point from the c.g. is also a stable point.

**Question 2:**

Let the random variable  $X$  have the distribution

$$X = \begin{cases} 6 & , \text{ prob } \frac{1}{2} \\ \frac{1}{3} & , \text{ prob } \frac{1}{2}. \end{cases}$$

Consider the product  $S_n = X_1, X_2, \dots, X_n$ , where the  $X_i$ 's are independent identically distributed according to  $X$ . How does  $S_n$  behave as  $n$  grows?

**Answer to question 2:**

This question was originally designed to lead into Parrondo's paradox, but was sufficiently challenging as it was. The correct answer is  $S_n \approx (\sqrt{2})^n$ . This can be obtained by taking the logarithm and using the law of large numbers. Thus

$$S_n^{\frac{1}{n}} \longrightarrow e^{E \ln X},$$

the geometric mean. Some people simply noted that the 6's and  $(\frac{1}{3})$ 's cancelled out, leaving a factor of 2 every two samples. Thus  $S_n \approx 2^{\frac{n}{2}}$ . It turns out that the expected value of  $S_n$ , given by  $ES_n = (\frac{19}{6})^n$ , has nothing to do with the behavior of the product.

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# Qualifying Examination 2001

## Logic Design

Giovanni De Micheli

January 2001

A Stanford student designs a complex digital circuit. (Assume the circuit is combinational for simplicity). He falls asleep once he has done.

His girlfriend plays him a trick. She adds a bubble to a gate. She leaves him a message saying so, without revealing where the bubble is, and leaves.

He wakes up, goes to Stanford, simulates exhaustively the circuit, and finds out that the output patterns have not changed (as compared to the previous evening).

His girlfriend sees him puzzled and says: "I did it for your own good!" What will he do? (... to the chip, not to his girlfriend...)

*The circuit has the same functionality as before. But this is bad news. It means that there is redundancy in the design.*

*A redundant design is inferior (as compared to an irredundant one) in terms of area (extra gates), power (dissipated by the redundant gates) and testability (the redundant net is not testable).*

*The right course of action is: i) search for the redundancy and ii) remove the redundancy.*

*There are various ways to search for the redundancy. The simplest is based on noticing that the redundant net is not testable. Indeed, any valued set by a test vector on that net produces the same output. Thus, an automated test pattern generator (ATPG) applied to the circuit would inform the designer that at least one net is untestable (for stuck-at faults). This net is where the bubble was inserted.*

*To remove the redundancy, the designer will: i) remove the entire gate with the bubble, ii) set the corresponding output net to either 1 or 0, iii) propagate the constant, i.e., simplify the gates in the fanout cone as a result of the chosen constant.*

*Finally, the student sends flowers to his girlfriend to thank her for the hint that allowed him to improve his circuit. Actually, she is a skilled professional designer.*

Comment: There are many ways to answer this question. At a minimum, students were expected to discover that the circuit had a redundancy and remove a redundant gate from a simple circuit example. Scores were given based on the thinking process, not necessarily on previous knowledge.

OFFICE MEMORANDUM ♦ STAR LABORATORY

March 20, 2001

To: Diane Shankle  
From: Tony Fraser-Smith  
Subject: Ph.D. Quals Question, January 2001

**Determining the Depth of the Ice on Europa**

As was the case last year, the student is given the following information concerning NASA's latest discoveries with respect to Europa, the fourth largest moon of Jupiter: (1) images acquired during recent spaceprobe flybys show a surface covered with ice (discolored ice in some places) and the layer of ice appears to be quite thick, and (2) magnetic and gravity measurements suggest very strongly that there is a liquid ocean beneath the ice. Liquid oceans are very unusual in the solar system, and the existence of one on Europa suggests the possibility of life. NASA has therefore placed the highest priority on a mission to Europa to see what can be learned about life in the ocean. First, however, NASA has to access the water under the ice, and even before it can reach the water it has to determine the thickness of the ice.

**Question:** What methods might NASA use to determine the thickness of the European ice? Remember that it may not be pure. Remember that it may be very thick. And remember that NASA has somewhat limited resources and there is no possibility whatsoever of a manned expedition to Europa.

**Answer.** To get full marks for this question, the student was expected to, first, discuss the many possible methods that might be used to determine the thickness of the ice and then, second, to discuss the most feasible appearing methods in greater detail.

Possible methods could include (1) flying a drill rig to Europa and using it to drill through the ice. But it should have been decided that this was infeasible due to the weight and size of the rig; (2) measuring the attenuation of a radio signal passing between a satellite orbiting Europa and the Earth as the satellite becomes occulted by Europa (i.e., passes behind it); (4) landing a probe on the surface of Europa and carrying out a seismic sounding experiment; (5) landing a probe on the surface and carrying out an acoustic sounding experiment; (6) landing a probe on the surface and having it melt its way through the ice; (7) landing a probe on the surface and carrying out an electromagnetic sounding experiment; (8) landing several probes at different distances apart on the surface and transmitting various kinds of signals between them to probe the surface.

The student was expected to consider what kind of frequencies would be best for the electromagnetic probing methods. For this they would have their attention drawn to the discolorations in the ice and the likelihood that the ocean had salts dissolved in it – in other words, the ice is probably somewhat contaminated with salts.

X-Sender: hector@db.stanford.edu  
Date: Thu, 15 Feb 2001 13:25:44 -0800  
To: Diane Shankle <shankle@ee.stanford.edu>  
From: Hector Garcia-Molina <hector@cs.stanford.edu>  
Subject: Re: Quals Question 2001

At 10:43 AM 2/15/01 -0800, you wrote:

I am still waiting for you to submit your Quals Question either by hard copy or email.

Please try to submit by 2/23/01.

Here is my question.  
hector

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Quals 2001 Question  
Hector Garcia-Molina

(a) What is a hash table? What is it used for?  
How can collisions be handled?

(b) Write pseudo-code to insert a new key into a hash table. Assume that open addressing is used to resolve collisions.

The hash table is implemented with an array X ranging from 0 to N. You are given a hash function h that you can call; the function return an integer between 0 and N. Your insert procedure takes as input a key. It returns a flag that is either:

OK: the value was successfully inserted  
DUP: the value was not inserted because it already exists in the table  
FULL: the table was full, no value was inserted.

**James F. Gibbons, 1/29/01 2:46 PM -0800, RE: Quals Meeting Today!**

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From: "James F. Gibbons" <gibbons@ee.stanford.edu>  
To: "Diane Shankle" <shankle@ee.stanford.edu>  
Cc: "Mary Cloutier" <cloutier@cis.Stanford.EDU>  
Subject: RE: Quals Meeting Today!  
Date: Mon, 29 Jan 2001 14:46:03 -0800  
X-Priority: 3 (Normal)  
Importance: Normal

Diane:

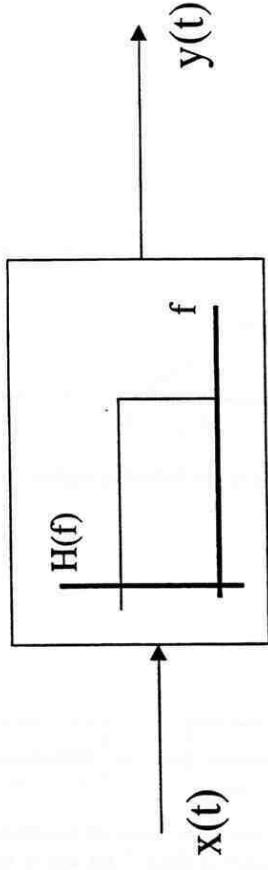
My Quals questions are always designed to provide plenty of room for people to go off in directions that allow me to explore how they think , how creative they might be, and so on. The specific questions I used to start discussion were:

1. What is an LED and how does it work? Lots of opportunity here to explore basic physical understanding of semiconductor physics and basic physical reasoning.
2. How would you make a semiconductor laser from an LED? Allows more sophisticated discussion of points in the first question, plus new ideas.
3. What is a solar cell and how does it work? This is entirely related to the discussion of the LED, but input and output variables are switched.
4. What are the factors that would set an ultimate limit on the power conversion efficiency of a solar cell? This requires thinking of the sun as a blackbody source, and understanding the relationship of its power spectrum to the operation of the solar cell. If a person makes it through this, then we can talk about other factors that limit the power conversion efficiency.

