

# Electrical Engineering

## Quals Questions

2004

Stephen Boyd

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Consider a cascade of 100 one-sample delays:



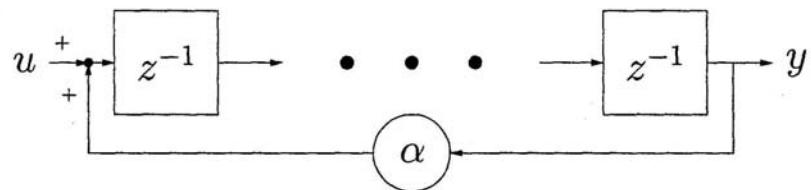
Express this as a linear dynamical system

$$x(t+1) = Ax(t) + Bu(t), \quad y(t) = Cx(t) + Du(t)$$

What are the eigenvalues of  $A$ ?

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Now we add simple feedback, with gain  $\alpha = 10^{-5}$ , to the system:



Express this as a linear dynamical system

$$x(t+1) = Ax(t) + Bu(t), \quad y(t) = Cx(t) + Du(t)$$

What are the eigenvalues of  $A$ ?

How different is the impulse response of the system with feedback ( $\alpha = 10^{-5}$ ) and without feedback ( $\alpha = 0$ )?

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To: Diane Shankle  
From: Tom Cover  
Subject: 2004 quals question

Diane,

Here are my 2 quals questions from 2004.

1. What is the minimum correlation 3 random variables can have with each other? Specifically, what is

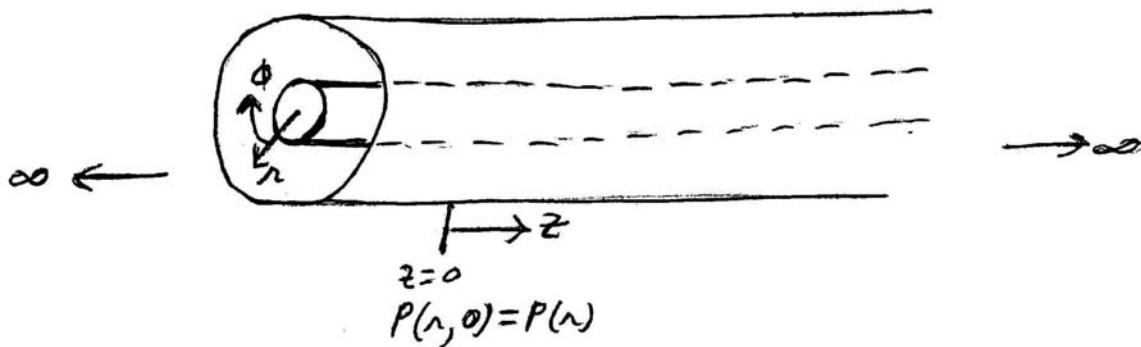
$$\max_{\{p_i\}} \min_{\substack{f(x_1, x_2, x_3) \\ EX_i=0, EX_i^2=1}} E[X_i X_j]$$

2. How would you use a fair coin to generate a random number  $X$  uniformly drawn from the unit interval?

Someone gives you a real number  $t$ . How many flips does it take to determine whether  $X < t$ ?

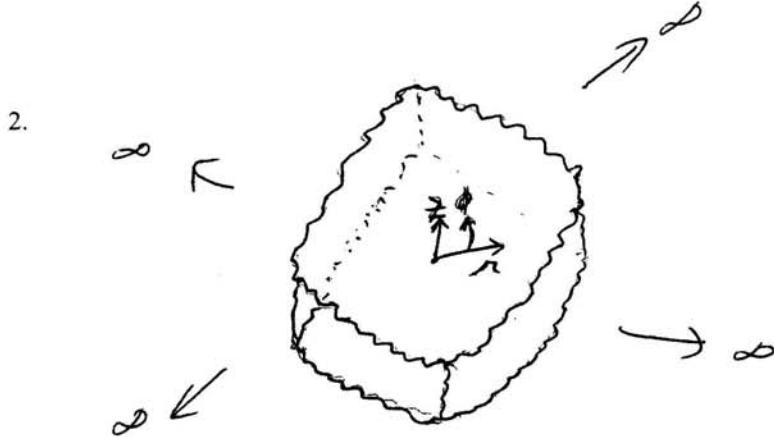
2004.pdf

1. Idealized Infinite Coaxial Line:
- Inner and Outer Conductors are perfect conductors
  - Dielectric is vacuum



- Consider a TEM wave traveling in  $+z$  direction (Sinusoidal, Continuous)  
[ Average Power Density in the TEM wave is ]  
[  $P(r, z)$  = Poynting Vector ]
- [Power Density at  $z = 0$  is  $P(r, 0) = P(r)$ ]

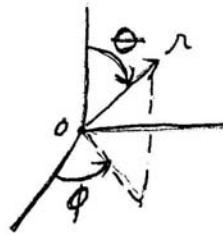
[? what is the  $z$  dependence of the average power density  
 $, P(r, z)$ , of the TEM wave in terms of  $P(r)$  and  $z$   
(Do Not find the  $r$  dependence).]



- 2 infinite parallel plates
  - . perfectly conducting
  - . vacuum between plates
- excite a TEM radially traveling wave at  $r = 0$   
using an omnidirectional source
- At large  $r$  power density in the TEM wave is  $P(r)$  = Poynting Vector
- Power density at  $r = 0$  is  $P(r) = P$

? what is the  $r$  dependence of the power density,  $P(r)$ , of the TEM wave in terms of  $P$  and  $r$ ?

3.



- Assume an idealized Isotropic radiator at  $r = 0$  in free space radiating an idealized TEM wave uniformly in all directions.
- Power radiated is  $P$  at  $r = 0$
- At large  $r$  the power density of the TEM wave is  $P(r)$  = Pointing vector

[? what is the  $r$  dependence of the power density  $P(r)$  of the TEM wave in terms of  $P$ ?]

4. Go Back to 1. Coaxial Line
- Add a lossy dielectric

? what is the  $z$  dependence of the power density  $P(r, z)$  in the coaxial line if the change in  $P(r, z)$  is 10 dB over a distance of 100 meters? (loss due to lossy dielectric)  
(? Is the wave still strictly TEM?? Why or why not?)

5. ? if add the same lossy dielectric same as in 4) between the plates of the 2 infinite parallel plates, what is the  $r$  dependence of  $P(r)$ ?
6. add lossy dielectric same as 4) around  $r = 0$  to  $\infty$  in the isotropic radiator problem 3.  
What is the  $r$  dependence of  $P(r)$ ?

# Qualifying Examination 2004

## Combinational Logic Design

Giovanni De Micheli

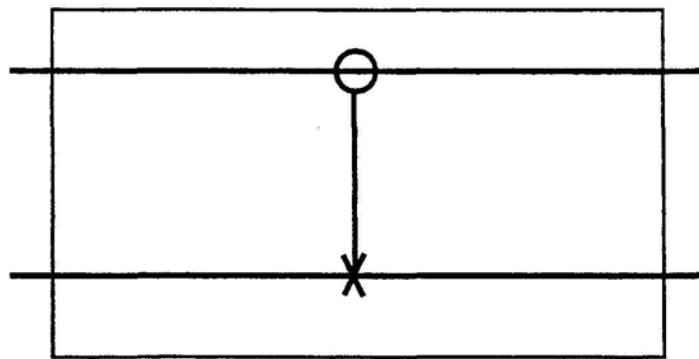
January 2004

Can you implement arbitrary combinational functions using 2-input ANDs and 2-input ORs as basic cells?

Can you implement arbitrary combinational functions using 2-input NANDs as basic cells?

Can you implement arbitrary combinational functions using 2-input MUXes as basic cells?

Show me how to realize a 2-input EXOR function with MUXES.



Consider the controlled controlled inverter (CCN) gate. Can you implement any combinational network using CCNs?

Can you recover the input values always from the inputs of a CCN from its outputs? How?

What are the advantages of reversible computation?

My qual questions: David Bill

PART I:

I would like you to think about an unconventional graph coloring problem. Feel free to ask for definitions, clarifications, hints, and to write on the white board -- or not.

Suppose you were to write a program that is given as input a finite undirected graph. The program should color the graph (i.e., assign numbers to the vertices) according to two rules:

- \* Two adjacent verticies must be colored the same.
- \* You must use as many colors as possible (where a color is "used" only if it is assigned to at least one vertex).

What would be a good algorithm to solve this problem, and what is its running time?

ANSWER:

Pick an uncolored vertex and a color that has not yet been used.

Use depth-first or breadth-first search to find all the vertices in the same connected component of the graph, and assign them the chosen color.

Repeat until all vertices are colored.

The algorithm can be made to run in time  $O(|V| + |E|)$  where  $V$  is the set of vertices and  $E$  is the set of edges.

PART II:

Let me change the problem. I'll give you the vertices at the beginning but no edges. Then, in the order I choose, I will give you a mixed sequence of two things: (1) a statement that there is an edge between two of the vertices you already have, and (2) a query asking the color of a vertex.

All your program has to do is answer the queries correctly. So, if I give you a bunch of edges and then ask for the colors of two vertices that are connected in the current graph, the colors you give me for the vertices better be the same; and, if the vertices are in different components, the colors better be different.

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What's a good algorithm to solve this problem, and how fast does it run?

ANSWER:

There are lots of algorithms. Here is the best one I know.

This can be solved efficiently by the union/find algorithm for disjoint sets.

Let's assume the vertices are numbered 1 ..  $|V|$  initially. The colors of the vertices are stored in an array indexed by the vertex numbers. Initially, the color of each vertex is just the vertex number.

Each time a new edge appears between two vertices with different colors, one of the vertices is re-colored by copying the color of the other vertex to its array entry.

To answer a color query for vertex  $i$ , look up  $\text{color}[i]$ . If it is  $i$ , that's the color. If not, look up  $\text{color}[\text{color}[i]]$ , etc. until you get to a vertex whose color is the same as the vertex number, and that's your color.

As described, the algorithm could be  $O(N |V|)$ , because the color chains could get as long as  $O(|V|)$ .

To make this fast, we want to keep the chains of colors short. This can be done by changing the color of the vertex whose color is shared by the smaller number of other vertices. This bounds the lengths of the color chains by  $O(\log N)$ . The total time to handle  $N$  new edges and queries will be  $O(N \log N)$ .

To make it even faster, use an idea called "path compression". Every time you look up the color of the node by going down the chain of colors, update everything with the color you found (so, the next time it's looked up, you get the answer directly instead of going down the chain again). With this optimization, the total cost of  $N$  queries becomes  $O(N \alpha(N))$  where  $\alpha(N)$  is an incredibly slowly growing function (the inverse of Ackerman's function). This is  $O(N)$  for practical purposes.

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Robert Dutton

ID# _____
<p>What is the <u>smallest value of capacitance</u> you could use for storage of information?</p>

The question is targeted at working towards a REAL circuit.

If folks were stuck, hint #1 is:

C defined as  $\Delta Q / \Delta V$

Then, given that equation, how much charge for how much  $\Delta V$ ?

How will you access that information and amplify it to a useable signal level?

Using the capacitance determined in part 1, now it's time to look at circuit to move charge on/off that capacitor and what are the limits imposed.

Most folks used a MOS "pass transistor" to access the capacitor.

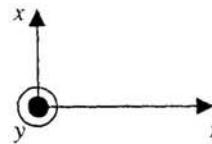
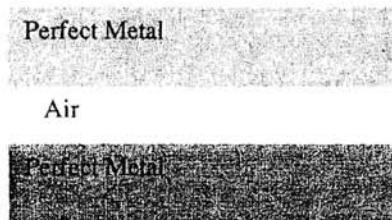
Then the questioning shifted to what are the limits that the MOS device--channel resistance and parasitic capacitances-- bring to the problem:

- Extra  $\Delta Q$  from capacitance (and channel charge for that matter)
- Channel resistance brings with it Thermal Noise and associated BandWidth

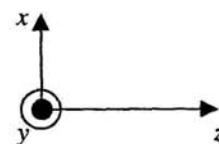
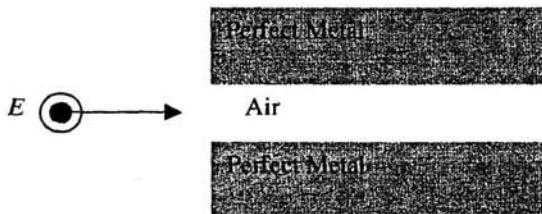
The circuits part of the problem moves into what are the other parasitics (I.e. Input capacitance, threshold voltage, offset if using differential etc.)

Shanhui Fan

Consider a parallel plate waveguide. The air region has a thickness of  $a$ . The metal is assumed to be perfect electrical conductor:



- (a) What are the boundary conditions for the electric fields at the metal-air interfaces?
- (b) Sketch the  $\beta \sim \omega$  diagram for the  $TE_1$  modes in this waveguide, where  $\omega$  is the angular frequency for the waves and  $\beta$  is the propagation constant. (i.e. the field varies along the  $z$ -direction as  $e^{-i\beta z}$ .)  $TE$  modes have the fields  $E_x$ ,  $E_z$  and  $H_y$  all equal to zero.
- (c) Sketch the field distribution for  $E_y$  as a function  $x$ , for the  $TE_1$  mode.
- (d) Sketch the  $\beta \sim \omega$  diagram for the  $TM_0$  modes in this waveguide.  $TM$  modes have the fields  $H_x$ ,  $H_z$  and  $E_y$  all equal to zero.
- (e) Sketch the field distribution for  $E_x$  as a function  $x$ , for the  $TM_0$  mode.



- (f) Consider a truncated waveguide with an entrance at  $z=0$ . A plane wave, with a wavelength in vacuum of  $100a$ , and with the electric field polarized along the  $y$ -direction, is incident upon the waveguide entrance. How does the intensity of the electric field in the waveguide at the waveguide center vary as a function of  $z$ ?

Scoring: (a) 1; (b) 2; (c) 1; (d) 2; (e) 1; (f) 3.

