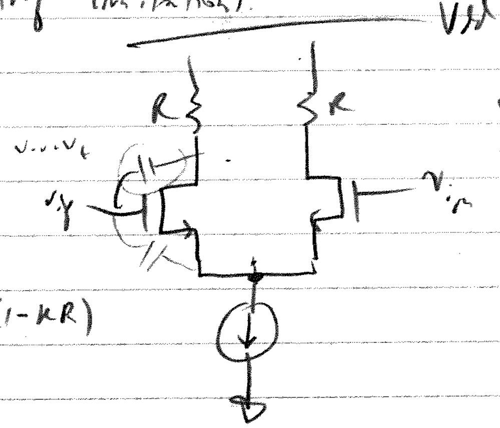
Alex Omid-Zohoor

Quals Solutions

Circuits

2000 - Wong

- voltage gain   diff pair.
- CM input range
- active load
- freq limitations.

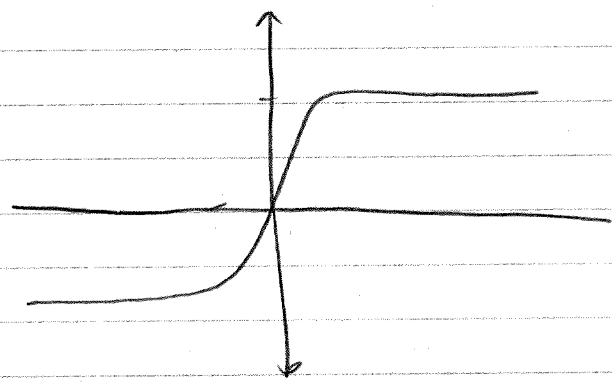


$\frac{v_{od}}{v_{id}} = -g_m R$

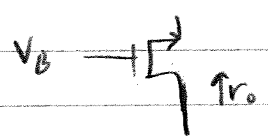
$C = (1 - kR)$

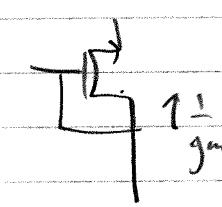
$k = -g_m$

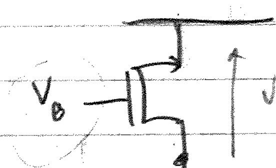
$\pm \sqrt{2}\, V_{ov}$



$V_t < V_{cm} < V_{dd} + V_t$

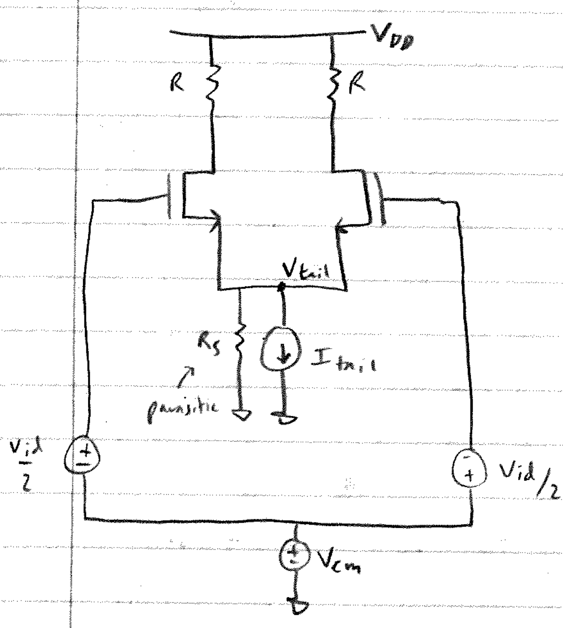






more output swing

high R high gain reduced swing	low R ratiometric	medium R
k ↑	k ↓	k ~
C ↑	C ↓	C ~



$A_v = -g_m R$

$A_{cm} = -\dfrac{g_m R}{1 + 2 g_m R_s}$

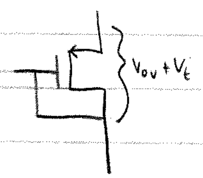
or $\min\left\{ V_{dd},\ V_{dd} - \frac{I_{tail}}{2} R + V_t \right\}$

if you assume V_{dd} is the max voltage available.

CM input range: $V_{tail} + V_t < V_{cm} < V_{dd} - \frac{I_{tail}}{2} R + V_t$

Active loads:

diode connected



$R_o = \left(\frac{1}{g_m} \| r_o\right) \approx \frac{1}{g_m}$

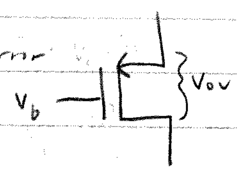
$\Rightarrow$ low gain

High bandwidth, High linearity.

(bootstrapping) $C = (1 - k C_{gd})$ small since $k \approx 1$

very low output swing

current mirror

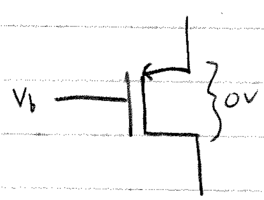


$R_o = (r_o \| r_o) \approx \frac{r_o}{2}$

$\Rightarrow$ high gain

low output swing

triode load



R_o medium

$\Rightarrow$ medium gain

high output swing

Sensitive to process variations (R depends on μC_{ox}, etc..)

2001 - Kovacs

Table : has sensor sensing something about a glowing cylinder.

- ruler, toilet paper, book, copper clad PCB, plastic ruler, full beverage can, paper, pen, hard cover book..

- Figure out what sensor was sensing

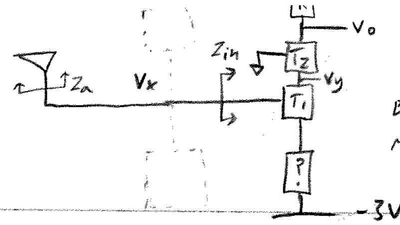
Exp 1: Sensor value vs. distance. Use ruler + pen to mark off points on toilet paper. 1, 2, 4, 6, 8
(or could actually just count squares)
See if readings fall off as $\frac{1}{r}$, $\frac{1}{r^2}$ or $\frac{1}{r^3}$
point source of EM radiation.

Exp 2: Place hard cover book between sensor + cylinder
(see if it's sensing visible light).

Exp 3: Place copper clad pcb between sensor + cylinder.
See if it's sensing EM radiation with low frequency (< GHz). PCB spacing ~1mm $\Rightarrow f < \sim 3\cdot10^{11} = 300\,GHz$.

~~To determine if it's sensing EM radiation:~~
Exp 4: if allowed, empty out beverage can + place it over the sensor (or surround cylinder with it). See if sensor reading drops sharply. Essentially surrounding the sensor with a faraday cage.

for T1 + T2



T1 + T2 identical
BJT: $V_{BE} = 0.7$ $\beta = 100$
MOS: $V_t = 0.5$ $k' = 100\mu A/V^2$

2000 Dutton

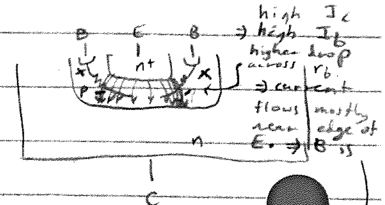
1. Explain strategy in choosing bias and select V_x and components to get small signal gain from antenna to V_o. For the box with "?" you can choose any component or short it.
2. What is small signal gain? How could you change circuit to improve it?
3. What is impedance seen by antenna looking into transistor? What can you do to make it a good match to antenna?
4. What are the dominant noise sources in the amplifier?

BJT: 1) LNA ⇒ want low noise, high gain.

r_b may decrease 50% as I_c is increased from 0.1mA to 10mA. Due to current crowding.

To minimize noise, minimize r_b ⇒ maximize I_c.

$$A_v = g_m R = \frac{I_c}{V_T} \cdot R = \frac{\frac{V_{DD}-V_o}{R}}{V_T} \cdot R = \frac{V_{DD}-V_o}{V_T} = \text{constant}.$$



high I_c ⇒ high I_b higher drop across r_b. ⇒ current flows mostly near E. ⇒ edge of B is effectively closer to E. ⇒ r_b decreases.

for npn: ~~forward active~~: $V_{BE} \geq 0.7$ $V_{BC} < V_{BC(ON)}$

⇒ for DC set $V_{BC} = 0$ ⇒ $V_o = 0V$ To keep T_2 on, need $V_y < -0.7V$ let ? be a short.

$$\Rightarrow I_c = \frac{3V - 0V}{R} \Rightarrow \text{choose } \boxed{I_c = 3mA} \Rightarrow \boxed{R = 1k\Omega}$$

$$I_B = \frac{I_c}{\beta} = 30\mu A \Rightarrow \frac{V_x - (-3V + 0.7V)}{r_b} = 30\mu A \Rightarrow \text{solve for } V_x.$$

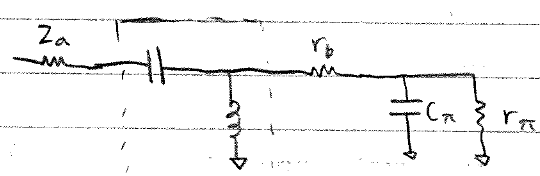
2) small signal gain is $-\frac{Z_{in}}{Z_a+Z_{in}} g_m R = -\frac{Z_{in}}{Z_a+Z_{in}} \frac{3V}{V_T} = -\frac{Z_{in}}{Z_a+Z_{in}} \frac{3V}{0.025V}$

$$\Rightarrow \boxed{A_v = -\frac{Z_{in}}{Z_a+Z_{in}} (120)}$$

could improve this by using an active load instead of R.

3) $Z_{in} = r_b + \left(\frac{1}{sC_\pi} \| r_\pi\right)$ (C_μ is bootstrapped completely by cascode).

L-match:



matching network.

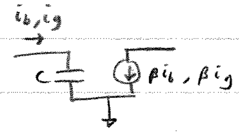
$\overline{v^2} = 4kTR$
$\overline{i^2} = \frac{4kT}{R}$

4) r_b is the dominant noise source in the amplifier. To first order, $F = \frac{r_b}{r_b + R_a} = 1 + \frac{r_b}{R_a}$

For bias network, choose large Resistors. $\Rightarrow \; i_n^2 = \frac{\ }{R}$ = small.
$\Rightarrow$ doesn't degrade noise performance of LNA.

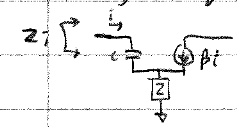
Impedance Matching - review EE314

High Freq. Small Signal Model :



$\beta i = g_m V_{gs,be} = g_m \, i \, \frac{1}{sC}$

$\Rightarrow \beta = \frac{g_m}{sC} \Rightarrow \beta = -j \frac{\omega_T}{\omega}$



$$Z_i = \frac{1}{sC} + (\beta + 1) Z = \frac{1}{sC} + \left(-j \frac{\omega_T}{\omega} + 1\right) Z$$

$\Rightarrow$ R $\Rightarrow$ C　　L $\Rightarrow$ R　　C $\Rightarrow$ - R negative R!

MOS　1) To minimize noise, maximize g_m (without changing W) $\Rightarrow$ maximize V_{ov} $\Rightarrow$ maximize V_X.

EE314 Intro to Noise Slide 55 $\rightarrow$ $\left(F = 1 + \frac{R_g}{R_a} + \frac{\gamma}{\alpha} R_a \, g_m \left(\frac{\omega}{\omega_T}\right)^2 \right)$　　$\omega_T = \frac{g_m}{C_{gs}} \propto \frac{g_m}{W}$

$\Rightarrow$ if we increase g_m by keeping ω_T constant, we degrade our noise factor.

$g_m = \mu C_{ox} \frac{W}{L} V_{ov}$ $\Rightarrow$ must increase V_{ov} while keeping W constant.

for MOS: Saturation: $V_{gs} \geq V_t$　$V_{gd} < V_t$　$\Rightarrow$ set $V_o = 0V$

need $V_g < -0.5V$　$\boxed{\text{let ? be an inductor.}}$

$-2.5V \leq V_X \leq 0V$ to keep T_1 and T_2 in saturation, want to maximize V_X.

To be safe let $\boxed{V_X = -0.5V.}$　$\Rightarrow$ $V_{ov} = V_X - V_{SS} - V_t = 2V$

$\Rightarrow$ $I_d = k' \frac{W}{L} V_{ov}^2$　set $\frac{W}{L} = 5$　$\Rightarrow$ $I_d = 100\mu A/V^2 \cdot 5 \cdot 4V^2 = \boxed{2mA = I_d}$

$\Rightarrow$ $\frac{V_{DD} - V_o}{R} = 2mA$ $\Rightarrow$ $R = \frac{3V}{2mA}$ $\Rightarrow$ $\boxed{R = 1.5 k\Omega}$　　$g_m = \frac{2I_d}{V_{ov}} = \frac{4mA}{2V} = 2mS$

2) $A_v = -\frac{Z_{in}}{Z_{in} + Z_a} g_m R$ $\Rightarrow$ $\boxed{A_v = -\frac{Z_{in}}{Z_{in} + Z_a} (3)}$ $\leftarrow$ lower gain

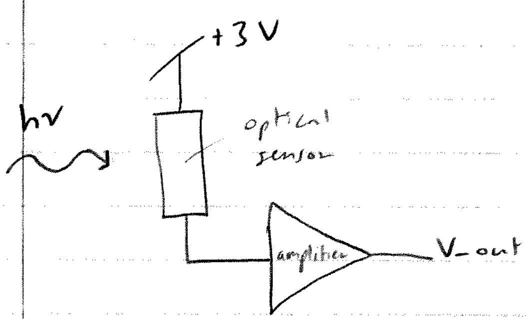
could <u>greatly</u> improve this by using an active load instead of R.

3) $Z_{in} = \frac{1}{sC_{gs}} + \omega_T L + j\omega L$　(C_{gd} is bootstrapped completely by cascode)

can choose degenerating inductance L such that $\omega_T L = R_a$. to get a good power match.

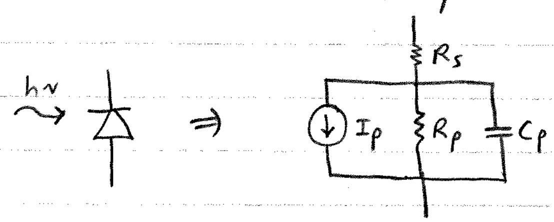
4) The dominant noise sources are gate noise and drain noise.

2002 - Dutton

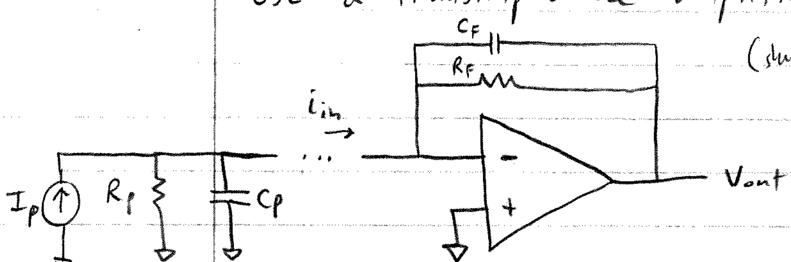


- Use reverse biased photodiode as sensor. photons striking the depletion region cause e^-, h^+ pairs to form through photogeneration. These carriers are swept across the depletion region, causing photocurrent I_p.

- Circuit model:



- Use a transimpedance amplifier to detect and amplify signal.



(shunt-shunt feedback)

$$\frac{V_{out}}{i_{in}} = -R_F\left(\frac{1}{1+sR_FC_F}\right)$$

$$\frac{V_o - V_i}{R_F \| C_F} = \frac{V_i}{R_p \| C_p}$$

$$A_{CL} \approx \frac{1}{f} = \frac{V_o}{V_i} = \left(\frac{1+sR_pC_p}{R_p} + \frac{1+sR_FC_F}{R_F}\right)\left(\frac{R_F}{1+sR_FC_F}\right)$$

$$= \frac{R_F + sR_pR_FC_p + R_p + sR_pR_FC_F}{R_p + sR_pR_FC_F}$$

$$= \frac{\frac{R_F+R_p}{R_p} + sR_F(C_p+C_F)}{1+sR_FC_F}$$

$$\Rightarrow A_{CL} = \frac{R_F+R_p}{R_p}\left(\frac{1+s\frac{R_FR_p}{R_F+R_p}(C_p+C_F)}{1+sR_FC_F}\right)$$

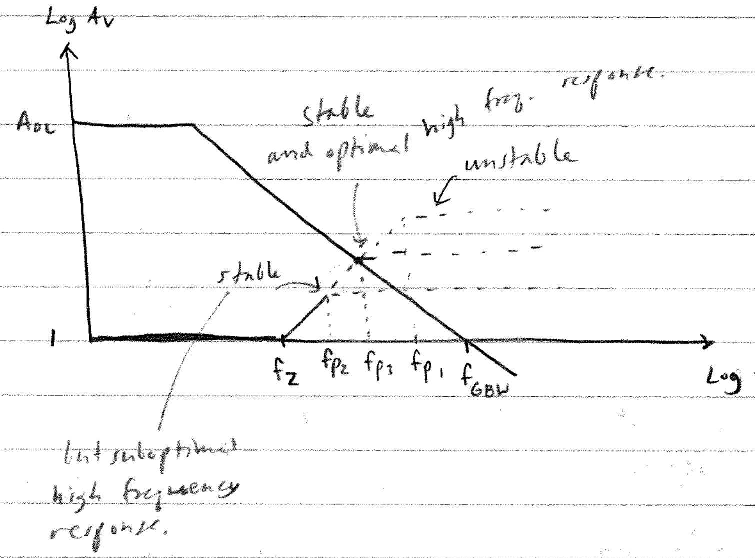
$$= \frac{R_F+R_p}{R_p}\,\frac{1+j\frac{\omega}{\omega_z}}{1+j\frac{\omega}{\omega_p}}$$

$$\omega_z = \frac{R_F+R_p}{R_FR_p(C_F+C_p)}$$

$$\omega_p = \frac{1}{R_FC_F}$$

typically $R_P >> R_F \Rightarrow A_{CL}(f << f_z) \approx 1$, $\omega_z < \omega_p$ since $R_P \| R_F \approx R_P$.

$$A_{CL}(f >> f_p) \approx \frac{C_F + C_P}{C_F}$$



$$\Rightarrow \frac{GBW}{f_p} = \frac{C_F + C_P}{C_F}$$

$$C_F = \frac{1}{4\pi R_F\, GBW}\left[1 + \sqrt{1 + 8\pi R_F C_P\, GBW}\right]$$

$$f_p \approx \sqrt{\frac{GBW}{2\pi R_F C_P}}$$

$\Uparrow$ in this analysis from TI, it seems that they assume $A_{CL} = \frac{1}{f}$.

Here is my solution: find f: DC: $\frac{R_P}{R_P + R_F}$

zero: $Z_F \to \infty \Rightarrow R_F \| \frac{1}{sC_F} = \frac{R_F}{1 + sR_F C_F} \to \infty \Rightarrow 1 + sR_F C_F = 0$

$\Rightarrow n_1(s) = 1 + sR_F C_F$ ← this is numerator of transfer function.

$$f = \frac{R_P \| C_P}{R_P \| C_P + R_F \| C_F} = \frac{\frac{R_P}{1 + sR_P C_P}}{\frac{R_P}{1 + sR_P C_P} + \frac{R_F}{1 + sR_F C_F}} = \frac{1 + sR_F C_F}{1 + sR_F C_F + \frac{R_F(1 + sR_P C_P)}{R_P}}$$

$$= \frac{1 + sR_F C_F}{1 + \frac{R_F}{R_P} + sR_F C_F + sR_F C_P} = \frac{1 + sR_F C_F}{\frac{R_F + R_P}{R_P} + sR_F(C_F + C_P)}$$

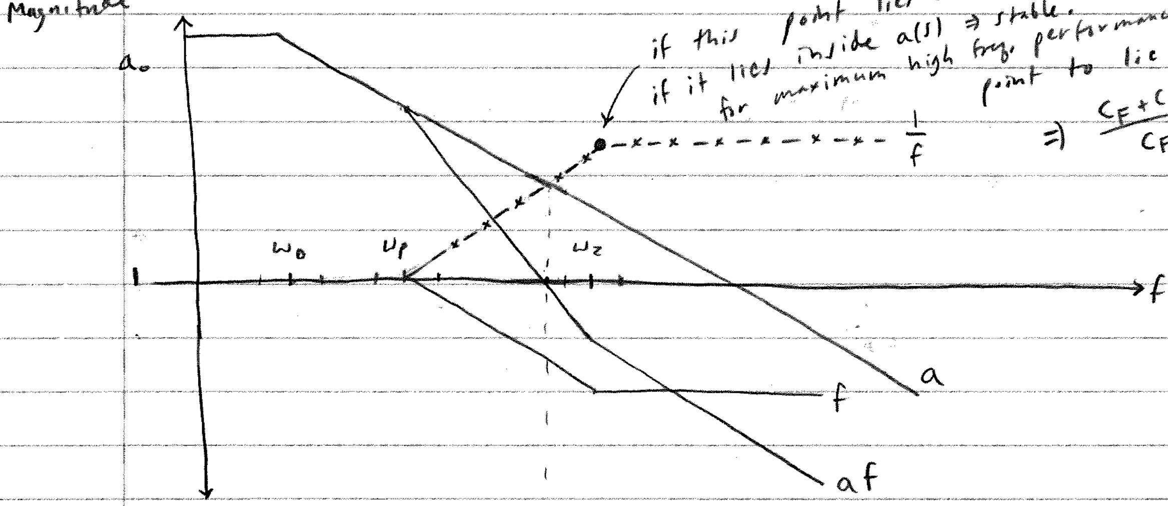
$$\Rightarrow \boxed{f = \frac{R_P}{R_F + R_P}\left(\frac{1 + sR_F C_F}{1 + s\frac{R_P R_F}{R_F + R_P}(C_F + C_P)}\right)}$$

$$\omega_p = \frac{1}{(R_P \| R_F)(C_F + C_P)} \qquad \omega_z = \frac{1}{R_F C_F}$$

typically $R_p >> R_F \Rightarrow f(0) = \frac{R_p}{R_p + R_F} \approx 1$, $R_p \| R_F \approx R_F$

$\Rightarrow \omega_p < \omega_z \qquad f(\infty) = \frac{C_F}{C_F + C_p}$

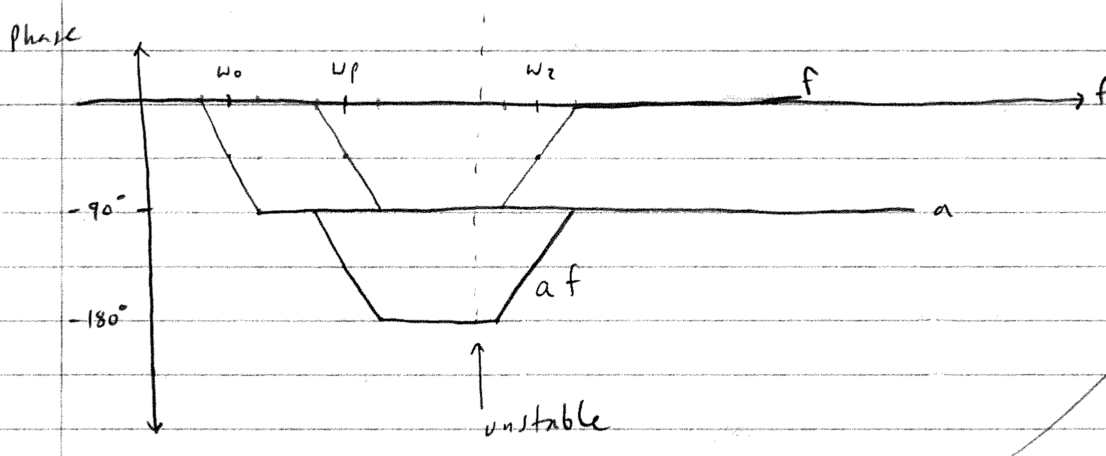
if this point lies outside of $a(s) \Rightarrow$ unstable.
if it lies inside $a(s) \Rightarrow$ stable.
for maximum high freq. performance want the point to lie exactly on $a(s)$.



$\Rightarrow \frac{C_F + C_p}{C_F} = \frac{GBW}{\omega_z}$

$= GBW \, R_F C_F$

$\Rightarrow$ solve for C_F.



$$C_F = \frac{1}{4\pi R_F GBW}\left[1 + \sqrt{(1 + 8\pi R_F C_p GBW)}\right]$$

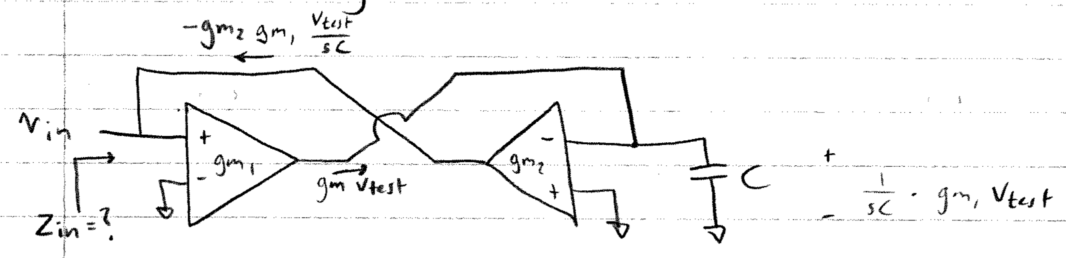
$f_z \approx \sqrt{\frac{GBW}{2\pi R_F C_p}}$ ← zero in feedback is pole in closed loop.

$\Rightarrow$ limits closed loop TIA bandwidth.

$\Rightarrow R_F \uparrow \Rightarrow$ gain $\uparrow \Rightarrow$ BW $\downarrow$

gain - bandwidth tradeoff.

2001 - Wooley



Use Blackman's. g_{m2} is feedback element.

$\Rightarrow Z_{(k=0)} = \infty$

$$i_{test} = g_{m1} g_{m2} \frac{V_{test}}{sC} \Rightarrow \frac{V_{test}}{i_{test}} = Z_{in} = \frac{sC}{g_{m1} g_{m2}}$$

$\Rightarrow$ looks inductive with $L_{eff} = \dfrac{C}{g_{m1} g_{m2}}$

2001 - Wong

1. Sketch schematic of cmos inverter + CS with active load

2. Compare small signal voltage gain assuming device sizes and bias currents are similar

3. Compare the bandwidth of the two circuits.

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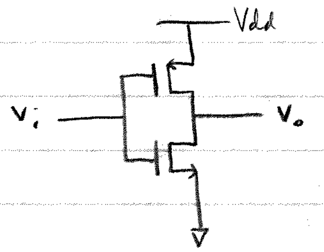
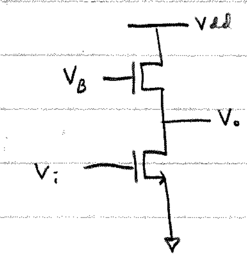
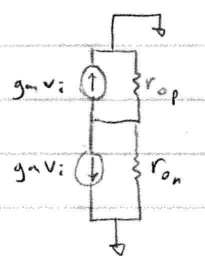
1. 



2. 

$\Rightarrow \; a_v = -(g_{m_n} + g_{m_p})(r_{op} \| r_{on}) \approx -g_m r_o$

CS: $a_v = -g_{m_n}(r_{op} \| r_{on})$ for current mirror load.

$\approx -\frac{1}{2} g_m r_o$

$C_{gd} \approx 0.2\, C_{gs}$

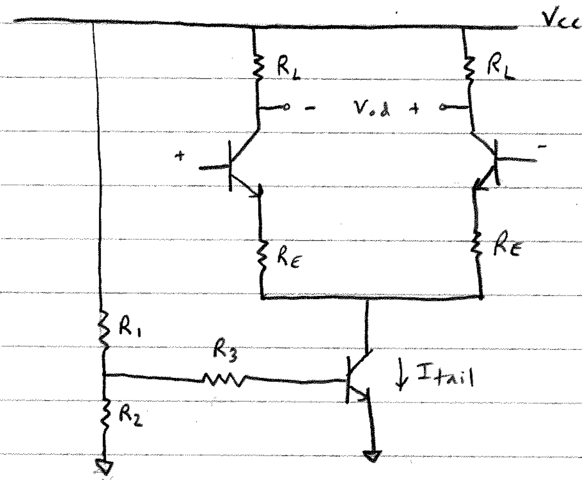
3. Inverter: $C_{in} = (C_{gd_p} + C_{gd_n})(1 - a_v) + 2C_{gs} \approx 2 g_m r_o \, C_{gd}$

CS: $C_{in} = (C_{gd})(1 - a_v) + C_{gs} \approx 0.5\, g_m r_o \, C_{gd}$

Assume same R_s $\Rightarrow$ $BW_{inverter} \approx \frac{1}{4} BW_{cs}$

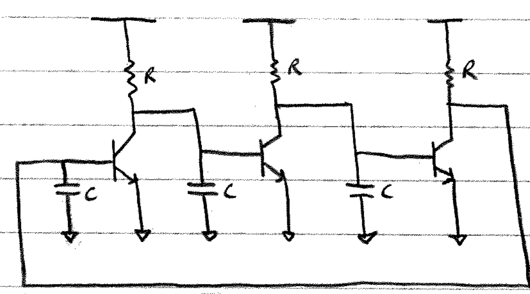
Kovacs 2002

1. Amplifier



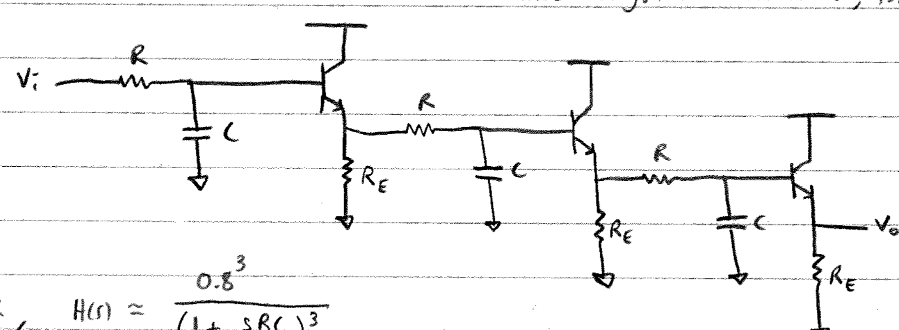
Advantages: Common mode rejection, linearity and bandwidth (Miller) can be traded off with gain by sizing R_L and R_E, high output swing, simple.
Disadvantages: Medium bandwidth, unable to drive resistive loads.

2. Oscillator



Advantages: simple, high tuning range (tune f by changing C or R).
Disadvantages: Worse phase noise performance than high Q tuned oscillators, sensitive to PVT variations (unlike crystal oscillators, for example)

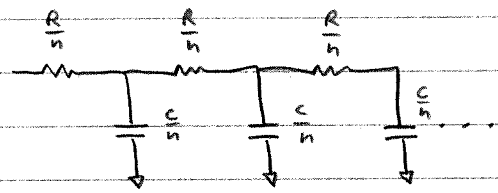
3. Filter:



if $R_E \gg R$, $H(s) \approx \frac{0.8^3}{(1+sRC)^3}$

$\Rightarrow$ 3rd order. pole at $f \approx \frac{1}{2\pi RC}$

Compare to simple passive RC ladder:



by ZVTC: $\sum \tau = \frac{C}{n} \sum_{i=1}^{n} \frac{iR}{n} = \frac{RC}{n^2} \frac{n(n+1)}{2} = \frac{RC}{2}\left(\frac{n+1}{n}\right)$

$\omega_{3dB} = \frac{2}{RC}\left(\frac{n}{n+1}\right) \Rightarrow \lim_{n\to\infty} \omega_{3dB} = \frac{2}{RC}$ ← frequency limited due to loading effects...

poles don't occur at same frequency ⇒ non-ideal rolloff.

With active buffers (common collector), loading effects are removed.

So for active cascaded implementation with common collector buffers:

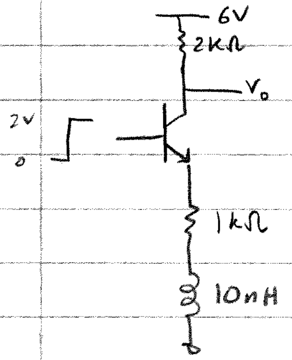
Advantages: simple to design, steep rolloff.

Disadvantages: possible instability, reduced headroom at output

$(V_{O_{CM}} = V_{I_{CM}} - (0.7V)\cdot 3)$

flexible, could instead choose high pass or low pass for each stage, so that overall, you could get a bandpass, highpass, or notch filter.

Wooley 2002



Sketch $V_o(t)$

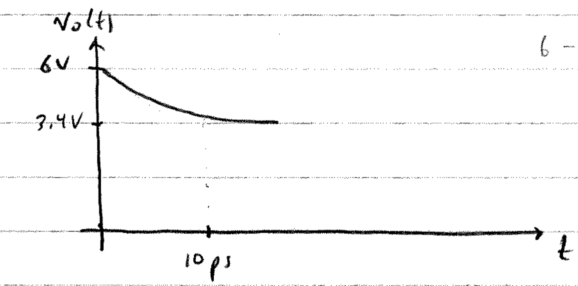
steady state: $V_L = 0$

$\Rightarrow V_E = 2V - 0.7V = 1.3V \Rightarrow I_c = 1.3mA$

$\Rightarrow V_o = 6V - (1.3 \cdot 2)V = 3.4V$

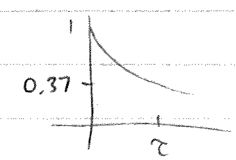
initially $V_o = 6V$, $V_E = 0V$

$V_L = L \frac{di}{dt}$ $\qquad \frac{L}{R} = \frac{10n}{1k} = 10 \cdot 10^{-12} = 10ps.$



$6 - 2.6\, e^{-t/\tau}$

$e^{-1} = \frac{1}{2.71}$



$t < 0 \qquad V_o = 6V, \quad i_c = 0A$

??

$V(t) = L \int_0^t i_L \, dt + V_o \quad \Rightarrow \quad i_L = 1.3mA - 1.3mA\, e^{t/\tau}$

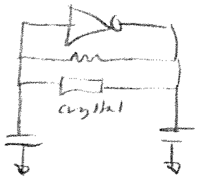
$\tau = \frac{L}{R} = 10ps.$

$V_o = 6 - 2.6\, e^{-t/\tau}$

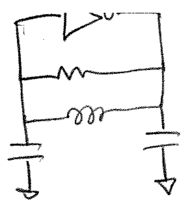
$i_L = 1.3(1 - e^{-t/\tau}) \Rightarrow 0.63$ after τ

??.

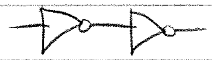
Pierce Oscillator



OR



Wong 2002

(1) 

(2) 

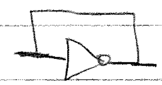
If the output is connected to the input how will the circuit behave?

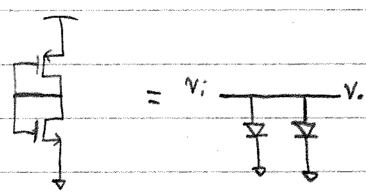
(3)  ← will this circuit oscillate if output is connected to input?

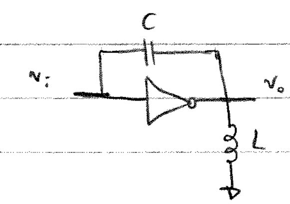
Bonus: what components will you add to make it oscillate?

1. ~~Buffer.~~ (digital) Latch

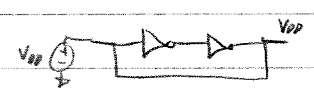
2. ring oscillator $f = \frac{1}{2 \cdot 3 T_d} = \frac{1}{6 T_d}$ where T_d is the delay of each inverter.

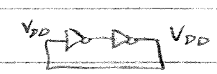
3.  No, feedforward through wire. $\Rightarrow \frac{V_o}{V_i} = 1$
is instantaneous (compared to T_d of inverter)





---

1. Latch (holds an input value) 

then open input: 

(value is held).

2. Ring oscillator

??. 3. Loop gain phase is: $\underset{\text{inversion}}{180^\circ} + \underset{\text{pole at input}}{90^\circ} = 270^\circ$ ← not sufficient to cause instability.

Wong 2004

1. $A_v = -g_{m_n}(r_{o_p} \| r_{o_n})$

2. Assume dominant pole at output:

$$A_v(s) = -g_{m_n} Z_{out} = -g_{m_n}\left(r_{o_p} \| r_{o_n} \| \frac{R}{2} \| \frac{1}{2sC}\right)$$

$$= -g_{m_n}\left(r_{o_p} \| r_{o_n} \| \frac{R}{2}\right)\left(\frac{1}{1+s(r_{o_p} \| r_{o_n} \| \frac{R}{2})2C}\right)$$

$\Rightarrow$ pole at $\omega_p = \frac{1}{(r_{o_p} \| r_{o_n} \| R)2C}$

$\Rightarrow$ lowpass response.

3. $$A_v(s) = -g_{m_n} Z_{out} = -g_{m_n}\left(r_{o_p} \| r_{o_n} \| \frac{R}{2} \| s\frac{L}{2}\right) = -g_{m_n}\frac{sL/2}{1+s\frac{L}{2(r_{o_p} \| r_{o_n} \| \frac{R}{2})}}$$

$\Rightarrow \omega_z = 0 \qquad \omega_p = \frac{2(r_{o_p} \| r_{o_n} \| R/2)}{L}$ $\Rightarrow$ high pass response.

4. $A_v(s) = -g_{m_n}\left(r_{o_p} \| r_{o_n} \| \frac{R}{2} \| \frac{1}{2sC} \| s\frac{L}{2}\right)$ let $r_{o_p} \| r_{o_n} \| \frac{R}{2} = R_o$

let $R_o \| \frac{1}{sC} \| sL = Z_{out} = \left(\frac{1}{R_o} + sC + \frac{1}{sL}\right)^{-1}$

$$= \left(\frac{sL}{sR_oL} + \frac{s^2RLC}{sR_oL} + \frac{2R_o}{sR_oL}\right)^{-1} = \frac{sR_oL}{s^2R_oLC + sL + 2R_o} = \frac{sR_o}{s^2R_oC + s + \frac{R_o}{L}}$$

$$\Rightarrow A_v(s) = -g_{m_n}\left(r_{o_p} \| r_{o_n} \| \frac{R}{2}\right)\frac{s}{s^2R_oC + s + \frac{R_o}{L}}$$

$\omega_z = 0 \qquad \omega_{p_{1,2}} = -1 \pm \sqrt{1 - 4R_o^2\frac{C}{L}} \qquad \omega_o = \frac{1}{\sqrt{LC}}$

$Q = \frac{R_o}{\sqrt{L/C}} = \omega_o R_o C$

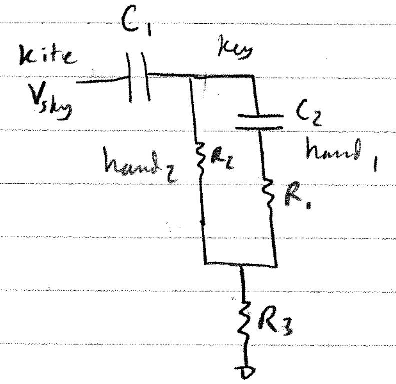
$\Rightarrow$ at $\omega = \omega_o = \frac{1}{\sqrt{LC}}$ $\quad A_v(j\omega_o) = -g_{m_n}\left(r_{o_p} \| r_{o_n} \| \frac{R}{2}\right)$

and $A_v(j\omega) < A_v(j\omega_o)$ for $\omega < \omega_o$ and $\omega > \omega_o$

$\Rightarrow$ band pass response.

Dutton 2003

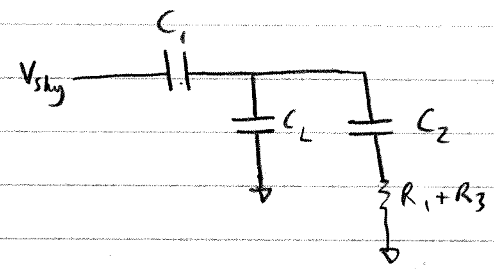
1) $Q = VC$



$C_2 >> C_1$ $\quad Q$ is conserved.

$$Q = V_1 C_1 = V_2 C_2 \Rightarrow V_2 = \frac{C_1}{C_2} V_1 \Rightarrow V_2 << V_1$$

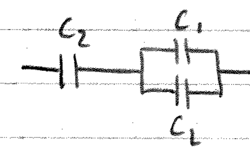
2)



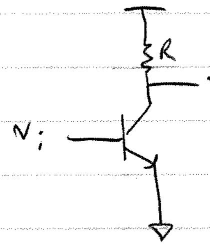
$Q_L = Q_1 \cdot \frac{C_L}{C_L + C_2}$ will charge with RC time constant $\tau = (R_1 + R_3)\left(\frac{C_2(C_1 + C_L)}{C_1 + C_2 + C_L}\right)$

simulated in LT spice ✓.

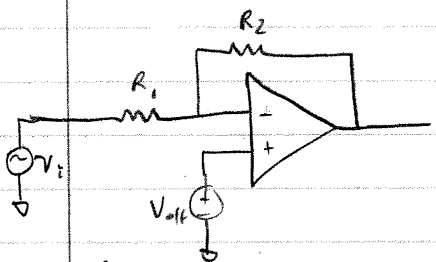
effective capacitance seen by $(R_1 + R_3)$ is



Kovacs 2003



$$A_v = -g_m R = -\frac{I_c}{V_T} R$$



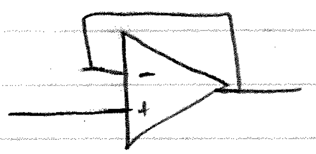
$$V_{out} = V_{off} - \frac{R_2}{R_1} V_i$$

$$\frac{V_{out} - V_{off}}{R_2} = -\frac{V_i - V_{off}}{R_1}$$

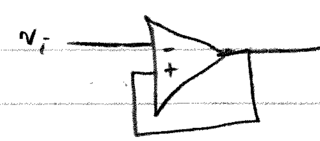
$$V_{out} - V_{off} = -\frac{R_2}{R_1} V_i + \frac{R_2}{R_1} V_{off}$$

$$V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{off} - \frac{R_2}{R_1} V_i$$

reduces max voltage swing at output.



voltage buffer



$V_{out} = (V_{out} - V_{in}) A$

Acts as a latch.

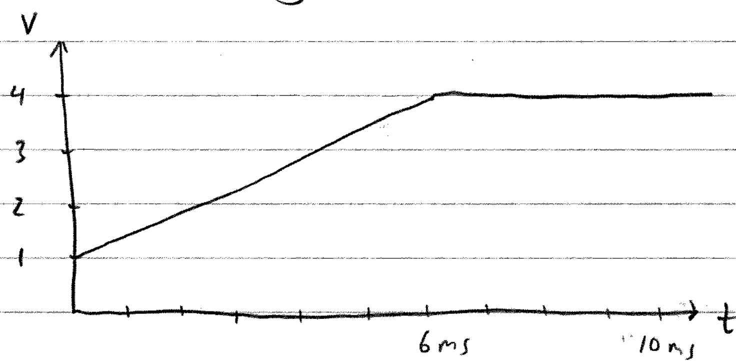
By superposition:

set $V_i = 0 \Rightarrow V_{o_1} = \left(1 + \frac{R_2}{R_1}\right) V_{off}$ (Non-inverting configuration)

set $V_{off} = 0 \Rightarrow V_{o_2} = -\frac{R_2}{R_1} V_i$ (Inverting configuration)

$$\Rightarrow V_o = V_{o_1} + V_{o_2} = \left(1 + \frac{R_2}{R_1}\right) V_{off} - \frac{R_2}{R_1} V_i$$

2004 - Wooley



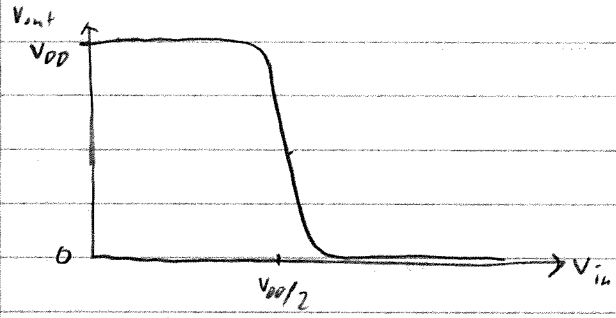
$$i = \frac{1V}{20\Omega} = 50mA$$

$$i = C\frac{dV}{dt} \Rightarrow V = \int \frac{i}{C} = \int \frac{50mA}{100\mu F} = t \cdot 500\ V/s = t \cdot 0.5\ V/ms$$

$$\frac{4V - 1V}{0.5\ V/ms} = 6ms$$ After 6ms, V_o remains constant, since

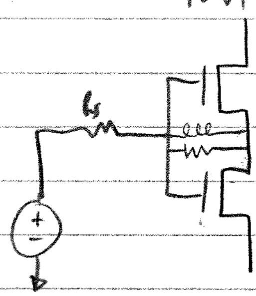
it acts as a charge conservation node.

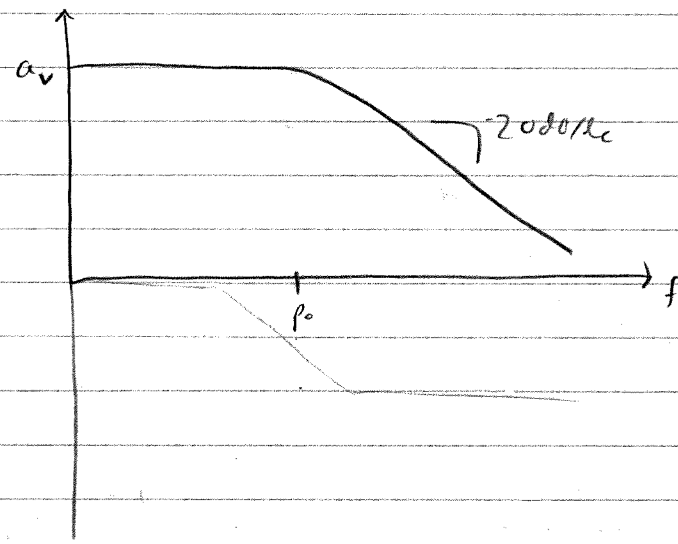
2005 - Wong



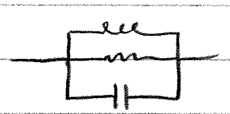
$a_{v_0} = -(g_{m_n} + g_{m_p})(r_{o_p} \| r_{o_n})$

$|a_v| = a_{v_0}\left(\frac{1}{(C_{gd_p} + C_{gd_n})(1 + a_v)\, R_s}\right)$





?

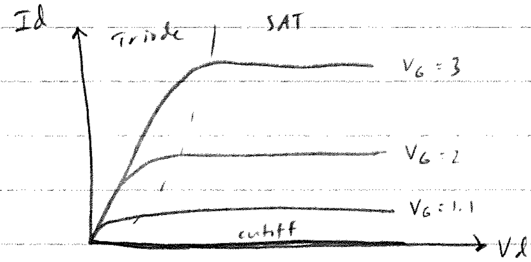


$w_0 = \frac{1}{\sqrt{LC}}$

2002 - shenoy

1.

2.



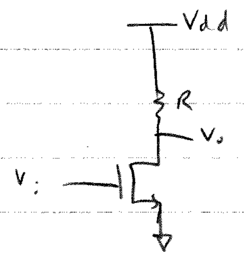
$V_T = 1\,V$

3. $I_d = \mu C_{ox} \frac{W}{L} \left(V_{gs} - V_T - \frac{V_{ds}}{2}\right) V_{ds}$

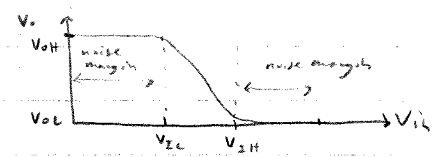
4. $g_m = \mu C_{ox} \frac{W}{L} (V_{gs} - V_T) = \frac{2 I_d}{V_{ov}} = \sqrt{2 \mu C_{ox} \frac{W}{L} I_d}$

5. (Yes, if you assume V_{ov} is constant) otherwise no, $g_m \propto \sqrt{I_d}$

6.



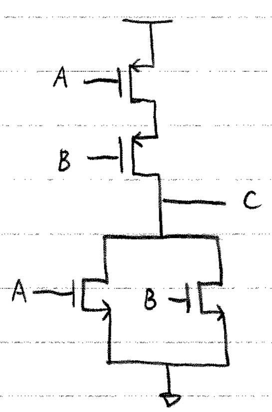
choose high R. ⇒ more gain
⇒ smaller "forbidden region" ⇒ higher noise margin.



7. replace R with pmos load to improve speed? Yes. with R, max current is $\frac{V_{dd}}{R}$. with pmos - max current can be higher ⇒ speed increases
→ can only increase this by decreasing R, ⇒ noise margin, speed tradeoff.

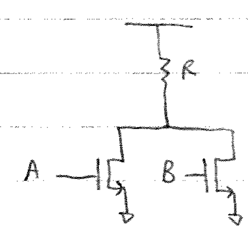
8. PMOS. ⇒ either n or p will be off when output is high or low ⇒ no static current.

9. NOR

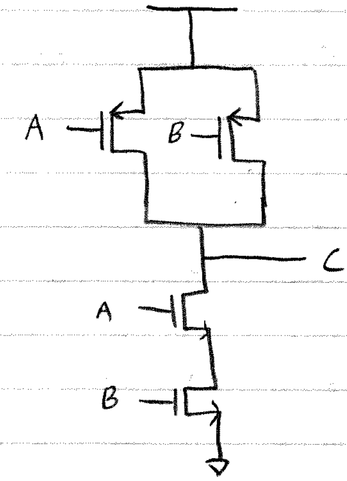


A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

alternatively:

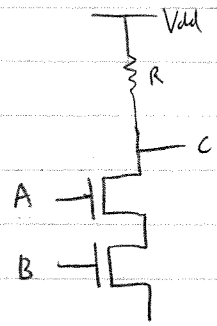


NAND :



A	B	C
0	0	1
0	1	1
1	0	1
1	1	0

alternatively :



1999 - Dutton

1) Impedance Match - Max Power Transfer
Narrowband Match - Low noise
LNA - Mixer - Filter ← Need some combination of these

2) MOS $I_{out} \sim (V_{in})^2$: Small 3rd order nonlinearity and moderate g_m.

Main considerations are gain and 3rd order nonlinearity:

for BJT: $I = I_s \left(\exp\left(\frac{V_g}{V_T}\right) - 1\right) = I_s \left(\frac{V_g}{V_T} + \frac{1}{2}\left(\frac{V_g}{V_T}\right)^2 + \frac{1}{3!}\left(\frac{V_g}{V_T}\right)^3 + \cdots \right)$
high g_m

3rd order term.

for MOS $I_{out} \sim (V_{in})^2$ mostly just have 2nd order term.
moderate g_m

for MOS $I_{out} \sim$ mostly just have 1st order term.
small g_m.

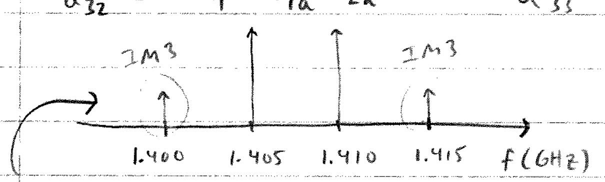
Nonlinearity: 2nd Order - HD - largely suppressed by differential topologies.
IM - doesn't matter for nearby signals.

3rd order - HD - not cancelled by differential topologies.
IM - nearby signals can have IM products that fall on the signal of interest.

$v_o = a_1 v_i(t) + a_2 v_i(t)^2 + a_3 v_i(t)^3$ if $v_i(t) = V_{1a}\cos(\omega_1 t) + V_{2a}\cos(\omega_2 t)$

$a_3 v_i(t)^3 = a_{32}\cos(\omega_1 t \pm 2\omega_2 t) + a_{33}\cos(2\omega_1 t \pm \omega_2 t)$

$a_{32} = \frac{3a_3}{4} V_{1a} V_{2a}^2$ $\quad a_{33} = \frac{3a_3}{4} V_{1a}^2 V_{2a}$



this IM3 product will land on our signal at 1.400 GHz.

3) $\boxed{\text{Gain}} \longrightarrow \boxed{\text{Gain}} \longrightarrow \boxed{\text{Mixer}}$

- Linearity is worse since input to mixer is larger
  ⇒ higher HD.
- Potentially lower NF, if Mixer NF is high.
- Requires very high Q bandpass filter since filtering is done at ~1.4 GHz, and bandwidth is ~ 10 MHz
  ⇒ Q ~ 141

Better Solution

✱ $\boxed{\text{Gain}} \longrightarrow \boxed{\text{Mixer}} \longrightarrow \boxed{\text{Gain}}$

- Linearity is better since input to mixer is small.
- Potentially higher NF (but not by much so long as first gain stage has reasonably high gain).
- Can use moderate Q filtering in first gain stage (LNA) and high performance lowpass filter in final gain stage.

Friis Equation:

$$F_{total} = 1 + (F_1 - 1) + \frac{(F_2 - 1)}{G_1} + \cdots + \frac{(F_n - 1)}{G_1 \cdot G_2 \cdots G_n}$$

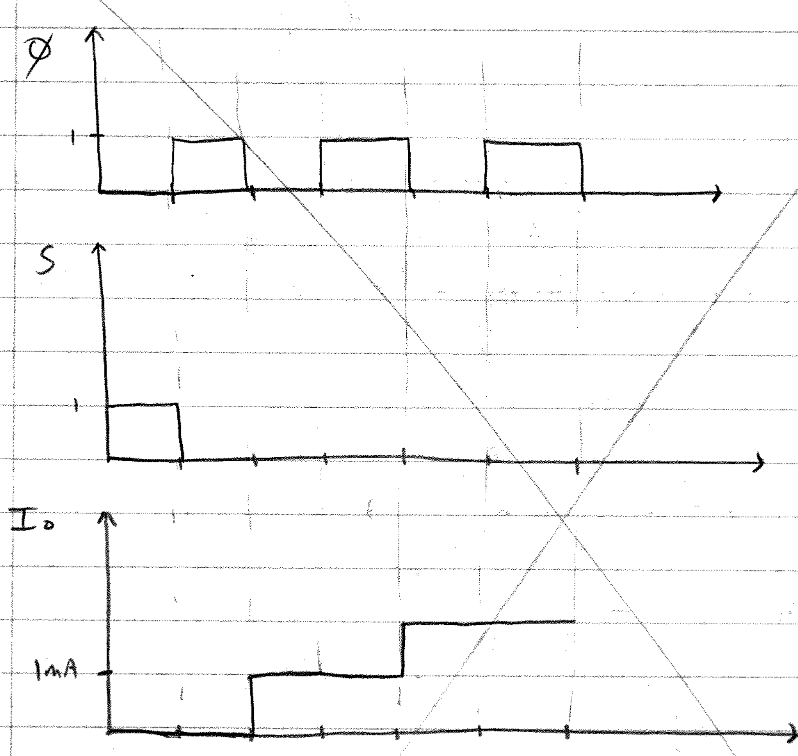
Where F is noise factor: $F = \frac{SNR_{in}}{SNR_{out}}$

Where $SNR = \frac{\text{signal power}}{\text{noise power}}$ $\qquad G = \text{power gain} = A_v^2$

Noise Figure $NF = 10 \cdot \log(F)$ $\quad \Rightarrow \quad F = 10^{NF/10}$

1999 - Wooley [redacted] Answer is wrong.

This circuit is an integrator. S resets the whole system.

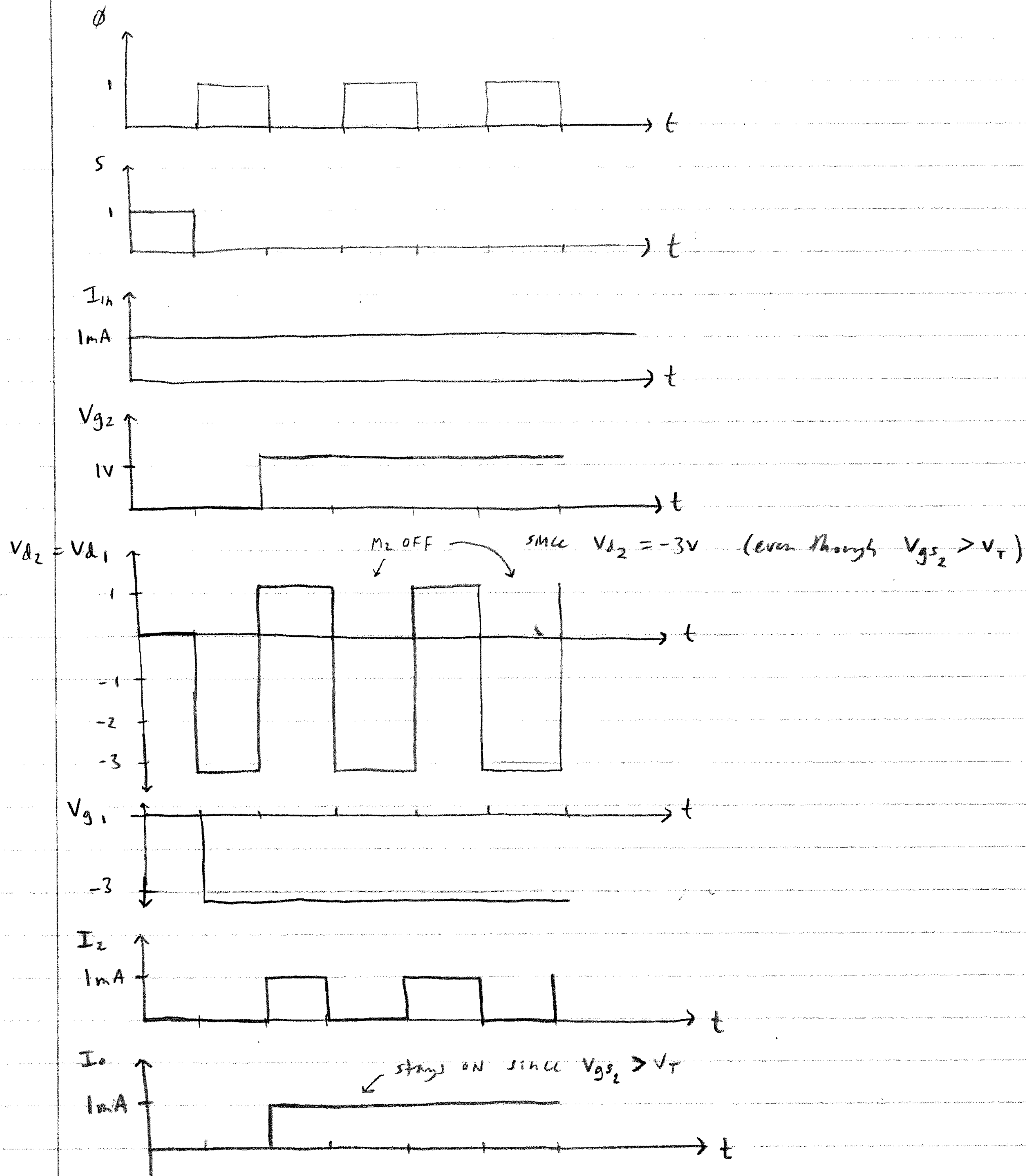


ϕ cycle	I_0	I_1
1	0	9
2	1	8
⋮	⋮	⋮
9	8	1
10	9	0
11	10	0
12	10	-1
13	11	-1
14	11	-2

? What happens here.

