OFFICE MEMORANDUM ◇ STAR LABORATORY

February 3, 1994

To: Gene Franklin/Diane Shankle

From: Tony Fraser-Smith

Subject: Ph.D. Quals Question, 1994

Electromagnetics

$\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$

$v_{ph} = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}$

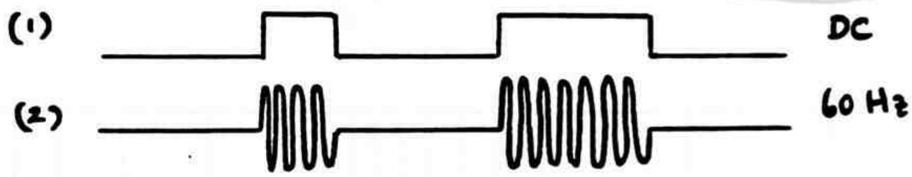
$v_g = \frac{d\omega}{d\beta} = \sqrt{\frac{8\omega}{\mu\sigma}}$

Following a brief review of the student's previous work on the propagation of electromagnetic fields in conducting media at low frequencies, in which it is established that both the attenuation and velocity (actually both the phase and group velocities) increase with increasing frequency, the following question is asked:

Question: You have gone down a deep mine and an accident has occurred in which a large amount of the overlying rock has filled up the elevator shaft and cut the telephone link with the surface. You want to send some kind of signal to the surface to let the rescuers know you are still alive. You look around and find (1) a considerable length of telephone wire strung along the side of the tunnel in which you are located, (2) a large battery (powering an emergency light), and (3) a light bulb that is still lit, indicating that some 60 Hz power is still available. What is the best way to send a signal to the surface with this equipment?

Answer: The wire first has to be arranged into some kind of antenna. Obviously doubling the wire back on itself and then connecting it to either the battery or the 60 Hz power will be ineffective, but a loop antenna or a straight dipole antenna configuration (the latter preferably with earthed ends) should be adequate.

The two ends of the wire can then be connected to either the battery or the 60 Hz supply. The student is advised to try generating a dot/dash pattern as is used in Morse code:



Concentrating first on the frequency content of the "DC" and "60-Hz" dot/dash patterns, we notice that the DC dot converts to a $\boxed{\text{sinc}}$ pattern centered on a frequency of 0 Hz in the frequency domain, whereas the 60-Hz dot converts to an equivalent sinc pattern centered on 60 Hz. Knowing that the attenuation increases with frequency, and not knowing the depth, or the conductivity, of the conducting material above the tunnel, we decide that it is safer to work with the DC dot/dash pattern.

Concentrating now on the DC dot/dash pattern, we first notice that the sharp edges are associated with the highest frequencies, which will be attenuated most rapidly. The dot and the dash will be converted to rounded pulses as they propagate upwards toward the earth's surface. In addition, since the dash is longer and thus has lower frequencies associated with it, it will probably reach the surface with a somewhat larger amplitude than the dot (assuming the dot and dash initial amplitudes are equal). $\boxed{\text{Dispersion}}$ also needs to be taken into account. The dot and the dash share some frequencies, which will reach the earth's surface at the same time. However, the dash has a lower average frequency content than the dot, so its shape will become distorted in such a way that the dot, which is assumed to be following the dash, will tend to catch up with it as time progresses. To keep the dot and dash clearly separate, their spacing needs to be increased.

OFFICE MEMORANDUM ◇ STAR LABORATORY

February 23, 1995

To: Diane Shankle

From: Tony Fraser-Smith   Electromagnetics

Subject: Ph.D. Quals Question, 1995

Question: Suppose an earthquake occurs in the earth at a depth of d meters (Figure 1). Although we have no idea what kind of electromagnetic signals, if any, are generated in the earth by earthquakes, let us assume that they do produce electromagnetic fields and that these fields have the same amplitude at all frequencies. Given the electrical conductivities (σ) shown in the figure, and given also that the skin depth δ at 1 Hz is 1.6 km for $\sigma = 0.1$ S/m, what frequencies are likely to be observed on the surface for earthquakes in California? Is there any delay? Are there any ways by which the seismic changes in the earth associated with earthquakes might generate electromagnetic signals?

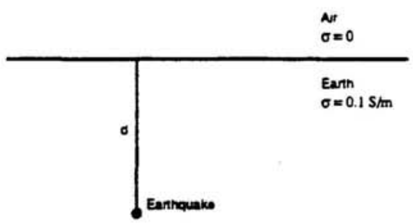


Figure 1. Earthquakes in California typically occur at depths (d, in meters) of around 10 km

Answer: After a brief discussion of good conductors and poor conductors, leading to the conclusion that the above problem must treat the earth as a good conductor ($\sigma/\omega\epsilon \gg 1$, where ω is the angular frequency and ϵ is the permittivity), the student must either remember or derive an expression for the attenuation of electromagnetic fields in a conducting medium in terms of the skin depth, i.e., $\delta = [2/(\omega\mu\sigma)]^{1/2}$, where μ is the permeability. In addition, the student should demonstrate some knowledge of how (1) the wave propagates with phase velocity $v = \omega/\beta = \omega\delta$, (2) it has a wavelength $\lambda = 2\pi\delta$, and (3) it is exponentially attenuated with attenuation constant $\alpha = 1/\delta$. Given the preceding information, it is easy to show that the phase velocity has a $\omega^{1/2}$ frequency dependence (*dispersion*!), meaning that it propagates more slowly as the frequency gets smaller.

With the information given in the problem, em signals with frequencies above 1 Hz will be severely attenuated as they propagate a distance of 10 km to the surface. On the other hand, for $f = 0.01$ Hz, it is easy to derive a skin depth of 16 km from the information given, showing that em signals will not be severely attenuated for frequencies less than about 0.01 Hz. Thus measurements at frequencies less than about 0.1 Hz appear most desirable if em signals from earthquakes are to be detected (more specifically, the frequency for which $\delta = 10$ km is close to 0.3 Hz).

Consideration of the phase (and group) velocity indicate delays of about 1 sec for the signals to reach the surface, which are not likely to be significant. Pressure changes in the earth associated with the earthquake seismic waves might produce electric charges through the piezoelectric effect, and the charge distributions might radiate em waves (very speculative). If the charge is on the surface the em waves will not be heavily attenuated and high frequencies may be observed.

$k \approx \beta + i\alpha$. $\delta = \frac{1}{\alpha}$

$v_{ph} = \omega/\beta = \omega\delta = \sqrt{\frac{2\omega}{\mu\sigma}}$

OFFICE MEMORANDUM ◇ STAR LABORATORY

January 26, 1996

To: Diane Shankle

From: Tony Fraser-Smith

Subject: Ph.D. Quals Question, 1996

Question: Suppose an earthquake occurs in the earth at a depth of $d = 10$ km (Figure 1). Although we have no idea what kind of electromagnetic signals, if any, are generated in the earth by earthquakes, let us assume that they do produce electromagnetic fields and that these fields have the same amplitude at all frequencies (Figure 2). Given the electrical conductivities (σ) shown in the figure, and given also that the skin depth δ at 1 Hz is 1.6 km for $\sigma = 0.1$ S/m, what frequencies are likely to be observed on the surface for earthquakes in California? Plot a figure equivalent to Figure 2 for the magnetic field measured on the surface. Are there any other ways in which the signal strength at the surface can be weakened in addition to absorption in the earth?

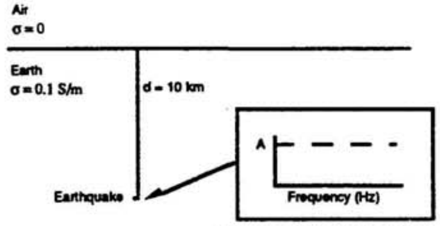


Figure 2: Frequency characteristic of the earthquake source.

Figure 1. Earthquakes in California typically occur at depths (d, in km) of around 10 km. Lacking information about the frequencies of the electromagnetic fields they may generate, we will assume they generate white noise (Figure 2; A is the amplitude of the magnetic component).

Answer: After a brief discussion of good conductors and poor conductors, leading to the conclusion that the above problem must treat the earth as a good conductor ($\sigma/\omega\epsilon \gg 1$, where ω is the angular frequency and ϵ is the permittivity), the student must either remember or derive an expression for the attenuation of electromagnetic fields in a conducting medium in terms of the skin depth, i.e. $\boxed{\delta = [2/(\omega\mu\sigma)]^{1/2},}$ where μ is the permeability. In addition, the student should demonstrate some knowledge of how (1) the wave propagates with phase velocity $v = \omega/\beta = \omega\delta$, and (2) it is exponentially attenuated with attenuation constant $\alpha = 1/\delta$. phase velocity has $\omega^{1/2}$ frequency dependence

Using the above information, the spectrum shown in Figure 3 can be derived by considering the attenuation at just a few specific frequencies around 1 Hz – say 0.01 Hz (skin depth 16 km) and 1 Hz. Obviously, the attenuation declines to zero at frequencies lower than about 0.03 Hz, at which the skin depth is just equal to 10 km, and increases to very high values at higher frequencies.

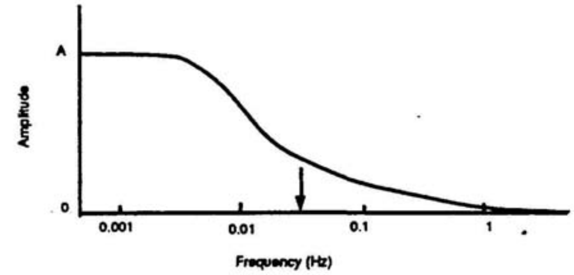


Figure 3. Spectrum (schematic) observed on the earth's surface. The arrow marks the frequency (0.03 Hz) at which the skin depth is just equal to 10 km, i.e., the depth to the source.

