Electrical Engineering

Quals Questions

2003

Stephen Boyd

Non-overlapping superposition property (NOSP).

u and v are scalar discrete-time signals defined for $t = 0, 1, 2, \ldots$

we say the signals u, v are *non-overlapping* if for each t, $u(t)v(t) = 0$

suppose A is a causal, time-invariant operator that satisfies $A(u + v) = A(u) + A(v)$ whenever u and v are non-overlapping

what can you say about A?

Ask me about anything that is not clear.

2003 Quals — J. Cioffi

TOTAL - 10 pts

Random Variables, Linear Systems, and Proability - 10 pts

A random variable x_1 takes the 2 values ± 1 with equal probability independently of a second random variable x_2 that takes the values ± 2 also with equal probability. The two random variables are sumed to $x = x_1 + x_2$, and x can only be observed after zero-mean Gaussian noise of variance $\sigma^2 = .1$ is added, that is $y = x + n$ is observed where n is the noise. You may use the function $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{u^2}{2}} du$ where $Q(\sqrt{10}) = 7.8 \times 10^{-4}$ and $Q(\sqrt{40}) = 1.3 \times 10^{-10}$.

a). x has how many possible values? (1 pt)

b). What are the means of x and y, $E[x] = ?$ and $E[y] = ?$ (1 pt)

c). What is the variance of x? (1)

d). What is the variance of y? (1)

e). What is the lowest probability of error in detecting x_1 given only an observation of y? (1 pt)

f). What is the lowest probability of error in detecting $x2$ given only an observation of y? (1 pt)

g). What is the lowest probability of error in detecting both x_1 and x_2 correctly given only an observation of y? (1 pt)

h). What is the lowest probability of error in detecting x_1 given an observation of y and a correct observation of x_2? (1 pt)

i). What is the lowest probability of error in detecting x_2 given an observation of y and a correct observation of x_1? (1 pt)

i

j). What is the lowest probability of error in any of parts e through f if $\sigma^2 = 0$? (1 pt)

Quals Solutions – Cioffi 2003

①.

a. $\underline{4}$ $\quad$ $\underline{\pm 1, \pm 3}$

b. $E[x] = E[y] = \underline{0}$

c. $E[x^2] = \underline{5} = var(x)$

d. $E[y^2] = 5 + .1 = \underline{5.1} = var(y)$

e. $P_{e,1} \leq 1.5Q(1/\sqrt{.1}) = (7.8 \times 10^{-4})1.5 = 1.2 \times 10^{-3}$

f. $P_{e,2} \cong .5\, Q(1/\sqrt{.1}) = 3.9 \times 10^{-4}$

g. $P_e = 1.5 \cdot Q(1/\sqrt{.1}) = 1.2 \times 10^{-3}$

h. $P_e = 1\, Q(1/\sqrt{.1}) = 7.8 \times 10^{-4}$

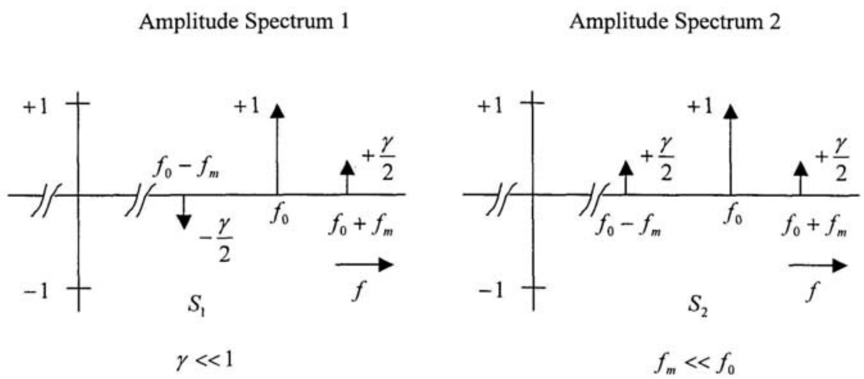
i). $P_e = 1 \cdot Q(2/\sqrt{.1}) = 1.3 \times 10^{-10}$

j). $P_e = 0$ all cases.

3	×	$x2 = 2$ $x1 = 1$
1	×	$x2 = 2$ $x1 = -1$
−1	×	$x2 = -2$ $x1 = 1$
−3	×	$x2 = -2$ $x1 = -3$

EE PhD Quals Question
Professor Donald C. Cox

January 2003



$S_i(t) = A_i(t) Cos[2\pi f_0 t + \phi_i(t)]$

- Questions about signals that are represented by the amplitude spectra above
- Horizontal axis is frequency
- Vertical axis is amplitude including sign
- Only + frequency part of spectra are shown
- The two amplitude spectra represent 2 different signals S_1 and S_2.
- You may ask questions to clarify your understanding of the spectra or problems.

Questions:

1) Which signal, S_1 or S_2, has the largest envelope variation in the time domain?

2) Which signal, S_1 or S_2, has the largest phase variation?
   2a) What is the phase variation of signal S_2?

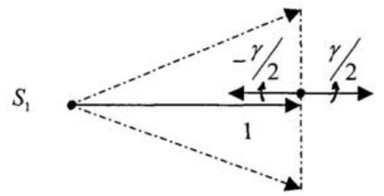
3) In a lab, how could you produce signals that have these amplitude spectra (or approximately these spectra)? Sketch a block diagram.

If there was no forward progress after 5 minutes the examinee would be asked a) can you write S_i in terms of sinusoids or exponentials. or b) Can you draw phasors for the signals, depending on what was being written or said.

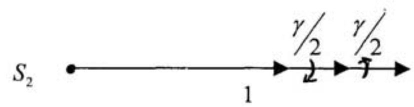
Solutions:

I. Most straight forward solution (no one started this way. Some proceeded this way after being asked b)

Phasors by inspection of spectra and assuming no negative frequencies:



Resultant of $f_0 + f_m$ and $f_0 - f_m$ is in quadrature with f_0



Resultant of $f_0 + f_m$ and $f_0 - f_m$ is in phase with f_0

1) Envelope variation of S_1 is $\sqrt{1+\gamma^2} - 1 \approx \frac{1}{2}\gamma^2$
   Envelope variation of S_2 is $(1+\gamma)-(1-\gamma) = 2\gamma$
   Envelope variation of S_2 > envelope variation of S_1
2) Phase variation $S_2 = 0$
   Phase variation of $S_1 \neq 0$
   Phase variation of S_1 > phase variation of S_2

II. Sums of real sinusoids assuming the spectra are one sided line spectra (no negative frequencies).

1) for S_1

$$x_1(t) = Cos 2\pi f_0 t + \frac{\gamma}{2}\left[Cos 2\pi(f_0 + f_m)t - Cos 2\pi(f_0 - f_m)t\right]$$

$$\omega = 2\pi f$$

$$x_1(t) = Cos\omega_0 t + \frac{\gamma}{2}\left[Cos\omega_0 t Cos\omega_m t - Sin\omega_0 t Sin\omega_m t - (Cos\omega_0 t Cos\omega_m t + Sin\omega_0 t Sin\omega_m t)\right]$$

$$x_1(t) = Cos\omega_0 t - \gamma Sin\omega_m t Sin\omega_0 t$$

$$x_1(t) = x_i Cos\omega_0 t - x_q Sin\omega_0 t = A_1(t) Cos[\omega_0 t + \phi_1(t)]$$
$$A_1(t) = envelope = \sqrt{x_i^2 + x_q^2} = \sqrt{1 + \gamma^2 Sin^2 \omega_m t}$$
$$A_1(t) \approx 1 + \frac{1}{2}\gamma^2 Sin^2 \omega_m t$$

Envelope variation of $S_1 \approx \frac{1}{2}\gamma^2$

For S_2

$$x_2(t) = Cos\omega_0 t + \frac{\gamma}{2}\left[Cos(\omega_0 + \omega_m)t + \frac{\gamma}{2} Cos(\omega_0 - \omega_m)t\right]$$
$$x_2(t) = Cos\omega_0 t + \gamma Cos\omega_m t Cos\omega_0 t$$
$$x_2(t) = [1 + \gamma Cos\omega_m t] Cos\omega_0 t = A_2(t) Cos[\omega_0 t + \phi_2(t)]$$
$$A_2(t) = 1 + \gamma Cos\omega_m t = envelope$$

Envelope variation $= 2\gamma$

and $2\gamma > \frac{1}{2}\gamma^2$ for $\gamma << 1$

II.

2) $\phi_1(t) = \tan^{-1}\frac{x_q}{x_i} = \tan^{-1}(-\gamma \sin \omega_m t)$

$$\phi_1(t) \approx -\gamma \sin \omega_m t$$
$$\phi_2(t) = \tan^{-1}\frac{0}{x_i} = 0$$

and $\phi_1(t) > \phi_2(t)$

III. Using complex exponentials and assuming one sided line spectra.

$$e^{j\omega_0 t} = Cos\omega_0 t + jSin\omega_0 t$$
$$\text{Re}\left[e^{j\omega_0 t}\right] = Cos\omega_0 t$$

1) $x_{ce1}(t) = e^{j\omega_0 t} + \frac{\gamma}{2}\left[e^{j(\omega_0+\omega_m)t} - e^{j(\omega_0-\omega_m)t}\right]$

$$x_{ce1}(t) = e^{j\omega_0 t}\left[1 + \frac{j\gamma}{j2}\left(e^{j\omega_m t} - e^{-j\omega_m t}\right)\right]$$
$$x_{ce1}(t) = e^{j\omega_0 t}[1 + j\gamma Sin\omega_m t]$$

$$x_1(t) = \mathrm{Re}[x_{ce1}(t)] = Cos\omega_0 t - \gamma Sin\omega_m t Sin\omega_0 t$$

Proceed as in II

$$x_{ce2}(t) = e^{j\omega_0 t}\left[1 + \gamma\left(\frac{e^{j\omega_m t} + e^{-j\omega_m t}}{2}\right)\right]$$

$$x_{ce2}(t) = e^{j\omega_0 t}[1 + \gamma Cos\omega_m t]$$

$$x_2(t) = \mathrm{Re}[x_{ce2}(t)] = [1 + \gamma Cos\omega_m t]Cos\omega_0 t$$

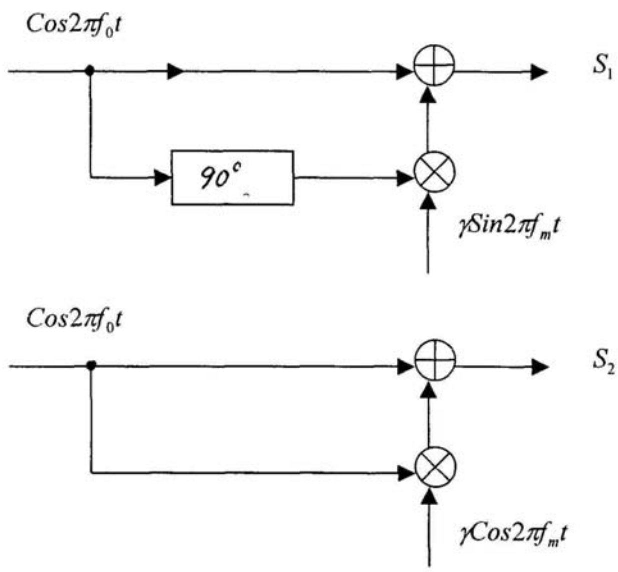
Proceed as in II

2) equations same as in II
phase variation same as in II

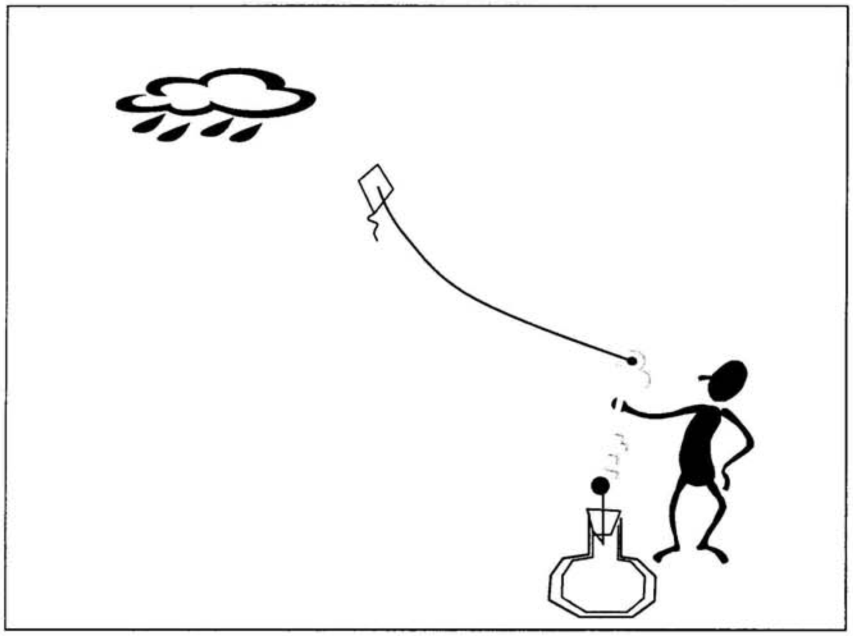
I, II, III

3)