See next page.

I_s	10	0	0	0	0	0
I_1	0	9	9	9	9	9
I_2	0	0	1	0	1	0



In this case $I_o \neq I_2$ due to the switching.
But since V_{gs_2} is held on C_{gs_2}, I_o remains constant
at 1mA after the first cycle.

2003 - Shenoy

1. a) Triode : $I_{ds} = \mu C_{ox} \frac{W}{L} (V_{gs} - V_t - \frac{V_{ds}}{2}) V_{ds}$

Sat : $I_{ds} = \frac{1}{2} \mu C_{ox} \frac{W}{L} (V_{gs} - V_t)^2$

2. a) $V_1 = \frac{V}{2}$ both will be ON

b) Saturation

c) $V_1 = \frac{V}{2}$??

3. $W_2 = \frac{1}{3} W$

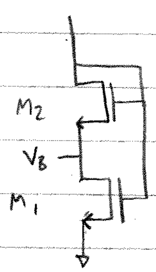
?

M_1 is sat, M_2 in triode

$$\frac{k'}{2}\left(\frac{W}{L}\right)_1 V_{ov}^2 = \frac{k'}{2}\left(\frac{W}{L}\right)_2 \left[2(2V_{ov}) V_{ov} - V_{ov}^2\right] \Rightarrow \left(\frac{W}{L}\right)_1 = 3\left(\frac{W}{L}\right)_2 \Rightarrow W_2 = \frac{1}{3} W_1$$

$$V_{dstr} = \left(\sqrt{k+1} - 1\right) V_{ov}$$

Sooch Cascode:



M2 must be in sat, M1 must be in linear.

$$\Rightarrow I_{d2} = I_{d1} \Rightarrow \frac{1}{2} \cancel{\mu C_{ox}} \left(\frac{W}{L}\right)_2 (V - V_B - V_T)^2 = \cancel{\mu C_{ox}} \left(\frac{W}{L}\right)_1 \left(V - V_T - \frac{V_B}{2}\right) V_B$$

let $k = \left(\frac{W}{L}\right)_2 / \left(\frac{W}{L}\right)_1$, $V - V_T = V_{ov}$

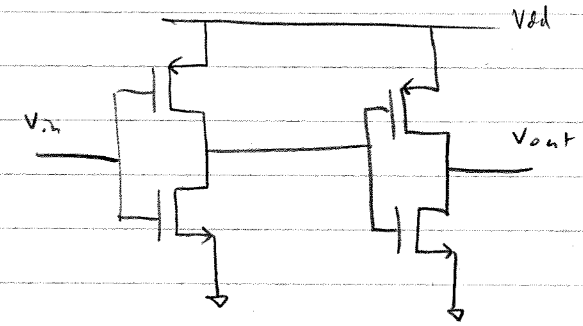
$$\Rightarrow k\left(V_{ov}^2 - 2V_B V_{ov} + V_B^2\right) = 2(V_B V_{ov}) - V_B^2$$

$$\Rightarrow k V_{ov}^2 - 2(k+1) V_B V_{ov} + (k+1) V_B^2 = 0$$

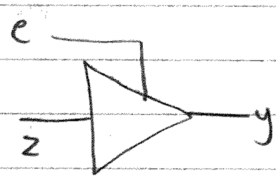
?

1999 - Wong

(1)

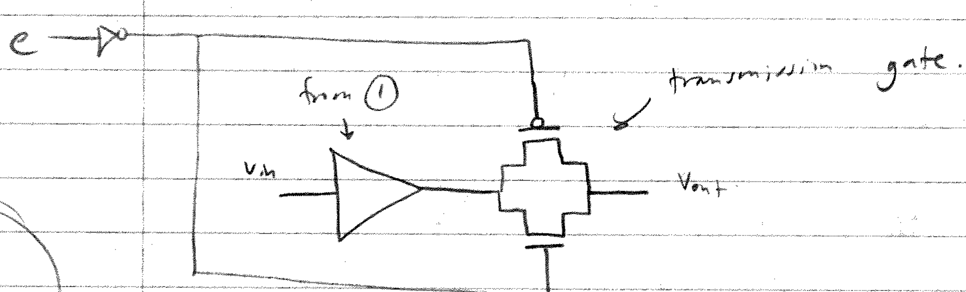


(2)



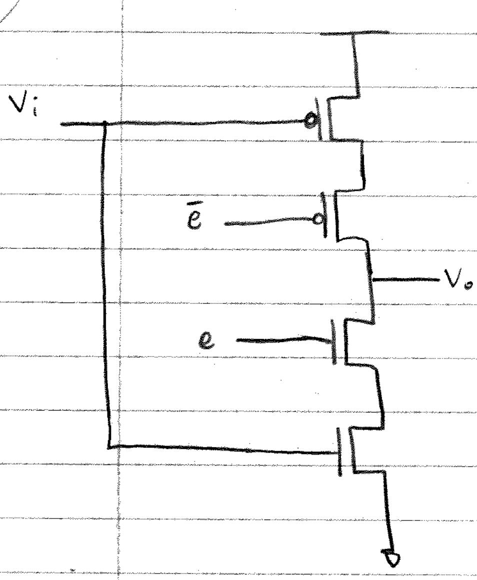
Digital design by Brown.

e	y
1	z
0	high impedance (y is floating)



?

(3)



2006 - Shenoy

1. a) $I_{ds} = \mu C_{ox} \frac{W}{L} (V_{gs} - V_t - \frac{V_{ds}}{2}) V_{ds}$

$g_m = \frac{\partial I_{ds}}{\partial V_{gs}} = \mu C_{ox} \frac{W}{L} V_{ds}$ $\quad V_{ds} < V_{GS} - V_T$

b) Gain is larger for Saturation mode:

$V_A >> V_{ds}$

Saturation: $g_{m_{sat}} = \mu C_{ox} \frac{W}{L} \underbrace{(V_{gs} - V_t)}_{> V_{ds}}$ $\quad R_{ds_{sat}} \approx r_0 \approx \frac{1}{\lambda I_{ds}} = \frac{V_A}{I_{ds}}$

Triode: $g_{m_{triode}} = \mu C_{ox} \frac{W}{L} V_{ds}$ $\quad R_{ds_{triode}} = \left(\frac{\partial I_{ds}}{\partial V_{ds}}\right)^{-1} \approx \frac{V_{ds}}{I_{ds}}$

$\frac{\partial I_{ds}}{\partial V_{ds}} = \mu C_{ox} \frac{W}{L} (V_{gs} - V_t - V_{ds}) \approx \mu C_{ox} \frac{W}{L} (V_{gs} - V_t)$ for $V_{ds} << V_{gs} - V_t$

$\Rightarrow R_{ds_{triode}} \approx \frac{1}{\mu C_{ox} \frac{W}{L} (V_{gs} - V_t)} \approx \frac{V_{ds}}{I_{ds}}$ for $V_{ds} << V_{gs} - V_t$

$\Rightarrow A_v = g_m R_{ds}$ $\quad g_{m_{sat}} > g_{m_{triode}}$ and $R_{ds_{sat}} > R_{ds_{triode}}$

$\Rightarrow A_{v_{sat}} > A_{v_{triode}}$.

2. a) ZVTC: $\quad f_{3dB} \approx \left(\sum_{i=1}^{3} R_i C_i\right)^{-1}$

$\Rightarrow$ dominant time constant is largest $R_i C_i$

$\Rightarrow$ should decrease the capacitor that has the largest RC time constant.

b) No. ZVTC can only be used to estimate the dominant pole. The individual RC time constants do not correspond to individual poles in the system.
(but if $R_{dom} C_{dom} >> R_i C_i \ \forall i$, $\frac{1}{R_{dom} C_{dom}} \approx f_{3dB}$.)

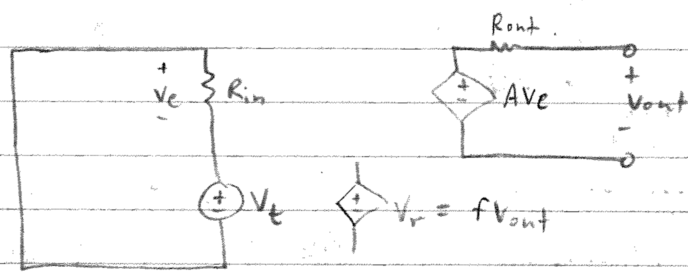
3. a) $V_{out} = A V_e$ $\quad V_e - V_{in} + f V_{out} = 0 \Rightarrow V_e = V_{in} - f V_{out}$

$V_{out} = A (V_{in} - f V_{out}) \Rightarrow V_{out} (1 + Af) = A V_{in} \Rightarrow \frac{V_{out}}{V_{in}} = \boxed{\frac{A}{1+Af} = G}$

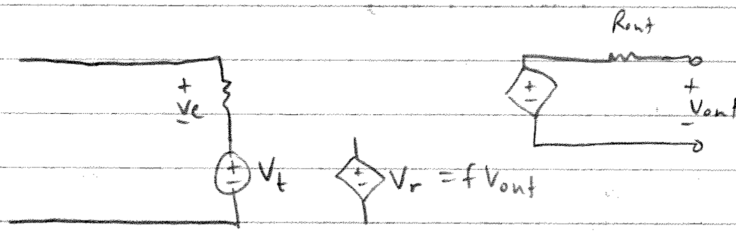
b) Blackman's Impedance Formula:

$$Z_{port} = Z_{port(k=0)} \frac{1 + R\,(\text{port shorted})}{1 + R\,(\text{port open})}$$

$$\Rightarrow R_{in,f\neq 0} = R_{in,f=0} \frac{1 + R\,(\text{port shorted})}{1 + R\,(\text{port open})}$$



$$R\,(\text{port shorted}) = -\frac{V_r}{V_t} = -\frac{-fAV_t}{V_t} = Af$$



$$R\,(\text{port open}) = -\frac{V_r}{V_t} = -\frac{0}{V_t} = 0$$

$$\Rightarrow R_{in,f\neq 0} = R_{in,f=0} \frac{1 + Af}{1 + 0}$$

$$\Rightarrow \boxed{R_{in,f\neq 0} = (1 + Af)\, R_{in,f=0}}$$

	Shunt	Series
Input	I	V
Output	V	I

OR (table above)

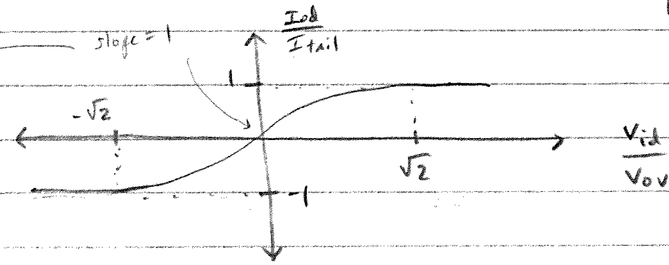
$\Rightarrow$ This is Series - Shunt

$$\Rightarrow \quad Z_i' = Z_i(1+T) \qquad Z_o' = \frac{Z_o}{(1+T)}$$

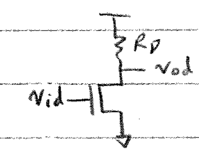
$$\Rightarrow \quad \boxed{R_{in,f\neq 0} = R_{in,f=0}(1+Af)}$$

4. a) $\dfrac{I_{od}}{I_{tail}} = \dfrac{V_{id}}{V_{ov}}\sqrt{1-\left(\dfrac{V_{id}}{2V_{ov}}\right)^2} \qquad \Rightarrow \quad \boxed{V_{ID} = \sqrt{2}\,(V_{GS0}-V_t)}$

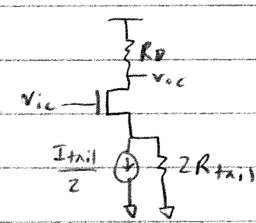
$\Rightarrow G_m = \dfrac{I_{tail}}{V_{ov}}$ slope = 1



b) $A_{dm} = \dfrac{v_{od}}{v_{id}} \approx -g_m R_D$ half circuit:



$A_{cm} = \dfrac{v_{oc}}{v_{ic}} \approx \dfrac{-g_m R_D}{1+g_m(2R_{tail})}$ half circuit:

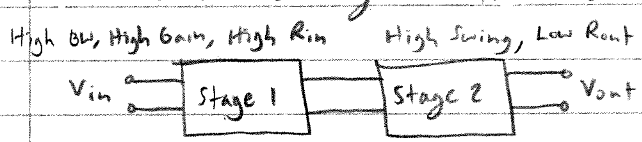


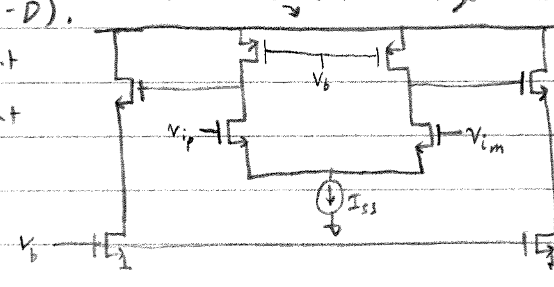
$CMRR = \left|\dfrac{A_{dm}}{A_{cm}}\right| \approx 1 + 2g_m R_{tail}$

$(=\infty$ if $R_{tail}=\infty$, i.e. ideal tail current source)

5. A basic 2-stage op-amp consists of a voltage amplifier (C-S) and a voltage buffer (C-D).

can add cascodes to C-S stage to increase gain.





1998 - Dutton

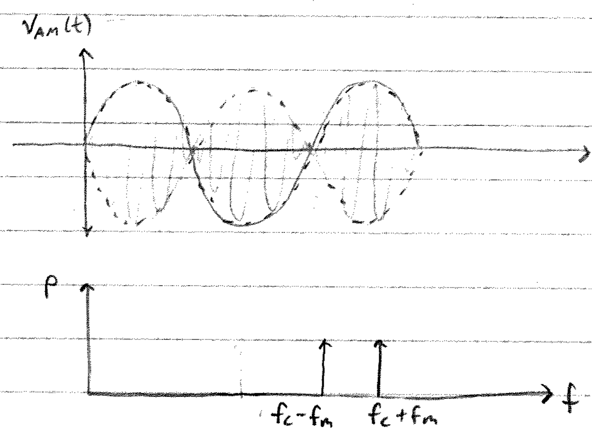
1. Ideal AM signal: for single tone

$$v_c(t) = V_c \sin(\omega_c t) \qquad v_m(t) = V_m \sin(\omega_m t)$$

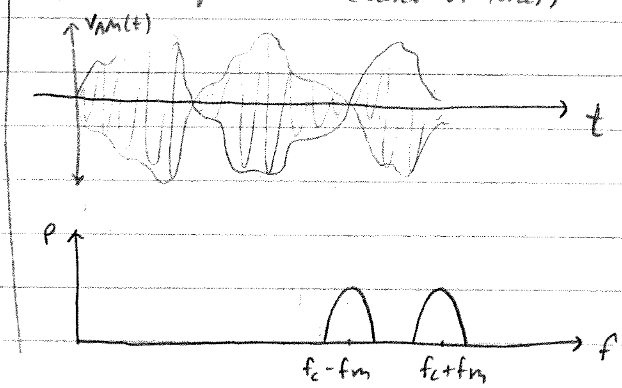
carrier modulating signal

$$\Rightarrow v_{AM}(t) = V_c V_m \sin(\omega_c t)\sin(\omega_m t)$$

$$= \frac{V_c V_m}{2}\left[\cos((\omega_c - \omega_m)t) - \cos((\omega_c + \omega_m)t)\right]$$



for multiple tone (band of tones)

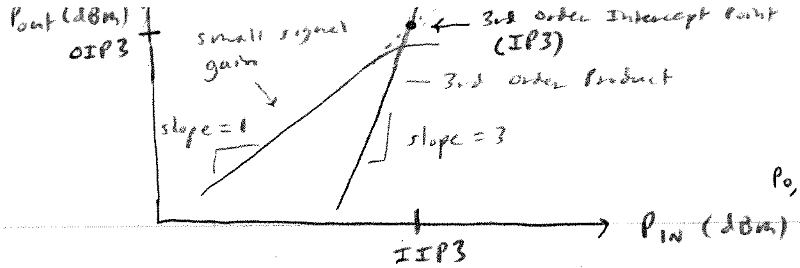


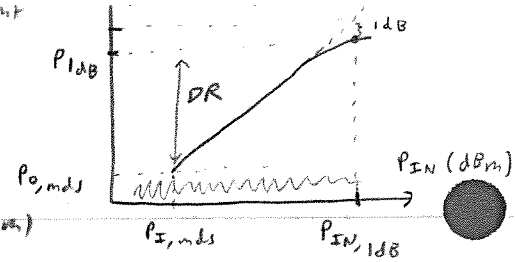
2. What are the limitations imposed by a real power amplifier on the ideal signal... how is the signal changed due to these real effects?

Give details of how a single transistor amplifier affects the results (for either MOS or BJT, what happens when the signals get too large?)

- MOS and BJT nonlinearity causes HD and IM.
  MOS $\propto V_{in}^2$ BJT $\propto \exp(V_{in}) = 1 + V_{in} + \frac{V_{in}^2}{2!} + \frac{V_{in}^3}{3!} + \ldots$

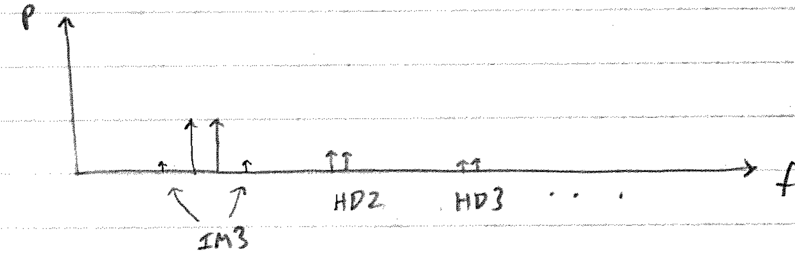
- Differential circuits help eliminate even harmonics, but we can't do this here since we have a single transistor amplifier.



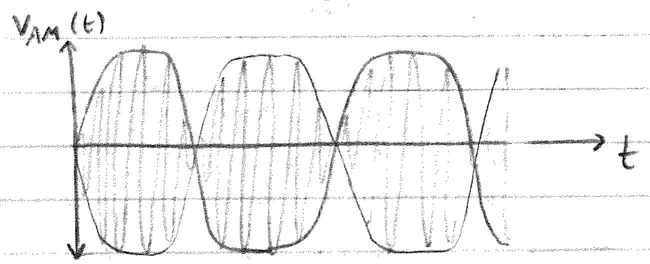


How does it look in frequency and time?

frequency:



If there are other signals being broadcast near your band, IM3 from your signal can interfere with theirs (if TX) or IM3 from their signals can interfere with your signal (if RX).



Gain compression ~~which~~ causes nonlinearity and HD.

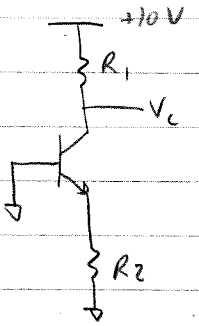
IM3 and HD3 will be worse for BJT than for MOS.

Ask Dutton ?

3. Give details of impact of the passive network (i.e. only finite gain is possible and what happens when the signals get too large) How does it look in frequency & time?

2004 - Shenoy

(1)



$V_c = V_{DD} = 10V$

BJT must be in cutoff.

(2) $g_m = \frac{I_c}{V_T} = \frac{q I_c}{kT}$ $\qquad I_c = I_s \exp\left(\frac{V}{V_T}\right) \Rightarrow \frac{\partial I_c}{\partial V} = \frac{1}{V_T} I_s \exp\left(\frac{V}{V_T}\right) = \frac{I_c}{V_T}$

$= \sqrt{2\mu C_{ox} \frac{W}{L} I_d}$

(3) $g_{m_{sat}} = \mu C_{ox} \frac{W}{L} (V_{gs} - V_t)$ $\qquad g_{m_{triode}} = \mu C_{ox} \frac{W}{L} V_{ds}$

(4) $\omega_t = \frac{g_m}{C_{gg}} = \frac{g_m}{C_{gs} + C_{gd}} \Rightarrow f_t = \frac{g_m}{2\pi (C_{gs} + C_{gd})} = \frac{3}{2} \frac{\mu V_{ov}}{L^2}$

(5) $L = 32\ nm \qquad f_t \approx 100\ GHz$

? check this.

(6) Can use ZVTC if there is a dominant pole.

$\Rightarrow f_{3dB} = \left(2\pi \sum_{i=1}^{n} R_i C_i\right)^{-1}$

(7) Can increase 3dB freq. by reducing L, increasing gm (burns more power). Want to increase gm by increasing Id (Vov) while keeping W constant, since increasing W will increase Cgs, Cgd, etc...

(8) Phase margin $\Rightarrow$ Bode criterion $\qquad PM = \phi\Big|_{|T(s)|=1} - (-180^\circ)$

Gain margin

T(s), RR $\qquad$ of Loop gain. $\qquad GM = |T(s)|\Big|_{\phi(s) = -180^\circ}$

(9) All closed loop poles should be in left half plane.

To assure stability you must meet the Bode criterion.

$\Rightarrow$ Phase margin $> 0^\circ$. In practical circuits, want Phase margin $> 30^\circ$ at least.

$|T(s)| < 1$ when $\phi(s) = -180^\circ$

2005 Shenoy

1. a) $(2.5 + V_{in} - V_T)^2 = (2.5 - V_{out} - V_T)^2$

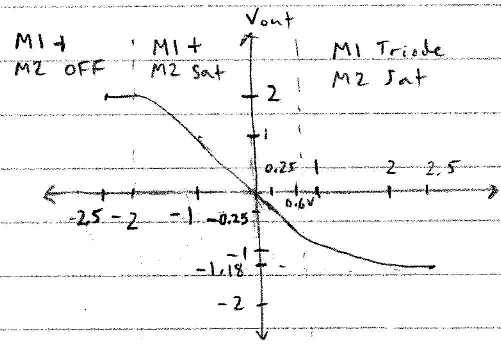
diode connected ⇒ M2 is always in SAT (or cutoff)

$V_{GD} < V_T$

when $V_{in} < -2$, M_1 off ⇒ $V_{out} = V_{dd} - V_T$

when $V_{in} > 0.25$ M_1 is in Triode.

$-2 < V_{in} < 0.25\,V$ M_1 is in Saturation.



$V_{in} = 2.5\,V$

$\Rightarrow \frac{1}{2}\cancel{\mu C_{ox}}\frac{W}{L}(2.5 - 0.5 - V_{out})^2$

$= \cancel{\mu C_{ox}}\frac{W}{L}\left(5 - 0.5 - \frac{V_{out} + 2.5}{2}\right)(V_{out} + 2.5)$

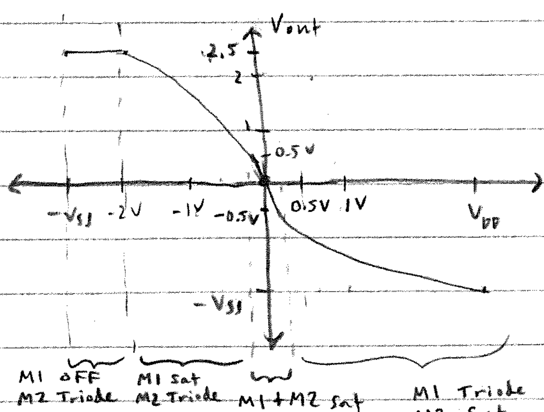
$\Rightarrow V_{out} = -1.18\,V$

b) $V_{out_{max}} = V_{DD} - V_T = 2V$

$\sqrt{8}$

$2\sqrt{2}$

2. a)



$2 + \sqrt{2}$

$2 - \sqrt{2}$

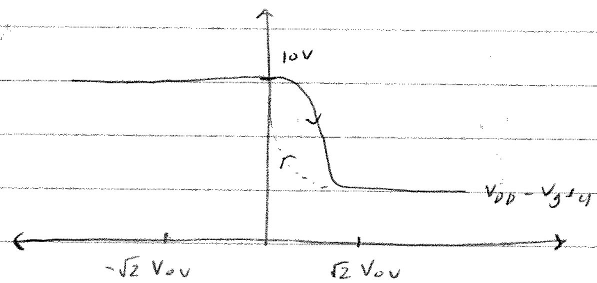
b) $V_{out_{Max}} = V_{DD}$

c) Gain is larger $-g_{m_1}(r_{o_1} \| r_{o_2})$ vs $-g_{m_1}(r_{o_1} \| \frac{1}{g_{m_2}}) \approx -\frac{g_{m_1}}{g_{m_2}} \approx -1$

slope of Vout vs. Vin curve $\left(\frac{V_{out}}{V_{in}}\right)_{MAX}$ is clearly larger at $V_{in} = 0$, since M1 + M2 are in SAT, but in this case, M2 is not diode connected.

Check this.

? (3.) a)



b) Gilbert cell. Used to cancel odd harmonics.

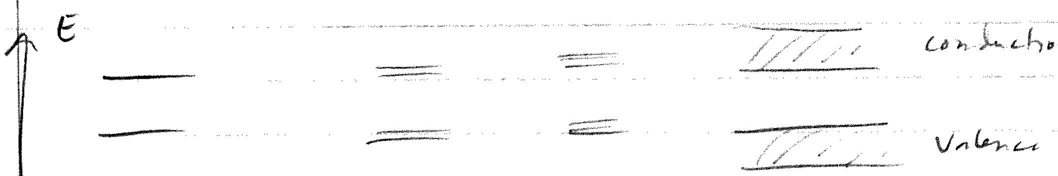
([redacted] says oscillator)

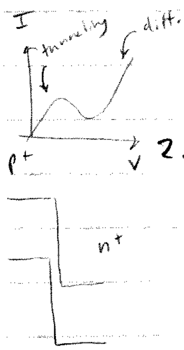
??

1995 - Dutton

1.) Lattice constants of semiconductors are on the order of 10 Å (5Å for Si) ⇒
which is comparable to the wavefunction of an electron.

Quantum Mechanics states that for an isolated atom, there are a finite number of energy levels that an electron can occupy. Thus, electron energy is quantized. When you bring many atoms together, the levels split and form what look like continuous bands:





2. light spectra of hydrogen gas. shows discrete lines corresponding to transition between quantized states,
Black body radiation, negative R of tunnel diode

3. electrons can only move as waves if the lattice is perfect (no vibrations ⇒ T = 0). Otherwise at T > 0, there is lattice scattering.

For example, for a resistor this will cause thermal noise: $i_n^2 = \frac{4kT}{R}$

Wave properties would be more observable in photonic devices

See next page.

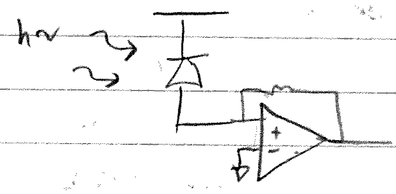
[redacted] answers.

1) semiconductors are composed of many atoms coming together. existence of energy bands and band gaps can only be explained by quantum mechanics.

? look up photoelectric effect.

2)

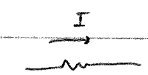
$h\nu$

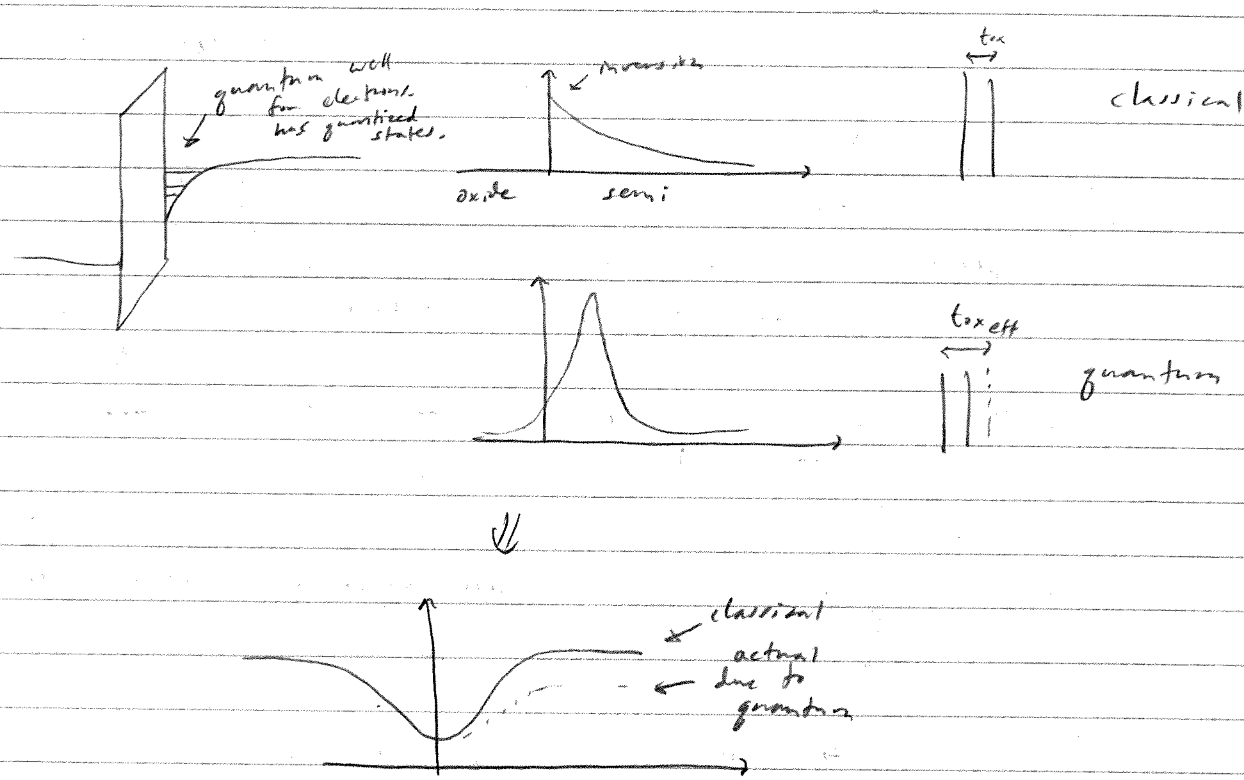


change ν until e^-, h^+ pairs are generated.
this shows that there is a bandgap.

OR photoelectric effect.

3. $I = q \cdot \frac{N}{t}$ ← # of e^-





???

1996 - Dutton

1) $0.1\,\mu m \times 0.1\,\mu m \qquad t_{ox} = 50\,\text{Å} = 5\,nm$

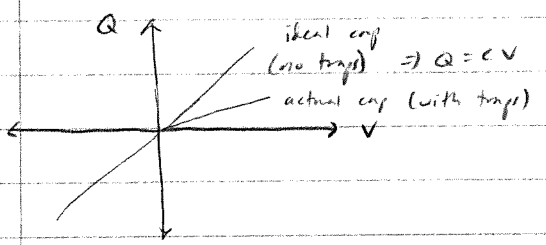
$$\rightarrow \quad C_{ox} = \frac{K\epsilon_0 A}{t_{ox}} \approx \frac{4 \quad 8.85\cdot 10^{-14}\,F/cm \quad (0.1)^2 \cdot 10^{-8}\,cm^2}{5\cdot 10^{-7}\,cm}$$

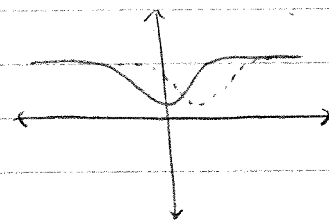
$$\approx 0.8 \cdot 8.85 \cdot 10^{-17}\,F = 7\cdot 10^{-17}\,f$$

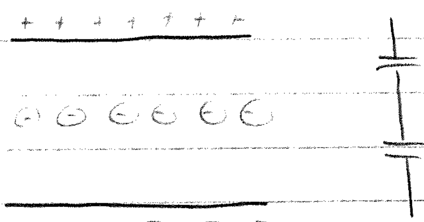
$$Q = C\overset{1V}{V} \Rightarrow \boxed{Q = 7\cdot 10^{-17}\,C}$$

$$\frac{Q}{q} = \frac{7\cdot 10^{-17}\,C}{1.6\cdot 10^{-19}\,C} \approx 4\cdot 10^{2} = \boxed{400\ e^-}$$

2)





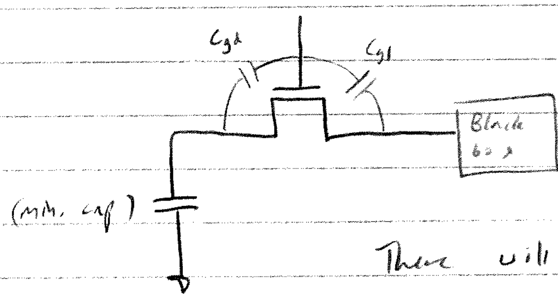


caps in series ⇒ overall C ↓

⇒ slope of Q-V curve decreases.

Charge moves onto traps through tunneling.

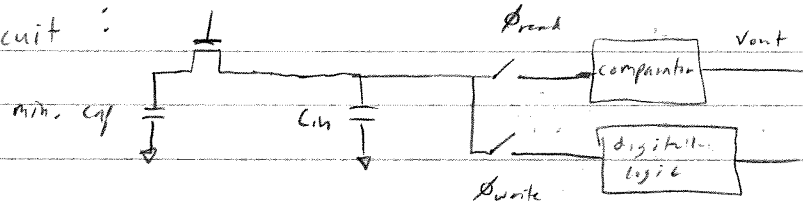
3)



need $C_{gd} \ll$ min cap.

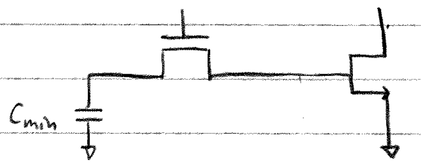
There will be charge sharing between min. cap and C_p. Also, I_{off} of pass transistor and I_{in} of Black box will continuously discharge min cap ⇒ there is a limit to how long the memory can hold its state.

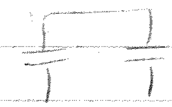
Black box circuit :



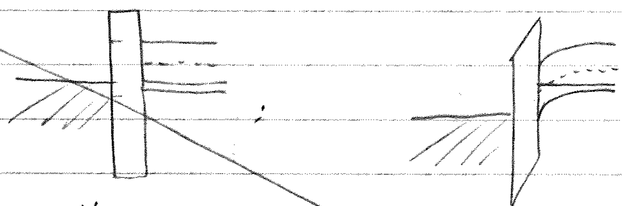
more detailed

for read :

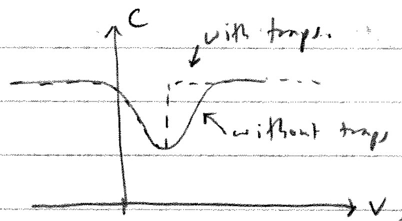




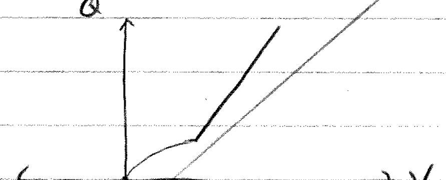
2)



$E_F > E_T$ at surface
⇒ traps fill with e^-s
⇒ since charge doesn't saturate, this is effectively like inversion, ⇒ $C \to C_{ox}$

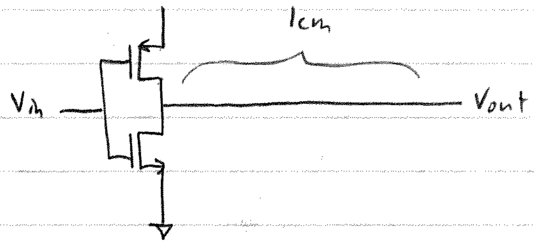


$Q = VG$



1994 Dutton

1.



$$\Rightarrow \frac{1}{\sqrt{LC}} = \frac{c}{\sqrt{\varepsilon_r \mu_r}}$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \qquad v_{prop} = \frac{1}{\sqrt{\varepsilon \mu}} = \frac{c}{\sqrt{\varepsilon_r \mu_r}}$$

For lossless transmission line: velocity factor $VF = \frac{v_{propagation}}{c} = \frac{1}{c\sqrt{LC}}$

$\Rightarrow VF \leq 1$

where $c = 3 \cdot 10^8 m/s$ $\quad$ ✱ $v_{propagation} = \frac{1}{\sqrt{LC}} < c$

$\frac{1}{\sqrt{\mu \varepsilon}}$

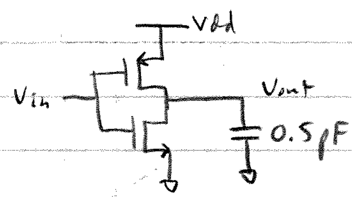
L is distributed inductance (Henries per unit length)

C is distributed capacitance (Farads per unit length)

$\Rightarrow$ Absolute fastest it could be without inversion is:

$$\frac{1cm}{c \cdot VF} > \frac{1cm}{c} = \frac{10^{-2} m}{3 \cdot 10^8 m/s} \approx 3.33 \cdot 10^{-11} s = \boxed{33.3 ps}$$

But with inversion it will actually take longer than this since the inverter will have a propagation delay.



$$t_{p_{HL}} = \ln(2) R_{eq_n} C_L$$

$$\approx 0.69\left(\frac{3}{4}\right) \frac{C_L V_{dd}}{I_{d_{sat_n}}} = 0.52 \frac{C_L V_{dd}}{(\frac{W}{L}) k_n' V_{D sat} (V_{dd} - V_T - V_{Dsat_n}/2)}$$

for $V_{dd} >> V_{d_{sat_n}}$ $\quad t_{p_{HL}} \approx 0.52 \frac{C_L}{(W/L)_n k_n' V_{D_{sat_n}}}$

$\Rightarrow$ we are limited by the time it takes to charge $C_L \approx 0.5 pF$.

to decrease t_p, could increase $\frac{W}{L}$, Vdd (since $V_{dsat} = V_{gs} - V_T = V_{dd} - V_T$), μC_{ox} or decrease V_T.

can only do this to an extent. After some point C_{gd_n} and C_{gd_p} (which get miller multiplied at the output by a factor of 2) will dominate 0.5pF and C_L ↑

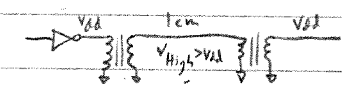
? → (What is the dynamic power dissipation of the Inverter (i.e. in the transition from high to low, when both M_n and M_p are in saturation, there is power dissipated in the channel resistance)

2. Energy requirements: Must charge wire (0.5 pF)

$$\Rightarrow \boxed{E_{min} = \frac{1}{2} C_w V_{dd}^2} = \frac{1}{2}(0.5\,pF)\,V_{dd}^2$$

↑ fundamental limit.

It will actually take more energy due to dynamic power dissipation in the parasitic resistance of the wire during the charging transient.

$P_w(t) = i(t) R_w$ can minimize $i(t)$ by transforming the voltage to high voltage. 

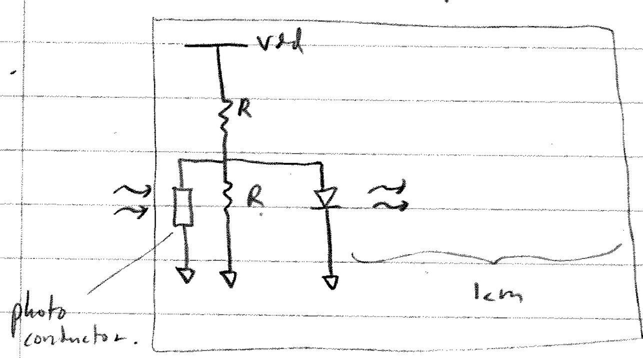
then transforming it back down.

After the inversion, $i_w = 0 \Rightarrow P_w = 0$

$\Rightarrow$ Zero static power in the wire.

But there is static power in the inverter due to finite gate leakage current and finite I_{off} (subthreshold current).

Could also do an optoelectronic based system:

1. 

Thus when V_{in} is low (no light incident), $R_{photoconductor}$ is high $\Rightarrow$ LED driven ON

$\Rightarrow$ V_{out} is high (light emitted).

When V_{in} is high (light incident), $R_{photoconductor}$ is low $\Rightarrow$ LED OFF

$\Rightarrow$ V_{out} is low (no light emitted).

Must have $R_{photoconductor_{Light}} \ll R \ll R_{photoconductor_{Dark}}$

propagation time limited by speed of light $\Rightarrow$ $\boxed{33.3\ ps}$

plus it will take time for photoconductor to respond to incident light.

due to transit time of carriers. (can speed up by decreasing length of photoconductor, increasing V across it, or decreasing carrier lifetime).

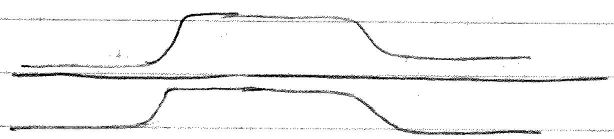
2. Absolute minimum energy requirement would depend on how many photons are required to consider the output high.

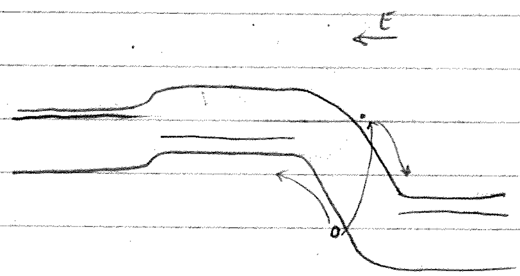
In this implementation, there is high static power since current is always flowing through $R_{photoconductor} \parallel$.

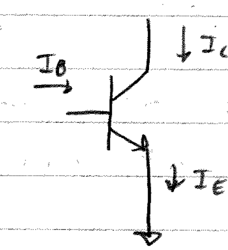
1994 - Wong

1. For $V_{CB} = 2V$, why is the base current negative at low V_{BE} ($V_{BE} < 0.4V$)? Hint $I_c = -I_B$ at low V_{BE}

$I_E = I_B + I_C$







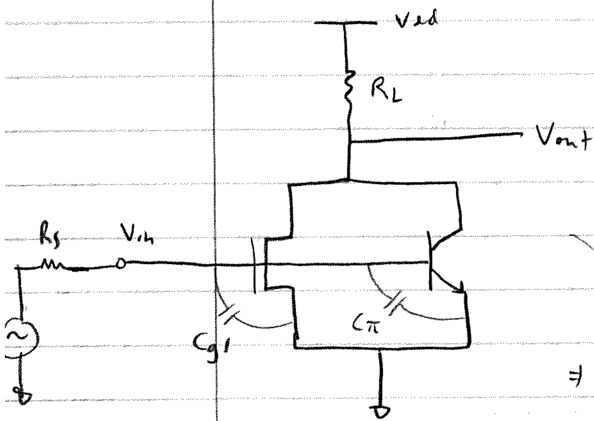
For high V_{BE}, many $\bar{e}$'s diffuse into the depletion region due to the lowered barrier and some recombine with holes. These holes must be replaced and thus I_B is positive.

For low V_{BE}, fewer $\bar{e}$'s diffuse into the depletion region,
$\Rightarrow$ Current is actually dominated by generation in the B-C depletion region.
These holes flow into the base and must be removed
($I_E = I_B + I_c$, we are given that $I_c = -I_B \Rightarrow I_E = 0$)
$\Rightarrow I_B$ is negative and equal to I_c.

2. For $V_{CB} = 3V$, why is the base current negative even for $V_{BE} = 0.6$ to $0.8V$? Hint: The negative base current is not observed at lower V_{CB} (2V). The negative base current tracks the collector current.

Similar to part 1), negative base current means excess holes are generated in the reverse biased base-collector junction. Since the negative base current tracks the collector current and only occurs at higher V_{CB}, it related to the collector current ($\bar{e}$s) going through a larger potential drop (high $\vec{E}$). The highly accelerated electrons can cause impact ionization and generate excess electron-hole pairs.

2006 - Dutton



for max small signal voltage gain, at what operating point should we bias the new device?

$$I_{out} = \frac{1}{2}\mu C_{ox} \frac{W}{L} (V_{in} - V_{Th})^2 + I_o e^{V_{in}/V_T}$$

$$\Rightarrow g_m = \mu C_{ox} \frac{W}{L} (V_{in} - V_{Th}) + \frac{1}{V_T} I_o e^{V_{in}/V_T}$$

$\Rightarrow$ want to maximize V_{in} such that MOS stays in sat and BJT stays in forward active.

$\Rightarrow$ $V_{dd} - IR_L > V_{in}$ Assume BJT current dominates.

$\Rightarrow$ $V_{dd} - I_o e^{V_{in}/V_T} R_L = V_{in}$ $\Rightarrow$ solve for V_{in}.

AC gain and bandwidth: $A_v = -(\underbrace{g_{m_{MOS}}}_{\mu C_{ox}\frac{W}{L}V_{ov} = 2I_d/V_{ov}} + \underbrace{g_{m_{BJT}}}_{I_c/V_T}) R_L$

$$f_{3dB} \approx \frac{1}{2\pi\left[(1+A_v)\left[C_{gd} + C_\mu\right] + C_{gs} + C_\pi\right] R_s}$$

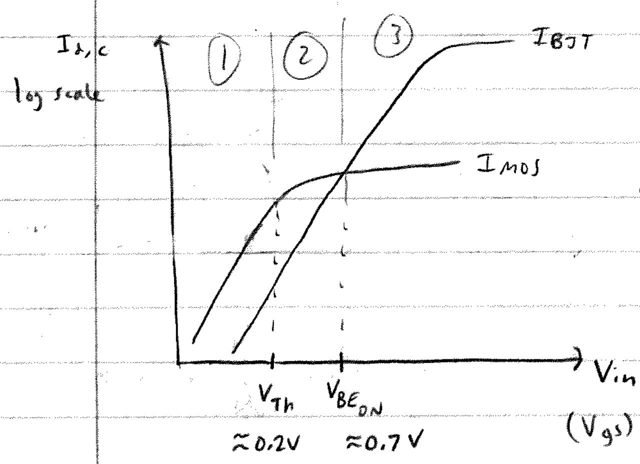
[redacted] $G_m = \frac{\partial I}{\partial V_{in}} = \frac{\partial I_D(MOS)}{\partial V_{in}} + \frac{\partial I_C(BJT)}{\partial V_{in}} = g_{m_{MOS}} + g_{m_{BJT}}$

~~Answer~~:

$= \mu C_{ox} \frac{W}{L} V_{ov} + \frac{I_c}{V_T}$

To maximize G_m, we should maximize $(I_c + I_{ds})$

$\Rightarrow$ we should bias in region 3.



AC gain: $a_v = -G_m R_L \approx -\frac{I_c}{V_T} R_L$

AC bandwidth: $\frac{1}{R_s(C_{gs} + C_\pi + (1+|a_v|)(C_{gd} + C_\mu))}$

In other words, this device is not very useful. In 3, it looks like a BJT with $C_\pi' = C_\pi + C_{gs}$, $C_\mu' = C_\mu + C_{gd}$. In 1 and 2, it looks like a MOSFET with $C_{gs}' = C_{gs} + C_\pi$ and $C_{gd}' = C_{gd} + C_\mu$. $\Rightarrow$ lower BW $\Rightarrow$ lower GBW product.

2004 - Dutton

$$C = \frac{\Delta Q}{\Delta V}$$

minimum Q is $q = 1.6 \cdot 10^{-19}$ C

maximum V is $V_{BR} \approx 10$ kV

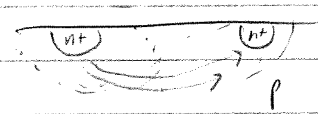
$\Rightarrow$ minimum C is $\sim \frac{10^{-19}}{10^4} = 10^{-23}$ F

1998 - Wong

1. What mechanisms are responsible for breakdown of MOSFET at large V_{DS}.

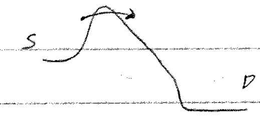
?

Punch through



depletion regions touch
⇒ current flows below channel.

How does punchthrough ~~work?~~
impact ionization?



DIBL ⇒ tunneling

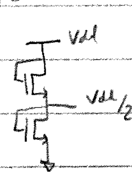
Reverse Breakdown of D-substrate diode

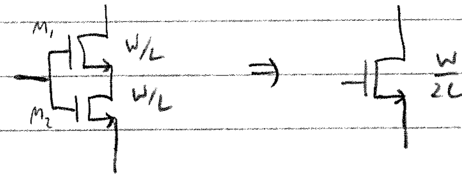
2. Given a MOSFET with $V_{BR} = 10V$, how can you modify the device or circuit to use it with $V_{DD} = 15V$

To reduce punchthrough, you can increase L, increase the doping of the bulk (to reduce size of S-D depletion regions), Decrease the doping of the drain (lightly doped drain) ⇒ lower $\vec{E}$ field for given V ⇒ higher V_{BR} between D-substrate.

To increase L at a circuit level, you can combine two transistors as such:

Or could divide Vdd by two as such:

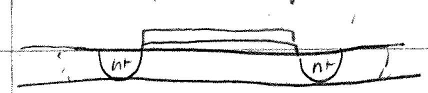




If M_1 is in SAT, M_2 must be in Linear.
Voltage drop across each transistor will be less.

?

To prevent punchthrough, could use SOI.



SiO_2

Now depletion regions can't touch below the channel.

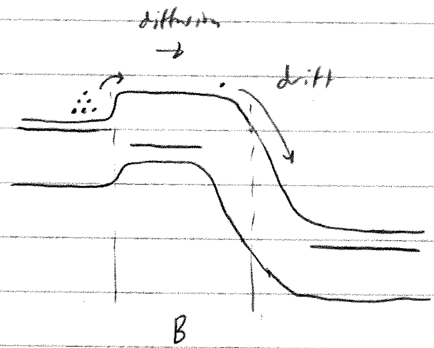
1997 - Wong

n+	p	n

n+	p	n+

Compare the transit time of carriers inside the base of a BJT and that along the channel of a MOSFET.

Assume BJT in forward Active + MOS in Saturation.



BJT will have $\tau_{BJT} = \tau_{diff} + \tau_{drift}$

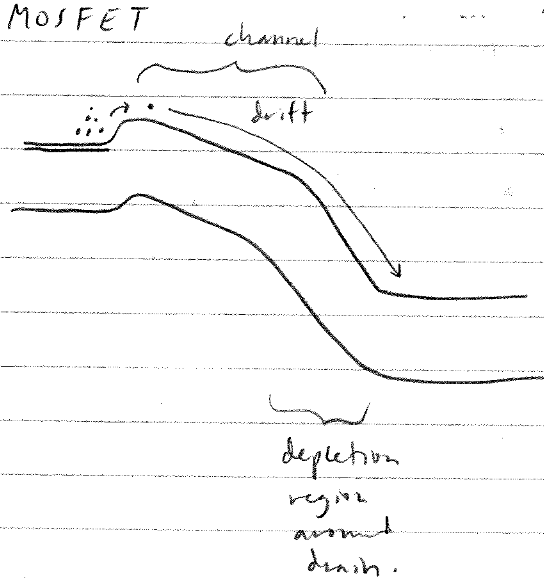
$v_d = \mu \vec{\mathcal{E}}$

mobility of depletion region (BC)

$\vec{\mathcal{E}}$ of BC depletion region. $\approx \frac{V_{CB}}{W_{dep}}$

diffusion is a much slower process than drift. Thus depending on the length of the base, τ_{BJT} could be greater than τ_{MOS}.

MOSFET



MOS will have $\tau_{MOS} = \tau_{drift}$

$v_d = \mu \vec{\mathcal{E}}$

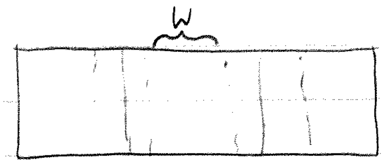
channel mobility

$\approx \frac{V_{DS}}{L}$

[redacted] Answer:

In base of BJT, $\acute{e}$'s are minority carriers & move by diffusion.

$$\tau_{t_{BJT}} = \frac{Q_B}{I_c} \simeq \frac{W^2}{2D_B} = \frac{W^2}{2\frac{kT}{q}\mu}$$ ← W is width of neutral base region.



↑ from EE216 notes.

⇒ for uniform base doping, $\tau_B > \tau_{MOS}$

for graded base doping, τ_B will be less, because there will be an internal electric field in the base which will aid the transport across the base.

$$\tau_{MOS} = \frac{L^2}{\mu V_{DS}} \qquad \tau = \frac{L}{v_d} = \frac{L}{\mu \mathcal{E}} = \frac{L^2}{\mu V_{ds}}$$

$$\tau_{t_{BJT}} = \frac{W^2}{2\left(\frac{kT}{q}\right)\mu_B} \qquad \tau_{MOS} = \frac{L^2}{V_{ds}\,\mu_{CH}}$$

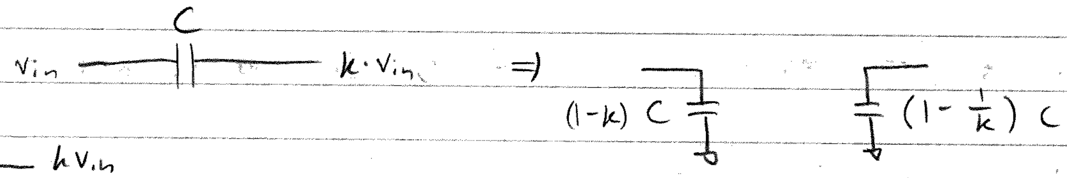
$\mu_{CH} < \mu_B$ due to surface scattering,

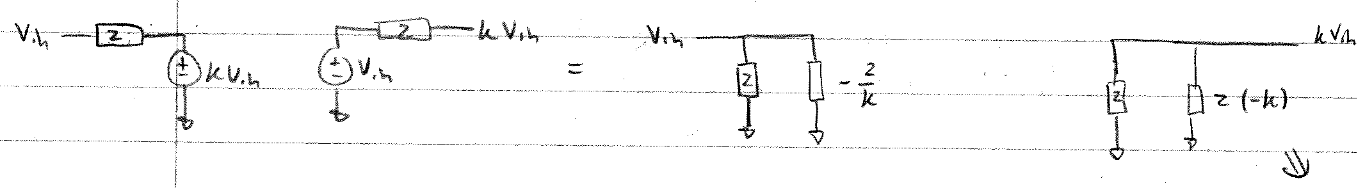
But $V_{ds} >> 2\left(\frac{kT}{q}\right)$ usually.

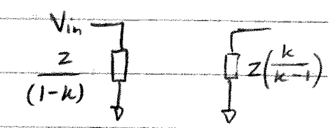
⇒ if $W_{Base} = L_{channel}$, $\boxed{\tau_B > \tau_{MOS}}$

1993 - Dutton

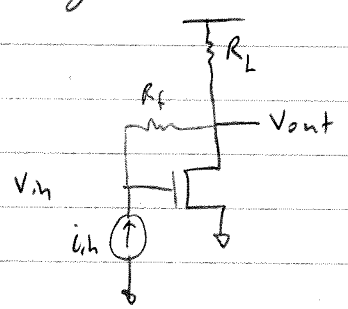
1. What is Miller Capacitance?



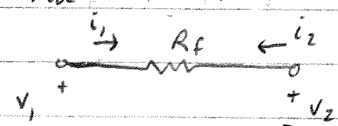




2. How does shunt-shunt feedback resistance effect the input and output impedance of a single stage transistor amplifier.



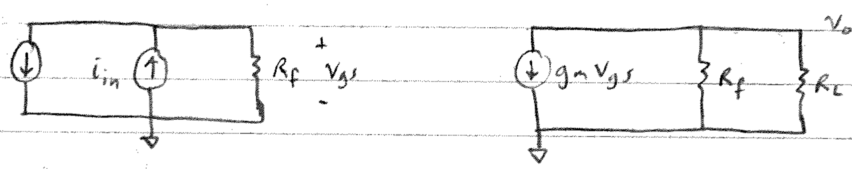
feedback network:



$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} = \frac{1}{R_f} \qquad y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0} = \frac{1}{R_f}$$

$$f = \left.\frac{i_1}{v_2}\right|_{v_1=0} = -\frac{1}{R_f}$$

$\Rightarrow \quad i_{fb} = -\frac{v_o}{R_f}$



$$\Rightarrow \quad a = \frac{v_o}{i_{in}} = -g_m R_f (R_f \| R_L) \qquad f = -\frac{1}{R_f} \quad \Rightarrow \quad af = g_m (R_f \| R_L)$$

$$A_{CL} = \frac{a}{1+af} = -\frac{g_m R_f (R_f \| R_L)}{1 + g_m (R_f \| R_L)} \approx -R_f \quad \text{for} \quad g_m (R_f \| R_L) >> 1$$

$$Z_i = \frac{R_f}{1 + g_m (R_f \| R_L)} \qquad Z_o = \frac{(R_f \| R_L)}{1 + g_m (R_f \| R_L)}$$

⇒ input and output impedance are decreased by $(1+T)$.

to compare to Miller, we must find $K = \left.\frac{V_{out}}{V_{in}}\right|_{CL}$

$$A_{CL} = \frac{V_{out}}{i_{in}} \Rightarrow K = \frac{V_{out}}{i_{in} \cdot Z_{in}} = \frac{A_{CL}}{Z_i} = -g_m(R_f \| R_L) = -T$$

$\Rightarrow$ V_i —R_f— V_o $\Rightarrow$

$$\frac{R_f}{1-K} = \frac{R_f}{1+T} = \frac{R_f}{1+g_m(R_f \| R_L)}$$

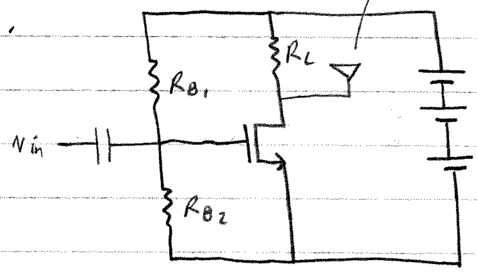
$$R_f\left(\frac{K}{K-1}\right) = \frac{R_f\,(g_m(R_f \| R_L))}{1+g_m(R_f \| R_L)} \approx R_f$$

$\Rightarrow$ This agrees with Miller for the input impedance, but not for the output impedance. $Z_o \simeq (R_f \| R_L) \neq \frac{(R_f \| R_L)}{1+T}$

Not sure exactly why this is, but it may have to do with the fact that in using the Z-port model, we neglect feed forward through R_f.

2007 - Shenoy

1. a) C-S.