In (f): exponential decay 1; being able to estimate the length of the exponential decay: 2.

**Ph.D. Quals Question**

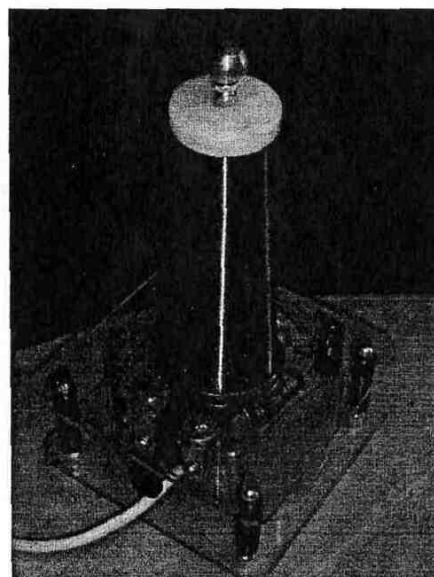
January 2004

A.C. Fraser-Smith

Space, Telecommunications and Radioscience Laboratory

**Compact Tesla Coil**

The figure below shows the compact Tesla Coil that was shown to each student. The white cord connects the coil to a 110 V power outlet and when the black knurled knob next to it is screwed in it moves two electrical contacts closer together and sparking takes place between them. At this time, if a coin held firmly in the fingers is brought toward the round aluminum ball at the top of the coil, sparks up to 1-2 inches long can be drawn out of the sphere. Obviously there is a very high voltage being generated on the sphere; this observation leads to the basic question asked of the students: what is the electrical engineering basis for the generation of this high voltage, given that the source voltage is 110 V?



Two hints were given: First, the students were told to remember Faraday's law of electromagnetic induction and then, second, an AM radio was turned on while the coil was sparking and it was shown the coil was generating radio interference across the entire AM band (i.e., covering many hundreds of kHz).

The answer to this question usually involved two steps: (1) the various components of the electronic part were identified, and then (2) a hypothesis for how the electronics worked was formulated.

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Points for (1) were awarded for identifying the many turns making up the red colored part of Tesla coil as the secondary of a **transformer**, with the two thick black coils at its base making up the primary. Obviously the much greater number of turns in the secondary will lead to higher voltages. During this inspection part of the test the students either noticed or had their attention drawn to the fact that one of the ends of the secondary coil was connected to the aluminum ball on the top and the other end, at the bottom, was connected to the green-colored socket on the right in the picture.; there was no direct electrical connection to any other part of the circuitry. At this time the students either noticed or had their attention drawn to the fact that the two ends of the primary disappeared into the circuitry containing the spark gap that was adjusted by means of the black knurled knob. Some students noticed that the wire comprising the secondary was much thinner than the wire for the primary, suggesting that the primary carried higher current.

Points for (2) were awarded for sensible explanations for how the coil works based on Faraday's law. These explanations most succinctly made use of Faraday's law in the form:

$$EMF = -\partial N/\partial t$$

where *EMF* indicates the induced emf, *N* is the magnetic flux threading the circuit, and *t* is the time. The transformer action discussed above is one part of this explanation, and it involves *N*. Another part, however, involves the  $\partial/\partial t$  term in the above equation. The noise produced by the Tesla coil in the AM radio radio indicates that high frequencies are involved, and high frequencies imply high  $\partial/\partial t$ , which in turn implies large emfs according to the Faraday equation. How are these high frequencies produced? This is where the students were expected to home in on the very noisy spark gap. The sparks were obviously very short lived and thus, eureka (for a Stanford EE student): the Fourier transform of an impulse is a function covering a wide range of frequencies in the frequency domain and the range of frequencies becomes larger as duration of the impulse gets smaller, thus the short-lived sparks give a big  $\partial/\partial t$  which helps produce the high voltages in the Tesla coil.

To put this question into perspective, the earliest demonstrations of radio waves (e.g., by their discoverer Heinrich Hertz) made use of spark gaps to generate the waves, and the earliest commercial transmitters were all mostly based on spark gap technology.

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X-Sender: hector@db.stanford.edu  
Date: Mon, 26 Apr 2004 10:39:40 -0700  
To: Diane Shankle <shankle@ee.Stanford.EDU>  
From: Hector Garcia-Molina <hector@cs.stanford.edu>  
Subject: Re: Quals Questions 2004 Overdue

At 10:17 AM 4/26/2004, you wrote:  
The Spring Quarter is coming to a close in less than seven weeks!  
Send in your Quals Question so I can mark you off my list!

Hector Garcia-Molina  
EE Quals Question 2004

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Consider two vectors A[1]...A[N] and B[1]...B[N]  
stored in two arrays. We want to compute the dot  
product defined as

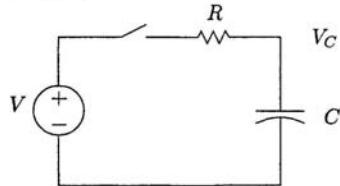
$$d = A[1]*B[1] + A[2]*B[2] + \dots + A[N]*B[N]$$

- (1) Write a statement (pseudo-code) to compute the dot product.
- (2) A sparse vector is one which contains very few non-zero  
values. If N is large, it is not effective to store a sparse  
vector in array, since a lot of space is wasted storing zeroes.  
Suggest an alterate representation for a sparse vector,  
which uses space proportional to the number of non-zero entries  
(not space proportional to N).
- (3) Write pseudo-code to compute the dot product when two  
vectors are represented using the data structure of Part (2).

**2003-2004 Electrical Engineering Qualifying Examination**

John Gill

The resistance  $R$ , capacitance  $C$ , and voltage  $V$  in the circuit shown below are independent random variables.



$$R \sim \text{Uniform}[(1 - \delta)R_0, (1 + \delta)R_0]$$

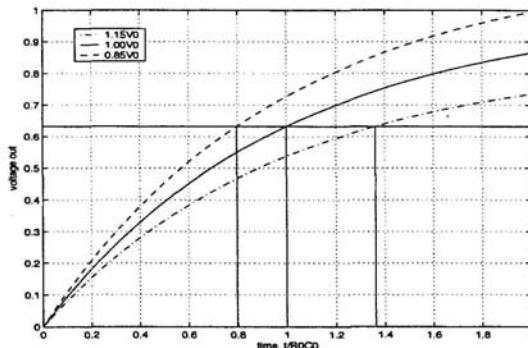
$$C \sim \text{Uniform}[(1 - \delta)C_0, (1 + \delta)C_0]$$

$$V \sim \text{Uniform}[(1 - \delta)V_0, (1 + \delta)V_0]$$

$$0 < \delta < 1$$

$$V_C(t) = V(1 - e^{-t/RC}) \quad (t > 0)$$

The *time constant* of this random RC circuit is defined to be the time  $T$  that is needed for the capacitor to charge to  $V_0(1 - e^{-1})$ . Examples of  $V_C(t)$  and  $T$  are shown in the following figure.



**Question 1** For this random circuit, the time constant is a random variable  $T$ . Find the conditional probability density of  $T$  given fixed values  $R$  and  $C$ .

**Question 2** Find the expected value of the random time constant  $T$ .

**Solution 1** Let  $V_1 = V_0(1 - e^{-1}) = 0.6321 V_0$  denote the nominal threshold voltage, that is, the voltage on the capacitor after one nominal time constant  $R_0 C_0$  when  $V = V_0$ . The time constant random variable  $T$  is a function of  $V$ ,  $R$ , and  $C$ ; it is the solution of the equation

$$V_C(t) = V(1 - e^{-T/RC}) = V_1.$$

Solving the equation is straightforward:

$$\begin{aligned} V(1 - e^{-T/RC}) = V_1 &\Rightarrow 1 - e^{-T/RC} = \frac{V_1}{V} \Rightarrow \\ e^{-T/RC} = 1 - \frac{V_1}{V} &\Rightarrow \frac{T}{RC} = -\ln\left(1 - \frac{V_1}{V}\right) \Rightarrow T = -RC \ln\left(1 - \frac{V_1}{V}\right) \end{aligned}$$

Since  $T$  is a monotonically decreasing function of  $V$ , the range of  $T$  is

$$-RC \ln \left( 1 - \frac{V_1}{V_0(1+\delta)} \right) \leq T \leq -RC \ln \left( 1 - \frac{V_1}{V_0(1-\delta)} \right).$$

If the random voltage  $V$  is too small—namely,  $V < V_1$ —then the capacitor voltage  $V_C(t)$  never reaches  $V_1$ . In this case  $T = +\infty$ . (The upper bound in the above equation is meaningless.) We assume from now on that  $\delta < e^{-1}$ .

Given the formula for  $T$ , there are two standard ways to find the pdf of  $T$ : find the cdf  $F_T(t)$  and differentiate, or express the pdf  $f_T(t)$  in terms of the pdf  $f_V(v)$  of  $V$ .

We can find the cdf  $F_T(t)$  by using its definition.

$$\begin{aligned} P\{T \leq t\} &= P\left\{-RC \ln \left( 1 - \frac{V_1}{V} \right) \leq t\right\} = P\left\{\ln \left( 1 - \frac{V_1}{V} \right) \geq -\frac{t}{RC}\right\} \\ &= P\left\{1 - \frac{V_1}{V} \geq e^{-t/RC}\right\} = P\left\{\frac{V_1}{V} \leq 1 - e^{-t/RC}\right\} = P\left\{V \geq \frac{V_1}{1 - e^{-t/RC}}\right\} \\ &= 1 - P\left\{V \leq \frac{V_1}{1 - e^{-t/RC}}\right\} = F_V\left(\frac{V_1}{1 - e^{-t/RC}}\right) = \frac{1}{2\delta V_0} \left( \frac{V_1}{1 - e^{-t/RC}} - V_0(1-\delta) \right) \end{aligned}$$

Finding the pdf is now an exercise in using the Chain Rule to differentiate the cdf.

$$\begin{aligned} f_T(t) &= \frac{d}{dt} \frac{1}{2\delta V_0} \left( \frac{V_1}{1 - e^{-t/RC}} - V_0(1-\delta) \right) = \frac{V_1}{2\delta V_0} \frac{d}{dt} \frac{1}{1 - e^{-t/RC}} \\ &= \frac{V_1}{2\delta V_0} \left( -\frac{1}{(1 - e^{-t/RC})^2} \right) \left( -\frac{e^{-t/RC}}{RC} \right) = \frac{V_1 e^{-t/RC}}{2RC\delta V_0(1 - e^{-t/RC})^2} \end{aligned}$$

The pdf of  $T = g(V)$  can be obtained directly from the pdf of  $V$  using a formula familiar to EE 278 students. Let  $v_1, v_2, \dots$  be the solutions of the equation  $t = g(v)$  and let  $g'(v_i)$  be the derivative of  $g$  evaluated at  $v_i$ . Then

$$f_T(t) = \sum_i \frac{f_V(v_i)}{|g'(v_i)|},$$

Since  $g(v) = -RC \ln(1 - V_1/V)$  is monotonically decreasing, there is at most one solution to the equation. By definition of  $T$ , the value of  $v$  corresponding to  $t$  satisfies  $v(1 - e^{-t/RC}) = V_1$ , hence  $v = V_1/(1 - e^{-t/RC})$ . Therefore

$$\begin{aligned} \frac{d}{dv} \left( \ln \left( 1 - \frac{V_1}{v} \right) \right) &= \frac{d}{dv} \left( \ln \left( \frac{V_1 - v}{v} \right) \right) = \frac{d}{dv} \left( \ln(v - V_1) - \ln v \right) = \frac{1}{v - V_1} - \frac{1}{v} \\ &= \frac{1}{v - v(1 - e^{-t/RC})} - \frac{1 - e^{-t/RC}}{V_1} = \frac{1}{ve^{-t/RC}} - \frac{1 - e^{-t/RC}}{V_1} \\ &= \frac{1 - e^{-t/RC}}{V_1 e^{-t/RC}} - \frac{1 - e^{-t/RC}}{V_1} = \frac{1 - e^{-t/RC} - e^{-t/RC}(1 - e^{-t/RC})}{V_1 e^{-t/RC}} = \frac{(1 - e^{-t/RC})^2}{V_1 e^{-t/RC}} \end{aligned}$$

Putting it all together, we find the pdf of  $T$ :

$$f_T(t) = \frac{f_V(v)}{|g'(v)|} = \frac{1}{2\delta V_0} \left( RC \frac{(1 - e^{-t/RC})^2}{V_1 e^{-t/RC}} \right)^{-1} = \frac{V_1 e^{-t/RC}}{2RC\delta V_0(1 - e^{-t/RC})^2}$$

We note with satisfaction and relief that both methods yield the same answer.

**Solution 2** In part 1 we found the formula for  $T$  as a function of  $R$ ,  $C$ , and  $V$ . Because  $R$ ,  $C$ , and  $V$  are independent,

$$E[T] = E\left[-RC \ln\left(1 - \frac{V_1}{V}\right)\right] = E[R]E[C]E\left[-\ln\left(1 - \frac{V_1}{V}\right)\right] = R_0 C_0 E\left[-\ln\left(1 - \frac{V_1}{V}\right)\right].$$

The uniform pdf for  $V$  leads to the following integral.

$$E[T] = R_0 C_0 E\left[-\ln\left(1 - \frac{V_1}{V}\right)\right] = \frac{R_0 C_0}{2\delta V_0} \int_{V_0(1-\delta)}^{V_0(1+\delta)} -\ln\left(1 - \frac{V_1}{v}\right) dv$$

The integrand is not defined when  $V < V_1$ , so  $E[T]$  is undefined (or infinite) when  $\delta \geq e^{-1}$ .