Students were asked if there could be any other factors in addition to absorption in the conducting medium that might reduce the strength of the earthquake signals as measured just above the earth's surface. The desired answer was some reference to reflection of the signals back down from the earth/air interface.

OFFICE MEMORANDUM ◇ STAR LABORATORY

February 6, 1997

To: Diane Shankle

From: Tony Fraser-Smith   Electromagnetics

Subject: Ph.D. Quals Question, 1997

Question: Explain how you might send an electromagnetic signal down to the center of the earth from the surface. Ignore the possibility that the inner core of the earth consists of molten iron and assume the earth is wholly a conducting material of conductivity $\sigma = 0.01$ S/m. Other possibly useful information: The radius of the earth is 6370 km, and at 1 Hz the skin depth (δ) for an electromagnetic wave propagating in a medium with a conductivity of 0.01 S/m is 5 km.

Answer: There are many different ways to answer the above question. An ideal answer would include most or all of the following: (1) A brief discussion of good conductors and poor conductors, leading to the conclusion that the above problem must treat the earth as a good conductor ($\sigma/\omega\epsilon \gg 1$, where ω is the angular frequency and ϵ is the permittivity). (2) Some technical discussion in which the student must either remember or derive an expression for the attenuation of electromagnetic fields in a good conductor in terms of the attenuation constant (α) or the skin depth (δ), where $\delta = 1/\alpha = \boxed{2/(\omega\mu\sigma)}^{1/2}$, and where μ is the permeability. (3) The student should demonstrate some knowledge of how the wave is exponentially attenuated (with attenuation constant $\alpha = 1/\delta$).

Good conduct is prerequi to use skin depth

Using the above information, the ideal answer would then include (4) a scaling of the skin depth information given at 1 Hz to derive the frequency corresponding to a skin depth of around 6370 km. The answer obtained is typically $\boxed{\text{around } 10^{-6} \text{ Hz,}}$ which is essentially dc as far as most electrical engineering students are concerned. The students are then asked if there are any electric or magnetic fields EMERGING from great depths in the earth, which would be subject to the same attenuation. This should lead into a discussion of (5) the earth's magnetic field, which is very nearly steady, but not quite, and (6) how there can be some very long term variations, which are consistent with the above frequency estimate. Finally, the students are told that there are some shorter term variations with periods around 1 Hz, and asked to explain them in the context of our discussion. The final conclusion (7) is that the variations either originate in the earth very close to the surface or they reach the earth's surface from above and thus are not subject to the same attenuation as that which takes place in the earth.

$$\frac{\sigma}{\omega\epsilon} = \frac{0.01}{1.\ 8\cancel{8}\times 10^{-12}} \qquad \epsilon_0 = 8.8\times 10^{-12}\ \mathrm{F/m}$$

OFFICE MEMORANDUM ◊ STAR LABORATORY

February 7, 1998

To: Diane Shankle

From: Tony Fraser-Smith *Electromagnetics*

Subject: Ph.D. Quals Question, 1998

Question: When we look at our AM/FM radio dials we find that the frequency range for the various AM stations is 500–1600 kHz. In addition, typical frequencies in the shortwave range, where radio signals from most of the countries in the world can be picked up, are 3 – 30 MHz. Despite the great numbers of stations operating in these frequency bands, the US operates a radio station that transmits at a center frequency of 76 Hz and Russia operates a similar station at 82 Hz. These are the only two such radio stations in the world. At these low frequencies it is very expensive to build and to operate transmitters, so why would the US and Russia bother?

Answer: There are many different ways to answer this question. An ideal answer would include most or all of the following: (1) Recognition that the US and Russia must use their low-frequency radio stations to communicate with their submarines (they have to use low frequencies for the signals to penetrate deeply enough into the sea water to reach the subs). (2) A brief discussion of how sea water is a good conductor, and that a measure of the ability of an electromagnetic wave to penetrate a good conductor is given by the skin depth δ, where $\delta = [2/(\omega\mu\sigma)]^{1/2}$, where where ω is the angular frequency, μ is the permeability, and where σ is the electrical conductivity. (3) The student should demonstrate some knowledge of how the wave is exponentially attenuated (with attenuation constant $\alpha = 1/\delta$).

At or around this stage the instructor gives the student the representative skin depth of 250 m for a 1 Hz electromagnetic wave penetrating sea water, and asks what the skin depth is for the US and Russian low-frequency radio signals.

Using the above information, the ideal answer would then include the following: (4) assuming a typical frequency of around 80 Hz, the skin depth is found to be $250/\sqrt{80} \approx 28$ m. This is probably not a very great depth for a submarine, so the 76 Hz and 82 Hz radio signals must be attenuated quite strongly as they penetrate the sea water down to the submarines. (5) Further, attenuation in the water is not the only loss mechanism – there will be substantial energy loss simply getting the signals into the sea water, due to reflection from the surface. (6) Another disadvantage would be the inability to send much information over a 76/82 Hz data link. In particular, it would not be possible to send a voice signal (in real time). The final conclusion is that the 76 and 82 Hz transmissions must work, or the US and Russia would not use them, but they must be difficult to detect at submarine depths and their ability to transfer information must be extremely limited.

Skin depth $\delta = \left(\frac{2}{\omega\mu\sigma}\right)^{1/2}$

Good Conductor approximation $\frac{\sigma}{\omega\epsilon} >> 0$

will it improve transmit range if transmitter tower is put under water?

skin depth $\delta = (\frac{2}{\omega\mu\sigma})^{1/2}$
Good conductor approximation $\frac{\sigma}{\omega\epsilon} >> 0$

Electromagnetics

OFFICE MEMORANDUM ◇ STAR LABORATORY

January 23, 1999

To: Diane Shankle

From: Tony Fraser-Smith

Subject: Ph.D. Quals Question, 1999

Penetration of Low-Frequency Electromagnetic Fields

The student is presented with a thin sheet of plastic insulation and asked about its electric and magnetic properties. He/she comes up with, or is led to, an electrical conductivity $\sigma = 0$, electrical permittivity $\epsilon = \epsilon_o$ (free space), and magnetic permeability $\mu = \mu_0$ (free space).

A strong horseshoe magnet is produced; the steel keeper is removed, and placed on the desk. The piece of plastic is placed over the keeper and the student asked if the magnet will attract the keeper through the plastic sheet. In the subsequent discussion the student is asked about the skin depth (δ) and inevitably he/she can write it down as $\delta = \sqrt{2/(\omega\mu\sigma)}$, where ω is the angular frequency, μ is the permeability, and where σ is the electrical conductivity. We discuss the significance of each of the factors in the expression for δ and its applicability. At this stage we will probably discuss the "good conductor" approximation $\sigma/\omega\epsilon \gg 0$ and how it applies at low frequencies (particularly very low frequencies). Needless to say, applicability of the good conductor approximation is a prerequisite for a material to have a skin depth.

$\omega \to 0$
$\sigma \to \epsilon$

The magnet is now brought up to the plastic sheet and it it seen that the keeper is strongly attracted through the plastic. This experimental result is discussed.

Next, a sheet of aluminum is produced and its electrical properties discussed (it is a good conductor, so σ is relatively large; $\mu \approx \mu_0$; $\epsilon \approx \epsilon_0$). Is the keeper attracted through the aluminum sheet? After the student arrives at an answer, a test is carried out and it is found that the keeper is attracted. A similar test is carried out with a sheet of copper, one of the best metallic conductors. Once again the keeper is attracted strongly through the metal. These results are then discussed in the context of the skin depth equation above. At this stage it is concluded that the low value of the frequency (in fact, $\omega \approx 0$) must be a crucial factor. For low frequency, skin depth is high

A thin sheet of steel is produced. The student is asked about its properties in the context of the skin depth equation. He/she is expected to come up with, or is led to, an electrical conductivity $\sigma \approx \sigma_{\text{copper}}, \sigma_{\text{aluminum}}$, electrical permittivity $\epsilon \approx \epsilon_o$ (free space), and magnetic permeability $\mu \gg \mu_0$ (free space). Will the magnet attract the steel keeper through the steel sheet? At this time the student is expected to fret over the fact that the high conductitivity and permeability values will reduce the skin depth but the frequencies involved are still extremely small, thus keeping the skin depth large. Given the previous results, it is hoped that the conclusion will be that the keeper is attracted. Experiment shows that the keeper is strongly attracted.

there is no magnetization kind of effect?

Finally, the student is asked if there was any means for preventing a low-frequency or DC magnetic field from penetrating through a material. High conductivity/high permeability materials might be briefly discussed, but it is hoped that reference will be made to the use of a superconducting material.

OFFICE MEMORANDUM ◇ STAR LABORATORY

January 21, 2000

To: Diane Shankle

From: Tony Fraser-Smith ACFS

Subject: Ph.D. Quals Question, January 2000

Probing the Surface of Europa

The student is told about NASA's latest discoveries on Europa, the fourth largest moon of Jupiter: (1) images acquired during recent spaceprobe flybys show a surface covered with ice (discolored ice in some places) and the layer of ice appears to be quite thick, and (2) magnetic and gravity measurements suggest very strongly that there is a liquid ocean beneath the ice Liquid oceans are very unusual in the solar system, and the existence of one on Europa suggests the possibility of life. NASA is therefore planning a mission specifically to Europa that will place a space probe in orbit around the moon (it will be called the Europa Orbiter) and which may land microprobes on the surface to learn more about the ice layer and – it is hoped – about the ocean underneath.

Question: Discuss what NASA might learn about the ice, and possibly ocean, by electromagnetic probing from a microprobe on the surface.