Make antenna out of wire. length should be $\sim \lambda/4$

for

$\lambda f = c \qquad \frac{3 \cdot 10^8}{3 \cdot 10^{10}} = 1\text{cm}$

For 30GHz, $\lambda = 1\text{cm}$

AM - 0.3-3 MHz ⇒ 1km - 100m

CB radio - 3-30MHz ⇒ 100m - 10m

FM - 30-300MHz ⇒ 10m - 1m

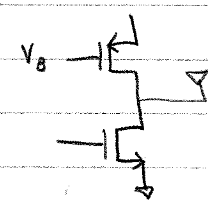
b) Goal with batteries is to maximize the output voltage swing ⇒ stack them in series to get highest possible rail.

c) Goal with resistors is to DC bias the MOSFET and to provide a load R_L for the MOSFET to achieve gain $-g_m R_L$.

d) Yes. With just a resistive load, the maximum gain achievable is $\approx -\frac{2V_{dd}}{V_{ov}}$ since R_L sets both the DC biasing and output load resistance.

$$\left|-g_m R_L\right| = \left|-\frac{2I_d}{V_{ov}} \cdot R_L\right| \quad \text{but} \quad I_d R_L < V_{dd} \Rightarrow \left|-g_m R_L\right| < \left|-\frac{2V_{dd}}{V_{ov}}\right|$$

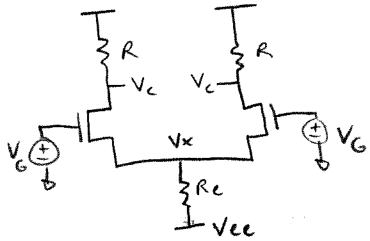
With a PMOS, we can create an active load:
Thus we can get higher gain: $-g_m(r_{o_n} \| r_{o_p})$



e) To increase 3dB frequency reduce any series resistance R_s at the input and if possible, the C_{gs} and C_{gd} of the MOSFETS since the dominant pole of this high gain amplifier will likely be:

$$\omega_{3dB} \approx \frac{1}{R_s(C_{gd})(1 + g_m(r_{o_n} \| r_{o_p}))}$$

due to Miller multiplication of C_{gd}.



2. a) $$\frac{V_x - V_{ee}}{R_e} = 2\, k_n' \left(\frac{W}{L}\right) (V_G - V_x - V_T)^2$$

$$\frac{V_x - V_{ee}}{R_e} = A\,(V_A - V_x)^2 = A\,(V_A^2 - 2V_A V_x + V_x^2) \qquad \text{let } B = AR_e$$

$$\Rightarrow V_{ee} = V_x - B\,(V_A^2 - 2V_A V_x + V_x^2)$$

$$\Rightarrow B V_x^2 - (2BV_A + 1)V_x + (BV_A^2 + V_{ee}) = 0$$

$$\Rightarrow V_x \approx \frac{(2BV_A+1) \pm \sqrt{4BV_A + 1 - 4BV_{ee}}}{2B}$$

$$\Rightarrow V_{ee} \uparrow \Rightarrow V_x \uparrow$$

OR qualitatively: $V_{ee} \uparrow \quad V_x = V_{ee} + 2 I_d R_e \uparrow \Rightarrow I_d \downarrow$

Cutoff when $V_G - V_x < V_T$

Saturation when $V_G - V_x > V_T$ and $V_C - V_G > -V_T$

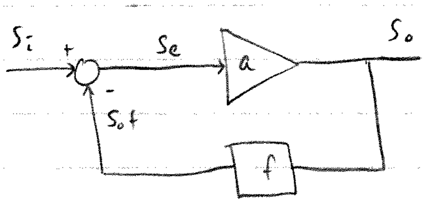
$V_G - V_C < V_T$

Triode when $V_G - V_x > V_T$ and $V_G - V_C > V_T$

$\Rightarrow$ as V_{ee} is increased $\boxed{\text{Triode} \rightarrow \text{Saturation} \rightarrow \text{Cutoff}}$

b) as V_{ee} is increased, V_x increases also.

when $V_{ee} = V_{cc}$, $V_x = V_{cc}$ since MOSFETs are in cutoff.

3. a) 

derive closed loop gain:

$$S_o = (S_i - S_o f)a = aS_i - afS_o$$

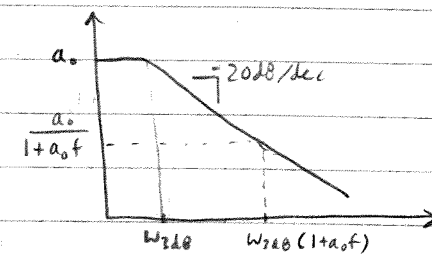
$$\Rightarrow (1+af)S_o = aS_i \Rightarrow \frac{S_o}{S_i} = \boxed{\frac{a}{1+af} = A}$$

b) derive closed loop 3dB frequency if $a(s) = \dfrac{a_o}{1+\frac{s}{\omega_{3dB}}}$

$$A = \frac{\left(\frac{a_o}{1+\frac{s}{\omega_{3dB}}}\right)}{1+\frac{a_o f}{1+\frac{s}{\omega_{3dB}}}} = \frac{a_o}{1+\frac{s}{\omega_{3dB}}+a_o f}$$

$$= \frac{a_o}{(1+a_o f)\left(1+\frac{s}{\omega_{3dB}\cdot(1+a_o f)}\right)} = \frac{A}{1+\frac{s}{\omega_{3dB_{CL}}}}$$

$$\Rightarrow \boxed{\omega_{3dB,CL} = (1+a_o f)\,\omega_{3dB}}$$



c) No. Gain bandwidth product is the same.

$$a_o \cdot \omega_{3dB} = \frac{a_o}{\cancel{(1+a_o f)}} \cdot \omega_{3dB} \cdot \cancel{(1+a_o f)}$$

2005 - Murmann

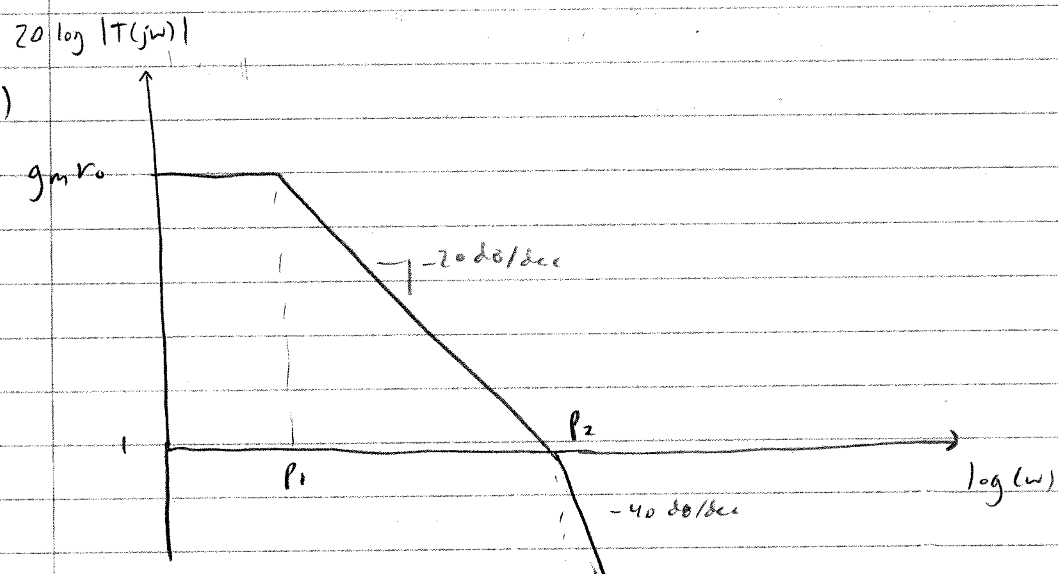
a) $I_{out} = (1+k) I_{in}$. Feedback used to match source voltages of the two right most transistors.
$\Rightarrow$ current amplifier.

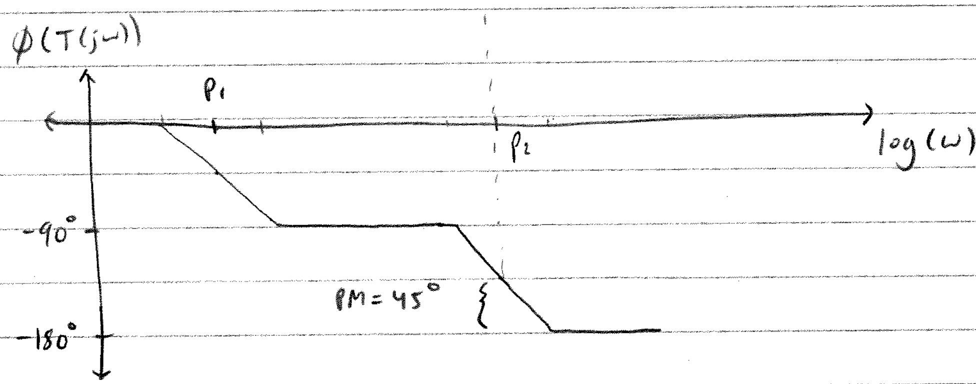
b). By inspection: $T(s) = -g_m \left(\frac{1}{g_m} \| \frac{1}{sC}\right)\left(g_m \left(r_o \| \frac{1}{sC}\right)\right)$

assuming nodes (1) and (2) are isolated

$$\Rightarrow T(s) = -g_m r_o \left(\frac{1}{(1+s r_o C)(1+s\frac{C}{g_m})} \right) \qquad \omega_{p_1} = \frac{1}{r_o C} \qquad \omega_{p_2} = \frac{1}{(\frac{1}{g_m})C}$$

c)





2006 - Murmann

A) a) $A_{vd} = -g_{m_9}\left(\frac{1}{g_{m_1}}\right)$ I_d is the same

$g_m = \mu C_{ox}\frac{W}{L}V_{ov} = \frac{2I_d}{V_{ov}} = \sqrt{2\mu C_{ox}\frac{W}{L}I_d}$

$\Rightarrow g_{m_9} = 3g_{m_1}$ $\Rightarrow$ $A_{vd} = -3$

b) $A_{vd} = -\frac{g_{m_9}}{1+g_{m_7}\left(-\frac{1}{g_{m_4}}\right)g_{m_1}} = -3\left(\frac{1}{1-3/2}\right) = \frac{3}{1/2} = 6$

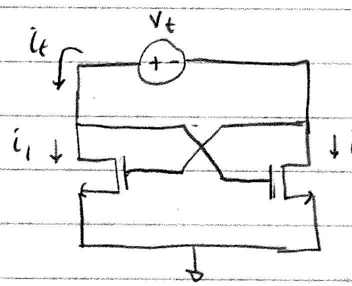
$\Rightarrow$ higher gain. but polarity of output is reversed.

B) positive feedback $\Rightarrow$ possible ~~instability~~, latch up

must have $|T(s)| < 1$ ~~for $\forall s$~~. for $s=0$

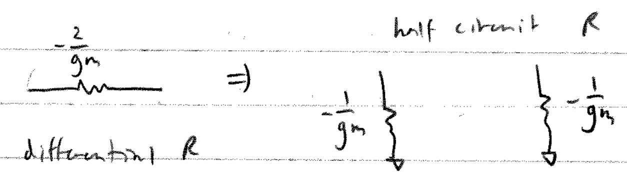
$T(0) = -\left(\frac{g_{m_4}}{g_{m_9}}\right)\left(-\frac{g_{m_4}}{g_{m_9}}\right) = \frac{4}{9}$ $\Rightarrow$ in this case, it should ~~be stable~~, not latch up

but it would if $4W \to 16W$ for example.



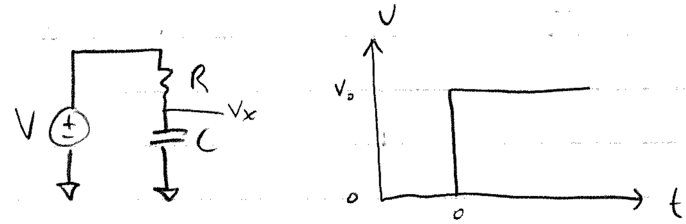
$\Rightarrow i_t = i_1 = -i_2$ $V_t = V_{gs_2} - V_{gs_1}$

$\Rightarrow g_m V_t = g_m V_{gs_2} - g_m V_{gs_1} = i_2 - i_1$
$= -2i_t \Rightarrow \frac{V_t}{i_t} = R_{in} = -\frac{2}{g_m}$



1993 - Meng

1. a)



Derive instantaneous power consumption of RC circuit as a function of time.

$$i(t) = C\frac{\delta V_x}{\delta t} = \frac{V - V_x}{R}$$

$$i(t) = \frac{V_0}{R} e^{-t/RC} \qquad P = i(t) \cdot V_0 = \frac{V_0^2}{R} e^{-t/RC}$$

b) What is total energy needed to charge cap from 0 to V_0.

$$\boxed{\frac{1}{2} C V_0^2} = \int_0^\infty P_c(t)\, dt$$

c) R effectively limits the amount of current that can flow into (charge) the cap. RC time constant determines the time it takes to charge C to V_0.

since $P = \frac{E}{t}$, P is effected by t_{charge} and thus R.

We are changing the voltage instantaneously, but do to the Resistor, the voltage across the capacitor cannot change instantaneously. The delay in the rise time of $V_c(t)$ is related to R.

Energy of the capacitor only depends on the charge required to develop a voltage of V_0 across it.

$Q = CV \Rightarrow E = \frac{1}{2} QV$

2. a) $E = \frac{1}{2} C V_{dd}^2 \Rightarrow$ if V_{dd} is reduced by factor of 2,

$\boxed{E \text{ is reduced by a factor of } 4}$

b) Time would increase. by a factor of ~2.

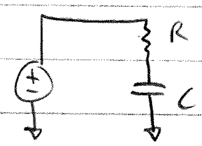
assuming $V_{dd} >> V_T$.

$I \propto V_{dd}^2$

$t = \frac{Q}{I}$ $Q = CV$ $I \propto V^2$ $\Rightarrow t_0 = \frac{Q_0}{I_0} \propto \frac{CV_0}{V_0^2}$

$\Rightarrow t_{new} \propto \frac{C\, V_0/2}{(V_0/2)^2} = 2 \frac{CV_0}{V_0^2} = 2 t_0$.

[redacted]'s Answers.



$V_c(t) = F.V. + (I.V. - F.V.)\, e^{-t/\tau}$

$= V_0 + (0 - V_0)\, e^{-t/RC}$

$= V_0 (1 - e^{-t/RC})$ $t \geq 0$

$i(t) = C \frac{dV_c}{dt} = C V_0 \frac{1}{RC} e^{-t/RC} = \frac{V_0}{R} e^{-t/RC}$ $t \geq 0$.

$P(t) = V_0 \cdot i(t) = \frac{V_0^2}{R} e^{-t/RC}$

$P_c(t) = i(t)\, V_c(t) = \frac{V_0^2}{R} (e^{-t/RC} - e^{-2t/RC})$

$P_R(t) = i^2(t)\, R = \frac{V_0^2}{R} e^{-2t/RC}$

$\Rightarrow$ $P(t) = P_c(t) + P_R(t)$

??? ← redo.

2004 - Meng

1. ~~$k'\left(\frac{W}{L}\right)(V_G - V_S - V_T)^2 = \frac{V_S + 1.5V}{1k\Omega} = \frac{3.5 - V_G}{10k\Omega}$~~

~~Assuming $V_D = V_G$~~

~~$\Rightarrow 3.5 - V_G = 10V_S + 15 \Rightarrow V_G = -11.5 - 10V_S$~~

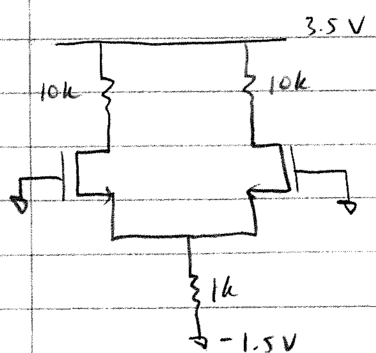
~~$k'\left(\frac{W}{L}\right)(-11.5 - 11V_S - 0.5)^2 = 5(-12 - 11V_S)^2 = \frac{V_S + 1.5}{1k\Omega}$~~

$V_S = -1V$ or $\cancel{0.5V}$ $\qquad I_{tail} = 0.5mA$

$I_D = 0.25mA$

$V_{GS} = 1V$ $\qquad V_{od_{cm}} = 1V$

$\Rightarrow V_{ov} = 0.5V \quad V_{DS} = 2V \Rightarrow$ saturation ✓



$k' = 0.2 mA/V^2 \quad \frac{W}{L} = 10 \quad V_T = 0.5$

find I_D and V_{GS}

$\frac{1}{2} 0.2 mA/V^2 (10) (-V_S - 0.5)^2 = \frac{V_S + 1.5}{2k}$

$\Rightarrow \frac{(-V_S - 0.5)^2}{1k} = \frac{V_S + 1.5}{2k}$

$\Rightarrow 2V_S^2 + 2V_S + 0.5 = V_S + 1.5 \Rightarrow 2V_S^2 + V_S - 1 = 0$

$\Rightarrow V_S = \frac{-1 \pm \sqrt{1+8}}{4} = -1V$ or $0.5V$

for $0.5V$, $I_d = \frac{2V}{2k} = 1mA \Rightarrow V_d = 3.5V - 10k \cdot 1mA = -6.5V$ ✗

for $-1V$, $I_d = \frac{0.5V}{2k} = 0.25mA \Rightarrow V_d = 3.5V - 10k \cdot 0.25mA = 1V$ ✓

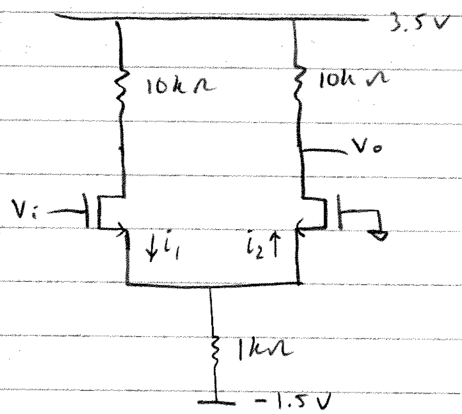
$\Rightarrow \boxed{V_{GS} = 1V} \quad \boxed{I_d = 0.25mA}$

??? redo.

(3) $$A_{CD} = \frac{(1k\Omega \parallel \frac{1}{g_{m_2}})}{(1k\Omega \parallel \frac{1}{g_{m_2}}) + \frac{1}{g_{m_1}}} = \frac{1}{3}$$

$$A_{CG} = g_{m_2}(10k\Omega) = 10$$

$$\Rightarrow \frac{v_o}{v_i} = A_{CD} \cdot A_{CG} = \boxed{\frac{10}{3}}$$

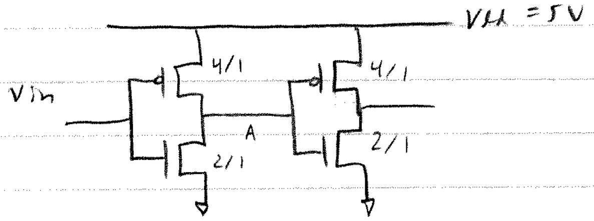


find $\frac{v_o}{v_i}$.

g

1994 - Meng

1.



$k_n' = 20\mu A/V^2 \qquad k_p' = 10\mu A/V^2$

① What is gate cap in standard CMOS techology? (for now Assume 10fF)

$C_{ox} \approx 8fF/\mu m^2$ for 32 nm technology.

② Gate delay from input to node A?

Assume transistor is in saturation during charging.

$I_{sat} = \frac{1}{2} k_n' \cdot \left(\frac{W}{L}\right)(V_{dd} - V_T)^2 = \frac{1}{2} 40\mu A/V^2 \cdot 25V^2 = 0.5mA$

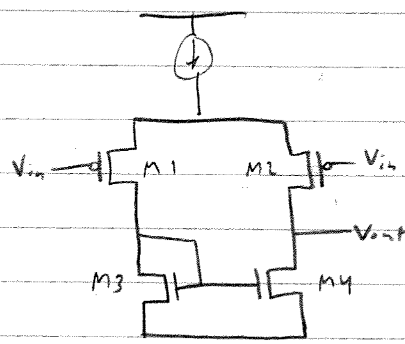
$$t_p \approx \frac{CV_{dd}}{I_{sat}} = \frac{5V \cdot 10fF}{0.5 \cdot 10^{-3}} = \frac{50 \cdot 10^{-15}}{5 \cdot 10^{-4}} = 0.1ns.$$

$$t_p = \frac{C\Delta V}{I} = \frac{C_{gs} V_{dd}}{\frac{1}{2} k_n' \frac{W}{L} (V_{dd} - V_T)^2} \approx \frac{2C_{gs}}{k_n' \left(\frac{W}{L}\right) V_{dd}}$$

$\left(\text{In reality, } t_p = \frac{3.4 C_{gs}}{k'\left(\frac{W}{L}\right) V_{dd}} \right)$

? ③ Design an output stage that aims at minimizing the delay of driving a capacitance 1000 times bigger than gate capacitance. Estimate the number of stages needed and the total delay.

??? Look this up.

2. 

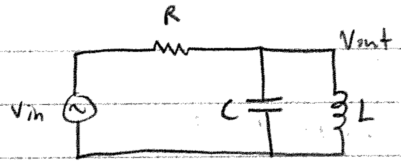
Assume the only offset contribution is from the mismatch in the W/L of the input transistors. Neglect transistor output resistance.

Params	↑	↓	Does not change
W_1, W_2	✓		
L_1, L_2		✓	
W_3, W_4			✓
L_3, L_4			✓
I		✓	
t_{ox}		✓	

1995 - Meng

1. a) Inductor: $V = L\frac{dI}{dt}$ Capacitor: $I = C\frac{dV}{dt}$

b)



$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}} = 1\,GHz$$

$$\Rightarrow LC = \left(\frac{1}{2\pi \cdot 10^9}\right)^2 \approx \frac{1}{40 \cdot 10^{18}} = 2.5 \cdot 10^{-20}$$

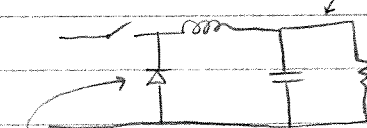
$\Rightarrow$ let $L = 2.5\,nH$ $C = 10\,pF$

$$Z_{max} = j\omega_0 L \,\|\, \frac{1}{j\omega_0 C} = \frac{j\omega_0 L \cdot \frac{1}{j\omega_0 C}}{j\omega_0 L + \frac{1}{j\omega_0 C}} = \frac{j\omega_0 L}{1 - \omega_0^2 LC}$$

$$\Rightarrow |Z_{max}| = \frac{L/\sqrt{LC}}{1-1} = \infty$$

$\leftarrow$ only theoretically, in reality L and C will have finite Q.

2. a) $V_{L_{ON}} = V_{in} - V_{on} - V_{out} = L\frac{dI_{L_{ON}}}{dt}$



assume pole of this RC lowpass is much lower than switching freq. Thus V_{out} remains ~ constant for T_{ON} and T_{OFF}

"Free wheeling" diode. See MIT 6.334 L1, 58

b) $V_{L_{OFF}} = -V_{D_{ON}} - V_{out} = L\frac{dI_{L_{OFF}}}{dt}$

c) In P.S.S. $\left\langle \frac{dI_L}{dt} \right\rangle = 0$ otherwise I_L would change from cycle to cycle $\Rightarrow$ violating P.S.S.

$$\Rightarrow \left(\frac{dI_{L_{ON}}}{dt}\right) T_{ON} + \left(\frac{dI_{L_{OFF}}}{dt}\right) T_{OFF} = 0$$

$D = \frac{T_{ON}}{T_{ON} + T_{OFF}}$

$$\Rightarrow (V_{in} - V_{on} - V_{out})\, T_{ON} = -(-V_{D_{ON}} - V_{out})\, T_{OFF}$$

$$\Rightarrow T_{ON} V_{in} = T_{ON} V_{ON} + T_{OFF} V_{D_{ON}} + (T_{ON} + T_{OFF}) V_{out}$$

if we further assume that $V_{ON} \approx V_{D_{ON}} \approx 0V$

$$\frac{V_{out}}{V_{in}} = \frac{T_{ON}}{T_{ON} + T_{OFF}} \Rightarrow \boxed{\frac{V_{out}}{V_{in}} = \frac{T_{ON}}{T_{total}}}$$

2008 - Meng

1. Synchronous Buck Converter

★ Inductor Volt-Second Balance :

integrate over one complete switching period.

$$v_L(t) = L\frac{di_L(t)}{dt} \Rightarrow \frac{1}{L}\int_0^{T_s} v_L(t)\,dt = i_L(T_s) - i_L(0)$$

In P.S.S. the net change in inductor current is zero from cycle to cycle.

$$\Rightarrow \quad 0 = \int_0^{T_s} v_L(t)\,dt$$

⇒ The total area (or volt-seconds) under the inductor voltage waveform is zero whenever the converter operates in steady state.

$$\Rightarrow \quad \langle v_L \rangle = \frac{1}{T_s}\underbrace{\int_0^{T_s} v_L(t)\,dt}_{0} = 0$$ ⇒ average inductor voltage is zero in P.S.S.

★ Capacitor Charge Balance :

integrate over one complete switching period.

$$i_c(t) = C\frac{dv_c(t)}{dt} \Rightarrow \frac{1}{C}\int_0^{T_s} i_c(t)\,dt = v_c(T_s) - v_c(0)$$

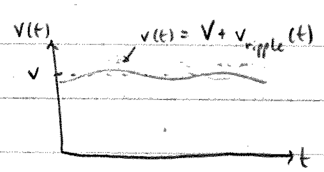
In P.S.S. the net change in capacitor voltage is zero from cycle to cycle.

$$\Rightarrow \quad 0 = \int_0^{T_s} i_c(t)\,dt$$

⇒ The total area (or charge) under the capacitor current waveform is zero whenever the converter operates in P.S.S.

$$\Rightarrow \langle i_c \rangle = \frac{1}{T_s}\int_0^{T_s} i_c(t)\,dt = 0$$ ⇒ average capacitor current is zero in P.S.S.

★ Small Ripple Approximation



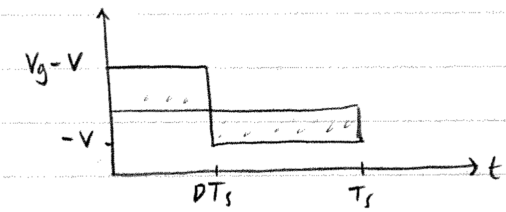
In a well designed converter, the output voltage ripple is small.

⇒ the waveforms can be easily determined by ignoring the ripple.

$$\|v_{ripple}\| << V \Rightarrow v(t) \approx V$$

a) ϕ_1: $V_{L_1} = V_g - V = L\dfrac{d(i_L)}{dt}$

ϕ_2: $V_{L_2} = -V = L\dfrac{d(i_L)}{dt}$

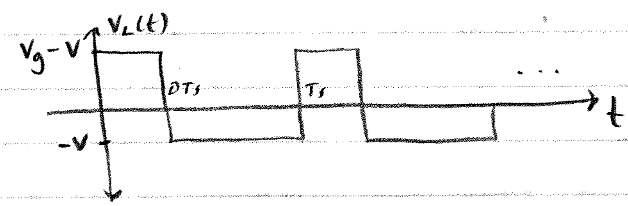


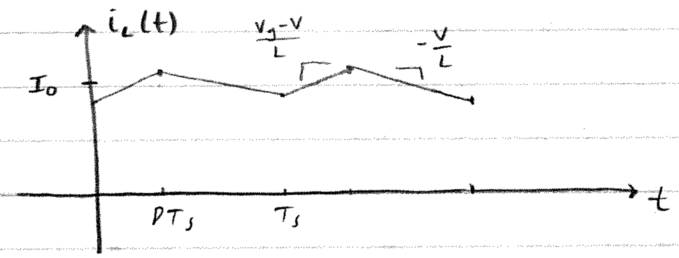
$\langle v_L \rangle = 0 \Rightarrow (V_g - V)DT_s + (-V)(1-D)T_s = 0$

$\Rightarrow DT_sV_g - \cancel{DT_sV} - VT_s + \cancel{VDT_s} \Rightarrow \boxed{V = DV_g}$

b) ϕ_1: $\dfrac{d(i_L)}{dt} = \dfrac{V_g - V}{L}$

ϕ_2: $\dfrac{d(i_L)}{dt} = -\dfrac{V}{L}$





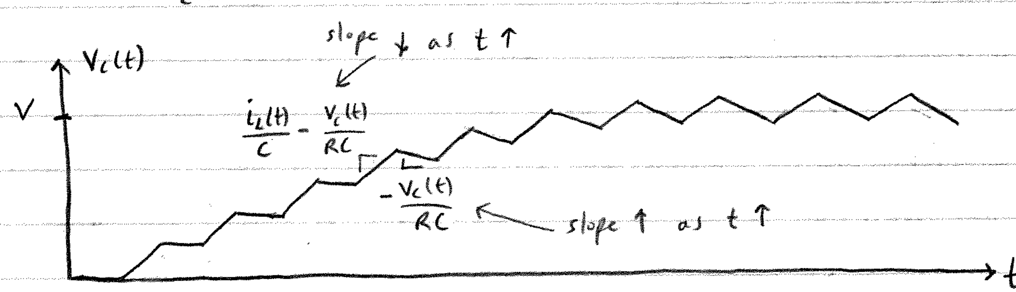
2. a) $v(t) = v_c(t)$

ϕ_1: $i_c(t) = -\dfrac{v_c(t)}{R} = C\dfrac{dv_c(t)}{dt} \Rightarrow \dfrac{dv_c(t)}{dt} = -\dfrac{v_c(t)}{RC}$

ϕ_2: $i_c(t) = i_L(t) - \dfrac{v_c(t)}{R} = C\dfrac{dv_c(t)}{dt} \Rightarrow \dfrac{dv_c(t)}{dt} = \dfrac{i_L(t)}{C} - \dfrac{v_c(t)}{RC}$

$v_c(0) = 0\,V \quad v_c(\text{final}) = V$

$\Rightarrow$



Assume $R_L = 0$

b) ϕ_1: $i_{c_1} = C\frac{dV_c}{dt} = -\frac{V}{R}$ $\qquad V_{L_1} = V_g$

ϕ_2: $i_{c_2} = C\frac{dV_c}{dt} = I_L - \frac{V}{R}$ $\qquad V_{L_2} = V_g - V$

$\Rightarrow V_g D\cancel{T} + (V_g - V)(1-D)\cancel{T} = 0$ $\qquad$ let $D' = 1-D$

$\Rightarrow V_g(D+D') = D'V \Rightarrow V_g = D'V \Rightarrow \boxed{V = \frac{1}{1-D}V_g}$

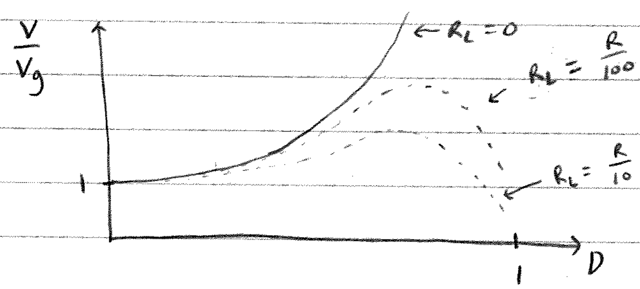
?? Not sure how to derive this.

If $R_L \neq 0$,

$$\boxed{V = \frac{1}{\frac{R_L}{R(1-D)} + 1 - D} V_g}$$

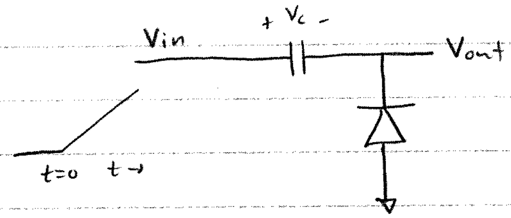
How is the steady state average output voltage V influenced by R_L and D?

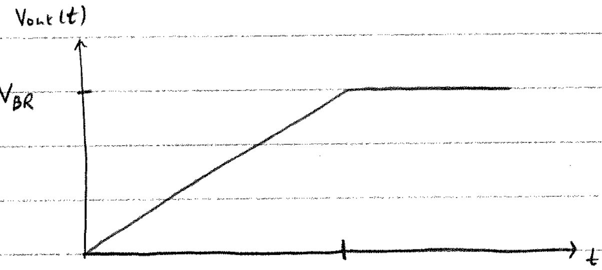
$D\uparrow \Rightarrow V\uparrow$, $\quad R_L \uparrow = V \downarrow$

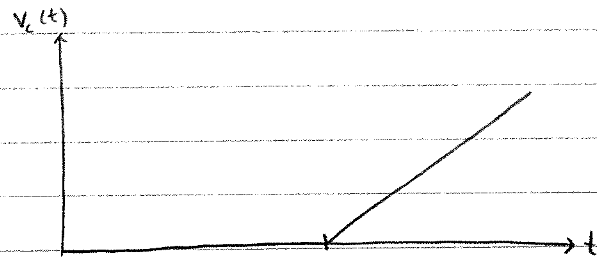


2010 - Murmann

1.





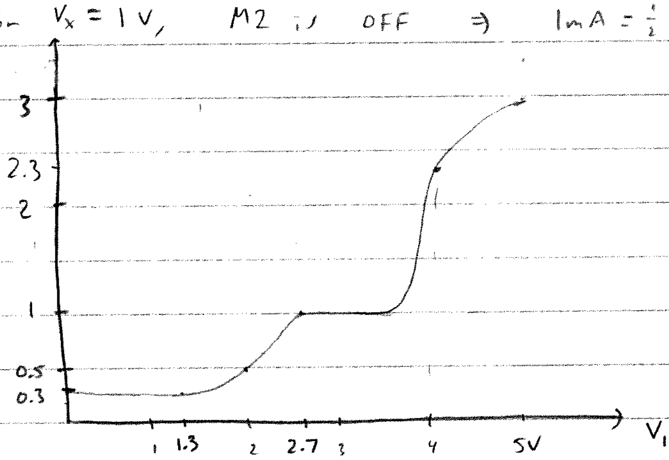


2. for $V_I = 2V$, $I_{d_1} = I_{d_2} = 0.5mA = \frac{1}{2}\, 4mA/V^2\, (2V - V_x - 1V)^2$

$\Rightarrow \quad 1V - V_x = \sqrt{\frac{1}{4}} \quad \Rightarrow \quad V_x = 1 - \frac{1}{2} \quad = \quad 0.5V$

for $V_I = 0V$, $1mA = \frac{1}{2}(4mA)(1 - V_x)^2 \Rightarrow V_x = 1 - \frac{1}{\sqrt{2}} \simeq 0.3V \Rightarrow$ M_1 turns on at $V_I = 1.3V$

for $V_x = 1V$, M2 is OFF $\Rightarrow$ $1mA = \frac{1}{2}(4mA)(V_I - 1 - 1)^2 \Rightarrow V_I = 2 + \frac{1}{\sqrt{2}} \simeq 2.7$



for $V_I = 4V$, $V_D = 5V - 1mA \cdot 2k\Omega = 3V \quad \Rightarrow \quad I_d = 1mA = \frac{1}{2}(4mA)\left((3 - V_x) - \frac{1}{2}(3 - V_x)\right)(3 - V_x)$

$\Rightarrow V_x = 2.3V$. and we know that V_x cannot rise above 3V,

since $V_D = 3V$

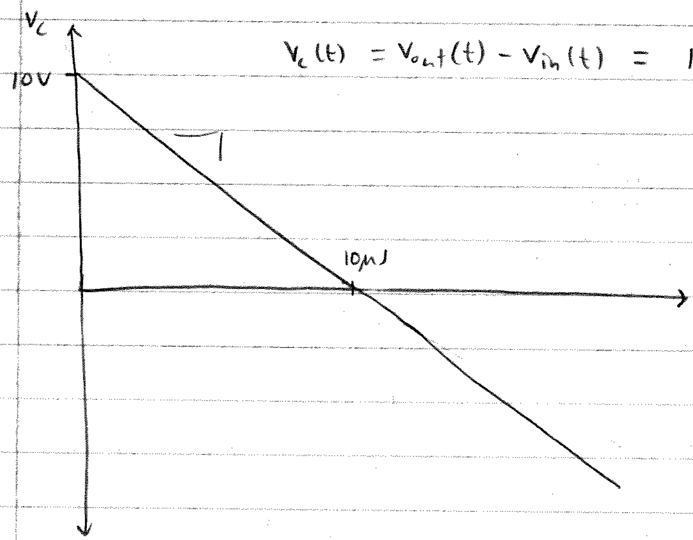
2007 - Murmann

at $t=0$, M1 is off $\Rightarrow$ 1mA flows through 1kΩ

$\Rightarrow V_{out}(0) = 11V \Rightarrow V_{in}(0) = 1V$ since $V_c(0) = 10V$

$$i = C\frac{dV}{dt} \Rightarrow \frac{dV}{dt} = \frac{i}{C} = \frac{1mA}{1nF} = 1\,V/\mu s$$

$$V_c(t) = V_{out}(t) - V_{in}(t) = 10V - 1\frac{V}{\mu s}\cdot t$$



$V_{in}(t) = 1V + 1\frac{V}{\mu s}\cdot t$ for $0 < t < 2\mu s$

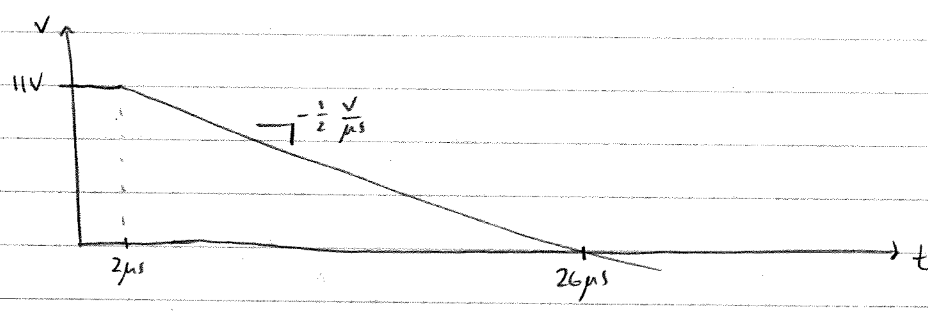
$\Rightarrow$ at $t = 2\mu s$, M1 turns on.

$V_{out}(t) = 11V \qquad 0 < t < 2\mu s$

$V_{out}(t) = 11V - (I_d \cdot 1k\Omega) = 11 - (V_{in} - 3) = 14V - V_{in}(t) \qquad t > 2\mu s$

$\Rightarrow V_c(t) = V_{out}(t) - V_{in}(t) = V_{out}(t) - (14V - V_{out}(t)) = 2V_{out}(t) - 14V = 10V - 1\frac{V}{\mu s}t$

$\Rightarrow V_{out}(t) = 12V - \frac{1}{2}\frac{V}{\mu s}t$ for $t > 2\mu s$



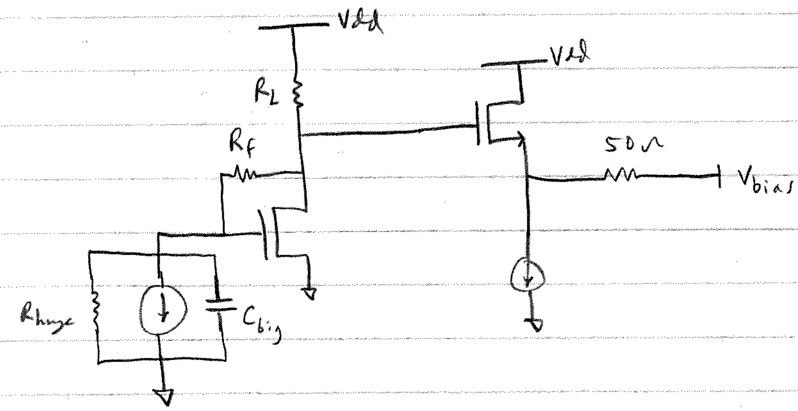
2008 - Murmann

1. $A_v = \frac{v_o}{v_i} = -g_m \left(2k\Omega \,\|\, 2k\Omega \,\|\, (2k\Omega + \underset{\infty}{r_o})\right) = -\underset{10mS}{g_m}(1k\Omega) = \boxed{-10}$

2. TIA. high R_o. Low R_{in}. $\frac{100mV}{100\mu A} = 1k\Omega$ gain

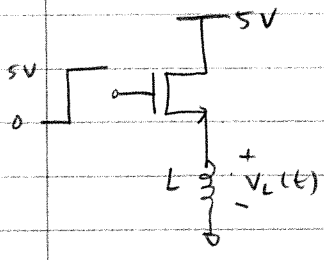
$R_f = 1k\Omega$

$R_L = 10k\Omega$



first order bandwidth expression: $\omega_{3dB} \approx \frac{1}{R_f \cdot C_{big}}$

1994 - Wooley



$V_T = 1V$

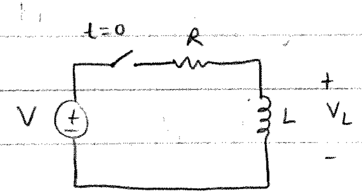
$V_L(t) = 0$ for $t < 0$

what happens to $V_L(t)$ for $t > 0$?

$V_L = L \frac{di_L}{dt}$ ⇒ current through inductor cannot change instantaneously.

M1 must always be in saturation or cutoff for $t > 0$, since $V_g = V_d = 5V$

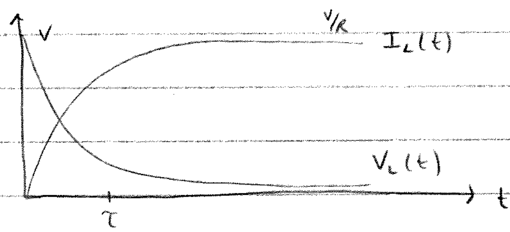
Consider the case of a basic LR circuit?



$V = IR + V_L = IR + L\frac{dI}{dt}$

at $t=0$, $V_L = 0$

⇒ $I_L(t) = \frac{V}{R}(1 - e^{-t/\tau})$ where $\tau = \frac{L}{R}$

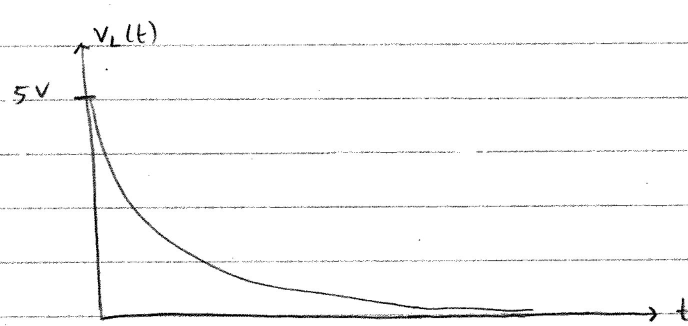


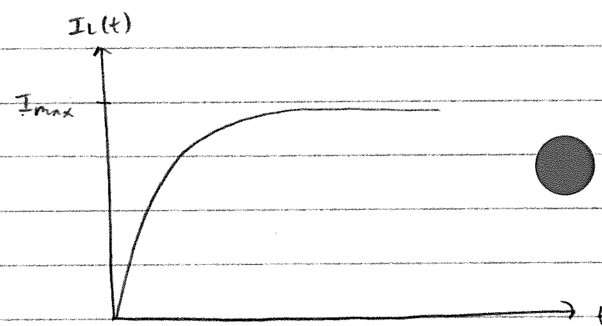
Initial Conditions: $V_L(0^-) = 0V$ $V_L(0^+) = L\frac{dI}{dt} = 5V$ if we assume a perfect step input ($\frac{dI}{dt} \to \infty$) ⇒ $V_L \to$ rail voltage.

$I_L(0) = 0A$

Steady State: $I_L(t)\Big|_{t\to\infty} = I_d = \frac{1}{2}\mu C_{ox}\frac{W}{L}(5V - 1V)^2$

$V_L(t) = L\frac{dI}{dt} = 0V$





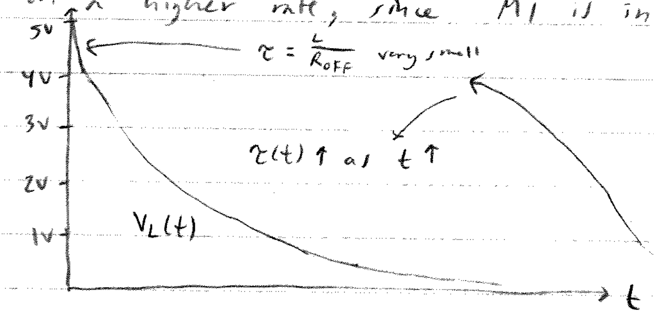
for $4V < V_L(t) < 5V$, M1 is in subthreshold

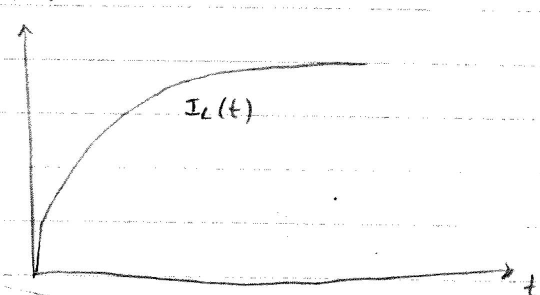
$\Rightarrow I_d = I_o e^{V_{gs}/\zeta V_T}$ where $V_T = \frac{kT}{q}$ + $\zeta > 1$ $V_{gs} = 5 - V_L(t)$

for $4 < V_L(t) < 5V$, M1 is in saturation

$\Rightarrow I_d = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{gs} - V_T)^2$ $V_{gs} - V_T = 5 - V_L(t) - 1 = 4 - V_L(t)$

$\Rightarrow$ initially $(4V < V_L(t) < 5V)$, we might expect current to increase at a higher rate, and voltage $V_L(t)$ to decrease at a higher rate, since M1 is in subthreshold.





We expect L to decay from 5V to 0V with a time dependent $\frac{L}{R}$ time constant $\tau(t) = \frac{L}{R} = \frac{L}{(1/g_m)} = g_m(t)\,L$

initially: subthreshold: $g_m = \frac{I_d}{\zeta V_T}$ where $\zeta > 1$. This is very large g_m. ??

then: saturation: $g_m = \sqrt{\mu C_{ox}\frac{W}{L}(2I_d)}$ $I_d \uparrow$ as $t \uparrow$

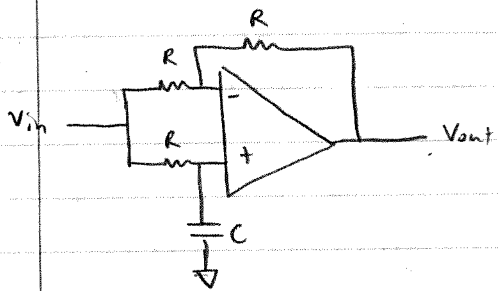
$\Rightarrow g_m \uparrow$ as $t \uparrow$ $\Rightarrow \tau(t) = g_m L \uparrow$ as $t \uparrow$

However in our simulations, we see that τ is shortest in subthreshold, which disagrees with our $\frac{1}{g_{m_{sub}}}$ assumption. Perhaps this just means that the $\frac{1}{g_m}$ assumption only applies for SAT, and in sub, R_{OFF} is very large. This makes intuitive sense. $\Rightarrow \frac{L}{R_{OFF}}$ should be very small.

In case this small signal model of $r = \frac{1}{g_m}$ does not apply, we could make a physical argument that as $V_L \downarrow$, $V_{gs} \uparrow$ and thus channel inversion becomes stronger $\Rightarrow$ more charge in the channel $\Rightarrow R_{channel} \downarrow$.

$$\frac{1}{1+j\omega RC} = \frac{1-j\omega RC}{1+\omega^2R^2C^2}$$

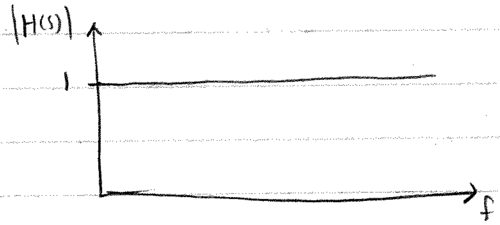
2001 - Lee

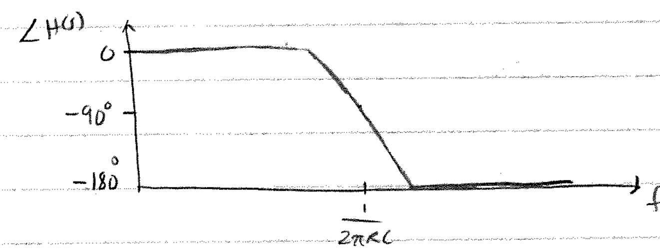


$$H(s) = \frac{1-sRC}{1+sRC}$$

$$|H(s)| = \frac{\sqrt{1^2+\omega^2R^2C^2}}{\sqrt{1^2+\omega^2R^2C^2}} = 1$$

$$\angle H(s) = \angle(1-sRC) + \angle\left(\frac{1}{1+sRC}\right)$$
$$= \angle(1-sRC) + \angle(1-sRC)$$
$$= \tan^{-1}\left(-\frac{\omega RC}{1}\right) + \tan^{-1}\left(-\frac{\omega RC}{1}\right) = 2\tan^{-1}(-\omega RC)$$





Response to step input:

can compute inverse Laplace transform of $\underbrace{\frac{1}{s}}_{\text{Laplace transform of } u(t)} \underbrace{\left(\frac{1-sRC}{1+sRC}\right)}_{H(s)}$

OR, could recognize that all single pole step responses have the form:

$$s(t) = A + Be^{-t/\tau} \quad \text{where } \tau = \frac{1}{\omega_p}$$

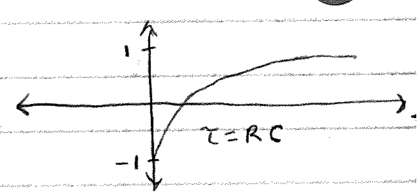
To find initial + final conditions, consider the circuit for $s=\infty$ and $s=0$:

at $t=0$, $s\to\infty$ $\Rightarrow \frac{1}{sC}\to 0$ $\Rightarrow$ circuit looks like inverting amp $\Rightarrow \frac{V_{out}}{V_{in}} = -1 \Rightarrow V_{out}(0) = -1$

at $t\to\infty$, $s\to 0$ $\Rightarrow \frac{1}{sC}\to\infty$ $\Rightarrow V_+$ is open, $\Rightarrow$ no current flow through any resistors. $\Rightarrow V_{out} = V_{in}$ $\Rightarrow \frac{V_{out}}{V_{in}} = 1$ $\Rightarrow V_{out}(\infty) = 1$

$\Rightarrow s(\infty) = A = 1 \qquad s(0) = A + B = -1 \Rightarrow B = -2$

$\Rightarrow s(t) = 1 - 2e^{-t/RC}$



2005 - Wooley

1. Identical BJTs. Large β_o + r_o $\Rightarrow$ I_c is independent of V_{ce}. Depends only on V_{be}.

$V_{be} \approx 0.7\text{V}$ $\qquad V_c = 0\text{V}$ at $t=0$ $\qquad V_s = V_{be} + V_c$

$\Rightarrow$ at $t=0$, $V_s = 0.7\text{V}$

Since the bases of all four transistors are connected, they must each pull an equal current I_c.

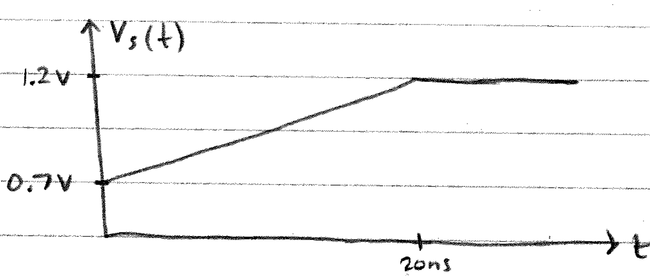
$\Rightarrow$ at $t=0$, $I_{c_1} = I_{c2} = I_{c3} = I_{c4} = 1\text{mA}$.

for $0 < t < 20\text{ns}$, $I_{c_1} = I_{c_2} = I_{c_3} = I_{c_4} = 1.25\text{mA}$

$\Rightarrow$ $i_c = I_{c_1} - 1\text{mA} = 0.25\text{mA}$ $\qquad i_c = C\frac{dV}{dt} \Rightarrow \frac{dV}{dt} = \frac{i}{c}$

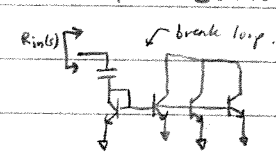
$\Rightarrow$ $V_c(t) = \frac{i}{c}\cdot t = \frac{0.25\text{mA}}{10\text{pF}}\cdot t \Rightarrow V_c(20\text{ns}) = \frac{0.25\text{mA}}{10\text{pF}}\cdot 20\text{ns} = 0.5\text{V}$

$\Rightarrow$



$g_{m_1} = \frac{I_c}{V_T} \approx \frac{1.25\text{mA}}{25\text{mV}}$

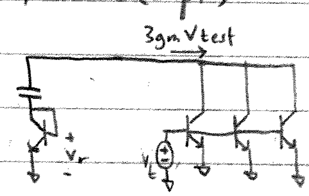
Note: There is negative shunt-shunt feedback



2. Small signal input impedance $R_{in}(s)\Big|_{\text{open loop}} = \left(\frac{1}{sC} + \frac{1}{g_{m_1}}\right)$

$\Rightarrow R_{in}(s)\Big|_{\text{open loop}} \approx \frac{1}{sC} + \frac{1}{g_{m_1}} \approx \frac{1}{s(10\text{pF})} + \frac{25\text{mV}}{1.25\text{mA}} \approx \frac{1}{s(10\text{pF})} + 20\Omega$

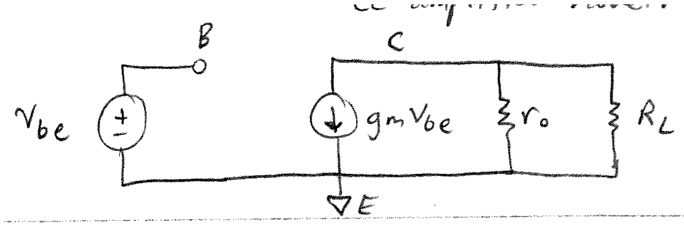
Find loop gain via RR analysis:



$V_r = -3g_m V_{test}\left(\frac{1}{g_m}\right) = -3V_{test}$

$T = RR = -\frac{V_r}{V_t} = 3$ $\qquad$ shunt-shunt feedback $\Rightarrow$ R_{in} is reduced by $1 + T = 4$

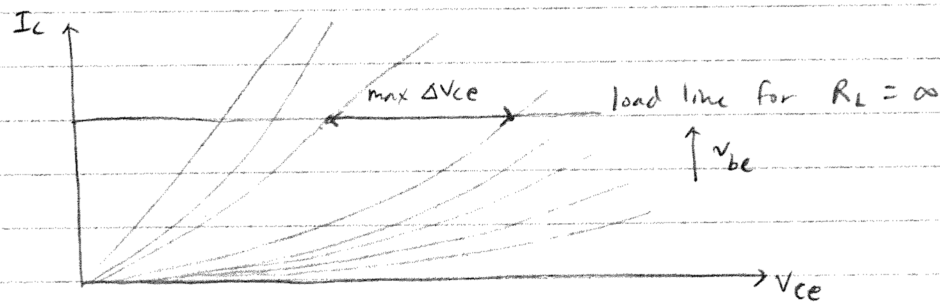
$\Rightarrow \boxed{R_{in}(s) = \frac{1}{4}\left(\frac{1}{sC} + \frac{1}{g_m}\right) = \frac{1}{s(40\text{pF})} + 5\Omega}$



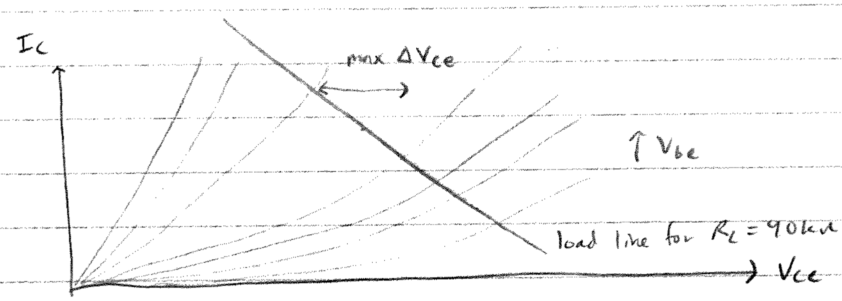
1995 - Lee

1). $g_m = \dfrac{\partial i_c}{\partial v_{be}}$; $r_o = \dfrac{\partial v_{ce}}{\partial i_c}$

$\Rightarrow g_m r_o = \dfrac{\partial v_{ce}}{\partial v_{be}} \approx \dfrac{\Delta v_{ce}}{\Delta v_{be}} = \dfrac{6V}{0.2V} = 30$



2.)



Load line is ~ perpendicular to IV curves at operating point $\Rightarrow r_o \approx R_L \Rightarrow g_m(r_o \| R_L) \approx \frac{1}{2} g_m r_o = 15$

2002 - Lee

1. $V_{out} = \frac{R_2}{R_1+R_2} V_{in}$

2. Yes R_1 can have a negative value. But if R_1's magnitude exceeds that of R_2, the net resistance seen by the driving source (R_1+R_2) is negative. Depending on the nature of the driving impedance (R_s, parallel L and C) this negative resistance can result in instability + oscillation. If we ignore this possibility,

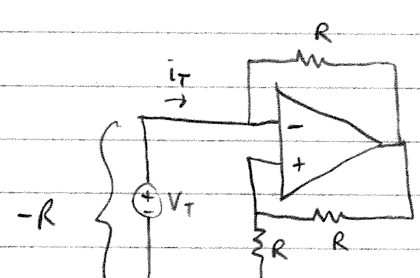
$V_{out} = \frac{R_2}{R_1+R_2} V_{in} = -K\, V_{in}$ where K is a constant.

$\Rightarrow$ the network produces an inversion in signal polarities.

3. Can a negative resistor be physically realized as an element with a total of no more than two terminals? Explain. [Clarification: There is a completely sealed black box, out of which only two wires emerge. Can there appear between those two terminals a negative resistance that functions forever?]