For completeness, we perform the integration. This was *not* a requirement for the problem. From integration by parts, tables of integrals, or long term memory we obtain

$$\int \ln x = x \ln x - x.$$

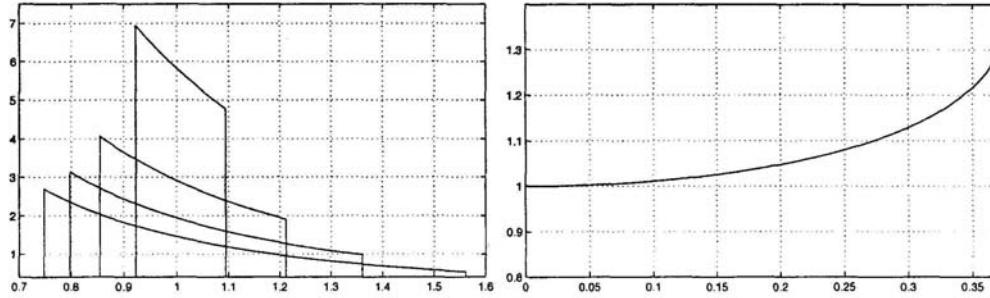
Next we find the *indefinite* integral needed for  $E[T]$  (additive constants can be ignored).

$$\begin{aligned} \int \ln\left(1 - \frac{V_1}{v}\right) dv &= \int \ln \frac{v - V_1}{v} dv = \int (\ln(v - V_1) - \ln v) dv \\ &= (v - V_1) \ln(v - V_1) - (v - V_1) - v \ln v + v = (v - V_1) \ln(v - V_1) - v \ln v. \end{aligned}$$

The final answer has a closed form but no obvious simplifications.

$$\begin{aligned} E[T] &= \frac{R_0 C_0}{2\delta V_0} \int_{V_0(1-\delta)}^{V_0(1+\delta)} -\ln\left(1 - \frac{V_1}{v}\right) dv = \frac{R_0 C_0}{2\delta V_0} (v \ln v - (v - V_1) \ln(v - V_1)) \Big|_{V_0(1-\delta)}^{V_0(1+\delta)} \\ &= \frac{R_0 C_0}{2\delta V_0} (V_0(1 + \delta) \ln(V_0(1 + \delta)) - V_0(1 - \delta) \ln(V_0(1 - \delta)) - (V_0(1 + \delta) - V_1) \ln(V_0(1 + \delta) - V_1) + (V_0(1 - \delta) - V_1) \ln(V_0(1 - \delta) - V_1)) \end{aligned}$$

The left graph shows the conditional pdfs of  $T$  given  $R = 1$ ,  $C = 1$  for  $\delta = 0.20, 0.15, 0.10, 0.05$  (left to right). The right graph plots the expected value of  $T$  as a function of  $\delta$  for  $0 < \delta < e^{-1}$ .



The pdf and mean of  $T$  are not defined for  $\delta \geq e^{-1}$ . It is somewhat surprising that as  $\delta \rightarrow e^{-1}$  the mean of  $T$  converges to a finite number  $\frac{1}{2}(e+1)\ln(e+1) - \frac{1}{2}(e-1)\ln(e-1) - \ln 2 = 1.2833$

Goldsmit

Lam

$$\text{Find } \gamma(t) \text{ for } \gamma(f) = \sum_{n=-\infty}^{\infty} \text{sinc}(f-n)$$

Consider a filter that is

- 1) linear
- 2) time-invariant
- 3) has impulse response  $h(t)$ , i.e.  
for input  $\delta(t)$ , the output is  $h(t)$

Show using these 3 properties that for  
any input  $x(t)$ , the filter output is  $x(t) * h(t)$

Let  $N(t)$  be a WSS AWGN random process with power spectral density  $\frac{N_0}{2}$  and zero mean

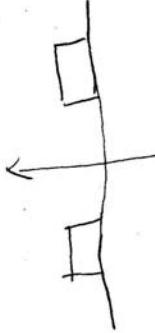
a) Find the autocorrelation function for

$\gamma(t) = N(t) \cos(2\pi f_c t + \theta)$  where  
 $\theta$  is uniform on  $[0, 2\pi]$  & indep. of  $N(t)$

$$\begin{aligned} \text{Is } \gamma(t) \text{ WSS?} \\ \gamma(t) &= E[N(t+\tau)N(t)] \\ &= R_N(\tau) = E[\cos(\omega_0 t + \theta) \cos(\omega_0(t+\tau) + \theta)] \\ &= R_N(\tau) \left( \frac{1}{2} + \frac{1}{2} \cos(2\pi f_c \tau) \right) \end{aligned}$$

b) Find the PSD of  
 $Z(t) = [N(t) * h(t)] \cos(2\pi f_c t + \theta)$  where

$$H(f) = \begin{cases} 5 & \text{if } |f| < B \\ 0 & \text{else} \end{cases}$$



**2004 PhD Quals Question**  
**J. S. Harris**

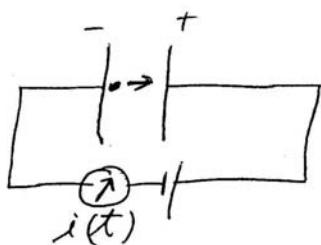
1. What is the Ideal diode or Shockley I-V equation for a p/n junction?
    - (a) What are the underlying assumptions to derive this equation?
  2. Draw the I-V characteristic and identify the key features?
  3. I now go into the lab and fabricate a p/n junction and measure it on my parameter analyzer. The I-V characteristic looks similar to the ideal diode equation, but there are some differences. Illustrate on your drawing what the commonly observed differences would be compared to the ideal diode and describe what these are due to?
  4. Most of the points you have made are understandable, but why do G-R centers dominate the I-V characteristics when they are 6-8 of orders of magnitude lower in concentration than the intrinsic semiconductor density?
    - (a) Are such centers equally effective in all semiconductors? Why or why not?
    - (b) When you draw in the deep level in the band picture, what does it represent? and how might this explain the effectiveness of traps in indirect bandgap semiconductors?
  5. What physical process causes the reverse breakdown characteristic? Can you illustrate the process on an energy band diagram.
    - (a) What is the minimum energy that the hot electron has to gain to cause generation of an electron-hole pair?
  6. Most of the time, the avalanche process is a parasitic that limits the voltage/power in diodes and transistors. Can you name any devices where we use this avalanche process advantageously or with design intent?
- 
- 
- 
- 
-

2004 Quals  
Steve Harris

The exam consisted of two questions. Typically 3-4 min were spent on the first question and 6 or 7 on the second.

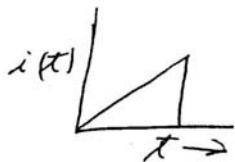
Question 1

at  $t=0$  a single electron begins traveling in a dc field.



Find  $i(t)$  as a function of time.

answer:



Some students thought the answer was a delta function. The hint was: "consider either induced charge or displacement current."

Question 2

A linear harmonic oscillator consisting of a positive and a negative charge lies in the plane of the paper.

A plane <sup>monochromatic</sup> wave is polarized along the x-axis and has a frequency which is equal to the natural frequency of the oscillator. The wave propagates into the paper.

$$+ \overrightarrow{ee} \rightarrow \overrightarrow{E}$$

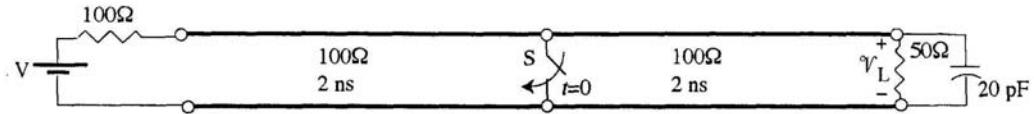
Find the force on the oscillator in the direction in which the plane wave is propagating. The oscillator has the equation:

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = -\frac{|e| E(t)}{m}$$

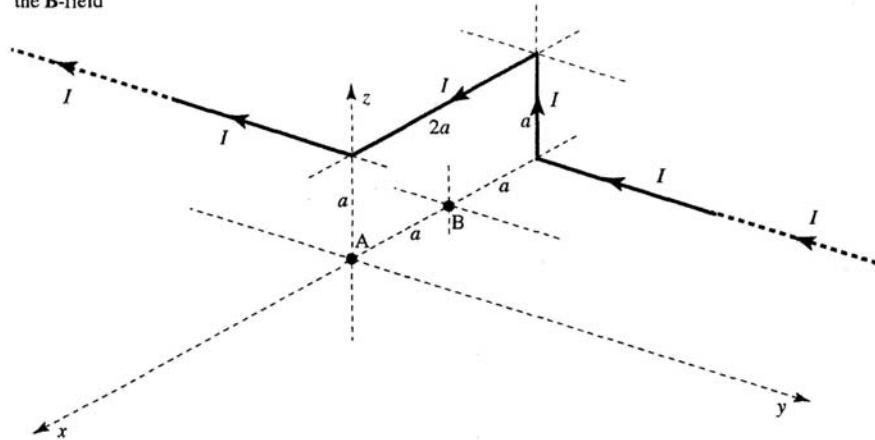
Either of two approaches were correct. 1) The wave excites the oscillator and losses energy and thereby imparts momentum to the oscillator. 2) ~~preferred~~  $|F| = |e|(\vec{V} \times \vec{B})$ , where  $\vec{V}$  is the velocity of the sinusoidally driven electron and  $\vec{B}$  is the magnetic field of the plane wave.

Find:  $|F| = \frac{|e|^2 |E|^2}{2m\gamma c}$ , where  $c$  = velocity of light in vacuum.  
— Exact, analytical answer not necessary.

Switch S has been open for a long time; closed at  $t=0$   
Determine and sketch  $\mathcal{V}_L(t)$

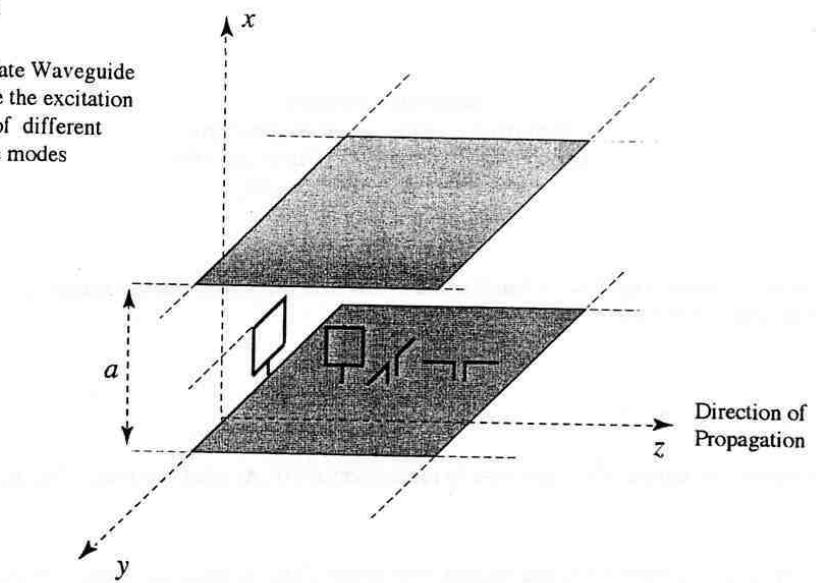


Determine the direction and magnitude of the **B**-field



Inan/2004/Q3

Parallel Plate Waveguide  
Investigate the excitation  
detection of different  
waveguide modes



**Stanford University**  
**Department of Electrical Engineering**  
**Qualifying Examination Winter 2003-04**  
**Professor Joseph M. Kahn**

**Q:** Let  $x(t)$  be a real signal that is bandlimited to less than  $B$  Hz. You can reconstruct  $x(t)$  from the samples taken at rate  $1/T$ :

$$x(nT), n = 0, \pm 1, \pm 2, \dots,$$

provided that  $1/T \geq 2B$ .

(a) Suppose you sample  $x^2(t)$  and wish to reconstruct  $x^2(t)$ . At what rate must  $x^2(t)$  be sampled?

(b) Suppose you sample  $x^3(t)$  and wish to reconstruct  $x^3(t)$ . At what rate must  $x^3(t)$  be sampled?

**A:** (a) Suppose  $x(t) \xrightarrow{FT} X(j\omega)$ . Then by the multiplication property of Fourier transforms:

$$x^2(t) = x(t) \cdot x(t) \xrightarrow{FT} \frac{1}{2\pi} X(j\omega) \otimes X(j\omega).$$

If  $x(t)$  is bandlimited to less than  $B$  Hz, then  $x^2(t)$  is bandlimited to less than  $2B$  Hz. Hence, the sampling rate must be doubled:  $1/T \geq 4B$ .

(b) Extending the arguments given above,

$$x^3(t) = x(t) \cdot x(t) \cdot x(t) \xrightarrow{FT} \frac{1}{2\pi} X(j\omega) \otimes X(j\omega) \otimes X(j\omega),$$

so that  $x^3(t)$  is bandlimited to less than  $3B$  Hz. Hence, one might conclude that the sampling rate must be increased by a factor of three:  $1/T \geq 6B$ . If we know, however, that we are sampling the cube of a signal bandlimited to less than  $B$  Hz, then we can sample  $x^3(t)$  at the same rate as we would sample  $x(t)$ , i.e.,  $1/T \geq 2B$ . We obtain the samples  $x^3(nT)$ . Taking the square root of these samples, we obtain the samples of  $x(t)$ , which are  $x(nT)$ . Using these samples, we reconstruct  $x(t)$ . Finally, we cube  $x(t)$  to obtain  $x^3(t)$ . This procedure cannot be used in part (a), because squaring destroys information on the sign of  $x(t)$ .

**Stanford University**  
**Department of Electrical Engineering**  
**Qualifying Examination Winter 2003-04**  
**Professor Joseph M. Kahn**

**Q:** Let a continuous-time linear time-invariant system be described by  $h(t) \xrightarrow{FT} H(j\omega)$ . Suppose that  $H(j\omega)$  is periodic in  $\omega$ , i.e.,  $H(j(\omega + \omega_0)) = H(j\omega)$ ,  $\forall \omega$ , for some  $\omega_0$ . Describe  $h(t)$  as precisely as you can, and show how to express  $h(t)$  in terms of  $H(j\omega)$ .

