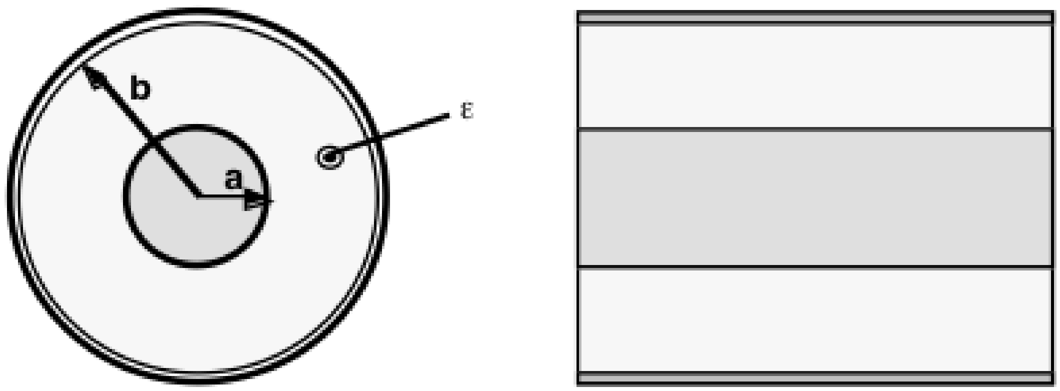
Velocity and losses in a coaxial transmission line

Consider a coaxial transmission line consisting of inner and out low-loss conductors separated by a low-loss dielectric having dielectic constant ε.



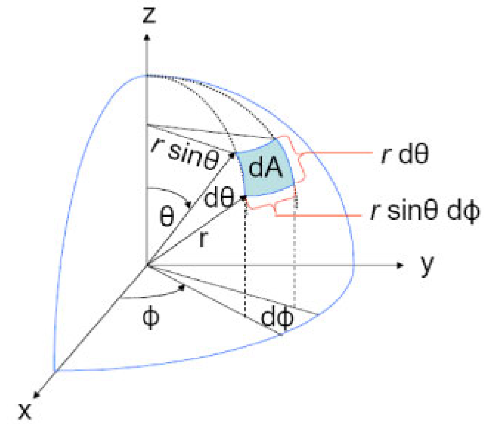
1. What is the velocity of propagation in such a transmission line?

2. What can you say about skin depth in the conductors, as a function of conductivity and frequency?

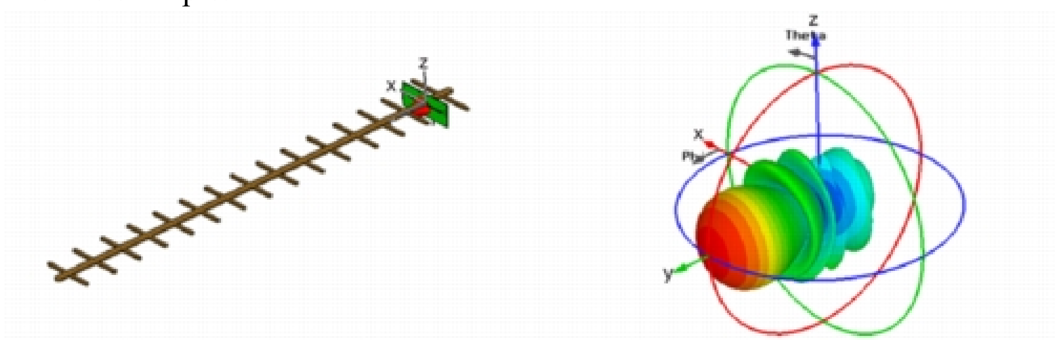
3. For a non-ideal transmission line (non-zero conductive and dielectric losses), what can you say about the behavior of conductive and dielectric losses as a function of frequency?

Power density of a radiating antenna

1. Consider a free-space radiator of electromagnetic waves. Show that the power flux (power per unit area) at a distance **r** is proportional to the inverse of the square of the distance, that is, $\mathbf{1/r^2}$. Use spherical coordinates.