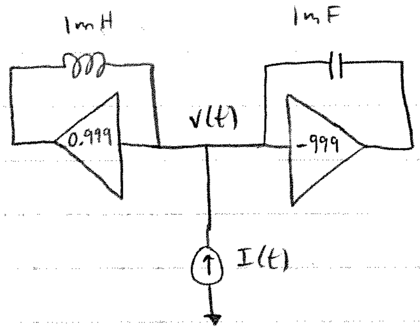
A negative resistor can supply energy $\left(P = I^2(-R) < 0\right)$.
$\Rightarrow$ there needs to be an energy source somewhere. Since it's not allowed to be in the box, then more than two terminals are needed.

e.g. rail voltages needed for opamp implementation:

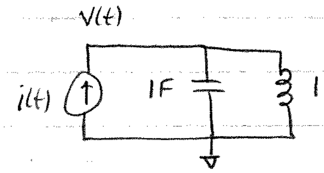


$$(V_T - i_T R)\frac{R}{R+R} = V_T$$
$$\Rightarrow 2V_T = V_T - i_T R$$
$$\Rightarrow V_T = -i_T R$$
$$\Rightarrow V_T/i_T = -R$$

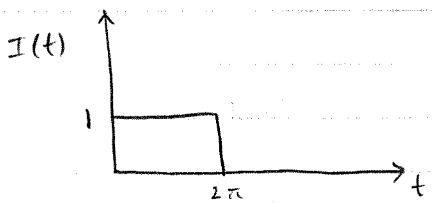
1997 - Lee



by Miller



$$Z(s) = \frac{V(s)}{I(s)} = \frac{\frac{1}{sC}\cdot sL}{\frac{1}{sC}+sL} = \frac{sL}{1+s^2LC} = \frac{s}{1+s^2}$$



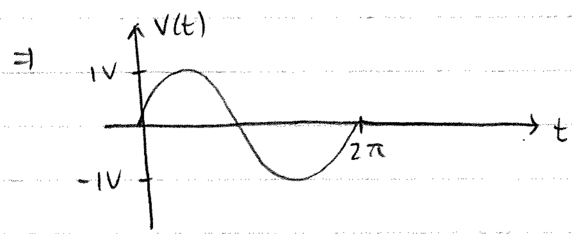
$$\Rightarrow I(t) = u(t) - u(t-2\pi)$$

$$\Rightarrow I(s) = \frac{1}{s} - e^{-2\pi s}\frac{1}{s} = (1-e^{-2\pi s})\frac{1}{s}$$

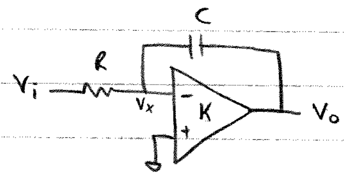
$$\Rightarrow V(s) = I(s)Z(s) = \frac{1}{1+s^2} - \underbrace{\frac{e^{-2\pi s}}{1+s^2}}_{0 \text{ for } 0<t<2\pi}$$

$$\Rightarrow V(t) = \sin(t) - \sin(t-2\pi)$$

$\Rightarrow$



2000 - Lee

1. 

Brute force Method:

$$\frac{V_i - V_x}{R} + \frac{V_o - V_x}{\frac{1}{sC}} = 0 \qquad V_o = -K V_x$$

$$\frac{V_i - V_x}{R} = \frac{(k+1)V_x}{\frac{1}{sC}} \Rightarrow \frac{V_i}{R} = \left(\frac{(k+1)}{\frac{1}{sC}} + \frac{1}{R}\right) V_x = \left(\frac{k+1}{\frac{1}{sC}} + \frac{1}{R}\right)\left(-\frac{V_o}{K}\right)$$

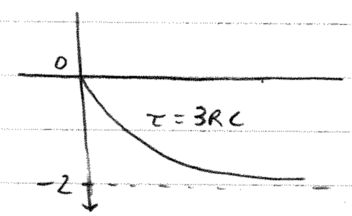
$$\Rightarrow V_o/V_i = -\frac{K}{R}\left(\frac{k+1}{\frac{1}{sC}} + \frac{1}{R}\right)^{-1} = \left(\frac{R(k+1) + \frac{1}{sC}}{\frac{1}{sC}\cdot R}\right)^{-1}\left(-\frac{K}{R}\right) = -\frac{K}{1+sRC(k+1)}$$

$$\frac{V_o}{V_i} = -\frac{2}{1+3sRC} = H(s) \qquad u(t) \Leftrightarrow \frac{1}{s}$$

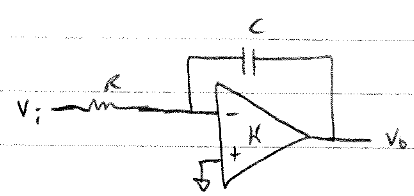
$$\Rightarrow V_o(s) = H(s)\frac{1}{s} = -\frac{2}{s(1+3sRC)} = \frac{c_1}{s} + \frac{c_2}{(1+3sRC)}$$

$$c_1 = -2 \quad c_2 = 6RC \Rightarrow V_o(s) = -\frac{2}{s} + \frac{6RC}{1+3sRC} = -\frac{2}{s} + \frac{2}{\frac{1}{3RC}+s}$$

$$\Rightarrow V_o(t) = -2u(t) + 2e^{-t/3RC}$$



Intuitive Method: Using Miller:

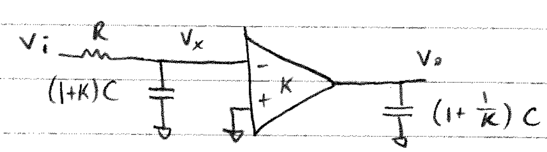


$$\frac{V_x}{V_i} = \frac{\frac{1}{s(3C)}}{\frac{1}{s(3C)} + R} = \frac{1}{1+3sRC}$$

⇓

$$V_o = -KV_x = -2V_x \Rightarrow \frac{V_o}{V_i} = \frac{V_o}{V_x}\frac{V_x}{V_i}$$

$$= -\frac{2}{1+3sRC} \quad ✓$$

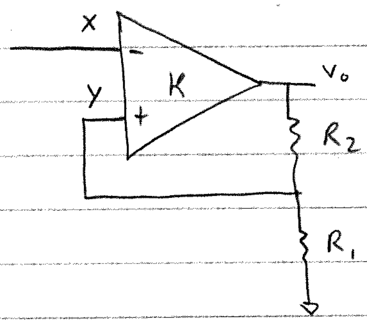


for step response, this is a one pole system.

The final value is DC ⇒ cap is open ⇒ $V_x = V_i$ ⇒ $V_o = -2V_i = -2V$

The initial value is 0 since cap is shorted. Time constant is $3RC = \tau$.

2.



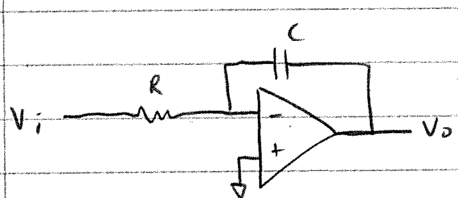
$$\frac{R_1}{R_1+R_2} = \frac{1}{K}$$

$$\Rightarrow V_y = \frac{V_o}{K}$$

$$V_o = K\,(V_y - V_x)$$

$$\Rightarrow V_o\left(1 - \frac{K}{K}\right) = -K V_x$$

$$\Rightarrow \frac{V_o}{V_x} = -\frac{K}{(1-1)} = -\infty \qquad \Rightarrow \text{positive feedback boosts gain to } \infty.$$

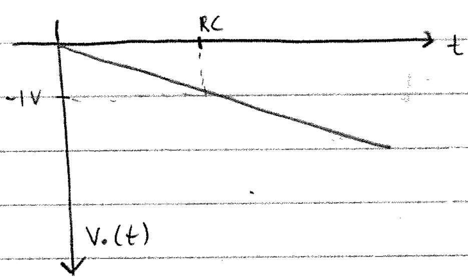


Brute force Method:

Integration with factor of $-\frac{1}{RC}$

$$\frac{V_i}{R} = -\frac{V_o}{\frac{1}{sC}} \Rightarrow \frac{V_o}{V_i} = -\frac{1}{sRC} = H(s)$$

$$V_o(s) = -\frac{1}{s^2 RC} \Rightarrow V_o(t) = -\frac{1}{RC}\,t = -\frac{t}{RC}$$



Another Method: use expression from Q1 and let $K \to -\infty$

$$\Rightarrow \left.\frac{V_o}{V_i}\right|_{K\to-\infty} = \left.-\frac{K}{1+(K+1)sRC}\right|_{K\to-\infty} = -\frac{1}{sRC}$$

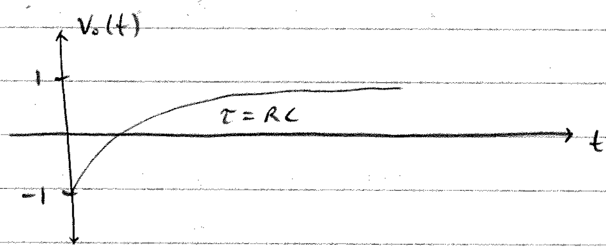
2001 - Lee (Continued)

$$V_o(s) = \frac{1}{s}\left(\frac{1-sRC}{1+sRC}\right) = \frac{1-sRC}{s(1+sRC)} = \frac{C_1}{s} + \frac{C_2}{1+sRC}$$

$$C_1 = 1 \qquad C_2 = \frac{1+1}{-1/RC} = -2RC$$

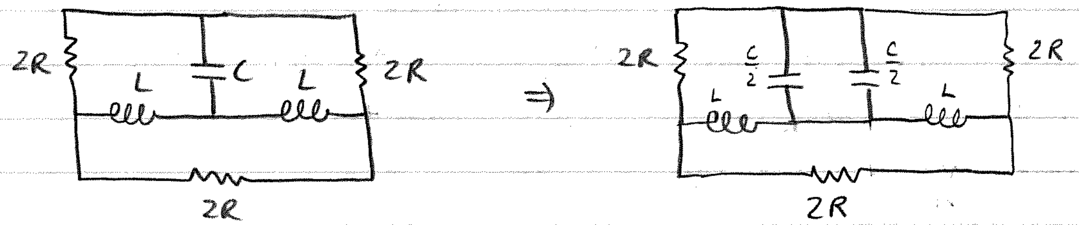
$$\Rightarrow V_o(s) = \frac{1}{s} - \frac{2RC}{1+sRC} = \frac{1}{s} - \frac{2}{\frac{1}{RC}+s}$$

$$\Rightarrow V(t) = u(t) - 2e^{-t/RC} = 1 - 2e^{-t/RC}$$



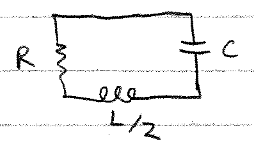
2004 - Lee

a)



Because we can specify the initial energies of the three energy storage elements independently (the initial inductor currents and capacitor voltage can all be chosen without conflict), there are three poles (three degrees of freedom).

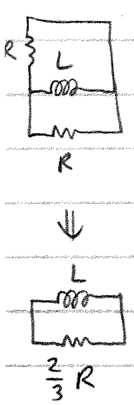
b) Symmetry enables simplified analysis. Suppose, for example we choose initial inductor currents of equal magnitude both flowing toward their common connection with the capacitor. By symmetry, the left and right halves of the circuit behave the same way. The voltage across the bottom resistor thus remains zero, so we can take it out of the network and we may also fold the left half onto the right half. Then the network becomes:



$$\frac{1}{\frac{sL}{2} + \frac{1}{sC} + R} = \frac{sC}{\frac{s^2LC}{2} + sRC + 1}$$

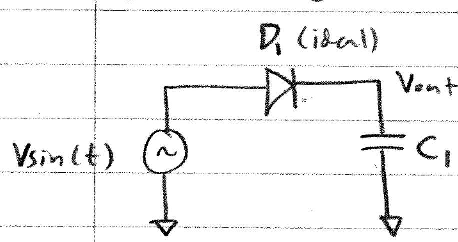
This second order network has poles at the roots of $\frac{s^2LC}{2} + sRC + 1$

$$\Rightarrow \boxed{p_{1,2} = \frac{-RC \pm \sqrt{R^2C^2 - 2LC}}{LC} = -\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{2}{LC}}}$$

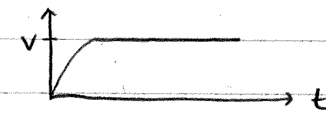


Now try a differential mode initial condition, using initial inductor currents that are equal in magnitude, but oriented so that one flows into the capacitor and the other out of it. This permits us to short across the capacitor (it never charges) together with the center of the bottom resistor. That decomposes the network into a simple LR network whose time constant is $\frac{3}{2}\frac{L}{R}$. The pole frequency is minus the reciprocal or $\boxed{p_3 = -\frac{2}{3}\frac{R}{L}}$

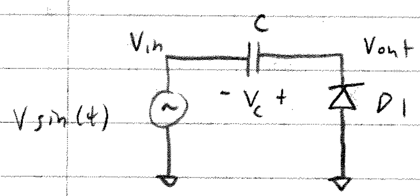
2005 - Lee

1. 

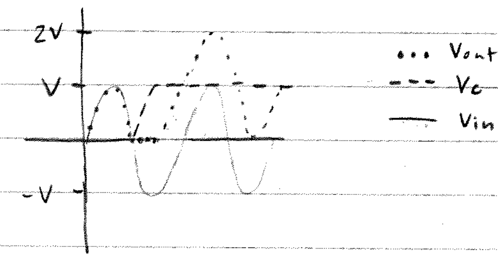
What is the steady state output voltage?



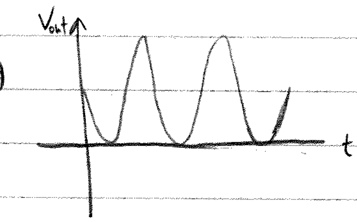
Ans: $\boxed{V}$ The diode charges C_1 up to V in the first quarter cycle. There is nothing to discharge the capacitor, so the output voltage stays at V forever. This circuit is a filtered halfwave rectifier.

2. 

Sketch the output voltage in steady state.

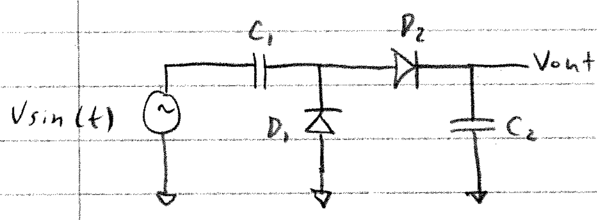
Ans: 

$V_{out} = V_{in} + V_c$

$\Rightarrow$ 

← in steady state.

The same three elements remain in series. The capacitor here charges up to V in the first negative quarter cycle, and stays charged forever. Note that the polarity of the capacitors voltage is such that it adds V to the sinusoidal drive. The output is thus still a sinusoid, just shifted upward by V, swinging between ground and 2V. This circuit is a level shifter (or clamp).

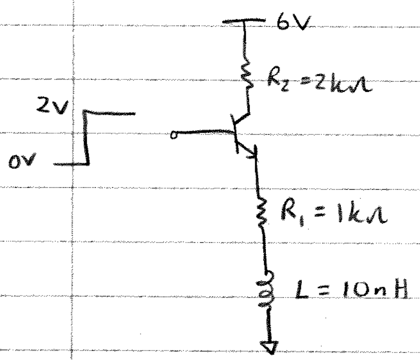
3. 

What is the steady state output voltage?

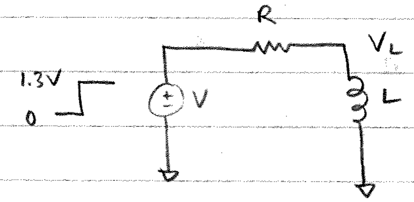
Ans: $\boxed{2V}$. On each negative going quarter cycle of the sinewave drive, the input capacitor charges up to V (left diode on, right diode off). On each positive going edge, some stored charge transfers to the output capacitor. Again, with nothing to discharge the output capacitor, this charge pumping continues until the output voltage is 2V (the peak output value of the previous circuit). This circuit is thus a voltage doubler.

If Wooley asks if you're done, say yes. Otherwise he won't give you the second part of the problem and you'll get a low score.

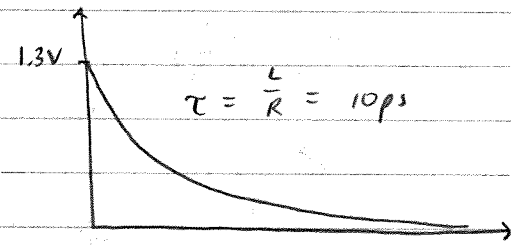
2002 - Wooley



$\Rightarrow$



$\Rightarrow V_L$



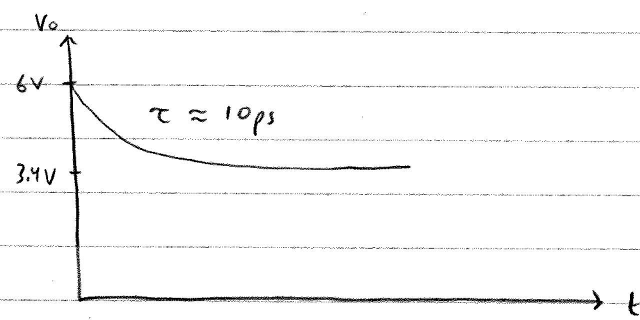
$\tau = \frac{L}{R} = 10ps$

initially, BJT is off $\Rightarrow V_{o_{initial}} = 6V$

at steady state, $V_e \sim 2 - 0.7 = 1.3V$ $\quad V_L = 0$

$\Rightarrow I = \frac{1.3V}{1k\Omega} = 1.3mA \Rightarrow V_{o_{final}} = 6V - 1.3 \cdot 2 = 3.4V$

$\Rightarrow$

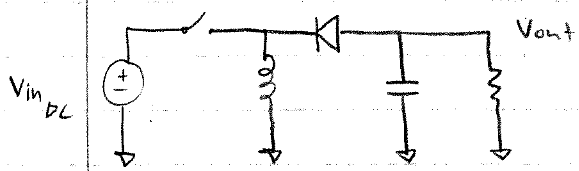


$\tau \approx 10ps$

We expect something like this.
In reality when we simulated this circuit, the τ was actually around 4ps.

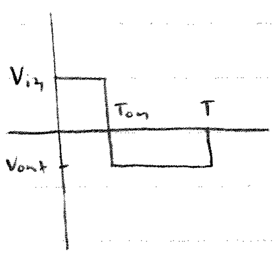
2003 - Lee

1.



Buck - Boost

Calculate the steady state DC ratio $\frac{V_{out}}{V_{in}}$ if the switch periodically closes for T_{on} then opens every T seconds. All elements are ideal $\Rightarrow$ zero diode drop, capacitor filters perfectly and the loading by the resistor may be neglected. Assume inductor current is never zero ($\Rightarrow$ continuous mode). Hint $\langle I_L \rangle = 0$

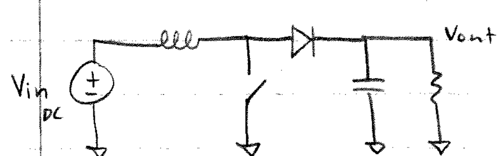


$$V_{in} T_{on} + V_{out} \overset{T - T_{on}}{T'} = 0 \Rightarrow \frac{V_{out}}{V_{in}} = -\frac{T_{on}}{T'}$$

$$\text{let } D = \frac{T_{on}}{T} \Rightarrow \frac{V_{out}}{V_{in}} = -\frac{T_{on}}{T - T_{on}} \Rightarrow \boxed{\frac{V_{out}}{V_{in}} = -\frac{D}{1-D}}$$

range is 0 to $-\infty$.

2.



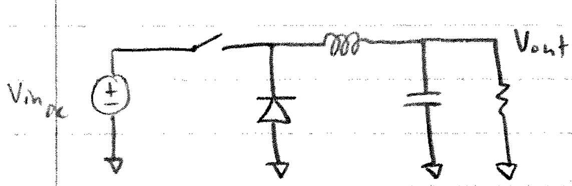
Boost

$$V_{in} T_{on} + (V_{in} - V_{out}) T' = 0$$

$$\Rightarrow V_{in} (T_{on} + T') = V_{out} T' \Rightarrow \frac{V_{out}}{V_{in}} = \frac{T_{on} + T'}{T'}$$

$$= \frac{T}{T - T_{on}} = \frac{1}{1 - \frac{T_{on}}{T}} = \frac{1}{1-D} \Rightarrow \boxed{\frac{V_{out}}{V_{in}} = \frac{1}{1-D}}$$ ← range is 1 to ∞

3.



Buck

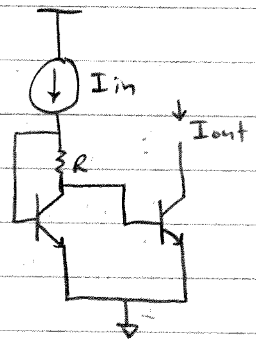
$$(V_{in} - V_{out}) T_{on} - V_{out} T' = 0$$

$$\Rightarrow V_{in} T_{on} = V_{out} (T' + T_{on}) = V_{out} (T)$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{T_{on}}{T} \Rightarrow \boxed{\frac{V_{out}}{V_{in}} = D}$$ ← range is 0 to 1

2007 - Lee

Plot the output current as a function of input current. Label any features of relevance.



$$I_{in} R = V_{be_2} - V_{be_1}$$

$$I_{in} = I_o e^{V_{be2}/V_T} \qquad I_{out} = I_o e^{V_{be1}/V_T}$$

$$\Rightarrow \frac{I_{in}}{I_{out}} = e^{(V_{be_2} - V_{be_1})/V_T} \Rightarrow V_{be_2} - V_{be_1} = V_T \ln\left(\frac{I_{in}}{I_{out}}\right)$$

$$= I_{in} R \Rightarrow \frac{I_{in}}{I_{out}} = e^{I_{in}R/V_T}$$

$$\Rightarrow I_{out} = I_{in} e^{-I_{in}R/V_T}$$

$\Rightarrow I_{out} \approx I_{in}$ for small I_{in}

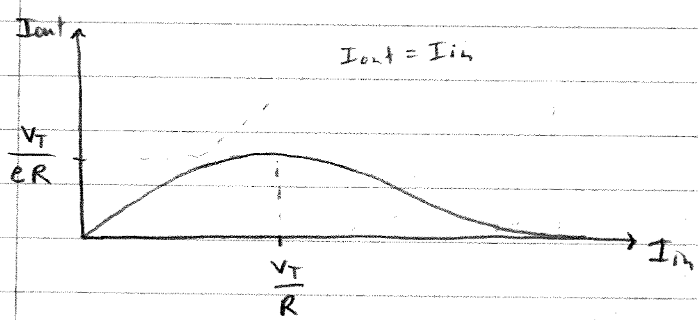
$I_{out} \approx 0$ for large I_{in}

$$\frac{\partial I_{out}}{\partial I_{in}} = I_{in}\left(-\frac{R}{V_T} e^{-I_{in}R/V_T}\right) + e^{-I_{in}R/V_T} = 0$$

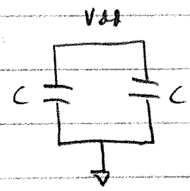
$$\Rightarrow -I_{in}\frac{R}{V_T} + 1 = 0 \Rightarrow I_{in} = \frac{V_T}{R}$$

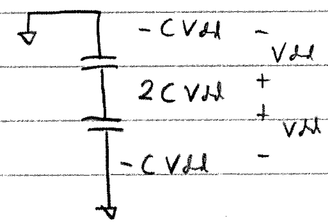
$\leftarrow$ slope $= 0$ at this point $\Rightarrow$ max of I_{out} vs. I_{in}

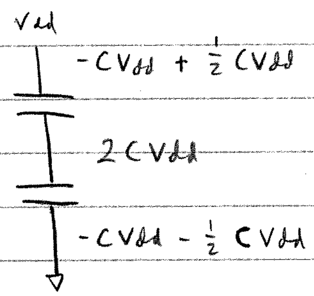
At this point, $I_{out} = \frac{V_T}{R} e^{-(V_T/R)(R/V_T)} = \frac{V_T}{eR}$

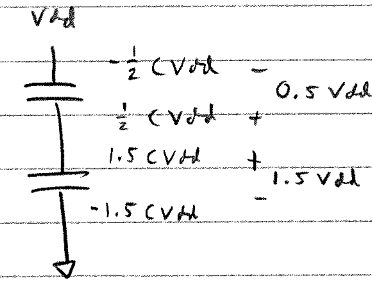


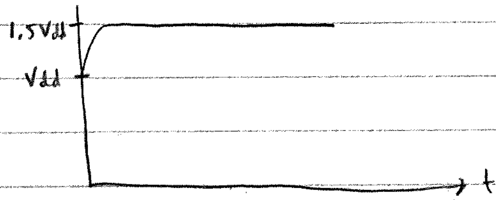
Wong 2010

1. 

$\Rightarrow$ initial 

final : 

$=$ 



2. $E_i = \frac{1}{2}(2C)V_{dd}^2 = CV_{dd}^2 \qquad E_f = \frac{1}{2}C\left(\frac{V_{dd}}{2}\right)^2 + \frac{1}{2}C\left(\frac{3V_{dd}}{2}\right)^2 = 1.25\,CV_{dd}^2$

3. if $C_{left} \ll C_{right}$, $V_{out_{max}} = 2V_{dd}$

4. ??

OR : 1) charge trapped on center node

2) $C = \frac{Q}{V_{DD}} \Rightarrow Q = CV_{DD}$

3) total Q on center node $= 2CV_{DD}$

4) charge redistributes.

5). $Q_1 + Q_2 = 2CV_{dd}$

$C_1 = \frac{Q_1}{V_{out}' - V_{DD}} \qquad C_2 = \frac{Q_2}{V_{out}' - 0}$

$Q_1 = C_1(V_{out}' - V_{DD}) \qquad Q_2 = C_2 V_{out}'$

$\Rightarrow C_1(V_{out}' - V_{DD}) + C_2 V_{out}' = 2CV_{dd}$

$\Rightarrow V_{out}'(C_1 + C_2) = 3CV_{dd} \Rightarrow V_{out}' = \frac{3C}{2C}V_{dd} = 1.5\,V_{dd}$

2005 - Dutton

1. 1) Gate delay
   2) Gate pitch
   3) I_{ON}/I_{OFF}

2. In general, scaling means $V_t \downarrow \Rightarrow V_{DD} \downarrow$ and also, there are increased short channel effects.

$$f_T = \frac{g_m}{C_{gg}} \approx \frac{3}{2}\frac{\mu V_{ov}}{L^2} \Rightarrow \text{as } L\downarrow,\ f_T\uparrow \text{ good.}$$

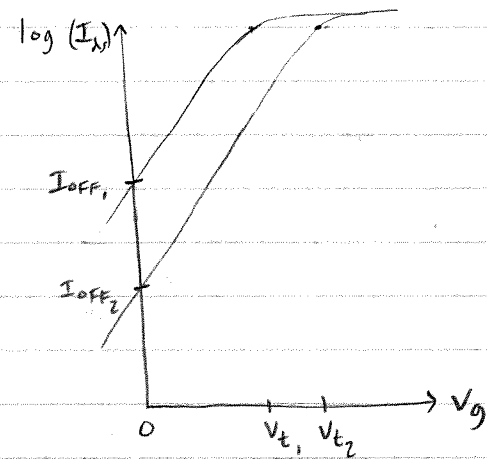
as $V_{DD}\downarrow$, voltage swing is reduced. Cascoded stages become less attractive.

Simple square law models cannot be used. Better to use a g_m/I_d approach.

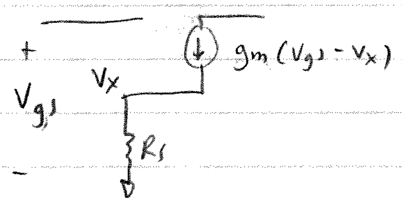
2009 - Dutton

1. Do lower V_t devices have lower "off currents" (for $V_{gs} = 0V$)?

No. Minimum S is 60mV/decade. $\Rightarrow$ the lower the V_t, the higher the "off current" for a given on current.



2. MOS



$$V_x = g_m(V_{gs} - V_x) R_s$$

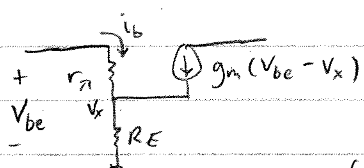
$$V_x (1 + g_m R_s) = g_m R_s V_{gs}$$

$$V_x = \frac{g_m R_s}{1 + g_m R_s} V_{gs}$$

$$\Rightarrow i = g_m V_{gs} \left(1 - \frac{g_m R_s}{1 + g_m R_s}\right) = \frac{g_m}{1 + g_m R_s} V_{gs}$$

$$\Rightarrow G_m = \frac{g_m}{1 + g_m R_s}$$

BJT



$$= \beta i_b = \beta \frac{(V_{be} - V_x)}{r_\pi}$$

$$V_x = \left(1 + \frac{1}{\beta}\right) g_m (V_{be} - V_x) R_E$$

$$\Rightarrow V_x = \frac{(1 + \frac{1}{\beta}) g_m R_E V_{be}}{1 + (1 + \frac{1}{\beta}) g_m R_E}$$

$$g_m (V_{be} - V_x) = g_m \left(\frac{V_{be}}{1 + (1 + \frac{1}{\beta}) g_m R_E} \right)$$

$$\Rightarrow G_m = \frac{g_m}{1 + (1 + \frac{1}{\beta}) g_m R_E} \Rightarrow \boxed{T}$$

BJT g_m is degraded slightly more since there is base current (vs. no gate current for MOS).

3. No. Intrinsic gate delay is the RC product τ of a MOSFET inverter driving its own gate capacitance.

Gate delay depends on R as well as C, and $R \propto \frac{1}{\mu}$

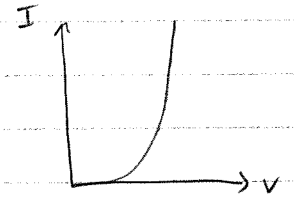
$\Rightarrow$ mobility matters. $\Rightarrow$ $\boxed{F}$

a) Yes.

b) Bias it in far right region by placing a resistive load $\|$ current source at the output + choosing $V_{control}$ such that the operating point is at point o

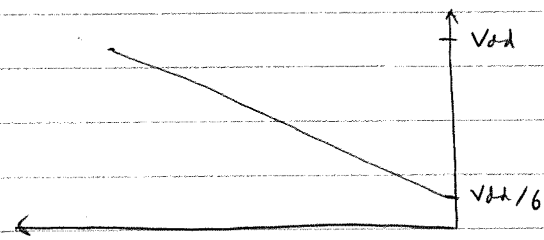
c) Limitations are finite output resistance $\left(\frac{\partial I_{out}}{\partial V_{out}} \neq 0\right)$, Limited output swing to keep the gain high (must stay in far right region). And high voltage operating point.

2010 - Dutton

A) 

PN diode, forward biased.

B) JFET as $V_{in}\downarrow$, depletion region grows $\Rightarrow$ channel narrows $\Rightarrow$ $R_{dev}\uparrow$ $\Rightarrow$ $V_{out}\uparrow$



2000 - Wooley

$V_T = 1V \quad \mu C_{ox} = 100\mu A/V^2 \quad \frac{W}{L} = 5 \quad A = 20 \quad R_{in} = ?$

$\frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{gs} - V_T)^2 = 1mA \Rightarrow 0.25mA\,(V_{gs} - 1V)^2 = 1mA$

$\Rightarrow V_{gs} - 1V = \sqrt{4} = 2V \Rightarrow V_{gs} = 3V, \quad V_{ov} = 2V$

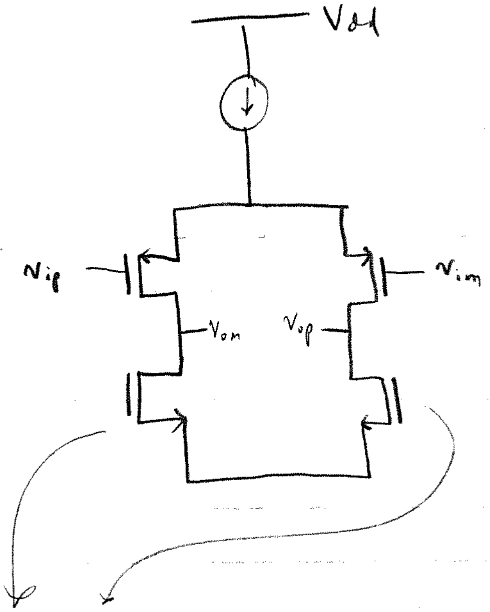
$g_m = \frac{2I_d}{V_{ov}} = 1mS$

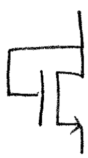
Using Blackman's Impedance Formula:

$Z_{k=0} = \frac{1}{g_m} \quad RR_{(short)} = -\left(\frac{-g_m R V_t A}{V_t}\right) = g_m R A$

$RR_{(open)} = 0 \Rightarrow Z = R_{in} = \frac{1}{g_m}\left(\frac{1 + g_m R A}{1 + 0}\right) = \frac{1}{g_m} + RA = \boxed{21k\Omega}$

2003 - Wong

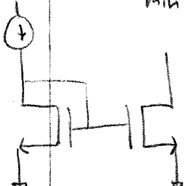


(1)  diode connected : High Linearity (if load was p-type) so not in this case.
Low Gain
Low Output swing

Linearity : If load was p-type, $A_v = -\frac{g_{mp1}}{g_{mp2}} = \frac{\sqrt{2\mu C_{ox}(W/L)_1 I_D}}{\sqrt{2\mu C_{ox}(W/L)_2 I_D}} = \sqrt{\frac{(W/L)_1}{(W/L)_2}}$ ⇒ ratiometric. Independent of process variations. (μ, C_{ox}, V_t)

Gain : $A_v = -\frac{g_{mp}}{g_{mn}}$

Output swing : $V_{out_{min}} = V_t + V_{ov}$

(2)  Current source load : High Gain
Medium Output swing.

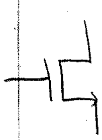
Gain : $A_v = -g_{mp}(r_{o_p} \| r_{o_n})$

Output swing : $V_{out_{min}} = V_{ov}$

$r_o = \frac{1}{\lambda I_D}$ $\lambda \propto \frac{1}{L}$ ⇒ $r_o \propto \frac{L}{I_d}$

but for fixed current, $L\uparrow \Rightarrow (\frac{W}{L})\downarrow \Rightarrow V_{ov}\uparrow$

⇒ higher gain ⇒ lower voltage swing.

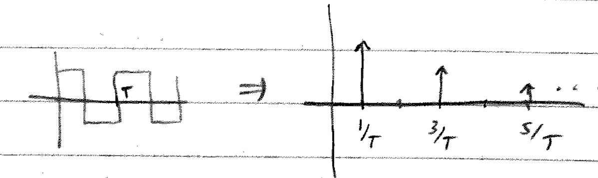
(3) V_B  Triode Load : Medium Gain (but susceptible to process variations)
High output Swing

Gain : $A_v = -g_{m_p}(r_{op} \| r_{o_n} \| R_{on})$ But $R_{on} = \frac{1}{\mu_n C_{ox}(\frac{W}{L})_2 (V_b - V_t)}$ ⇒ susceptible to process variations (μ, C_{ox}, V_t)

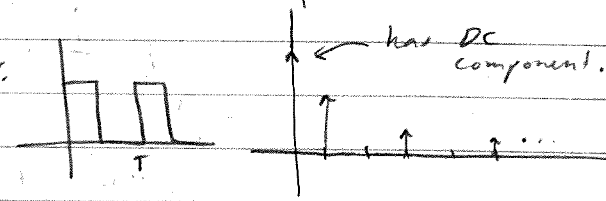
Output Swing : $V_{out_{min}} = 0$

2009 - Wong

1. Power spectrum of square wave is:



Power spectrum of positive square wave is:

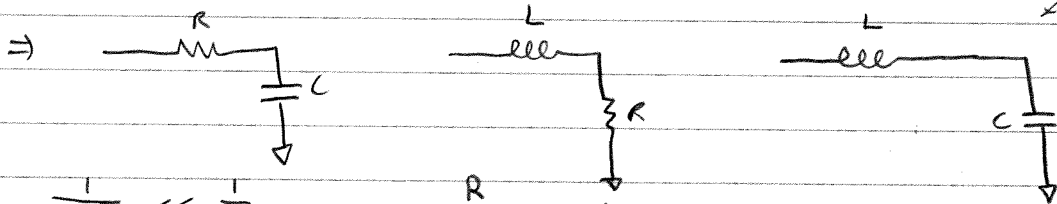


In this case, we want to extract only the DC component.

$\Rightarrow$ use a LPF with $f_p \ll \frac{1}{T}$

except for peaking at resonant frequency, this filter has a low pass behavior.

$\Rightarrow$



$\frac{1}{2\pi RC} \ll \frac{1}{T}$ $\qquad$ $\frac{R}{2\pi L} \ll \frac{1}{T}$ $\qquad$ $\frac{1}{2\pi\sqrt{LC}} \ll \frac{1}{T}$

$2\pi RC \gg T$ $\qquad$ $2\pi \frac{L}{R} \gg T$ $\qquad$ $2\pi\sqrt{LC} \gg T$

2. If the output has to drive a heavy load, how would you modify the circuit?

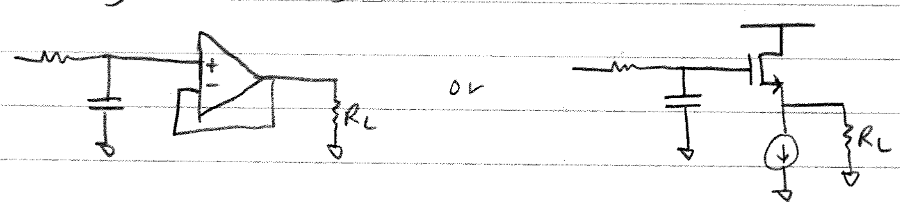
Heavy load $\Rightarrow$ small R_L

R-C : Won't work. R_L will increase BW.

R-L : Will work. R_L will decrease BW.

LC : ω_0 stays the same. Peaking reduced $\left(Q = \frac{R}{\sqrt{L/C}} = \omega_0 RC\right)$

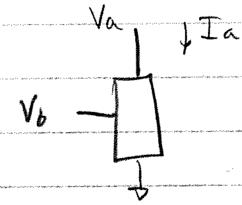
Also could add unity gain voltage buffer capable of driving the heavy load.



there will be a level shift here of $V_{ov} + V_t$.

2007 - Dutton

$$I_a = \left(V_b + \frac{V_a}{10}\right)^2$$

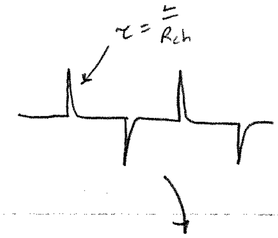


$$g_m = \frac{\partial I_a}{\partial V_b} = 2\left(V_b + \frac{V_a}{10}\right)$$

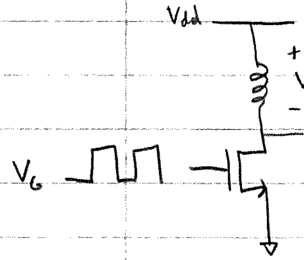
$$g_{ds} = \frac{\partial I_a}{\partial V_a} = \frac{2}{10}\left(V_b + \frac{V_a}{10}\right)$$

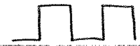
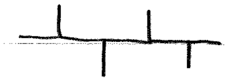
$$\Rightarrow g_m r_o = \frac{g_m}{g_{ds}} = 10$$ ← intrinsic gain.

considering finite channel resistance

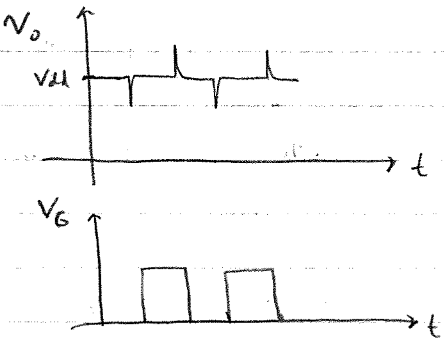


2006 - Kovacs



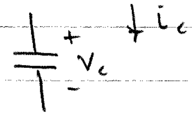
$V_L = L \frac{di}{dt}$ i_{ds}  $\Rightarrow$ V_L 

$V_o = V_{dd} - V_L$ $\Rightarrow$ V_o



Capacitor + Inductor Basics:

Capacitor:



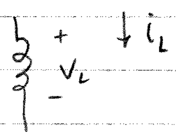
$$i = C \frac{dV}{dt}$$

$$V = \frac{1}{C}\int_{t_0}^{t} i\,dt + V(t_0)$$

$$P = iV = CV\frac{dV}{dt}$$

$$E = \int P\,dt = \int CV\,dV = \frac{1}{2}CV^2$$

Inductor:

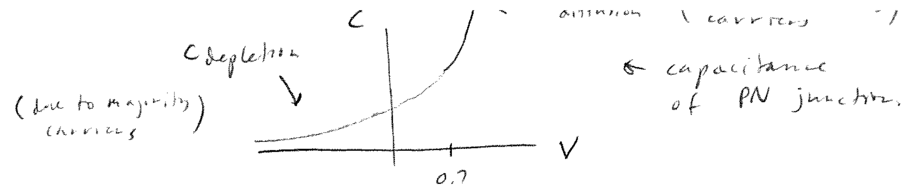


$$V = L\frac{di}{dt}$$

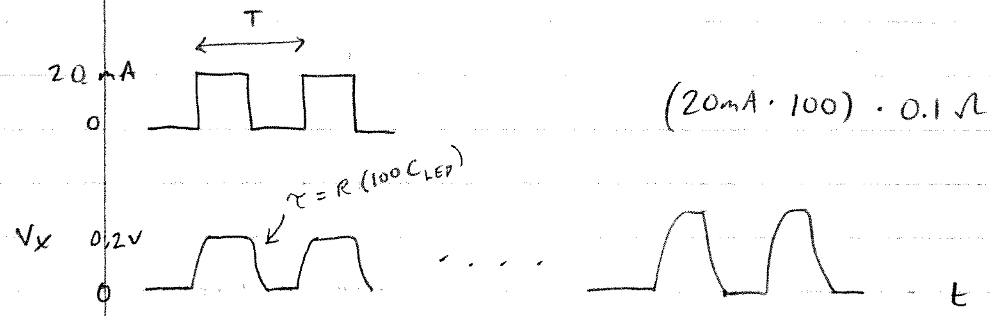
$$i = \frac{1}{L}\int_{t_0}^{t} V\,dt + i(t_0)$$

$$P = iV = Li\frac{di}{dt}$$

$$E = \int P\,dt = \int Li\,di = \frac{1}{2}Li^2$$



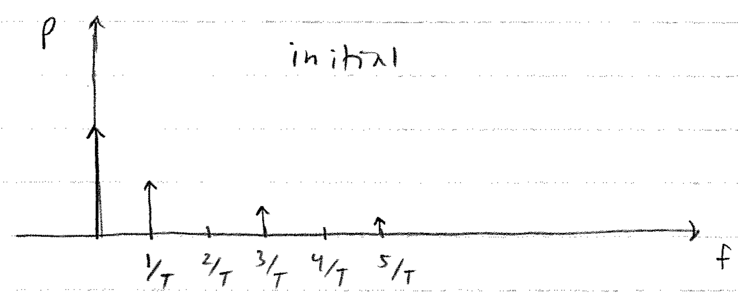
2005 - Kovacs

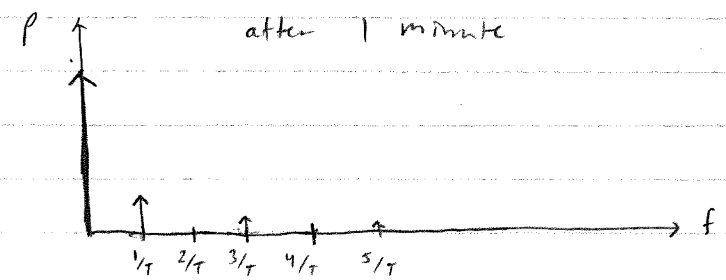


$(20mA \cdot 100) \cdot 0.1\,\Omega = 0.2V$

as time passes, Resistor heats up $(P = I^2R)$
and as $T\uparrow$, $R\uparrow$. As $R\uparrow$, V_x increases
and $\tau = R(100\,C_{LED})$ increases. ⇒ output is smoother and
has increased power (especially in its DC component)

⇒ Power spectrum:



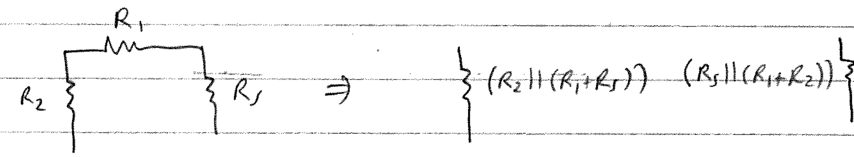


2009 - Poon

1. Series - Series

2. $f = \dfrac{R_S R_2}{R_1 + R_2 + R_S}$

3. loading :



$$\Rightarrow a = -g_{m_1} (r_{o_1} \| r_{o_3}) (-g_{m_5} r_{o_5}) \frac{g_{m6}}{1 + g_{m_6} (R_S \| (R_1 + R_2))}$$

$$\Rightarrow a = \frac{g_{m1} g_{m5} g_{m6} (r_{o_1} \| r_{o_3}) r_{o5}}{1 + g_{m_6} (R_S \| (R_1 + R_2))}$$

4. $R'_{out} = \big(1 + g_{m_6} (R_S \| (R_1 + R_2))\big) (1 + af)$

How to find f :

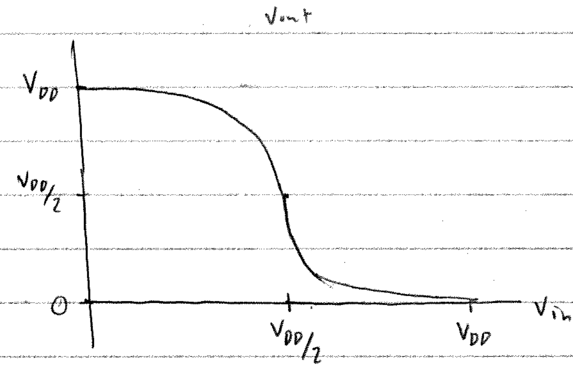
If feedback is shunt at input, short input feedback node and calculate feedback current.
If feedback is series at input, open input feedback node and calculate the feedback voltage.
If feedback is shunt at output, drive feedback network with a voltage source.
If feedback is series at output, drive feedback network with a current source.

How to find loading effects :

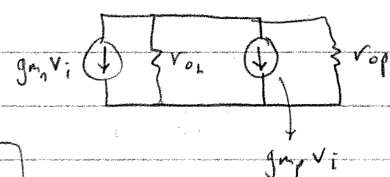
If feedback is shunt at input (output), short input (output) feedback node to find feedback loading on output (input). If feedback is series at input (output), open the input (output) feedback node to calculate the output (input) feedback loading.

2005 - Wong

1.



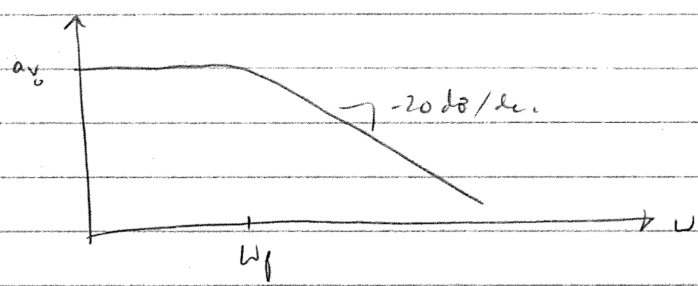
2. N + P in sat ⇒



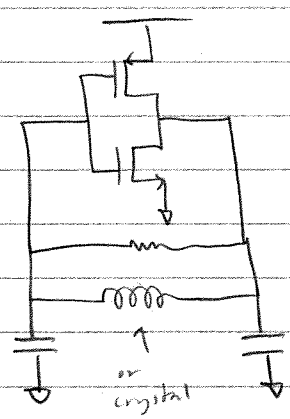
$$\Rightarrow a_v = \boxed{-(g_{mn}+g_{mp})(r_{on} \| r_{op})}$$

3. Dominant pole due to miller multiplied Cgd + Cgs at input.
Assume Rs at input.

$$\Rightarrow \omega_p = \frac{1}{R_s\left(2C_{gd}\left(1+(g_{mn}+g_{mp})(r_{on}\|r_{op})\right)+2C_{gs}\right)}$$



4.

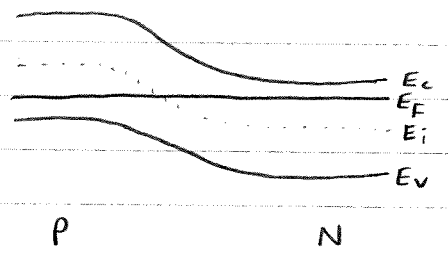


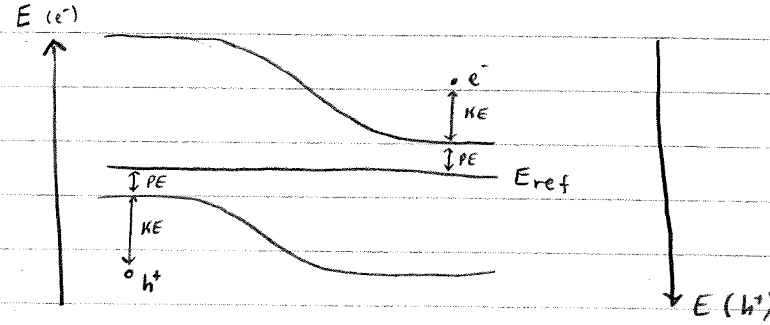
5. $\omega = \frac{1}{\sqrt{LC}}$ amplitude determined by R? Vdd?

Devices

1998 - Harris 11/25

1. Draw an energy band diagram for a p/n junction.
What does the vertical axis represent (Total E, KE, or PE)?

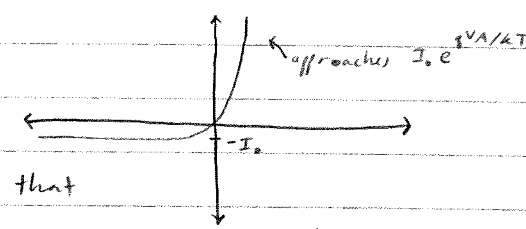




The vertical axis is total energy. E_c and E_v are lines of PE and zero KE for electrons and holes, respectively. ← X
KE is the energy above E_c for electrons and below E_v for holes.

2. Draw the I-V characteristic for the p/n junction and explain what physically happens in terms of the band diagram above when you apply a bias to the junction. What is the "turn on voltage"?

Ideal PN Junction : $I = I_0 (e^{qV_A/kT} - 1)$



At zero bias there is an electrostatic barrier (V_{bi}) that provides a balance between drift and diffusion of carriers in the junction. Under forward bias the barrier is reduced, favoring carrier diffusion over drift. The carriers have a Boltzmann distribution in energy, thus the number of carriers which exceed the remaining barrier (are injected as minority carriers into the opposite conductivity region) is proportional to $\exp[(V_A - V_{bi})/kT]$, providing an exponential dependence of current on voltage. Under reverse bias, the barrier is increased, reducing the injection of carriers and favoring drift. However, the concentration of minority carriers can only go to zero and the number which reach the p/n junction is limited by diffusion from the bulk, which remains essentially constant with bias, producing a saturation current independent of bias. The threshold voltage V_{th} is just the extrapolation from the high forward current region to zero current and is usually about $\frac{2}{3} E_g/q$ (at Room temperature) implies forward bias.

3. What happens to the injected carriers? What energy do the photons have if the recombination is radiative?

The injected carriers are minority carriers, which are at a higher concentration than their thermal equilibrium concentrations and they are surrounded by a far larger number of majority carriers, thus they recombine (either radiatively (band to band) or non radiatively) to move toward their equilibrium value. If the recombination is radiative, then the photon energy, $h\nu$, is equal to E_g.

4. How is it possible to get $h\nu = E_g$ photons emitted when we've only had to apply $V_{th} \sim \frac{2}{3}\frac{E_g}{q}$ volts bias to get forward current to flow? Is this a violation of Conservation of Energy and the Laws of Thermodynamics?

The minority carriers only require about $\frac{2}{3}\frac{E_g}{q}$ to be injected because these are the carriers farthest up (highest KE) in the Boltzmann distribution, so the "added" energy is KE from the lattice.

5. What would happen if the diode was placed in thermal isolation and forward biased but with a window where photons could escape? Could I make a refrigerator with such a diode?

Yes. If the diode were placed in a thermally isolated environment and forward biased with only radiative recombination occuring, the diode initially loses energy, thus it's temperature drops and it serves as a refrigerator. (Later external source will provide energy to it)

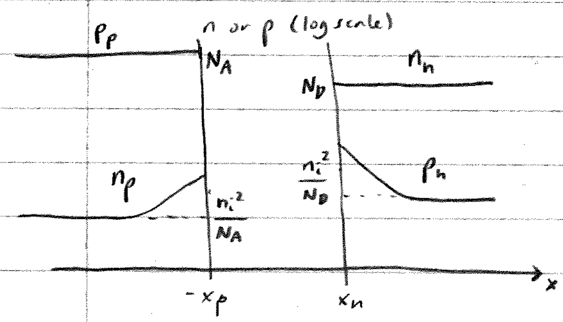
6. What would the ideal "asymptotic" IV characteristic look like and what would be the values of V_{th} and T?

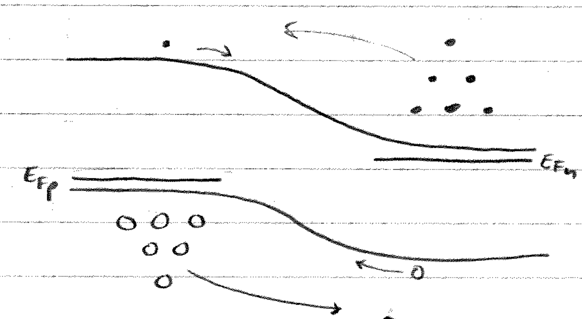
Ideally, the asymptotic IV characteristic would have zero current until reaching $V_A = \frac{E_g}{q}$ whereupon the current increases extremely rapidly, since I is proportional to $\exp[(V_A - V_{th})/kT]$ and in the limit, T is going to 0K.

7. Do you think the same thing will happen with a Schottky barrier (M-S diode)? Why or why not?

No. Because forward current is a result of majority carriers from the semiconductor being injected as hot ($KE = \Phi_b$) into the metal, which has no bandgap, thus all of the KE is dissipated by scattering and thermal energy is deposited into the metal. $\Rightarrow$ we can't use this as a refrigerator.

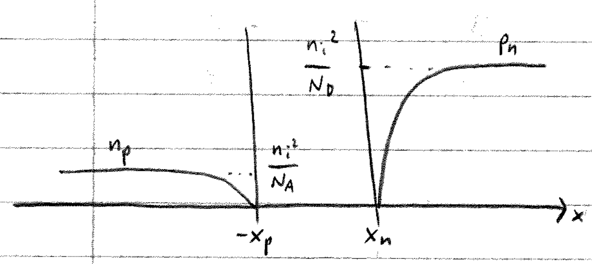
Relevant Basics: PN Junction

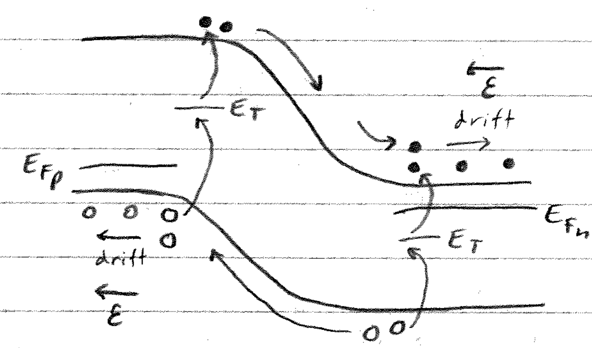




excess minority holes recombine here with majority electrons. These electrons are replaced by electrons from the contact through drift.

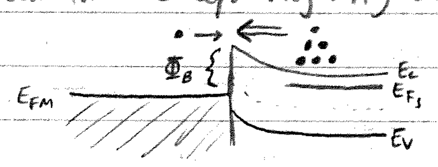
$$\Delta n_p(-x_p) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)$$
$$\Delta p_n(x_n) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$





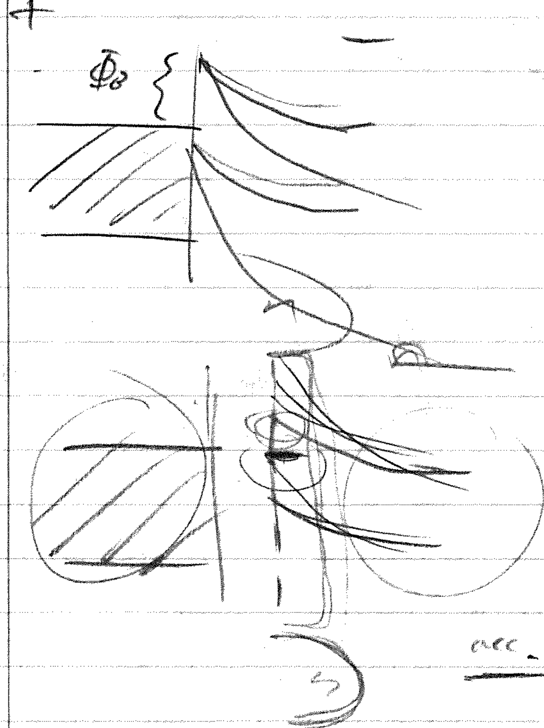
minority electrons that diffuse into depletion region are replaced through generation. This creates a slight excess of majority carriers near the junction which creates an $\vec{\mathcal{E}}$ field that sweeps majority carriers towards the contacts.

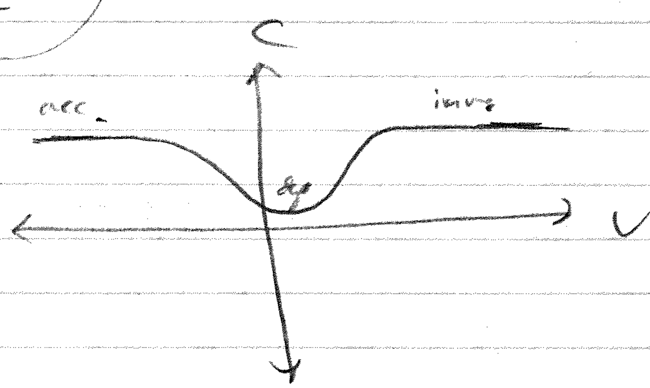
MS Diode: Forward bias:

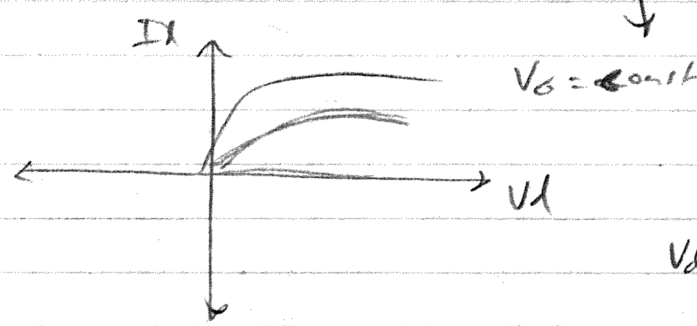


$\Rightarrow$ no recombination of injected carriers on metal side.