To get full marks for this question, the student was expected to draw attention, at some time during the discussion, to techniques other than electromagnetic probing that might be used. Obviously it would be impractical to drill through a thick layer of ice, given the logistics involved, but acoustic or seismic signals might be used to measure the thickness of the ice.

The student was then expected to consider what kind of frequencies would be best for electromagnetic probing. For this they would have their attention drawn to the discolorations in the ice and the likelihood that the ocean had salts dissolved in it – in other words, the ice is probably somewhat contaminated and "salty." With this information, they would first look to see when the ice could be considered to be a good or poor conductor. For this they could start with the quantity $\sigma/\omega\epsilon$, which is a measure of the relative magnitude of the conduction current to the displacement current in a medium (σ is the electrical conductivity, ω is the angular frequency, and ϵ is the permittivity), and discuss the "good conductor" approximation $\sigma/\omega\epsilon \gg 1$. Alternatively, they could start with the transition frequency $\omega_c = \sigma/\epsilon$, below which frequency the medium acts as a good conductor.

The Europan ice probably has a conductivity $\sigma < 4$ S/m, where 4 S/m is typical for sea water on Earth, and a permittivity of around $\epsilon = 81 \times 8.85 \times 10^{-12}$ F/m. Out of this the students should conclude that they are likely to be dealing with the good conductor approximation for frequencies less than 890 MHz. Knowing that high frequency electromagnetic waves are usually rapidly attenuated in conducting materials, the student should conclude that NASA would have to use the lowest possible frequencies, and that they would be operating with ice that acted electromagnetically as a good conductor.

In the subsequent discussion the student is either asked about or derives the skin depth (δ) and inevitably he/she can write it down as $\delta = \sqrt{2/(\omega\mu\sigma)}$, where μ is the permeability. We discuss the significance of each of the factors in the expression for δ and its applicability.

Obviously the frequencies required for penetration through several km of slightly salty ice will be on the order of $\boxed{\text{1 Hz or less.}}$

We end by discussing how we might send out a pulse of these low frequency radio waves through the ice and obtaining echoes back that could give us an indication of the depth of the ice. The important thing here is for the student to realize that there is dispersion, with the higher frequency components of the pulse travelling faster than the lower frequency components, and also $\boxed{\text{frequency-dependent absorption,}}$ with the higher frequency components suffering greater absorption. These two effects change the shape of the radiated pulse and degrade the accuracy of the measurement of ice thickness.

Is this a general property of conductor?

OFFICE MEMORANDUM ◇ STAR LABORATORY

March 20, 2001

To: Diane Shankle

From: Tony Fraser-Smith

Subject: Ph.D. Quals Question, January 2001

Determining the Depth of the Ice on Europa

As was the case last year, the student is given the following information concerning NASA's latest discoveries with respect to Europa, the fourth largest moon of Jupiter: (1) images acquired during recent spaceprobe flybys show a surface covered with ice (discolored ice in some places) and the layer of ice appears to be quite thick, and (2) magnetic and gravity measurements suggest very strongly that there is a liquid ocean beneath the ice. Liquid oceans are very unusual in the solar system, and the existence of one on Europa suggests the possibility of life. NASA has therefore placed the highest priority on a mission to Europa to see what can be learned about life in the ocean. First, however, NASA has to access the water under the ice, and even before it can reach the water it has to determine the thickness of the ice.

Question: What methods might NASA use to determine the thickness of the Europan ice? Remember that it may not be pure. Remember that it may be very thick. And remember that NASA has somewhat limited resources and there is no possibility whatsoever of a manned expedition to Europa.

Answer. To get full marks for this question, the student was expected to, first, discuss the many possible methods that might be used to determine the thickness of the ice and then, second, to discuss the most feasible appearing methods in greater detail.

Possible methods could include (1) flying a drill rig to Europa and using it to drill through the ice. But it should have been decided that this was infeasible due to the weight and size of the rig; (2) measuring the attenuation of a radio signal passing between a satellite orbiting Europa and the Earth as the satellite becomes occulted by Europa (i.e., passes behind it); (4) landing a probe on the surface of Europa and carrying out a seismic sounding experiment; (5) landing a probe on the surface and carrying out an ~~acoustic sounding experiment~~; (6) landing a probe on the surface and having it melt its way through the ice; (7) landing a probe on the surface and carrying out an electromagnteic sounding experiment; (8) landing several probes at different distances apart on the surface and transmitting various kinds of signals between them to probe the surface.

The student was expected to consider what kind of frequencies would be best for the electromagnetic probing methods. For this they would have their attention drawn to the discolorations in the ice and the likelihood that the ocean had salts dissolved in it – in other words, the ice is probably somewhat contaminated with salts.

OFFICE MEMORANDUM ◊ STAR LABORATORY

January 30, 2002

To: Diane Shankle

From: Tony Fraser-Smith

Subject: Ph.D. Quals Question, 2002

Penetration of Low-Frequency Electromagnetic Fields

The student is presented with three thin metal plates and asked about their electric and magnetic properties. Two of the plates are aluminum, with one about four times the thickness of the other. The other plate is steel; it is about the same thickness as the thinner aluminum plate. The student is very briefly asked about electrical conductivity σ, electrical permittivity ϵ ($= \epsilon_o$ in free space), and magnetic permeability μ ($= \mu_0$ in free space). Inevitably they can write the formula for skin depth $\delta = \sqrt{2/\omega\mu\sigma}$, i.e., the distance over which an electomagnetic wave of angular frequency ω will propagate through a conductor before its amplitude declines to $1/e$ of its initial value.

A strong horseshoe magnet is produced; the steel keeper is removed, and placed on the desk. The student is asked to check the metal plates to see if they are magnetic (the steel plate is) and then asked if the magnet will attract the keeper through the three different plates. In the subsequent discussion involving skin depth (δ) the student will need to recognize that ω is very small whereas σ is quite large for metals and μ is also quite large for the steel plate. Having discussed the situation and hopefully with the student having demonstrated some ability to think as an engineer, we carry out a test and find that the keeper is attracted in all cases, but the strength of attraction decreases as follows: (1) thin aluminum plate, strongest; (2) thick aluminum plate; (3) steel plate (weakest). This is exactly what would be expected if the skin depth was a major factor in the attractive force.

The student is now asked if there is any other reason for the strength of attraction between the magnet and its keeper being weaker when the thicker aluminum sheet is used. At this stage the student should realize that the distance between the magnet and the keeper is greater for the thicker sheet and that as a result the attraction could be weaker for that reason alone. A this time he/she might question the skin depth argument, but the situation can be rescued by the student comparing the attractive forces when the thin aluminum and the steel plates are used. Since they are the same thickness the different compositions as clearly an important factor.

Finally, the magnet is moved around on one side of the thin aluminum plate and it is seen how the keeper tracks the motion on the other side of the plate. The point here is that there has to be a component of attractive force parallel to the surface of the plate for the keeper to move and this requires the magnet to be offset relative to the keeper.

Monday, March 10, 2003

To: Diane Shankle

From: Tony Fraser-Smith

Subject: Ph.D. Quals Question, 2003

Impact-Triggered Flashing-Light Device

This question was prompted by a company recruiting open house at Stanford. One of the more technically-oriented companies handed out a clear plastic ball with the company's logo on its outside and a spherical plastic insert at its center that would flash for about 30 s (at about 10-20 flashes/s) after the ball was bounced on the ground. One of these balls was taken apart and the plastic insert removed. The insert was then split in half and the electronic "innards" removed. The figure below shows the two halves of the plastic insert and the electronic part. A pen shows the scale. The students had the ball and its operation described, after which they were shown the parts of the plastic insert exactly as they appear in the figure and asked how the electronic part worked. They were allowed to fiddle around with the parts shown as much as they liked, but not allowed to dismember the electronic part (since it was required for other exams).

The answer to this question usually took place in two stages: (1) the various components of the electronic part were identified, and then (2) a hypothesis for how the electronics worked was formulated. For this latter part the students were asked to write down a circuit and to base their hypothesis entirely on what they could identify in the electronic component.

Points for (1) were awarded for identifying two LEDs, one on each side of the green circuit board (left and right in the figure), a small cylindrical spring with a fixed metal rod sticking up along its axis (between the two LED's in the figure), and two small silver oxide button batteries connected in series (underneath the circuit board in the figure). There was sometimes discussion of the black dot that can be seen on the front right of the circuit board, and sometimes it was asserted – probably correctly – that there had to be a "chip" hidden underneath it, but once again the students were instructed to come up with a hypothesis for how that electronics worked based on the components they could see. A point was specifically awarded for recognition that there had to be some source of power for the device to work, since, rather incredibly, not all students could identify the batteries.