2. An isotropic radiator radiates equal power in all directions. A directive antenna enhances power in a desired direction by reducing transmission in other directions. Directivity is defined as the ratio $D = P_{dir}/P_{iso}$, as shown by directive radiation patterns such as this example:



(a) Yagi Antenna Model

(b) Yagi Antenna 3D Radiation Pattern

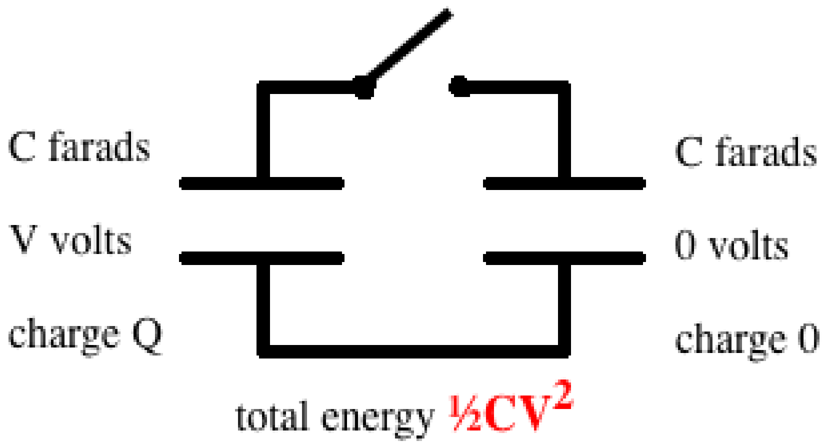
(c) Yagi Antenna Azimuth Plane Pattern

(d) Yagi Antenna Elevation Plane Pattern

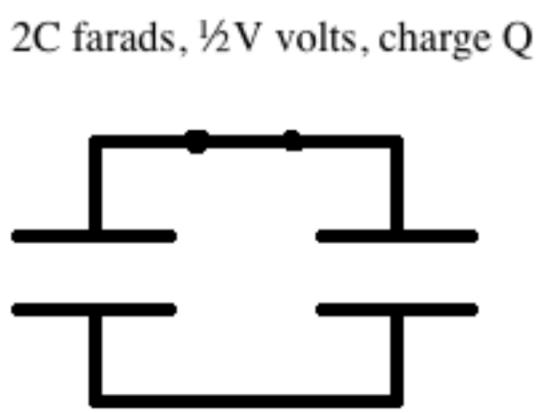
What is the integral over all azimuth and elevation angles of the directivity of an isotropic antenna, and of a directive antenna?

The capacitor paradox:

Consider a capacitor C charged to a voltage V. The charge stored in the capacitor is $Q = CV$ and the energy stored in the capacitor is $1/2CV^2$.



1. Now close the switch to share the charge with the second, equal capacitor. Since charge is conserved and the voltage on both capacitors must be equal, what is the resulting voltage?



2. Now what is the total energy stored in both capacitors?

3. If this is not equal to the original stored energy, what might be some reasons for the difference?

Electromagnetics Solutions 2014

Coaxial transmission line:

$v_p = c/\sqrt{\varepsilon}$

$\delta = \sqrt{(2/\mu\sigma\omega)}$

Conductive loss α $\sqrt{f}$

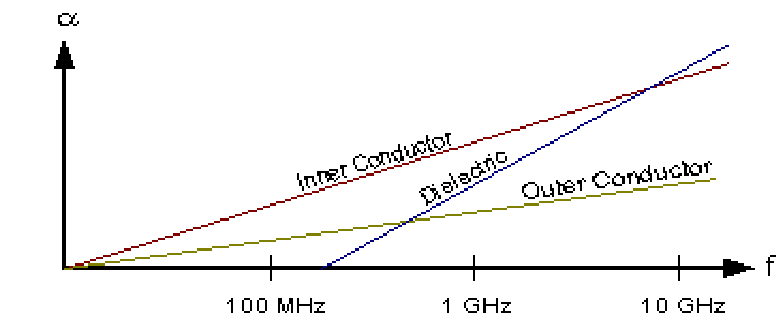
Dielectric loss α f

Cable attenuation is the sum of the conductor losses and the dielectric losses per the following equations.