S_1



Stanford PhD Quals Exam (Dutton Question)



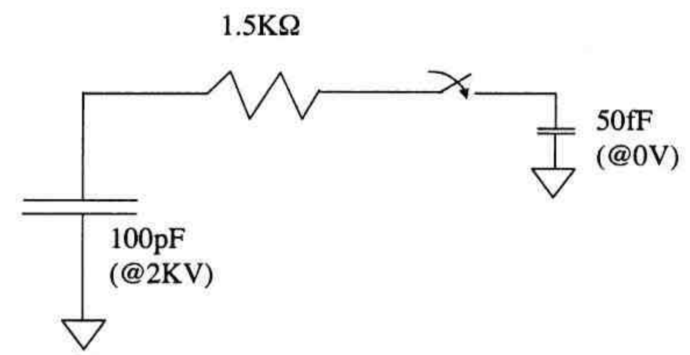
Ben Franklin did a famous experiment 250 years ago. He flew a kite on a rainy day

He performed TWO experiments:

1) While holding the key using an insulator on his hand, he then touched his bare hand to the key and felt something (a potential…say on the order of 200V)
2) Next, with his bare hand OUT of the picture (and still using an insulator to hold the key) he touched the key to a Lyden Jar*

Using your understanding of circuits (equivalent circuits) try to explain the results of both experiments quantitatively. That is, why does he feel only 200V and how much charge can he transfer onto the Lyden Jar (how long does it take etc.)?

*Lyden Jar--a 0.3cm thick glass jar, coated on the inside and outside with metal…the two metal layers do not touch.



1.5KΩ
50fF
(@0V)
100pF
(@2KV)

Was it really like this? (TBD)

THE KITE EXPERIMENT. In 1752, when Franklin attached a key to a kite string and drew electric sparks from the key, he became the first person to identify lightning as an electrical discharge.

Abbas El Gamal

Problem 1 – Time to Break a Record

Let the historical scores of some individual sport, e.g., the long jump, be an infinite sequence $X_1, X_2, \ldots$ of independent and identically distributed continuous random variables. A record is achieved in jump $k > 1$ if $X_k > X_1, X_2, \ldots, X_{k-1}$ (we don't consider the first score to be a record).

Let N be the index of the first record. Find the expected value of N?

Problem 2 – Random Process

Consider the discrete-time random process $X_1, X_2, \ldots$ with X_1 having probability density function (pdf)

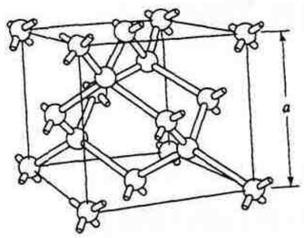
$$f_{X_1}(x) = \begin{cases} 2x, & \text{if } 0 \leq x \leq 1, \\ 0, & \text{otherwise,} \end{cases}$$

and X_{n+1} uniformly distributed on the interval $(1 - X_n, 1]$, given $X_1, X_2, \ldots, X_n$.

Is this process stationary ? Justify your answer.

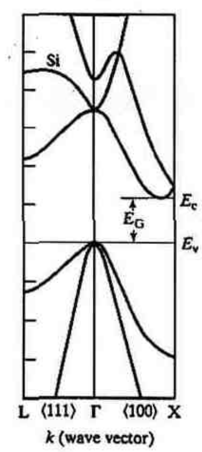
Shanhui Fan

Si has the following lattice structure:



(a) How many atoms are there per primitive unit cell?
(b) Sketch the atom configuration on the (110) surface.

Below is the band structure for Si:



(c) How many conduction band minima are there?
(d) Is a Si sample doped with P atoms a p-type or a n-type semiconductor? (Hint: P is in group V)
(e) For the sample described in (d), suppose the dopant concentration for the P atoms is 10^{16} cm^{-3}, and the dopant level is in the gap and is 0.05eV from the band edge, what is the density of the majority carrier at T = 300K? Why? (Note: T=300K represents an energy scale of 0.025eV. At 300K, the effective density of states for both the conduction and the valence band is approximately 10^{19} cm^{-3})

2003 Quals questions of G. Franklin.

I showed the candidate a laboratory device for magnetic levitation. A small ball bearing was suspended about 1 cm from a black cylinder. Typical questions went like this.

1. How does this device work?

   (a) Magnetic levitation

2. Is feedback control needed?

   (a) yes to achieve balance between gravity and magnetic forces.

3. What is the actuator?

   (a) A solenoid magnet.

4. What is the sensor?

   (a) A beam of infrared light is shined across the ball and the intensity of the received light is a function of the position.

5. How does the control work?

   (a) If the ball is too close to the magnet, the ...eld is made weaker; if the ball is too far away, the ...eld is made stronger.

6. How many dimensions of control are needed?

   (a) Only one. the ball is stable in the transverse directions because of the shape of the solenoid ...eld. (The candidate was asked to sketch the ...eld, at this point.)

7. What is the qualitive form of the transfer function?

   (a) $G(s) = \frac{A}{s^2 \text{ ¡ } a^2}$. (the candidate was given hints about doing a small signal model and linear dependance on the magnet and ball position.)

8. What form of controller transfer function would you suggest?

   (a) A lead network, such as $D(s) = \frac{Ts + 1}{\alpha Ts + 1}$ (No one got this far in the 12 minutes.)

OFFICE MEMORANDUM
SPACE, TELECOMMUNICATIONS

STANFORD UNIVERSITY
& RADIOSCIENCE LABORATORY

Monday, March 10, 2003

To: Diane Shankle

From: Tony Fraser-Smith

Subject: Ph.D. Quals Question, 2003

Impact-Triggered Flashing-Light Device

This question was prompted by a company recruiting open house at Stanford. One of the more technically-oriented companies handed out a clear plastic ball with the company's logo on its outside and a spherical plastic insert at its center that would flash for about 30 s (at about 10-20 flashes/s) after the ball was bounced on the ground. One of these balls was taken apart and the plastic insert removed. The insert was then split in half and the electronic "innards" removed. The figure below shows the two halves of the plastic insert and the electronic part. A pen shows the scale. The students had the ball and its operation described, after which they were shown the parts of the plastic insert exactly as they appear in the figure and asked how the electronic part worked. They were allowed to fiddle around with the parts shown as much as they liked, but not allowed to dismember the electronic part (since it was required for other exams).

The answer to this question usually took place in two stages: (1) the various components of the electronic part were identified, and then (2) a hypothesis for how the electronics worked was formulated. For this latter part the students were asked to write down a circuit and to base their hypothesis entirely on what they could identify in the electronic component.

Points for (1) were awarded for identifying two LEDs, one on each side of the green circuit board (left and right in the figure), a small cylindrical spring with a fixed metal rod sticking up along its axis (between the two LED's in the figure), and two small silver oxide button batteries connected in series (underneath the circuit board in the figure). There was sometimes discussion of the black dot that can be seen on the front right of the circuit board, and sometimes it was asserted – probably correctly – that there had to be a "chip" hidden underneath it, but once again the students were instructed to come up with a hypothesis for how that electronics worked based on the components they could see. A point was specifically awarded for recognition that there had to be

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Date: Tue, 01 Apr 2003 19:20:17 -0800
From: Ji m Gibbons <gibbons@cis.stanford.edu>
X-Accept-Language: en,pdf
To: Diane Shankle <shankle@ee.Stanford.EDU>
Subject: Re: Quals Question 2003

Diane:

     That's it....!!

Diane Shankle wrote:

> This is what I have!
>
>     I had three questions, asked in this order:
>
>         1.  What is a light-emitting diode and how does it work?
>
>         2. What is a semiconductor diode laser and how does it work?
>
>         3. A man is driving late at night on a country road in Nevada.  He does
> not know where he is.  He has some chest pains so he pulls over to the
> side of the road and dials 911.  The operator answers and asks where he
> is so he/she knows where to send the paramedics.  He of course says " I
> don't know".  How can he be found?  Hint if necessary:  Just keep
> talking and we will find you.
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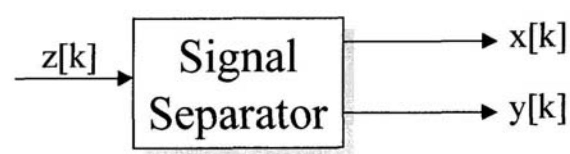
Signal Separation

You are given a random sequence z[k], which is the sum of two white noise random sequences x[k] and y[k],

$$z[k] = x[k] + y[k]$$

We want to separate z[k] into its components x[k] and y[k].



Give necessary and sufficient conditions for the joint pdf $p_{xy}(x,y)$ that assure that x[k] and y[k] can be recovered perfectly.

some source of power for the device to work, since, rather incredibly, not all students could identify the batteries.

Points for (2) were awarded for sensible circuits and hypotheses, which left room for some interesting discussions. The following outline several of these discussions:

1. Quite commonly, there was discussion of the circuit arrangement of the two LEDs: were they connected in parallel or series? Here it was helpful to know that an LED requires about 0.8 V to light up, and the silver oxide batteries that appear to power the electronic unit are connected in series and provide about 3 V. These facts led to the conclusion that the LEDs were most probably connected in series rather than in parallel.
2. Two LEDs connected in parallel with a battery power supply could not have a simpler circuit. However, what triggers the flashing light? Obviously some kind of triggering mechanism (or accelerometer) is required and equally obviously this must be the spring component. Most students who identified this component as the trigger argued that it would be displaced as the ball bounced on the ground and that it would touch the vertical metal rod along its axis, thus providing the electrical contact to initiate flashing of the LEDs. But what maintains the flashing? At this point some students argued plausibly that the spring would keep on vibrating and touching the metal rod, thus keeping the flashing going until the vibration of the spring became damped.
3. Some students argued that the spring was actually an inductor and the flashing of the LEDs corresponded to the electrical oscillation frequency of the spring's (or coil's) inductance in parallel with the stray capacitance of the circuit. This led to further consideration of the appropriate angular resonance frequency of an LC circuit: $(LC)^{-0.5}$. Here reasonable assumptions for the values of L and C led to frequencies in the MHz or even GHz range. At this stage most students would dismiss this particular explanation for the flashing.