**A:** Define  $G(j\omega)$ , which is one period of  $H(j\omega)$ :

$$G(j\omega) = \begin{cases} H(j\omega) & \omega_1 \leq \omega < \omega_1 + \omega_0 \\ 0 & \text{otherwise} \end{cases}.$$

Then we can express  $H(j\omega)$  as the periodic extension of  $G(j\omega)$ :

$$H(j\omega) = G(j\omega) \otimes \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0).$$

Using the multiplication property of Fourier transforms, and defining  $g(t)$  as the inverse Fourier transform of  $G(j\omega)$ , we have:

$$h(t) = \frac{2\pi}{\omega_0} \cdot g(t) \cdot \sum_{n=-\infty}^{\infty} \delta\left(t - n\frac{2\pi}{\omega_0}\right).$$

In summary,  $h(t)$  is an impulse-sampled version of the inverse Fourier transform of one period of  $H(j\omega)$ .

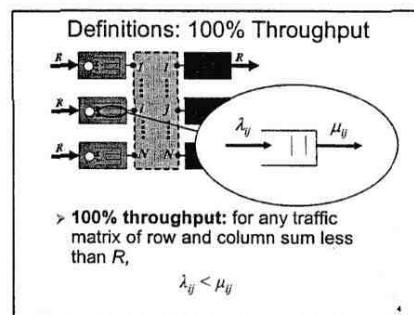
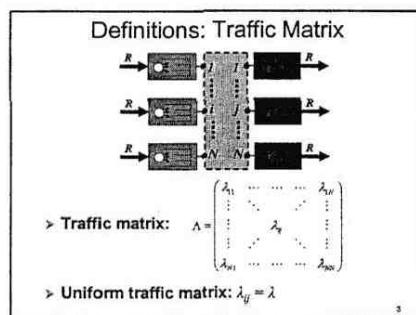
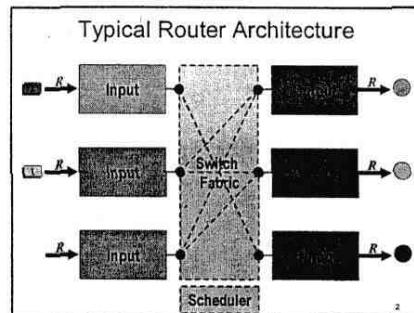
If you were allowed to build any piece of hardware, what would you do to get round your software/algorithm problem?

**Using Load-Balancing To Build High-Performance Routers**

Isaac Keslassy

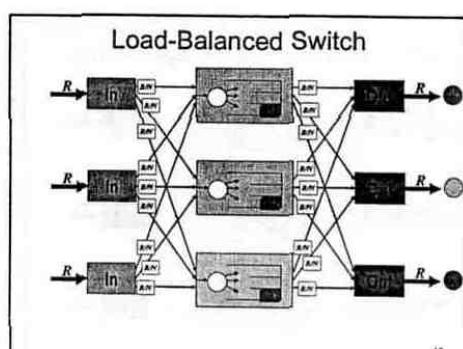
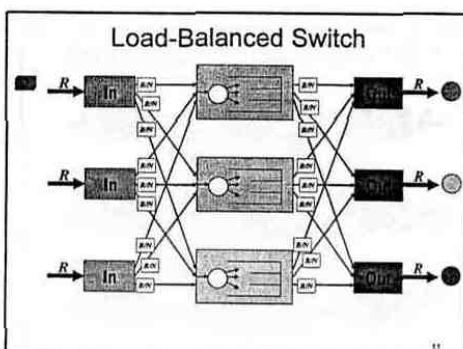
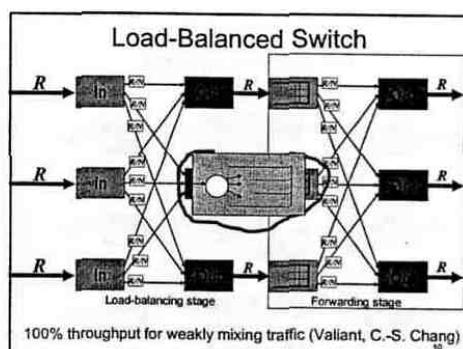
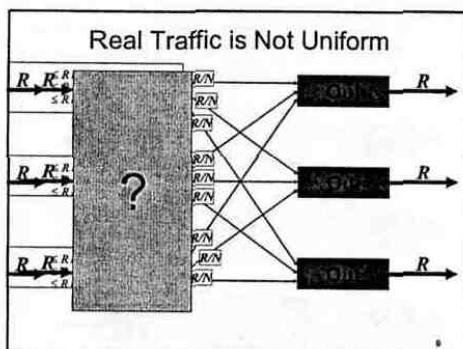
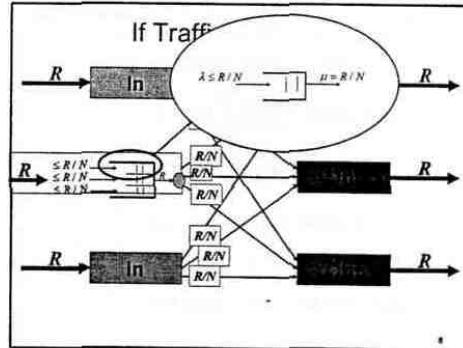
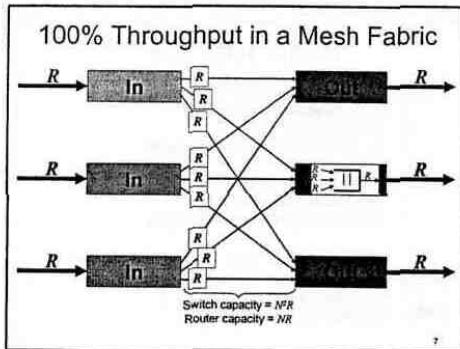


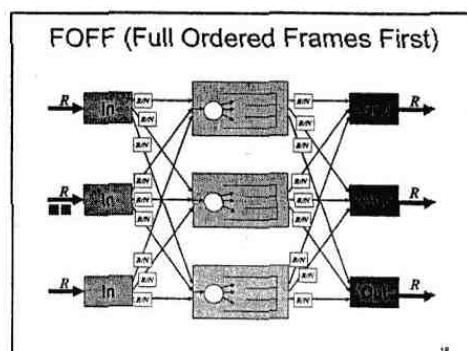
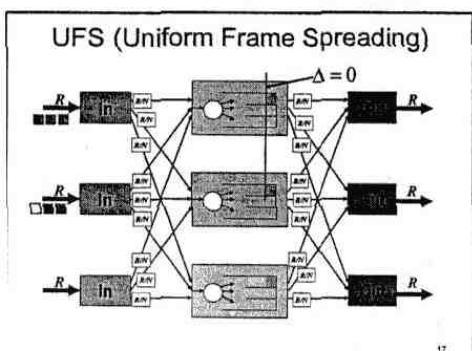
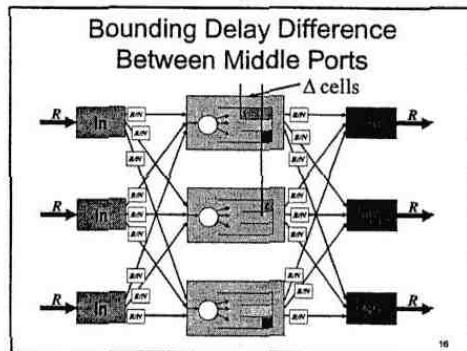
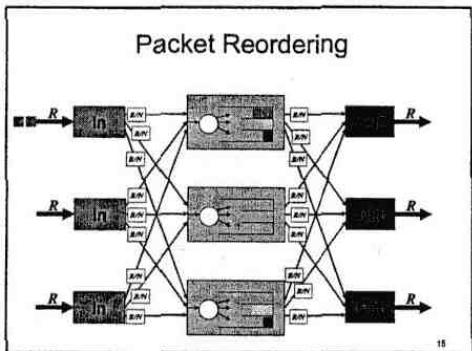
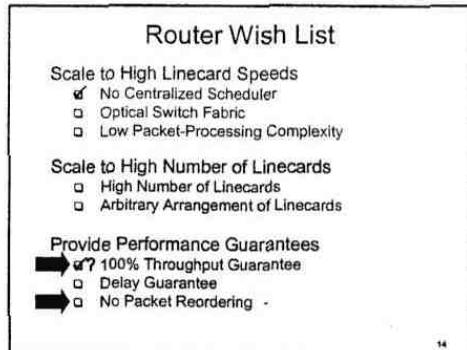
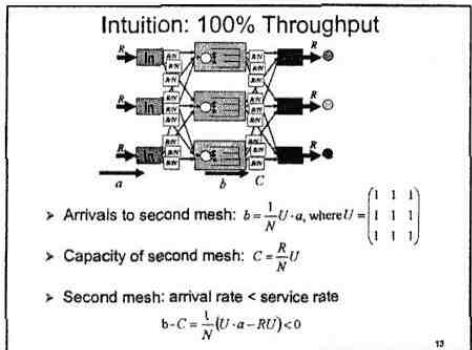
P.h.D. Oral Examination  
Department of Electrical Engineering  
Stanford University



- Router Wish List**
- Scale to High Linecard Speeds
    - No Centralized Scheduler
    - Optical Switch Fabric
    - Low Packet-Processing Complexity
  - Scale to High Number of Linecards
    - High Number of Linecards
    - Arbitrary Arrangement of Linecards
  - Provide Performance Guarantees
    - 100% Throughput Guarantee
    - Delay Guarantee
    - No Packet Reordering

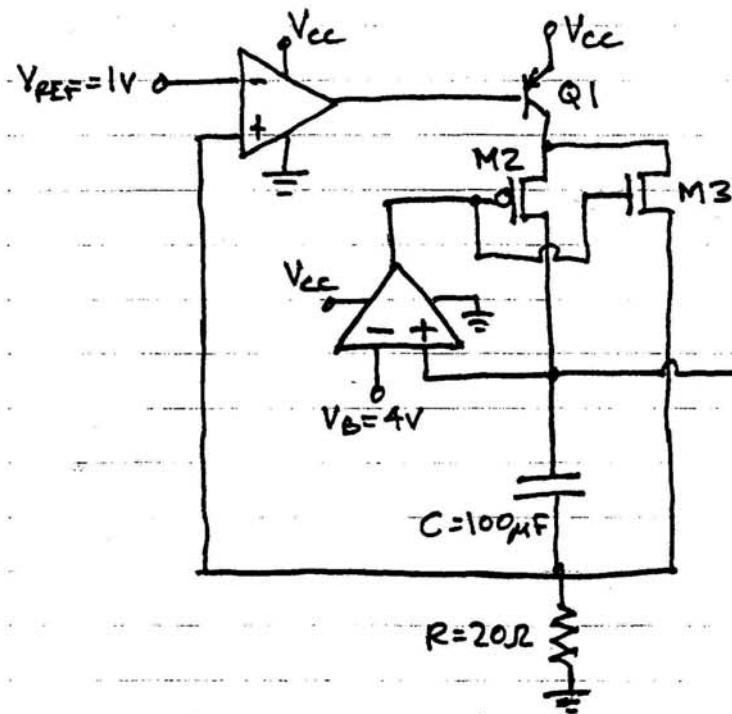
- Stanford 100Tb/s Router**
- > "Optics in Routers" project
  - > <http://yuba.stanford.edu/or/>
  - > Some challenging numbers:
    - > 100Tb/s
    - > 160Gb/s linecards
    - > 640 linecards





BRUCE WOOLEY

EE QUALS 03-04



$$Q1: B \gg 1$$
$$V_{BE(on)} = 0.7V$$

$$M2: V_T = -0.5V$$

$$M3: V_T = +0.5V$$

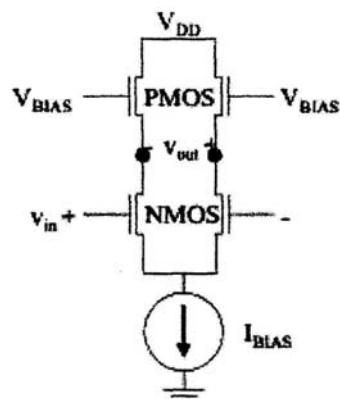
OP AMPS:

$$0 \leq V_{out} \leq V_{CC}$$

$$|I_{out(max)}| = 5mA$$

Plot what happens to  $V_O$  with time when  $V_{CC}$  steps from 0 to 5V.

2004 Qualifying Exam  
Simon Wong



What is the small signal voltage gain of the amplifier ?

What is the small signal voltage gain of the amplifier (versus frequency) if a resistor  $R$  and a capacitor  $C$  are connected (in parallel) across the output terminals ?

What is the small signal voltage gain of the amplifier (versus frequency) if a resistor  $R$  and an inductor  $L$  are connected (in parallel) across the output terminals ?

What is the small signal voltage gain of the amplifier (versus frequency) if a resistor  $R$ , a capacitor  $C$  and an inductor  $L$  are connected (in parallel) across the output terminals ?

3) Suppose you observe

$$Y = X + N = \sum_{i=1}^n b_i \Phi_i + N,$$

where the  $b_i$ 's are iid with

$$b_i = \begin{cases} 1 & \text{w.p. } 1/2 \\ -1 & \text{w.p. } 1/2, \end{cases}$$

independent of  $N \sim \mathcal{N}(0, I_{m \times m})$ .

1. What is the optimal estimate of  $b_i$  based on  $Y$  in the sense of minimizing the bit error probability  $\Pr(\hat{b}_i(Y) \neq b_i)$ ?
2. Are these estimates also optimal in the sense of minimizing the block error probability  $\Pr(\hat{b}_1(Y) \neq b_1 \text{ or } \hat{b}_2(Y) \neq b_2 \dots \text{ or } \hat{b}_n(Y) \neq b_n)$ ? Explain.
3. What is the optimal estimate of  $X$  based on  $Y$  in the sense of minimizing  $E \| \hat{X}(Y) - X \|_2^2$ ?