Points for (2) were awarded for sensible circuits and hypotheses, which left room for some interesting discussions. The following outline several of these discussions:

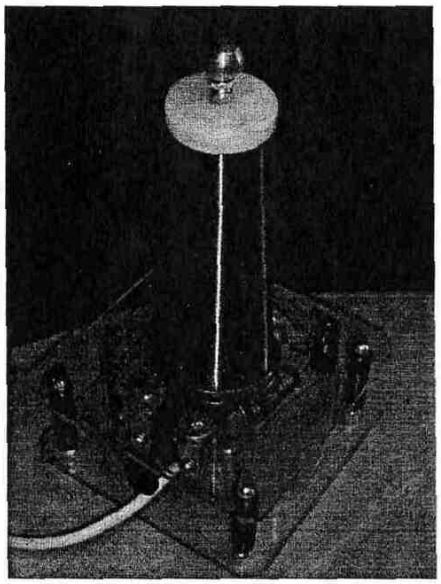
1. Quite commonly, there was discussion of the circuit arrangement of the two LEDs: were they connected in parallel or series? Here it was helpful to know that an LED requires about 0.8 V to light up, and the silver oxide batteries that appear to power the electronic unit are connected in series and provide about 3 V. These facts led to the conclusion that the LEDs were most probably connected in series rather than in parallel.
2. Two LEDs connected in parallel with a battery power supply could not have a simpler circuit. However, what triggers the flashing light? Obviously some kind of triggering mechanism (or accelerometer) is required and equally obviously this must be the spring component. Most students who identified this component as the trigger argued that it would be displaced as the ball bounced on the ground and that it would touch the vertical metal rod along its axis, thus providing the electrical contact to initiate flashing of the LEDs. But what maintains the flashing? At this point some students argued plausibly that the spring would keep on vibrating and touching the metal rod, thus keeping the flashing going until the vibration of the spring became damped.
3. Some students argued that the spring was actually an inductor and the flashing of the LEDs corresponded to the electrical oscillation frequency of the spring's (or coil's) inductance in parallel with the stray capacitance of the circuit. This led to further consideration of the appropriate angular resonance frequency of an LC circuit: (LC)^-0.5. Here reasonable assumptions for the values of L and C led to frequencies in the MHz or even GHz range. At this stage most students would dismiss this particular explanation for the flashing.

In general, identification of the various components comprising the electronic component of the flashing ball and reasonable hypotheses for the way it worked (along with intelligent comments on why there were two LEDs and two batteries, instead of one of each) would lead to a perfect score.

Ph.D. Quals Question
January 2004
A.C. Fraser-Smith
Space, Telecommunications and Radioscience Laboratory

Compact Tesla Coil

The figure below shows the compact Tesla Coil that was shown to each student. The white cord connects the coil to a 110 V power outlet and when the black knurled knob next to it is screwed in it moves two electrical contacts closer together and sparking takes place between them. At this time, if a coin held firmly in the fingers is brought toward the round aluminum ball at the top of the coil, sparks up to 1–2 inches long can be drawn out of the sphere. Obviously there is a very high voltage being generated on the sphere; this observation leads to the basic question asked of the students: what is the electrical engineering basis for the generation of this high voltage, given that the source voltage is 110 V?



Two hints were given: First, the students were told to remember Faraday's law of electromagnetic induction and then, second, an AM radio was turned on while the coil was sparking and it was shown the coil was generating radio interference across the entire AM band (i.e., covering many hundreds of kHz).

The answer to this question usually involved two steps: (1) the various components of the electronic part were identified, and then (2) a hypothesis for how the electronics worked was formulated.

Points for (1) were awarded for identifying the many turns making up the red colored part of Tesla coil as the secondary of a **transformer,** with the two thick black coils at its base making up the primary. Obviously the much greater number of turns in the secondary will lead to higher voltages. During this inspection part of the test the students either noticed or had their attention drawn to the fact that one of the ends of the secondary coil was connected to the aluminum ball on the top and the other end, at the bottom, was connected to the green-colored socket on the right in the picture.; there was no direct electrical connection to any other part of the circuitry. At this time the students either noticed or had their attention drawn to the fact that the two ends of the primary disappeared into the circuitry containing the spark gap that was adjusted by means of the black knurled knob. Some students noticed that the wire comprising the secondary was much thinner than the wire for the primary, suggesting that the primary carried higher current.

Points for (2) were awarded for sensible explanations for how the coil works based on Faraday's law. These explanations most succinctly made use of Faraday's law in the form:

$$EMF = -\,\partial N/\partial t$$

where EMF indicates the induced emf, N is the magnetic flux threading the circuit, and t is the time. The transformer action discussed above is one part of this explanation, and it involves N. Another part, however, involves the $\partial/\partial t$ term in the above equation. The noise produced by the Tesla coil in the AM radio radio indicates that high frequencies are involved, and high frequencies imply high $\partial/\partial t$, which in turn implies large emfs according to the Faraday equation. How are these high frequencies produced? This is where the students were expected to home in on the very noisy spark gap. The sparks were obviously very short lived and thus, eureka (for a Stanford EE student): the Fourier transform of an impulse is a function covering a wide range of frequencies in the frequency domain and the range of frequencies becomes larger as duration of the impulse gets smaller, thus the short-lived sparks give a big $\partial/\partial t$ which helps produce the high voltages in the Tesla coil.

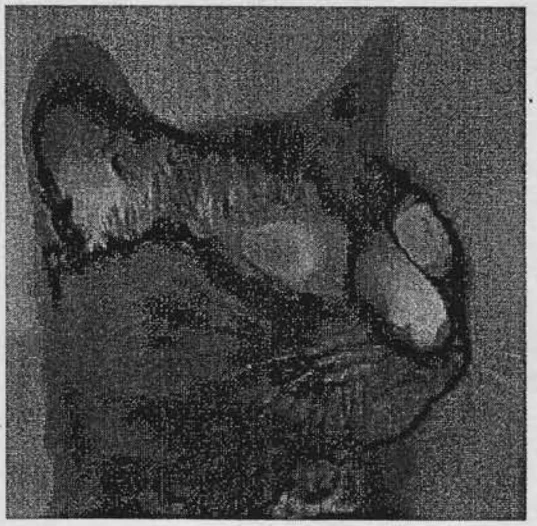
To put this question into perspective, the earliest demonstrations of radio waves (e.g., by their discoverer Heinrich Hertz) made use of spark gaps to generate the waves, and the earliest commercial transmitters were all mostly based on spark gap technology.

Ph.D. Quals Question
January 2005
A.C. Fraser-Smith
Space, Telecommunications and Radioscience Laboratory

BLACKBODY RADIATION

To start, the figure below was shown to the students and attention drawn to its label: "A Cat in Infrared." They were asked, "Do you have any comment about the label on this figure?"

A Cat in Infrared



At this stage, most students began discussing blackbody radiation, which is good, but once again their attention was drawn to the label on the figure and asked if they could comment on it. The expected answer was "Oh, we cannot see infrared. The picture must have been prepared by using some process that converts the infrared radiation emitted by a cat to some visible representation." This usually led to a brief discussion of possible conversion processes and how the cat's ears and eyes in the above figure are hot and its nose cold. The use of night vision 'scopes was sometimes discussed at this stage (good). After this introductory start the students were asked specific questions about blackbody radiation. For example, "why is the term 'blackbody' used?" Importantly, students were expected to know that blackbody radiation depends only on the temperature of the source.

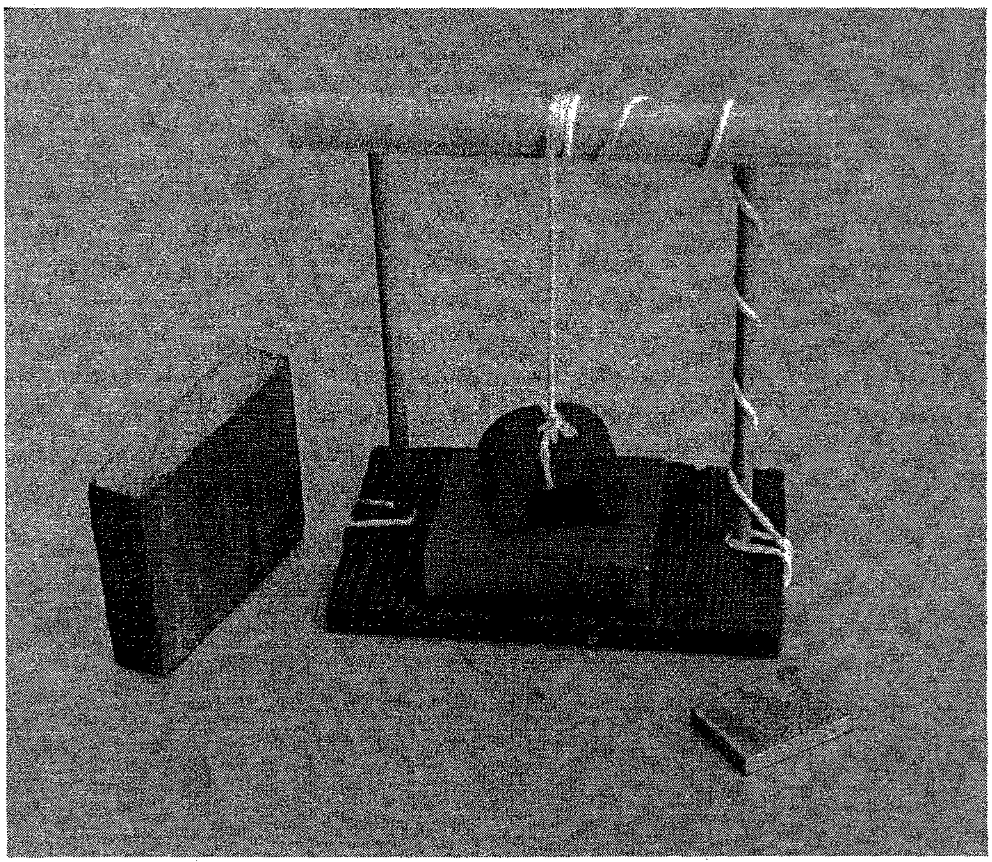
A schematic chip layout was now shown to the students and they were asked how blackbody radiation might be used in an electrical engineering context to diagnose chip problems. Most knew that heating of modern microprocessors is a major problem (good), but once again they were directed to the chip layout. Here they were expected to point out that hot spots (as revealed by blackbody radiation – in this case infrared – measurements) might indicate shorting or some other form of circuit failure, and cold spots and regions might indicate an open circuit preventing current from flowing. This was all hypothetical, but logical. Some students pointed out that chip features are now of nanometer scale, whereas typical infrared wavelengths are of micrometer scale, so the diagnostics technique will lack resolution: another important point.