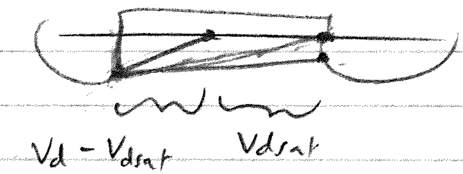
1996 Harris



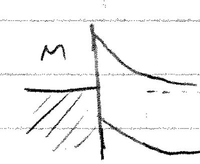




$V_{dsat} = V_{gs} - V_t$



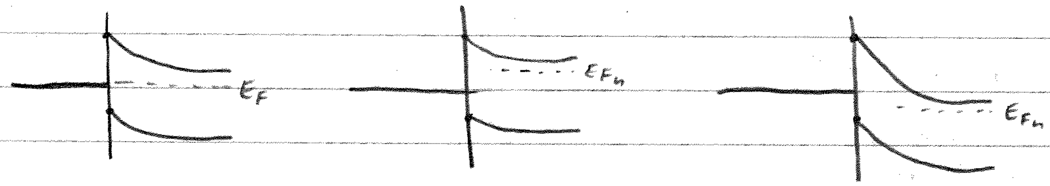
A: 1. If I deposit a metal layer onto a semiconductor to form an MS junction:



A. What occurs at the interface or junction?
Carriers move ⇒ sheet of charge forms at metal interface and depletion region forms, ⇒ band bending.

B. Draw an energy band diagram and explain what happens when a voltage is applied to the junction.

+ V_A − Forward Bias

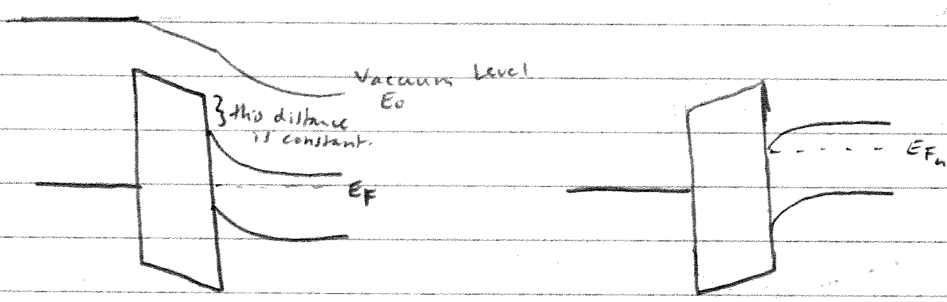


$V_A = 0$ $V_A > 0$ $V_A < 0$

2. If I now insert a thin insulator between the metal and semiconductor:

C. What does the insulator do? Prevents current flow

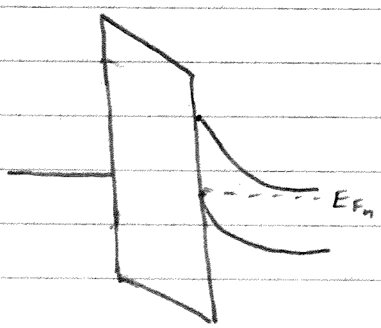
Draw an energy band diagram. Explain what happens with bias:



$V_A = 0$ $V_A > 0$

accumulation

Pierret pg. 575

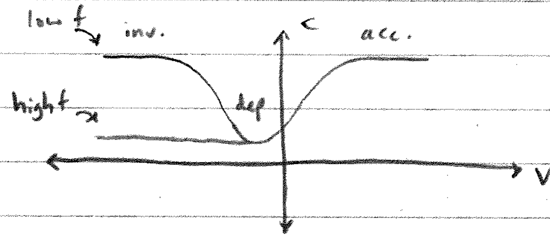


$V_A \ll 0$

inversion.

D. Since there is no current flow, an IV curve is not meaningful. What measurement might be used to characterize an MIS structure and explain what you expect it to look like?

C-V measurement.



E. What causes the inversion layer to form? minority carriers. (which are formed by thermal generation).

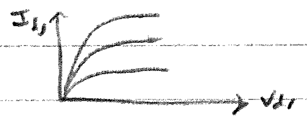
3. I can utilize this structure to form a transistor:

F. What is the basis of forming such a transistor: like MOSFET

G. From where do carriers originate that form the inversion layer?

they mostly come from the source + drain.

H. Draw IV characteristics.



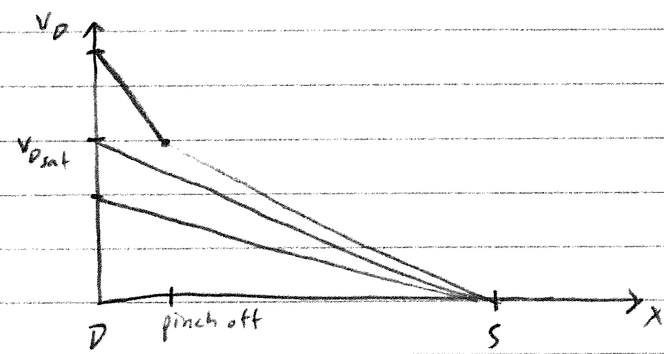
I. What are V_{th} and V_{dsat}.

V_{th} : $p_n = n_o$ V_{dsat}: voltage where channel pinches off at drain.

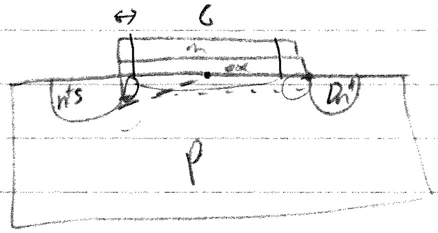
↖ when inversion layer starts to form.

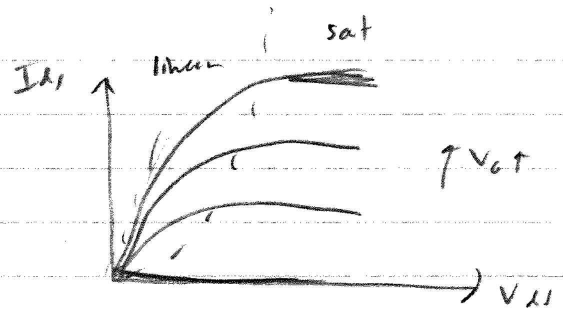
J. What does the potential look like along the channel for

$V_D < V_{Dsat}$ and $V_D > V_{Dsat}$



1997 - Harris

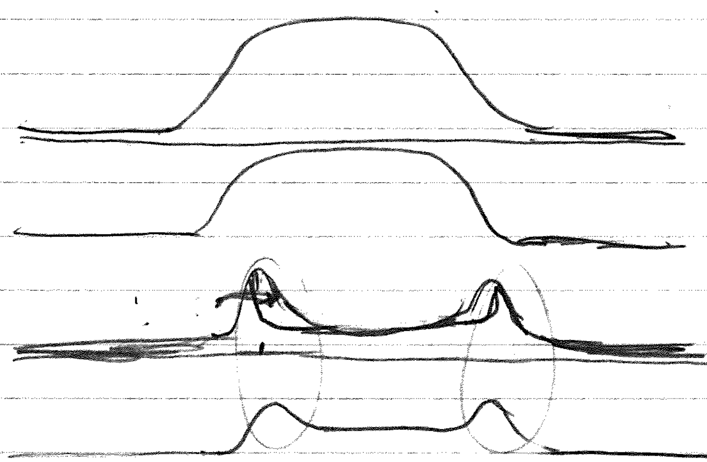




$V_{ds} < V_{gs} - V_{th}$

$V_{ds} > V_{gs} - V_{th}$ $\quad V_{gd} < V_{th}$

$I_d \propto V_{ds}$



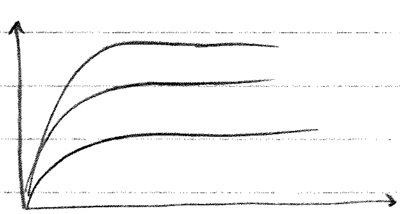
$Q = CV$ $\qquad V = \frac{Q}{C}$

~~100mV = 1~~

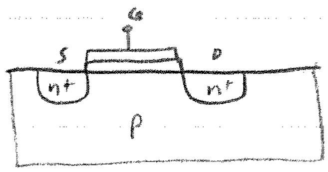
$100\,meV = \frac{1eV}{C}$

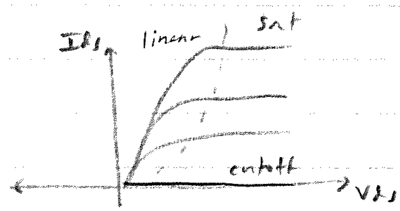
$C = \frac{1}{100}$

100 times less.



1. Sketch cross-section of MOSFET. Draw IV curves. Describe 2 or 3 most important regions and how these are related to the physics within the device.

A. 



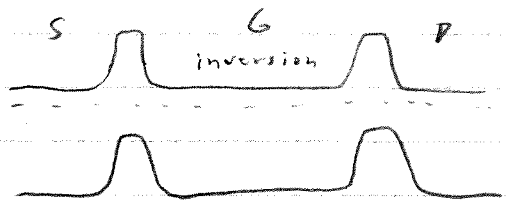
2. What happens if gate does not overlap Source and Drain regions?

A. Neglecting fringing fields, the transistor does not function.

3. Can an inversion region still form under gate with applied V_{gs}?

A. Yes. This is a MOS cap. The device first goes into deep depletion and eventually forms an inversion layer under the gate from thermal generation of carriers.

4. Draw band diagram assuming a gate bias creates an inversion layer.



5. If I make the p regions narrow, I claim I can make a new type of transistor. What might be the important physical mechanisms operative in such a device?

A. Tunneling. If I apply a positive bias to D, an electron can tunnel from S to channel and then from channel to D. Thus get I_{ds}.

6. I would like to make a device that could operate at room temp. If I could make the gate (and hence inversion region) so small that the tunneling of a single electron into this small region increases the potential by 100 meV

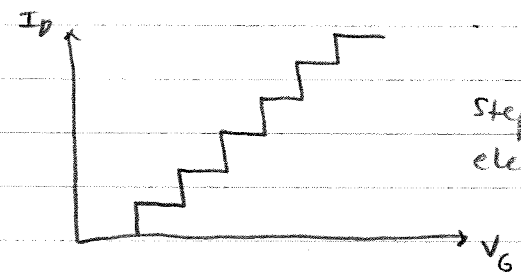
how small would this have to be (for instance compared to the gate capacitance of a MOSFET)?

$C = \frac{dQ}{dV} = \frac{1.6 \cdot 10^{-19}}{0.1} = 1.6 \cdot 10^{-18}\,F$ which is very small compared to a typical MOSFET gate $C_g = 10^{-15}\,F/\mu m^2$ or $1\,fF/\mu m^2$

$Q = CV \Rightarrow C_{gate} = \frac{q}{100\,mV}$

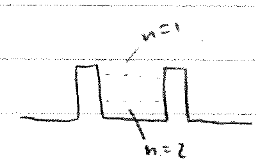
charge of e^-: q $100\,mV$

7. what might you guess the I-V characteristic to look like if I had such a small structure and current transport limited by tunneling of single electrons.



Steps corresponding to single electron tunneling.

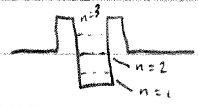
$V_G = V_{G_1}$



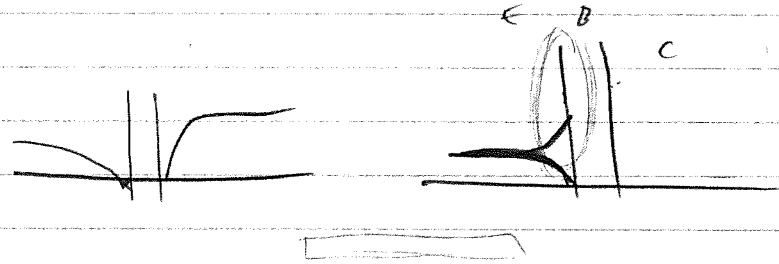
when an e^- tunnels into the well, the energies of the allowable states increase $\Rightarrow$ other e^-'s can't tunnel into the well.

If you apply a larger gate voltage V_G, the well becomes deeper $\Rightarrow$ there are more available states in the well. $\Rightarrow$ can have more electrons in the well simultaneously.

$V_G = V_{G_2} > V_{G_1}$



1994 - Harris



1. What are the advantages of MOSFET + associated technology to BJTs.

Lower power, simpler fabrication technology, natural device isolation, higher integration levels, high input impedance and no gate leakage ⇒ charge storage, complementary architecture.

2 What areas has the bipolar transistor remained important?

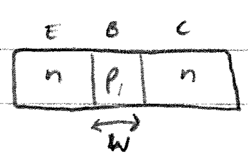
Analog and A/D circuits, microwave and high speed digital circuits, BiCMOS circuits, high power devices.

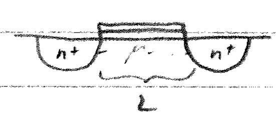
3. Why has bipolar remained faster.

It has been easier to control the base width to very small dimensions by ion implantation and epitaxy than the source drain spacing for MOSFETs by lithography. As devices are scaled to very small dimensions, parasitic capacitances no longer scale and the high current drive capability and lower voltage operation of bipolar transistors enables them to operate at higher speeds.

$\tau = \frac{W^2}{2D_B}$

$= \frac{W^2}{2\mu V_T}$





but BJTs are still faster since ~~MOSFET speed is~~ limited by parasitic capacitance.

$\tau = \frac{L}{v_{th}} \Rightarrow \tau = \frac{L}{\mu \mathcal{E}} = \frac{L^2}{\mu V_{CE}}$

$\tau = \frac{L^2}{\mu V_{ds}}$

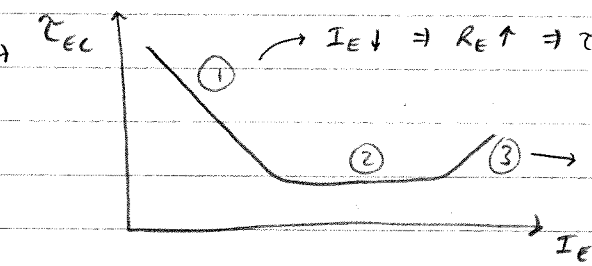
$\mathcal{E} = \frac{V}{L}$

since $\mu_B > \mu_{CH}$ ← due to lattice scattering, lower carrier concentration

~~$\tau_{BJT} < \tau_{MOS}$~~ but $V_T << V_{ds} \Rightarrow \tau_{BJT} > \tau_{MOS}$

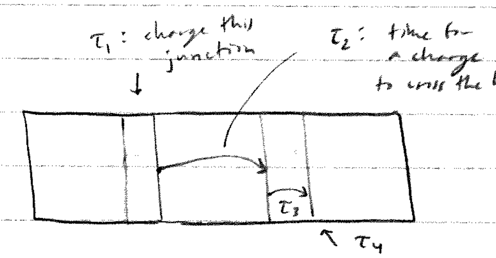
The high speed performance of a BJT varies with emitter current as follows: How is this related to the physics of the device operation?

transit time from emitter to collector



① → $I_E \downarrow \Rightarrow R_E \uparrow \Rightarrow \tau_1, \tau_4 \uparrow$ takes long time to charge CB and EB junction.

③ → kirk effect



$\tau_1 = R_E C_{j_{BE}}$ $\quad R_E = \frac{dV_{BE}}{dI_E}$, $I_E = I_0 e^{qV_{BE}/kT}$

$\Rightarrow R_E = \frac{kT}{qI_E} = \frac{V_T}{I_E}$

$\tau_2 = \frac{W^2}{2D_B}$

$\tau_3 = \frac{W_c}{2V_{sat}}$

$\tau_{EC} = \tau_1 + \tau_2 + \tau_3 + \tau_4$ $\quad \tau_4 = R_C C_{j_{CB}}$ $\quad R_C = \frac{dV_{CB}}{dI_C} = \frac{V_T}{I_C}$

τ_3 : time for charge to drift across CB depletion width : ~

τ_4 : time to charge CB junction

? kirk effect :

1993 - Harris

(1) What are the properties of the "ideal" semiconductor and why?

1. Bandgap - 1.2-1.5eV (direct for optical devices)
   - Too Small (<1.2eV): large n_i, inability to control n & p, poor operating temperature range, large leakage currents, Low breakdown voltage (for FETS and BJT)

   - Too Large (>1.5eV): insulator (unable to ionize impurities and obtain free carriers), poor ohmic contacts, high power dissipation and high V_{TH} for FET, BJT, diodes and lasers.

2. Naturally Forming Oxide - Good insulator, passivates (makes nonreactive to air) the surface, has no interface states (trap states), thermally stable, good high temperature diffusion + ion implantation barrier, easily etched, has good etch selectivity to semiconductor. $K = 3.9$ (for Si, $K = 11.7$)

3. Easily Grown - and prepared as large area wafers that are physically robust, thermally and chemically stable, compatible with both metals and insulators, able to be controllably doped n or p over a broad range and to high densities, able to be semi-insulating and have a lower dielectric constant (<2)

4. High Mobility - for both e^- and h^+ (and equal for complementary circuits), low resistivity for diodes, lasers, BJT, etc., high v_{dsat} which is reached at lower electric fields for high speed and low power dissipation.

5. High Thermal Conductivity - and small thermal expansion coefficient and one that matches to its oxide and metals.

6. Heterojunction Possibilities - to be able to fabricate higher performance lasers, HBTs, MODFETs, OEICs, QW devices.
   optoelectronic quantum well.

7. Temperature Independence - of properties.

8. High Breakdown Field - for high power devices

?

low effective mass
⇒ lower density of states
⇒ less tunneling

9. Small and Equal Effective Mass - Better tunneling for low resistance ohmic contacts, lower I_{th} for lasers, and higher current in tunnelling devices.

10. Short/Long Carrier Lifetime - Short for high speed devices, Long for CCDs, solar cells, and high power devices.

11. Widely Abundant, Low Cost, Non-Toxic material.

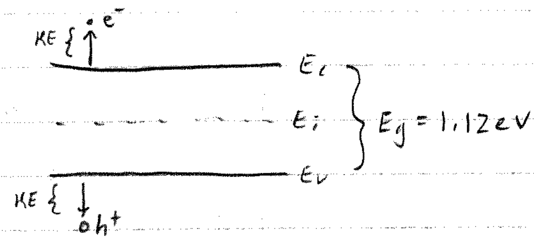
(2) If you could choose one property to be temperature independent, which would you choose?

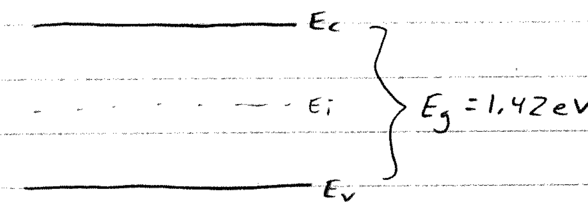
Bandgap - ($E_g(T) = E_g(0) - \frac{\alpha T^2}{T+\beta}$, $E_g(0), \alpha, \beta$ are constants)
Temperature variation in bandgap causes threshold variation in FETs and HBTs, wavelength of lasers, and leakage currents in all devices.

Mobility - a high, temperature independent mobility for both $\bar{e}$ and h^+ makes complementary, high speed circuits much easier to design.

2005 - Harris

1.





Si : $m_n/m_o = 0.26$
$m_p/m_o = 0.39$

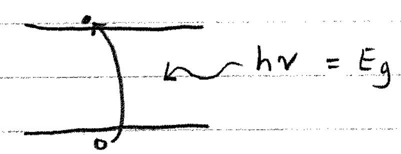
GaAs : $m_n/m_o = 0.068$ ⇒ much higher μ_n
$m_p/m_o = 0.5$ ⇒ lower μ_p
⇒ worse for CMOS (digital logic)
but good for high frequency (MMIC)

Indirect bandgap.

Direct bandgap.
Wider bandgap ⇒ less sensitive to heat, resistance to radiation damage (good for space electronics), highly resistive in pure state with high dielectric constant ⇒ ideal electrical substrate (provides natural isolation).

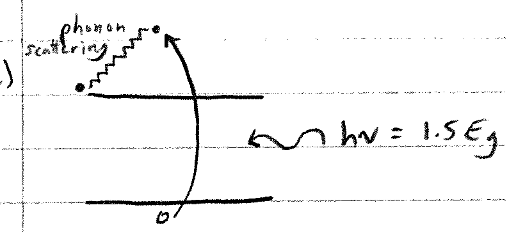
a) Vertical axis is total Energy.

b)



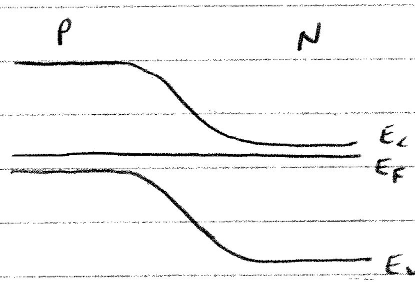
band to band generation of e^- h^+ pair.

c)

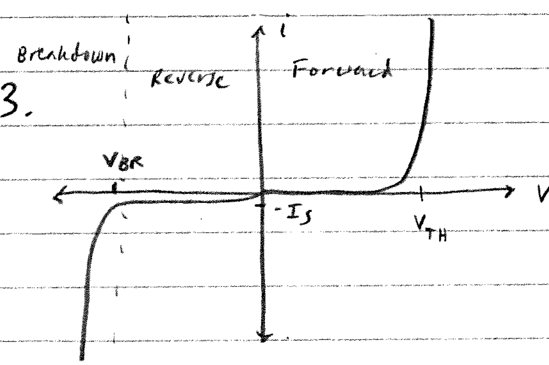


d) Excess energy is transferred to lattice as heat (phonons).

2.



3.

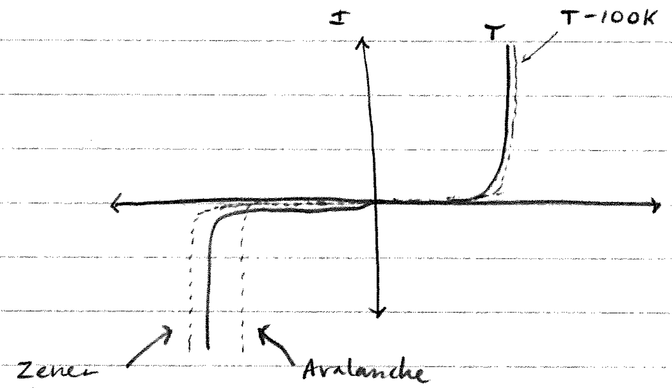


a) Forward : Barrier lowering ⇒ diffusion of majority carriers
⇒ exponential. $I = I_s\left(\exp\left(\frac{qV_A}{kT}\right) - 1\right)$

Reverse : Drift of minority carriers, Barrier increases but doesn't matter.
⇒ saturates. $I_s = qA\left(\frac{D_P}{L_P}\cdot\frac{n_i^2}{N_D} + \frac{D_N}{L_P}\frac{n_i^2}{N_A}\right)$

Breakdown : Due to zener breakdown (if voltage is low) heavily doped
or Avalanche breakdown (if voltage is high). lightly doped

b)



T↓ ⇒ KE of majority carriers ↓

Forward: V_{TH} vs. T : $-2mV/°C$

Reverse: $I_s \propto T$

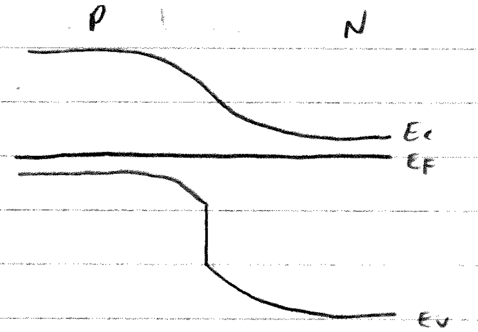
Breakdown: Zener: T↑ V_{BR} ↓

Avalanche: T↑ V_{BR} ↑

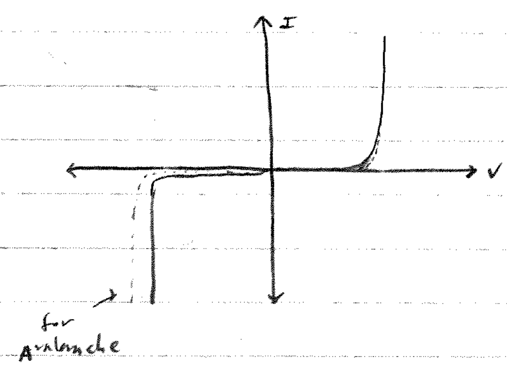
T↓ ⇒ D_p and D_n ↓ and n and p ↓

⇒ I_s ↓

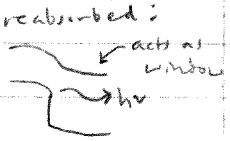
4. a)



b) Reverse current I_s decreases, since n_i decreases (E_g larger) on n-side. Forward current reduced by factor of 2 (e^- current dominates, h^+'s see large barrier). Possible increase in V_{BR} for avalanche since E_G is higher on n side.



or for an LED to prevent photons from being reabsorbed:

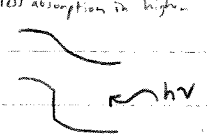


c) Yes. This would be advantageous in a HBT. If you made the emitter of an npn HBT a wider bandgap material, you could heavily dope the base (which would reduce the negative effects of Base width modulation and r_B which leads to current crowding) without sacrificing emitter efficiency:

$$\gamma = \frac{I_{E_n}}{I_{E_n} + I_{E_p}}$$

Since holes from base would see a large barrier due to the hetero structure.

or for a solar cell. could allow more light to reach the depletion region ⇒ increase solar efficiency, less absorption in high-

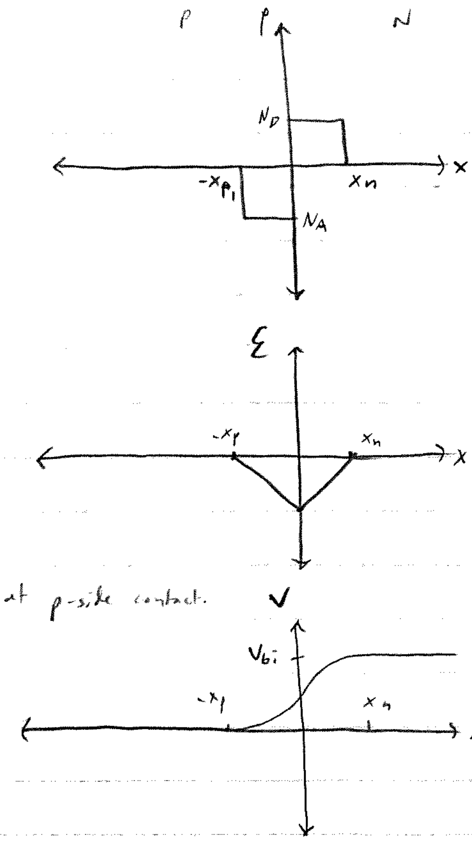


2003 - Harris

1. What is the depletion approximation for a pn junction?

- depletion region is fully depleted of mobile charge carriers.
- therefore there is no recombination in the depletion region.
- net charge density outside of depletion region is zero.

2.



$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$

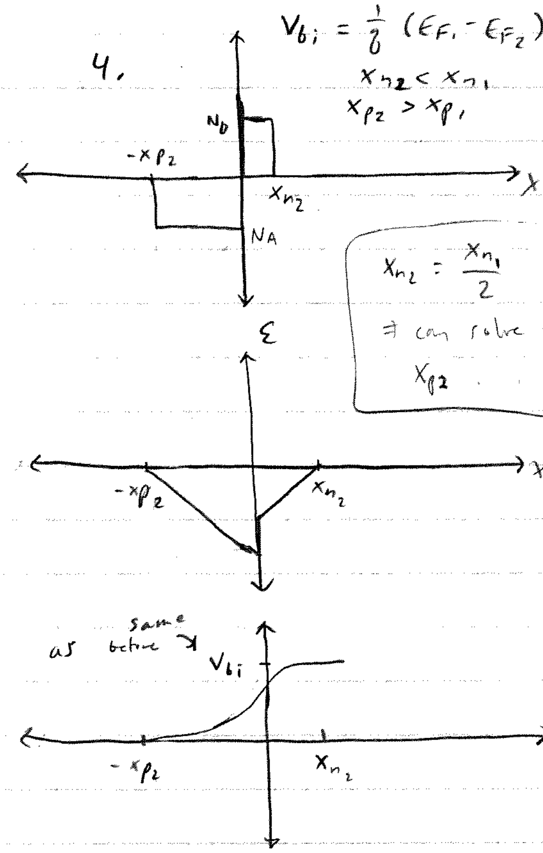
let V=0 at p-side contact.

4.

$V_{bi} = \frac{1}{q}(E_{F_1} - E_{F_2}) = \text{same}$

$x_{n_2} < x_{n_1}$

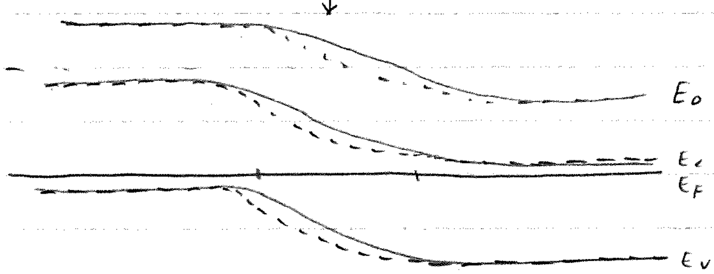
$x_{p_2} > x_{p_1}$



$x_{n_2} = \frac{x_{n_1}}{2}$

∴ can solve for x_{p_2}.

3.

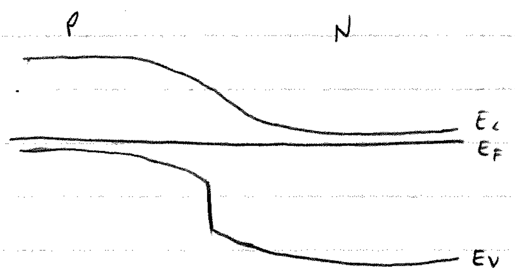


5.

——— original

- - - - with sheet charge.

6.



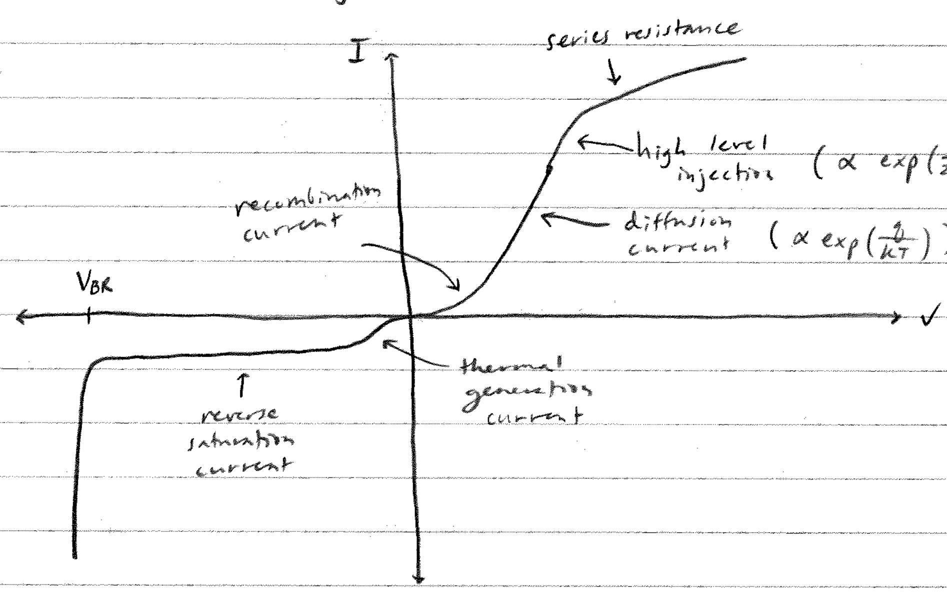
2004 - Harris

1. a) $I = I_s \left(\exp\left(\frac{qV_A}{kT}\right) - 1 \right)$ $I_s = qA \left[\frac{D_n n_p}{L_n} + \frac{q D_p p_n}{L_p} \right]$

- underlying assumptions:
  - no recombination or generation in depletion region.
  - low level injection.
  - $\mathcal{E} = 0$ in quasineutral regions (Depletion approximation)
  - assume drift current of minority carriers at the edge of the depletion region is zero. ?
  - (law of the junction holds: $np = n_i^2 \left(\exp\left(\frac{qV_A}{kT}\right)\right)$)

2.

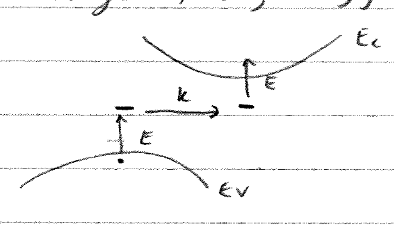
3.

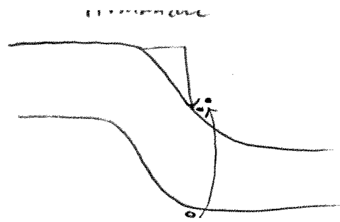


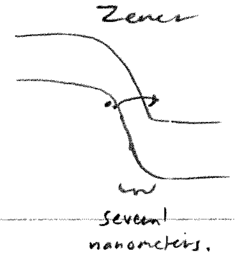
4. Because their concentration is comparable to the minority carrier concentration ⇒ their current dominates for low voltage.

a) No. They are more effective when E_T is close to E_i. (close to center of the band). And more effective for indirect semiconductors. Thermal generation and recombination in indirect semiconductors requires energy and momentum simultaneously. With added trap states, you can get generation/recombination through various changes in only energy or only momentum.

b)







5. Avalanche and Zener (tunneling) breakdown.

a) E_g.

6. "Zener" diodes for voltage references.

1995 - Harris

1. LED is a forward biased PN Junction. injected minority carriers recombine radiatively with majority carriers

2. Different Bandgaps, Doesn't have to be direct bandgap, but indirect is much less efficient. Indirect materials must incorporate an optically efficient trap.

3. The laser is also a p/n junction diode operating in the same basic manner as the LED except that it is inside a Fabry-Perot cavity which provides optical feedback and when the number of injected carriers is high enough (threshold current), a population inversion is reached and this combined with the feedback produces stimulated emission and an optical oscillator or laser. Below threshold, the laser is an LED — it emits only spontaneous, not stimulated photons.

population inversion: density of electrons in conduction band is larger than in the valence band.

Degenerately dope both sides and forward bias the PN Junction. Then a region in the depletion region will have population inversion.

4. The laser output is controlled by optical feedback of the Fabry-Perot cavity, which only occurs over a very narrow angular range. A laser can only be made with a direct band gap material because the trap transition saturates and cannot achieve a population inversion.

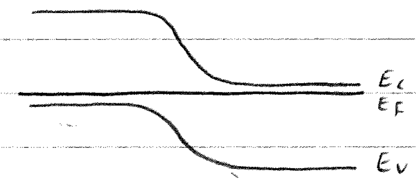
5. Yes. LED - changes color due to change in E_g.
Laser changes only slightly due to thermal expansion of optical cavity.

5. The laser output is polarized along either the horizontal or vertical plane of the laser. Since the cavity controls the output either the cavity shape or reflectivity of the mirrors is different for TM vs. TE polarized light. And one mode is strongly favored.

6.

2001 - Harris

1, a)



e) forward - high level injection
reverse - breakdown.

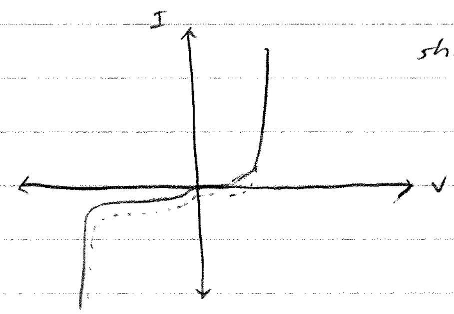
f) KE of electron or hole must be greater than E_G

2. b) V_{th} : $-2mV/°C$ $T\downarrow$ $V_{TH}\uparrow$

V_{BR} : $T\downarrow$ $V_{BR}\uparrow$ zener $T\downarrow V_{BR}\downarrow$ Avalanche

dI/dV : ?

3. a)



shine light ⇒ shifts current down

$I_t = I_{dark} - I_{ph}$

b) producing energy. $I \cdot V < 0$

c) Solar cell.

d) Use narrow base diode.

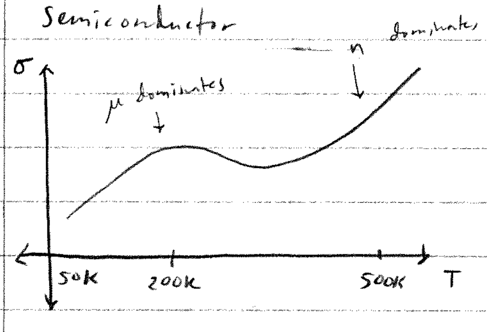
e) mainly drift of minority carriers. Yes there is a contribution from the depletion region, which is the generation current from thermal generation inside the depletion region.

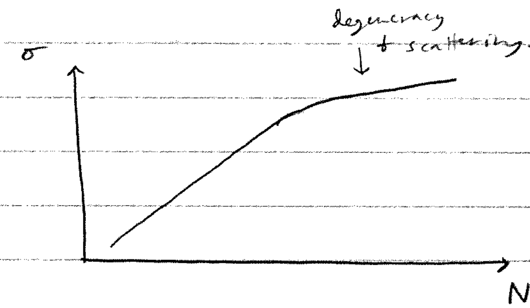
f) heterojunction.

2000 - Harris

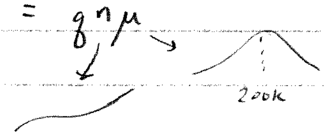
1. Conductivity

2. Semiconductor

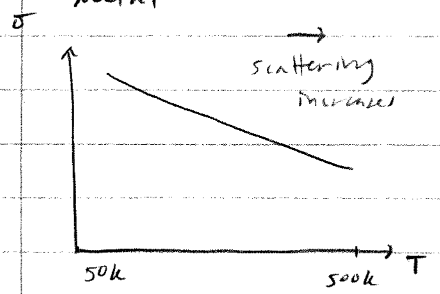


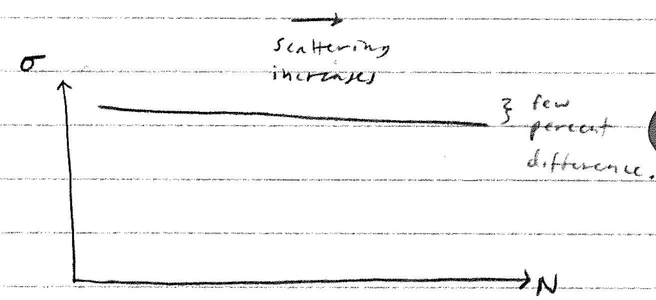


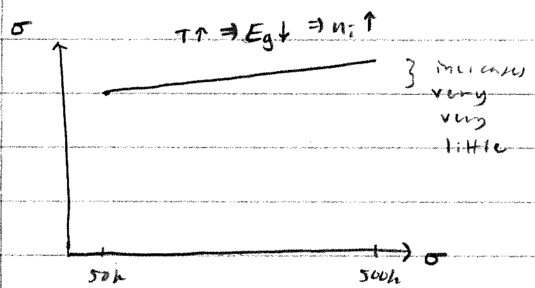
$\sigma = q n \mu$

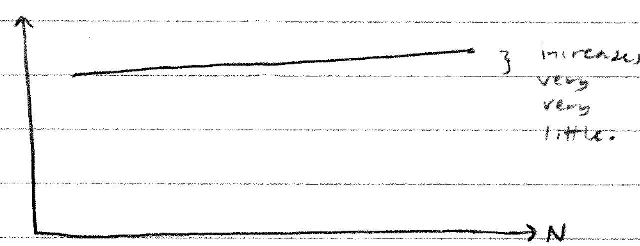


Metal





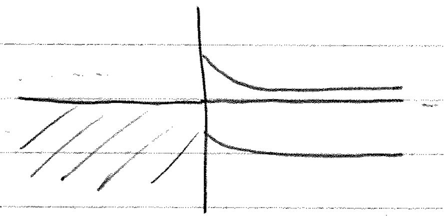


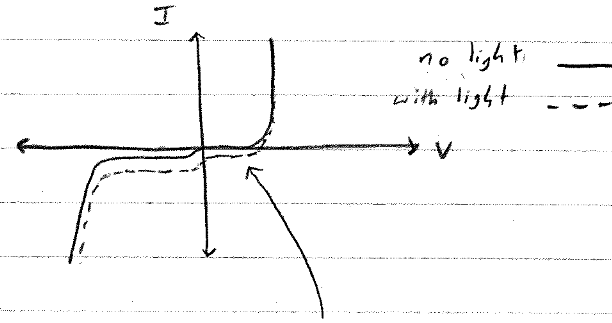


3. ideal properties of semiconductor

2002 - Harris

1, a)



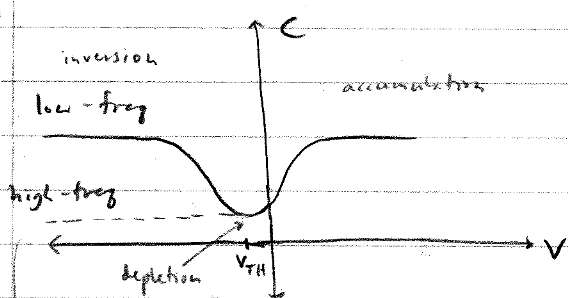


b) generation in depletion region ⇒ IV curve shifts down.

2, a) prevents current from flowing, and since there is a voltage drop across the insulator, you also reduce the band bending in the semiconductor.

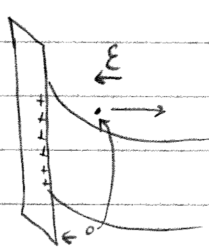
b) C-V measurement, Threshold voltage, doping, interface charge, thickness of oxide, V_{bi}

c)

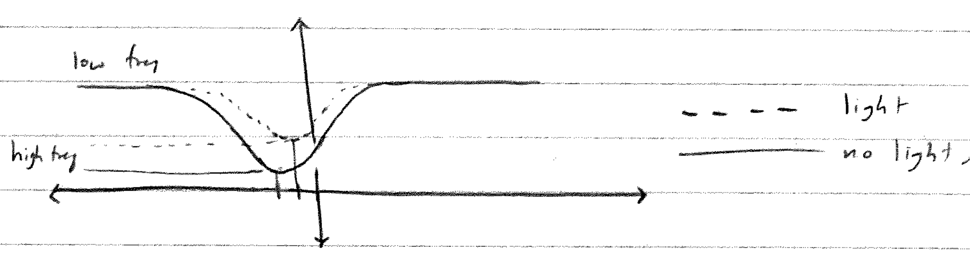


minority carriers can't respond fast enough.

d)



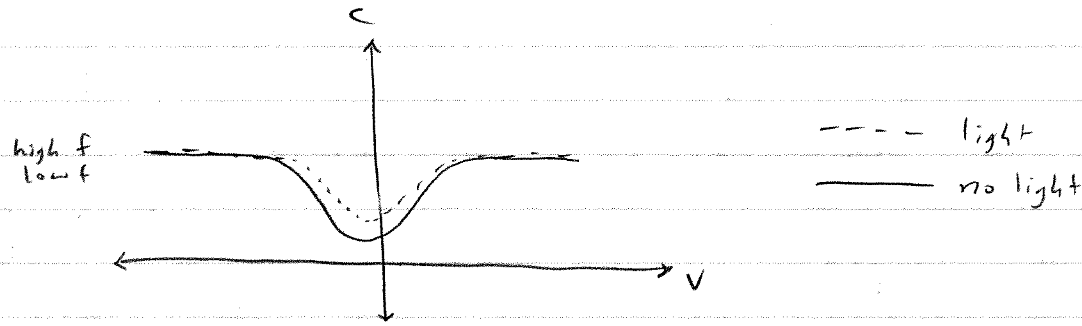
$Q = VC \Rightarrow C = \frac{Q}{V} \Rightarrow C\uparrow$ for depletion



when V is very negative or very positive, there is no depletion region, so the generated carriers just recombine and have no effect.

$|V_{TH}|$ decreases, $C_{depletion}$ increases.

3. Yes.



with light: $|V_{TH}|$ decreases, C_{dep} increases.

No difference between low + high freq, because now have source + drain to provide minority carriers.

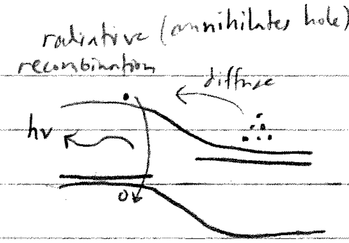
IV - above V_{TH} — roughly the same.

below V_{TH} - subthreshold current increases due to weak channel formed by h^+ by gate from photogeneration.

(can't turn MOSFET off very well).

2008 - Harris

1. Forward biased PN junction.



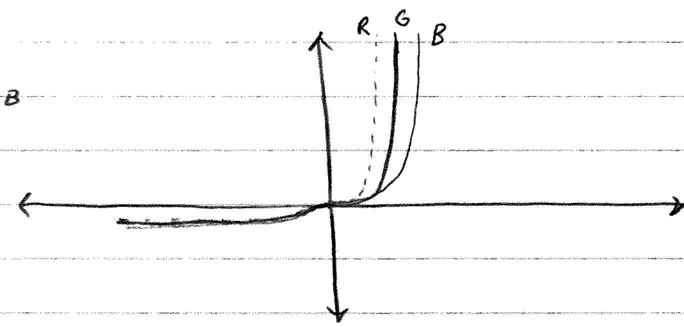
2. Different bandgap energy E_G $\lambda = \frac{1.24}{E}$ $E_g \lesssim E_{photon} \lesssim E_g + kT$

No. Doesn't have to be direct bandgap semiconductor (though these are much more efficient). If it's an indirect bandgap semiconductor, it must incorporate an optically efficient trap.

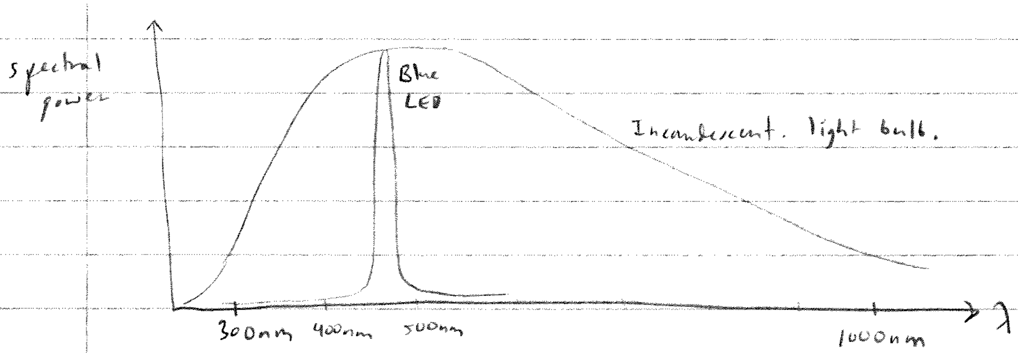
3. Can mix Red Green and Blue LED's to get white light. (Or, you can use a phosphor material to convert monochromatic light from a blue or UV LED to broad spectrum white light (much like how a flourescent light bulb works).)

4. $V_{TH} \approx \frac{2}{3} E_G$ $\lambda \propto \frac{1}{E_G} \Rightarrow f \propto E_G \Rightarrow E_{G_R} < E_{G_G} < E_{G_B}$

$\Rightarrow V_{TH_R} < V_{TH_G} < V_{TH_B}$

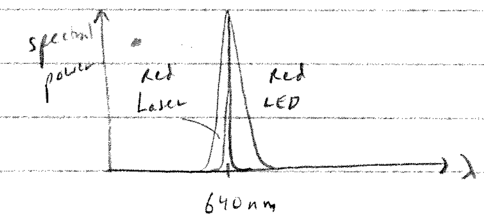


5. $\lambda_R \approx 640nm$ $\lambda_G \approx 550nm$ $\lambda_B \approx 470nm$



6. Kinetic energy of carriers (KT)

7. Temperature decreases $\Rightarrow E_G \uparrow \Rightarrow V_{TH} \uparrow$ and $\lambda \downarrow$ $(f \uparrow)$
also, the total integrated spectral power may decrease since there will be fewer energetic carriers available as $t \to \infty$.

9. 

Laser spectrum is much narrower.

8. The laser is also a p/n junction diode operating in the same basic manner as the LED except that it is inside a Fabry-Perot cavity which provides optical feedback and when the number of injected carriers is high enough (threshold current), a population inversion is reached and this, combined with the feedback produces stimulated emission and an optical oscillator or laser. Below threshold, the laser is an LED — it emits only spontaneous, not stimulated photons.
The laser output is strongly directional because the output is largely controlled by the optical feedback from the Fabry Perot cavity, which only occurs over a very narrow angular range. Photons not emitted directly down the optical axis experience no feedback and gain.

10. Yes. The LED output will "change color" $(\lambda \downarrow)$ since $E_G \uparrow$ as $T \downarrow$. The laser output is controlled by the optical feedback, hence the fabry-perot cavity resonant wavelength. Since the linear thermal expansion coefficient is very small compared to the bandgap shift, the laser output remains at the same wavelength until there is insufficient gain at this wavelength and then the laser either stops oscillating or it "jumps" to a different wavelength where there is another Fabry-Perot resonance and sufficient gain to oscillate at this new wavelength.

2001 - Pease

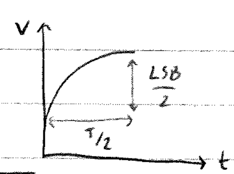
1. - Scaling ⇒ increased precision and speed.
   - Digital circuits are less sensitive to noise.
   - Analog design is more specialized ⇒ doing a digital design can be more readily acheivable.
   - Digital designs can sometimes be flexible (e.g. FPGA) ⇒ can change the design to implement a fix or improve performance through software rather than hardware.
   - for storage digital can be exactly reproduced, and doesn't degrade over time.
2. - Sampling rate
   - Power dissipation
   - Resolution (number of bits)
   - Accuracy (LSB)
   - Quantization Error
   - Linearity (DNL, INL)
   - Dynamic Range
3. Compares continuous input to a number of discrete levels (with a comparator). Based on these comparisons, a digital representation of the input is generated as the output.

   Ideally, if the two inputs to a comparator are the same, the output will equal $\frac{V_{cc}}{2}$ which is an undetermined state (neither high nor low). (But in most practical comparators, finite mismatch will cause the output to either go high or low.)

4. Yes, the Heisenberg uncertainty principle does set a limit on the combination of high speed and high resolution.

specifically, $\Delta p \Delta x \geq \hbar$ can alternatively be stated as $\Delta E \Delta t \geq \hbar$

in the context of an ADC, $\Delta E = \frac{(\frac{LSB}{2})^2}{R} \cdot \frac{T}{2}$ $\quad \Delta t = \frac{T}{2}$ $\Rightarrow \Delta E \Delta T = \frac{1}{R}(\frac{LSB \cdot T}{4})^2 \geq \hbar$



$f_{samp} = \frac{1}{T}$

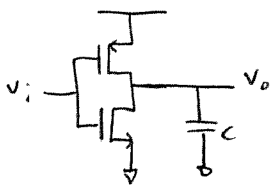
$LSB = V_{FS}\, 2^{-N}$

$$\Rightarrow \boxed{2^N \cdot f_{samp} \leq \frac{V_{FS}}{4\sqrt{\hbar R}}}$$

Thermal noise limits resolution, Jitter noise limits conversion speed.

[illegible] ⇒ must tradeoff speed to improve resolution

2002 - Pease

1 

2 $\frac{1}{2} C V_{dd}^2 + \frac{1}{2} C(-V_{dd})^2 = C V_{dd}^2$ is dissipated in the mosfets during the clock transitions.

since $V_{gs} \downarrow$ for smaller devices. since $V_{TH} \downarrow$ for smaller devices.

3 per cycle: $C \downarrow$ $V_{dd} \downarrow$ $\Rightarrow$ $P \downarrow$

but f will increase $\Rightarrow$ power may actually increase since there will be more cycles.
also leakage current will increase as we scale down. So off current will increase and more static power will be dissipated.

4 2nd law of thermodynamics:

in order to do work, you must dissipate power (increase entropy).
But to make a battery, we must reduce entropy. So we must find materials that have low entropy (high initial potential energy).
And find ways to efficiently extract energy.

Doesn't benefit from scaling like IC's because you fundamentally need a certain amount of volume to store a certain amount of energy.

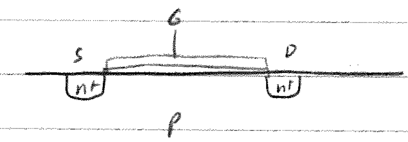
Could also try to use supercapacitors. But these tend to degrade after ~100 cycles.

$\downarrow$

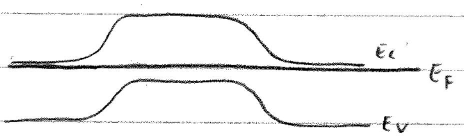
Some people are trying to use graphene for this.

2007 - Harris

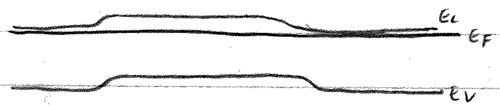
1. A.



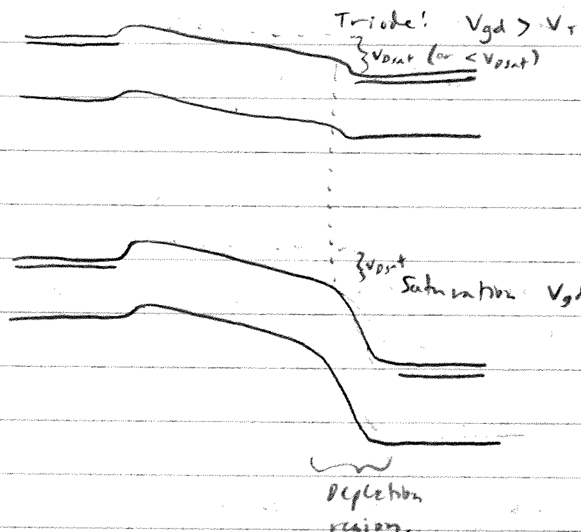
B.



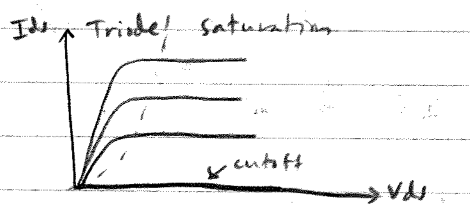
C.



D.



E.



cutoff: see B

Triode: see D. current is due to drift of carriers in channel. thus $I_d = A \cdot \mu \vec{E} \propto V_{ds}$ since channel does not pinch off.

Saturation: see D. current saturates since after pinch off, there is simply a reverse biased p-n junction. The drop across the channel from S to pinch off point is V_{dsat}. This voltage remains constant and determines the current I_{ds}.

2000 - Pease

1. Make more money (smaller gate pitch ⇒ more chips per wafer)
Increase speed, reduce supply voltage, reduce power consumption, reduce size, Moore's Law.

$f = \frac{I}{CV}$

$C \sim \frac{1}{k^2}$

$I \sim \frac{1}{k}$

$\Rightarrow f \sim k$

2. Speed $\propto \frac{1}{L}$ since $I_{ds} \propto \frac{1}{L}$ $\left(\tau = \frac{CV}{I} \Rightarrow f_{max} = \frac{I}{CV}\right)$

energy per clock cycle = $\underbrace{C}_{\text{interconnect}} V_{dd}^2$ ⇒ (only changes if V_{dd} reduces since $V_{TH} \downarrow$ with scaling)

3. R only affects the charging time and not the total energy stored (dissipated). Since $\underbrace{E}_{\text{per cycle.}} = CV^2$

4. No. This is equivalent to increasing the R, $E = CV^2$ regardless.

5. 1) atomic size (for thickness of oxide, length of channel, etc.)
2) Gate leakage (due to quantum tunneling)
3) Subthreshold current.

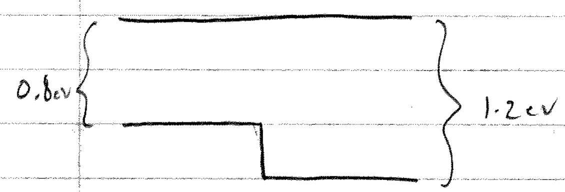
Thermodynamic limit: $\Delta Q = E_{min} = kT \ln(2)$

Heisenberg uncertainty: $\Delta p \cdot \Delta x > h$

$\Delta E \cdot \Delta t > h$ ⇒ $f_{max} \approx 25\ THz$

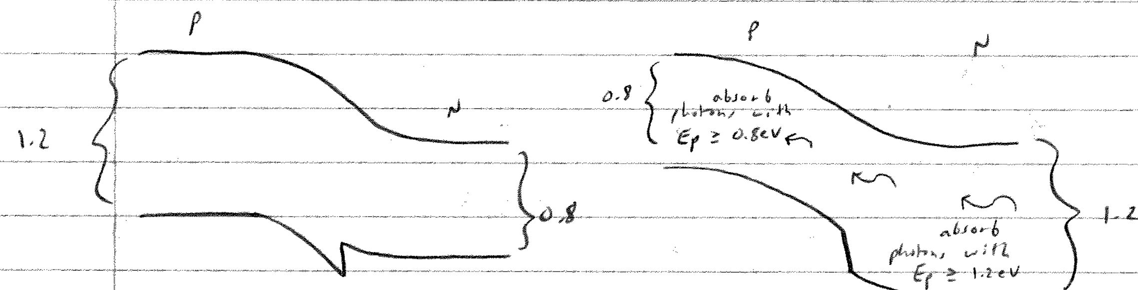
$\Delta x_{min} \sim 1.5nm$

Pre 1993 - Harris

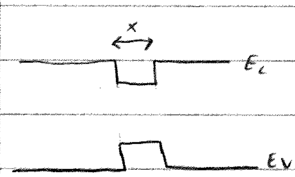


Heterojunction: junction of two materials with different bandgaps (but typically with matching lattice constants)

Could be used for HBT: increases emitter efficiency.
Could be used for LED or solar cell: wide band gap material acts as a transparent window to allow photons in/out without causing generation.



assume $\chi_1 = \chi_2$ in these sketches.



what happens as x gets small? it will behave as a quantum well ⇒ energy states will become discrete rather than continuous.

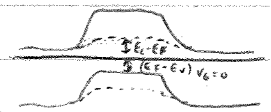
can use these structures to make a laser. trap charges in quantum well to get population inversion.