In what follows assume  $\{\Phi_i\}_{i=1}^n$ ,  $\Phi_i \in \mathbb{R}^m$ , to be a given deterministic *orthonormal* set and  $X = \sum_{i=1}^n b_i \Phi_i$ , where  $b_i \in \{-1, 1\}$ .

- 2) Suppose you observe  $X$ . Can you unambiguously tell me what the  $b_i$ 's are ? How ?

Problem

Jennifer widow

=====

Towers of Hanoi

There are three posts,  $P_1$ ,  $P_2$ , and  $P_3$ . Post  $P_1$  starts with a tower of  $N$  disks on it,  $D_1, D_2, \dots, D_N$ , of strictly decreasing size from bottom to top. The goal is to move all  $N$  disks from post  $P_1$  to post  $P_2$ , possibly via post  $P_3$ , subject to:

- (1) At most one disk may be moved at a time.
- (2) No disk may ever be placed on top of a smaller disk.

Specifically, write a general procedure:

Move({disk1, disk2, ..., diskM}, post1, post2, post3)

that emits a sequence of instructions for moving disk1, disk2, ..., diskM from post1 to post2, possibly via post3. To solve the original problem we call:

Move({D\_1, D\_2, ..., D\_N}, P\_1, P\_2, P\_3)

Each emitted instruction is of the form "move  $D_i$  from  $P_j$  to  $P_k$ ".

Hint on request:

Use recursion-- note that Move() can be called with any set of disks and any parameter ordering of the three posts.

Additional/alternate problems:

(A1) Write a function that takes an argument  $N$  and returns the number of moves required to solve the problem with  $N$  disks.

(A2) What is the computational complexity of the problem (in #disks)?

Solution

Move({disk1, disk2, ..., diskM}, post1, post2, post3):  
If  $M=1$  then emit "move [disk1] from [post1] to [post2]"  
Else  
    Move({disk2, disk3, ..., diskM}, post1, post3, post2)  
    Move({disk1}, post1, post2, post3)  
    Move({disk2, disk3, ..., diskM}, post3, post2, post1)

(A1)  $f(1) = 1; f(N > 1) = 2*f(N-1) + 1$

(A2) Exponential in  $N$ :  $O(2^N)$  Specifically,  $f(N) = 2^N - 1$

---

Sachy Weissman

*Quals Question*

- 1) Suppose I give you a collection of vectors, or signals,  $\{\Phi_i\}_{i=1}^n$ , where  $\Phi_i \in \mathbb{R}^m$ . What do we mean by " $\{\Phi_i\}_{i=1}^n$  is an orthonormal set"? What does it imply about the relationship between  $n$  and  $m$ ?

# **EE Qualifying Exam Questions**

**January 12-16, 2004**

**Yoshi Yamamoto**

**In optical communication systems, the laser transmitter output is on-off modulated to send a logical zero signal by “off pulse” and logical one signal by “on pulse”. In a receiver side, the attenuated signal is detected by a photon counter and the received signal level is compared to the threshold value to decide which logical state was sent.**

- 1. If a noise-free photon counter is available and we require a bit error rate of  $P_e=10^{-9}$ , how many photons on average must be received?**

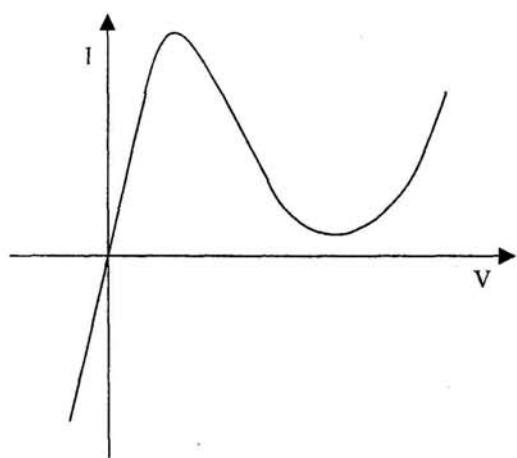
**[Hint: A received signal has the Poisson distribution of photons,  
 $P(n) = e^{-\bar{n}}(\bar{n})^n/n!$ ]**

- 2. In practice, a received signal is enhanced by a fiber amplifier before the photon counting detection. Why is a fiber amplifier useful?**

2. (1 point)

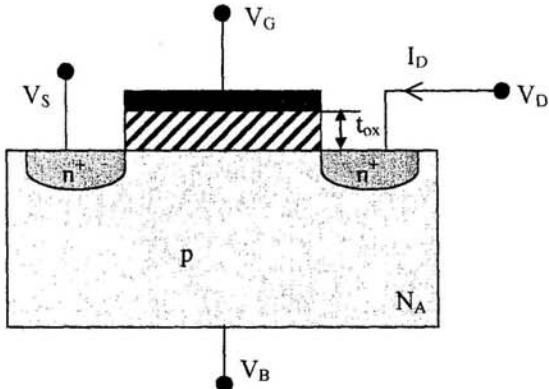
Which semiconductor device has the following I-V characteristics?

Explain different regions of operation.



**1. MOSFET physics (9 points).** Questions (a) and (c) are worth two points each; all other questions are worth 1 point each.

Consider an n-type MOSFET, as shown in the following figure:



The four terminal voltages  $V_S$ ,  $V_D$ ,  $V_G$ , and  $V_B$  correspond to source, drain, gate, and bulk (substrate) bias voltages, respectively, and  $I_D$  is the drain current.

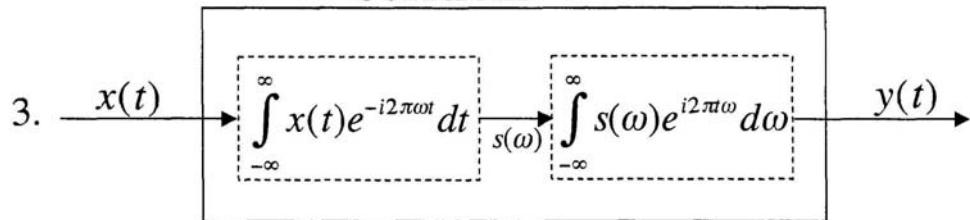
For questions (a) to (e), assume that  $V_B = V_S = 0$ .

- Discuss the dependence (if any) of the MOSFET threshold voltage  $V_T$  on the following parameters: substrate doping  $N_A$ , gate oxide thickness  $t_{ox}$ , channel length  $L$ , and channel width  $w$ .
- Sketch  $I_D$  versus  $V_D$  for a couple of different  $V_G$ . Indicate different regions of MOSFET operation on the plot.
- Is there a difference in the carrier transport mechanism through the MOSFET operating above- and subthreshold? Discuss the dependence of  $I_D$  on  $V_G$  and on  $V_D$  for both regions of operation (above and sub-threshold).
- Assume that the MOSFET is biased in such a way that it operates above threshold and in the saturation. You slowly start increasing  $V_G$ , but the remaining three terminal voltages remain constant. Does the drain current change and why?
- Repeat the question (d), but for the MOSFET operating sub-threshold.

For questions (f) to (g), do not assume that  $V_B = V_S = 0$ .

- The MOSFET is in the sub-threshold regime. You start slowly increasing  $V_S$ , but the remaining three terminal voltages remain constant. How does this affect the drain current  $I_D$  and why? Assume that the transistor remains in the same operating regime.
- Repeat the question (f), but for a decrease in the bulk voltage  $V_B$ .

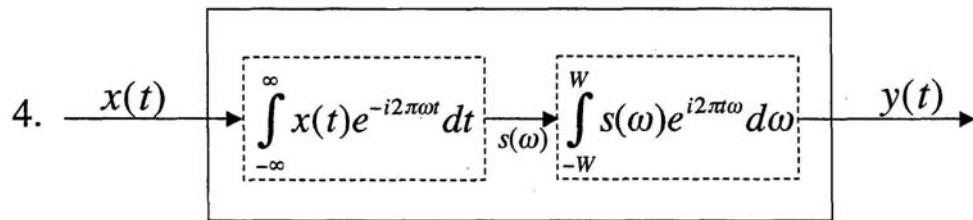
## Solutions



Is this system linear? yes

Is this system time invariant? yes

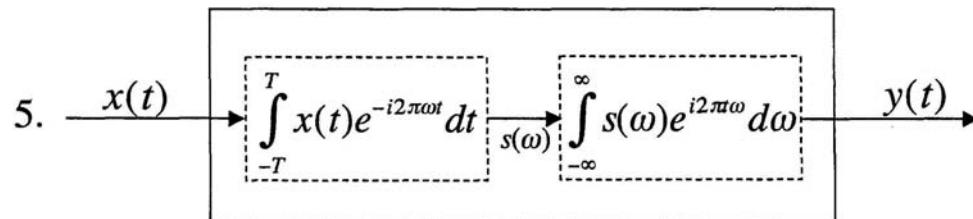
What is the impulse response?  $\delta(t) \rightarrow \delta(t)$



Is this system linear? yes

Is this system time invariant? yes

What is the impulse response?  $\delta(t) \rightarrow 2W \operatorname{sinc}(2Wt)$



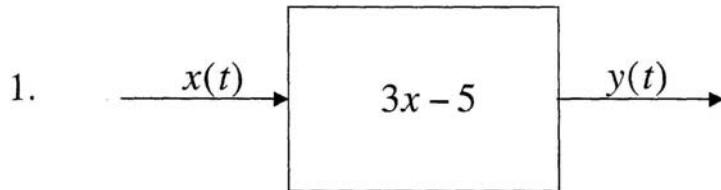
Is this system linear? yes

Is this system time invariant? no

What is the impulse response?

$$\delta(t - t_0) \rightarrow \begin{cases} \delta(t - t_0) & -T < t_0 < T \\ 0 & \text{else} \end{cases}$$

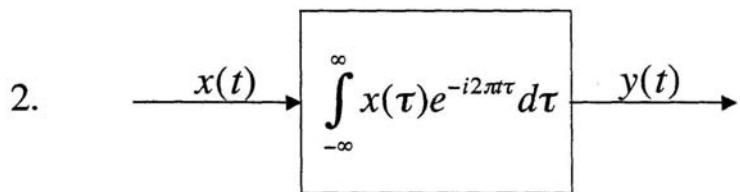
## Solutions



Is this system linear? no

Is this system time invariant? yes

What is the impulse response?  $\delta(t) \rightarrow 3\delta(t) - 5$

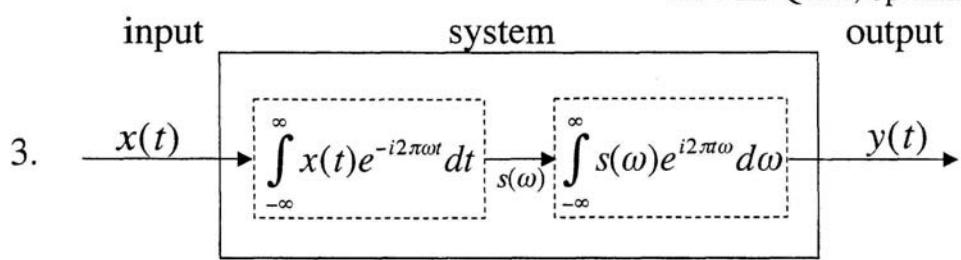


Is this system linear? yes

Is this system time invariant? no

What is the impulse response?  $\delta(t - t_0) \rightarrow e^{-i2\pi t t_0}$

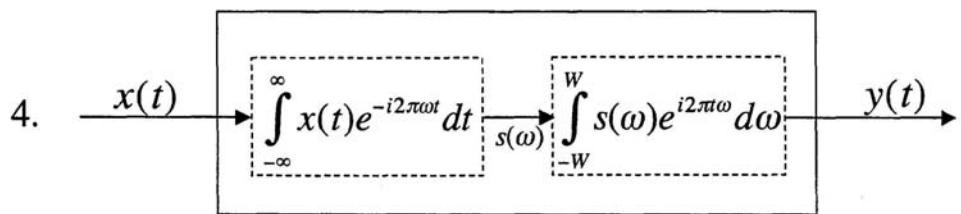
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Is this system linear?

Is this system time invariant?

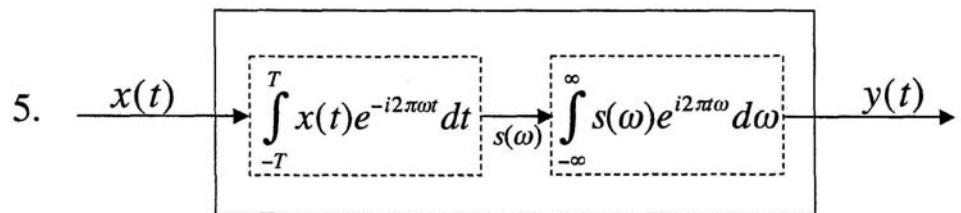
## What is the impulse response?



Is this system linear?

Is this system time invariant?

What is the impulse response?

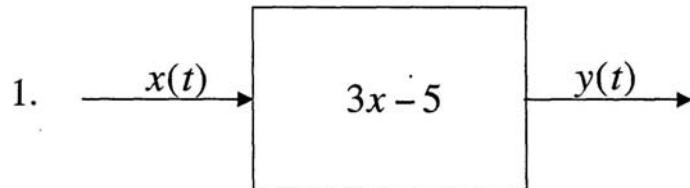
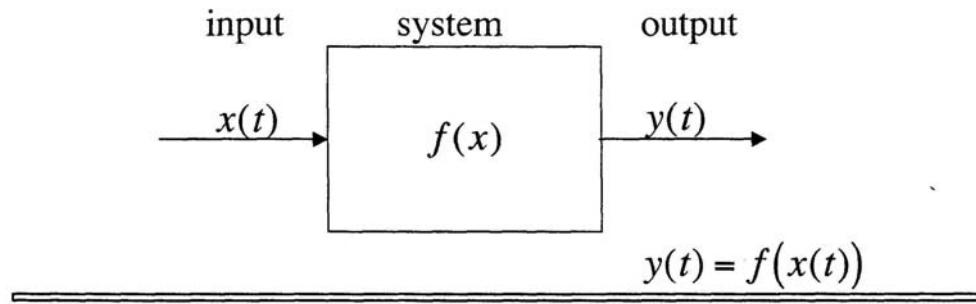


Is this system linear?