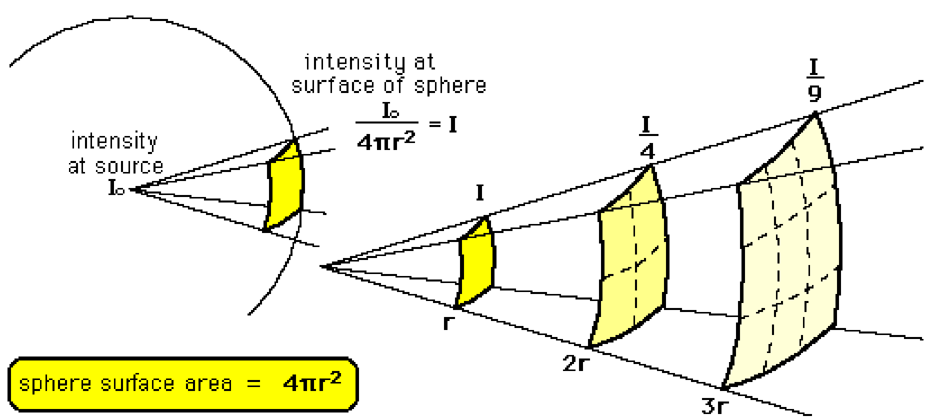
$$\alpha_{conductors} = \alpha_c = \frac{11.39}{Z} * \sqrt{f} * \left| \frac{\sqrt{\rho_{rd}}}{d} + \frac{\sqrt{\rho_{rD}}}{D} \right| \quad \frac{dB}{m}$$

$$\alpha_{dielectric} = \alpha_{diel} = 90.96 * f * \sqrt{\varepsilon_r} * \tan(\delta) \quad \frac{dB}{m}$$

ρ_r = 1 for copper, 10 for steel



Inverse square law and directivity:

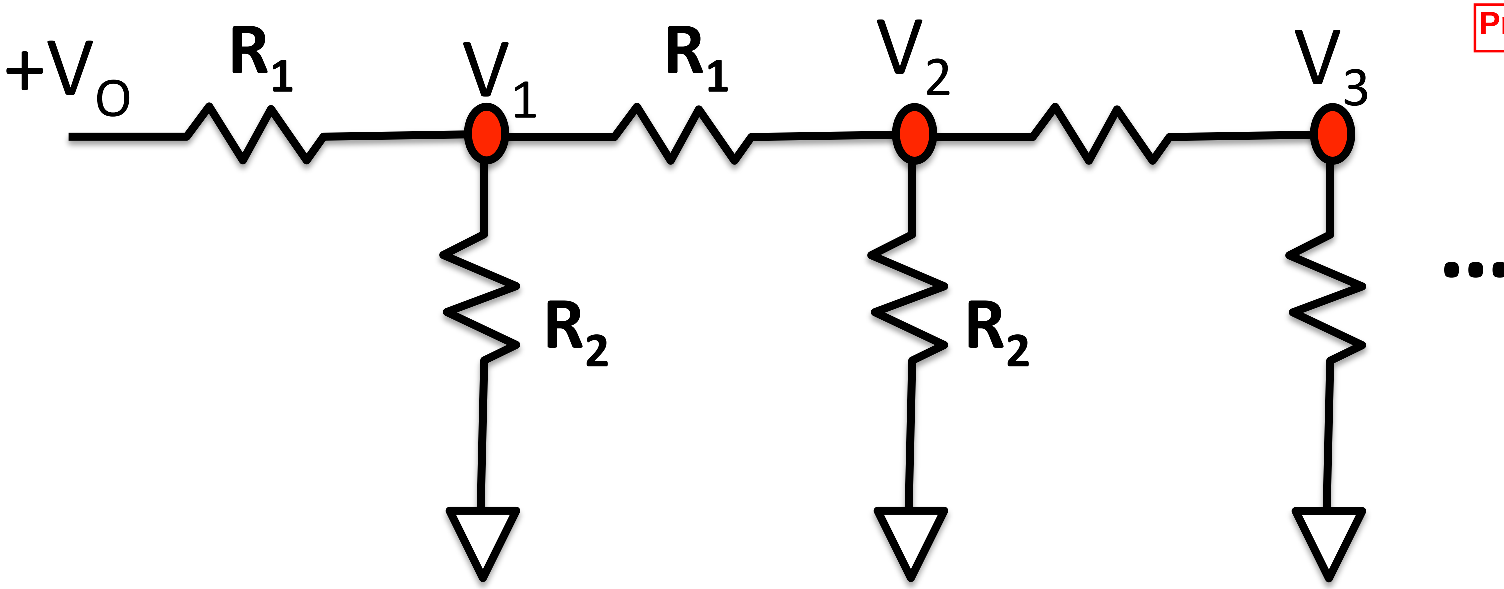


Directivity integrated over a full sphere is *unity* for all antennas, isotropic or directive (no power is generated in the antenna).

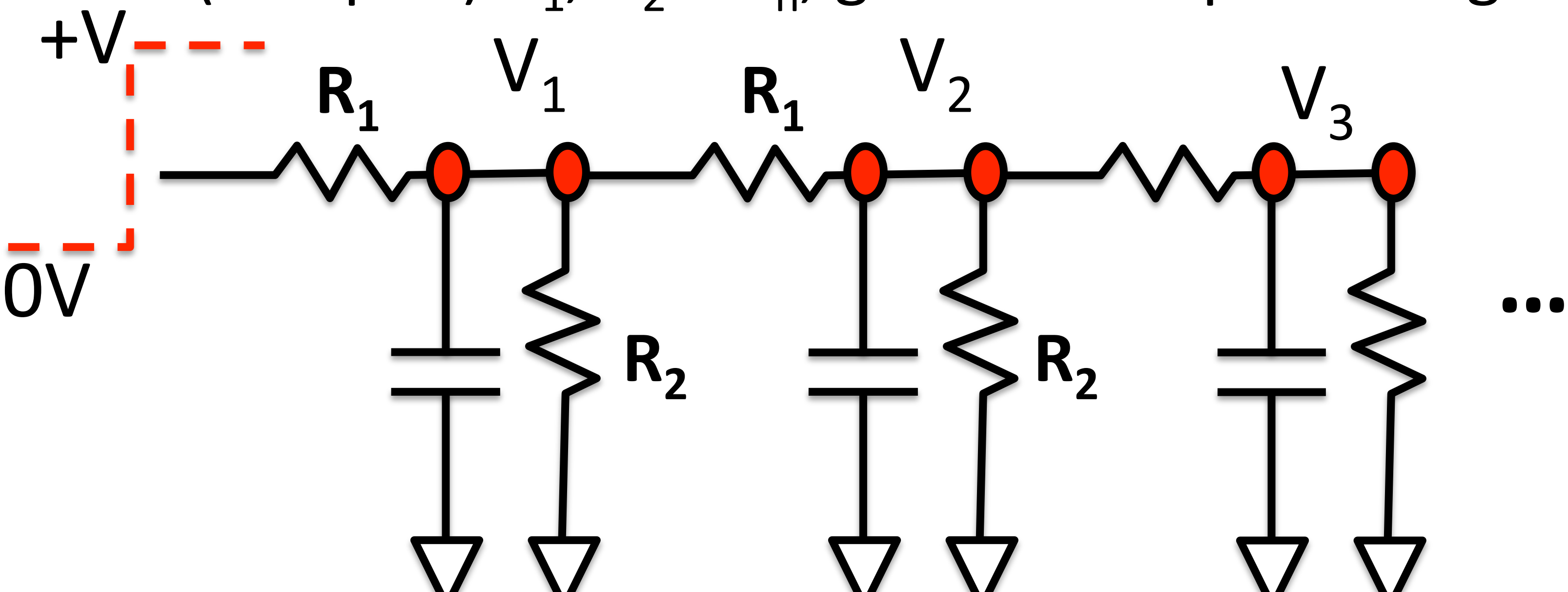
Capacitor paradox:

Total initial energy is $^1/_2CV^2$. After switch is closed, charge is divided equally between capacitors, so voltage is $^1/_2V$ and stored energy is $^1/_4CV^2$. The remaining energy is either *dissipated* in the resistance of the circuit or the switch, or if the circuit is lossless, is *radiated* from the circuit because of the current pulse, or is *stored* in magnetic fields of the connecting wires.