In general, identification of the various components comprising the electronic component of the flashing ball and reasonable hypotheses for the way it worked (along with intelligent comments on why there were two LEDs and two batteries, instead of one of each) would lead to a perfect score.

X-Sender: hector@db.stanford.edu
Date: Tue, 01 Apr 2003 11:28:52 -0800
To: Diane Shankle <shankle@ee.Stanford.EDU>
From: Hector Garcia-Molina <hector@cs.stanford.edu>
Subject: Re: Quals Question 2003

At 09:15 AM 4/1/2003 -0800, you wrote:
> Still waiting for all the Quals Questions to be turned in.!

I may have already sent my question in, but just in case,
here it is again...

hector

Hector Garcia-Molina
EE Quals Question 2003
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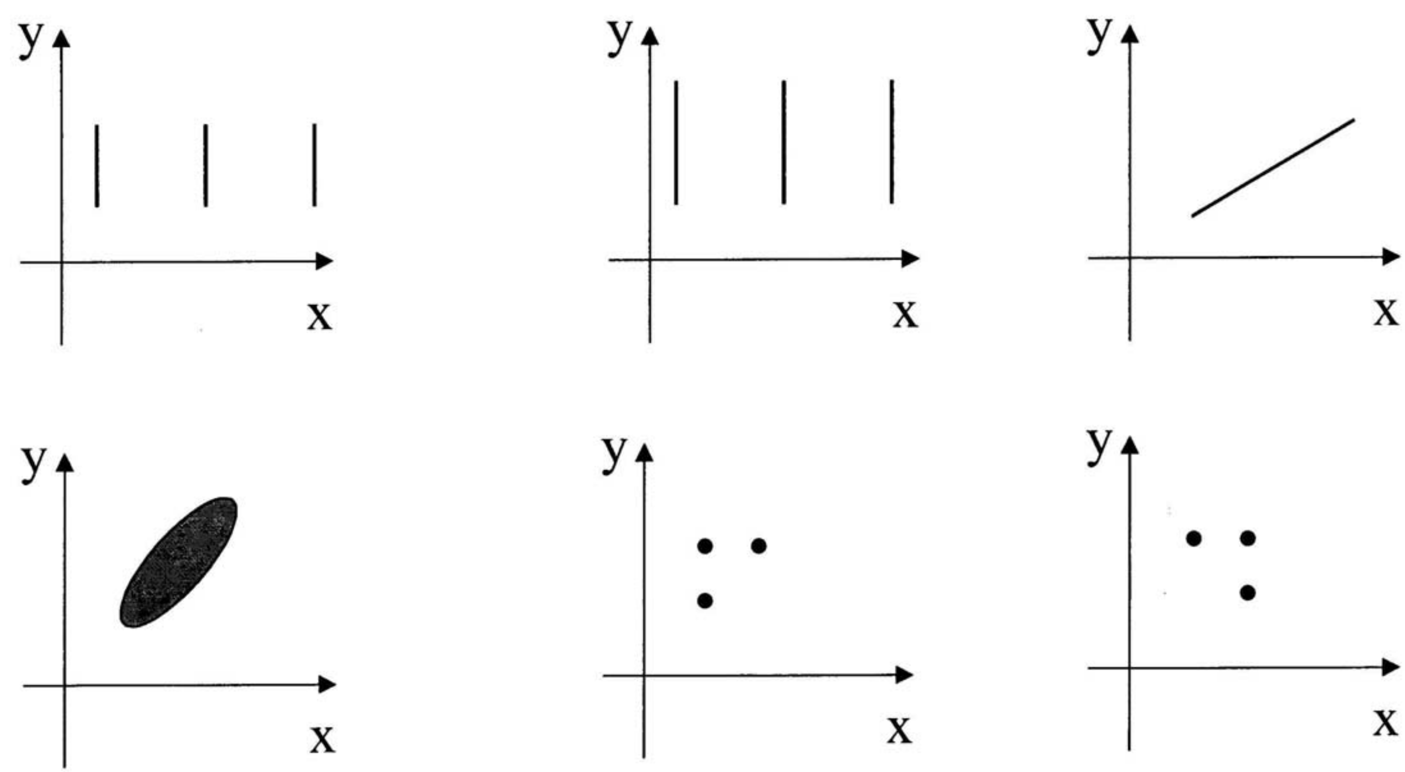
(1) Define briefly what a binary search tree is.

(2) Write a procedure for searching a binary tree.
There is a global variable T that points to the root of the tree.
(If the tree is empty, T is null.)
The procedure takes as a parameter the value to search for,
and returns a pointer. If the value was found in the tree, the
returned pointer identifies the node holding the value.
If not, the returned pointer is null.

(3) Write a procedure for inserting a new value into the same tree.
The input parameter is the value to be inserted.
No value is returned.
Duplicates are allowed in the tree, so a new node is created
even if the value is already in the tree.

(4) If there are N nodes in the tree,
What is the worst-case number of nodes that must be inspected
in a search? What is the average case?

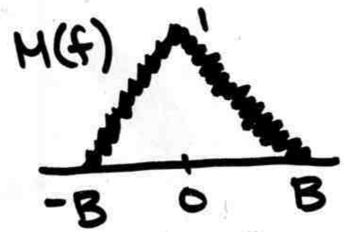
Which of these pdfs allow x and y be recovered without loss?



① Andrea Golds

$$m(t) \rightarrow \otimes \xrightarrow{y(t)} \boxed{\begin{array}{c}\text{Ideal}\\ \text{Sampler}\\ f_s\end{array}} \xrightarrow{z(t)}$$

$\uparrow \cos 2\pi f_c t$



$B \ll f_c$

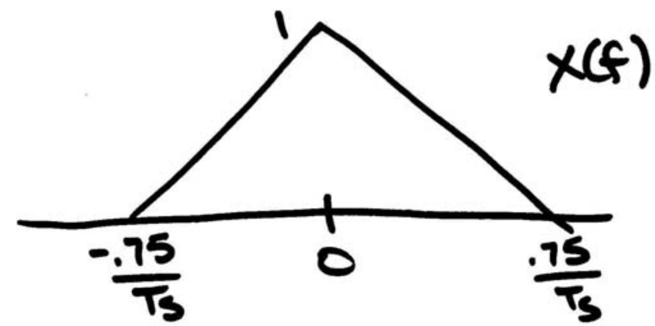
ⓐ Sketch $Y(f)$

ⓑ For $f_s = f_c$, Sketch $Z(f)$
Can $m(t)$ be obtained from $z(t)$?

ⓒ For what values of $f_s < f_c$, if any,
can $m(t)$ be obtained from $z(t)$ with a LPF?

$\begin{cases} f_c - kf_s = 0 \\ f_c \geq 2B \end{cases}$ $f_c = k \cdot \frac{f_c}{\boxed{\frac{f_s}{B}}}$

(2) Let $x(t)$ be a continuous-time signal with



Define the discrete-time signal $x[n] = x(nT_s)$

What is the DFT of $x[n]$, $n=0,1,\dots,7$?

January 2003: R.M. Gray's Qualifier Question

The points mentioned below are simply guidelines, the actual scores depended both on success at finding and explaining solutions and on demonstrated creativity or adaptivity in solving the problems.

$$f_X(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$
$$Z = \text{floor}(X)$$
$$W = X - Z = X \bmod 1$$

- Find the probability mass function (pmf) $p_Z(n) = \Pr(Z = n)$.

Solution

This was intended as an easy basic problem to get people started. It count 0.5 points if done without help.

$$\begin{aligned} p_Z(n) &= \int_n^{n+1} f_X(x)\,dx \\ &= \int_n^{n+1} e^{-x}\,dx \\ &= e^{-n} - e^{-(n+1)} \\ &= e^{-n}(1 - e^{-1});\ n = 0, 1, 2, \ldots \end{aligned}$$

- Find the pdf $f_W(w)$ for the random variable W.

Solution

This part counted roughly 2.5 points. The first two questions together were considered very basic and totaled 3 points.

To find a pdf, first find the cumulative distribution function (cdf) $F_W(w) = \Pr(W \leq w)$ and then take the derivative, $f_W(w) = dF_W(w)/dw$.

$$\begin{aligned} F_W(w) &= \Pr(W \leq w) \\ &= \sum_{n=0}^{\infty} \Pr(W \leq w \text{ and } Z = n) \\ &= \sum_{n=0}^{\infty} \Pr(n < X \leq n + w) \\ &= \sum_{n=0}^{\infty} F_X(n + w) - F_X(n) \end{aligned}$$

so that differentiation yields

$$\begin{aligned} f_W(w) &= \sum_{n=0}^{\infty} f_X(n + w) \qquad (1) \\ &= \sum_{n=0}^{\infty} e^{-(n+w)} \\ &= e^{-w} \sum_{n=0}^{\infty} e^{-n} \\ &= \frac{e^{-w}}{1 - e^{-1}};\ 0 \leq w < 1 \end{aligned}$$

Note that the domain of definition is an important part of the answer.

Several people just wrote down something like (1) but were not able to prove it or explain wl it should be true (there were several variations). Simply stating the result got a point if it wa stated correctly, but full credit and continuing to the other problems required either a proof or sound explanation.

This pdf is a truncated and suitably normalized exponential pdf. Many students lost time l working out the the details of the cdf before taking the derivative. It is much quicker to use the fa that differentiation and integration are inverse operations and hence not waste the time actual evalutating the integral only to differentitate it.

The problem could also be done using approximation arguments without the cdf by using tl approximation that $f_W(w) \approx \Pr(w \le W < w + \Delta)$ to write

$$\begin{aligned}
f_W(w)\Delta &\approx \Pr(w \le W < w + \Delta) \\
&= \sum_{n=0}^{\infty} \Pr(w \le W < w + \Delta \text{ and } Z = n) \\
&= \sum_{n=0}^{\infty} \Pr(n + w \le X \le n + w + \Delta \text{ and } Z = n) \\
&= \sum_{n=0}^{\infty} \Pr(n + w \le X \le n + w + \Delta) \\
&\approx \sum_{n=0}^{\infty} f_X(n + w)\Delta
\end{aligned}$$

This works because as with the cdf you are working with actual probabilities and hence tot probability works.

This can also be done with more work by using conditional probabilities. For example, fro Bayes rule

$$f_W(w) = \sum_{n=0}^{\infty} f_{W|Z}(w|n)p_Z(n).$$

The pmf p_Z is known, but to solve the problem this way requires finding the conditional p $f_{W|Z}(w|n)$. Several people who tried this route got it wrong or guessed and could not justify This is worked through later in these solutions.

Define now a new random variable (called a *quantizer*)

$$Q = Z + a,$$

where $0 \le a < 1$ is a fixed number, and define the *error*

$$\epsilon = Q - X.$$

- Assume you know $E(W)$ and σ_W^2. What choice of a will minimize the mean squared er $E(\epsilon^2)$ and what is the resulting minimum mean squared error?

Solution

This problem was considered the core of the question and counted 4 points. As above, a gu had to be justified to get full credit.