Is this system time invariant?

What is the impulse response?

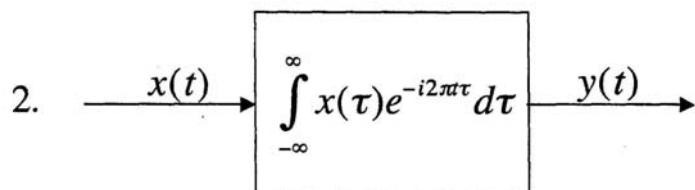
These problems all concern systems of the following type:



Is this system linear?

Is this system time invariant?

What is the impulse response?



Is this system linear?

Is this system time invariant?

What is the impulse response?

To: Julius Smith <jos@ccrma.stanford.edu>  
From: Diane Shankle <shankle@ee.stanford.edu>  
Subject: Re: Quals Questions 2004

Cc:

Bcc:

Attachments:

---

Here was my exam this year:

1. How may we show that a system is linear?
2. Show that superposition implies scaling for rational scalars.
3. Under what conditions does superposition imply scaling for real scalars?

Julius

---

Julius O. Smith III <jos@ccrma.stanford.edu>  
Assoc. Prof. of Music and (by courtesy) Electrical Engineering  
CCRMA, Stanford University  
<http://www-ccrma.stanford.edu/~jos/>

To: Krishna Shenoy <shenoy@stanford.edu>  
From: Diane Shankle <shankle@ee.stanford.edu>  
Subject: Re: 2004 Quals questions.

Cc:

Bcc:

Attachments:

---

A series of questions, with accompanying circuit diagrams, were presented and short verbal and/or written equations were typically offered as solutions. Some completed all questions, some did not.

- 1) A BJT with one resistor between the collector and +10 V and one resistor between emitter and ground was shown. The base was grounded. What is the collector voltage?
- 2) For a BJT, how is transconductance related to collector current?
- 3) For a MOSFET, how is transconductance related to drain current?
- 4) What is the unity-gain frequency (ft) of a MOSFET with a typical small signal model (Cgs and Cgd in place)?
- 5) In a modern CMOS process, what is a typical gate length and ft?
- 6) How can you determine the high-frequency 3dB frequency for a circuit?
- 7) How can you increase the 3 dB frequency?
- 8) For a feedback amplifier, discuss the important figures of merit for stability.
- 9) How can you assure stability?  

---

To: saraswat <saraswat@cis.stanford.edu>  
From: Diane Shankle <shankle@ee.stanford.edu>  
Subject: Quals Questions 2004  
Cc:  
Bcc:  
Attachments:

---

My question was:

What are the ultimate dimensional limits to device scaling? What is the minimum time a device will need to switch its state? What will be the corresponding energy requirement?

Krishna

Stanford University  
Department of Electrical Engineering

EE Quals. Jan 12-16, 2004.

1. Ecologists want to maintain the balance between robins and worms on an island. Suppose that there are initially  $R$  robins and  $W$  worms. The strategy employed by the ecologists is this: Every day, they sample an animal (robin or worm) at random and add one of the *opposite* type.

$$\begin{matrix} R_n \\ W_n \end{matrix}$$

Let  $R_n, W_n$  be the number of robins and worms at the end of the  $n^{\text{th}}$  day. Let  $f_n = \frac{W_n}{W_n + R_n}$  be the corresponding fraction of worms.

- $$R_{n+1} = \begin{cases} R_n + f_n & f_n \\ R_n & 1-f_n \end{cases}$$
- a. What is  $F_{n+1} = E \left[ \frac{W_{n+1}}{W_{n+1} + R_{n+1}} \right]$  in terms of  $R_n$  and  $W_n$ ?
- $$W_{n+1} = \begin{cases} W_n & f_n \\ W_n + 1-f_n & 1-f_n \end{cases}$$
- b. Show that if  $f_n > 0.5$  then  $F_{n+1} < f_n$ ; conversely, if  $f_n < 0.5$  then  $F_{n+1} > f_n$ . Use this to support the ecologists' belief that eventually the proportion of robins and worms is equal. ✓

$$f_{n+1} = \begin{cases} \frac{W_n}{W_n + R_n} & f_n \\ \frac{1+f_n}{1+W_n+R_n} & 1-f_n \end{cases}$$

$$\frac{1-f_n + W_n}{1+W_n+R_n}$$

$$f_n > 0.5$$

$$W_n > R_n$$

$$\frac{1-f_n}{1} < \frac{W_n}{W_n + R_n}$$

$$f_n = \frac{W_n}{W_n + R_n} > F_{n+1}$$

chan.pdf

2. Farmer Frank initially has 99 cows and 1 bull. Before going to the market every week, he samples an animal at random and buys one more of the same type. Because of the large number of cows initially, his friend, engineer Ernie, says that this strategy of buying animals will dramatically increase the fraction of cows. The fraction of cows will eventually become 1, Ernie says.

Do you think Ernie is right? Justify.

$$f_n > \frac{1}{2}$$

~~$f_{n+1} = f_n$~~

$$R_n \quad C_n \quad C_{n+1} = \begin{cases} C_n \\ C_{n+1} \end{cases}$$

$$b_n \quad b_{n+1} = \begin{cases} b_n + 1 \\ b_n \end{cases}$$

$$\frac{b_n}{C_n + b_n} \quad \frac{b_n}{C_n + b_n}$$

$$f_{n+1} = \frac{C_n}{1+b_n+C_n} \quad \frac{b_n}{C_n+b_n} \quad f_n = \frac{99}{100} \quad \frac{100}{101}$$

$$= \frac{1-C_n}{1+b_n+C_n} \quad \frac{1}{100} \quad f_n = \frac{99}{100} \quad \frac{99}{100} = \frac{99}{100}$$

$$E f_{n+1} = E \left[ \frac{f_n + C_n}{1+b_n+C_n} \right] = f_n \quad \frac{99 + 1}{100} = \frac{99}{100}$$

Paulraj : Quals Questions 2004

Consider the Least Squares problem  $\mathbf{Ax} = \mathbf{b}$

Let  $\mathbf{A} = [ \mathbf{a}_1 \mathbf{a}_2 \dots \mathbf{a}_m ]$  is collection of  $m$  different vectors each of dimension  $n \times 1$ , with  $n >> m$

We assume  $\mathbf{Ax} = \mathbf{b} = \mathbf{b}_0 + \mathbf{e}$  where  $\mathbf{b}$ ,  $\mathbf{b}_0$  and  $\mathbf{e}$  are  $n \times 1$  vectors.  $\mathbf{e}$  is the error vector.  $\mathbf{b}_0$  solves  $\mathbf{Ax} = \mathbf{b}_0$  exactly

The problem is find  $\hat{\mathbf{x}}$  such that we minimize  $\|\mathbf{Ax} - \mathbf{b}\|_2$

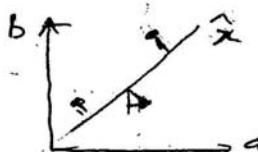
1 D picture of LS



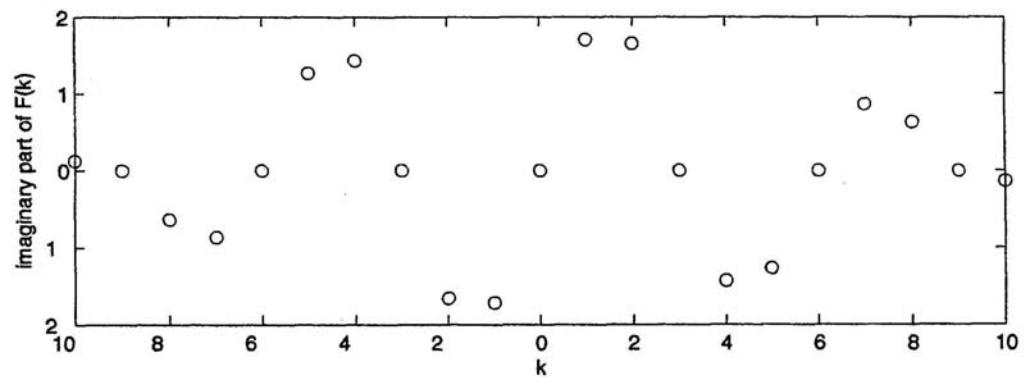
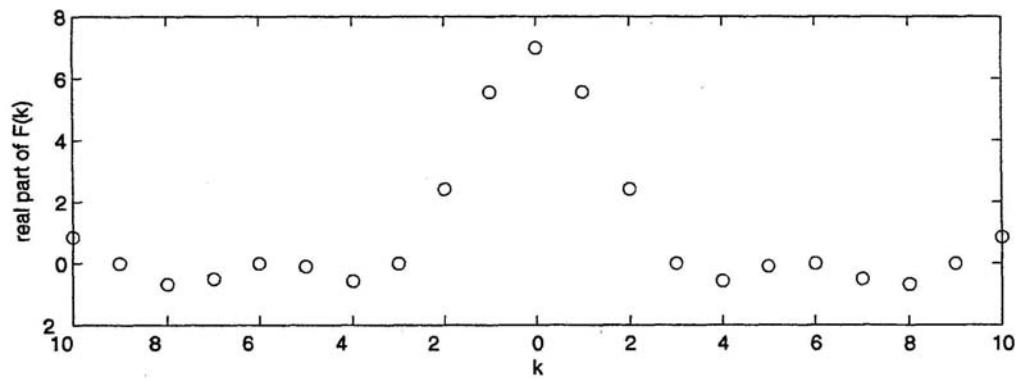
- Draw a diagram to illustrate the LS approximation problem  $\|\mathbf{Ax} - \mathbf{b}\|_2$
- What is the LS solution for  $\hat{\mathbf{x}}$
- What happens if  $m=n$
- What happens if  $m < n$

If there are errors on both sides of the equation i.e.  $\mathbf{Ax} = (\mathbf{A}_0 + \mathbf{E}) \mathbf{x} = \mathbf{b} = (\mathbf{b}_0 + \mathbf{e})$  and  $\mathbf{E}$  the error in  $\mathbf{A}$ .  $\mathbf{A}_0$  and  $\mathbf{b}_0$  solve  $\mathbf{A}_0 \mathbf{x} = \mathbf{b}_0$  exactly.

1-D picture is



- Draw a diagram to illustrate this TLS problem  $\|(A \mathbf{x} - \mathbf{b})\|_2$
- What is the TLS solution for  $\hat{\mathbf{x}}$



*Brad Osgood*

**EE Qualifying Exam  
January 2004**

**A true story**

Professor Osgood and a graduate student were working on a discrete form of the sampling theorem. This included looking at the DFT of the discrete rect function

$$f[n] = \begin{cases} 1, & |n| \leq \frac{N}{4} \\ 0, & -\frac{N}{2} + 1 \leq n < -\frac{N}{4}, \quad \frac{N}{4} < n \leq \frac{N}{2} \end{cases}$$

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The grad student, ever eager, said 'Let me work this out.' A short time later the student came back saying 'I took a particular value of  $N$  and I plotted the DFT using MATLAB (their FFT routine). Here are plots of the real part and the imaginary part (attached).' Professor Osgood said, 'That can't be correct.'

How would you resolve this confusion and why do you think it arose?



2003-2004 PhD Qualifying Examination in Device area

Yoshio Nishi, Professor (research)

1. Draw ideal silicon MOS diode band diagram where work functions for metal and silicon are equal.
2. Incorporate work function difference between metal and silicon.
3. Draw silicon gate MOS diode band diagram for both  $n^+$ -poly on p-Si and  $p^+$ -poly on p-Si, and discuss how C-V characteristics will be looking like as you approach ultimate limit of oxide thickness.
4. Discuss what will be the factors considered and their implications to MOSC-V characteristics and how you can measure them, if gate oxide is grown in less clean environment and also silicon-silicon dioxide interface is not ideal.
5. Discuss implications of 4 for MOSFET characteristics, i.e.  $V_{th}$ , field effect mobility, on-current, leakage current, CMOS inverter operations

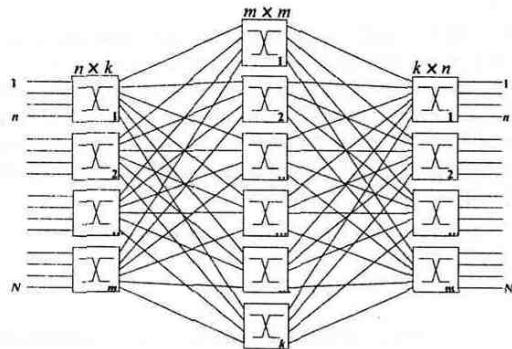
**Q2: Majority element in an array.**

We say that  $m$  is the “majority value” of an array  $A[1, 2, \dots, n]$  has if  $m$  appears more than  $n/2$  times in array  $A$ . (Note that not every array has a majority element.)

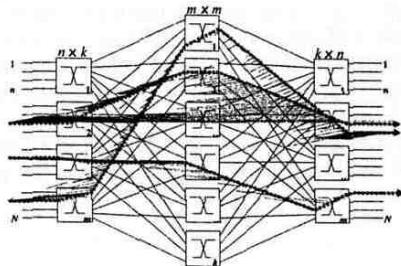
- a) Describe an algorithm that will return the majority element of an array.
- b) Can you think an  $O(n)$  time algorithm that returns the majority element of an  $n$ -element array?

**Q1: An Algorithm for a Switching Network**

Consider the  $N$ -input and  $N$ -output switching network shown below. New connections are added one at a time between a single unoccupied input and a single unoccupied output.



For example, here is the same switching network with three connections.