Boundary conditions for Poisson's Eq:

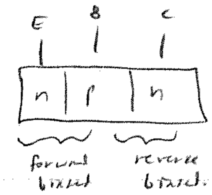
$$\nabla \cdot \mathcal{E} = \frac{\rho}{K_s \varepsilon_0} \qquad \boxed{\frac{\partial \mathcal{E}}{\partial x} = \frac{\rho}{K_s \varepsilon_0}} \quad \text{for 1-D.}$$

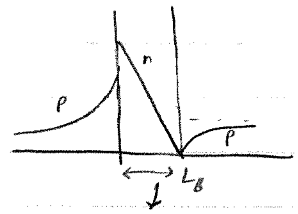
- for MOSFET what's the limit to band bending in depletion?

$(E_c - E_F) > (E_F - E_v)_{V_G=0}$



- How does NPN BJT work?



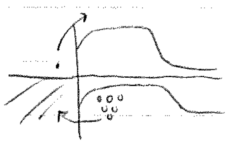


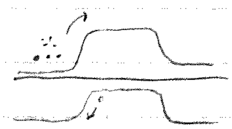
assume L_B < diffusion length.

Emitter efficiency: $\gamma = \dfrac{I_{Ep}}{I_{Ep} + I_{En}}$

determined by doping of base relative to doping of emitter.

- Why not metal-p-n





$$\gamma = \frac{I_{En}}{I_{En} + I_{Ep}}$$ $\Rightarrow$ terrible emitter efficiency for metal-p-n

1996 - Pease

1\. MOSFET : Digital + Analog CMOS.

BJT : High performance analog and MMIC/RF. Not often used as switch due to base current.

JFET : Voltage controlled resistor.

2\. Zero off current. Zero gate leakage. High gain.
High switch speed (fast turn on + off transient).

small gate pitch ⇒ more profit!

3\. Gate leakage : tunneling due to thickness of gate. but there will always be some chance of tunneling (quantum). set by physics.

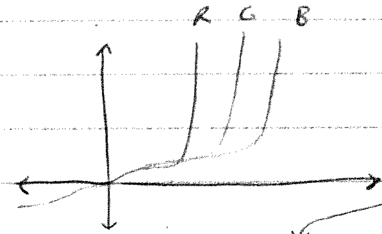
Switching speed : determined by technology (gate length, mobility, parasitic capacitance, inter-connect parasitics, isolation)

4\. Yes. Much better on/off ratio. OFF current of a relay is very close to zero. Roger Howe makes these.
Can be lower power than MOSFET for switching.
Can tolerate larger temperature variation than MOSFET.

problem is mechanical reliability / endurance.
has finite number of switching cycles.

2010 - Harris

1.



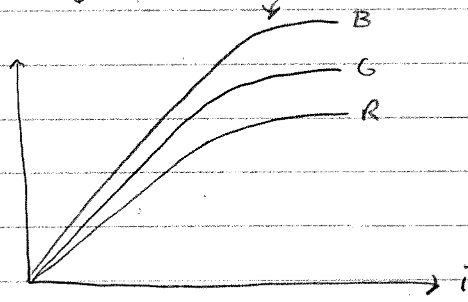
L saturates at high i,
since as $i\uparrow \Rightarrow T\uparrow \Rightarrow \tau_r \downarrow$
$\Rightarrow$ nonradiative recombination (through traps) increases since $\tau_r \downarrow$

$\tau_r = \dfrac{1}{v_{th}\,\sigma \cdot N_T}$ $\Rightarrow$ proportion of radiative recombination decreases.

$\frac{3}{2}kT$

2.

Light Intensity W/cm²



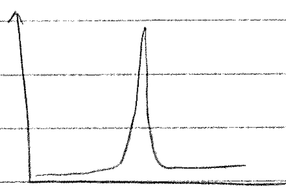
$\lambda = \dfrac{1.24}{E}$

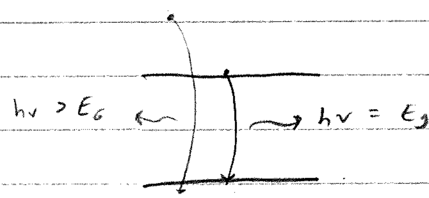
$\Rightarrow E = \dfrac{1.24}{\lambda}$

$\Rightarrow$ Blue light has higher energy photons than green & red light.

3.

spectrum





why is $\lambda_{max} = \dfrac{1.24}{E_G}$

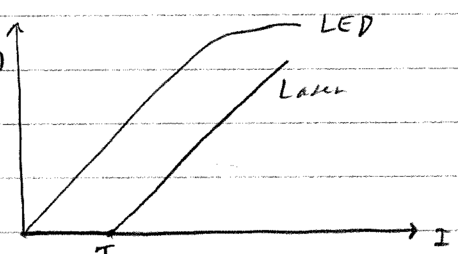
since you can't have carriers in Bandgap $\Rightarrow$ minimum energy difference from recombination is E_G.

4.

5. For laser: must heavily dope to achieve population inversion, Must use direct bandgap material.
Need a Fabry-Perot resonance cavity.

6.

L (intensity)



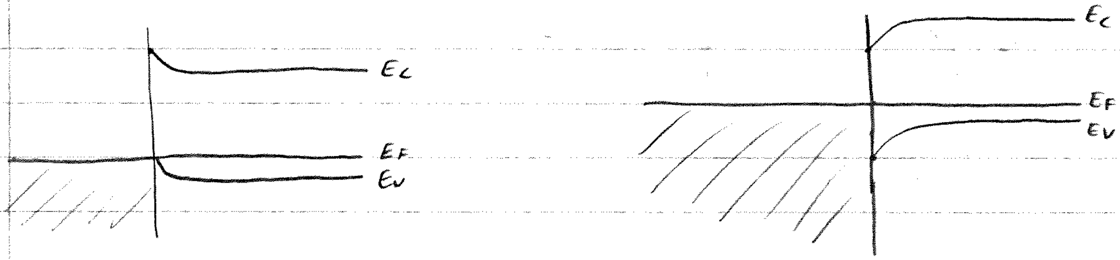
I_{TH} $\rightarrow$ needed to achieve population inversion.

7.

8. Below I_{th}, the laser acts as a simple LED (only spontaneous emission, not stimulated emission). Thus, you can use it as a solar cell.
For solar cell, want to maximize diffusion length to collect carriers.
(for LED + laser you want recombination. For a solar cell, you don't).

2003 – Vuckovic

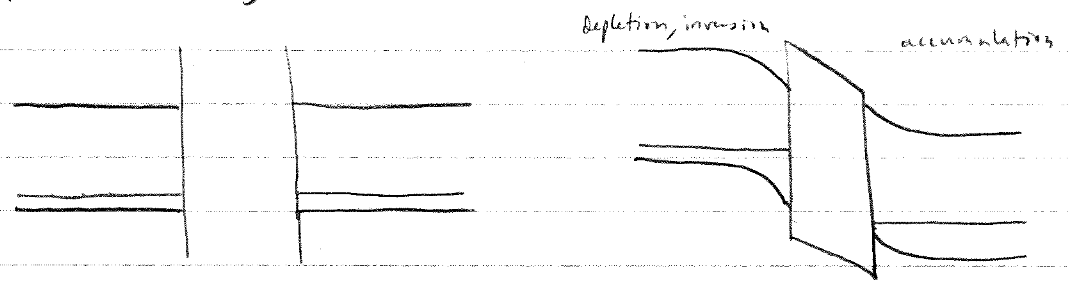
1. - $\Phi_m > \Phi_s \Rightarrow$ ohmic (Contact)
   - $\Phi_M < \Phi_s \Rightarrow$ rectifying (diode)



$\Phi_M > \Phi_s$ Ohmic $\qquad\qquad$ $\Phi_M < \Phi_s$ Rectifying.

2. Inversion : Diagram 2 , Bias point d
   Accumulation : Diagram 4 , Bias point a
   Depletion : Diagram 3

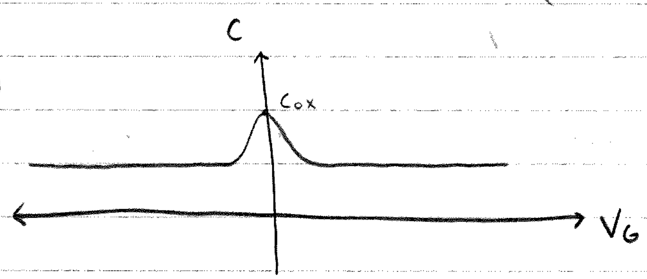
3.



$V_G = 0$ $\qquad\qquad$ $V_G < 0\,V$

(mirror image for $V_G > 0V$) due to symmetry.

$\Rightarrow$

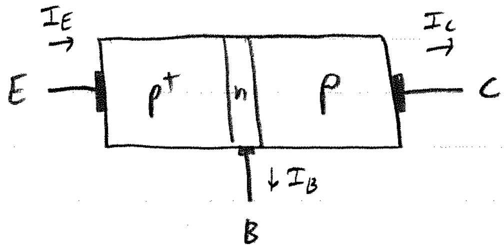


4. Intuitively, this should behave like a PN diode.

$V_C = V_E \Rightarrow V_{BE} = V_{CE} \Rightarrow$ BJT can only operate in

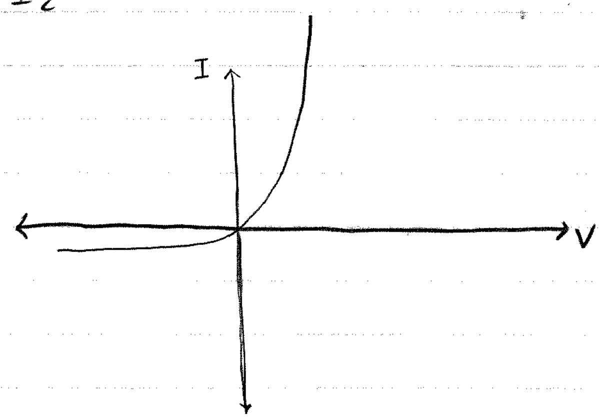
			E-B	C-B
	Saturation	$(V > 0) \Rightarrow$	Forward Biased	Forward Biased
OR	Cutoff	$(V < 0) \Rightarrow$	Reverse Biased	Reverse Biased.

$$\beta = \frac{I_c}{I_B} \qquad \alpha = \frac{I_c}{I_E}$$



$$I_E = I_B + I_c$$

$$I = I_B = I_E - I_c$$



Ebers Molls: $I_E = I_{Fo}(e^{V_{EB}/V_T} - 1) - \alpha_R I_{Ro}(e^{V_{CB}/V_T} - 1)$

$I_c = \alpha_F I_{Fo}(e^{V_{EB}/V_T} - 1) - I_{Ro}(e^{V_{CB}/V_T} - 1)$

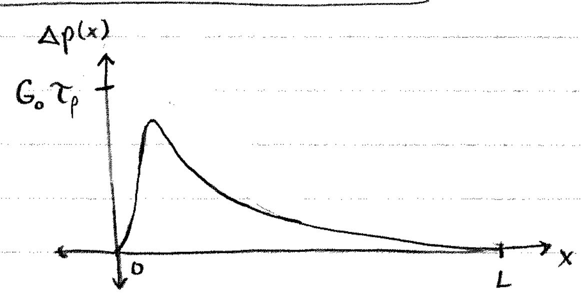
$\Rightarrow$ in this problem, $V_{EB} = V_{CB} = V$

$$\Rightarrow \quad I = I_E - I_c = \left[I_{Fo}(1-\alpha_F) + I_{Ro}(1-\alpha_R)\right](e^{V/V_T} - 1)$$

5. $$\frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L(x) \Rightarrow \boxed{0 = D_p \frac{\partial^2 \Delta p_n(x)}{\partial x^2} - \frac{\Delta p_n(x)}{\tau_p} + G_0 e^{-\alpha x}}$$

$$\boxed{\text{Boundary Conditions: } \Delta p_n(0) = \Delta p_n(L) = 0}$$

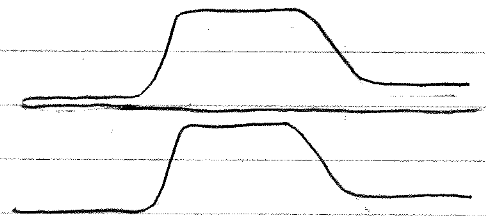
Rough Sketch:



2006 - Peumans

1\.

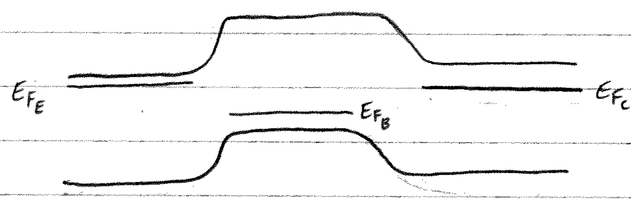
n^+ p n



equilibrium.

Linear region $\Rightarrow$ Saturation $\Rightarrow$ BE and BC both forward biased.

$\Downarrow$



Current gain: $\beta = \frac{I_c}{I_B} = \frac{\alpha_{dc}}{1-\alpha_{dc}}$

$\alpha_{dc} = \gamma \alpha_T$ $\gamma = \frac{I_{E_n}}{I_E}$ emitter efficiency

$\alpha_T = \frac{I_{C_n}}{I_{E_n}}$ base transport factor.

Can improve current gain β by:

decrease base doping $\Rightarrow$ $N_E >> N_B$ $\Rightarrow$ $\gamma = \frac{1}{1 + \frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E}}$ $\uparrow$

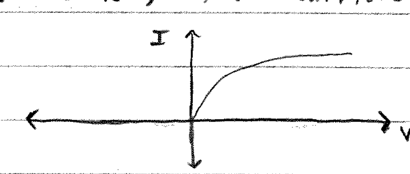
decrease base width $\Rightarrow$ $W << L_B$ $\Rightarrow$ $\alpha_T = \frac{1}{1+\frac{1}{2}\left(\frac{W}{L_B}\right)^2}$ $\uparrow$

$\Rightarrow$ $\beta_{dc} = \frac{1}{\frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$ $\uparrow$

2\. Strange because it seems to violate conservation of energy. But it actually doesn't. The extra energy comes from the KE (due to kT) of the carriers.

? electron gun

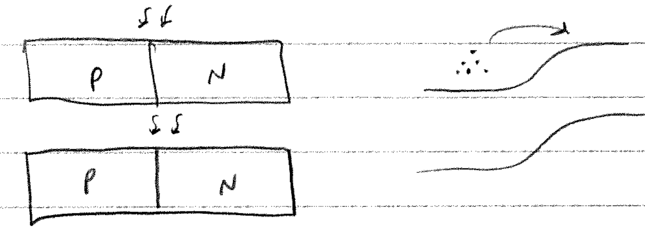
3\. $I = \sqrt{\frac{2Vq}{m_e^*}} \times n \times q = \sqrt{\frac{2Vq}{m_{e_0}}} \times n \times q$



?

2007 - Peumans

1. Yes, due to KE of carriers.

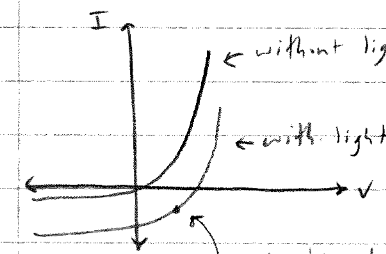
2. 

sunlight → heat → $T\uparrow \Rightarrow kT\uparrow \Rightarrow KE\uparrow$

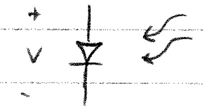
No

2008 - Penmans

1.

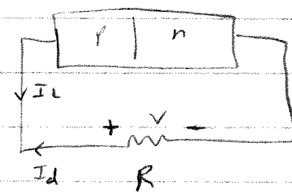


maximize IV.



What are the physical constraints between light in and e^- out.
(physical effects that must be considered)

- absorption coefficient α (and spectrum)
- diffusion length
- length of p and n region.
- width of depletion region.

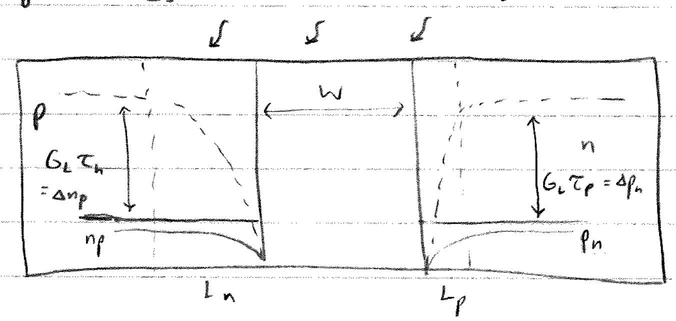


$I_{total} = I_L - I_d$

$V_{oc} = \frac{kT}{q} \ln\left(\frac{I_L}{I_s}\right)$ $I_{sc} = I_s e^{V_{oc}/V_T}$

— without light
--- with light.

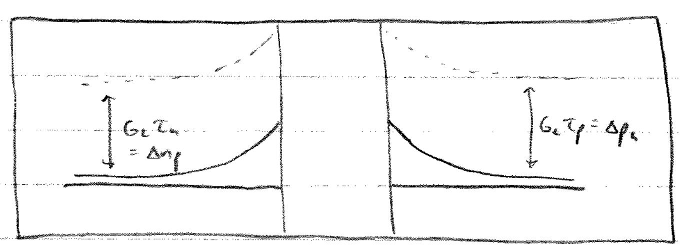
reverse biased:



$L_n > L_p$ for Si since $\mu_n > \mu_p$

$L_n = \sqrt{D_n \tau_n}$
$= \sqrt{\left(\mu_n \frac{kT}{q}\right) \tau_n}$

"optimum bias": slight forward bias



What determines V_{max}? $V_{oc} = \frac{kT}{q} \ln\left(\frac{I_L}{I_s}\right) \Rightarrow T, I_L$ and I_s.

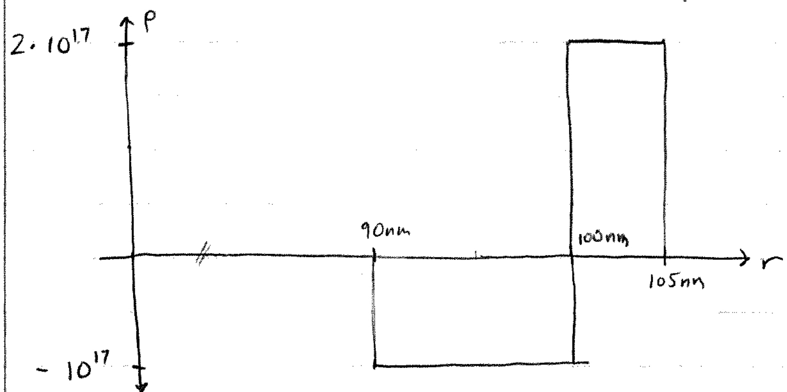
How can you get more voltage? Increase T, light intensity, or decrease I_s.

2006 - Howe

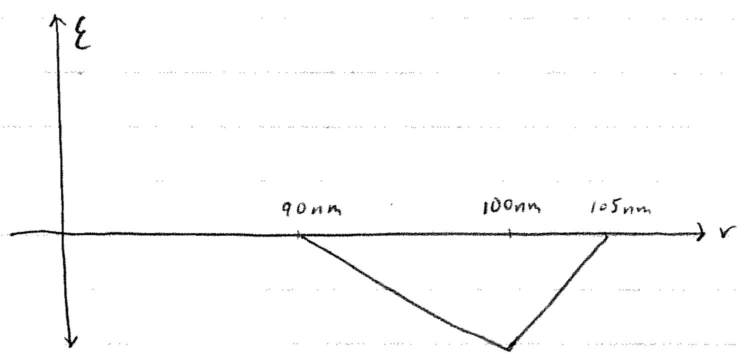
a) Assume: $x_p \ll R$ ⇒ can approximate with a 1-D model.

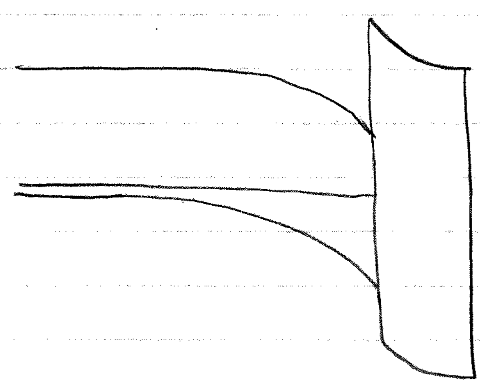
If you assume the holes are somehow able to leave the wire. ⇒ assume $\mathcal{E}$ field outside wire = 0.

$$\rho_{ox} \approx \frac{10^{11}\,cm^{-2}}{t_{ox}} \quad \text{since} \quad t_{ox} \ll R \Rightarrow \rho_{ox} = \frac{10^{11}\,cm^{-2}}{5\cdot 10^{-7}\,cm} = 2\cdot 10^{17}\,cm^{-3}$$



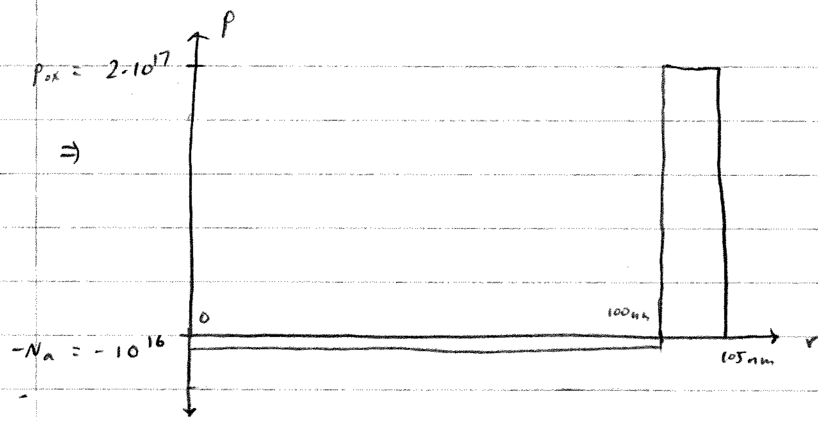
$x_p = 10nm \ll R = 100nm$ ✓

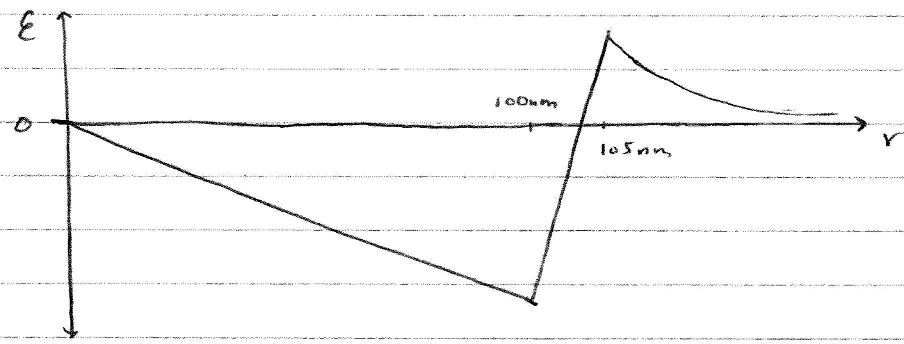


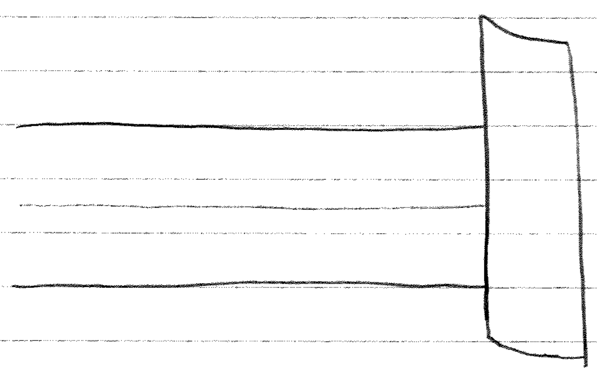


If $N_a = 10^{16}\,cm^{-3}$, depletion region will extend all the way to the center of the cylinder, since $R \cdot N_a = 10 \cdot t_{ox} \cdot N_a < t_{ox}\rho_{ox} = 20\, t_{ox} N_A$.

⇒



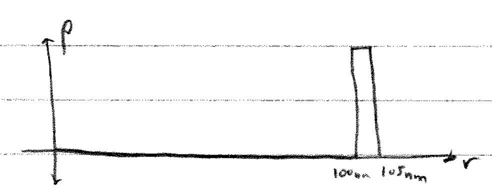


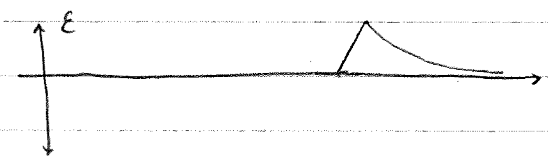


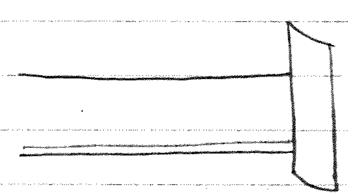
?

OR If you assume that holes cannot be removed, ℰ outside the wire is positive + ℰ field at R is 0 due to Gauss's Law.

⇒







2006 Vuckovic

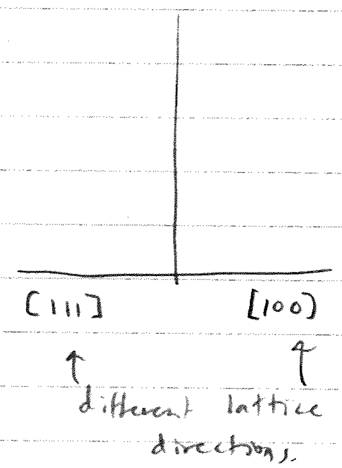
2. Calculated with quantum mechanics. Assume wave function of electrons in a crystal lattice have a Bloch form

$$\psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\vec{r}} u_k(\vec{r})$$ ← periodic since crystal lattice is periodic.

Apply periodic boundary conditions.

Along the curves, we have the allowable energy states for various momentum values. Probability of electrons is zero in band gaps and higher in valleys than in peaks. Probability distributions of electrons in space in such materials is periodic.

$\vec{k}$ - wave vector $(\vec{k} = \hbar \cdot p)$



These graphs basically show the allowed states for electrons and holes at different momentums for different lattice directions.

a) 1.42 eV

b) $\boxed{L}$ $m^* = \dfrac{1}{\hbar^2 \cdot \frac{d^2E}{dk^2}}$ $\Rightarrow$ sharper curve $\Rightarrow$ lower m^*.

c) $\cancel{10^{18}}$ cm^{-3} dopants will be fully ionized.

(but actually $N_c = 4 \cdot 10^{17}$ so this doping is actually degenerate. So the concentration of electrons in the conduction band should be less than $10^{18} cm^{-3}$)

d) $n = N_c \exp\left(\frac{E_F - E_C}{kT}\right)$

assume $E_F = E_D$ $\Rightarrow$ $n_L = N_c \cdot \exp\underbrace{\left(\frac{1.4 - 1.71}{kT}\right)}_{\approx 10^{-5}} \approx 10^{13}$

if we assume $n_\Gamma \approx 10^{18}$, $\Rightarrow$ $\frac{n_L}{n_\Gamma} \approx 10^{-5} \Rightarrow 0.001\%$

dopant momentum is uniform in $\vec{k}$ space, since the location is fixed (Heisenberg uncertainty principle).

e) GaAs has a direct bandgap
Si has an indirect bandgap.

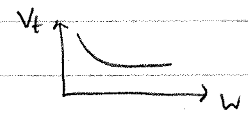
2004 Vuckovic

1. a) $N_A \uparrow$, $V_T \downarrow$

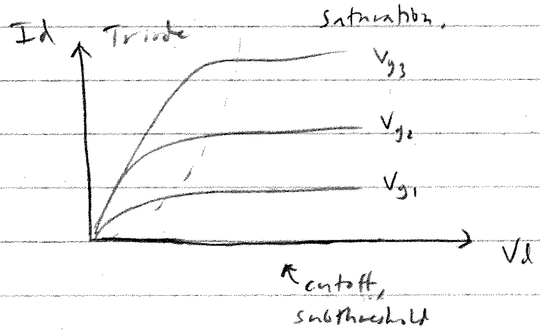
$t_{ox} \uparrow$, $V_T \uparrow$

$L \downarrow$, $V_T \downarrow$

$W \downarrow$, $V_T \uparrow$



b)



c) Subthreshold : diffusion. Some energetic electrons surmount barrier + diffuse into "channel" region where they are swept across to the drain.
above subthreshold : drift.

Subthreshold : $I_D = I_0 e^{q\left(\frac{V_g - V_t}{kT}\right)} \left(1 - e^{-qV_D/kT}\right)$

	Sub	above sub
$V_g \uparrow$	$\uparrow$	$\uparrow$
$V_d \uparrow$	$\uparrow$	$\uparrow$

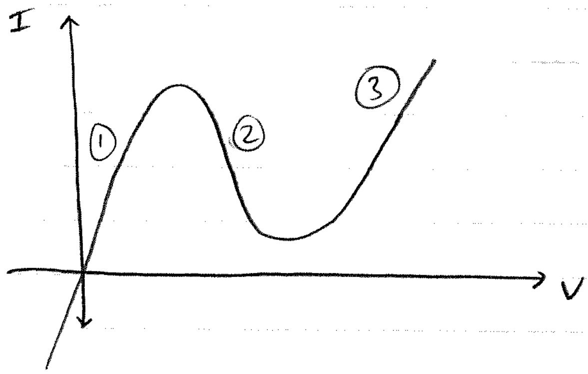
d) Yes. $V_g \uparrow \Rightarrow I_d \uparrow$ because you have more inversion layer charge in the channel. + the more charge you have in the channel, the greater your drift current.

e) Yes. ← ? look into this.

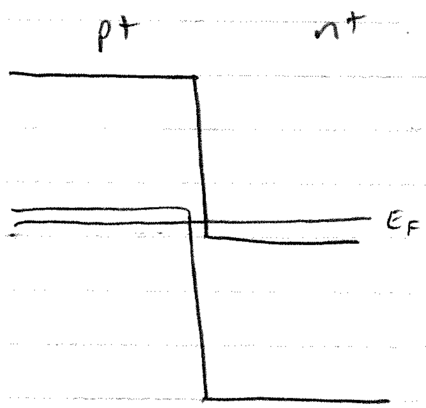
f) $I_D \downarrow$ because barrier to diffusion is increased. $(V_{GS} \uparrow)$

g) $I_D \downarrow$ because barrier to diffusion is increased $(V_{BS} \downarrow)$

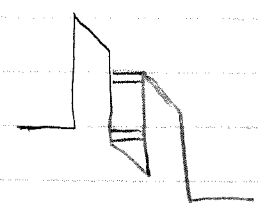
2.



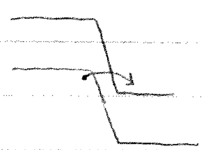
Tunnel Diode



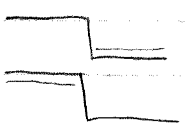
degenerate p-N



1 Tunneling

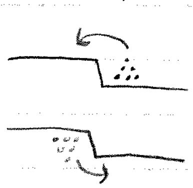


2



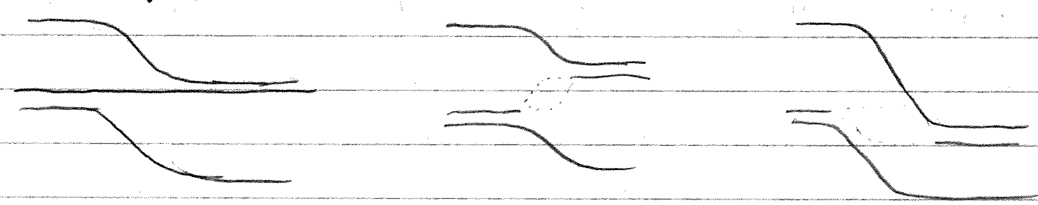
Can't tunnel anymore.

3 Diffusion Current. (Forward biased PN)



2003 - Peumans

(1) P $V=0$ N $V>0$



(2) $I = I_0 \left(e^{qV/kT} - 1\right)$

$I_0 = qAn_i^2 \left(\frac{D_p}{L_p}\frac{1}{N_D} + \frac{D_n}{L_n}\frac{1}{N_A}\right)$

(3) $I_{total} = I_{diff} - I_{ph}$ when you shine a light on it.

(4) $\eta = \frac{P_{output}}{P_{input}} = \frac{I_{max} V_{max}}{(\text{light Intensity}) \cdot A}$ ∴ solar cell efficiency.

(5). ~10% (actually typically 20-40%)

1993 - Pianetta

(1) n-type GaAs

(1) N_D, Shallow

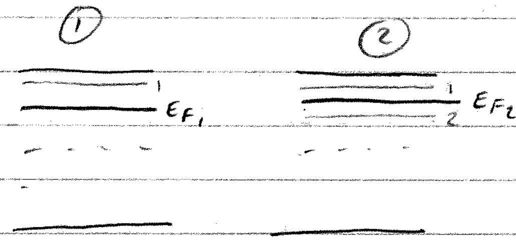
(2) N_D, 0.25 eV below E_c and Shallow.

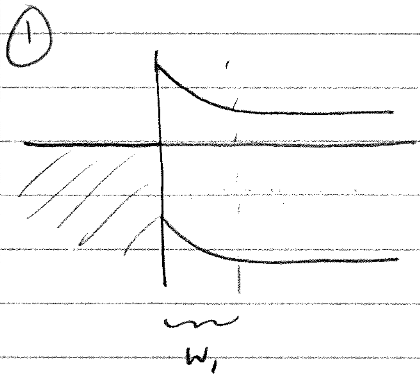
Just look at conductivity at 300K.

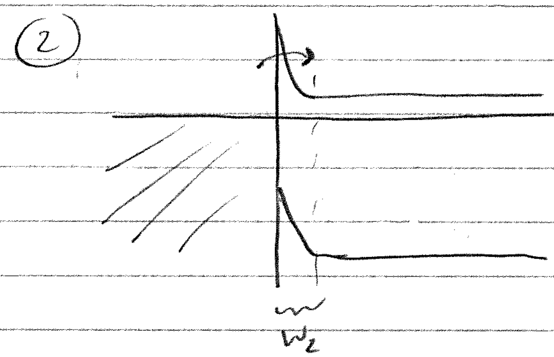
$\Rightarrow$ for (2), doping is 10 kT below E_c $\Rightarrow$ very little ionization but shallow dopants ionize $\Rightarrow$ will have higher conductivity.

(1) will have higher conductivity since its dopants will ionize.
Can't tell the difference between (1) + (2) in terms of just conductivity.

(2) $E_{F_2} > E_{F_1}$







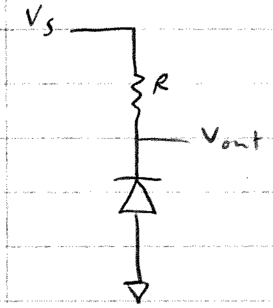
$W_2 < W_1$ because there are more dopants in (2) $\Rightarrow$ shorter depletion region.

More tunneling $\Rightarrow$ $I_2 > I_1$

$\Rightarrow$ You can distinguish (1) from (2) by doing a contact resistance measurement. ($I_2 > I_1$ for a given $V_{applied}$)

- For low doping CV measurement would measure depletion width
- For high dopings tunneling occurs $\Rightarrow$ measure current.

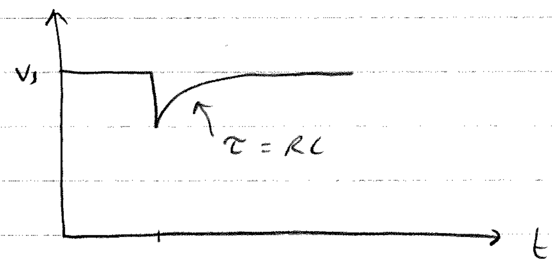
2002 - Miller



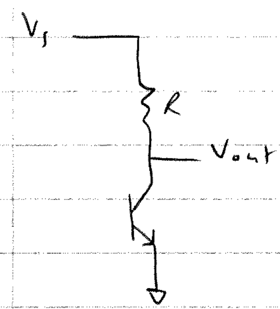
dark: $V_{out} = V_s - I_0 R$

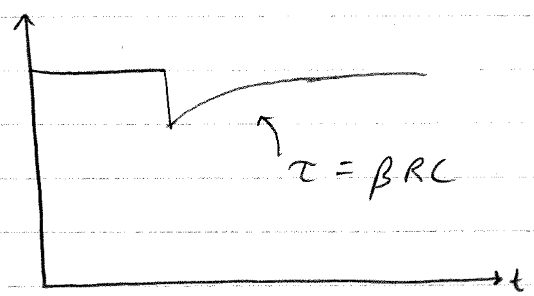
$I_0 \approx 0 \Rightarrow V_{out} = V_s$

Light pulse: $V_{out} = V_s - I_L R$



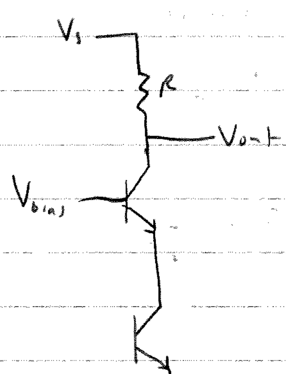
Diode





Miller-effect.

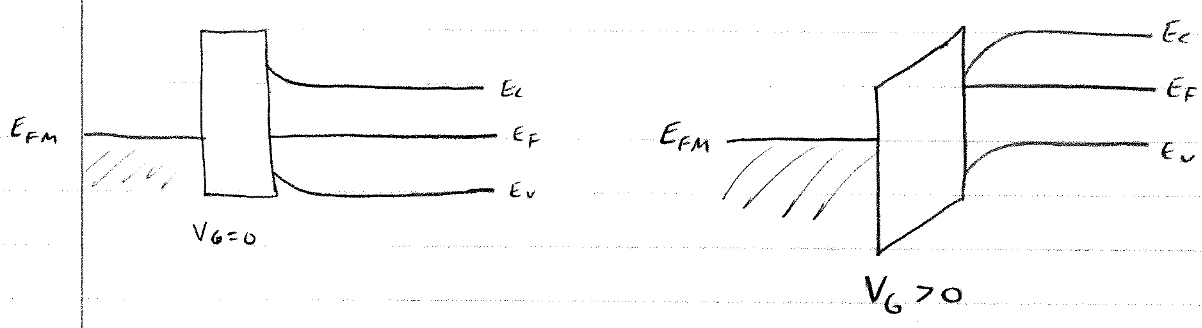
possible solution: use cascode.



1995 - Pianetta

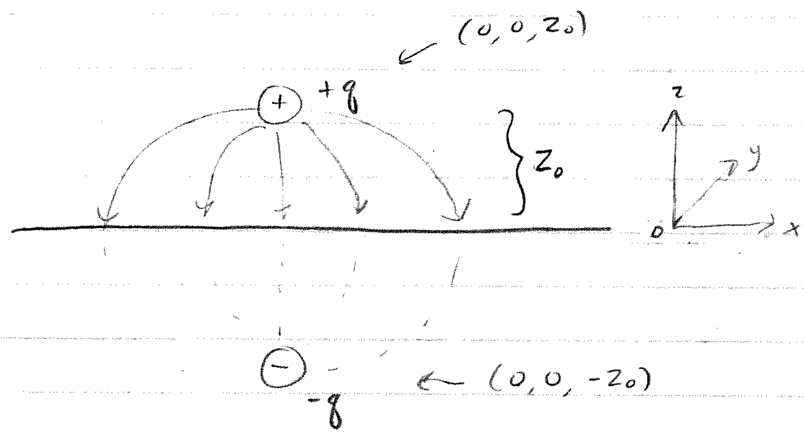
1. a) $1\text{Å} = 10^{-10}m = 0.1nm \Rightarrow 100\text{Å} = 10nm$

Doesn't mention doping ⇒ assume intrinsic:
Similar to MOS. Assume $\Phi_M > \Phi_{SM}$

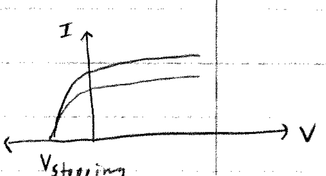


b) Assuming a conducting inversion layer is formed

b)



IV of photoelectric effect



$qV_{stopping} = h\nu - \Phi_{Si}$

potential in cartesian coordinates: (let $V=0$ for $z=0$)

$$V(x,y,z) = \frac{(q/4\pi\epsilon_0)}{\sqrt{x^2+y^2+(z-z_0)^2}} - \frac{(q/4\pi\epsilon_0)}{\sqrt{x^2+y^2+(z+z_0)^2}}$$

$$E_z(x,y,0) = -\frac{\partial}{\partial z}\left(V(x,y,z)\right)\Big|_{z=0} = -\frac{2(q/4\pi\epsilon_0)\,z_0}{(x^2+y^2+z_0^2)^{3/2}}$$

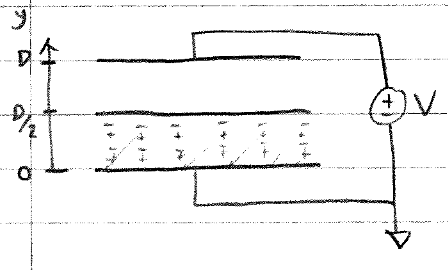
$$\sigma_{induced}(x,y) = -\frac{(q\,z_0/2\pi)}{(x^2+y^2+z_0^2)^{3/2}}$$

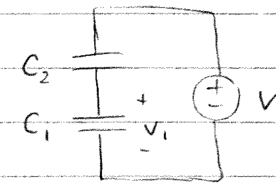
$$\Rightarrow \sigma_{induced}(r) = \frac{-q z_0 r}{(r^2+z_0^2)^{3/2}} \Rightarrow \text{falls off as } \frac{1}{r^2}$$

electrons collect mostly right below probe tip with circular symmetry.

1996 - Pianetta

1\.





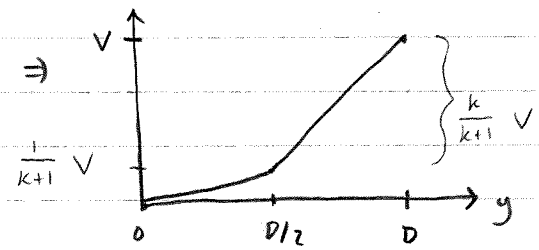
$$E = -\nabla V$$

$$\Rightarrow \quad V(y) = -\int E(y)\,dy$$

$$E(y) = \begin{cases} E_1, & 0 < y < D/2 \\ E_2, & D/2 < y < D \end{cases}$$

$E_1 < E_2$ due to dielectric (dielectric constant $k > 1$)

$\Rightarrow$



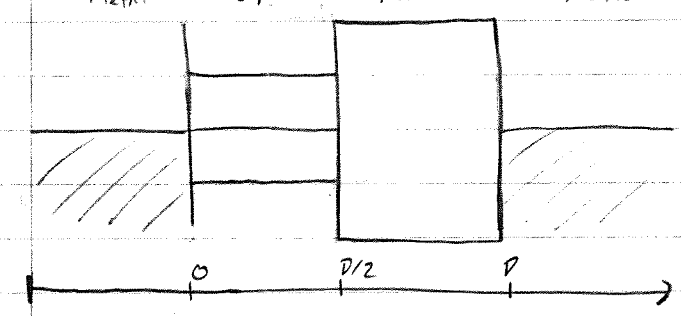
$$C_1 = \frac{k\epsilon_0 A}{(D/2)} \qquad C_2 = \frac{\epsilon_0 A}{(D/2)}$$

$$V_1 = \frac{C_2}{C_1 + C_2} V = \frac{1}{k+1} V$$

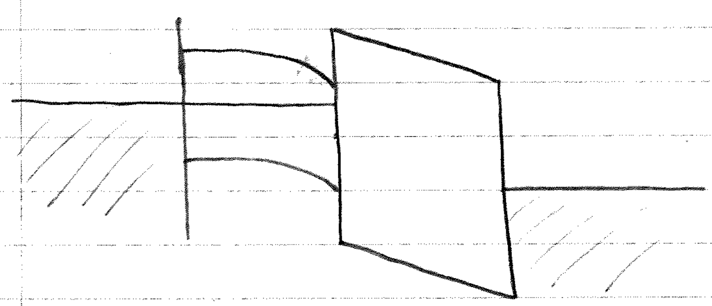
$$V_2 = \frac{C_1}{C_1 + C_2} V = \frac{k}{k+1} V$$

2\.

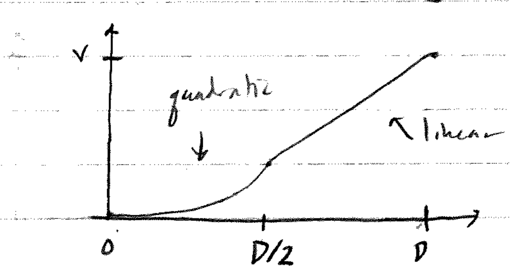
Assume $\Phi_s = \Phi_M$ for simplicity

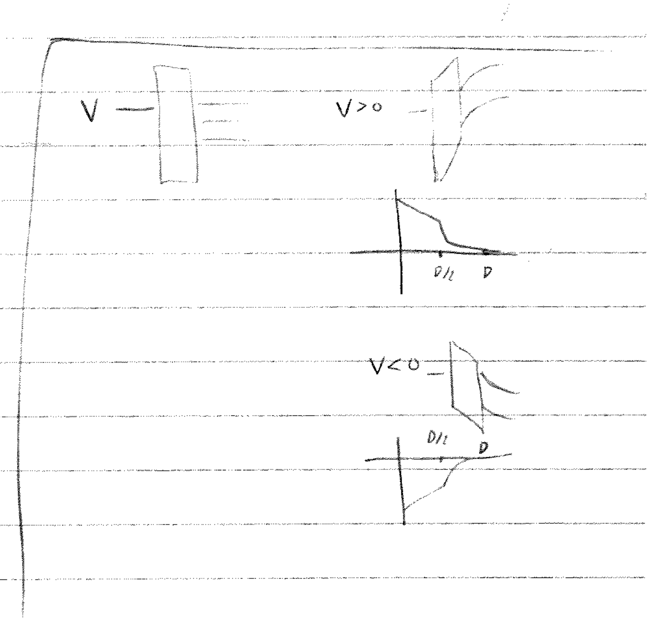


$V = 0$



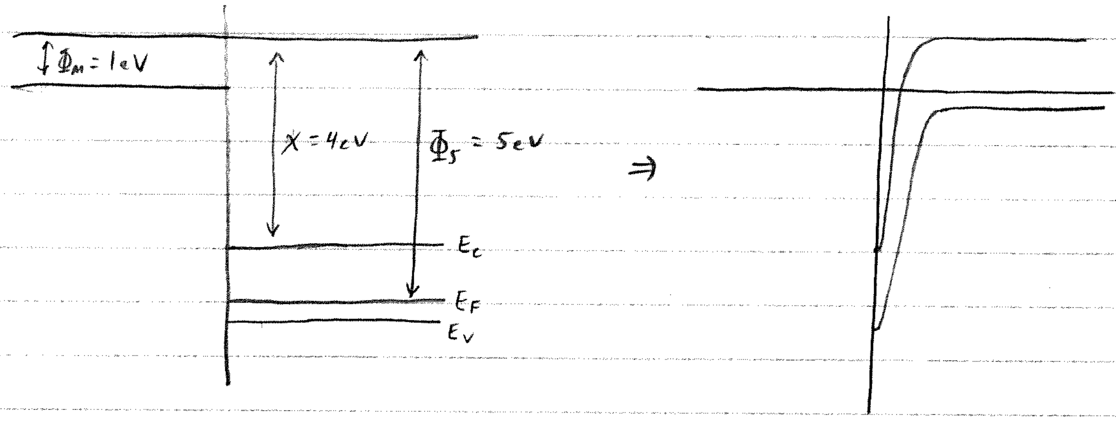
$V > 0$



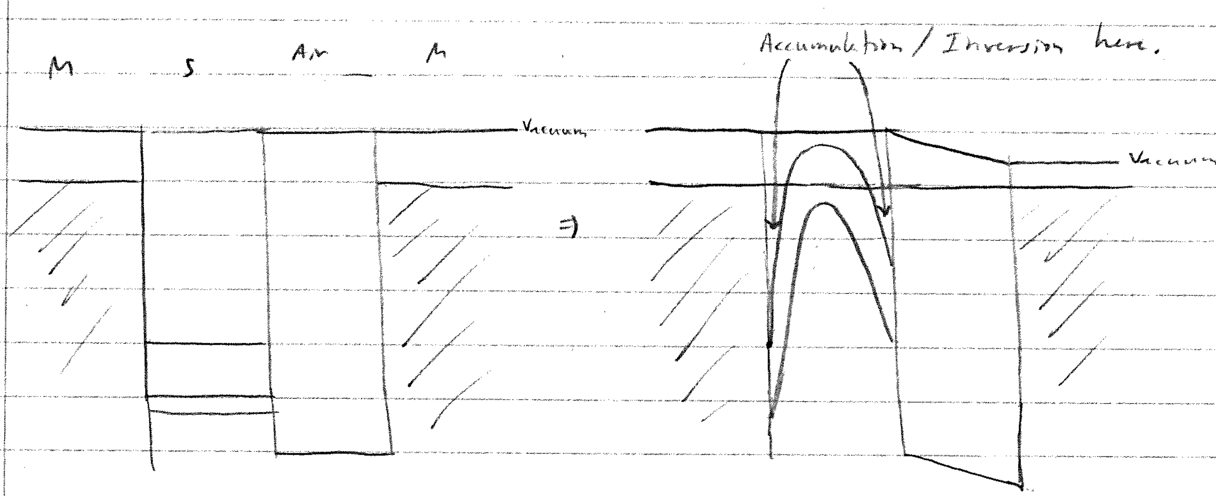


probably will have degenerate semiconductor at the interface and possibly a quantum well.

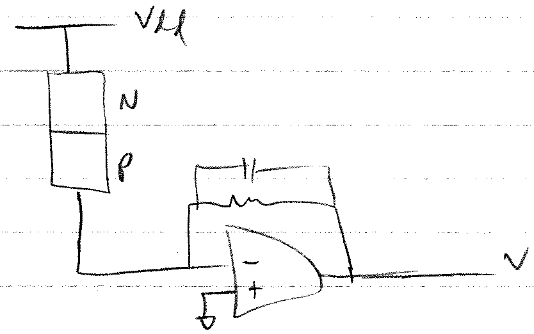
3.

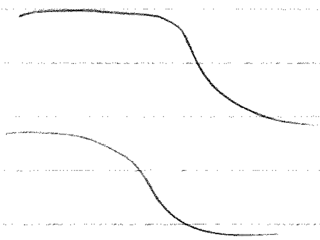


The band diagram shows very strong bending and even possible accumulation depending on the type of semiconductor. There were no other expected answers to this part, primarily this part was to allow students to give me their ideas on what might happen in such a structure which was similar to a familiar device, but with very different physical constants.



2000 - Pianetta



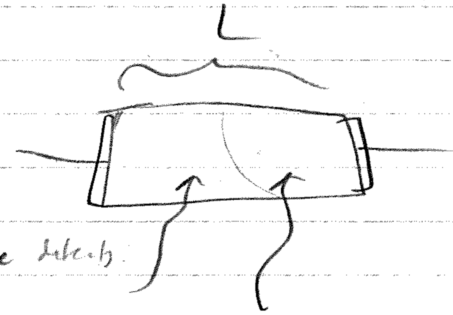


BJT,

10^{10} /s $\qquad 1e = 1.6 \cdot 10^{-19}$ C $\cdot 10^{10} = 1.6 \cdot 10^{-9}$ C/s

1.6 nA

$h\nu$

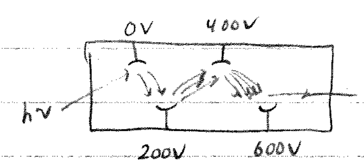


~~A~~ $\quad D_n >> L$

pure, no traps, few surface defects.
impurities,

$\alpha \downarrow$

1. PN Junction (reverse biased), Silicon film between two ohmic contacts, PIN Diode, BJT, MOSFET, Photomultiplier → (by impact ionization) avalanche diode.



2. $\approx 10^{10} \cdot 10^{-19} = 10^{-9}$ A $\Rightarrow$ 1 nA

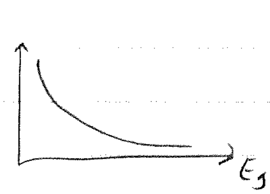
3. 1 nA is small $\Rightarrow$ should have dark current no more than 10% of the photocurrent. $\Rightarrow$ if 1V is applied

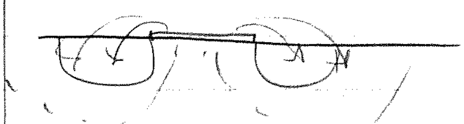
$R_{light} \approx \frac{1V}{1nA} = 10^9 \Omega \qquad R_{dark} \approx \frac{1V}{0.1nA} = 10^{10} \Omega$

Long carrier lifetime, Want them to be collected by electrodes, Want $L << D_n$, antireflection coating at surface, reflecting coating at back, ohmic contact, heat sink, Low impurities.

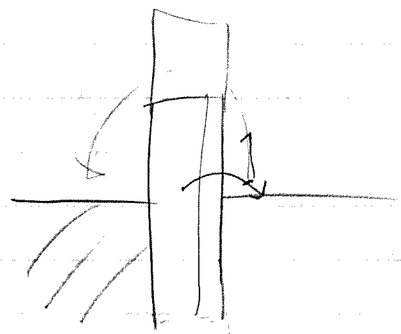
1999 - Pianetta

high $k \Rightarrow$ Low $E_g \Rightarrow$ low V_t





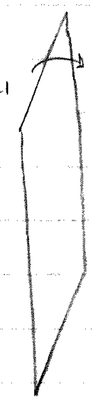
$\vec{E}$ field constant to avoid breakdown.



Barrier decreased

A. $E_g \downarrow \Rightarrow \Lambda \Rightarrow$ Increased Tunneling $\Rightarrow$ Increased Gate Leakage Current.
+ Increased thermionic emission
+ hot carrier injection

$E_g \uparrow \Rightarrow$ Tunneling reduced. $\Rightarrow$ Decreased Gate Leakage Current.



$V \uparrow \Rightarrow$ more tunneling $\Rightarrow$ leakage Current $\uparrow$

$T \uparrow \Rightarrow$ leakage Current $\uparrow$ since carriers are more energetic $\Rightarrow$ more of them see the thin barrier as shown in the drawing on the left.

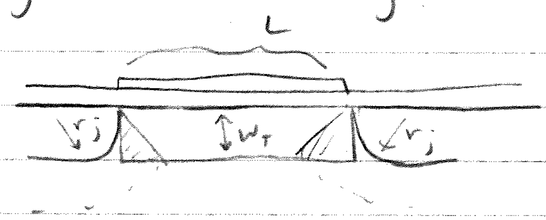
Tunneling is a function of barrier height and thickness. height $\downarrow$, Tunneling $\uparrow$
thickness $\downarrow$, Tunneling $\uparrow$

Need to know operating voltage (FN tunneling) and operating temperature (increased thermionic emission and $T \uparrow \Rightarrow E_g \downarrow$)

B. $$V_T = V_{(\Phi_S = 2\Phi_F)} = 2\Phi_F + \frac{K_S X_o}{K_o}\sqrt{\frac{4qN_A}{K_S\epsilon_o}\Phi_F}$$

For scaling, we generally want to keep the $\vec{\mathcal{E}}$ field in the oxide constant (breakdown limitations).

When lateral dimensions are made smaller ($L\downarrow$), the source and drain assist in depleting the region under the gate:



$$\Delta V_{T\,(\text{short channel})} = -\frac{qN_A W_T}{C_o}\frac{r_j}{L}\left(\sqrt{1+\frac{2W_T}{r_j}} - 1\right)$$

$\Rightarrow$ get unwanted short channel effects.

$$L_{min} = 0.4\left[r_j X_o (W_S + W_D)^2\right]^{1/3} \qquad X_o \text{ in } \text{Å}$$

$\Rightarrow$ To achieve smaller L_{min}, we must decrease X_o (thinner dielectric).

$C_{ox} = \dfrac{K_o\epsilon_o A}{X_o}$ inversely proportional to oxide thickness.

$V_T \propto X_o$ roughly proportional to oxide thickness.

Problem with thinner oxides is more tunneling. Can solve this by either going to infinite bandgap or by increasing the thickness of the dielectric and then solving the resulting problems with lowered capacitance and increased threshold voltage by using a high K dielectric ($K_o\uparrow$).

2002 - Pianetta

1. What determines the mobility of a semiconductor and explain the temperature dependence.

$$\mu_n = \frac{q\tau_{cn}}{m_n^*} \qquad \mu_p = \frac{q\tau_{cp}}{m_p^*}$$

where τ is mean time between collisions and m^* is effective mass.

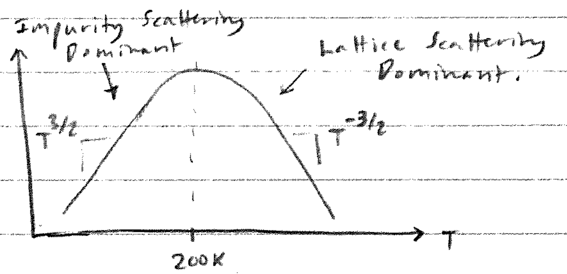
for low $\vec{E}$, $v_d = \mu \vec{E}$ where v_d is drift velocity.

Mobility depends on effective mass, lattice scattering (atoms displaced by lattice vibrations. Since e^-'s behave as waves, ~~they do not~~ scatter from a perfect lattice.), impurity scattering, (defect scattering: vacancies, interstitials, dislocations, grain boundaries.) and Temperature

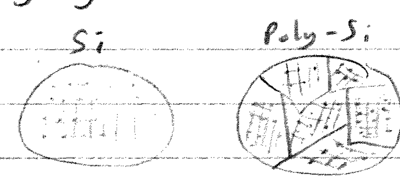
$$\mu_I \propto \frac{T^{3/2}}{N_I} \qquad N_I = N_d^+ + N_a^- \quad \text{(Total doping (not effective doping))}$$

$$\mu_L \propto T^{-3/2}$$

$$\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L} \quad \text{Mathiessen's Rule.} \qquad \left(\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L} + \frac{1}{\mu_D} \leftarrow \text{defects}\right)$$



3. What determines the mobility of polycrystalline silicon vs. single crystalline silicon?



Scattering at grain boundaries.

Poly-Si can be deposited at lower temperatures than Si.

polycrystalline has lower μ than crystalline silicon.

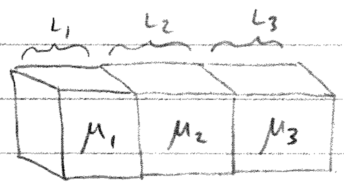
2. Given three different materials: Si, Ge, GaAs. Can you explain why their mobilities are different?

$$m^* = \frac{1}{\hbar^2 \left(\frac{\partial^2 E}{\partial^2 k}\right)} \Rightarrow$$ since Si, Ge, and GaAs have different (E-k) energy bands, they will all have different effective masses.