$\epsilon = Q - X = Z + a - X = a - W$ so the mean squared error is

$$\begin{aligned} E(\epsilon^2) &= E([a - W]^2) \\ &= a^2 - 2aE(W) + E(W^2) \end{aligned}$$

so from calculus the best a should be the one for which the derivative of this expression is 0. i.e., for which

$$2a - 2E(W) = 0$$

or

$$a = E(W)$$

with resulting mean squared error

$$E(\epsilon^2) = E(W^2) - E(W)^2 = \sigma_W^2.$$

The zero derivative condition only guarantees a stationary point, but the second derivative of the squared error is 2, which is positive, and hence the stationary point is a minimum. It was OK to simply observe that a quadratic in a is convex and hence a zero derivative is enough.

Altentatively,

$$\begin{aligned} E(\epsilon^2) &= (a - E(W))^2 + E(W^2) - E(W)^2 \\ &= (a - E(W))^2 + \sigma_W^2 \\ &\geq \sigma_W^2 \end{aligned}$$

with equality if $a = E(W)$.

• Find $E(W)$, the conditional expectation, and $E(W|Z)$.

Solution

There are several ways to evaluate $E(W)$. One way is to use the definition and integration by parts, which can get messy, at least when I do it. Another way is to find the transform (moment generating function) of W and differentiate. This also can be messy. A third way is to observe $E(W) = E(X - Z) = E(X) - E(Z)$. An exponential density with parameter 1 has mean and variance 1, so that $E(X) = 1$. Thus we all need is $E(Z)$. A geometric random variable has mean $1/p$, and hence $E(Z) = 1/(1 - e^{-1}) - 1 = e^{-1}/(1 - e^{-1})$. Thus $EW = (1 - 2e^{-1})/(1 - e^{-1})$.

The conditional expectation can be found from the conditional pdf $f_{W|Z}(w|n)$, which is the derivative of the conditional cdf

$$\begin{aligned} F_{W|Z}(w|n) &= \Pr(W \leq w | Z = n) \\ &= \frac{\Pr(W \leq w \text{ and } n < X \leq n+1)}{\Pr(n < X \leq n+1)} \\ &= \frac{\Pr(n < X \leq n + w)}{p_Z(n)} \\ &= \frac{F_X(n + w) - F_X(n)}{p_Z(n)};\ 0 \leq w < 1 \end{aligned}$$

Taking the derivative yields

$$\begin{aligned} f_{W|Z}(w|n) &= \frac{e^{-(n+w)}}{p_Z(n)} \\ &= \frac{e^{-w}}{1 - e^{-1}};\ 0 \leq w < 1 \end{aligned}$$

So that $f_{W|Z}(w|n) = f_W(w)$ and W and Z are independent! This also implies that $E(W|Z) = E(W)$.

2003 PhD Quals Question
J. S. Harris

1. What is the Depletion Approximation for a p/n junction?

2. Can you draw the charge distribution, electric field and potential for an abrupt p/n junction under the depletion approximation?

3. Can you draw an energy band diagram for the p/n junction, including the vacuum level?

4. I'm now a graduate student just starting my experimental work and I try to make my first p/n junction. I find that it isn't like the ideal one we have just discussed, but it has a plane of positive charge right at the p/n junction interface which is exactly $1/2(N_d x_n)$ of the original depletion region. If we now go back to your original drawings for the idealized p/n junction, using a different color pen, draw in the charge distribution, electric field and potential for the new situation.

5. Again, using a different color, draw the energy band diagram, including vacuum level, for the p/n junction with interface charge.

6. My skills have now imporved and I can make a good p/n heterojunction in which the bandgap of the n-region is 1.5 eV and that of the p-region is 1.0 eV and both materials have exactly the same electron affinity. Draw the energy band diagram for this p/N heterojunction, including the vacuum level. Why is there a discontinuity in the valence band and not the conduction band?

X-Sender: horowitz@vlsi.stanford.edu
Date: Thu, 06 Feb 2003 16:31:04 -0800
To: Diane Shankle <shankle@ee.Stanford.EDU>
From: Mark Horowitz <horowitz@Stanford.EDU>
Subject: Re: Quals Question 2003

Quals question:

Have a picture of an inverter driving a capacitor to Gnd. Also on the board is a simple model of the current through a transistor, Ids= k(Vgs -Vtheff)

Look at the power supply current, where does the current flow?
What is the energy comsumed in this circuit
Capacitor is lossless. Where is the power dissipated
Write an equation for delay
Can you reduce the supply and not change the delay?
To minimize power, what should the threshold be?
What is the ratio of the static to dynamic power?

At 10:23 AM 1/31/03 -0800, you wrote:

> Quals Question 2003
>
> Please send a copy of your Quals Question or you can email the question.
> Thanks,
> Diane Shankle
> Packard 165
> MC:9505
> (650) 723-3194
> Fax:(650) 723-1882

X-Sender: khuri-ya@loki.stanford.edu (Unverified)
Date: Fri, 31 Jan 2003 12:02:24 -0800
To: Diane Shankle <shankle@ee.Stanford.EDU>
From: Pierre Khuri-Yakub <khuri-yakub@Stanford.EDU>
Subject: Re: Quals Question 2003

Diane,
My question was: "How do the characteristics of a transmission line change if it is supported on a vibrating membrane".
Pierre
At 10:23 AM 1/31/2003 -0800, you wrote:

> Quals Question 2003
>
> Please send a copy of your Quals Question or you can email the question.
> Thanks,
> Diane Shankle
> Packard 165
> MC:9505
> (650) 723-3194
> Fax:(650) 723-1882

Prof. B. (Pierre) T. Khuri-Yakub
E. L. Ginzton Laboratory, room 11
Stanford University
Stanford, CA 94305-4088
e-mail: khuri-yakub@stanford.edu
Phone: 650-723-0718
Fax: 650-725-2533
http://piezo.stanford.edu

To: Gregory Kovacs <kovacs@cis.stanford.edu>
From: Diane Shankle <shankle@ee.stanford.edu>
Subject: Re: Quals Question 2003
Cc:
Bcc:
X-Attachments:

> Greg Kovacs Quals Question 2003
>
> The student was shown a schematic for a basic, no-tricks common-emitter amplifier using an NPN bipolar transistor. They were asked to determine the gain of the circuit from the information provided, which was fully adequate.
>
> If finished in time, the student was shown an inverting op-amp circuit with a small-signal input signal and an offset voltage added to the positive op-amp terminal. The student was told the amplifier and all components were ideal, and asked to describe the output signal as accurately as possible.
>
> If finished in time, the student was shown an op-amp voltage follower and a configuration with the positive input terminal tied to the output. The student was asked to comment in general terms on the behavior of the two circuits.

Student Name: ______________________________

This question will explore your understanding of performance, power, and energy consumption trade-offs in modern processors. It will probably help you to write down the basic equations that calculate the execution time (T), consumed power (P), and consumed energy (E) when running some application on a processor:

T = Execution Time =

P = Power Consumption =

E = Energy Consumption =

a) What is the impact of decreasing the processor clock frequency on T, P, and E? What is the impact of decreasing the processor power supply on T, P, and E? Explain briefly.

b) Assume an application that is easily parallelizable and that you are not interested in performance improvement. How can the use of two processors (parallel processing) help you achieve significant reductions in P and E? Explain briefly.

c) In terms of performance, RISC instructions sets are typically preferred over CISC instructions sets: compilers can produce better executable code for RISC and it should be easier to build a high frequency RISC processor. From the point of view of power consumption, how would you compare RISC and CISC instruction sets?

d) Modern superscalar processors use speculative execution based on a number of prediction techniques (branch prediction, memory dependence prediction, etc). What is the likely impact of speculation on T, P, and E? Explain briefly.

Date: Tue, 06 May 2003 09:54:37 -0700
From: Monica Lam <lam@cs.stanford.edu>
X-Accept-Language: en-us, en
To: Diane Shankle <shankle@ee.Stanford.EDU>
Subject: Re: Quals Question 2003

Diane Shankle wrote:

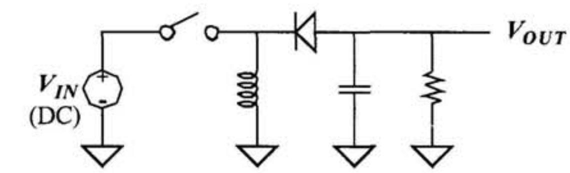
> Still waiting for all the Quals Questions to be turned in.!
>
> Quals Question 2003
>
> Please send a copy of your Quals Question or you can email the question.
> Thanks,
> Diane Shankle
> Packard 165
> MC:9505
> (650) 723-3194
> Fax:(650) 723-1882

The quals question was:

What is garbage collection? And how is garbage collection implemented?
What are the advantages and disadvantages of the different algorithms.

Problem 1: Consider the following circuit:

FIGURE 1. Network for Problem 1



Calculate the steady state, DC ratio V_{OUT}/V_{IN}, if the switch periodically closes for a time T_{on} (then opens) every T seconds. All elements are ideal (e.g., zero diode drop), the capacitor filters perfectly, and loading by the resistor may be neglected. Assume the inductor current is never zero. Hint: You may wish to consider the average voltage across the inductor.

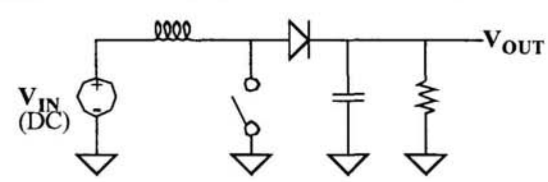
Answer: An ideal inductor acts as a DC short, but more than half of the students who uttered this statement at one point seemed unaware that a completely equivalent statement is that the **average** voltage across an inductor is zero. When the switch is closed, the inductor voltage is V_{IN}, causing a linearly growing inductor current. When the switch opens, this current is pulled out of the output network, causing V_{OUT} to go negative. Because the diode is on during this time, the voltage across the inductor is V_{OUT}, which may be inferred from the problem statement to be constant. Combining what we know, we can quickly write:

$$V_{in}T_{on} + V_{out}(T - T_{on}) = 0 \Rightarrow \frac{V_{out}}{V_{in}} = -\frac{T_{on}}{T - T_{on}} = -\frac{D}{1 - D}, \quad \text{(EQ 1)}$$

where we have denoted by D the *duty ratio* T_{on}/T. The output voltage is thus negative in sign, and larger than the input voltage for D greater than 0.5.

Problem 2: Now permute the inductor, diode and switch as follows, and recompute the relationship between input and output voltage:

FIGURE 2. Network for Problem 2



Answer: Following an exactly analogous approach, we obtain:

$$V_{in}T_{on} + (V_{in} - V_{out})(T - T_{on}) = 0 \Rightarrow \frac{V_{out}}{V_{in}} = \frac{T}{T - T_{on}} = \frac{1}{1 - D}, \quad \text{(EQ 2)}$$

so this circuit does not invert polarity, but boosts voltage for all values of D.

To:
From: Diane Shankle <shankle@ee.stanford.edu>
Subject: Fwd: Re: Quals Question 2003
Cc:
Bcc:
X-Attachments:

From: Bruce Lusignan <lusignan@ee.Stanford.EDU>
Subject: Re: Quals Question 2003

Quals Question:

We're transmitting at 800 MHz with 1 Watt power output and as omni-directional antenna. The receiver has a 500 Deg.K noise temperature and a 1-foot diam. parabolic antenna. The useful range is 100 miles.