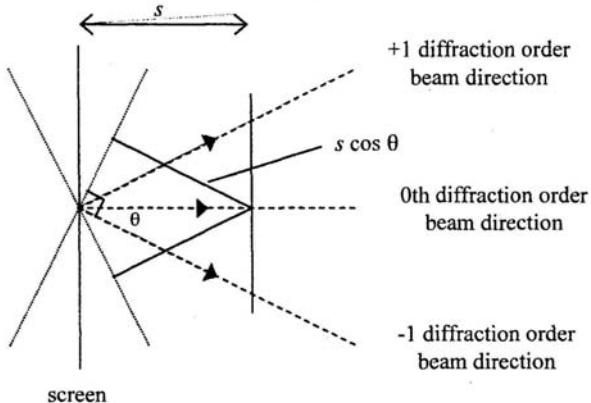


- Write down  $N$  as a function of  $n$  and  $m$ .
- In general, is  $k$  larger, smaller, or equal to  $n$ ? Explain.
- What is the minimum value of  $k$  (as a function of  $n$ ) such that a new connection can always be added when its input and output are unoccupied?
- Do you think the value of  $k$  depends on whether connections are all together, or one at a time (in an arbitrary order)?

---

### Supplementary questions

The answer as to what happens at —— and multiples of that distance is called Talbot self-imaging. At the distance, all the diffraction orders have the same relative phase as they did at the screen, and the intensity pattern is therefore the same as it would be at the screen itself. Hence an image of the screen would appear if we put a piece of white card at this distance. To see how this happens at this specific distance, see the figure below.



In this figure, we see the first diffraction orders (both positive and negative) and the "straight through" beam (i.e., the zeroth diffraction order). The path length for the +1 (or -1) diffraction order phase front to hit the center of the observing screen at distance  $s$  is

Hence the path length difference between that and the zero order path is

Now, the diffraction angle  $\theta$  is

and so we have

Hence, for the first order diffraction beams to be in phase with the zeroth order beam, we must have

## Solution

### Main questions

Huygens's principle states that each point on the surface of a phase front of a wave can be regarded as a source of spherical wavefronts, and the subsequent phase fronts can be constructed from the envelope formed by adding this together.

[If the examinee does not know Huygens's principle, it will be given to them so they can demonstrate their reasoning in the rest of the question.]

- a. Only one slit.

The light from a narrow slit diffracts out in all directions, and so a screen at a very large distance would show essentially uniform illumination. [Note that, though the standard result for a slit of finite width is technically a "sinc" function, because the slit is stated as being much narrower than a wavelength, there really are no zeros in the actual pattern, because we never get to a condition of destructive interference between light from different parts of the slit.] Some students reasoned from their knowledge that there is a Fourier transform relationship between the field at the slits and the field in the far distance (or far field), reasoning that the slit is like an impulse, which transforms to a uniform distribution.

- b. Two slits.

For two slits, this experiment is known as Young's two slit experiment. The result is that we see an interference pattern of alternating bright and dark stripes on a screen at a very large distance. This can be deduced from the notion of expanding circular phase fronts from the two narrow slit sources and the resulting interference at a screen at a very large distance. Again, some students reasoned from Fourier transforms to deduce that the transform of two impulse functions leads to a cosinusoidal transform, which also explains the alternating bright and dark stripes.

- c. A very large number of slits.

The result at a screen at a very large distance will be a set of relatively narrow beams at equally spaced angles. There are several ways this can be deduced. The standard way an optics person would likely go at this would be to consider the screen as a diffraction grating, and look at the directions that give constructive interference between the beams. Those knowing the Fourier transform relationship between the fields could reason that the Fourier transform of a set of equally spaced impulses (or delta functions) is a set of equally spaced impulses (or delta functions). Another way of deducing the result is to consider the superposition of the interference patterns of slits of separation  $d$  and those of slits of separation  $3d$ , and those of separation  $5d$ , etc., thereby gradually building up the pattern for all the slits. The only angles for which these all add up in phase are the angles of strongest interference in the two beam case. This leads to a sharpening of the interference pattern from the two beam case, but no change in the position of the interference peaks. Again, this is equivalent to a set of beams at specific, equally spaced angles.

**Supplementary questions**

In the region moderately close to the screen, for the case where we have a very large number of equally spaced slits, something special happens at a distance

behind the screen. What is it, and can you describe why it is seen at this distance? (The same thing can also happen at integer multiples of this distance.)

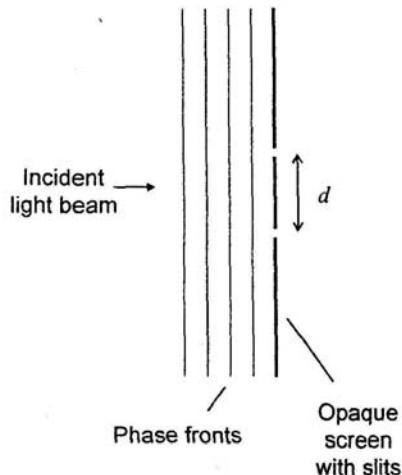
a

## EE Ph. D. Qualifying Exam Question 2004

David Miller

Preparatory question: Can you state Huygens's principle for wave propagation?

Consider an opaque screen with one or more slits in it. We shine a plane, monochromatic (single wavelength) light beam perpendicular to the screen from one side. Each of these slits is very narrow compared to a wavelength,  $\lambda$ . When there is more than one slit, all the slits are parallel, and are equally spaced a distance  $d$  apart, where  $d \gg \lambda$ . A screen with two slits is shown in the figure.

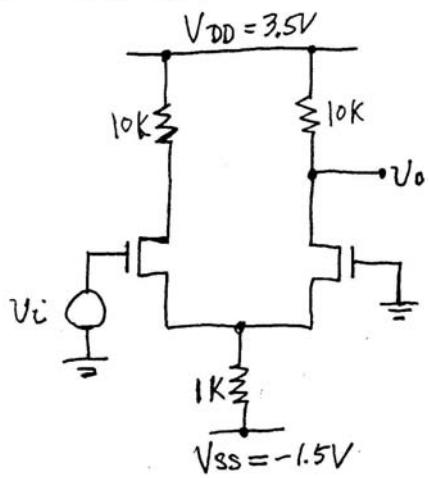


In each case below, describe qualitatively what the distribution of light intensity looks like on the other side of the screen at a very large distance from the screen, e.g., on another, white screen very far away on the right.

- Only one slit.
- Two slits.
- A very large number of slits.

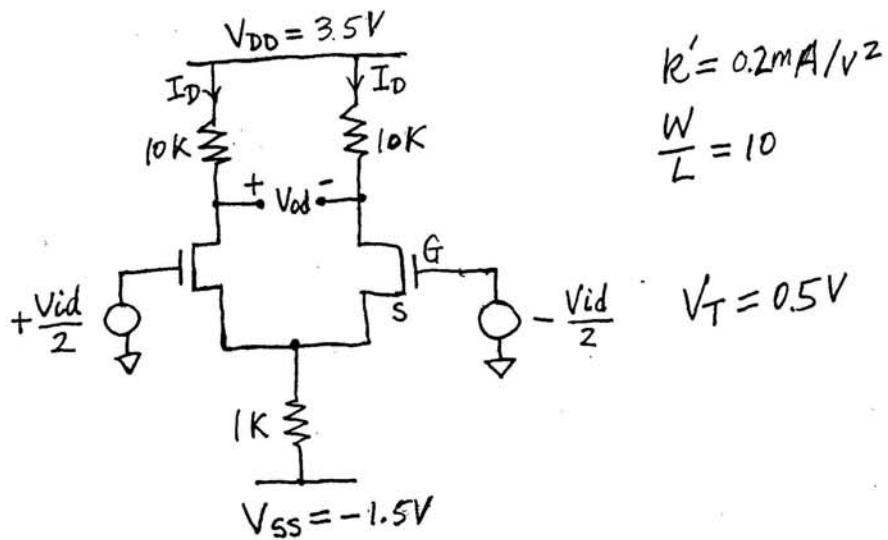
[If you finish these parts of the question, supplementary questions will be asked.]

(3) If the circuit is used as a single-ended amplifier, as shown below, find its small-signal voltage gain ( $v_o/v_i$ ).



TERESA MENZ  
2004

- (1) For the differential amplifier shown below, calculate its DC bias current ( $I_D$ ) and the gate-source voltage  $V_{GS}$ . Assume that there is no mismatch in the two transistors.



EE 352  
Fall 2004

To: Bruce Lusignan <lusignan@ee.Stanford.EDU>  
From: Diane Shankle <shankle@ee.stanford.edu>  
Subject: Re: Quals Questions 2004 Overdue  
Cc:  
Bcc:  
Attachments:

Bruce Lusignan

Topic: Space Communications

There is an Orbiter 1,000 miles above the surface of Mars. The Mars-Rover on the surface can communicate to the Orbiter with 100 Watts transmit power and a 0 dB gain antenna. The Orbiter has a gain of 20 dB, a receiver with 500 degree Kelvin noise temperature. The frequency used is 2 GHz.

With these values the data rate possible is 10,000 bits/sec.

(1) Now change the Rover gain from 0 to 10 dB. What data rate can be transmitted now?

(ans. 100.000bits/sec)

(2) With 0 dB Rover gain again, we now change the Orbiter receive Temp to 1,000 degrees Kelvin. What data rate can be transmitted now?

(ans. 5,000 bits/sec)

(3) 0 dB Rover gain and 500 deg temp again, we now change the frequency from 2 to 1 GHz. How much data rate now?

(ans, 40,000 bits/sec...but Orbiter and Rover antennas have to be bigger to maintain same gain.)

(4) How big is the Orbiter Antenna at 2 GHz?.... at 1 GHz?

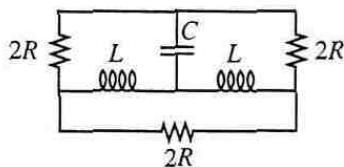
(ans at 2 GHz about 0.5 to 0.6 M diameter depending on efficiency..... at 1 GHz 1.0 to 1.2 M diameter.)

(5) Change the modulation from a efficiency of 7 dB Eb/No to 4 dB Eb/No, what is the data rate improvement?

(ans raises it from 10,000 bits/sec to 20,000 bits/sec.)

*SOLUTIONS to Prof. Tom Lee's qual question of 2004*

FIGURE 1. Network for problem



- a) How many poles does this network possess? State your assumptions.

*Because we can specify the initial energies of the three energy storage elements independently (the initial inductor currents and capacitor voltage can all be chosen without conflict), there are three poles (three degrees of freedom).*

- b) Provide expressions for the pole(s).

*Symmetry enables simplified analysis. Suppose, for example, we choose initial inductor currents of equal magnitude, both flowing toward their common connection with the capacitor. By symmetry, the left and right halves of the circuit behave the same way. The voltage across the bottom resistor thus remains zero, so we can take it out of the network. And we may also fold the left half onto the right half (or, if we wish, bisect the circuit; both will yield the same answer). Then the network becomes:*

FIGURE 2. Network after common-mode decomposition



*This second-order network has poles at the roots of*

$$s^2 \frac{LC}{2} + sRC + 1. \quad (\text{EQ 1})$$

*Now try a differential-mode initial condition, using initial inductor currents that are equal in magnitude, but oriented so that one flows into the capacitor, and the other out of it. This permits us to short across the capacitor (it never charges), together with the center of the bottom resistor. That decomposes the network into a simple LR network, whose time constant is  $(3/2)L/R$ . The pole frequency is (minus) the reciprocal, or  $(-2/3)R/L$ .*

Common blunder: Trying to supply an input by placing a voltage source across an element. *Doing so shorts out that element, changing the network!* Use a current source instead, or cut a wire and connect the voltage source to the cut ends. Better still, recognize that you don't need an input to find poles, because they are the natural frequencies of the network! Just supply some initial energy and then get out of the way. With the right choice of initial conditions, you can identify individual modes, saving you the problem of having to find the roots of a third-order polynomial (impossible to do in 12 minutes, so you shouldn't even try).

NAME: Christos Kozylakis

Modern CISC processors (e.g. the superscalar Pentium 4 or Athlon designs) are not as CISC as they used to be. Often, a RISC core lies at the heart of the processor and a "front-end" translates a CISC instruction to a sequence of one or more RISC-like micro-instructions.



- 1) Designers of CISC processors are faced with the decision of placing an instruction cache at location (A) or at location (B). Discuss the advantages and disadvantages of each alternative, particularly with respect to wide superscalar microarchitectures.

A particular CISC processor has a cache only at location (A). Its RISC core could achieve CPI of 1.0, if it was given a steady stream of micro-instructions. However, the front-end takes 5 cycles (unpipelined) to translate a CISC instruction. On the average, each CISC instruction translates to 3 RISC microinstructions.

A (B) cache is proposed as an alternative. On a hit, this cache can produce 1 translation per cycle. Misses take 12 cycles. What hit rate would you need for the (B) cache in order to operate the RISC core at its peak performance? What speedup does this give you over the original processor with the cache in location (A)?

Tip: assume that both alternatives have sufficient buffering at proper locations in order to stream-line execution.

To: Pierre Khuri-Yakub <khuri-yakub@stanford.edu>

From: Diane Shankle <shankle@ee.stanford.edu>

Subject: Re: Quals Questions 2004 Overdue

Cc:

Bcc:

Attachments:

---

Pierre Khuri-Yakub

I asked the students to explain the operation of a capacitor motor.

Pierre

## Summary of Contributions

## ➤ Packet-Switch Scheduling

- ▶ I. Keslassy and N. McKeown, "Analysis of Scheduling Algorithms That Provide 100% Throughput in Input-Queued Switches," *Proceedings of the 39th Annual Allerton Conference on Communication, Control, and Computing*, Monticello, Illinois, October 2001.
  - ▶ I. Keslassy, M. Kodialam, T. V. Lakshman and D. Stiliadis, "On Guaranteed Smooth Scheduling for Input-Queued Switches," *Proceedings of IEEE Infocom '03*, San Francisco, California, April 2003.
  - ▶ I. Keslassy, R. Zhang-Shen and N. McKeown, "Maximum Size Matching is Unstable for Any Packet Switch," *IEEE Communications Letters*, Vol. 7, No. 10, pp. 496-498, Oct. 2003.
  - ▶ I. Keslassy, M. Kodialam, T. V. Lakshman and D. Stiliadis, "On Guaranteed Smooth Scheduling for Input-Queued Switches," submitted to *IEEE/ACM Transactions on Networking*.