A good candidate will finish three of these questions in something under 10 minutes. A great candidate will get close to answering all four. An average candidate will do reasonably well on two of them. A poor candidate typically will not do well on any of them.

I hope this gets close to what you need. I would like to know what you do with these pieces of information so I know how much of this I can use next time.

- **Given:** ideal lowpass filter with input signal  $x(t)$  and output signal  $y(t)$ .

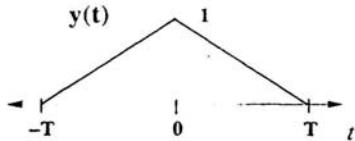


- Suppose that  $x(t)$  is bounded, such that  $|x(t)| < 1$  for all  $t$ .
- **Show that the amplitude of  $y(t)$  can nevertheless be arbitrarily large, i.e.,  $y(t)$  is unbounded.**



Quals Question 2000-2001: Goldsmith

**Problem 1:** Let  $Y(f) = X^2(f)$ , where  $y(t)$  is as shown below.

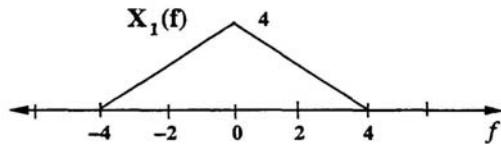


- (a) What is  $x(t)$ ?
- (b) What is  $Y(f)$ ?

**Problem 2:** Let

$$x_2(t) = \sum_{i=-\infty}^{\infty} x_1(t - iT),$$

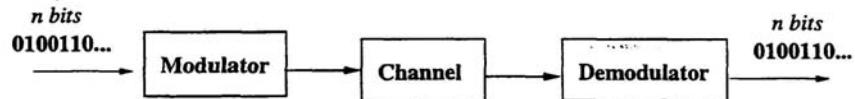
where  $X_1(f)$  is as shown below. Find the Fourier series coefficients for  $x_2(t)$  for  $T = 2$  seconds.



**Problem 3:** What is the Nyquist sampling rate of the following signals:

- (a)  $\text{sinc}(t) * \text{sinc}(0.5t) * \text{sinc}(0.1t)$ .
- (b)  $\cos(2\pi t/T_0)\text{rect}(t/T_0)$ .

**Problem 4:**



In the above figure, the channel does not introduce any noise or distortion. Design a modulator and demodulator to send  $n$  bits over this channel in time  $T$  for any  $n$  and  $T$ . In practice what would limit the data rate  $R = n/T$  for this system?

# January 2001 Quals

R.M. Gray

Note: 9.5 points of my scoring were for the first three questions. The fourth question was held in reserve and one person reached it and solved it for a perfect 10.

A system  $\mathcal{S}$  has as input a real-valued discrete-time (DT) signal  $x_0, x_1, x_2, \dots, x_{N-1}$ . The output of the system is another DT signal  $y_0, y_1, y_2, \dots, y_{N-1}$ . The output is determined by the input according to the following equations:

$$\begin{aligned} y_0 &= x_0 - x_{N-1} + c \\ y_1 &= x_1 - x_0 + c \\ y_2 &= x_2 - x_1 + c \\ &\vdots \\ y_{N-2} &= x_{N-2} - x_{N-3} + c \\ y_{N-1} &= x_{N-1} - x_{N-2} + c \end{aligned}$$

where  $c$  is a real constant.

- (1) Is the system  $\mathcal{S}$  linear?

*Solution:*

This question was intended to get started with some basic systems concepts in a possibly unfamiliar setting, a finite-length sequence. The basic definition of linearity is unchanged, however. The system is indeed linear if the constant  $c$  is set to 0, otherwise it is not. For example, if inputs  $\{x_0^{(i)}, x_1^{(i)}, \dots, x_{N-1}^{(i)}\}$ ,  $i = 1, 2$ , yield outputs  $\{y_0^{(i)}, y_1^{(i)}, \dots, y_{N-1}^{(i)}\}$ ,  $i = 1, 2$ , then the linear combination input  $\{ax_0^{(1)} + bx_0^{(2)}, \dots, ax_{N-1}^{(1)} + bx_{N-1}^{(2)}\}$  yields output  $\{ay_0^{(1)} + by_0^{(2)} + (a+b)c, \dots, ay_{N-1}^{(1)} + by_{N-1}^{(2)} + (a+b)c\}$ , which is not  $\{ay_0^{(1)} + by_0^{(2)}, \dots, ay_{N-1}^{(1)} + by_{N-1}^{(2)}\}$  unless  $c = 0$ .

- (2) Henceforth assume  $c = 0$ . A sequence  $u_0, u_1, \dots, u_{N-1}$  is an *eigenfunction* and  $\lambda$  is the corresponding *eigenvalue* of  $\mathcal{S}$  if putting  $u_0, u_1, \dots, u_{N-1}$  into the system yields an output of  $\lambda u_0, \lambda u_1, \dots, \lambda u_{N-1}$ . What are the eigenvalues and eigenvectors of  $\mathcal{S}$ ?

*Solution:*

Solution requires finding  $u_n$  and  $\lambda$  solving

$$\begin{aligned} \lambda u_0 &= u_0 - u_{N-1} \\ \lambda u_1 &= u_1 - u_0 \\ \lambda u_2 &= u_2 - u_1 \\ &\vdots \\ \lambda u_{N-2} &= u_{N-2} - u_{N-3} \\ \lambda u_{N-1} &= u_{N-1} - u_{N-2} \end{aligned}$$

This is a linear difference equation with constant coefficients which simplifies to  $u_n = u_{n-1}/(1 - \lambda)$  for  $n = 2, 3, \dots, N - 1$  and  $u_{N-1} = u_0/(1 - \lambda)$ . The usual guess is  $u_n = az^n$ , which in this case

becomes  $1 = z^{-1}/(1 - \lambda)$  for  $n = 2, 3, \dots, N - 1$  and  $z^{N-1} = 1/(1 - \lambda)$ , or  $z = 1/(1 - \lambda)$  and  $z^N = 1$ , i.e.,  $z = 1/(1 - \lambda)$  is an  $N$ th root of 1, e.g., for any integer  $k$  and any constant  $a$

$$\begin{aligned} z &= e^{j2\pi k/N} \\ z &= \frac{1}{1 - \lambda} \\ u_n &= ae^{j2\pi kn/N}; n = 1, 2, 3, \dots, N - 1 \\ \lambda &= 1 - e^{-j2\pi k/N} \end{aligned}$$

Note that one possible eigenvalue ( $k = 0$ ) is 0.

Alternatively, just note that  $u_0 = u_{N-1}/(1 - \lambda)$ ,  $u_1 = u_0/(1 - \lambda) = u_{N-1}/(1 - \lambda)^2, \dots$ ,  $u_k = u_{N-1}/(1 - \lambda)^{k+1}$  all the way up to  $u_{N-1} = u_{N-1}/(1 - \lambda)^N$ , which can only be if  $1 - \lambda$  is a root of unity.

Once you have the eigenvalues, the eigenfunctions are given by picking an arbitrary  $u_{N-1}$  and using the above formula, i.e.,  $u_k = u_{N-1}/(1 - \lambda)^{k+1}$ . Alternatively, you can recognize this as a DFT example and recall that eigenfunctions must be complex exponentials of the form  $e^{j2\pi kn/N}$ . Other methods used included writing the matrix equations and solving for the determinant of  $H - \lambda I = 0$ , which works but takes more skill than I have at linear algebra under stress. Some used DFT ideas to get the answer.

- (3) Suppose you are given the output sequence  $y_0, y_1, \dots, y_{N-1}$ . Find a simple formula for determining the inputs  $x_0, x_1, \dots, x_{N-1}$ .

*Solution:*

This can be tackled by matrix methods, but that way runs into trouble because of the 0 eigenvalue. Fourier series methods can also be used, but the simplest way is the straightforward approach. Rewriting the basic equations with the  $y$ 's on the right,

$$\begin{aligned} x_0 &= y_0 - x_{N-1} \\ x_1 &= y_1 - x_0 \\ x_2 &= y_2 - x_1 \\ &\vdots \\ x_{N-2} &= y_{N-2} - x_{N-3} \\ x_{N-1} &= y_{N-1} - x_{N-2} \end{aligned}$$

A little thought should convince you that this would be trivial if you knew  $x_{N-1} = r$ , say, since then you could simply iteratively compute the  $x_n$ , i.e.,  $x_0 = y_0 + r$  and

$$x_n = x_{n-1} + y_n, n = 1, 2, \dots, N - 1.$$

By using this formula in itself to replace  $x_{n-1}$ , it turns out that

$$x_n = \sum_{i=0}^n y_i + r,$$

i.e., you just sum outputs to get the input (which makes sense since the outputs came from differences). Since this formula works for  $n = N - 1$ ,

$$x_{N-1} = r = \sum_{i=0}^n y_i + r$$

which can only be true if  $\sum_{i=0}^n y_i = 0$ , but this *must* be true for any output to this system. To convince yourself of this, just add up the right hand sides of the equations defining the systems, everything cancels.