(a) What's the range if we reduce power to 1/2 watt?..{ 100/ sq. root 2}

(b) How big do we have to make the parabolic receiver to get back to the same range, 100 miles?..{1 x sq. root 2 feet}

(c) We change the noise temperature to 250 Deg.K, (from 500). How much is the range now?..{ 100 x sq. root 2 miles}

(d) Back at 500 deg.K. We now get 3 dB of rain absorption. How much is the range now? { a factor of La=2 reduction in signal received PLUS a reduction in the effective noise temperature: Teff = 500+ 290(La - 1)/ La = 500 +145 = 645. The range is less by sq. root of (2 x 645/ 500).}

(e) Now the original signal changes from analog, needing 10 kHz bandwidth and 10 dB C/N, to digital, needing 5 kBit/sec and 7 dB Eb/No. What is the range now? WE have a 3 dB (factor of 2) improvement in sensitivity and another 2 in information rate.. a factor of 4 in all. The range increases sq. root of 4. I.e. it doubles to 200 miles.}

(f) A bad multipath blocks the signal, about how far do we have to move to get out of the null. {about half of a wavelength c/f = 3 x 10 **8 / 800 x 10**6.. 0.37 m about half of that.}

(g) We now change the radio frequency from 800 to 80 MHz. What about the range? { With omni on one end and fixed size antenna on the other, the range doesn't change. The reduction in receive antenna gain is balanced by reduction in "free space" loss.}

Qualifying Examinations

January 2003
Professor Nick McKeown

1. **Expectation.**

Explain intuitively why expectation is linear.

2. Randomized Quicksort.

Consider a set of n numbers, S, that we wish to sort using the following randomized algorithm.

> **Input: The set of numbers, S.**
> **Output: Elements of S in increasing order.**
>
> **Steps:**
>
> **1. Choose y uniformly and at random from S.**
> **2. Compare every element with y to find: S_1, the set of elements smaller than y, and S_2, the set of elements larger than y.**
> **3. Recursively sort S_1 and S_2. Output S_1, y, S_2.**

In this question, we're going to find out how long the algorithm takes to run. We'll define the expected running time to be the *expected number of comparisons* needed to sort the set S. Let $S_{(1)}$ be the smallest element of S, $S_{(n)}$ the largest, and $S_{(i)}$ the ith smallest.

(a) Consider two elements $S_{(i)}$ and $S_{(j)}$. How many times can they be compared to each other by the algorithm?

(b) Define X_{ij} to be equal to 1 if elements $S_{(i)}$ and $S_{(j)}$ are compared by the algorithm, and 0 otherwise. Write down an expression for the total number of comparisons in terms of X_{ij}.

(c) Find an expression for the expected number of comparisons as an expression of $E[X_{ij}] \equiv p_{ij}$. .

In what follows, we'll find an expression for p_{ij}.

(d) Draw a binary decision tree, rooted on y, that shows the sets evolving at each stage of the recursion. Show the set with smaller elements to the left. What order do we need to read out the elements in the tree to output them in sorted order?

(e) Pick two elements from S at random. Under what conditions will they be compared during the execution of the algorithm?

(f) Consider the list, γ, of all the elements in S found by reading the elements in the tree, starting at the root y and traversing the tree from left to right along each row. Now pick two elements from γ, say $S_{(i)}$ and $S_{(j)}$. Let's figure out the probability that they are compared by the algorithm. i.e. the probability that one is in the sub-tree of the other. Let $S_{(k)}$ have rank such that $i \leq k \leq j$, and let $S_{(k)}$ appear earliest in γ of the elements in the range $S_{(i)}$ to $S_{(j)}$. If k is equal to neither i or j, will i and j appear in the same sub-tree as each other?

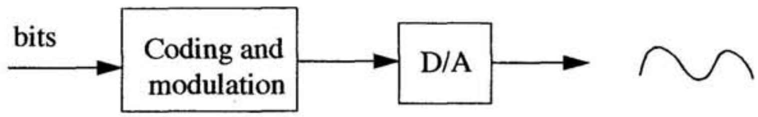
(g) If k is equal to either i or j, will they appear in the same sub-tree as each other?

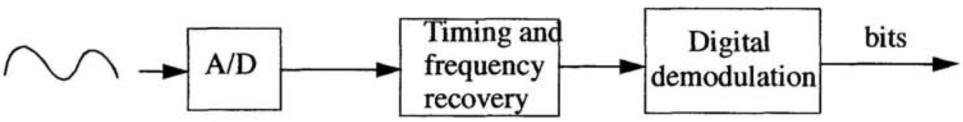
(h) Given that there are $j Đ i + 1$ elements in the range $S_{(i)}$ to $S_{(j)}$, and we know that i and j will be compared iff either one of them is first in list γ, what is p_{ij}?

(i) Prove that the expected total number of comparisons equals $2n\sum_{k=1}^{n}\frac{1}{k}$.

from TERESA Meng
Quals 2003

Consider a standard communication system as follows:





1. Since the receiver clock is not synchronized with the transmitter clock, can you suggest a design at both the transmitter (perhaps using some training bits) and the receiver that will provide a means to perform sample timing recovery?

2. If the signal has been frequency-converted to a carrier frequency at the transmitter, can you suggest a design at both the transmitter and the receiver that will provide a means to perform the carrier **frequency offset** recovery?

Ph. D. Qualifying Exam 2003

David Miller

Describe qualitatively how a rainbow is formed.

Some questions you might consider as you deduce how a rainbow is formed include:

Relative to one another, where are the rain, the sun and the observer when the observer perceives a rainbow?

If an observer perceives a circle or a circular arc of light, what does that mean about the rays of light arriving at the observer's eye?

What form would we choose as a reasonable approximation to the shape of a raindrop?

What happens when a ray of light from a specific direction hits a raindrop?

Why is the angle of the blue arc different from the angle of the red arc (and hence why does the rainbow separate colors)?

If this question is completed, supplementary questions will be asked.

Quals question solution

David Miller, January 2003

In this question, as the examiner I was much more interested in how the examinee approached the question than whether they got all the way through to the actual and complete answer. I did not expect any quantitative answer, and in fact, I would regard even a complete qualitative solution of this problem as being impossible in the time allotted. What I was most looking for was to see the student construct hypotheses creatively, and then examine those hypotheses critically themselves (possibly with some proding from the examiner). This combination of creativity and critical thinking is a very important one in research.

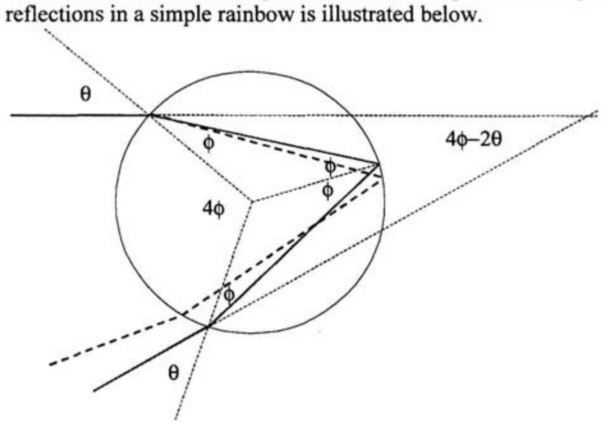
For completeness, here is the full solution to the rainbow problem. It does not require any physics beyond middle school or high school general science, but it is quite tricky. Many of the explanations of rainbows found in popular sources are substantially incomplete or even wrong. For example, rainbows do not involve total internal reflection (though they do involve internal reflection).

A rainbow is formed when the sun is behind the observer and the rain is in front. The rainbow arc lies on a circle whose center is along the line from the sun through the observer.

When an observer sees a circle or circular arc, it means that the rays hitting the eye are coming in at a particular angle relative to the center of the arc.

Raindrops are, to a good first approximation, spherical.

The reason we see a rainbow is that, for a given color, the light refracted through the raindrop and reflected out comes out predominantly at a particular angle. The general form of the reflections in a simple rainbow is illustrated below.



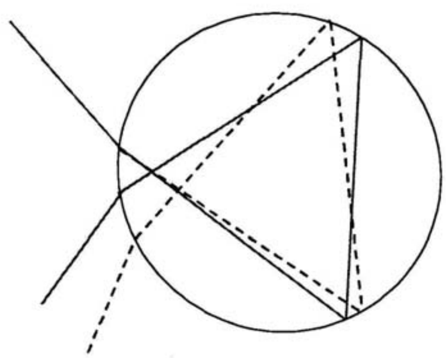
The heavy solid lines show the kind of path taken by a "red" (solid) ray through the drop as it (i) refracts as it enters the drop, changing its angle from the incident angle θ to the refracted angle ϕ, (ii) partially bounces off of the back of the drop, and (iii) refracts out of the drop again. As shown from the angles in the diagram, the total angular change is $\pi-4\phi+2\theta$.

The relationship between incident and refracted angles is given by Snell's Law

where is the refractive index of the drop (assuming the air has a refractive index of 1).

Refractive index increases as the wavelength gets shorter, so its refracted angle is relatively smaller, and so a "blue" (dashed) ray takes a path as shown by the heavy dashed line, with a corresponding smaller cone angle as perceived by an observer, which is why the blue arc of a rainbow lies inside the red arc of a rainbow.

A second rainbow is formed by light rays making a second reflection inside the drop, as illustrated below. This rainbow is seen at larger angles in particularly bright rainbows, and has an inverted color order. Note that now the relative angles of the red and blue arcs are reversed, with the blue arc corresponding to a larger cone angle to the observer.



The reason why there is an apparent single angle of arc for any given wavelength is much more subtle. To see this it is necessary to understand that there is a dominant angle (or small angular range) that accounts for a large fraction of all the reflected rays. The reason why there is a dominant angle is that, for any given color, there is a maximum to the angular deviation $\pi-4\phi+2\theta$. Because the maximum is a turning point, there is a range of incident ray positions on the sphere (and hence a range of incidence angles relative to the spherical surface) that all give a similar deviation angle. To find that maximum angle, one could differentiate the expression $\pi-4\phi+2\theta$ with respect to θ and find where that derivative is zero.

Formally, substituting using Snell's Law, we would have

Formally carrying through this calculation (which the examinee is not expected to do) would lead, for a refractive index of ~1.33, to the angle θ ~59.4 degrees, with a corresponding refracted angle ϕ ~40.2 degrees, and a resulting deviation angle from the original ray of ~ 42 degrees, which is therefore the approximate cone angle of the rainbow arc (depending of course on wavelength because the water refractive index depends on wavelength).

Hence, the bands of colors seen are not the only angles at which those colors are reflected back, but merely the dominant angles. There is also reflection at essentially all smaller angles. In fact if one looks very carefully at a bright rainbow, one will see that the entire inside of the rainbow arc is actually somewhat bright with white light.

A good site discussing rainbows is http://www.unidata.ucar.edu/staff/blynds/rnbw.html

Excellent pictures of rainbows, showing secondary rainbows and supernumerary arcs are to be found at http://www.jal.cc.il.us/~mikolajsawicki/rainbows.htm

2003 PhD Qualifying Exam.

Professor Yoshio Nishi

This is to test students in the area of device and device physics, by going through several layers of questions, from fundamental to advanced levels. I asked students to use white board for whatever needed either to derive equations, figures or diagrams.

First question was to draw the drain current-voltage characteristics for long channel nMOSFET, followed by basic operation principle by using band diagram. Most of students did pass this test in 2 minutes.
Then, I drew the drain current voltage characteristics with drain current humps for unknown MOS device structure on a white board, and asked what structure(s) could possibly cause such characteristics. Most of students needed some hint, and came across the idea of floating substrate effects on MOSFET on insulating substrate where the substrate of transistor is partially depleted.
Next question was to ask students possible temperature dependence of the current hump phenomena. Students who understand electron scattering cross section and the impact ionization well answered this question correctly.
I also asked students to draw equivalent circuits for the structure with parasitic bipolar transistor.
Next question was to combine size effects of the phenomena, and what possible structure would be free from this phenomena. Only a couple of students reached the last question and answered well.
The last question was to draw the band diagram for very thin gate and thin body SOI MOSFET, and ask surface quantization and possible carrier re-population in different valleys in the momentum space.