Also, Si, Ge, and GaAs will have different lattice properties such as atomic size and lattice constant which will lead to different amounts of lattice scattering, and thus different values of τ_c.

$\mu = \frac{q\tau_c}{m^*} \Rightarrow$ Si, Ge, and GaAs will have different mobilities.

4.



and $\rho_1 = \rho_2 = \rho_3 = \rho$

and $n_1 = n_2 = n_3 = n >> p$

If we assume $L_1 = L_2 = L_3 = L$ and $A_1 = A_2 = A_3 = A$

then $\rho = \frac{1}{\sigma} = \frac{1}{q(\mu_n n + \mu_p p)} \approx \frac{1}{q(\mu_n n)}$ $\quad R = \frac{\rho L}{A}$

$$\Rightarrow R_t = R_1 + R_2 + R_3 = \left(\frac{L}{qA}\left(\frac{1}{\mu_{n_1}} + \frac{1}{\mu_{n_2}} + \frac{1}{\mu_{n_3}}\right)\right) = \frac{L}{qA}\left(\frac{1}{\mu_t}\right)$$

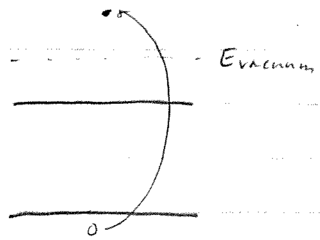
$$\Rightarrow \boxed{\frac{1}{\mu_t} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3}}$$

2001 - Pianetta

1. X-ray detector using Si. How does it work?

- Photoelectric Effect. Since X-rays have such high energy, the generated carriers will have large kinetic energy beyond vacuum level. And carriers can also cause impact ionization.

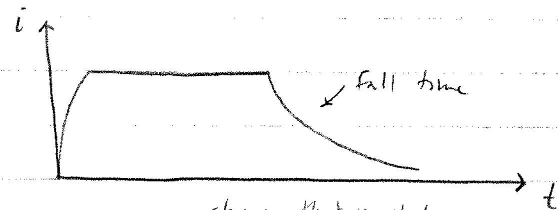
In order to conserve energy, # of e^- h^+ pairs must be less than X-ray energy divided by bandgap of Si.



$$E_{x\text{-ray}} = \frac{1.24}{\lambda}$$

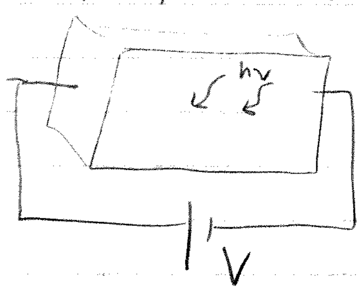
2. Can use PN, PIN, Photoconductor, or Phototransistor (BJT)

3. Expected signal:



For PIN
to reduce fall time, decrease C. $\Rightarrow$ Q $\downarrow$ (charge that must be removed)
increase width of intrinsic region for PIN

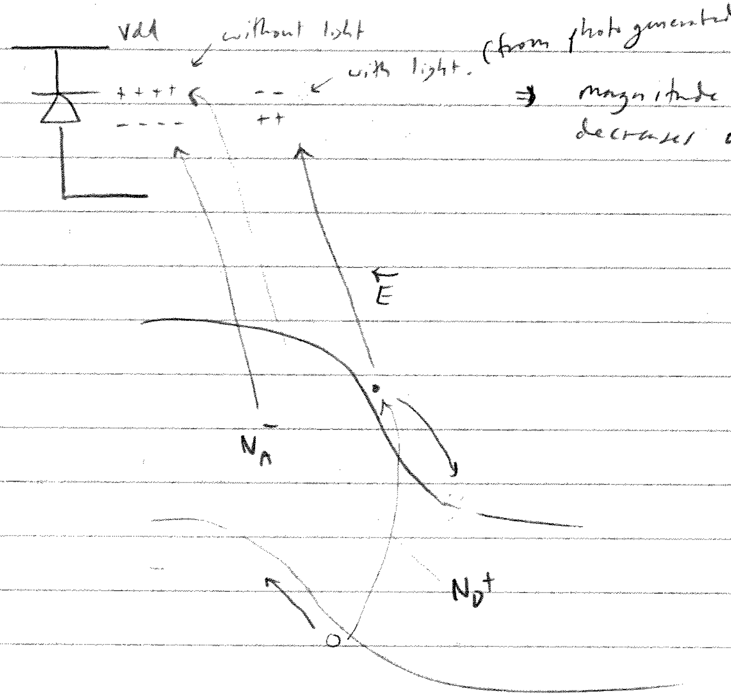
For Photoconductor, to reduce fall time, increase bias V.
$v_d = \mu \mathcal{E} \propto V \Rightarrow$ carriers removed faster. OR decrease carrier lifetime so excess carriers are more quickly eliminated via recombination.



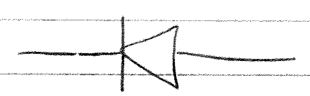
$$f_{max} = \frac{1}{\left(\begin{array}{c}\text{carrier transit}\\ \text{time across } W_I\end{array}\right)} \simeq \frac{1}{(W_I / v_{sat})}$$

but you can't make W_I arbitrarily small. Eventually, the RC time constant associated with the internal series R and junction capacitance $\left(C_j = \frac{K_s \epsilon_0 A}{W_I}\right) \propto \frac{1}{W_I}$ will limit the response of the diode.

For PN and BJT to decrease fall time,
you should also decrease C_{dep} ⇒ Q ↓
⇒ decrease doping to increase W_{dep} ⇒ C_{dep} ↓

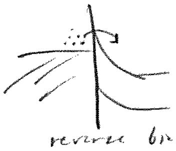


⇒ magnitude of reverse bias decreases with light.

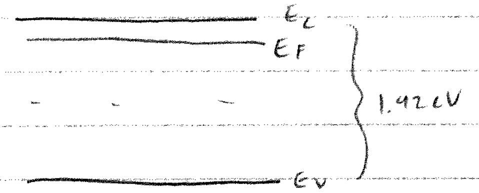


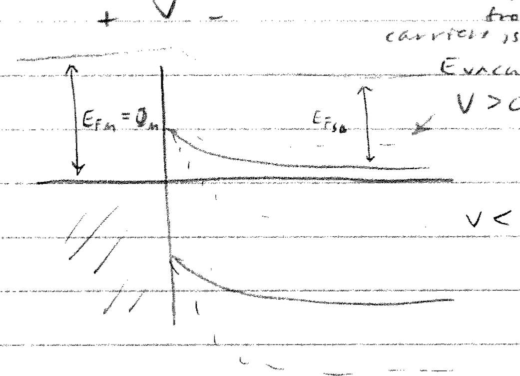
Called majority carrier device. (recombination + diffusion current from minority carriers is negligible)

Pre - 1993

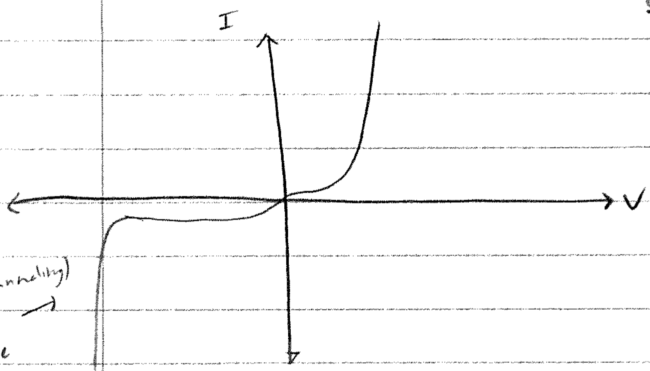


1)





$\Phi_B = \Phi_m - \chi$ $\Phi_s = \Phi$

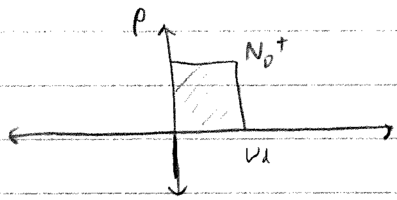


zener (tunneling) or avalanche breakdown

2) derive width of depletion region:

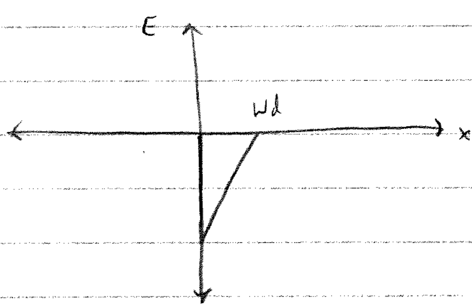
$$\frac{dE}{dx} = \frac{\rho}{\varepsilon}$$

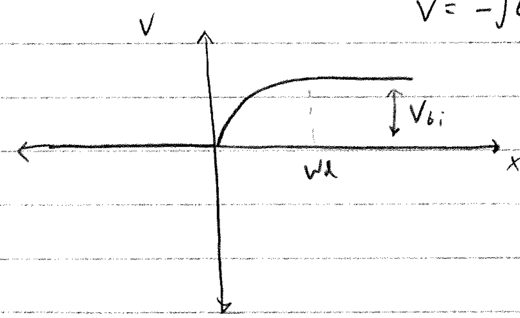
$$E = -\frac{dV}{dx}$$



$$\frac{\rho}{k_0 \epsilon_0} = \frac{\partial E}{\partial x} \leftarrow \text{Poisson's Eq.}$$

$$V = -\int E\,dx$$



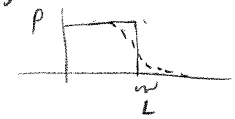


$$V_{bi} = \frac{1}{q}\left(E_{F_m} - E_{F_{SB}}\right) = \frac{1}{q}\left(\Phi_m - \Phi_s\right)$$

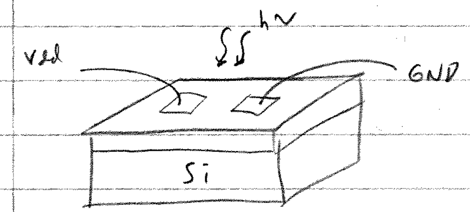
if you know doping N_D, you can solve for Wd.

OR: $W_d = \sqrt{\dfrac{2\epsilon_s (V_i - V_a)}{q N_d}}$ $\epsilon_s = \epsilon_r \epsilon_0$ for GaAs

Debye length describes how abrupt a junction is.

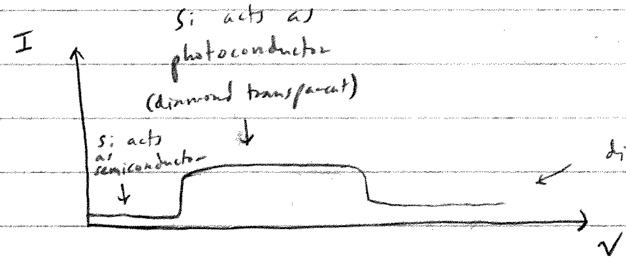


Pre-1993 - Pianetta



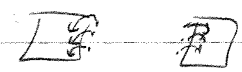
diamond is an insulator.

but diamond can become a semiconductor when sufficient # of carriers are excited.



diamond absorbs the photon ⇒ carriers aren't generated in Si. (Diamond is opaque)

But only the photocarriers generated right next to the contacts can contribute to current.

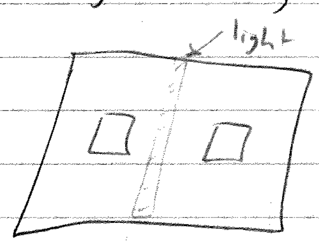


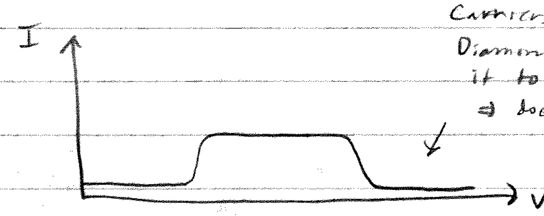
$$\rho = \frac{h}{\lambda} \qquad \lambda = \frac{1.24}{E_g}$$

?

$$I_L = \rho \frac{q \tau_p G_L}{\tau_t} \left(1 + \frac{\mu_p}{\mu_n}\right) A \cdot L$$

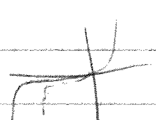
Now if light is only shown over this area:





Carriers generated in Diamond can't make it to contacts. ⇒ doesn't contribute to current.

metal - n-GaAs. When you apply negative bias, the fermi level will decrease at the surface and the donors ionize ⇒ ionized impurities contribute positive charge at interface ⇒ depletion width decreases ⇒ more current, higher σ ⇒ tunneling at lower applied voltage. When you apply positive voltage, nothing really changes



- Electron tunneling - direct tunneling $t_{ox} < 3nm$ FN tunneling $t_{ox} > 5nm$

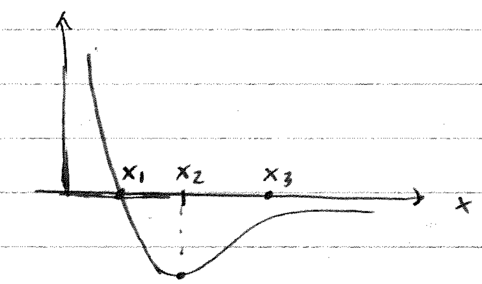
1998 - Wang

$KE \propto kT$ $\uparrow$

$KE + U$

$KE\uparrow$ $U\downarrow$

U(x) ← potential energy between 2 atoms,



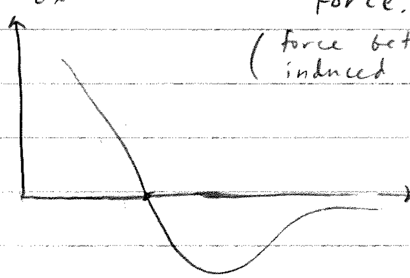
2 atoms

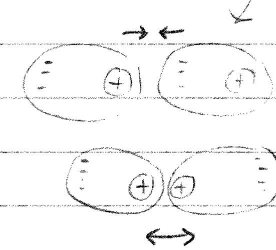
When $T = 0K$

$x = x_2$ since the system will go to the minimum energy state. (point where $F = 0 \Rightarrow$ stable point)

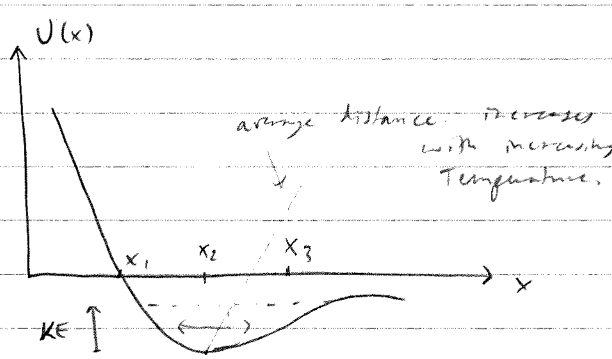
$KE = \frac{3}{2} kT$

$F = -\frac{\partial U}{\partial x}$ ← Van der Waals Force. (force between induced dipoles)





When $T\uparrow$, what happens to x_{avg}?



$F = -\frac{\partial U}{\partial x}$

$\Rightarrow F_{left} > 0$, $x < x_2$

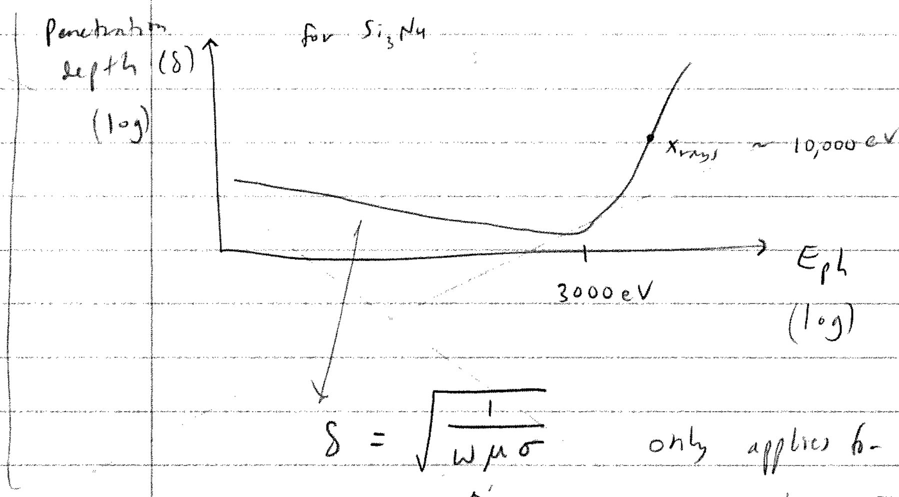
$F_{right} < 0$, $x > x_2$

$|F_{right}| < |F_{left}|$

since the slope is lower on the right side, x_{avg} will be greater than x_2 when $T\uparrow$

This is the mechanism for thermal expansion.

2008 - Pianetta

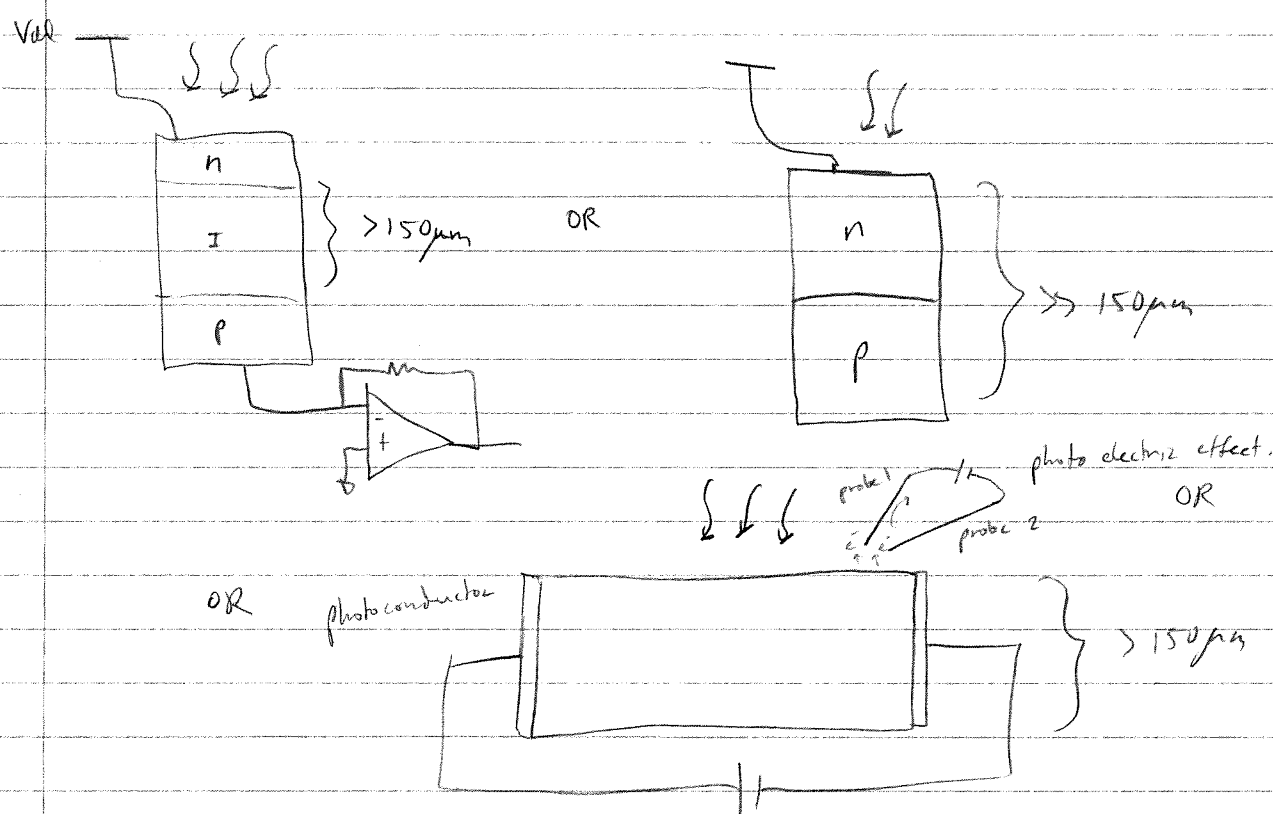


$$\delta = \sqrt{\frac{1}{\omega \mu \sigma}}$$ only applies for good conductors.

frequency — when $\sigma \gg \omega \varepsilon_r \varepsilon_0$

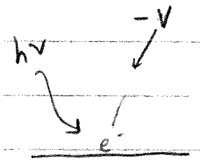
we actually care about α, not δ.

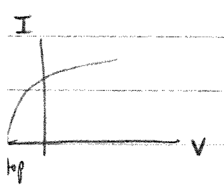
How would you use a Si device to efficiently detect photons with an energy as high as 10,000 eV? Note that the $\frac{1}{e}$ penetration depth of such a photon is about 150 µm versus a few µm for visible light photons.



Could you use such a device to measure the energy of an individual photon?

OR use photoelectric effect to measure $V_{stopping}$





Yes, using the fact that the electron created by the high energy photon has a very high kinetic energy which will be converted into a cascade of secondary electrons (due to impact ionization) and can be swept out of the device as a current pulse.

What determines the response time of the detector?

Concepts that should be understood include the time required to sweep the electrons out of the depletion region and the RC time constant of the device.

$-qV_{stopping} = h\nu - \Phi_s$;

How could you make the detector faster?

Builds on answer to previous question including reducing sweep out time with higher voltage, reducing R and C. Ideas for reducing C include shaping the electrodes to have a small back electrode that can still collect the charge.

Can reduce R by reducing contact resistance, n^+ and p^+ wells. OR can use a semiconductor with higher μ such as GaAs.

2005 - Pianetta

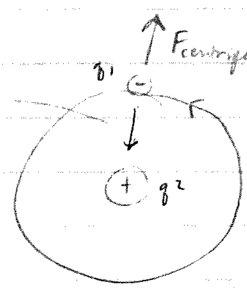
Consider a simplified model of an atom with a positive core and with the outer or valence electrons modeled as a spherical shell of charge. Also consider one of the core electrons with a binding energy E_b. If I were to remove charge from the outer valence shell, for example, by oxidation of the atom, how does the binding energy of the core electron change?

$$F = \frac{-kq_1q_2}{r^2} \qquad W = \int_{\infty}^{r_0} F(r)\,dr = -kq_1q_2 \int_{\infty}^{r_0} \frac{1}{r^2}\,dr$$

$$= -kq_1q_2 \left(-\frac{1}{r} \Big]_{-\infty}^{r_0} \right) = \frac{kq_1q_2}{r_0}$$

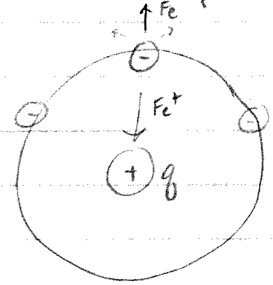
$$F_{centripetal} = \frac{mv^2}{r}$$

$$F_e = \frac{kq_1q_2}{r^2}$$



$$Fe = F_c$$

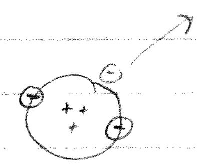
Fe^- ← due to electron shielding.



⇒ when an electron is removed, $Fe^- \downarrow$

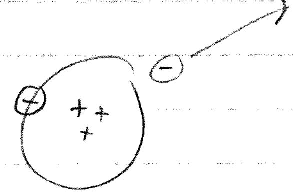
⇒ $\boxed{r_0 \downarrow}$

$q_1 = e^- \quad q_2 = e^-$



for removing the first e^-: $E_b = W = q_1 \cdot \Delta V = q_1 \int_{r_0}^{\infty} \vec{E}(r)\,dr = q_1 \int_{r_0}^{\infty} \frac{q_2}{4\pi\epsilon_0 r^2}\,dr = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r_0}$

for removing the 2nd e^-: $q_1 = e^- \quad q_2 = 2e^- \quad \Rightarrow E_b \uparrow$



1999 - Wang

1) How do you express or describe the momentum of an electron in a crystalline solid?

Crystal momentum is a momentum-like vector associated with electrons in a crystal lattice. It is defined by the associated wave vectors k of the lattice:

$$\boxed{p_{crystal} = \hbar k} \qquad p = \frac{h}{\lambda} \qquad k \equiv \frac{2\pi}{\lambda}$$

DeBroglie

True momentum, or average momentum, is given by:

$$\langle p_x \rangle = \int_{-\infty}^{\infty} \psi^* \frac{\hbar}{i} \frac{\partial \psi}{\partial x} \, dx$$

← for momentum in x-direction for the 1-D case.

2) Is it true that the property of such an electron will not change if its wave vector is increased by a reciprocal lattice vector? If yes, what is the underlying physical law or theorem?

Yes. Bloch's theorem:

Consider a Bravais Lattice with translational symmetry vector $\mathbf{a}$
$\Rightarrow$ $U(\mathbf{r})$ is periodic. $(U(\mathbf{r}) = U(\mathbf{r}+\mathbf{a}))$.

$$\Rightarrow \psi(\mathbf{r}+\mathbf{a}) = e^{i\mathbf{k}\mathbf{a}}\psi(\mathbf{r}) \iff \psi(\mathbf{r}) = e^{i\mathbf{k}\mathbf{r}} u(\mathbf{r})$$

unit cell wavefunction with same periodicity as potential.

where $u(\mathbf{r}) = u(\mathbf{r}+\mathbf{a})$

Reciprocal lattice is the set of all vectors $\mathbf{K}$ that satisfy $e^{i\mathbf{K}\mathbf{a}} = 1$

$\Rightarrow$ if $\psi(r+a) = e^{ika}\psi(r)$, then increasing wave vector by a reciprocal lattice vector yields:

$$\psi(r+a) = e^{i(k+K)a}\psi(r) = \underbrace{e^{iKa}}_{\text{equals 1 by definition}} e^{ika}\psi(r) = e^{ika}\psi(r)$$

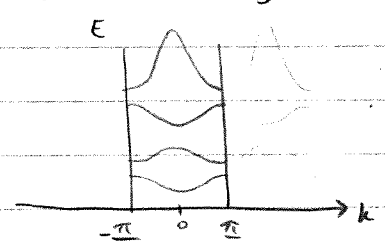
$\Rightarrow$ wavefunction stays the same.

3) Show that the real (or average) momentum of the electron indeed does not change if its wave vector is increased by a reciprocal lattice vector.

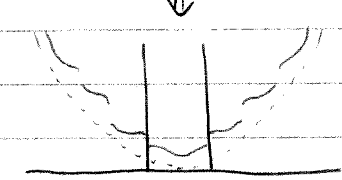
Although one may expect momentum to change, since crystal momentum ($\hbar k \rightarrow \hbar(k+K)$) changes. The real (average) momentum must not change, since the wavefunction stays the same.

$$\langle p_x \rangle = \int_{-\infty}^{\infty} \psi^* \frac{\hbar}{i} \frac{\partial \psi}{\partial x} dx$$ ← this is for the 1-D case for momentum in the x-direction, but the 3-D case also depends only on wavefunction.

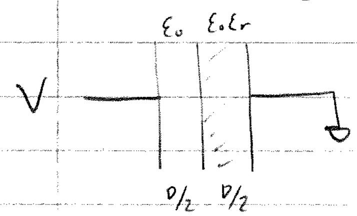
This is also why E-k diagram repeats with period $\frac{2\pi}{a}$.



⇕ identical representations

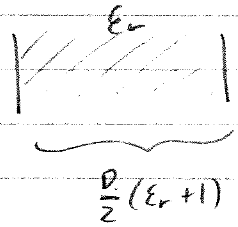


2003 - Wang



$$C_1 = \frac{2\varepsilon_0 A}{D} \qquad C_2 = \frac{2\varepsilon_r\varepsilon_0 A}{D} = \varepsilon_r C_1$$

$$C_{eff} = \frac{\varepsilon_r}{1+\varepsilon_r} C_1 = \frac{\varepsilon_r\varepsilon_0 A}{\frac{D}{2}(\varepsilon_r+1)}$$

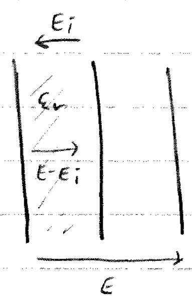


$$F = QE = Q\frac{\sigma}{\varepsilon_r\varepsilon_0}$$

$$= Q\cdot\frac{Q}{A\varepsilon_r\varepsilon_0} = \frac{Q^2}{A\varepsilon_r\varepsilon_0}$$

$$Q = CV = \frac{\varepsilon_r\varepsilon_0 A}{\frac{d}{2}(\varepsilon_r+1)}\cdot V \Rightarrow \cancel{F = \frac{\varepsilon_r\varepsilon_0 A V^2}{\left(\frac{D(\varepsilon_r+1)}{2}\right)^2}}$$

don't think this is right.



$$\varepsilon_r(E-E_i) = E$$

$$V = \frac{d}{2}(E-E_i) + \frac{d}{2}E$$

$$= \frac{d}{2}\cdot\frac{E}{\varepsilon_r} + \frac{d}{2}E = \frac{d}{2}\left(1+\frac{1}{\varepsilon_r}\right)E$$

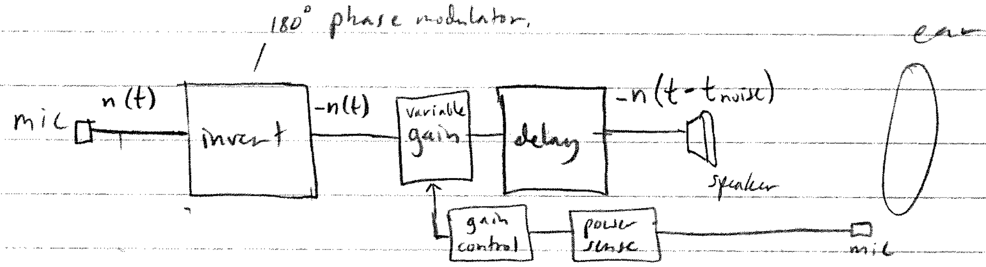
$$Q = CV = V\cdot\left(\frac{1}{1+\frac{1}{\varepsilon_r}}\right)\frac{2\varepsilon_0 A}{d}$$

$$F = \frac{QE}{2} = \frac{1}{2}V\left(\frac{1}{1+\frac{1}{\varepsilon_r}}\right)\frac{2\varepsilon_0 A}{d}\cdot V\cdot\frac{2}{d}\left(\frac{1}{1+\frac{1}{\varepsilon_r}}\right) = \boxed{\frac{2V^2\varepsilon_0 A}{(1+\frac{1}{\varepsilon_r})^2 d^2} = F}$$

$$\left(\text{for } \varepsilon_r = 1,\quad F = \frac{QE}{2} = \frac{CVE}{2} = \frac{\varepsilon_0 A V^2}{2d^2}\right)$$

2006 - Pease

$$t_{audio} = \frac{d_2}{v_{sound}} < t_{noise}$$



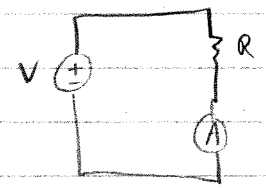
$$t_{noise} = \frac{d_1}{v_{sound}} = \frac{d_1}{343\,m/s} \approx \frac{5cm}{343\,m/s} \approx \frac{15cm}{1000\,m/s} = \frac{15 \cdot 10^{-2} m}{1000\,m/s} = 150\mu s$$

if we assume that the inversion block response is much faster than t_{noise} (~150μs), we can create a wave that will destructively interfere with the noise waveform before it hits the ear. In order for this to work, we must ensure that the volume gain is 1 for this system. So we should have some means of adjusting gain perhaps with feedback. Adjust gain until the volume sensed near the ear is minimized.

2005 - Pease

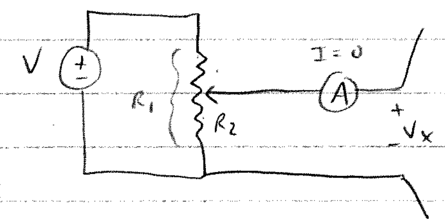
1. $1V = \frac{1J}{1C}$

2. Voltmeter:

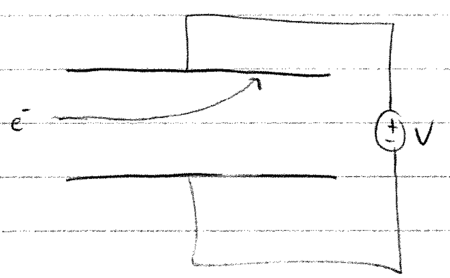


$V = IR$

want R to be large.



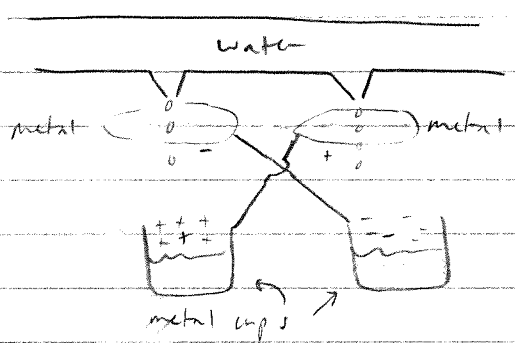
$\frac{V}{R_1} = \frac{V_x}{R_2}$



measure distance of deflection. This will be related to V.

3. No.

4.



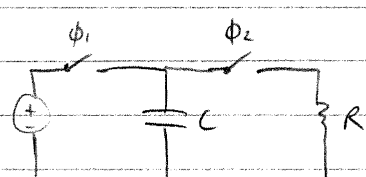
positive feedback.

limitations: may have ESD sparks between cups.

5. Because the cups only form a capacitor which discharges.

6. Devise a voltmeter with infinite resistance. $(> 10^{14} \Omega)$

Use C with ~~thick dielectric~~ (large tox)



measure discharge time of C. This should be related to voltage.

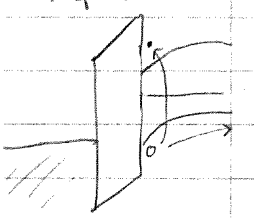
Input resistance of a cap should be similar to that at the gate of a MOSFET which is typically $> 10^{12} \Omega$ (due to leakage through dielectric, tunneling, etc.)

Pre 1993 - Pease

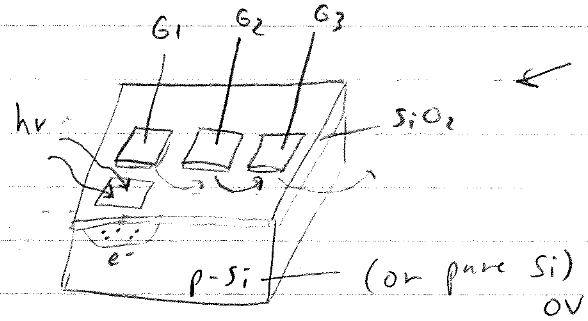
1. How does a TV Camera Work?, Discuss problems of getting good collection efficiency, interconnecting and addressing of cells, diffusion lengths, and capacitor storage times.

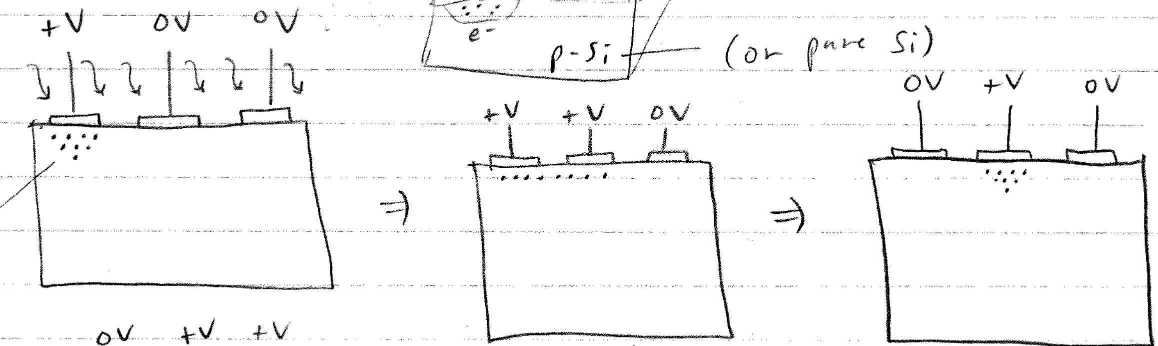
CCD.

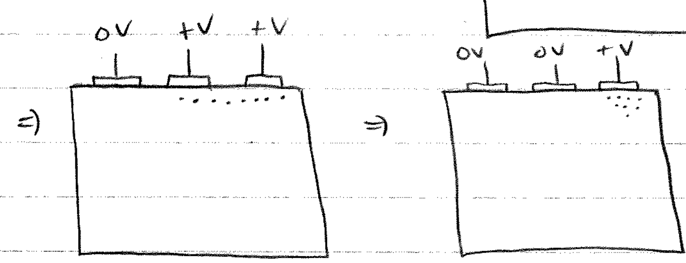
MOS is biased in deep depletion.



accumulated charge is proportional to light intensity.







⇒ read out pixels row by row or column by column.

Collection efficiency:

- Quantum efficiency: absorption length $\propto \frac{1}{\lambda}$

  ⇒ short wavelength photons absorbed very near surface. long wavelength photons absorbed deeper in bulk where the pixel gate potential well is relatively shallow, rather than in the depletion region. ⇒ these resulting photoelectrons may migrate to a neighboring pixel (charge diffusion ⇒ image blurring).

  - pure silicon is shiny (reflective), especially for short wavelength (blue) photons. This can be reduced by using an antireflection coating like MgF_2.

Diffusion Lengths: Doesn't really matter except for low wavelength photons. But some surface recombination can degrade the charge transfer and thus the readout efficiency.

Capacitor Storage times: Purer Si ⇒ higher R ⇒ higher storage time. But more expensive. There should also always be finite leakage current due to trap assisted tunneling through the oxide.

2. a) Explain how a microwave heats a chicken.

Microwaves use EM waves (2.45 GHz) to cause polarized molecules (H_2O) to oscillate, generating heat. Generates heat within the material.

b) Compare with infrared heating.

Infrared heating heats the material's surface first and then heat moves inward. This is slower and less energy efficient.

c) Do microwaves have higher or lower frequency than infrared?

Lower. Microwaves ~ 2.45 GHz. Infrared ~ 1-400 THz.

d) Can you devise a method for quick cooling the chicken using the method that microwaves use to heat it? Not sure...

3. Why is a very long wave used for sonar?

Skin depth $\delta = \left(\frac{2}{\omega\mu\sigma}\right)^{1/2} \propto \frac{1}{\sqrt{\omega}} \Rightarrow$ longer wave $\Rightarrow$ lower frequency $\Rightarrow$ greater skin depth $\Rightarrow$ wave can travel farther.

4. Define an active device:

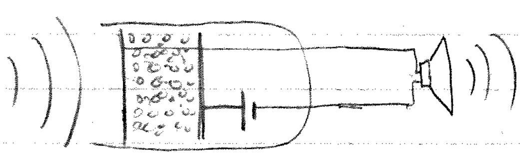
Any type of circuit component with the ability to electrically control electron flow. Also, active devices are capable of delivering power to the circuit (power gain greater than one).

Active: Vacuum Tubes, Transistors, Silicon controlled Rectifiers, TRIACs.

Passive: Resistors, capacitors, inductors, transformers, diodes.

5. Can you use telephone speakers and microphones for power amplification?

Yes. Telephone microphones are carbon microphones which consist of 2 metal plates separated by granules of carbon. When pressure (i.e. from a sound wave) is applied to the mic, the granules of carbon are pressed together, decreasing resistance.



Thus, since the speaker power comes from the battery, it is possible to achieve power amplification.

6. a) If there is a current of $100\ A/cm^2$ in a copper wire, what is v_d?

$I = q n v_d A \Rightarrow v_d = \left(\frac{I}{A}\right)/(nq)$ for copper, $n \simeq 8.483 \cdot 10^{22}\ cm^{-3}$ $q = 1.6 \cdot 10^{-19}\ C$

$\Rightarrow \boxed{v_d = 7.37 \cdot 10^{-5}\ m/s}$

b) how long must you wait for the current pulse to traverse the wire?

Much less than that implied by the drift velocity. The wire acts as a transmission line. So $v_{propagation} = \frac{1}{\sqrt{LC}} \approx 0.75\,c$ typically.

c: speed of light; L: unit inductance; C: unit capacitance

Since wire is full of carriers, you don't need to wait for a carrier on the far left side of the wire to drift to the far right before the signal arrives.

7. What are good properties of SiO_2?

Naturally forming oxide on Si, very few interface traps, and dangling bonds. Chemically stable in air ($\Rightarrow$ passivates Si surface). Good insulator ($E_g = 9eV$).

Good high temperature diffusion and ion implantation barrier.

Easily etched. Has good etch sensitivity to semiconductor (Si).

Reasonably high dielectric constant ($k = 3.9$)

8. Why greater conductance for shorter channels?

$G = \frac{1}{R} \Rightarrow G = \frac{A}{\rho L} \propto \frac{1}{L}$

9. a) What is the mechanism that limits speed in todays VLSI chips?

propagation delay / transit frequency $\Rightarrow$ minimum channel length.

Also thermal dissipation. Power dissipation / Area is increasing.

$\Rightarrow$ increased operating temperature. Acceptable range: $-55°C - 125°C$ ($220K - 400K$)

$°C = K+273$

b) Estimate transit time in a MOS transistor:

$$\tau = \frac{L}{v_d} = \frac{L}{\mu \mathcal{E}} = \frac{L}{\mu \frac{V_{ds}}{L}} = \frac{L^2}{\mu V_{ds}} \Rightarrow \boxed{\tau = \frac{L^2}{\mu V_{ds}}}$$

c) What benefits on speed can be expected when you go to liquid N_2 temperature (77K) in MOS and Bipolar circuits? What problems would you expect?

- Benefits - reduced lattice scattering, reduced thermal noise, higher μ than $T > 300K$

- Problems - increased impurity scattering. process variations. $\Rightarrow$ higher V_{TH}, V_T

V_{TH} ($-1.2mV/°C$) $\uparrow$, R ($1\%/°C$) $\downarrow$, $V_T = \frac{kT}{q}$ ($-2mV/°C$) $\uparrow$? check this.

problems: BJT - lower current since carriers are less energetic $\Rightarrow$ fewer can diffuse.

10. Is a transformer an active device? If not, why?

No. No power gain. $\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s} \Rightarrow P_{in} = P_{out}$ for ideal transformer.

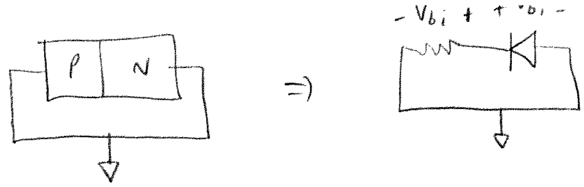
11. MOSFET: $g_m = \mu C_{ox} \frac{W}{L} V_{ov} = \sqrt{2\mu C_{ox} \frac{W}{L} I_d}$ BJT: $g_m = \frac{I_c}{V_T}$

12. Do you increase g_m by increasing ϵ_{ox}?

Yes. $g_m = \mu C_{ox} \frac{W}{L} V_{ov} = \mu \frac{\epsilon_{ox} A}{t_{ox}} \frac{W}{L} V_{ov} \propto \epsilon_{ox}$.

Is there a limit? not sure...

Yes. $k \propto \frac{1}{E_G} \Rightarrow$ at some point for high k, E_g will be too low and leakage current will become too large.



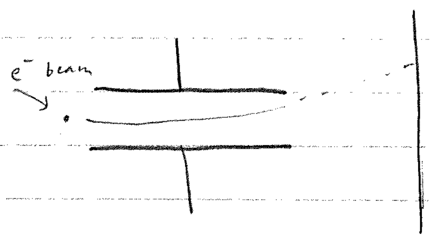
13. Will an electron beam directed right over a PN junction (short circuited) deflect the beam?

I think yes, due to the local electric field set up by V_{bi}.

14. a) A 10eV x-ray is incident upon the bottom plate of a 0.001μF open circuited capacitor. The photon excites an electron from the bottom plate to the top plate. What is the voltage across this cap?

$Q = V \cdot C \Rightarrow V = \frac{Q}{C} = \frac{1.6 \cdot 10^{-19}}{1 \cdot 10^{-9}} = 1.6 \cdot 10^{-10} \Rightarrow \boxed{V = 0.16\,nV}$

b) How would you measure this voltage? describe the experiment in detail and find the fundamental physical limiting factor.



detect e^-, measure deflection, $\Rightarrow$ calculate $\vec{E}$

$\Rightarrow$ calculate voltage.

thermal noise on the capacitor plates? Could possibly cool the capacitor to low temp (liquid N_2 = 77K)

2007 - Pease

- A cathode ray oscilloscope operates by focusing and deflecting a beam of free electrons onto a screen. Outline the limitations to the sensitivity, resolution and speed. You may assume the electron lens is free of aberrations.

Speed: Time for beam to hit screen - almost instantaneous.
Flourescent screen off → on - almost instantaneous.

per pixel { response time CRT : $< \mu s$
LCD : 2-5 ms

refresh rate CRT : limited by screen area and speed of lens circuitry.
ideally want 60Hz refresh rate per pixel (detectable by eye).

Resolution: Size of pixel is determined by beam width which is determined by beam aperture, and it is also determined by packing density of flourescent molecules.
And by the resolution of the voltage you apply to the deflection lens.

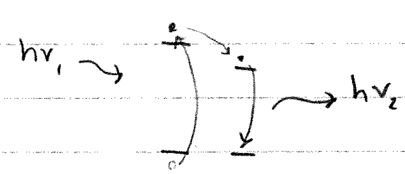
Eyes also have a limitation: Rayleigh criterion. Human eye can distinguish 1 μm distance. $\theta_R = 1.22 \frac{\lambda}{d}$ ← wavelength of light. / diameter of eye's lens diameter

⇒ resolution is limited by the eye, not the CRT.

Sensitivity: Limited by quantum efficiency (η) of the flourescent screen. # of electrons you need to focus on the screen to emit a photon from the flourescent molecule. Some energy will be dissipated as heat.

Contrast of CRT >> LCD
↳ doesn't have true blacks.

Flourescent Molecule:



e.g. Alamar Blue.

green ⇝ ☐ ⇝ red/orange

2008 - Pease

1. Why do heat sinks have fins? To maximize surface area.

2. Express the area advantage of the above finned structure over an unfinned structure in terms of h, L_c, L_W.

$$A_{unfinned} = L \cdot Z \qquad A_{finned} \overset{\text{assuming } n \text{ fins.}}{=} n(2h + L_c)Z = \left(\frac{L}{L_W + L_c}\right)(2h + L_c)Z$$

3. Choose h, L_c, L_W. Want L_c and L_W small so you have more fins. Want h large so you have longer fins.

i.e. to maximize $A_{finned} = \left(\frac{L}{L_W + L_c}\right)(2h + L_c)Z$

from Wikipedia: efficiency $\eta = \dfrac{\tanh\left(\sqrt{\frac{2h_f \cdot h}{k \cdot L_W}}\right)}{\sqrt{\frac{2h_f \cdot h}{k \cdot L_W}}}$

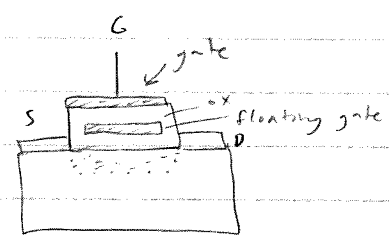
where h_f is a convection coefficient of the fins. (depends on the material of fins and fluid)

without fins: $A = n(L_c + L_W)Z$

with fins: $A = n(L_c + 2h)Z$

$\Rightarrow$ must have $\boxed{h > \frac{L_W}{2}}$

How does flash memory work?



FLASH

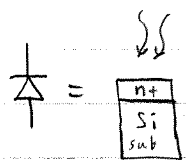
write: if you apply $V_G > 0$

inversion layer forms + electrons tunnel onto floating gate. $\Rightarrow V_{Th} \uparrow$.

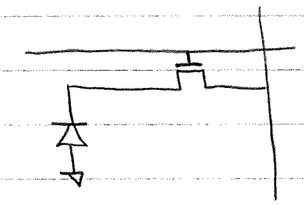
2010 - Pease

1\. 2009 Nobel Prize in Physics awarded for "the invention of an imaging semiconductor circuit — The CCD"

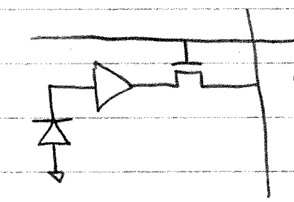
a) CCD or CMOS sensor (Active Pixel Sensor or Passive Pixel Sensor)



PPS: - 1 transistor per pixel
- small pixel, large fill factor
(% of pixel area that is sensitive to light)
- But, slow, low SNR
- readout speed limited by row transfer time



APS: - 3-4 transistors per pixel
- fast, high SNR
- But, larger pixel, lower fill factor
- column readout time limits readout speed.



- As technology scaled below 0.5 μm, APS pixel size and fill factor were no longer a problem ⇒ current technology of choice.

elaborate...

b) The CCD was invented at Bell Labs in 1969 as a new type of memory storage device. Though it did not prove to have a future in memory storage, the CCD gave rise to an explosion in digital imaging, with the first CCD based video cameras appearing in the early 1970s. Although CCDs are to some extent now supplemented by competing technologies, their use in applications ranging from digital cameras to the Hubble space telescope has completely transformed image processing.

c) 1. <u>Low power</u>. CCDs consume as much as 100 times more power than an equivalent CMOS sensor.
For CCD, entire array is switching all the time.
(high C, high V, and high f result in high $P = CV^2 f$)

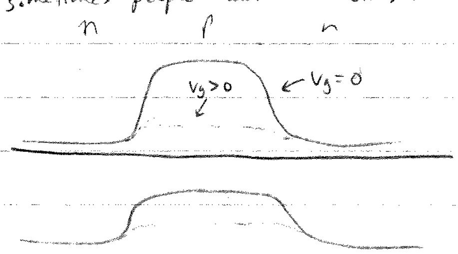
2. <u>Low Cost</u>. CMOS chips can be fabricated on just about any standard silicon production line, so they are much cheaper than CCDs which require special processing.

3. <u>Faster</u>. For CCDs, there is a tradeoff between transfer speed and transfer efficiency. This is not the case for CMOS sensors, since each pixel is read individually. (local $Q \to V$ conversion for APS)
⇒ don't have to worry about transfer efficiency.

4. <u>Circuit Integration</u>. Digital logic circuits, clock drivers, counters, and analog to digital converters can be placed on the same Si chip and at the same time as the photodiode array for CMOS sensors. (You can't do this for CCD's). This enables CMOS sensors to participate in the process shrinks that move to smaller linewidths with a minimum of redesign, in a manner similar to IC's.

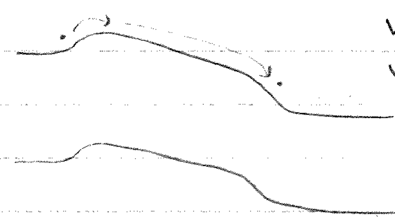
2007 - Nishi

1. (100) is standard surface orientation for Si (highest μ)
sometimes people also use (110).



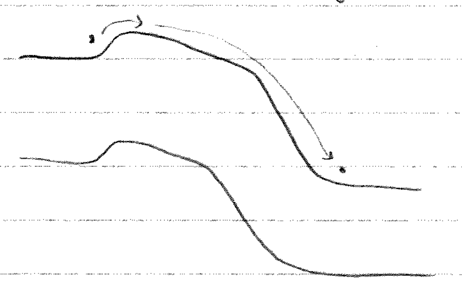
$V_d = 0$

Triode

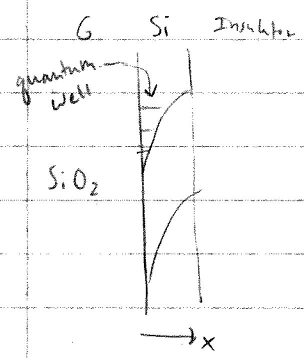


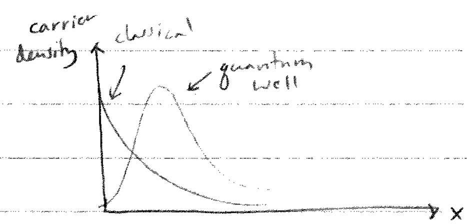
$V_d > 0$
$V_{gd} > V_t$

Saturation $V_d > 0$
$V_{gd} < V_t$



current is due to drift.
Thus, Id does not increase
with increasing Vd in saturation
since this voltage just drops
across the drain depletion region.
$\Rightarrow \vec{\mathcal{E}}$ in channel is ~ the same.

2. 



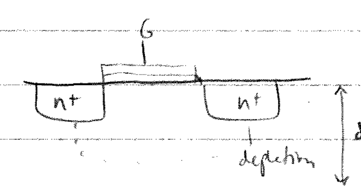
Volume inversion

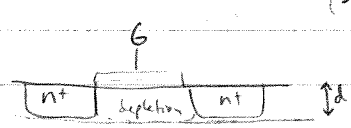
$\Rightarrow$ carriers are mostly away from the surface
$\Rightarrow$ less surface scattering
$\Rightarrow$ $\boxed{\text{increased mobility.}}$

$t_{ox_{eff}} \uparrow \Rightarrow C_{ox} \downarrow$ but $\mu \uparrow$

$\Rightarrow$ $I_d \uparrow$ for given $(V_g - V_t)$ since $\mu \uparrow$ (but $V_t \uparrow$ so you need higher V_g)
(see below)

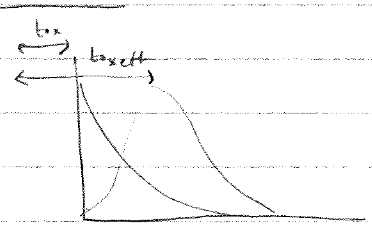
classically:





$\Rightarrow$ as $d \downarrow$, $V_t \downarrow$
since less charge is needed to create the depletion region,
since the depletion region is smaller.

But: For quantum:



So for very small d, $\boxed{V_t \uparrow}$ since $t_{ox_{eff}} \uparrow$

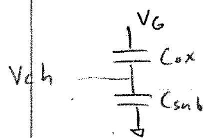
$I_{dsub} \uparrow$ since $C_{ox} \downarrow$  and $\mu \uparrow$

$$I_{d_{sub}} \approx \frac{W}{L} \alpha_e C_{ox} \left(\frac{kT}{q}\right)^2 (n-1)\, e^{q(V_{gs}-V_T)/nkT} \left(1 - e^{-qV_{ds}/kT}\right)$$

Simplified: from Razavi $I_{D_{sub}} = I_0\, e^{(V_{GS}/\zeta V_T)}$ where $V_T = \frac{kT}{q}$ $\quad \zeta > 1$

↑ possibly due to capacitive divider

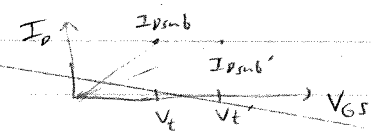
$I_{D_{sub}} \uparrow$ since $C_{ox} \downarrow$ since $t_{ox} \uparrow$



similar to V_{Base}

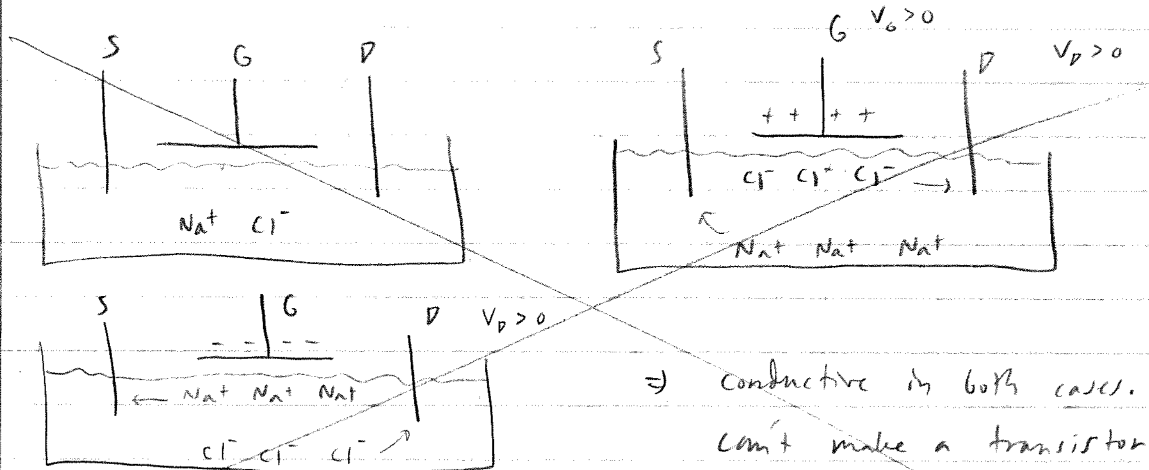
$V_{ch} = \frac{C_{ox}}{C_{sub} + C_{ox}} V_G \downarrow \Rightarrow \zeta \uparrow$

~~Can also think about this as $V_t \uparrow \Rightarrow$ for a given V_{gs}, $I_{D_{sub}}$ should decrease.~~



$I_G \downarrow$ since $t_{ox_{eff}} \uparrow \Rightarrow$ less tunneling.

3.



~~$\Rightarrow$ conductive in both cases. can't make a transistor.~~

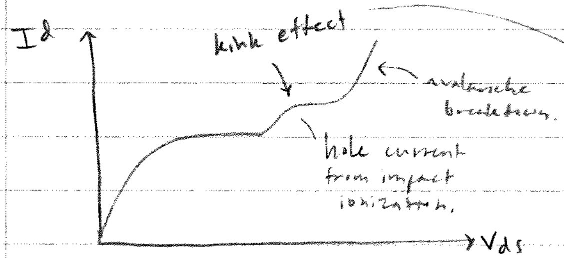
~~Should behave like a conductor with low mobility and zero leakage current.~~

Should still assume solid state. $\Rightarrow$ ions = dopants

carriers have higher mass, higher volume $\Rightarrow$ lower mobility.

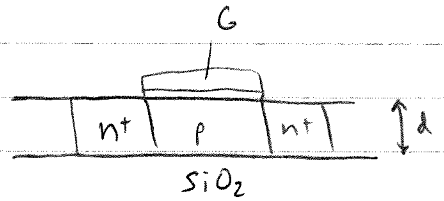
2003 - Nishi

1.

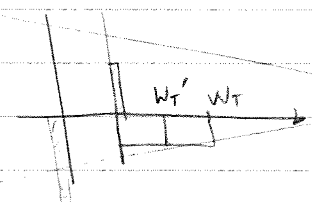


only happens in partially depleted enhancement mode SOI MOSFET operated in strong inversion. (also called floating substrate effect).