Note that this solution works for *any* value of  $r$ , i.e., the answer is not unique. This is because the system is not invertible, which is equivalent to the existence of the 0 eigenvalue.

- (4) This problem was held in reserve for those who did everything in a whiz and already had a 9 or 9.5 depending on accuracy and how they tackled things. Only one person got to this problem (and solved it for a perfect 10).

Suppose you are given a Fourier series for  $x_0, x_1, \dots, x_{N-1}$ , i.e., a representation of the form

$$x_n = \sum_{k=0}^{N-1} a_k e^{j2\pi n \frac{k}{N}}; \quad n = 0, 1, \dots, N-1$$

Find a formula for the output energy  $\mathcal{E} = \sum_{k=0}^{N-1} |y_k|^2$  in terms of the  $a_k$ .

*Solution:*

Here the simplest way is to use Parseval's theorem and to recognize that the Fourier series for the output follows from that of the input and the fact that the operation is a cyclic difference or delay, i.e.,

$$\begin{aligned} y_n &= \sum_{k=0}^{N-1} b_k e^{j2\pi n \frac{k}{N}} \\ &= x_n - x_{n-1} \\ &= \sum_{k=0}^{N-1} a_k e^{j2\pi n \frac{k}{N}} - \sum_{k=0}^{N-1} a_k e^{j2\pi(n-1) \frac{k}{N}} \\ &= \sum_{k=0}^{N-1} a_k (1 - e^{j2\pi k/N}) e^{j2\pi n \frac{k}{N}} \end{aligned}$$

so that

$$b_k = a_k (1 - e^{j2\pi k/N}).$$

(Or just recognize this is the DFT shift theorem.)

From the discrete version of Parseval's theorem (which can be derived in a few lines if forgotten)

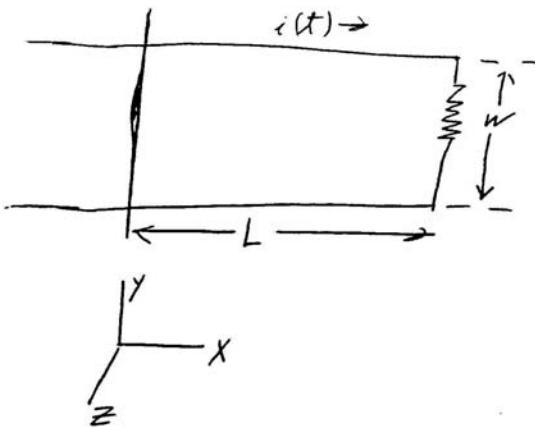
$$\begin{aligned} \mathcal{E} &= \sum_{k=0}^{N-1} |y_k|^2 \\ &= N \sum_{k=0}^{N-1} |b_k|^2 \\ &= N \sum_{k=0}^{N-1} |a_k|^2 |1 - e^{j2\pi k/N}|^2 \\ &= 4N \sum_{k=0}^{N-1} |a_k|^2 \sin^2\left(\frac{\pi k}{N}\right) \end{aligned}$$

*Comments:* Other questions along the way might have occurred, e.g., expressing the system in matrix notation, relating eigenfunctions to eigenvectors, asking about the inverse and determinant if the system matrix, and questions about basic Fourier series in this context, including inversion and Parseval's relation.

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**2001 PhD Quals**  
**James Harris**

1.
  - a) Can you first draw the energy band diagram for a p/n junction?
  - b) What does the vertical axis represent? Can an electron occupy a point above the conduction band minimum or below the valence band maximum? What is the Total Energy, Potential Energy, Kinetic Energy?
  - c) Can you draw the I-V characteristic for this diode and tell me what is physically happening in your band diagram that explains the forward and reverse bias characteristics?
  - d) Why is the forward current exponential with voltage?
  - e) What happens when the applied electric field becomes quite high?
  - f) What is the condition for an electron or hole to create an additional electron-hole pair in this process?
2.
  - a) If we take your I-V characteristic to be that at room temperature (300°K) and I now reduce the temperature to 100°K, how do you expect the forward and reverse characteristics to change and why?
  - b) How are the threshold,  $V_{th}$ , slope,  $dI/dV$  and breakdown voltage affected?
  - c) What are the sources of the forward and reverse currents and upon what do they depend?
3. What if I now shine light on the diode.
  - a) What does this look like on your I-V diagram? Can we use this phenomena in any useful way?
  - b) Is there anything different or unusual about the 4<sup>th</sup> quadrant region that you have drawn the characteristic?
  - c) Is there any device that uses this property or region of operation?
  - d) If we are using this device as a photodiode, we want to both generate and collect e-h pairs efficiently and to minimize the dark current (what we've been calling the reverse saturation current). How might we minimize the reverse saturation or dark current?
  - e) What is the source of the reverse leakage current? Is there any contribution from the depletion region? Why or why not?
  - f) If the source of all the dark current in a photodiode is outside the depletion region, is there anything from a device physics and design point of view that you can think of to reduce the dark current?



- a) A sliding bar moves to the right on metallic rails with the equation of motion

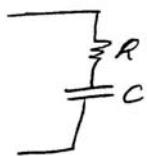
$$x = t + t^2$$

Assume a constant magnetic field  $\bar{B} = B_0 \bar{a}_z$ .

Find the current  $i(t)$  as a function of time.  
What physical idea do you use to determine the direction of current flow?

- b) Now let  $\bar{B} = B_0 t \bar{a}_z$ . Repeat part (a).  
What direction does the current flow? Will it reverse direction?

- c) move a free conducting bar (No rails) at constant velocity in the  $x$ -direction. Discuss the charge distribution inside the bar.
- d) as in part (c) except the motion is sinusoidal in the  $x$ -direction. Is there a current in the bar?
- e) Replace  $R$  by



write the differential Eq. for  $i(t)$ .

Solution

Quals 2001  
S.E. Harris

$$\oint \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \int \vec{B} \cdot d\vec{s}$$

with  $i(t)$  clockwise  $d\vec{s}$  is into paper

a)  $\oint \vec{E} \cdot d\vec{l} = - \int \vec{B} \cdot d\vec{s} = - B_0 L [L - (x + t^2)] = - B_0 L x + B_0 w t + B_0 w t^2$

$\frac{\oint \vec{E} \cdot d\vec{l}}{L \text{ clockwise}} = - \frac{d\Phi}{dt} = - (B_0 w + 2B_0 w t) \quad \text{current flow is c.e.}$

by Lenz's Law, opposes linkage changes

b)  $\oint \vec{E} \cdot d\vec{l} = - B_0 x L [L - (x + t^2)] = - B_0 w L x + B_0 w t^2 + B_0 w t^3$

$$\begin{aligned} \frac{\oint \vec{E} \cdot d\vec{l}}{L} &= + B_0 w L - 2B_0 w t + 3B_0 w t^2 \\ &= B_0 w [L - 2t - 3t^2] \end{aligned}$$

→ flow is clockwise at small  $t$ , & c.e. at large  $t$

c) positive & negative charge collects at  $-y$  &  $+y$ , respectively.

d) charge motion is sinusoidal in the  $\vec{a}_y$  direction.

e)  $R \frac{di}{dt} + \frac{i}{c} = \frac{dV(t)}{dt}$

$$R \frac{di}{dt} + \frac{i}{c} = \underbrace{\frac{dV(t)}{dt}}_{\text{generated EMF}}$$

**Mark Horowitz, 2/15/01 11:01 AM -0800, Re: Quals Question 2001**

**1**

X-Sender: horowitz@vlsi.stanford.edu  
Date: Thu, 15 Feb 2001 11:01:41 -0800  
To: Diane Shankle <shankle@ee.stanford.edu>  
From: Mark Horowitz <horowitz@stanford.edu>  
Subject: Re: Quals Question 2001

Quals questions:

Assume we want to generate a field specifier for a 64 bit machine. This is a unit that generates all constants that have a contiguous sequence of '1's. How many bits would it take? How would you encode it.