Oyekunle
Olukotun

Parallelism and Locality

1. How do parallelism and locality get used in modern microprocessor designs?

Assumptions

- 20 issue OOO processor, unlimited window size
- Perfect branch prediction
- 4 MB F.A. 1-word line cache, single cycle access
- 100 cycle main memory access
- Nonblocking cache
- No structural hazards

1. Program mallocs and initializes a 1 MB linked list data structure
2. Program runs a C loop:

```
for (p=head; p!=NIL; p = p->link)
    ++(p->value);
```

```
        J       test
loop:   LW      R5, 0(R4)
        ADDI    R5, R5, #1
        SW      R5, 0(R4)
        LW      R4, 4(R4)
test:   BNEZ    R4, loop
```

Assume 100,000 iterations with this data set

1. X and Y are in main memory
2. C loop:

```
for (i=0; i < 100,000; i++)
    Y(i) = a*X(i) + Y(i);
```

```
foo:  LD     F2, 0 (R1)      ; load X(i)
      MULTD  F4, F2, F0      ; multiply a*X(i)
      LD     F6, 0 (R2)      ; load Y(i)
      ADDD   F6, F4, F6      ; add a*X(i) + Y(i)
      SD     0 (R2), F6      ; store Y(i)
      ADDI   R1, R1, #8      ; increment X index
      ADDI   R2, R2, #8      ; increment Y index
      SGTI   R3, R1, #100000; test if done
      BEQZ   R3, foo         ; loop if not done
```

2. Which loop is faster? Why? How much faster?

Brad Osgood

Electrical Engineering Qualifying Exam
Winter 2003

This is a question about sampling and interpolation for periodic, bandlimited signals.

Smith says: I have a signal which is bandlimited and periodic. So to reconstruct it from sampled values I apply the sampling formula with a high enough sampling rate and I get the usual infinite series of sinc functions that gives me the interpolation.

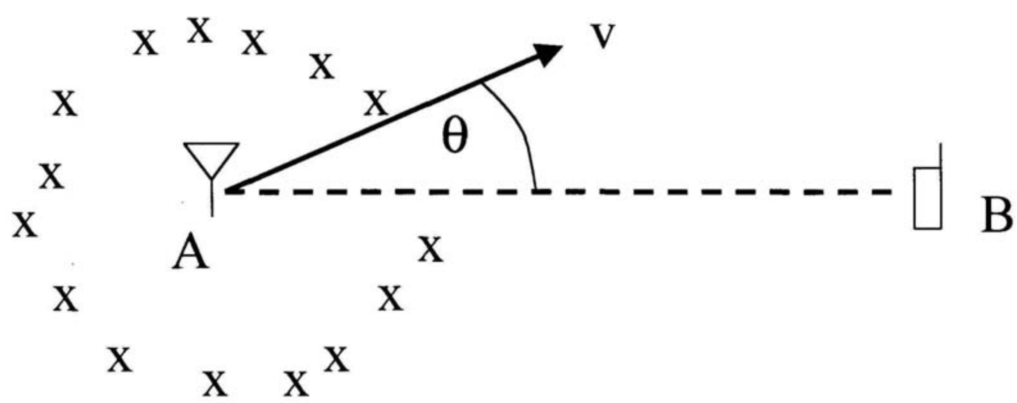
Jones says: Sure, but if you *know* the signal is periodic shouldn't you need only a *finite* number of samples? Shouldn't your infinite series become a finite sum?

Help Smith and Jones to resolve their dilemma. Work with a single sinusoid of period 1 if you wish.

Quals Question – 2003

A. Paulraj

Consider a mobile transmitter "A" moving with a speed 'v" and angle 'θ'. The mobile is surrounded by uniform (on average) random amplitude scatterers. The mobile emits a constant amplitude CW signal with radian frequency $2\pi f$. The transmitted signal bounces off the random amplitude scatterers that surrounds the mobile and arrive at a receiver 'B'. We assume no direct path between the Tx and the Rx. See Fig 1.



Questions:

1. Sketch the signal waveform at the receiver
2. Sketch the power spectrum of the received signal
3. How will received waveform and power spectrum change if

- The mobile velocity ‘v’ is doubled
- The mobile direction ‘theta” changes from + 30 to – 90 degrees

4. Is the received signal ‘stationary’?

5. What is an analytical expression for the power spectrum and the autocorrelation function of the received signal

6. How is the ‘stationarity’ property related to the ‘uncorrelated’ scattering property

X-Sender: quate@eemail.Stanford.EDU
Date: Fri, 17 Jan 2003 14:21:45 -0800
To: Diane Shankle <shankle@ee.Stanford.EDU>
From: cal quate <quate@Stanford.EDU>
Subject: My questions on the Quals Jan '03

Diane
Here are the questions that I asked on the Quals for Jan '03:

1) a-What is the change in resonant frequency of a simple LC circuit when C is changed by a small amount?
b- How much energy is required to make a small change in the capacitance?

2) A thin optical lens is illuminated by a plane wave at normal incidence.
Describe the properties of the wave after passing through the lens. What is the beam diameter at the focal plane??

cal quate
rm12 Ginzton

Date: Thu, 3 Apr 2003 13:47:40 -0800
Subject: Re: Quals Question 2003
From: Krishna Saraswat <Saraswat@stanford.edu>
To: Diane Shankle <shankle@ee.Stanford.EDU>

Quals question 2002 - Prof. Krishna Saraswat

Q1. We make two MOS capacitor on Si, one with large gate dielectric thickness and the other with ultrathin gate dielectric so that there is appreciable amount of gate leakage. What will be the measured capacitance-voltage characteristics in each case?

Hint. How does leakage change the charge induced in the semiconductor? What is the circuit model of a leaky capacitor?

Q2. For the MOS capacitor with ultrathin gate dielectric if we increase the size of the gate electrode what will be the measured capacitance per unit area?

Hint. Will the gate electrode be equipotential in the presence of leakage through the dielectric realizing the fact that electrode material has finite resistivity? How does the leakage through the dielectric change the potential of the gate electrode from center to the edge if the contact is made at the center?

X-Sender: quate@eemail.Stanford.EDU
Date: Fri, 17 Jan 2003 14:21:45 -0800
To: Diane Shankle <shankle@ee.Stanford.EDU>
From: cal quate <quate@Stanford.EDU>
Subject: My questions on the Quals Jan '03

Diane
Here are the questions that I asked on the Quals for Jan '03:

1) a-What is the change in resonant frequency of a simple LC circuit when C is changed by a small amount?
b- How much energy is required to make a small change in the capacitance?

2) A thin optical lens is illuminated by a plane wave at normal incidence.
Describe the properties of the wave after passing through the lens. What is the beam diameter at the focal plane??

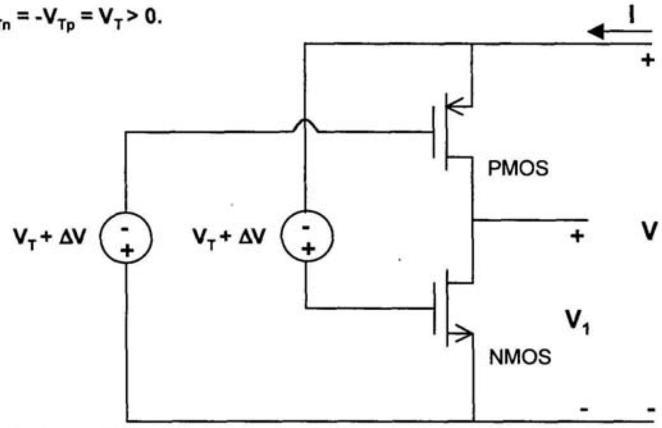
cal quate
rm12 Ginzton

Question 1

For an n-channel MOSFET, what is I_{DS} in a) triode and b) saturation mode?
Express in terms of: V_{DS} , V_{GS} , V_T , W, L, μ_n and C_{ox}.

Question 2

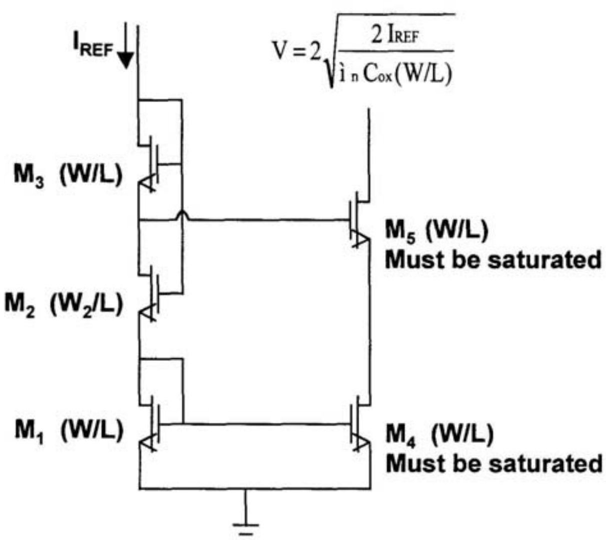
The NMOS & PMOS I-V characteristics are complementary (symmetrical), $\Delta V > 0$, and $V_{Tn} = -V_{Tp} = V_T > 0$.



a) For V > 0, what is V_1 as a function of V?
b) For $0 < V < 2\Delta V$, is the NMOS in triode or saturation?
c) For V < 0, what is V_1 as a function of V?

Question 3

What is W_2 (in terms of W) such that M_4 and M_5 operate in saturation mode?



Question 1

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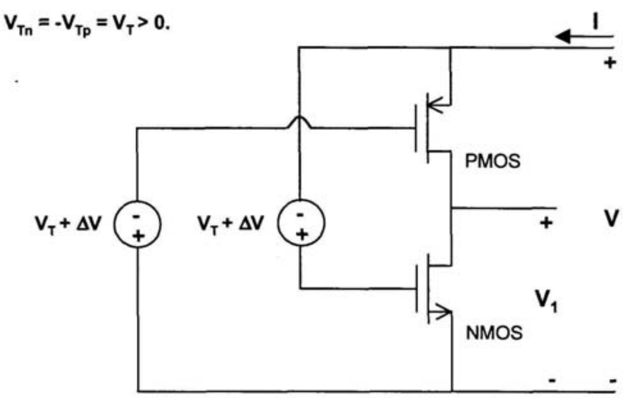

a) For V > 0, what is V_1 as a function of V?

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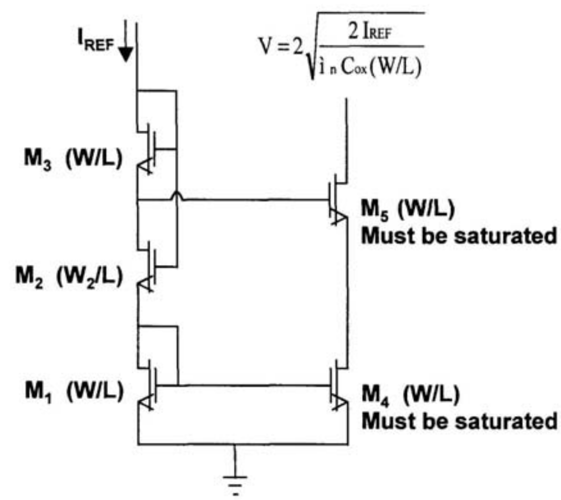
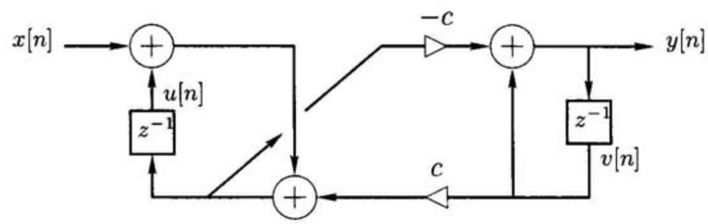


EE Quals Problem
January 13-17, 2003
Julius Smith

Consider the system diagram below:



Assuming the system has an oscillating impulse response, find the frequency of oscillation as a function of coefficient c.