Prof. Dutton



Determine (and plot) V_1, V_2 ...V_n, given a dc input voltage Vo

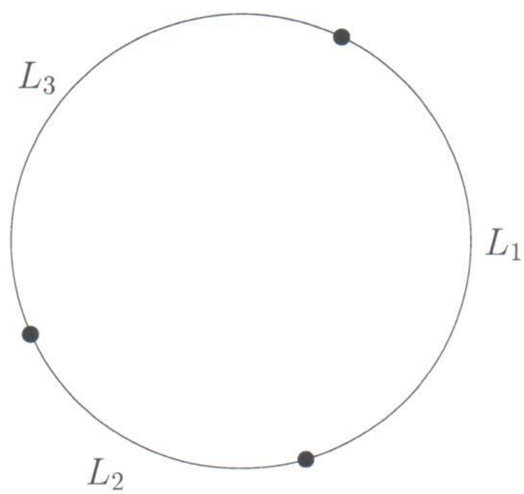


Determine (and plot) the Step response for V_1, V_2 ...V_n,
As a function of time. Do this for both 0->+V and +V->0

Abbas El Gamal 2014 Quals

A circle and three random points

Three points are randomly and independently selected on a circle of unit radius. Denote the lengths of the three arcs resulting from this selection by L_1, L_2, and L_3.



Cond prob. + symmetry 2
Work out 2

4 1. What is the conditional probability $\mathsf{P}\{L_3 > L_2 | L_2 > L_1\}$?

6 2. Find the correlation coefficient $\rho_{X,Y}$ between $X = (L_2 - L_1)$ and $Y = (L_3 - L_2)$.

Def 1
Symm 2
Work out 3.

Hint: Define $\mathrm{Var}(L_1)$
$\mathrm{Cov}(L_1, L_3)$

Ph.D. Quals Question
January 13-17, 2014
A.C. Fraser-Smith
Department of Electrical Engineering
Stanford University

The Geiger Counter

The students enter the examiner's office to find the device pictured below sitting on the table in front of them – clicking irregularly and with a small red light flashing as it does so. The screen typically reads 0.008 mR/hr or less. They are asked what they think it is and what it is doing.

Front Back

At this early stage it is not particularly critical for them to know what it is, but they can at least notice that it says "radiation alert" on both its back and front. At this stage, if they are in doubt, the examiner points out that it is a *Geiger counter* and enquires about what radiation it is detecting. The student really should show some awareness about the radiation that is being detected, since it consists of electrical charges and high-frequency radiation photons: alpha, beta, or gamma (and x-ray) radiation, i.e., helium nuclei, electrons, and gamma (and x-ray) photons. When this has been straightened out, the examiner goes on to point out that the key sensor in the counter is a Geiger-Müller tube and that although the technology dates back to the early 1920's it is still the most widely used technology for radiation detectors. However, it has one severe weakness: the counter cannot distinguish between the different forms of radiation. Positive particles, negative particles, and high-energy photons all produce the same output from the counter. It cannot distinguish between them.

While this discussion is going on the examiner will have placed the counter on a piece of innocuous-appearing rock, which increases the clicking to a level about five times the previous background level, or more (> 0.030 mR/hr). The rock contains granite, which is naturally radioactive, but at a level that is considered harmless.