Finally, students were asked if they have heard about cosmic microwave radiation (CMR), which is understood to have originated shortly after our universe originated according to the "big bang" theory. It was pointed out that the gases doing the radiating would be extremely hot. Next, it was pointed out that the measured spectrum of CMR corresponded to that of a black body at a temperature of around 3 K. How could the "extremely hot" be reconciled with the measured 3 K? Students went off in a number of different directions, but many pointed out that the CMR was being emitted by gases traveling at very high speed away from the earth and that it would be subject to a doppler shift that could go some way to accounting for the low frequency of the 3 K CMR. The key points were to convert from temperature to a frequency point of view and then to realize that the CMR sources were traveling away from the observer and thus the CMR would undergo a doppler shift to lower frequencies.

Ph.D. Quals Question
January 2006
A.C. Fraser-Smith
Space, Telecommunications and Radioscience Laboratory

Faraday's Law

The figure below shows the experimental arrangement that was shown to each student. The key item is a powerful horseshoe magnet (red) that is hanging by a piece of string from a wooden suspension. When the students first see it, it has a steel "keeper" across its two ends. This is the silvery colored piece of metal sitting off to the right in the figure; the instructor or student has to really wrestle with that piece of metal to disconnect it from the magnet – thus emphasizing the strength of the magnet. The student is next asked to identify the object to the left in the figure. Many say "wood," whereupon the instructor reminds them that they are electrical engineering students and in most cases they amend their description to "insulator" or better, "poor conductor." Next, they are presented with the heavy, reddish object shown here under the magnet. Better prepared. most students will say "it is copper, a good conductor." The instructor next pokes the magnet to set it in oscillation, without either the wood or copper block beneath it, and asks the student to watch its motion, which can go on, seemingly unimpeded, for a long time. Next he places the "poor conductor" under it, without any noticeable change in the magnet's oscillations. He then asks the student to put the copper under it, whereupon it stops. The student is asked to get it oscillating again, but without much success. Quite noticeably, its motion is heavily damped. Now begins the academic part: the instructor asks the student to explain what is going on from an electrical engineering point of view, with emphasis on the basic laws involved.



Points were awarded for (1) simply working out what was going on from an EE point of view, i.e., the moving magnet was creating a time-varying magnetic field in the conductor – as well as in the insulator (wood) – which produced eddy currents whose magnetic fields opposed the motion creating them. Then (2) the wood produced a negligible effect because the eddy currents were inhibited by the lack of conductivity and were small/negligible, while the eddy currents could flow relatively freely in the copper.

Further points were awarded for (3) identification of Faraday's Law as the physical basis for the hypothesized link between time-varying magnetic field in the conductor and the induced eddy currents. This identification most succinctly made use of Faraday's law in the form:

$$EMF = -\,\partial N/\partial t$$

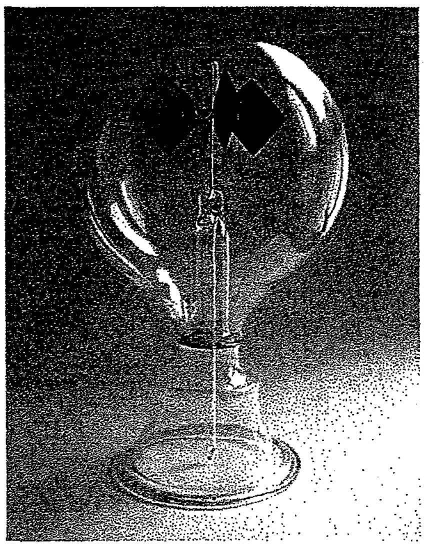
where *EMF* indicates the induced emf, N is the magnetic flux threading the circuit, and t is the time. In addition, since the pendulum motion of the magnet was most clearly opposed by the currents in the conducting material, it was expected that the student would mention (4) Lenz's Law (essentially the minus sign in the above equation). Finally, (5) in response to a query from the instructor about why the motion of the magnet was opposed and not assisted by the eddy currents, some mention of conservation of energy led to the final assignment of points for this part.

To end this brief exam, the instructor held the wooden stand down firmly on the table and asked the student to remove the copper block without touching or otherwise disturbing the magnet. To cut a long story short, this can only be done by slipping the copper block out from under the magnet at an extremely slow rate, i.e., so that the rate of change of magnetic field in the block is so slow that the induced currents are negligible. Slipping the block out quickly leads to a substantial disturbance of the magnet. At this stage the final points were awarded based on the student's recognition that the disturbance of the magnet was simply another aspect of the original problem. A student who laughed and said "you are having me on – there is no way I can avoid disturbing the magnet except by taking the copper away very slowly" got full marks for this part.

Ph.D. Quals Question
January 2007
A.C. Fraser-Smith
Space, Telecommunications and Radioscience Laboratory

Crookes Radiometer

The picture below shows the device that was placed on the table in front of each student being examined. It is usually considered merely a "conversation piece" nowadays, but when it was first invented in 1873 by a famous British experimental physicist, Sir William Crookes, it stimulated a number of scientific studies by eminent physicists, including James Clerk Maxwell, Osborne Reynolds, and Albert Einstein. This fact is drawn to the student's attention with the comment that "this kind of scientific interest probably indicates that there is more to this device than meets the eye." The students are asked if they have seen such a device before (most students have not) and they are asked if they know what it is called. The examiner prefers the name *Crookes radiometer* for it, but *light mill* and *solar engine* are also used.


Crookes Radiometer (from Wikipedia)

The most important part of the radiometer is a rotor to which four vanes are attached, each of which is blackened on one side and white on the other. The rotor is balanced on a vertical support and it is free to turn with very little friction. When a light is shone on the radiometer (or when it is placed in the Sun), the rotor and its four vanes begin to rotate, with the black surfaces moving away from the light and the white toward it. If the light is turned off the rotor soon begins to slow down and it is obvious that there is air or gas inside the clear glass bulb creating a drag on the vanes. At this stage the students are told that the air inside the bulb has been pumped out to create a partial vacuum but not a "perfect" vacuum (i.e., no gas at all).

Following this introduction, which only takes a short time, the student is asked how the device works.

The first and most obvious explanation is that the light pushes the black sides of the vanes away from the source of light – presumably because the light is absorbed on those sides but reflected from the white sides. Students considering this possibility are asked to work out the momentum imparted to the vanes by a single photon. It should immediately become apparent that a reflected photon (white side) imparts twice the momentum that is absorbed (black side). Thus the white sides should be pushed away from the light source, a conclusion clearly in conflict with observation. At this stage the student should dismiss this possible mechanism of operation.

The second explanation that should be looked into at this stage is based on the presence of gas in the bulb. Following up on the likelihood that the black sides of the vanes absorb more light than the white sides and are therefore warmer, the student here can draw upon the gas law, PV = nRT, to predict an increased PV in the gas close to the black sides of the vanes as the T of the gas increases. This increased PV should "push" the black sides away from the region of increased PV, thus causing the rotor to turn. This is not in fact the whole story, but it is good enough for some marks. The instructor will probably ask what the vanes are made of at this stage. After some period of puzzlement the student will possibly conclude that they must be made of insulating material, since materials that are good conductors of heat would soon lead to the white sides being the same temperature as the black sides and there would be no net force on the vanes.

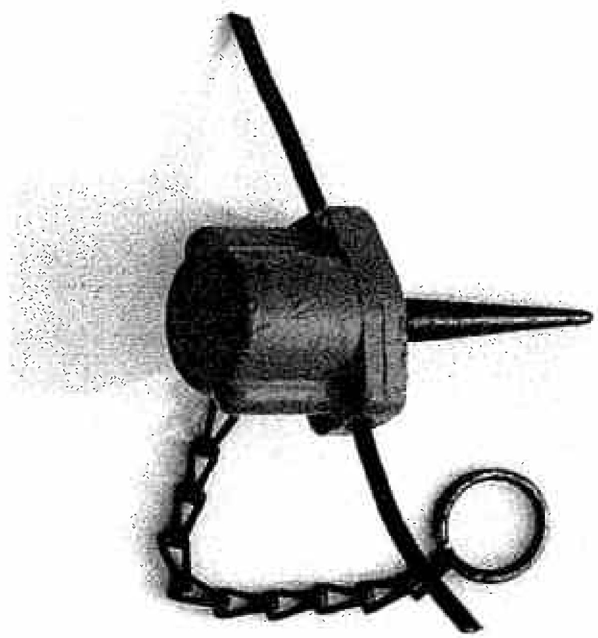
A possible third mechanism is increased electromagnetic radiation away from the warm black sides of the vanes, as compared with the cooler white sides. This can lead to lengthy discussion but it is usually dismissed on the grounds that the bulb has to contain some gas for the rotor to rotate as demonstrated and thus this third possible mechanism must be secondary to the gas heating mechanism.

Scoring for this question consisted generally of 4 points for a scientifically-valid consideration of the first mechanism, with 4 more points for the second mechanism, and 2 discretionary points for such items as the third mechanism or questions relating to the composition of the vanes and its effect on the heating of the gas.

Ph.D. Quals Question
January 2008
A.C. Fraser-Smith
Space, Telecommunications and Radioscience Laboratory

The Geophone

The picture below shows the device that was placed on the table in front of each student being examined. It is a geophone, a device that is used in large numbers by oil companies and other Earth resources companies to prospect for valuable minerals. What it does is measure Earth vibrations, i.e., it converts vibration of the ground into a voltage that can be measured by an appropriate instrument (e.g., a voltmeter) attached to the wires emerging from the device. The picture below doesn't show its scale; the ring on the chain is just large enough to hold a quarter. The student is shown the device, encouraged to shake it (there is clearly something loose inside its body) and asked how they think it works.