X-Sender: olav@loki.stanford.edu (Unverified)
Date: Fri, 31 Jan 2003 14:58:20 -0800
To: Diane Shankle <shankle@ee.Stanford.EDU>
From: Olav Solgaard <solgaard@Stanford.EDU>
Subject: Re: Quals Question 2003

Content-Type: text/html; charset="us-ascii"
X-MIME-Autoconverted: from 8bit to quoted-printable by leland3.Stanford.EDU id h0VMwQc23340

Diane:

Here is my questions and answers.

olav

1. What is this?
A retroreflector.
2. What is its function?
It reflects light back in the direction from which it came.
3. How does it do that? Show how a 2-D retroreflector works.
Light incident on a 90-degree corner in 2-D will be reflected back in the direction it came. This can be seen by using the law of reflection on the two surfaces making up the corner.
4. How can that principle be extended to 3-D?
Consider the projection of the incoming light beam on two planes each orthogonal to one of the planes of the corner and to each other. In each of these planes, the projection will be retroreflected, which means that that the ray must be retroreflected in 3-D.
5. Retroreflectors are sometimes referred to as cat eyes. Why?
Cats eyes act as retroreflectors. An eye or a sphere can act as a retroreflector if the light that is incident on it is focused on the far side of the eye or sphere such that it has normal incidence on the surface where it is focused. Under these conditions, which are satisfied in good eyes and spheres of index ~1.5, some light will be retroreflected.
6. Cats and many other nocturnal animals have a reflecting layer right behind their retina that substantially increases the amount retroreflected light. Why?
The amount of retroreflection depends on the reflections from the far side of the eye. This is high in nocturnal hunters who have developed a reflecting layer beneath their retinas to double the amount of absorbed light.
7. How accurate must the angles of a corner cube be?
The diffraction angle is roughly lambda/(beam radius), so the angles of the corner should deviate from 90 degrees by less than substantially less than this value.
8. What would I observe if I was to send a spatially coherent light beam of radius 1cm onto this retroreflector?
The beam will cover several corner-cube reflectors, and the reflected light will form a diffraction pattern much like a grating. The total area of the pattern will be determined by the diffraction angle from the individual corner-cubes, while the size of each maximum will be determined by the diffraction angle of the illuminated spot.

9. How can you make a light modulator using a corner cube?

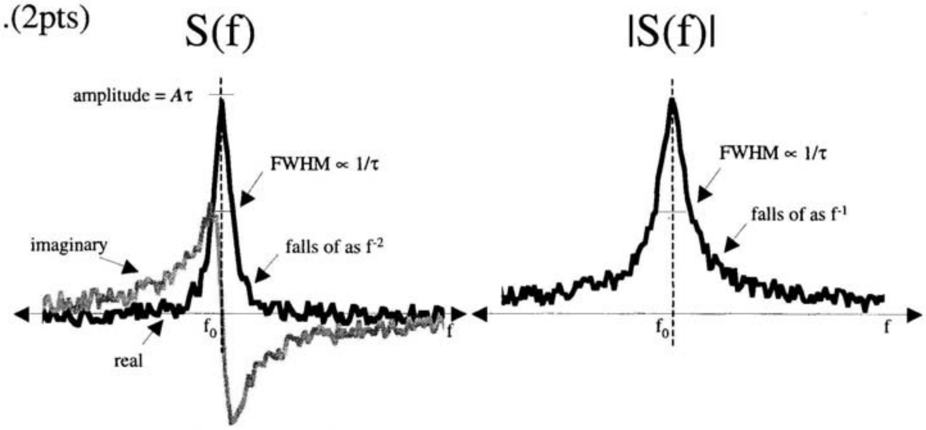
Any reasonable answer is accepted.

At 10:23 AM 1/31/2003 -0800, you wrote:

> Quals Question 2003
>
> Please send a copy of your Quals Question or you can email the question.
> Thanks,
> Diane Shankle
> Packard 165
> MC:9505
> (650) 723-3194
> Fax:(650) 723-1882

Solutions

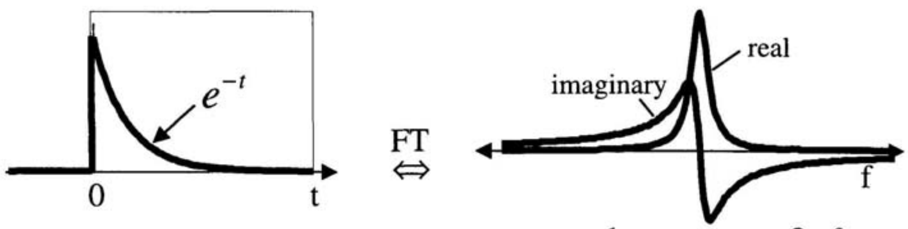
1.(2pts)



2. (2pts) The additive noise term in S(f) is zero mean complex WGN. While the noise in |S(f)| is white, it is, in general, not Gaussian. The noise is well approximated as zero mean WGN where the signal component is large, Rayleigh distributed where the signal is small, and obeys Rician statistics for intermediate values.

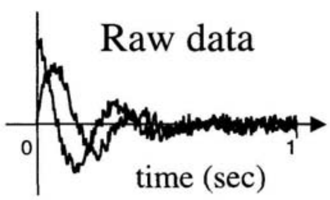
3. (4 pts) Algorithms #1and #2 yield unbiased estimates of $\boldsymbol{A}$, however the estimator variances are quite high since information available from s(t), $t \neq 0$, is ignored. Note: the var($\hat{A}_2$) is sqrt(2) less than var($\hat{A}_1$), and both algorithms yields estimates independent of uncertainties in f_0 and τ. Algorithm #3 should yield a better estimate, depending on the value of M. Choosing M involves tradeoffs among estimator bias, variance, and robustness to modeling errors. If M is too small, the algorithm will underestimate $\boldsymbol{A}$. If M is too big, the estimator will be unnecessarily noisy (note: if M is large, $\hat{A}_3$ approaches $\hat{A}_2$). Given that the FWHM of the spectral peak is $1/\pi\tau$, the best choice of M would be around this width with an additional margin added to account for the uncertainties in f_0 and τ.

This problem concerns the following Fourier Transform pair:



$$\frac{1}{1+(2\pi f)^2} - j\frac{2\pi f}{1+(2\pi f)^2}$$

Consider the following measurement:



Signal model

$$s(t) = Ae^{-t/\tau}e^{j2\pi f_o t} + n(t)$$

where A, τ, and f_0 are real constants, and $n(t)$ is complex zero mean white Gaussian noise

Let S(f) be the Fourier transform of s(t).

1. Sketch S(f) and |S(f)|.

2. What are the noise statistics of S(f) and |S(f)|?

Solutions (cont.)

4. (2pts) There are a variety of choices for improved estimators. For example, $\hat{A}_3$ ignores all information contained in the imaginary part of S(f). Can you come up with a modified version of $\hat{A}_3$ that uses *all* available information? Another natural choice is to select nominal values for τ and f_0 , say τ' and f_0', then compute the linear least squares estimate of $\boldsymbol{A}$.

$$\hat{A}_3 = \frac{2}{\tau'} \int s(t) e^{-t/\tau'} e^{-j2\pi f_0' t} dt$$

A key question is how well the estimator performs when τ' and/or f_0' are inaccurate.

The optimum least squares estimate for $\boldsymbol{A}$ involves first estimating the nonlinear parameters τ and f_0, then solving the resulting linear least squares solution. A difficulty that may arise is that the estimation of the nonlinear parameters will be noise sensitive, and this approach may also not be robust to signal modeling errors (e.g. Gaussian instead of exponential signal decay, etc).

X-Sender: bvr@bvr.pobox.stanford.edu (Unverified)
Date: Fri, 31 Jan 2003 16:47:18 -0800
To: Diane Shankle <shankle@ee.Stanford.EDU>
From: Benjamin Van Roy <bvr@Stanford.EDU>
Subject: Re: Quals Question 2003

Hi Diane,

Here's my question.

Best wishes,

Ben

Suppose that when busses arrive, they arrive at multiples of five minutes. Suppose a bus arrives with probability p at any of these times, independently of what's happened in the past. What is the expected amount of time you have to wait if you arrive just after t=0? What is the average time between consecutive busses? If you arrive just after t=0, what is the expected amount of time between the most recent bus, which you missed, and the next bus to arrive? Why is this different from your previous answer? If you have a watch with only a minute hand and no second hand, and you know nothing about the distribution of inter-arrival times, how would you estimate the average inter-arrival time? How does this quantity relate to how long you'd usually have to wait for a bus?

At 10:23 AM 1/31/2003 -0800, you wrote:

> Quals Question 2003
>
> Please send a copy of your Quals Question or you can email the question.
> Thanks,
> Diane Shankle
> Packard 165
> MC:9505
> (650) 723-3194
> Fax:(650) 723-1882

$\frac{5}{p}$ $\frac{5}{p} + \frac{5}{p}\cdot\left(\frac{1}{p} - 1\right) = \frac{10}{p} - 5$

2002-2003 EE Ph.D. Qualifying Exam
Question area: *Electronic Devices*
Examiner: *Jelena Vuckovic*

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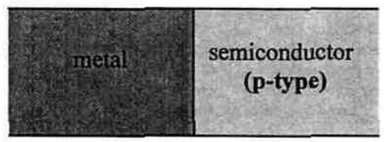
1. <u>Metal-semiconductor (MS) junction</u> *(3 pt)*

A junction is formed directly between a metal and a p-type semiconductor, as shown in Fig. 1a. The workfunction of semiconductor can either be smaller than the workfunction of metal ($\Phi_m > \Phi_s$, Fig. 1b), or larger than it ($\Phi_m < \Phi_s$, Fig. 1c).

- What is this structure used for? Discuss both the cases of $\Phi_m > \Phi_s$ and $\Phi_m < \Phi_s$.
- Choose one of these two cases ($\Phi_m > \Phi_s$ or $\Phi_m < \Phi_s$) and sketch the equilibrium band diagram after the junction is formed.