We encode the constant using the starting and ending location of the string of '1's. How would you build the hardware to decode this. You can use a normal decoder or a thermometer decoder.

How would you build a thermometer decoder. Don't worry about speed at first, just get the logic right.

How might you make the decoder faster. Can you estimate its speed?

At 10:43 AM 2/15/01 -0800, you wrote:

I am still waiting for you to submit your Quals Question either by hard copy or email.  
Please try to submit by 2/23/01.

Happy Thursday,  
Diane

---

**Printed for Diane Shankle <shankle@ee.stanford.edu>**

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Subject: Re: Quals Question 2001  
Date: Fri, 16 Feb 2001 16:32:06 -0700  
From: Greg Kovacs <kovacs@cis.stanford.edu>  
To: "Diane Shankle" <shankle@ee.stanford.edu>

The student was asked to sit at a table with several objects on it. He or she was told that a device on the table was a sensor of some kind, and that it was sensing something about a glowing cylinder also on the table. Also on the table were various items (paper, pen, hardcover book, sheet of copper-clad printed circuit board, toilet paper roll, full beverage can, and a plastic ruler). The student was told that he or she could touch and use these items at will. The student was allowed to move and examine everything on the table, except for the sensor (because it is delicate). It was stated that the sensor could detect something to do with (emitted by) the glowing cylinder, and the student was asked to try to figure out what (as specifically as possible) was being sensed (e.g., what the sensor detects from the glowing rod). The various objects on the table could be used to carry out a variety of simple experiments to obtain information about the signal and the sensor.

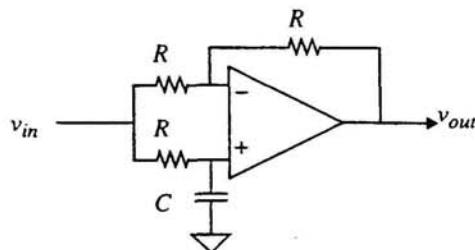
Sorry - I thought I had submitted it.

Best,  
Greg

>I am still waiting for you to submit your Quals Question either by hard copy  
>or email.  
>Please try to submit by 2/23/01.  
>  
>Happy Thursday,  
>Diane  
>

1) This first problem concerns the 3dB bandwidth of the following circuit. To find it, first sketch  $v_{out}(j\omega)/v_{in}(j\omega)$  of the system, being sure to consider both magnitude and phase. You may assume that the op-amp is ideal in all respects (zero input current, zero output impedance, infinite bandwidth, etc.). Label all important features of your sketches, paying particular attention to the asymptotic behavior of the circuit at DC and at infinite frequency:

**FIGURE 1. Op-amp with RC network**



Outline of solution: Brute force can be used to deduce that the transfer function is

$$H(s) = \frac{1 - sRC}{1 + sRC}. \quad (1)$$

The frequency response of this function has a magnitude of unity at all frequencies, while the phase goes from 0 at DC to  $-180^\circ$  at infinitely high frequency. A surprisingly common error was an attempt to plot the entire frequency response with a single smooth curve passing through zero in going from 1 to  $-1$ , rather than magnitude and phase separately, as is required in general, owing to the need to accommodate complex quantities. (How do you plot a gain of  $1 + j$  on a scale that has only real numbers???)

If brute force is not used, the same behavior is readily deduced. First, recognize that the capacitor is an open circuit at DC. Hence,  $v_+ = v_{in}$ . If the op-amp's ideality includes infinite gain, then an added assumption that the output voltage is bounded allows us to state that  $v_+ = v_- = v_{in}$ . No currents flow through any resistor, and the output voltage then equals the input voltage.

At infinite frequency, the capacitor is a short and the circuit degenerates to a simple inverting amplifier.

There is only one capacitor, and therefore a maximum of one pole. The phase shift tells us that we must have that pole, and a right half-plane zero besides (there's no other way to get  $-180^\circ$  and also have the magnitude equal unity at DC and infinite frequency). Furthermore, the pole and zero frequencies must be equal in magnitude, or else the frequency response magnitude could not equal unity at both extremes of frequency.

This circuit is a phase shifter, and ideally leaves the magnitude untouched from DC to daylight because of its infinite bandwidth.

2) Now sketch the unit step response for the same circuit. Again, be sure to label all relevant features of your sketch.

Again, one may use brute force and compute the inverse Laplace transform of

$$\frac{1}{s} \left( \frac{1 - sRC}{1 + sRC} \right). \quad (2)$$

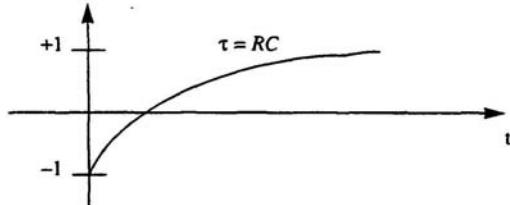
where the  $1/s$  factor is the Laplace transform of a unit step.

Another way is to recognize that all single-pole step responses are necessarily of the form

$$A + Be^{-\frac{t}{\tau}}. \quad (3)$$

The reason? One pole means one natural frequency, and therefore one exponential term. To accommodate initial and final conditions, we have constants  $A$  and  $B$ .

To find the initial and final values is easy. For a step response, these are just the magnitude of the transfer function at infinite and zero frequency, respectively. Think of the physical meanings of the initial and final values of the step response. The response to the infinitely fast transition of the step is the response to infinite frequency. Similarly, the asymptotic response to the flat portion of the step is simply the response to DC. Here, those values are  $-1$  and  $1$ . Hence, the step response starts at  $-1$  and asymptotically approaches  $+1$ :



Many who carried out the inverse transforming correctly were nonetheless bothered by their answer because of the instantaneous drop to  $-1$  at  $t = 0^+$ . Just remember, an infinitely fast rise is perfectly okay as long as there is infinite bandwidth to support it. Here, that infinite bandwidth certainly exists.

Edward J. Mccluskey  
 EJMC OVIS JAN 2001

Q1. Write Truth Map or equation for 2-bit multiplier

$$F = L_1' A + L_2 B$$

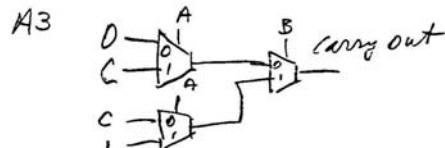
Q2. Draw CMOS circuit for 2-bit multipl.



Q3. Draw circuit for Carry Out of full adder using only 2-bit muxes

Q4. Draw circuit for D Latch using only 2-bit muxes

A4



Q5. Design 4 input, 3 output Combinational Circuit

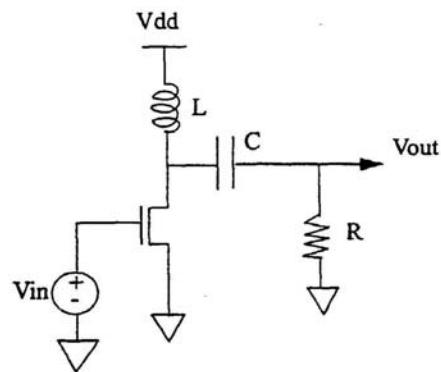
Output is binary number equal to the weight (# of 1's) in input number ab

A5

| cd | 00  | 01  | 11  | 10  |
|----|-----|-----|-----|-----|
| 00 | 000 | 011 | 010 | 001 |
| 01 | 001 | 010 | 011 | 010 |
| 11 | 010 | 011 | 101 | 011 |
| 10 | 001 | 010 | 011 | 000 |

TERESA MENG  
2001

What is the maximum output power of the following amplifier? What is its maximum "power efficiency"? Your answer may depend on the assumptions you make regarding R, L, C, and Vin.



7

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What is the reason that the more advanced the technology, i.e. smaller geometry, the more difficult the design of power amplifiers? Can you think of some methods to eliminate this problem?