The examiner then proceeds to ask his **key exam question**: With the information that has been given about the counter, how does the student, a potential electrical engineer, think the counter works? In several cases the photoelectric effect was mentioned, but his was easily dismissed as an explanation since it involves photons hitting a surface, but the Geiger counter also responds to energetic charged particles. In the following, for simplicity, we will lump these photons together with the energetic charged particles and call them all "particles." The thinking process should have proceeded very roughly as follows:

(1) Since the response of the Geiger counter is independent of the form of the radiation, i.e., whether it is photons or particles, much less whether the particles are positively or negatively charged, about the only thing relevant seems to be the energy of the particles/photons. Given that the sensor is a tube, it should be concluded that the particles' energy leads to them ionizing some gas particles (of an assumed low pressure inert gas) in the tube through collisions.

(2) Since the Geiger counter responds to single particles, of various energies but which in macroscopic terms are not very substantial, it seems necessary for some amplification of the ionization to take place for it to be detected. At this stage it would be natural for the student to enquire if there was a voltage applied to the tube (yes; some hundreds of volts DC) or this information was given as a hint. Making the assumption (correct) that a high (positive) voltage was applied to an electrode inside the tube and that its metal walls were grounded, the electric field inside the tube would accelerate the original ionization electrons, producing even more ionization, thus creating an "avalanche" of electrons. Some students were aware of *avalanche photodiodes*, solid state devices that amplify an initial small burst of ionization on similar principles. These avalanches would be detected as a measureable pulse of current passing between the tube's electrode and its metal walls.

(3) The ions produced as part of the ionization in the gas would also be accelerated by the electric field, in the opposite direction to the electrons of course, but being much heavier they will not produce much additional ionization. Instead, they will tend to remain in the region around the electrode, creating a positive space charge that will help prevent the electrons reaching all the way to the electrode. Thus the ions act to quench the generation of avalanches.

Note: the circular patch in the back of the counter is a thin mica window that allows low energy particles to penetrate into the Geiger-Müller tube. It is protected from being punctured by a thin metal mesh.

Qualifying Exam, Jan 2014

Examiner: Sachin Katti

A graph is said to be 2-connected if deleting any one of its edges does not disconnect the graph. (A connected graph means that there exists a path between any two nodes).

a) Describe an algorithm to check if a graph is 2-connected?
b) What is the running time of your algorithm?
c) A well-known fact about 2-connected graphs is that there are two edge disjoint paths between any two nodes (edge disjoint paths means that the paths do not have any edges in common). Use this fact to design an algorithm to check if a graph is 2-connected that runs in O(E+V) time? (E = # of edges, V = # of nodes)

EE Qualifying Exam
January 2014

Lost in Translation?

For your research you'll need to develop the ability to translate scientific and mathematical statements from the language used in books and articles from another field to that of your own. Consider this theorem, copied verbatim from a mathematics text:

Theorem *If (ζ_ν) are the finite Fourier coefficients of the sequence (z_ν) and if we subject the (z_ν) to the cyclic transformation*

$$\begin{aligned}
z_0' &= a_0 z_0 + a_1 z_1 + a_2 z_2 + \cdots + a_{k-1} z_{k-1} \\
z_1' &= a_{k-1} z_0 + a_0 z_1 + a_1 z_2 + \cdots + a_{k-2} z_{k-1} \\
&\vdots \\
&\vdots \\
z_{k-1}' &= a_1 z_0 + a_2 z_1 + a_3 z_2 + \cdots + a_0 z_{k-1},
\end{aligned}$$

then the finite Fourier coefficients of the new sequence (z_ν') are

$$\zeta_\nu' = \zeta_\nu f(e^{2\pi i\nu/k}),$$

where

$$f(z) = a_0 + a_1 z + \cdots + a_{k-1} z^{k-1}.$$

This is actually a standard result in digital signal processing. Make the translation. Why is the result true?