(a)

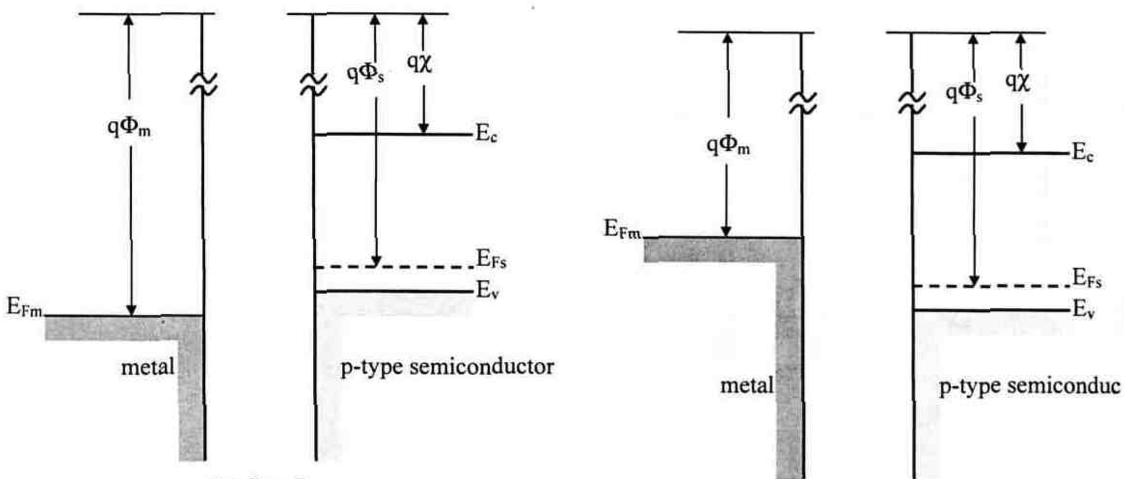


Fig. 1 (a) A junction between a metal and a p-type semiconductor.
Energy band diagrams of the metal and the semiconductor before the junction is formed for **(b)** $\Phi_m > \Phi_s$; **(c)** $\Phi_m < \Phi_s$.

2. Metal-oxide-semiconductor (MOS) capacitor *(2pt)*

The ideal MOS capacitor high-frequency capacitance-voltage (C-V) relation and the band diagrams corresponding to various bias conditions are shown in Fig. 2.

- Identify the band diagram (1-4) corresponding to the inversion condition?
- Same for the accumulation.
- Same for the depletion.
- Identify the bias points (a-d) corresponding to inversion and to accumulation.

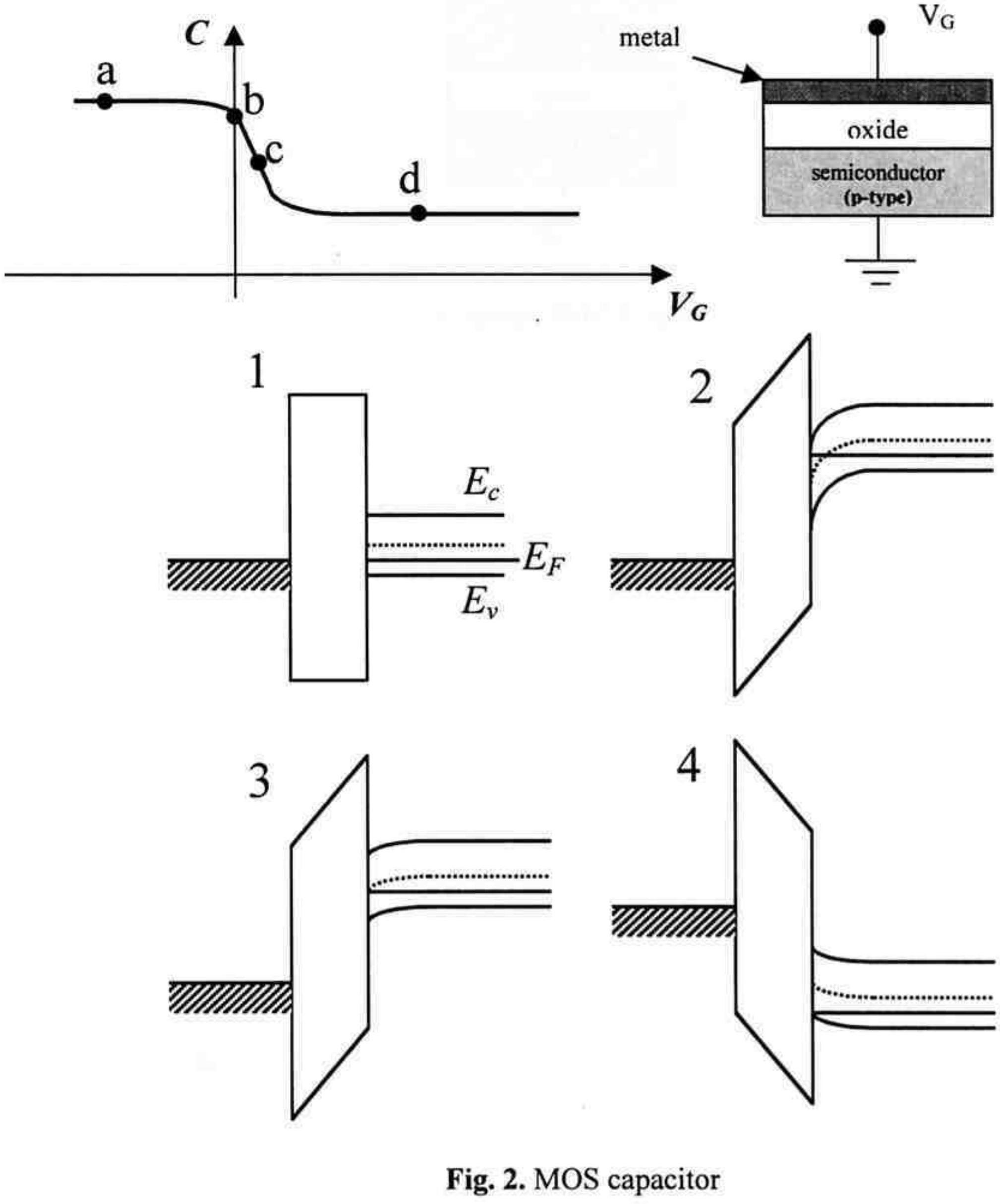


Fig. 2. MOS capacitor

3. <u>SOS capacitor</u> *(1 pt)*

What happens to the high-frequency capacitance-voltage (C-V) relation from Fig. 2 when you replace the metallic gate in a standard MOS capacitor with semiconductor, as shown in Fig. 3? Assume that the semiconductors on both sides of the oxide are identical and composed of the same p-type non-degenerate silicon used in the MOS structure from Fig. 2. Sketch the expected shape for the high frequency C-V relation of this SOS capacitor.

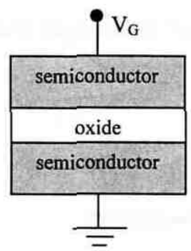


Fig. 3. SOS capacitor

4. <u>BJT as a two-terminal device</u> *(2 pt)*

What happens to a bipolar-junction (BJT) transistor when you connect its emitter (E) and collector (C) terminals together, as illustrated in Fig. 4?
Starting from the simple model of a BJT transistor, derive the I-V relationship of this two-terminal device.

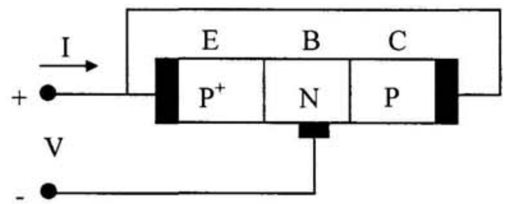


Fig. 4. BJT as a two-terminal device

5. **Nonuniform carrier generation** *(2pt)*

n-type Si sample of the length L is illuminated. The generation rate of electron-hole pairs is dependent on position as $G(x)=G_0e^{-\alpha x}$ (the generation rate is expressed in the units of [electron-hole pairs/(m^3s)]). Assume that the surface recombination at x=0 and x=L is very strong (i.e., the minority carrier concentration at these surfaces approaches its equilibrium value).

- Can you roughly sketch the excess carrier concentration $\Delta p(x)$ that you would expect as a solution of this problem?
- Write down the differential equation and boundary conditions (at x=0 and x=L) needed to solve the excess carrier concentration $\Delta p_n(x)$ within the sample.

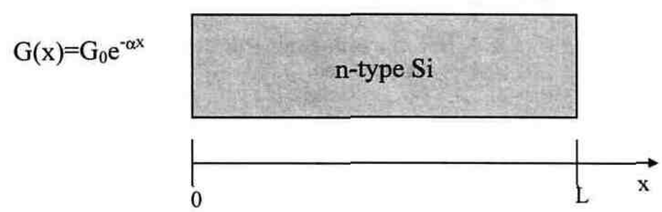


Fig. 5. Position dependent generation rate

What happens to a bipolar-junction (BJT) transistor when you connect its emitter (E) and collector (C) terminals together, as illustrated in Fig. 4?
Starting from the simple model of a BJT transistor, derive the I-V relationship of this two-terminal device.

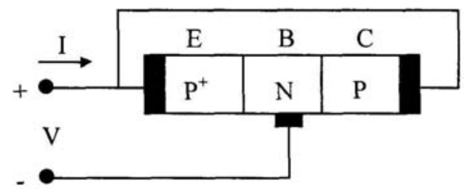


Fig. 4. BJT as a two-terminal device

5. <u>Nonuniform carrier generation</u> *(2pt)*

X-Sender: sxwang@sxwang.pobox.stanford.edu (Unverified)
Date: Tue, 04 Feb 2003 16:59:17 -0800
To: Diane Shankle <shankle@ee.Stanford.EDU>
From: Shan Wang <sxwang@ee.Stanford.EDU>
Subject: Re: Quals Question 2003

Hi Diane,
My question is as follows. -Shan Wang

2003 EE Quals Question by Shan Wang
Consider two metal plates in parallel. Between the two plates are a uniform air gap and a dielectric material with a known dielectric constant. If
an voltage U is applied to the two metal plates, derive an expression for the force between the two plates.

At 10:23 AM 1/31/2003 -0800, you wrote:

> Quals Question 2003
>
> Please send a copy of your Quals Question or you can email the question.
> Thanks,
> Diane Shankle
> Packard 165
> MC:9505
> (650) 723-3194
> Fax:(650) 723-1882

****

Shan X. Wang
Associate Professor of Materials Science & Engineering
and of Electrical Engineering
Director, Stanford Center for Research on Information Storage Materials (CRISM)

Email: sxwang@ee.stanford.edu
Phone: 650-723-8671
Fax: 650-736-1984
http://crism.stanford.edu/~sxwang
http://mse.stanford.edu/faculty/wang.html
Mail address:
Geballe Laboratory for Advanced Materials
McCullough Building, Room 351
476 Lomita Mall
Stanford University
Stanford, CA 94305-4045
Administrative Associate:
Chuck Rasmussen
McCullough, Rm. 355
phone number: 723-0698
chuckr1@stanford.edu.

```
s1 = GetNext(S1)
while s1 <> DONE do
  Reset(S2)
  s2 = GetNext(S2)
  while s2 <> DONE do
    if s2 = s1 then return(s1)
    s2 = GetNext(S2)
  s1 = GetNext(S1)
return(DONE)
```

2003 Qualifying Exam
Simon Wong

Construct a differential amplifier out of the circuit shown; define the input and output terminals; complete the necessary connections.

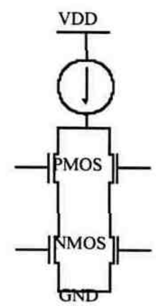
Solution:

Inputs : PMOS gates

Outputs : PMOS and NMOS drain(s) (differential or single-ended output depending on load configuration)

Possible NMOS load configurations : diode connected; gates DC biased, current mirror

Discuss the tradeoffs of various configurations.



2003 EE Qualifying Examination
Yoshi Yamamoto

1. Explain I-V characteristics of a pn junction and define a diode's differential resistance R.

2. When $R \gg R_S$ (R_S : source resistance), explain the current noise at three bias points: $V \gg V_T$, $|V| \ll V_T$ and $V \ll -V_T$, where $V_T = k_B T/q$.

3. When $R \ll R_S$, how is the above result modified?