## EE Ph. D. Qualifying Examination, January 2001

Question by Prof. David Miller

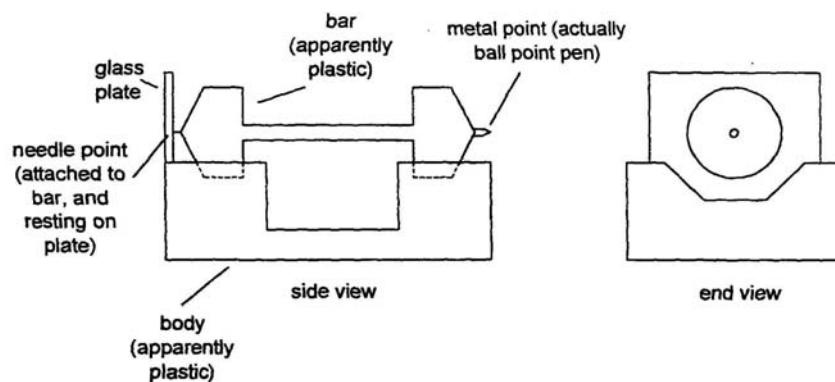
The written question given to the students was as follows.

"Deduce how this device works, that is, how does it stably levitate the bar? You should attempt to draw a simple diagram, on this piece of paper, of the internal structure of the device that explains the operation principle."

You may pick up or move the device or its parts.

If you complete this question, supplementary questions will be asked to fill the remaining time."

The device looked approximately as in the following picture.



The bar is levitated by some means to be determined by the student, and sits in mid air, resting against the glass plate on the needle point. The bar can be lifted out of the device, and moved about or rotated.

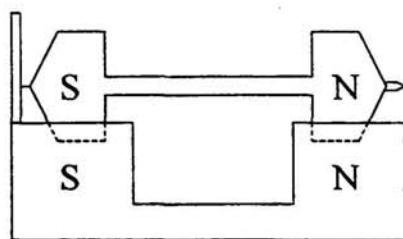
If students were getting stuck, hints were given, though hints reduced the final possible score.

### Solution

The obvious (and correct) reasonable assumption by the student is that this works somehow by magnetism. There are several possibilities that might explain the levitation, though only one basic hypothesis survives a few simple experiments.

There are two simple tests one can perform on this system: (i) Turn the bar around so the ball point is at the glass plate end; (ii) Move the bar sideways so that the needle point end "lump" on the bar is above the right end of the base. (The bar can also be spun, which eliminates some other possibilities quite simply.)

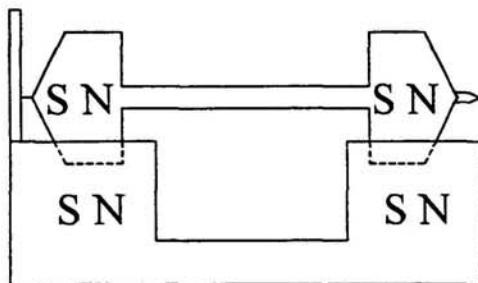
Most students started with the simple hypothesis of one magnet in the base and one in the bar, as below, with poles arranged to repel.



If the bar is turned around (swapping the N and S ends), the bar is no longer levitated, consistent with this hypothesis. But if the bar is moved sideways so that its left end is above the right end of the base, the bar is repelled rather than attracted, inconsistent with this hypothesis.

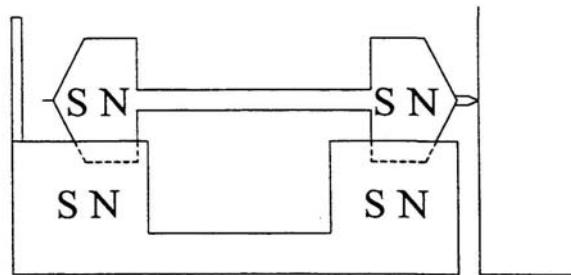
Another simple hypothesis is that there are two magnets in the bar, each with one pole on the axis and another on the outside (i.e., we move from one pole to the other as we move along a radius), and poles in the base arranged to repel the appropriate outside poles on the bar. Such arrangements can satisfy the consequences of either (i) turning the bar around or (ii) moving the bar so its left end is above the right end of the base, but not both, by similar logic to the first above hypothesis.

The correct answer, which is the only hypothesis that any student came up with that does satisfy both tests, is below. There are two magnets in the base and two in the bar, each oriented in the same way with north and south poles along the horizontal axis. (The choice of North and South here is of course arbitrary, though their relative positions are not.)

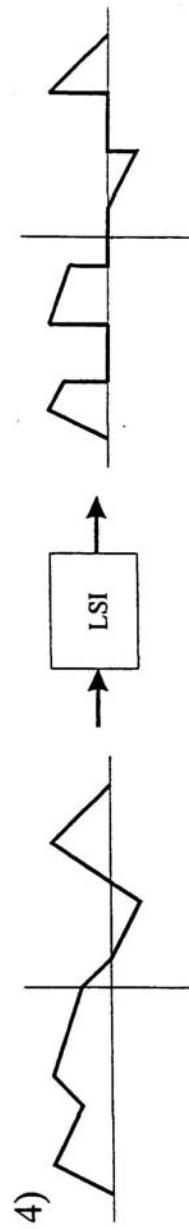
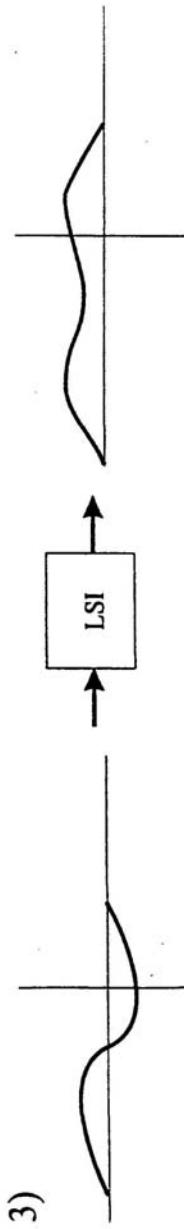
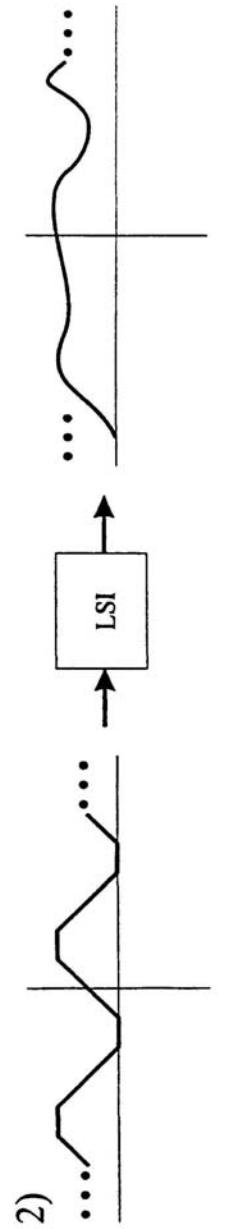
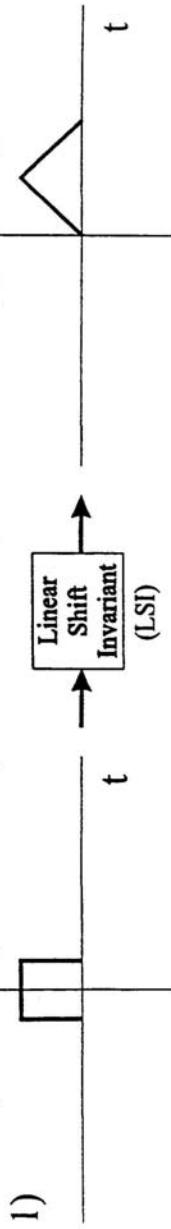


Note that the magnets in the base and the bar have to be slightly offset so that there is a slight force to push the needle point against the plate, and this point was asked as a supplementary question for those who got this far.

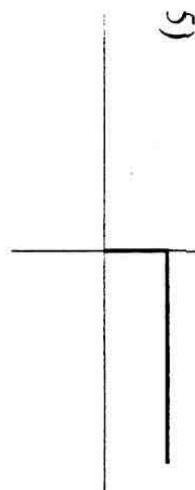
As a further supplementary question for students who got this far, students were asked whether, using possibly other objects on the desk, they could get the bar to be levitated and be resting on the ball point end rather than the needle end. This could be achieved using the side of a clock that was sitting on the desk, as shown below.