A Geophone

Scoring for this question consisted generally of 6 points for a scientifically-valid consideration of the way vibrations of the device are converted to a voltage, with 4 more points for a reasonable discussion of its actual frequency response as compared with what users might consider an ideal response.

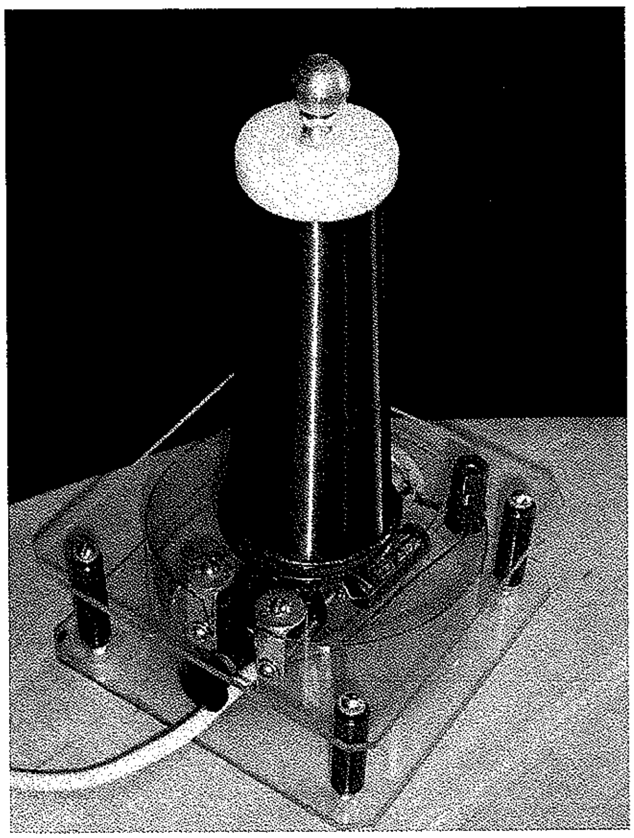
A number of students thought the spike sticking out to the right in the picture above was an antenna. This is not an unreasonable assumption but the spike is really just that and it is meant to hold the device firmly in place on the ground (implications for frequency response?). There was no penalty for making the antenna assumption. The device contains a cylindrical magnet suspended by leaf springs and free to move along the device's axis. The magnet is surrounded by a coil and its motion produces a voltage in the coil through Faraday's Law. It is an analog, not digital, device, like the vast majority of sensors in EE and it only measures motion in one direction (dimension) – in this case, in the vertical direction. We would three such devices to measure a full 3D response.

Turning now to the frequency dependence, we have a weight suspended on a spring (actually springs in the geophone case). Most students quoted an analogy with an LC circuit, which was good. Now the problem is this: such a mechanical device, or its electrical analog, has a well-defined resonance frequency and possibly not much response apart from that resonance. Obviously users of the geophone would like to have a device that has some breadth to its frequency response. Some discussion of how the frequency response of the device might be broadened was therefore appropriate at this stage. Resistance added in an LC circuit; perhaps some damping in the actual spring/magnet setup. Returning to Faraday's Law, and the fundamental aspects of the device's response to ground motions, it is important to notice that the voltage induced in the coil surrounding the magnet varies as the rate of change of magnetic field and thus the device responds preferentially to higher-frequency ground vibrations. Finally, the spike! The device needs to be closely coupled to the ground to measure the ground vibrations properly. It is not hard to imagine a situation where the device is sitting loosely on the ground, responding to low-frequency motions but not responding at all to higher frequency motions.

Ph.D. Quals Question
January 2009
A.C. Fraser-Smith
Space, Telecommunications and Radioscience Laboratory

Compact Tesla Coil

The figure below shows the compact Tesla Coil that was shown to each student. The white cord at the bottom connects the coil to a 110 V power outlet and when the black knurled knob next to it is screwed in it moves two electrical contacts closer together and sparking takes place between them. At this time, if a coin held firmly in the fingers is brought toward the round aluminum ball at the top of the coil, sparks up to 1–2 inches long can be drawn out of the sphere. Obviously there is a very high voltage being generated on the sphere; this observation leads to the first question asked of the students: (1) what is the electrical engineering basis for the generation of this high voltage, given that the source voltage is 110 V?



Two hints were given: First, the students were told, or guided, to remember Faraday's law of electromagnetic induction and then, second, an AM radio was turned on while the coil was sparking and it was shown the coil was generating radio interference across the entire AM band (i.e., covering many hundreds of kHz).

The answer to this question usually involved two steps: (i) identification of the key components of the device, and then (ii) a hypothesis for how these components generated the high voltage.

Points for (i) were awarded for identifying the many turns making up the red colored part of Tesla coil as the secondary of a **transformer,** with the two thick black coils at its base making up the primary. Obviously the much greater number of turns in the secondary will lead to higher voltages. During this inspection part of the test the students either noticed or had their attention drawn to the fact that one of the ends of the secondary coil was connected to the aluminum ball on the top and the other end, at the bottom, was connected to the green-colored socket on the right in the picture.; there was no direct electrical connection to any other part of the circuitry. At this time the students either noticed or had their attention drawn to the fact that the two ends of the primary disappeared into the circuitry containing the spark gap that was adjusted by means of the black knurled knob. Some students noticed that the wire comprising the secondary was much thinner than the wire for the primary, suggesting that the primary carried higher current.

Points for (ii) were awarded for sensible explanations for how the coil works based on Faraday's law. These explanations most succinctly made use of Faraday's law in the form:

$$EMF = -\,\partial N/\partial t$$

where *EMF* indicates the induced emf, N is the magnetic flux threading the circuit, and t is the time. The transformer action discussed above is one part of this explanation, and it involves N. Another part, however, involves the $\partial/\partial t$ term in the above equation. The noise produced by the Tesla coil in the AM radio indicates that high frequencies are involved, and high frequencies imply high $\partial/\partial t$, which in turn implies large emfs according to the Faraday equation. How are these high frequencies produced? This is where the students were expected to home in on the very noisy spark gap. The sparks were obviously very short lived and thus, eureka (for a Stanford EE student): the Fourier transform of an impulse is a function covering a wide range of frequencies in the frequency domain and the range of frequencies becomes larger as duration of the impulse gets smaller, thus the short-lived sparks give a big $\partial/\partial t$ which helps produce the high voltages in the Tesla coil.

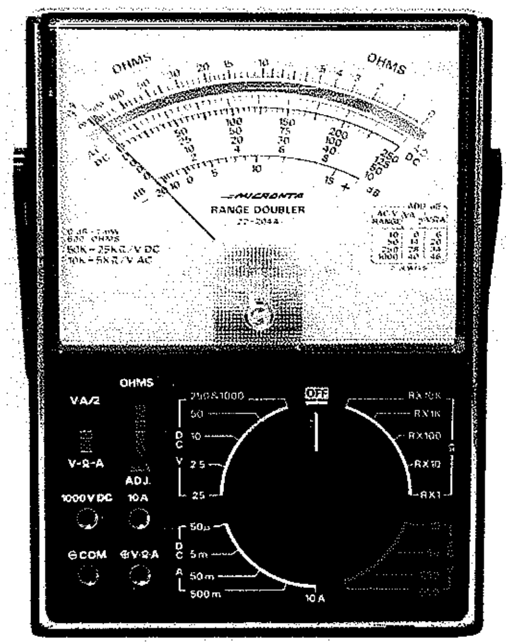
To put this question into perspective, the earliest demonstrations of radio waves (e.g., by their discoverer Heinrich Hertz) made use of spark gaps to generate the waves, and the earliest commercial transmitters were all mostly based on spark gap technology.

Finally, a small fluorescent bulb was brought near the Tesla Coil while it was in operation, whereupon it began to glow. Thus the second, minor, question: (2) why does the bulb glow when it is not even connected to the Coil? A hint was given that the bulb contained gas at low pressure along with some mercury vapor. Here it was sufficient to point out that the strong em fields being produced by the Tesla Coil could ionize the gases in the bulb, generating em radiation in the UV and visible ranges. The UV radiation would make the phosphor coating on the inside of the bulb begin to glow.

Ph.D. Quals Question
January 11-15, 2010
A.C. Fraser-Smith
Department of Electrical Engineering
Stanford University

Multimeter

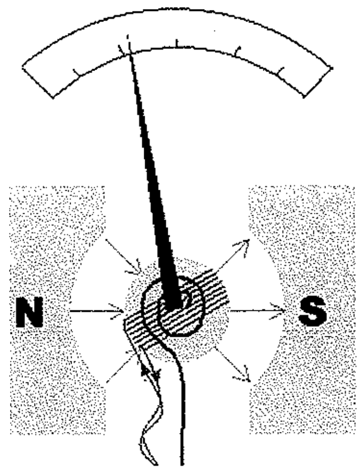
The figure below shows the analog multimeter that was shown to each student. Two leads were also on hand – sometimes inserted in the two sockets on the bottom left (in picture) – sometimes not, but terminating in two pointed probes. The multimeter was not plugged in to a power socket (it was quite clearly not intended to be plugged in) and it had no obvious power source. There was no easily removable panel on the back for a battery; in fact, the back was attached by two not very obvious screws. The students were told that the multimeter was about 40 years old and still worked perfectly, enabling its user to measure DC current, DC voltage and resistance. There were also a few switch settings for measuring AC voltage, but the student was told not to bother about AC. **First question**: Look inside the multimeter in the region where the pointer pivots and explain what electromagnetic principles and mechanical tricks are involved in making it work. One hint was given: Start with its measurement of current.