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## Summary of Contributions

## ➤ Scheduling in Optical Networks

- > I. Keslassy, M. Kodialam, T. V. Lakshman and D. Stiliadis,  
 "Scheduling Schemes for Delay Graphs with Applications to Optical  
 Packet Networks," to appear in *Proceedings of IEEE HPSR '04*,  
 Phoenix, Arizona, April 2004.

## ➤ Scheduling in Wireless Networks

- I. Keslassy, M. Kodialam and T. V. Lakshman, "Faster Algorithms for Minimum-Energy Scheduling of Wireless Data Transmissions," *Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt '03)*, INRIA Sophia-Antipolis, France, March 2003.

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## Summary of Contributions

## ➤ Router Buffer Sizing

- G. Appenzeller, I. Keslasy and N. McKeown, "Sizing Router Buffers," submitted to *ACM SIGCOMM '04*.

## ➤ Image Classification

- > I. Keslassy, M. Kalman, D. Wang, and B. Girod, "Classification of Compound Images Based on Transform Coefficient Likelihood," Proceedings of the International Conference on Image Processing (ICIP '01), Thessaloniki, Greece, October 2001.

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Merci !



5

Thank you.

Bad Packet Schedule					
	T+1	T+2	T+3	T+4	
Tx Group A	Tx LC A1	A1	X2	B1	B2
	Tx LC A2	B2	X1	A2	B1
Tx Group B	Tx LC B1	B1	B2	A1	A2
	Tx LC B2	A2	B1	B2	A1

Group Schedule				
	T+1	T+2	T+3	T+4
Tx Group A	AB	AB	AB	AB
Tx Group B	AB	AB	AB	AB

Good Packet Schedule					
	T+1	T+2	T+3	T+4	
Tx Group A	Tx LC A1	A1	A2	B1	B2
	Tx LC A2	B2	B1	A2	A1
Tx Group B	Tx LC B1	B1	B2	A1	A2
	Tx LC B2	A2	A1	B2	B1

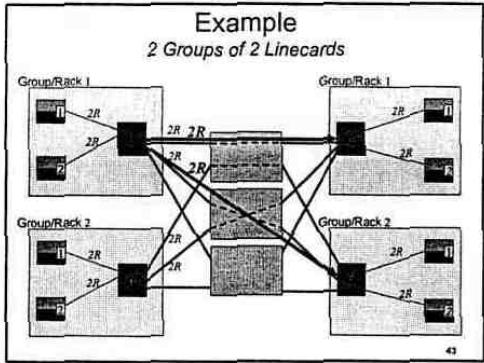
➤ Theorem: There exists a polynomial-time algorithm that finds the correct packet schedule.

Router Wish List				
Scale to High Linecard Speeds				
➤ No Centralized Scheduler				
➤ Optical Switch Fabric				
➤ Low Packet-Processing Complexity				
Scale to High Number of Linecards				
➤ High Number of Linecards				
➤ Arbitrary Arrangement of Linecards				
Provide Performance Guarantees				
➤ 100% Throughput Guarantee				
➤ Delay Guarantee				
➤ No Packet Reordering				

Summary				
➤ The load-balanced switch				
➤ Does not need any centralized scheduling				
➤ Can use a mesh				
➤ Using FOFF				
➤ It keeps packets in order				
➤ It guarantees 100% throughput				
➤ Using the hybrid electro-optical architecture				
➤ It scales to high port numbers				
➤ It tolerates linecard failure				

Summary of Contributions				
➤ Load-Balanced Switch				
➤ I. Keslassy and N. McKeown, "Maintaining Packet Order in Two-Stage Switches," <i>Proceedings of IEEE Infocom '02</i> , New York, June 2002.				
➤ I. Keslassy, S.-T. Chuang, K. Yu, D. Miller, M. Horowitz, O. Solgaard and N. McKeown, "Scaling Internet Routers Using Optics," <i>ACM SIGCOMM '03</i> , Karlsruhe, Germany, August 2003. Also in <i>Computer Communication Review</i> , vol. 33, no. 4, p. 189, October 2003.				
➤ I. Keslassy, S.-T. Chuang and N. McKeown, "A Load-Balanced Switch with Arbitrary Number of Linecards," to appear in <i>Proceedings of IEEE Infocom '04</i> , Hong Kong, March 2004.				
➤ I. Keslassy, C.-S. Chang, N. McKeown and D.-S. Lee, "Maximizing the Throughput of Fixed Interconnection Networks," in preparation.				

So will the 100Tb/s switch/router work?  
What would cause this approach to fail?



### Number of MEMS Switches

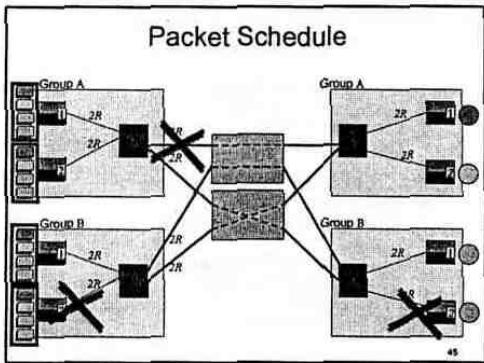
➢  $G$  groups,  $L_i$  linecards in group  $i$ ,  $L = \max_i(L_i)$ ,  
 $N = \sum_{i=1}^G L_i$

➢ **Theorem:**  $M=L+G-1$  MEMS switches are sufficient for bandwidth.

➢ Examples:

- $L = G = \sqrt{N} \Rightarrow M \approx 2\sqrt{N}$
- $N = 640, L = 16, G = 40 \Rightarrow M = 55$

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### Rules for Packet Schedule

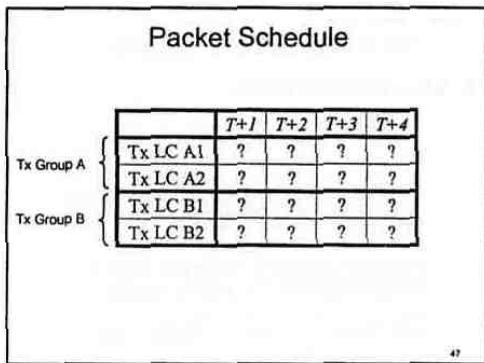
At each time-slot:

- Each transmitting linecard sends one packet
- Each receiving linecard receives one packet
- (MEMS constraint) Each transmitting group  $i$  sends at most one packet to each receiving group  $j$  through each MEMS connecting them

In a schedule of  $N$  time-slots:

- Each transmitting linecard sends exactly one packet to each receiving linecard

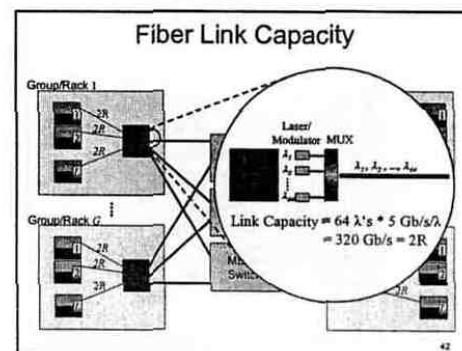
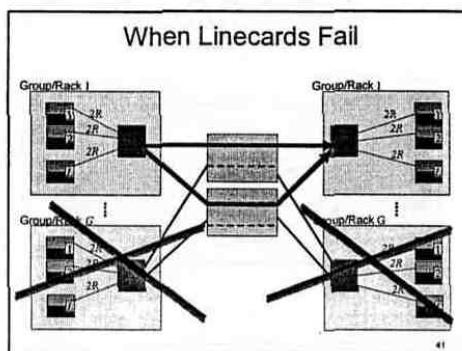
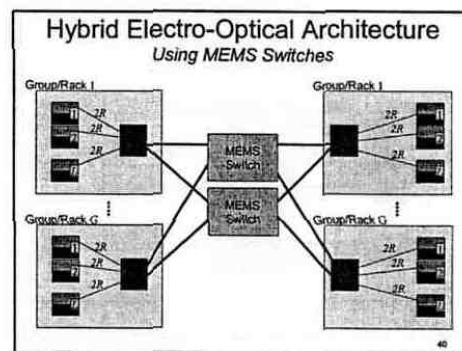
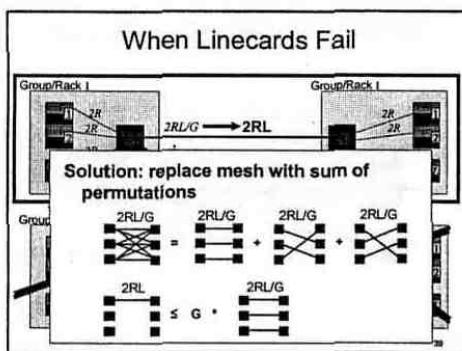
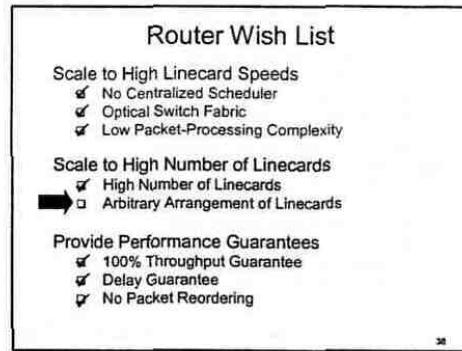
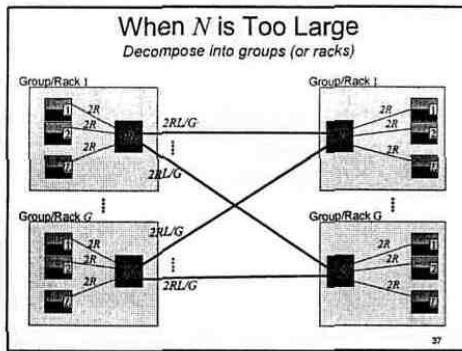
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### Packet Schedule

	T+1	T+2	T+3	T+4
Tx Group A	Tx LC A1	A1	A2	B1
	Tx LC A2	B2	A1	A2
Tx Group B	Tx LC B1	B1	B2	A1
	Tx LC B2	A2	B1	B2

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### Router Wish List

- Scale to High Linecard Speeds
  - No Centralized Scheduler
  - Optical Switch Fabric
  - Low Packet-Processing Complexity
  
- Scale to High Number of Linecards
  - High Number of Linecards
  - Arbitrary Arrangement of Linecards
  
- Provide Performance Guarantees
  - 100% Throughput Guarantee
  - Delay Guarantee
  - No Packet Reordering

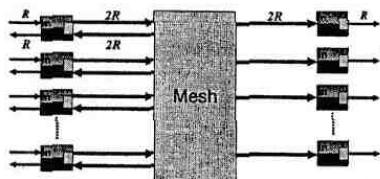
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### Scaling Problem

- For  $N < 64$ , an AWGR is a good solution.
- We want  $N = 640$ .
- Need to decompose.

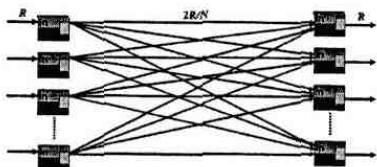
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### A Different Representation of the Mesh



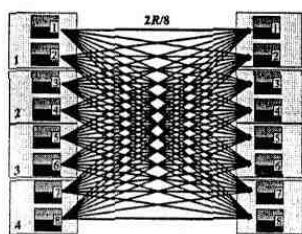
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### A Different Representation of the Mesh



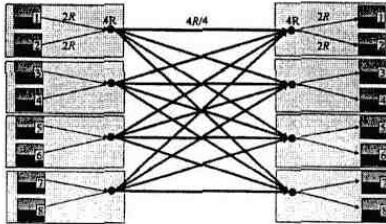
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### Example: N=8



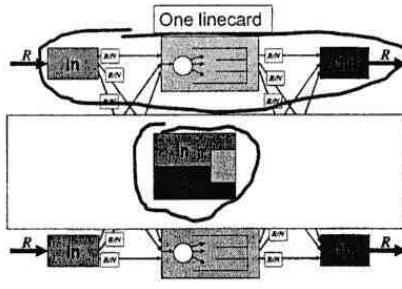
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### When $N$ is Too Large Decompose into groups (or racks)

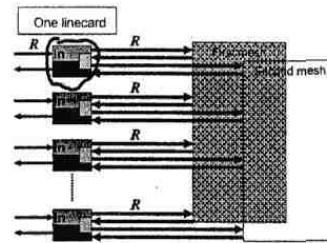


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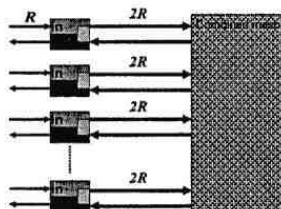
### From Two Meshes to One Mesh



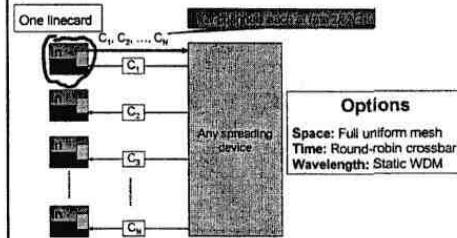
### From Two Meshes to One Mesh



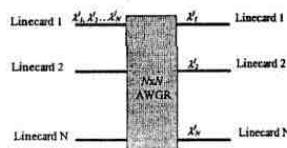
### From Two Meshes to One Mesh



### Many Fabric Options



### AWGR (Arrayed Waveguide Grating Router) A Passive Optical Component



- Wavelength  $i$  on input port  $j$  goes to output port  $(i+j-1) \bmod N$
- Can shuffle information from different inputs

### Static WDM Switching: Packaging