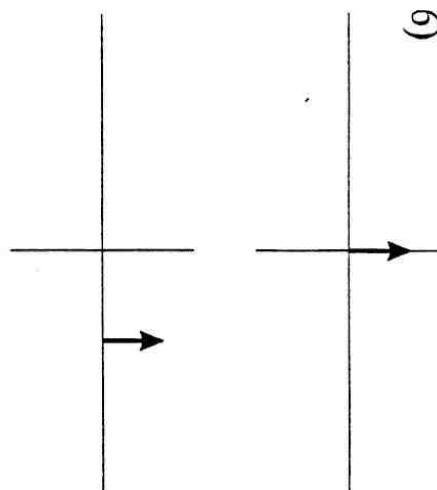
For each system below, indicate if it is possible for the output function to occur, given the input function.



5)



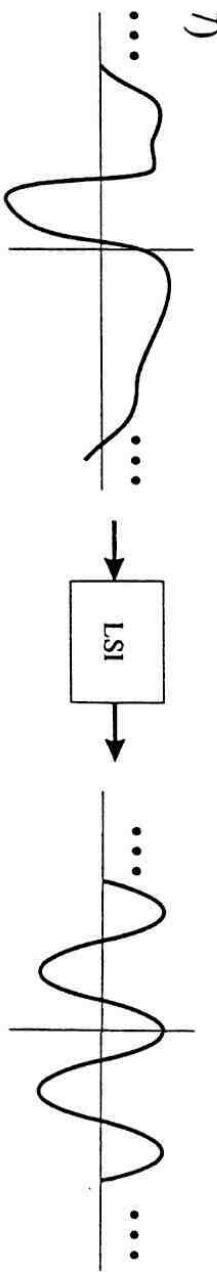
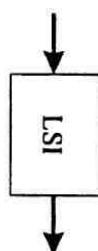
6)



Case a



Case b



7)

7. Yes, narrow bandpass filter would do it.

6. No

5. Yes, impulse response is the derivative of the step response.

4. No

3. No

2. No

1. Yes, impulse response is a shifted rectangle.

### Quals Solutions

Nishimura

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Brad Osgood

A friend wants to find an explicit, if approximate, formula for the spectrum of a complicated signal. His idea is to approximate the signal by a polynomial and find the Fourier transform. Comment on this method. Can you *find* the Fourier transform of a polynomial? What other methods might your friend consider?

Fabian Pease, 1/29/01 1:59 PM -0800, Re: Quals Meeting Today!

1

From: "Fabian Pease" <pease@cis.Stanford.EDU>  
To: "Diane Shankle" <shankle@ee.stanford.edu>  
Subject: Re: Quals Meeting Today!  
Date: Mon, 29 Jan 2001 13:59:21 -0800  
X-Priority: 3

Thanks Diane:  
Here's my Question(s):

1. During the last 30 years digital circuitry has been increasingly doing tasks previously done with analog circuitry. Why?
2. List some of the parameters used to describe the technical performance of an analog-to-digital converter (ADC).
3. Outline how an ADC works. What happens when the 2 inputs to a comparator are the same?
4. What are some of the fundamental limits to ADC performance; for example does the Heisenberg uncertainty principle set a limit to the combination of high speed and high resolution? How might the various forms of noise limit speed and resolution?

Fabian  
-----Original Message-----

Piero Pianetta

Question:

Describe how you would make a detector for x-rays using silicon and how it works?  
Also describe the signal that one would expect to see from such a detector.

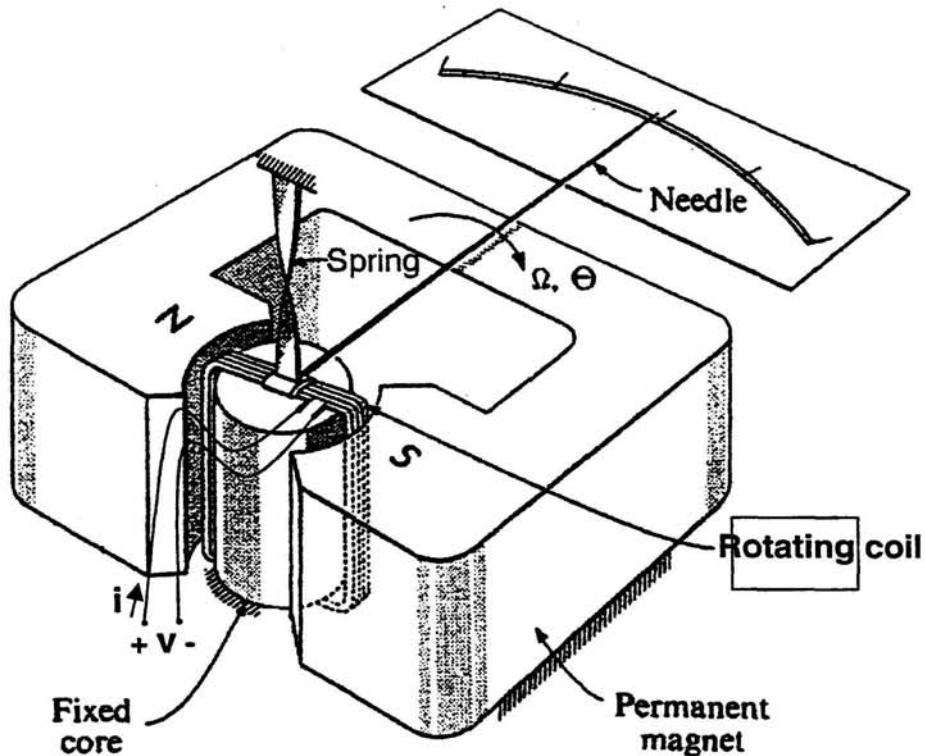
Answer:

It is important to first recognize that photon detectors whether for x-rays or visible light generate electron hole pairs via the photoelectric affect. These can be detected by using silicon that is in a PN, PIN, phototransistor or photoresistor configuration. The important point here is that the absorption of the photon results in an electron hole pair in the silicon that results in current flow in these structures.

In addition, since x-ray photons are much more energetic than visible photons, the emitted electron will have significant amounts of kinetic energy that will in turn create a cascade of additional electron hole pairs. In order to conserve energy, it must be recognized that the number of electron hole pairs will be less than the x-ray energy divided by the band gap of silicon (in reality, the current is a factor of 2-3 smaller but this was an acceptable approach).

The signal from such a detector would be a pulse whose rise and fall times are determined by the capacitance of the device and how quickly the electrons could be swept out of the device. The capacitance of the device could be lowered by going from a PN to a PIN structure—a calculation of the capacitance was also helpful. Higher voltages could be used if a resistive structure were employed thus sweeping the electrons more quickly through the device. In addition, smaller contacts could be used to lower the capacitance if a strategy for collecting the electrons could be developed.

## D'Arsonval Galvanometer



1. Model this device as a LTI system with input  $i$  (current) and output  $\theta$  (needle angle). Assume the torque induced in the coil by current  $i$  is independent of  $\theta$ .



Hint: What is the equivalent electrical circuit?

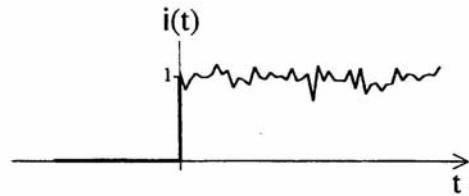
Daniel Spier

2. (a) Sketch the response,  $\theta(t)$ , to the following input:

$$i(t) = (1 + n(t))H(t), \text{ where}$$

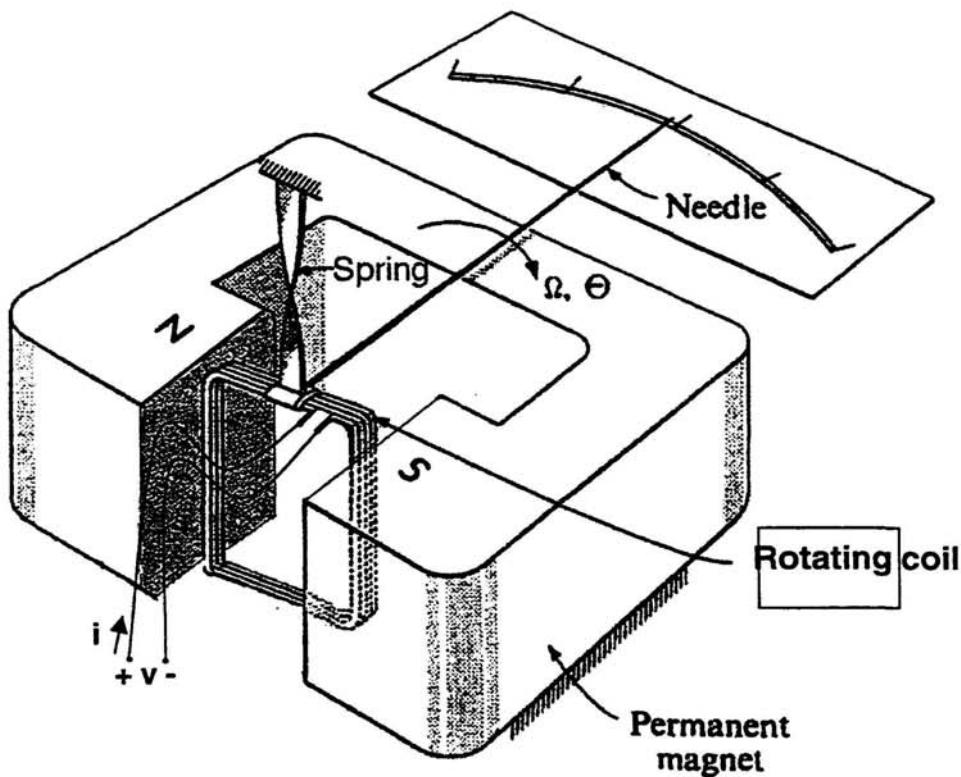
$H(t)$  is a step function

$n(t)$  is white Gaussian noise:  $\mu=0, \sigma=0.1$



(b) In designing the device, how much damping would you use and how would you implement it?

## Low-Cost Galvanometer



3. In this low-cost design, the torque,  $T$ , on the coil induced by input current  $i$  is no longer independent of  $\theta$ . Rather:

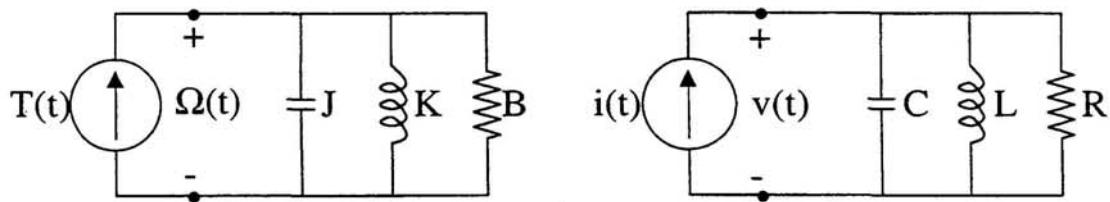
$$T(t) \propto i(t) \cos \theta$$

How would you analyze this system?

## D'Arsonval Galvanometer

### Solutions

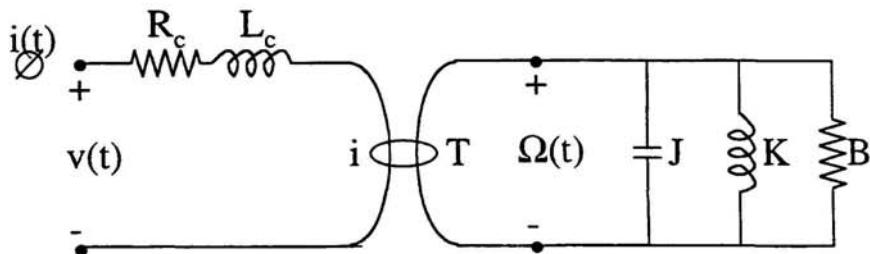
1. (3pts) This device is well modeled as a simple 2nd order linear system and is analogous to a parallel LRC circuit



with the following corresponding variables:

|                            |          |       |
|----------------------------|----------|-------|
| torque $T$                 | $\times$ | $i$   |
| angular frequency $\Omega$ | $\times$ | $v$   |
| needle angle $\theta$      | $\times$ | $w$   |
| spring constant $K$        | $\times$ | $1/L$ |
| inertia $J$                | $\times$ | $C$   |
| damping $B$                | $\times$ | $1/R$ |

Note: the full model including coil inductance leads to a 3rd order LTI system with the galvanometer acting as a transducer converting electrical to mechanical energy.



# D'Arsonval Galvanometer

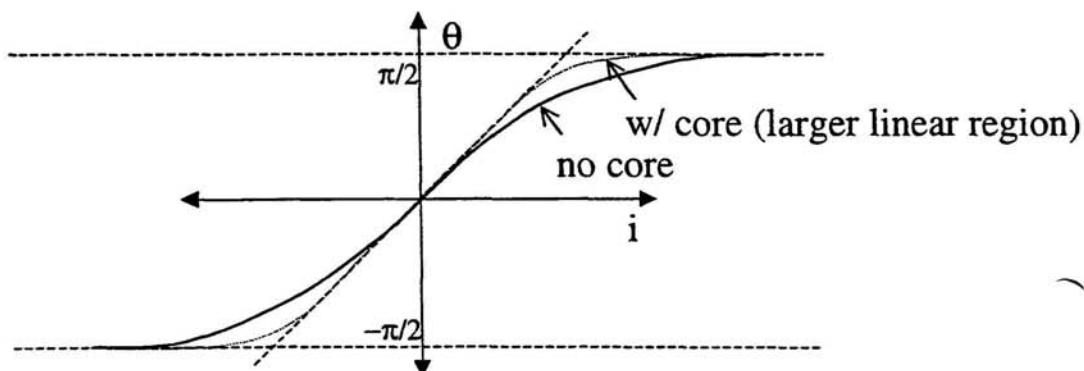
## Solutions

2. a) (3pts) Response to  $i(t) = (1+n(t))H(t)$  is given by the step response plus Gaussian noise. Note, the noise is not white but rather has predominant energy at the resonant frequency and attenuated higher frequencies (see frequency response).

b) (1pts) System critically damped if  $B=2\sqrt{JK}$ . Damping is usually added by either increasing the air resistance of the needle or surrounding the coil with non-ferrous conducting material in order to exploit the eddy currents induced by the motion of the coil in the magnetic field.

3. (3pts) Full dynamic analysis is much more difficult than linear case (system is still time invariant). We can however make several observations:

- System natural frequency is unchanged (computed with  $i(t)=0$ ). In fact if we view  $T(t)$  as input rather than  $i(t)$ , system is LTI as before.
- For small  $\theta$ , system is LTI as before.
- Can easily compute the steady state response since effects of  $J$  and  $B$  drop out. Steady state output will look something like:



**len tyler, 2/15/01 3:27 PM -0800, Re: Quals Question 2001**

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1

X-Sender len@nova.stanford.edu  
Date: Thu, 15 Feb 2001 15:27:02 -0800  
To: Diane Shankle <shankle@ee.stanford.edu>  
From: len tyler <len.tyler@stanford.edu>  
Subject: Re: Quals Question 2001

Suppose that you are transported to an isolated island populated by people with no exposure to modern thought: these people believe that the world is flat, that their island is floating, and that the sun, moon, planets, and stars reel about the Earth each day. What evidence could you adduce on the basis of observations that you could perform on the island that actually the Earth spins on its axis while orbiting the sun once each year?

>I am still waiting for you to submit your Quals Question either by hard copy  
>or email.  
>Please try to submit by 2/23/01.  
>  
>Happy Thursday.  
>Diane

---

G. Leonard Tyler, Prof. (650) 723 3535  
Packard EE Building, Room 331 (650) 544 6421 alt  
350 Serra Mall (650) 723 9251 FAX  
Stanford University, California  
94305-9515 len.tyler@stanford.edu

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Printed for Diane Shankle <shankle@ee.stanford.edu>

cal quate, 1/29/01 2:31 PM -0800, Re: Quals Meeting Today!

1

X-Sender: quate@EE.Stanford.EDU (Unverified)  
Date: Mon, 29 Jan 2001 14:31:54 -0800  
To: Diane Shankle <shankle@ee.stanford.edu>  
From: cal quate <quate@Stanford.EDU>  
Subject: Re: Quals Meeting Today!