Simple analog multimeter

The answer to this question usually involved two steps: (i) identification of the key components of the device, and then, closely related to this first step, (ii) an explanation for how these components worked to give a measurement.

A number of students immediately recognized that they were dealing with a galvanometer; some even mentioned a D'Arsonval galvanometer, which was encouraging, but which did not necessarily lead to a correct explanation for how it worked! Although the complete mechanism surrounding the pivot of the pointer was not particularly easy to see, all its important features could be seen. With a little prodding from the examiner, if necessary, the student usually – but not always – ended up sketching out something approximating the mechanism shown below:



Sketch of a galvanometer mechanism. The red wires carry the current to be measured; it passes through a coil wound around the (steel) cylinder holding the pointer. The green object is a restoring spring. N and S indicate the two poles of a magnet.

Following identification of the components, the examiner looked for some discussion of the force on wires or coils carrying current in a magnetic field. He then looked for some discussion of the curved shape of the faces of the magnet, which combined with the pivoting steel (!) cylinder holding the pointer, would lead to a uniform magnetic field surrounding the coil, independent of its angular position. In other words, the torque on the coil would be dependent only on the strength of the current passing through the coil. The restraining spring would balance this torque and lead to the current reading.

The student was then given a **second question**: If the operation of the meter depended on current passing through the coil, how did it measure resistance? Around this time, as a hint, the examiner would switch the multimeter to a resistance setting and demonstrate how the pointer would move toward its zero setting (on the right side of the dial) when the two probes were touched. The knurled knob labeled "Ohms" in the figure above enabled the pointer to be moved exactly to its zero setting. At this time the better students would carefully, and sometimes not so carefully, inspect the multimeter and declare with authority that there had to be a battery inside, since some source of energy was required to move the pointer, even though there was no obvious way of inserting such battery. The examiner would confirm this fact and then again ask how resistance was measured. The answer: the preparatory step, touching the probes together and adjusting the resistance measurement to zero, established a full-scale deflection for a known internal resistance. When the unknown resistance is placed in series in the circuit the deflection is less than full scale due to the reduced current flowing and the calibrated scale can indicate the resistance.

Ph.D. Quals Question
January 11-15, 2010
A.C. Fraser-Smith
Department of Electrical Engineering
Stanford University

Multimeter

The figure below shows the analog multimeter that was shown to each student. Two leads were also on hand – sometimes inserted in the two sockets on the bottom left (in picture) – sometimes not, but terminating in two pointed probes. The multimeter was not plugged in to a power socket (it was quite clearly not intended to be plugged in) and it had no obvious power source. There was no easily removable panel on the back for a battery; in fact, the back was attached by two not very obvious screws. The students were told that the multimeter was about 40 years old and still worked perfectly, enabling its user to measure DC current, DC voltage and resistance. There were also a few switch settings for measuring AC voltage, but the student was told not to bother about AC. **First question**: Look inside the multimeter in the region where the pointer pivots and explain what electromagnetic principles and mechanical tricks are involved in making it work. One hint was given: Start with its measurement of current.

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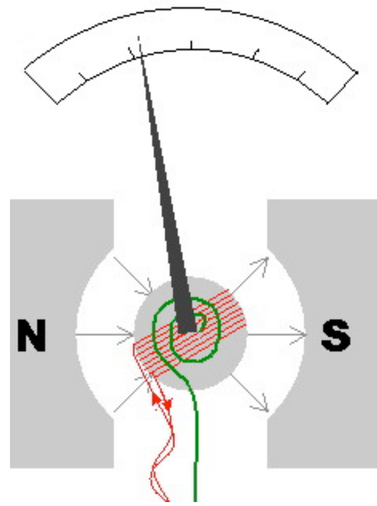
A number of students immediately recognized that they were dealing with a galvanometer; some even mentioned a D'Arsonval galvanometer, which was encouraging, but which did not necessarily lead to a correct explanation for how it worked! Although the complete mechanism surrounding the pivot of the pointer was not particularly easy to see, all its important features could be seen. With a little prodding from the examiner, if necessary, the student usually – but not always – ended up sketching out something approximating the mechanism shown below:



Sketch of a galvanometer mechanism. The red wires carry the current to be measured; it passes through a coil wound around the (steel) cylinder holding the pointer. The green object is a restoring spring. N and S indicate the two poles of a magnet.

Following identification of the components, the examiner looked for some discussion of the force on wires or coils carrying current in a magnetic field. He then looked for some discussion of the curved shape of the faces of the magnet, which combined with the pivoting steel (!) cylinder holding the pointer, would lead to a uniform magnetic field surrounding the coil, independent of its angular position. In other words, the torque on the coil would be dependent only on the strength of the current passing through the coil. The restraining spring would balance this torque and lead to the current reading.

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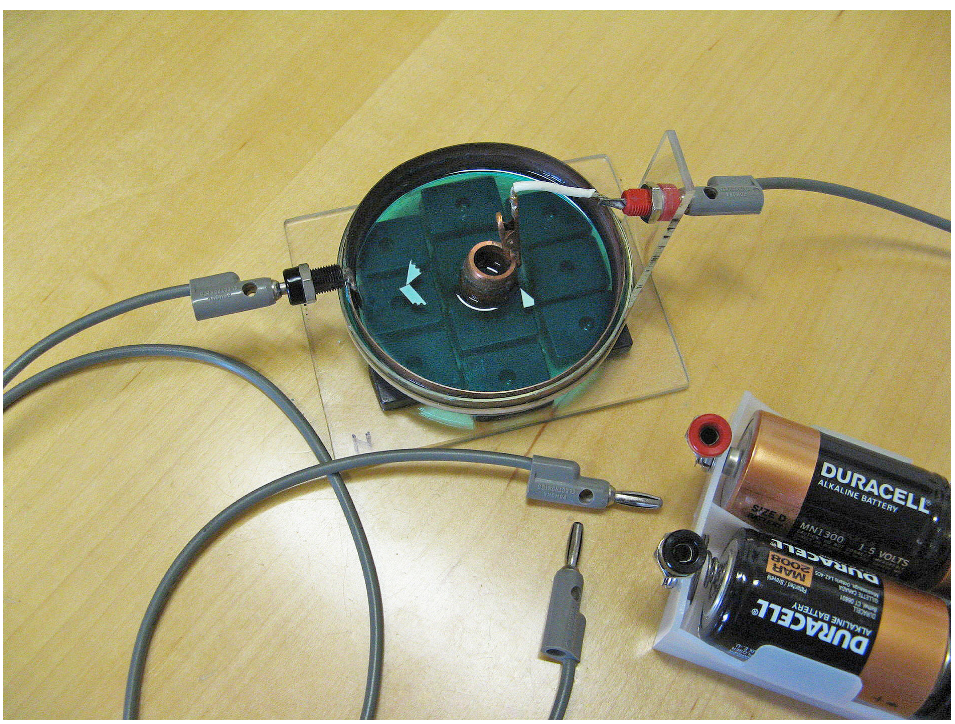
Ph.D. Quals Question
January 10-14, 2011
A.C. Fraser-Smith
Department of Electrical Engineering
Stanford University

Liquid Cyclotron

The figure below shows the liquid cyclotron that was shown to each student. It consisted of a little plastic dish (actually a petri dish) containing distilled water, "laced" with some copper sulphate to make it electrically conducting, and with a copper ring inside its outer circumference and a much smaller copper ring at its center. Both of these copper rings are immersed the copper sulphate solution and both can be separately connected to a battery pack (4 D-cell batteries, giving 6 V), part of which can be seen at the bottom right (the connections to the inner and outer copper rings can also be seen). The little dish sits on top of a number of black rectangular items with holes in their centers that are glued to an underlying piece of flat plastic sheet. The examiner assembled this demonstration equipment and, originally for his own information, marked the enigmatic "N" on the plastic sheet, which can be seen in the photo.

For the next step in the exam process the examiner connected the wires to the battery pack, thus driving a current between the copper rings, which we might now refer to as electrodes. Lo and behold, the copper sulphate solution begins to rotate. This rotation is made more obvious by some small pieces of white plastic floating on top of the solution, which can be seen in the photo and which rotate along with the fluid. It appears that the fluid rotates more quickly near the central electrode. Next, the examiner swaps the wires to the battery pack. The rotating solution slows down, stops, and reverses its direction of rotation. At this stage the examiner asks the students what is going on and states that he is prepared to answer any questions about the equipment.



Liquid cyclotron demonstration

One key question that the examiner was looking for was "what are the black rectangular items?" This generally indicated that the student was already on the right track. The answer was that they were common refrigerator magnets and they were glued down with their poles all oriented in the same direction. The "N" indicated that all the N poles were pointing upward, although that is not an important issue regarding how the system works. In a few cases the gluing issue was addressed: unless the magnets are glued down they immediately clump up, due to the attraction between their various poles, and will not lie flat. This is actually an extremely important feature rarely discussed in textbooks: magnets are not held together just by the magnetic fields of their component structures (e.g., "domains").

Once the magnet situation was understood it was usually deduced that there must be a largely vertical magnetic field penetrating through the solution.

The examiner rather expected that the students would be knowledgeable about the blue tint of the solution, but students nowadays seem to have had less exposure to chemistry and thus are not aware that copper sulphate is the only common salt giving a blue solution. At all events, the examiner would volunteer this information and mention that its presence made the ordinary distilled water it was dissolved in electrically conducting (unlike distilled water itself). Because it was not an important issue for understanding how the solution rotated, it was only occasionally pointed out that the reason copper sulphate was used instead of ordinary salt, or any other salt, was to minimize the electrolytic interactions between the copper electrodes and the conducting solution.