Solution: Let's see, "finite Fourier coefficients ..." They must be talking about the DFT, and the ζ's are the "coefficients." The indexing goes from 0 to $k-1$ so it looks like the DFT of order k. We must have $\underline{z} = (z_0, z_1, \dots, z_{k-1})$ as the input and $\underline{\zeta} = (\zeta_0, \zeta_1, \dots, \zeta_{k-1})$ as the output on applying the DFT, i.e.,

$$\underline{\zeta} = \mathcal{F}\underline{z}, \quad \zeta_\nu = \sum_{n=0}^{k-1} z_n e^{-2\pi i n\nu/k}.$$

We want to find $\underline{\zeta}' = \mathcal{F}\underline{z}'$ when $\underline{z}'$ is obtained from $\underline{z}$ by the indicated "cyclic transformation." We can write that relationship between the z_ν' and the z_ν as a matrix product:

$$\begin{pmatrix} z_0' \\ z_1' \\ z_2' \\ \vdots \\ z_{k-1}' \end{pmatrix} = \begin{pmatrix} a_0 & a_1 & a_2 & \dots & a_{k-1} \\ a_{k-1} & a_0 & a_1 & \dots & a_{k-2} \\ a_{k-2} & a_{k-1} & a_0 & \dots & a_{k-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_1 & a_2 & a_3 & \dots & a_0 \end{pmatrix} \begin{pmatrix} z_0 \\ z_1 \\ z_2 \\ \vdots \\ z_{k-1} \end{pmatrix}$$

The matrix, call it A, is *circulant*, and we know that the system

$$\underline{z}' = A\underline{z}$$

(being tine invariant) can be written as a convolution

$$\underline{z'} = \underline{h} * \underline{z}$$

where $\underline{h}$ is the first column of the matrix (convolution with the impulse response),

$$\underline{h} = \begin{pmatrix} a_0 \\ a_{k-1} \\ a_{k-2} \\ \vdots \\ a_1 \end{pmatrix}.$$

Then by the convolution theorem

$$\underline{\zeta'} = \mathcal{F}\underline{z'} = (\mathcal{F}\underline{h})(\mathcal{F}\underline{z}) = (\mathcal{F}\underline{h})\,\underline{\zeta},$$

so we have to see if the product on the right-hand side is the translation of what the author wrote, i.e.,

$$\zeta'_\nu = \zeta_\nu f(e^{2\pi i\nu/k}).$$

The DFT of $\underline{h}$ is

$$\mathcal{F}\underline{h}[\nu] = \sum_{n=0}^{k-1} \underline{h}_n e^{-2\pi i n\nu/k} = \sum_{n=0}^{k-1} a_{k-n} e^{-2\pi i n\nu/k}.$$

Note here that we have to regard $\underline{h}$ (as well as the other inputs and outputs) as being periodic of period k, so $a_k = a_0$, etc. According to this we have

$$\zeta'_\nu = \zeta_\nu \sum_{n=0}^{k-1} a_{k-n} e^{-2\pi i n\nu/k} = \zeta_\nu(a_0 + a_{k-1}e^{-2\pi i\nu/k} + a_{k-2}e^{-2\pi i 2\nu/k} + \cdots + a_1 e^{-2\pi i(k-1)\nu/k}).$$

Doesn't look quite like what the author has. But we can write

$$e^{-2\pi i n\nu/k} = e^{-2\pi i n\nu/k} e^{2\pi i k\nu/k} = e^{2\pi i(k-n)\nu/k}$$

and that brings the sum above into the form

$$\begin{aligned}
\zeta'_\nu = \zeta_\nu \sum_{n=0}^{k-1} a_{k-n} e^{-2\pi i n\nu/k} &= \zeta_\nu(a_0 + a_{k-1}e^{-2\pi i\nu/k} + a_{k-2}e^{-2\pi i 2\nu/k} + \cdots + a_1 e^{-2\pi i(k-1)\nu/k}) \\
&= \zeta_\nu(a_0 + a_{k-1}e^{2\pi i\nu(k-1)/k} + a_{k-2}e^{2\pi i\nu(k-2)/k} + \cdots + a_1 e^{2\pi i\nu/k}) \\
&= \zeta_\nu f(e^{2\pi i\nu/k}),
\end{aligned}$$

with

$$f(z) = a_0 + a_1 z + \cdots + a_{k-1}z^{k-1}.$$

Just as the author wrote!

Pease's Quals question 2014.

Here is a 'device'*; (in fact a 0.1mF capacitor with black tape disguising it)* .

Using the inexpensive meter provided tell me what you can about the device.