Diane;

My Questions:

- 1 - A parallel plate capacitor is half filled with a dielectric. Is there a force on the dielectric when a voltage is applied across the plates.
- 2 - What is the size of a capacitor when the addition of a single electron will change the stored energy by  $kT$ , where  $kT$  is the thermal energy.
- 3 - With the relation  $B=\mu_0\mu H$  given, please discuss a method for measuring  $\mu$ .
- 4 - Light is propagating from left to right through a dielectric crystal. If an E field is applied across the crystal, will the light propagation be changed in any way?

cal quate

=====

14:31:54 PM 1/29/2001 2000 -0800

## **EE Qualifying Exam Questions**

**January 22 – 26, 2001**

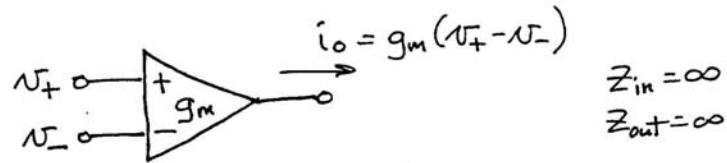
**Yoshi Yamamoto**

**What are thermal noise, quantum noise and 1/f noise? You can choose any one of the following systems and describe the origins of those fluctuations.**

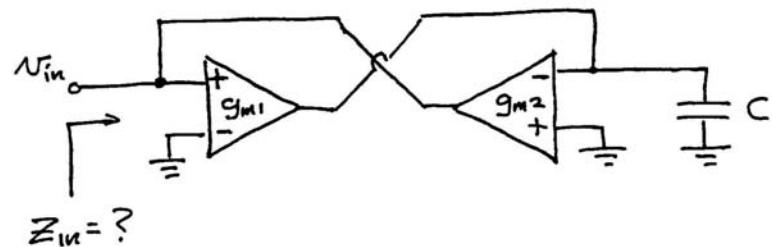
- [1] Simple macroscopic conductor
- [2] Mesoscopic conductor under ballistic regime
- [3] pn junction under either forward or reverse bias
- [4] Tunnel junction
- [5] Laser/maser
- [6] Parametric oscillator
- [7] Mechanical oscillator

Bruce Woolley  
QUALS QUESTION 00-01

Ideal Transconductance



What is  $Z_{in}$  for the following circuit?



Simon Wong, 2/16/01 5:28 PM -0800, Re: Quals Question 2001

1

X-Sender: swong@marco.stanford.edu  
Date: Fri, 16 Feb 2001 17 28:42 -0800  
To: Diane Shankle <shankle@ee.stanford.edu>  
From: Simon Wong <wong@ee.stanford.edu>  
Subject: Re: Quals Question 2001

2001 Qualifying Exam Question  
Simon Wong

1. Sketch the schematic of
  - a) a CMOS Inverter, and
  - b) a Common Source Amplifier with an Active Load.
2. Compare the Small Signal Voltage Gain of the two circuits assuming the device sizes and the bias currents are similar.
3. Compare the Bandwidth of the two circuits.

At 11:08 AM 2/15/2001 -0800, you wrote:

I am still waiting for you to submit your Quals Question either by hard copy or email.  
Please try to submit by 2/23/01.

Happy Thursday,  
Diane

---

Printed for Diane Shankle <shankle@ee.stanford.edu>

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To: Diane Shankle <shankle@ee.stanford.edu>  
Subject: Re: Quals Meeting Today!  
Date: Mon, 29 Jan 2001 20:43:32 -0800  
From: Jennifer Widom <widom@DB.Stanford.EDU>

Jennifer Widom 2001 EE quals questions with sample solutions

=====

Consider a binary tree with values in each node. That is, each node N of the tree has:

N.value: an integer  
N.left: the root of the left subtree, or NULL  
N.right: the root of the right subtree, or NULL

Every node has either two children or zero (i.e., N.left = NULL iff N.right = NULL), but trees need not be balanced.

=====

(1) Write a recursive function Sum(T) that returns the sum of all values in the binary tree rooted at T. Do not use any global variables.

```
Sum(T):  
    if T.left = NULL then return(T.value)  
    else return(T.value + Sum(T.left) + Sum(T.right))
```

=====

(2) Write a recursive function Height(T) that returns the length of the longest path from the root of the binary tree rooted at T to a leaf. Do not use any global variables.

```
Height(T):  
    if T.left = NULL then return(0)  
    else return(1 + max(Height(T.left), Height(T.right)))
```

=====

(3) Write a recursive function MinTwo(T) that returns the two smallest values in the binary tree rooted at T. You may assume the tree contains at least 2 (therefore 3) nodes, and that each value in the tree is unique. Do not use any global variables.

```
MinTwo(T):  
    // local variable temp has type set of integers  
    if T.left = NULL then return({T.value})  
    else begin  
        temp := MinTwo(T.left) UNION MinTwo(T.right) UNION {T.value};  
        return({min(temp), min(temp - min(temp))})  
    end
```

$$y_k = (1+\alpha) y_0 + n_{ik}$$

$$y_0 = \$10$$

Solution:

$$a). \quad y_k = (1+\alpha)^k y_0 + \sum_{i=1}^k n_i (1+\alpha)^{k-i} \quad \sigma = .25 \\ \sigma^2 = .0625$$

b). interest

$$\text{daily } \frac{\alpha}{100} = .1\% \text{ if } \alpha = .001$$

$$\text{annually } (1+\alpha)^{250} = 1.2839 \Rightarrow 28.39\%$$

$$c). \quad E[y_k] = (1+\alpha)^k y_0 = 10(1.2839) = \$12.84$$

Gaussian

$$d). \quad E[y_k^2] = (1+\alpha)^{2k} y_0^2 + \underbrace{\sum_{i=1}^k \sigma^2 (1+\alpha)^{2(k-i)}}_{\text{variance}}$$

$$\text{var}(y_k) = \sigma^2 \left[ \frac{(1+\alpha)^{2k} - 1}{(1+\alpha)^2 - 1} \right]$$

$$\sigma_{y,k}^2 = \sigma^2_{y,0} + (\alpha)^2 + \sigma^2$$

$$\sum_{i=0}^{k-1} (1+\alpha)^{2i} \sigma^2 = \frac{(1+\alpha)^{2k} - 1}{(1+\alpha)^2 - 1} = 324 \sigma^2$$

$$\text{std} = 18.6 \sim \$4.50$$

$$e). \quad \Pr\{y_0 < y_0\} = \Pr\{n_0 > .01\} = Q\left[\frac{.01}{.0625}\right] = Q\left[\frac{.01}{.0016}\right];$$

$$\Pr\{y_0 < y_0\} \Rightarrow E[y_{10}] = \$11.00 \quad \text{For } 250$$

$$= \Pr\{\text{noise} < -\$1.00\} = Q\left[\frac{1}{\sigma_{10} = 3.176}\right]$$

$$\text{var}(y_{10}) = \sigma^2 \left[ 10.1107 \right] \quad 3.17916 \quad -10dB$$

$$\Pr\{y_{250} < y_0\} = \Pr\{\text{noise} < -2.38\} \quad Q\left[\frac{2.38}{4.50}\right] \approx \boxed{.2}$$

$$\Pr\{y_{250} > \left(\frac{2.84}{3.1791}\right) y_0\} = y_2 \quad \boxed{e). \quad (1+\alpha)^{250} = 2} \\ \alpha = .0028$$

**cal quate, 1/29/01 2:31 PM -0800, Re: Quals Meeting Today!**

**1**

X-Sender: quate@EE.Stanford.EDU (Unverified)  
Date: Mon, 29 Jan 2001 14:31:54 -0800  
To: Diane Shankle <shankle@ee.stanford.edu>  
From: cal quate <quate@Stanford.EDU>  
Subject: Re: Quals Meeting Today!

Diane;

My Questions:

- 1 - A parallel plate capacitor is half filled with a dielectric. Is there a force on the dielectric when a voltage is applied across the plates.
- 2 - What is the size of a capacitor when the addition of a single electron will change the stored energy by  $kT$ , where  $kT$  is the thermal energy.
- 3 - With the relation  $B=\mu_0\mu_r H$  given, please discuss a method for measuring  $\mu_r$ .
- 4 - Light is propagating from left to right through a dielectric crystal. If an E field is applied across the crystal, will the light propagation be changed in any way?

cal quate

=====

To: Diane Shankle <shankle@ee.stanford.edu>  
Subject: Re: Quals Meeting Today!  
Date: Mon, 29 Jan 2001 20:43:32 -0800  
From: Jennifer Widom <widom@DB.Stanford.EDU>

Jennifer Widom 2001 EE quals questions with sample solutions:

=====

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  end