The next step in understanding the liquid cyclotron was for the students to make use of the force equation:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Where **F** is the force on charge q, **E** is the electric field, **v** is the charge velocity, and **B** is the magnetic field. Obviously **E** will be radially directed and, with a little more thought, strongest near the central electrode. With the resulting v's of the charge carriers all radially directed, the **v x B** forces will lead to circular motion – in the same direction for both positive and negative charges. The rate of circular motion will be strongest near the central electrode. Finally, interaction (collisions) between the charge carriers and neutral particles will produce circular motion of the fluid itself. Reversing the voltage supply will reverse the direction of **E** and thus reverse the direction of motion of the fluid.

There are a few tricky points in comparing this liquid cyclotron with the conventional cyclotron discussed in, say, physics classes describing particle accelerators. In particular, there are both positive and negative charges being driven around inside the region of magnetic field, and then there are their interactions with the neutral fluid, making the circular motion of the charged particles visible but also slowing it down.

Ph.D. Quals Question
January 23-27, 2012
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Super Capacitors

Quals students were told that the advent of electric cars has led to a strong interest in the development of "super" or "ultra" capacitors, which might just conceivably replace batteries or, more likely, given current progress, be used in conjunction with batteries. The students were warned that my questioning would be directed toward the development of these high-capacity capacitors and that it would largely involve basic electromagnetism, or more particularly, basic electricity. To start with they were asked to describe Gauss's theorem relating the surface integral of the electric field **E** over a closed surface to the enclosed charge Q:

$$\int \mathbf{E}.\mathbf{ds} = Q/\varepsilon_0$$

Next, when we apply this to the surface of a plane conducting plate of infinite extent carrying surface charge σ we obtain the electric field perpendicular to the surface:

$$E = \sigma/\varepsilon_0$$

This same expression also applies to the electric field close to the surface of any conductor, with σ the surface charge density in the immediate vicinity.

For two parallel conducting planes of area A separated by a distance d carrying equal but opposite charges Q, -Q, the electric field has the same form as above (we will assume it is everywhere perpendicular to the surfaces of the plates, with no fringing) and the capacitance of the plates is

$$C = Q/V = \sigma A/(\sigma d/\varepsilon_0) = \varepsilon_0 A/d$$

Although all isolated conducting bodies have some capacitance, these capacitances are generally very small and measured in microfarads or micromicrofarads. [To illustrate, the capacitance of a sphere of radius R is given by $C = 4\pi\,\varepsilon_0 R$ and for a sphere as large as the Earth, with R = 6370 km, we only have C = 708 μF]. Once again in general, large capacitances can only be achieved by using two close conductors bearing opposite charges (this arrangement is called a **capacitor**) and here the above equation for the capacitance between two plates is representative. Not only that, most attempts to produce very large capacitances are based on the conducting plate model. We will now consider how "super" or "ultra" capacitors might be produced using this model.

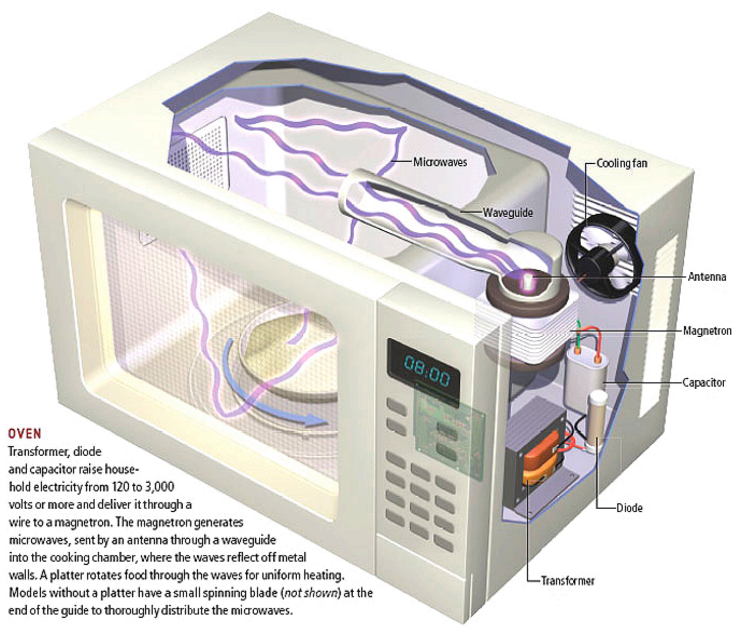
Obviously, we can make C large by increasing A and decreasing d, and this is what is done in conventional capacitors. However, these conventional capacitors are not "super" nor "ultra." **Background material**: The best of these conventional capacitors is the electrolytic capacitor, invented around 1890, which can have a capacitance in the mF range. In these capacitors a very thin (small d) layer of non-conducting aluminum oxide is formed between two aluminum plates containing an electrolyte or an electrolytesoaked paper spacer (the electrolyte can have many different compositions, but borax has been successfully used in the past).

In a “nano” world, increasing A is difficult without increasing the size of the capacitor whereas decreasing d can lead to small capacitors and this is in tune with our present “nano” world. However, we have to remember that $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m (value given to students) and so even d = 1 nm and A = 1 m^2 (is this a reasonable value?) will leave C less than 1 F. At this stage the students were told that there were now commercial electrolytic supercapacitors (or ultracapacitors) available with capacitances on the order of a few farads or more. How did they think this was achieved (given what we have already discussed)? What was looked for here in the wrap-up was a suggestion that the area A needed to be increased. The existing supercapacitors make use of carbon materials with large effective areas due to their porosity; areas of as much as 250 m^2 are quoted. Small d’s (yes, on the order of 1 nm) are achieved by producing electrical double layers. There will be much research done on these capacitors over the next few decades because of the large commercial interest in their applications.

Microwave Ovens: EE Quals Question 2013
A. C. Fraser-Smith

(1) I start by stating that we are going to talk about how microwave ovens work and I then ask what is actually meant by ‘microwaves’ here? Are we dealing with ultrasound or electromagnetic waves? Micro implies something small – how small?”

It is not all that simple distinguishing between the use of ultrasound or em waves in a microwave oven simply by looking at one. The prohibition against putting metal objects inside was considered a clue, as was the grill (assumed to be metal) in the window. Having been told that em waves were used is not a particularly bad answer. Turning to the “micro,” many students state that it relates to the wavelength of the waves, i.e., they are around 10^(-6) of a meter. At this stage they are told that the frequency of operation of typical commercial microwave ovens is 2.45 GHz – some students know this! So, what is the wavelength (in free space), assuming, say that the frequency is ~3 GHz? A quick computation of $\lambda = c/f$ gives $\lambda = 0.1$m or 10 cm. Obviously the “micro” is a misnomer. A few students knew that when it first became possible to generate microwaves in the late 1930’s the wavelengths were “micro” in comparison with the wavelengths of the radio waves in common use at the time. (2 points)



(2) Given that there is some device within a microwave oven that generates microwaves and radiates them into the chamber of the oven, what, in your opinion, are the important features of the oven from the electromagnetic point of view?

The device generating the microwaves is a magnetron with an attached antenna. Many students knew this but it was treated as being knowledge and not considered

part of the exam. However, the following were important. (i) The microwaves enter the oven's "cavity" and it obviously must have metal walls to contain the radiation. Plastic, i.e., non-conducting, walls would allow it to leak out, reducing the oven's efficiency and making the radiation a safety hazard. (ii) The window in the oven's door contains a metal mesh that can be seen though but which prevents the microwaves from passing through. (iii) the holes in the mesh are roughly 1 mm in size, which is much less that the ~10 cm wavelength of the radiation. The mesh is seen as a solid metal sheet. Light waves can easily penetrate, though, since their wavelengths are much less than ~ 1mm. (iv) With its radiation confined to a metal cavity, without much loss, it must be undesirable to operate a microwave oven empty. Often the instructions for the ovens include a warning about this use. (v) Pointed, or sharp-edged, metal objects will have voltages induced in them that might lead to sparking at the points and edges, causing them to become a fire-hazard. (vi) Although the oven's cavity is not necessarily a resonant cavity its em fields will have some form of standing wave pattern. Thus the oven may not heat uniformly. We can guess that the spacing between the hot spots will be about half a wavelength (two peaks of electric field per wavelength), or around 5 cm. This is why the ovens often have a rotating plate on the bottom (and in some cases there is a metal "paddle" rotating where the microwaves come in, to help disperse them).
(5 points)

(3) How do you think the microwaves heat food?

It is common knowledge that the microwave heating involves the water molecules in the food but from this point the process becomes murky. Students thought the microwaves might resonate with the molecules and/or cause them to rotate vigorously. Some thought the microwaves would heat the food from the inside! We investigated these possibilities after the water molecule was described as being dipolar with a dipole moment (negative on the oxygen end and positive on the end where the two hydrogen atoms are bound). The students were also told that although it is possible to set up internal resonances in individual water molecules the frequencies involved are outside the microwave range. With this information we decided: (i) that the water molecules would first rotate in one direction to line up with the electric field of the wave and then rotate in the opposite direction as the electric field oscillated. There would be no net rotation and the molecule would feed energy into the water and its surroundings by oscillating backwards and forwards and interacting with the surrounding molecules as it did so, i.e., heating by collisions. (ii) we dismissed the concept of the microwaves heating food from the inside out, since they would have to penetrate into the food from the outside, heating it and losing intensity as they penetrated. Nevertheless, (iii) because food is unlikely to be a good conductor, the skin depth at microwave frequencies could well be on the order of a wavelength and the food should be heated relatively uniformly. (3 points)