SOI prevents CMOS Latchup and punchthrough



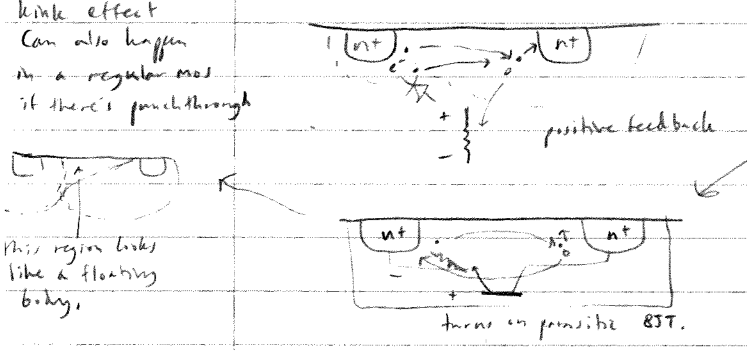
p substrate is floating. Not connected to GND



~~$W_T' = d$ since d is so small.~~

~~⇒ we call the substrate partially depleted.~~

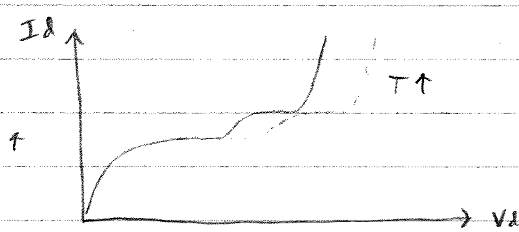
kink effect can also happen in a regular mos if there's punchthrough



for SOI

since substrate is floating, holes generated from impact ionization are collected by the source rather than the bulk GND connection. This cause the hump.

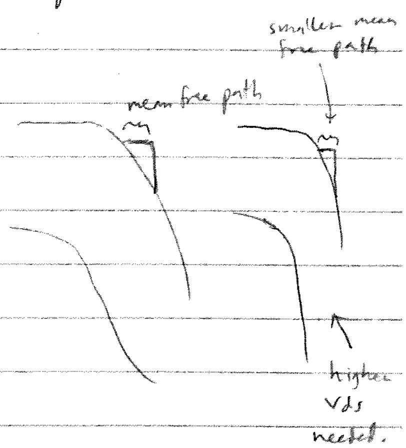
2. T dependence.



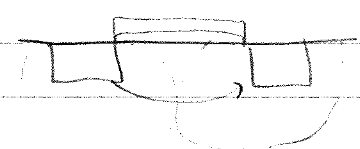
T↑ ⇒ lattice scattering ↑

⇒ mean free path ↓

⇒ need higher Vd for impact ionization.



3. Parasitic BJT :



punch through

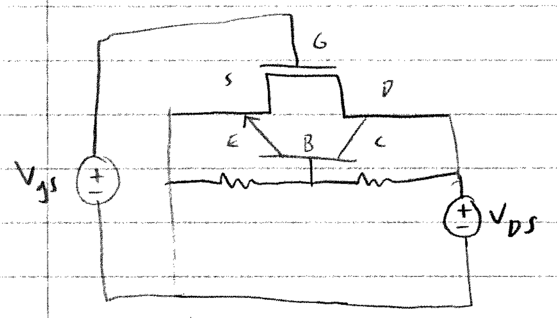
~~⇒ nonideal parasitic BJT~~

~~effective base width is 0.~~

$I_D \propto V_D^2$

⇒ for $V_g = 0$, $I_D \neq 0$

for punchthrough, not parasitic BJT.



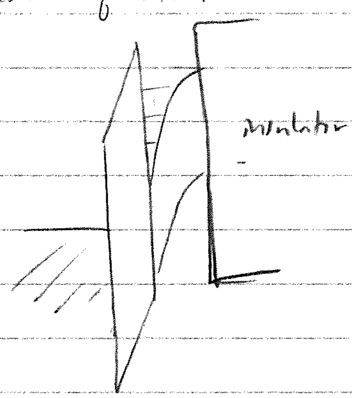
4. How do you prevent punch through?

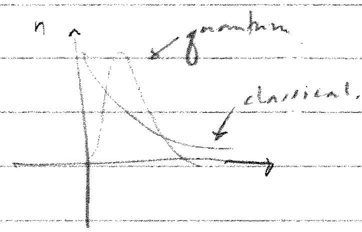
use long channel, heavily dope substrate, use SOI,

5. Size effects: Vds at which hump occurs is smaller.

also as you get very small, there can be tunneling from source to drain.

6. Last question





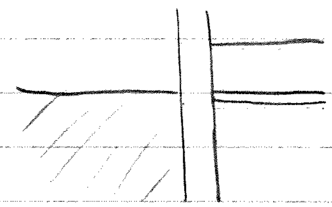
surface quantization / Volume inversion

2008 - Nishi

1.

$V_G = 0$

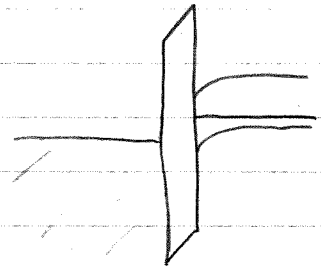
M or p-type



⇒

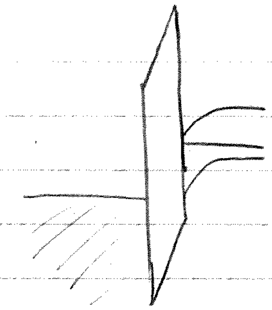
Depletion

$V_G > 0$



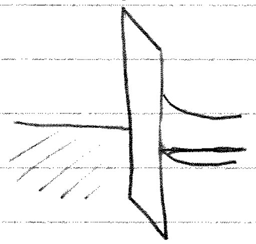
Inversion

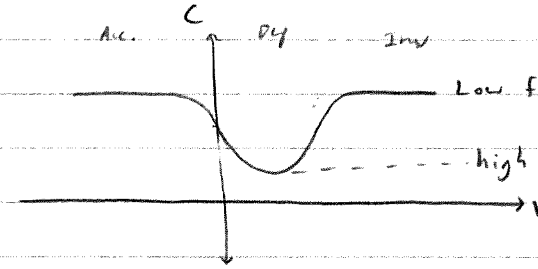
$V_G >> 0$



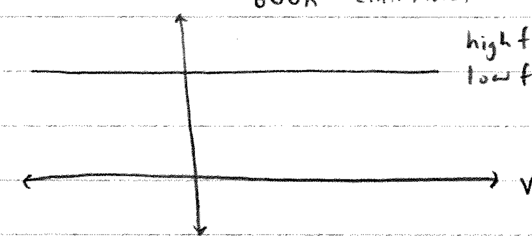
Accumulation

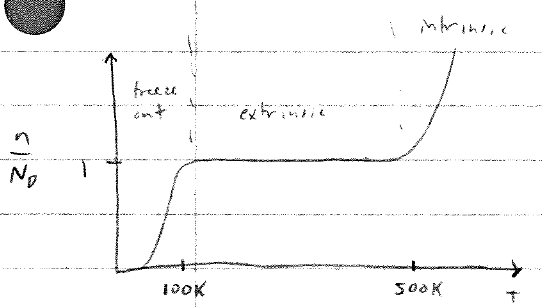
$V_g < 0$

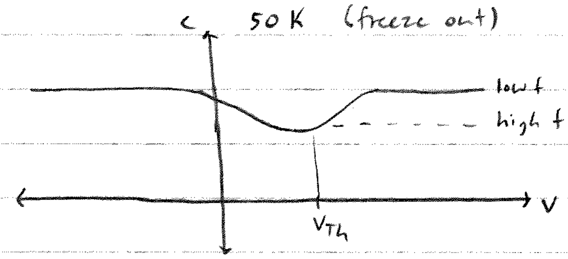




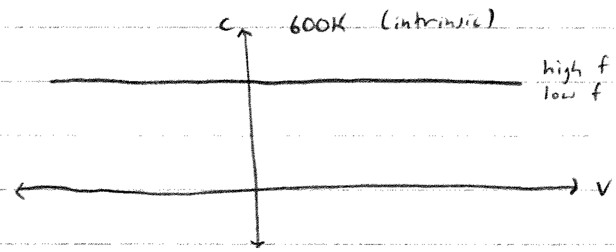
2. a) Highly Doped:

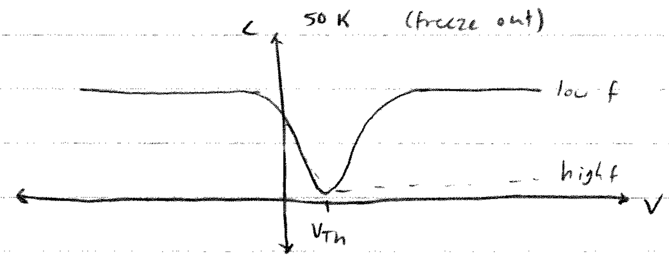




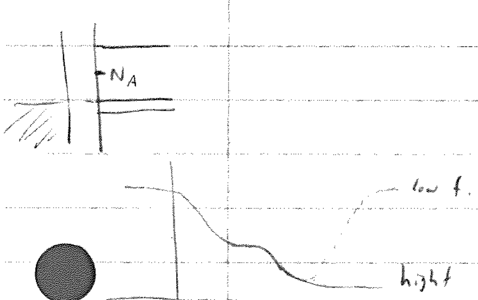


b) Lightly Doped:

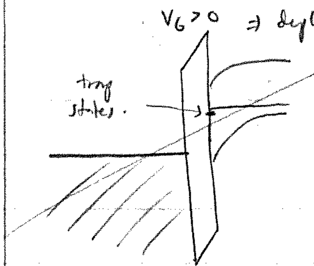


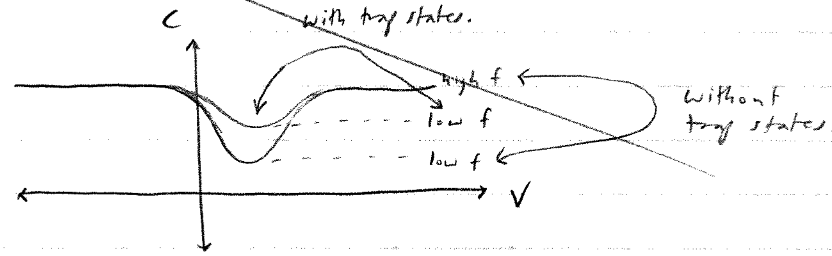


Extra practice:

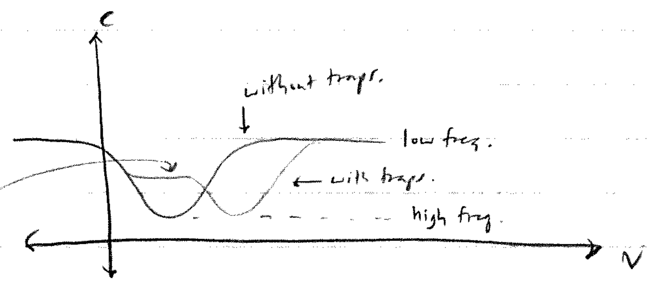


3. ~~Depletion capacitance will increase, since in depletion, these states will fall below the fermi level ⇒ they will fill with charge and contribute to the depletion charge. ⇒ the depletion width will decrease and the depletion capacitance will increase.~~





from EE216 HW



$V_{th} \uparrow$

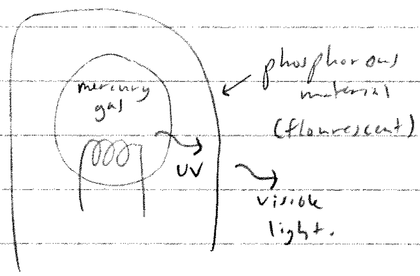
$\phi_s = 2\phi_f$. ← this is only accomplished by band bending in the silicon. ⇒ the max depletion W will be the same with or without traps, so $C_{depl_{min}}$ will be the same with or without traps.

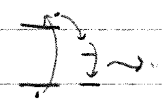
e^-'s that would form inversion layer instead fill the traps. ⇒ $V_{Th} \uparrow$.

1995 - Saraswat

Sensor in the box is a photodiode.

How does a flourescent light work?

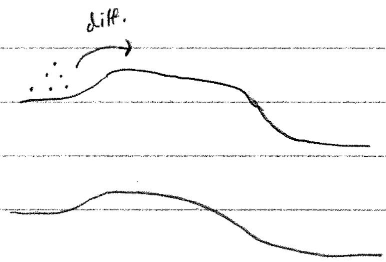




2006 - Nishi

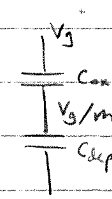
1. MOSFET, low V_g + low V_d, I_{diff} dominates,

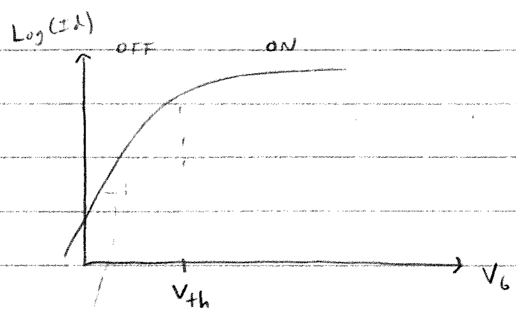
$$I_{drift} = \underbrace{n}_{\text{small, since } V_g \text{ small}} q \underbrace{\vec{E}}_{\text{small since } V_d \text{ small}} \mu \ll I_{diff}$$



$$I_{sub} = \mu_o \frac{W}{L} C_{ox} (V_T)^2 (m-1) \exp\left(\frac{V_g - V_{th}}{m V_T}\right)\left(1 - \exp\left(-\frac{V_D}{V_T}\right)\right)$$

where $V_T = \frac{kT}{q}$ $\quad m = 1 + \frac{C_{depl}}{C_{ox}}$





$$\text{S-factor} = \text{subthreshold slope} = \frac{dV_g}{d(\log(I_d))} = S_t = 2.3 \frac{kT}{q} m = 2.3 \frac{kT}{q}\left(1 + \frac{C_{depl}}{C_{ox}}\right) \quad \frac{V}{\text{decade-Amp}}$$

$\frac{1}{\log(e)}$

↑ holding V_D constant.

$C_{ox} \gg C_{depl}$

$$S_{t_{min}} = 2.3 \frac{kT}{q}(1 + 0) = \frac{60\,mV}{\text{decade}} \text{ at } T = 300K$$

to decrease S-factor, you should decrease t_{ox}, decrease substrate doping, use a high K dielectric.

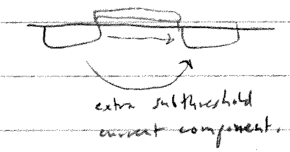
Typical S-factor is $80 - 120 \frac{mV}{\text{decade}}$

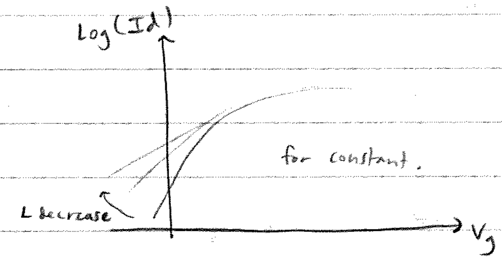
2. Long channel: $T\uparrow$ $S_t\uparrow$ $\quad \varepsilon_{Si}\uparrow$ $S_t\uparrow$

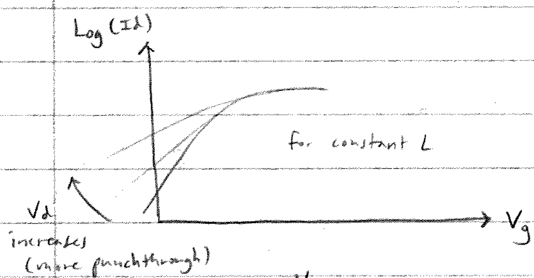
$t_{ox}\downarrow$ $S_t\downarrow$ $\quad \varepsilon_{ox}\uparrow$ $S_t\downarrow$

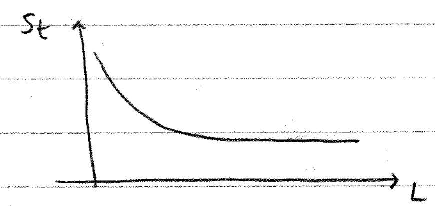
$N_{bulk}\downarrow$ $S_t\downarrow$

Short Channel : S-factor will degrade (increase) if you have punch through









3. $S_t < 60 \frac{mV}{decade}$, what would you question about that?

1) structure of device. (e.g. could be an electro-nano-mechanical relay) or IMOS

2) main carrier transport mechanism (we assume diffusion for our theory) impact ionization or tunneling might allow a better S_t.)

3) temperature of operation.

(if you decrease temperature, S decreases)

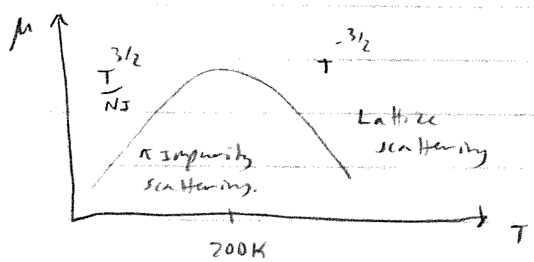
2009 - Nishi

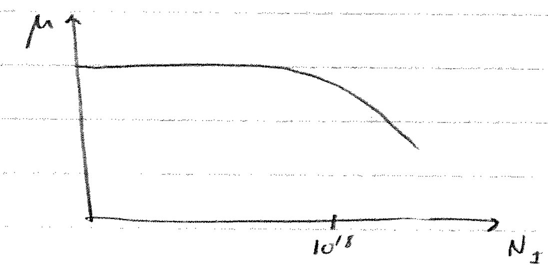
1. $\mu = \frac{q\tau}{m^*}$ τ is mean free time $\tau = \frac{1}{P}$ ← probability of scattering

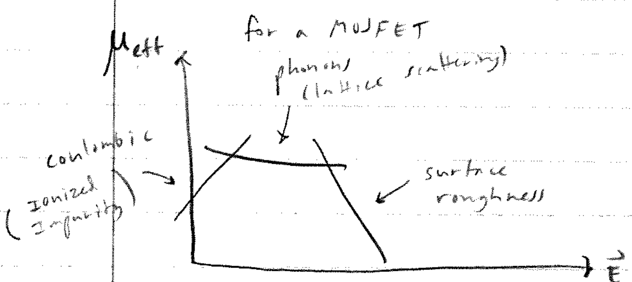
2. $\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \cdots$

$P = P_1 + P_2 + \cdots$ if the mechanisms are independent.

$\Rightarrow \frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \cdots$ if the mechanisms are independent







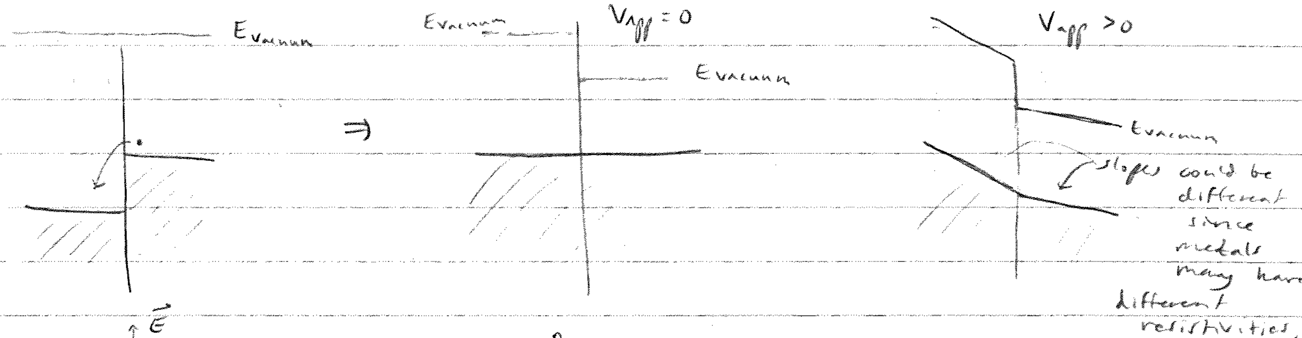
3. If size of the semiconductor is not large compared to the mean free path. Then you have ballistic transport.
So $\mu \neq \frac{q\tau}{m^*}$, since you can't use this statistical model which depends on mean free path. There is a high probability that many electrons will travel through the semiconductor without scattering at all.

If $\vec{E}$ is very high, $v \neq \mu \vec{E}$, $v = v_{sat} = v_{th}$

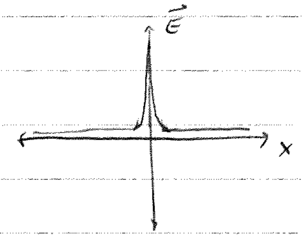
Also if the different scattering mechanisms are not independent

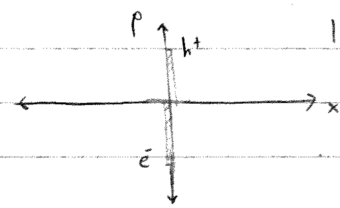
2010 - Nishi

1.

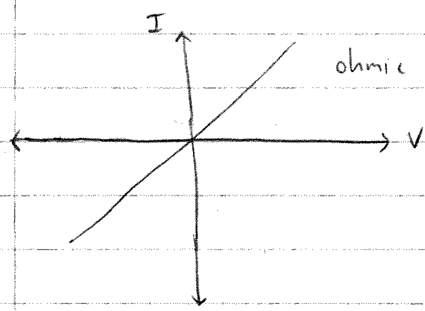


Thomas Fermi Field





localized sheet charge near interface.

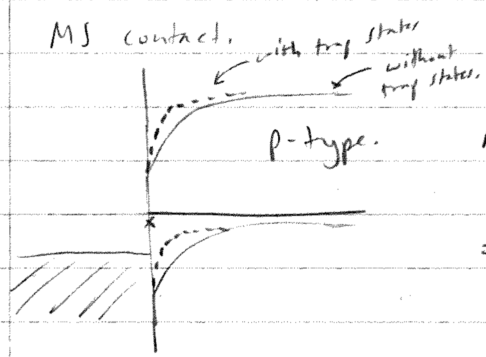


Note: For p-type, only deep trap states that are acceptor-like will have an effect ($W_{dep}\downarrow \Rightarrow C_{dep}\uparrow$, $I_{tunnel}\uparrow$)
For n-type, only deep trap states that are donor-like will have an effect ($W_{dep}\downarrow \Rightarrow C_{dep}\uparrow$, $I_{tunnel}\uparrow$)

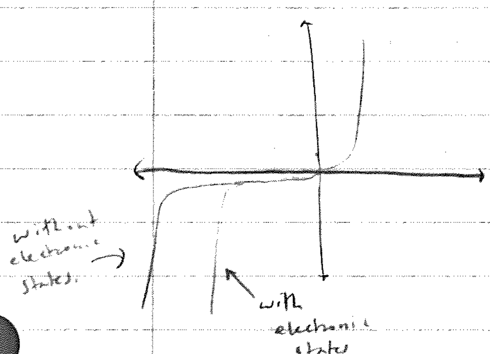
2. MS contact.

(Actually, for MOS-C, both types have an effect. ---

3.



Assuming acceptor-like states at midgap:
states are ionized in depletion.
⇒ decreases depletion width ⇒ more tunneling
⇒ I ↑

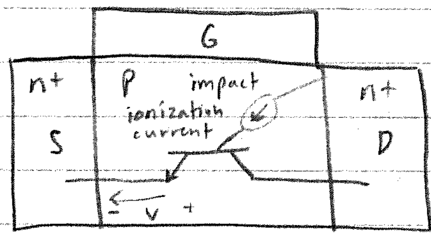


Assuming donor-like states, it doesn't matter.

Assuming donor-like states at midgap:
states are ionized in depletion.
⇒ decreasing depletion width ⇒ more tunneling
⇒ I ↑.

1996 - Saraswat

1. Body of B is floating ⇒ the source junction can become forward biased, turning on a parasitic npn bipolar transistor between source and drain, in addition to the normal MOS transistor. This parasitic BJT can amplify impact ionization current (base current).



(floating body effect)

2. Will this phenomenon take place in a PMOS transistor?

Yes, but the extent will be smaller, because β of a pnp is smaller than β of an npn since $\mu_p < \mu_n$

$$\beta_{DC} = \frac{1}{\frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2} \qquad \beta_{pnp} < \beta_{npn}$$

for npn: $D_E = D_p = \mu_p \frac{kT}{q}$ $\quad D_B = D_n = \mu_n \frac{kT}{q} \Rightarrow \frac{D_E}{D_B} = \frac{\mu_p}{\mu_n} < 1 \Rightarrow \beta$ higher

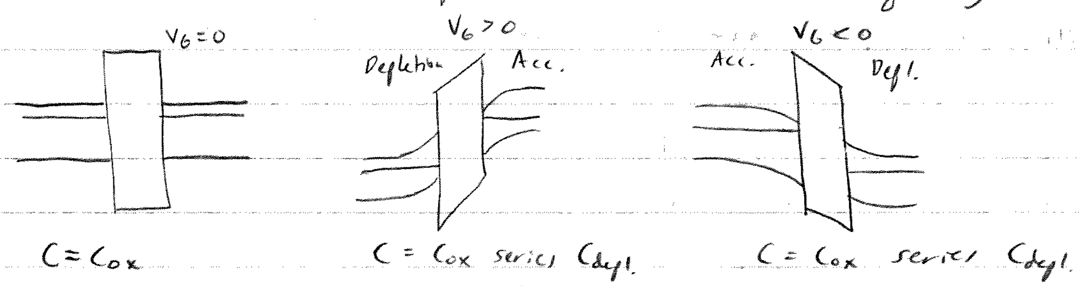
for pnp: $D_E = D_n = \mu_n \frac{kT}{q}$ $\quad D_B = D_p = \mu_p \frac{kT}{q} \Rightarrow \frac{D_E}{D_B} = \frac{\mu_n}{\mu_p} > 1 \Rightarrow \beta$ lower

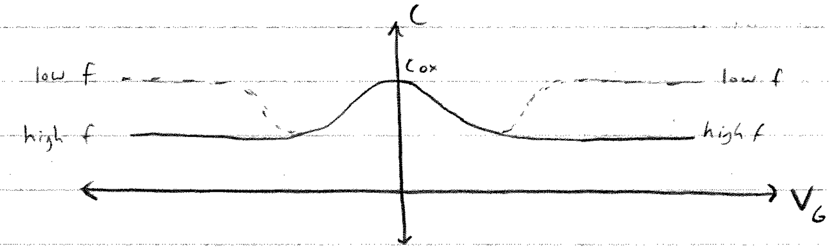
3. The point defects act as generation-recombination centers, thus reducing the lifetime of minority carriers. This reduces the β of the parasitic BJT and hence the excess current disappears.

i.e. $L_B \downarrow \Rightarrow \beta_{DC} \downarrow$

1998 - Saraswat

1. At zero bias, the MOS capacitor will be in the flatband condition with gate oxide capacitance C_{ox} as the total capacitance. As the potential across the device is changed, the gate electrode will also be subjected to band bending similar to the substrate. When the substrate is depleted, the gate will be in accumulation and vice versa. The total capacitance will be C_{ox} in series with the Si depletion capacitance. As a result the CV characteristics for positive bias will be a mirror image of the characteristics for negative bias and will look like a bell shaped curve for high frequency measurement.

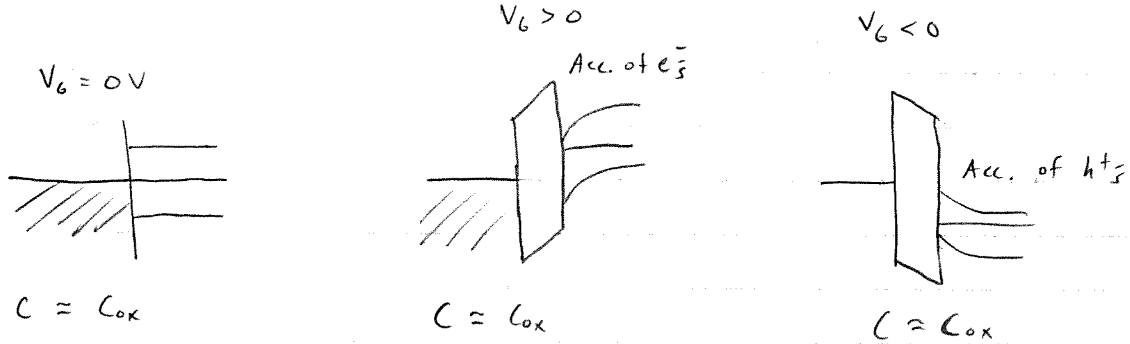


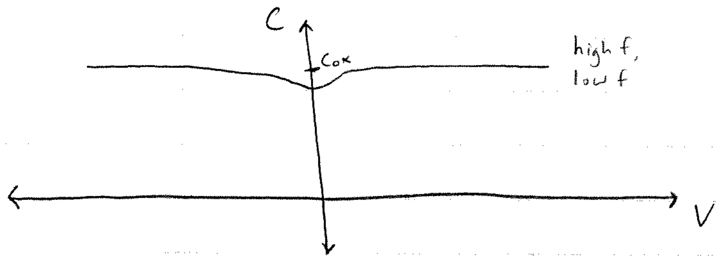


2. At zero bias, the MOS cap will be in flatband condition with gate oxide capacitance, C_{ox} as the total capacitance and Si will be intrinsic with E_F at midgap. As the bias is made +V or −V, the Fermi level at the surface will shift towards the conduction or valence band respectively. In each case, the surface will be accumulated with electrons or holes. Carrying this argument further, the C-V characteristics will look similar to the case of Q1. (I think this just means that it will also be symmetric. In this case, the bell should be upside down.)

Seems that he means dopants are uniformly distributed in band

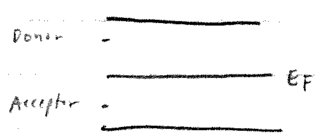
(see next page)



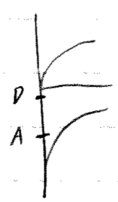


$+ \; N_D = N_A$

if E_F is at midgap ^ donor levels should be symmetric about E_F

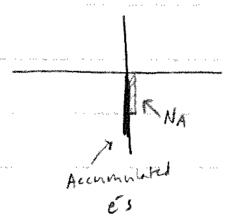


in this case you can have simultaneous accumulation and depletion.



N_D ionized e^-s

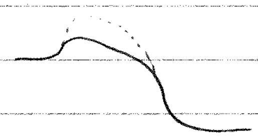
N_A depleted



2000 - Saraswat

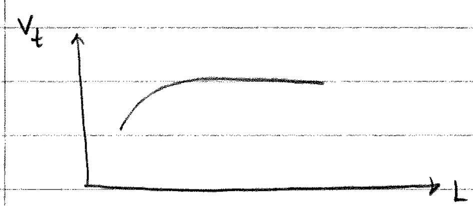
As $L \downarrow$: 500nm $\rightarrow$ 5nm

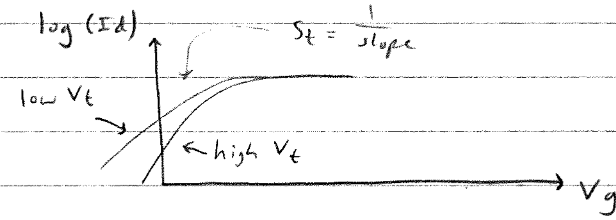
1. DIBL



$\Rightarrow I_{sub} \uparrow$, $V_T \downarrow$

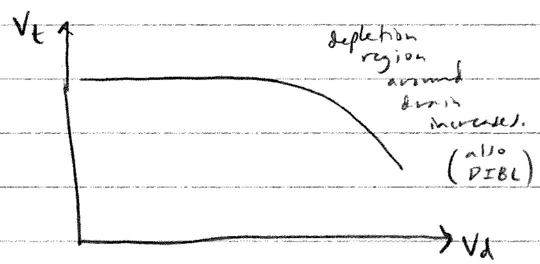
2.

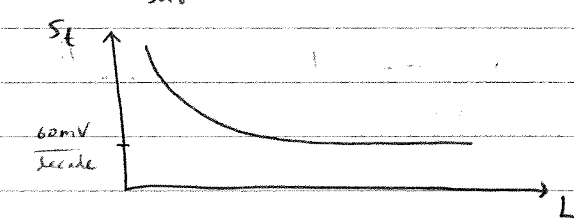




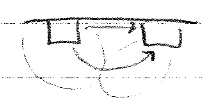
for low V_t S factor increases for $V_t \downarrow$

$\Rightarrow I_{sub} \uparrow$



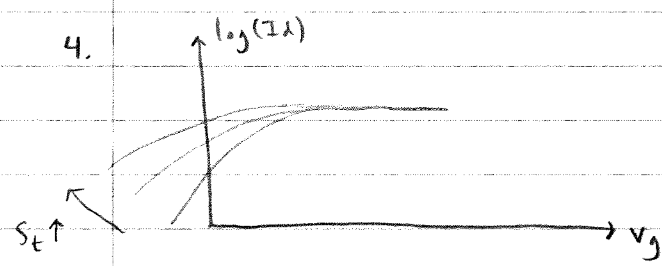


3. punchthrough



$S_t \uparrow$, $I_{sub} \uparrow$, $I_d \propto V_D^2$ at low V_g.

4.



as $S_t \uparrow$, Can't control Id with V_g.

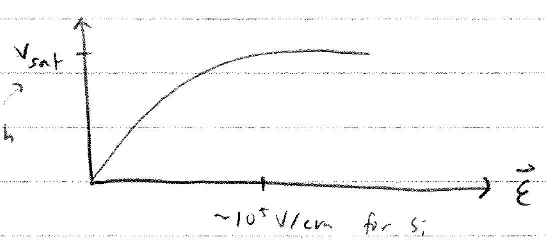
V_g becomes less important. $I_{sub} \uparrow$

average velocity by magnitude

$\frac{1}{2} m V_{th}^2 = \frac{3}{2} kT$

$= V_{th}$

5.



v_{drift} saturates

$I_d \propto (V_g - V_t)^2$ is no longer valid.

now $I_d \propto (V_g - V_t)$

$\Rightarrow$ linear rather than square dependence on $(V_g - V_t)$

6. $R_{channel} \uparrow$ $\quad R = \frac{\rho l}{A} \propto \frac{\rho l}{l^2} \propto \frac{\rho}{l}$

7. $R_{contact} \uparrow$ since area of contact is smaller. $R \propto \rho$

ρ is limited by limit of solid solubility which has already been reached.

mean free path.

8. But as $L\downarrow$, can have ballistic transport if $L < l$

$\Rightarrow$ almost no scattering. $\Rightarrow$ v_d can exceed $v_{sat} = v_{th}$

$\Rightarrow$ higher g_m, I_d.

9. as $L\downarrow$, $\vec{E}\uparrow$ $\Rightarrow$ hot carrier effects + breakdown.

because in practice, people don't scale voltage down as $L\downarrow$ (in order to keep making performance gains).

?? elaborate here.

	K	E_g
SiO_2	4	9
HfO_2	25	6

EOT is ~ 1.2nm in modern processes

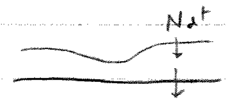
but t_{ox} is ~10nm since it uses high k.

2000 - Saraswat

As $t_{ox} \downarrow$; 100nm → 1nm

1. harder to deposit → uniformity issue + reliability issue. ⇒ easier to breakdown

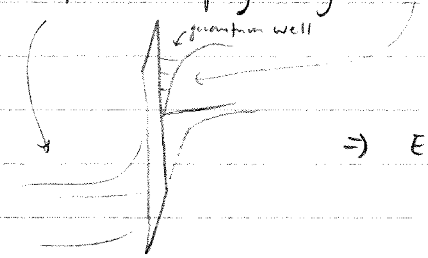
2. dopant penetration from gate into oxide or channel.

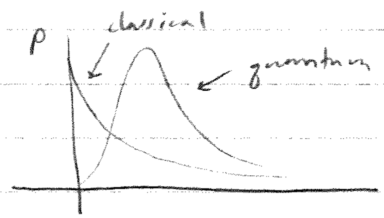


oxide ⇒ $I_{gate} \uparrow$ due to trap assisted tunneling + V_t will shift.

channel ⇒ decrease channel mobility due to increased surface scattering.

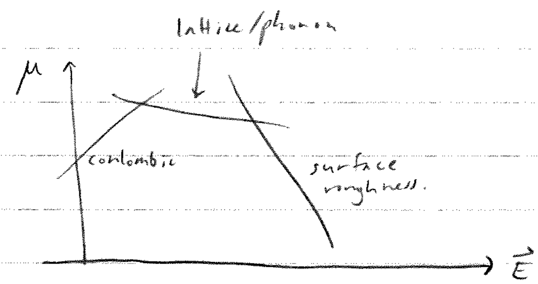
3. Quantization Effect + depletion in Poly-Si gate





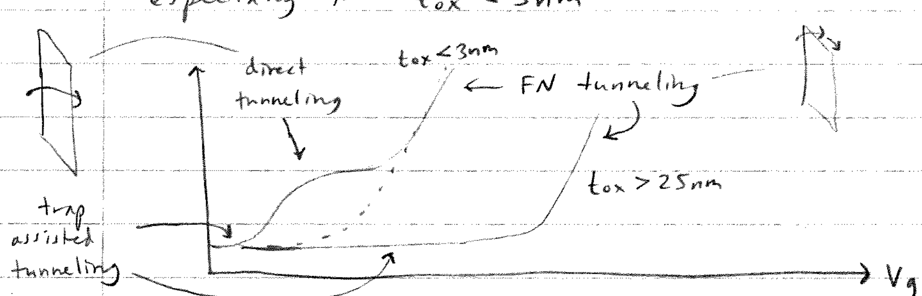
$\Rightarrow$ EOT $\uparrow \Rightarrow C_{ox} \downarrow \Rightarrow I_d \downarrow \Rightarrow g_m \downarrow$

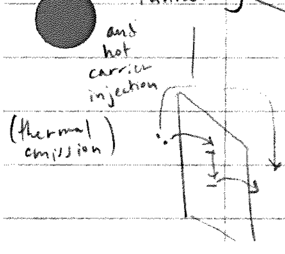
4.



5. direct tunneling, even at small $\vec{E}$ $\quad J_{DT} \propto e^{-t_{ox}}$

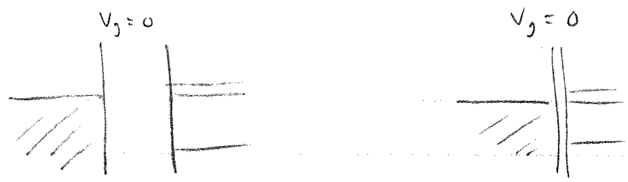
especially for $t_{ox} < 3nm$





2003 - Saraswat

1.

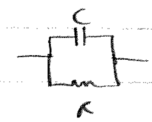


$C_{ox_1} = \frac{\varepsilon_0 A}{t_{ox_1}}$ $\quad$ $C_{ox_2} = \frac{\varepsilon_0 A}{t_{ox_2}} > C_{ox_1}$

$C = \frac{Q}{V}$

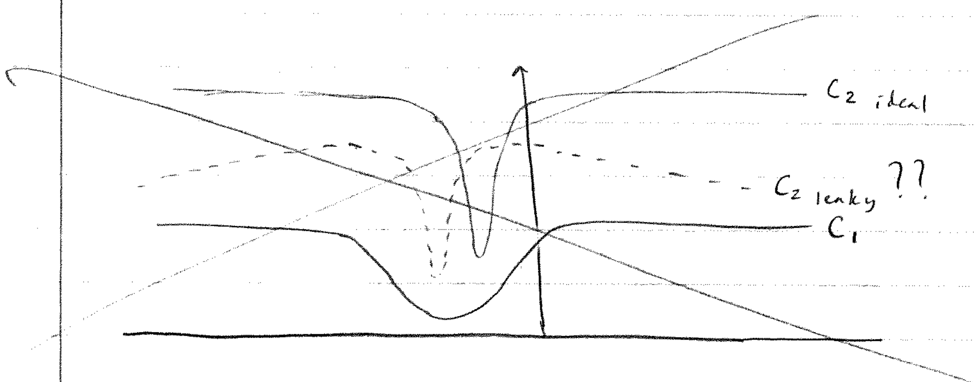
$\Rightarrow$ leakage will take some Q away

$\Rightarrow$ decrease C.

can model as 

$\Rightarrow \cancel{(sC + \frac{1}{R}) = sC'}$

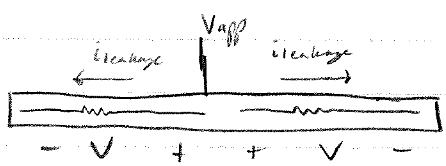
$\Rightarrow \cancel{C' = C + \frac{1}{sR}}$??



see next page.

if $|V| \uparrow$, $J_{tunnel} \uparrow \Rightarrow C \downarrow$

2.

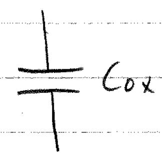


$\Rightarrow$ voltage at the edges will be less than Vapp

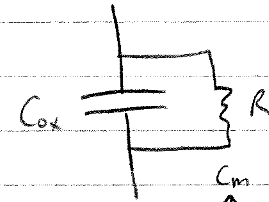
?? $\Rightarrow$ in inversion + accumulation, the capacitance per unit area will increase. compared to if there were no parasitic resistance in the contact.

1.

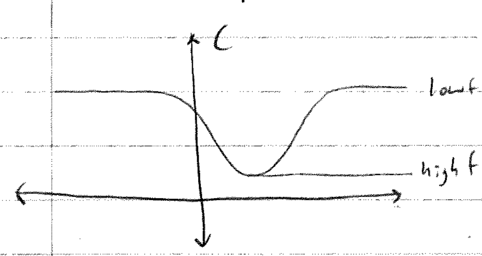
non-leaky

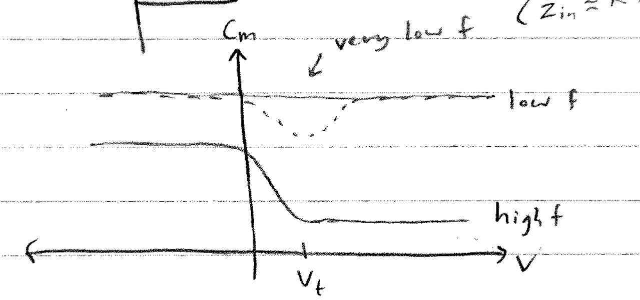


leaky



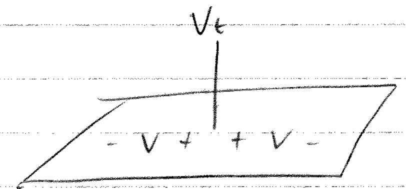
$$Z = \frac{R}{1+sCR} = \frac{1}{sC_m}$$ ← measured C

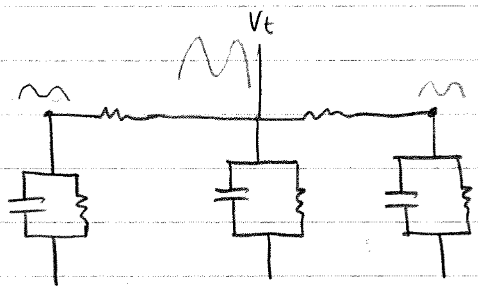


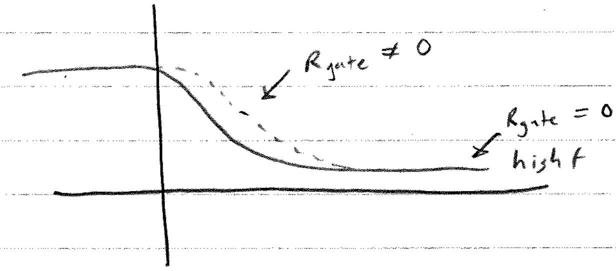


2. Consider high f measurement.

⇒ from 0V to V_t, C_m should increase





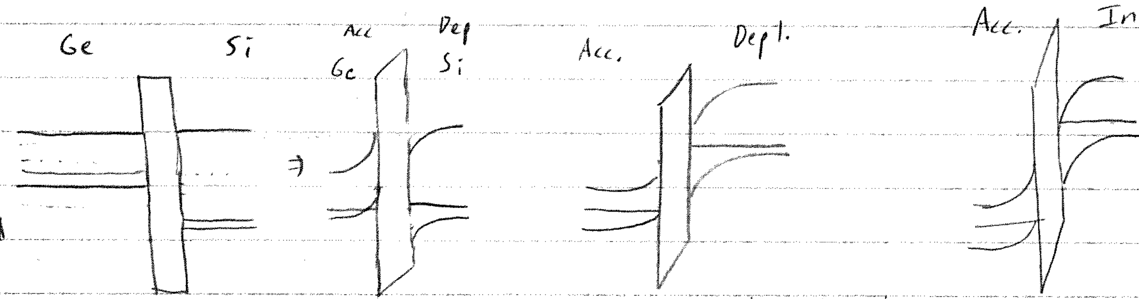


2006 - Saraswat

(K of SiO_2 is 4)

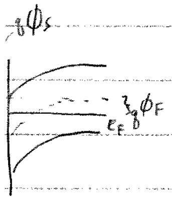
$\chi \approx 4eV$	$\chi \approx 4eV$	$\chi \approx 4eV$
- $K = 12$	- $K = 16$	- $K = 12$
Si - 1.12 eV	Ge - 0.66 eV	GaAs - 1.42 eV
- $n_i = 10^{10} cm^{-3}$	- $n_i = 2 \cdot 10^{13} cm^{-3}$	- $n_i = 2 \cdot 10^{6} cm^{-3}$

Assume $\Phi_{s_{Ge}} = \Phi_{s_{Si}}$



$V_G = 0 \Rightarrow C = C_{ox}$ $\quad$ $V_G > 0 \quad C = \left(\frac{1}{C_{ox}} + \frac{1}{C_{dep_{Si}}}\right)^{-1}$ $\quad$ $V_G >> 0 \quad C = C_{ox}$

$V_G < 0 \Rightarrow C = \left(\frac{1}{C_{ox}} + \frac{1}{C_{dep_{Ge}}}\right)^{-1}$ $\quad$ $V_G << 0 \Rightarrow C = C_{ox}$



$E_{g_{Ge}} < E_{g_{Si}} \Rightarrow$ Ge needs less band bending to reach depletion.

$\Rightarrow V_{T_{Ge}} < V_{T_{Si}}$ (however the workfunction difference shifts our whole CV curve to the left)

$W_{MAX} = \sqrt{\frac{2K\varepsilon_0}{qN_A}(2\phi_F)}$ where $2\phi_F = \phi_s$ total band bending $\Rightarrow W_{MAX_{Si}} > W_{MAX_{Ge}}$

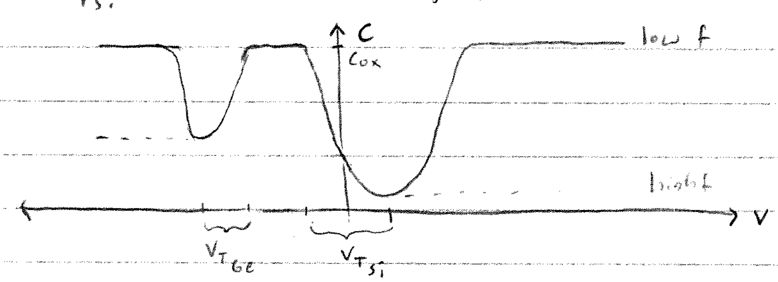
Assuming the difference in ϕ_s is greater than the difference in K.

$E_F - E_i = kT \ln\left(\frac{N_D}{n_i}\right) \Rightarrow$ for $\frac{(E_F-E_i)Si}{(E_F-E_i)Ge} < 1$

$\Phi_F = E_i - E_F$ or $E_F - E_i$

$\Rightarrow C_{dep_{Ge}} > C_{dep_{Si}} \Rightarrow$ without light, our C-V is:

$W_{max} \propto \sqrt{2\phi_F}$

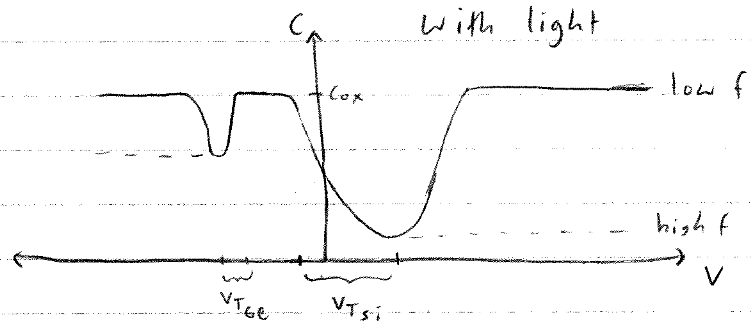


When light shines: $E = \frac{1.24}{\lambda} = \frac{1.24}{1.3} = 0.95\,eV$

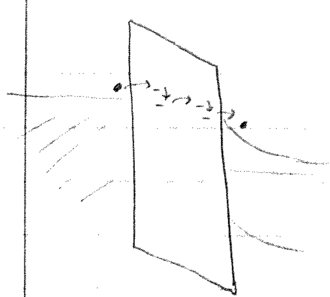
$\Rightarrow E_{G_{Ge}} < E_{ph} < E_{G_{Si}} \Rightarrow \bar{e}\ h^+$ pairs will be generated

in Ge, but not Si. This will increase the depletion capacitance of Ge since the generated $\bar{e}$'s in and around the depletion region will drift towards the SiO_2 interface and create a sheet of charge. This sheet of charge $\Rightarrow W_{dm} \downarrow$ will also lower the threshold voltage $V_{T_{Ge}}$. The Si CV won't change.

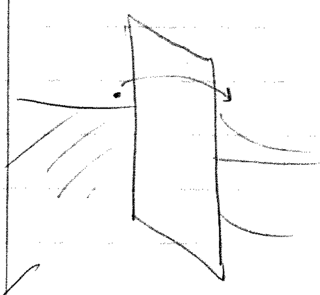
$\Rightarrow$ The CV will look like:



2007 - Saraswat

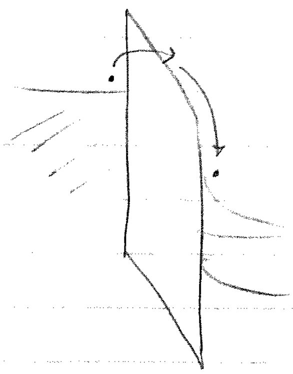


Small V_g Trap assisted tunneling, relatively constant + independent of V_g.



small V_g, for $t_{ox} < 3nm$ also have direct tunneling current

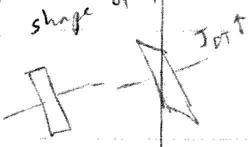
$J_{DT} \propto e^{-t_{ox}}$



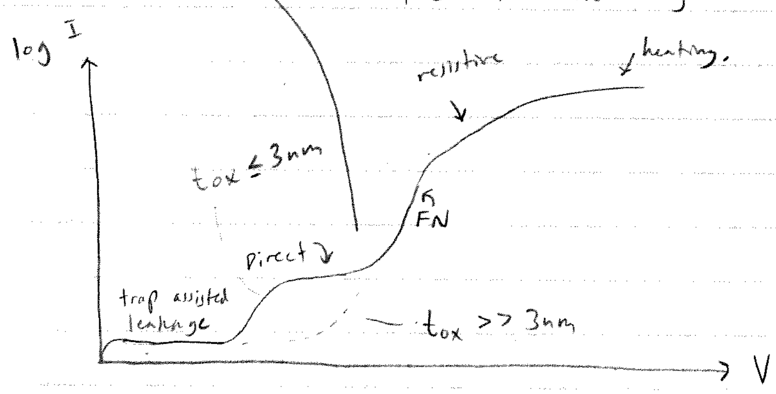
medium V_g, FN tunneling.

$J_{FN} \propto e^{V_g}$

for direct tunneling, J_{DT} depends on both the height and shape of the barrier.



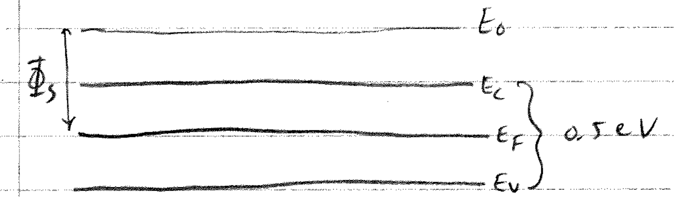
At very high current, there may be destructive breakdown. Current may become limited by the resistance of the Si bulk. At very very high current, the Si bulk may heat up + resistance will increase.



for very high V, there may also be destructive breakdown of the oxide.

2006 - Wong

1.



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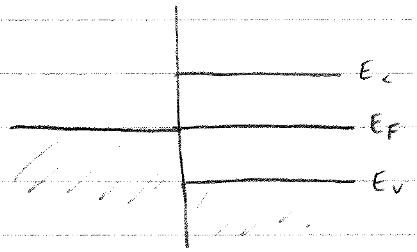

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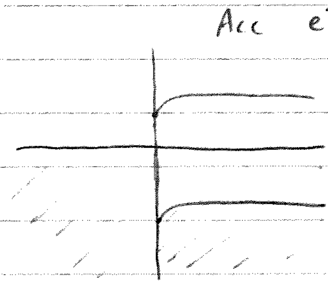

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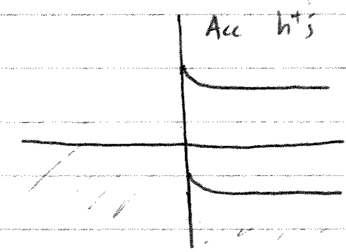
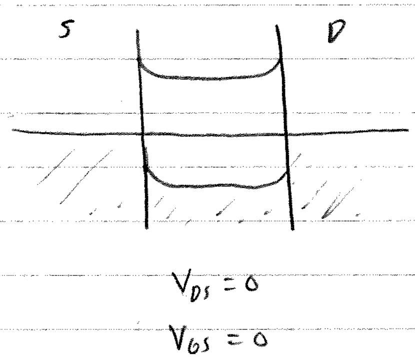
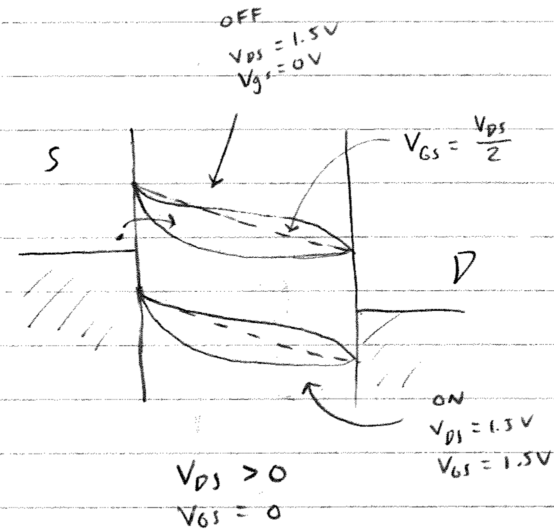
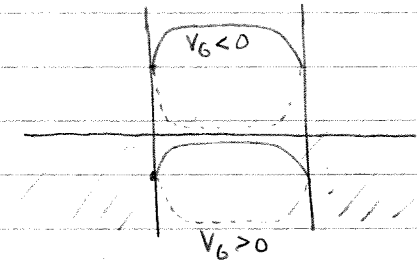


5.





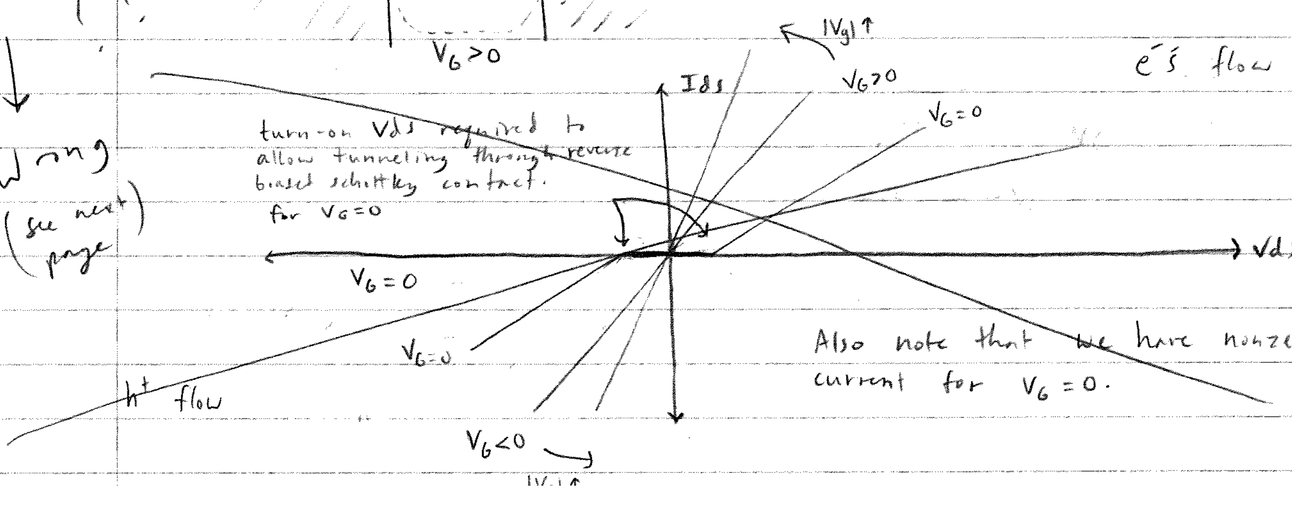


$V_{GS} > 0 \Rightarrow$ n-channel (e^- acc.)

$V_{GS} < 0 \Rightarrow$ p-channel (h^+ acc.)

Maybe ask Wong. ??

Wrong (see next page)



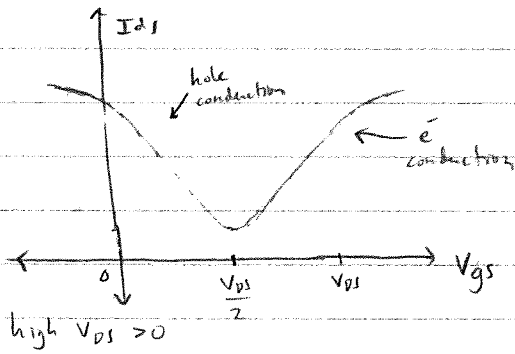
Also note that we have nonzero current for $V_G = 0$.

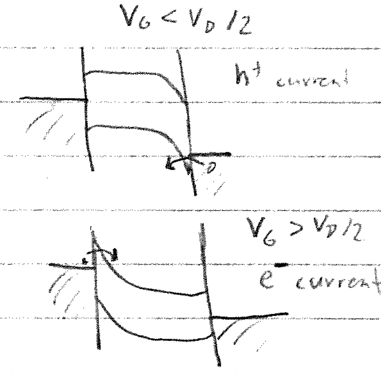
For ambipolar, S-D contacts must be metal. So that the source can supply both e^-'s and h^+'s

(n^+ source can only supply e^-'s. p^+ source can only supply h^+s)

6. Could improve contacts by using a heavily doped (degenerate) n^+ well at the S and D. ⇒ the depletion region will be very narrow, and direct tunneling can occur even at $V_{DS} = 0$. Thus the contacts become ohmic.

7.



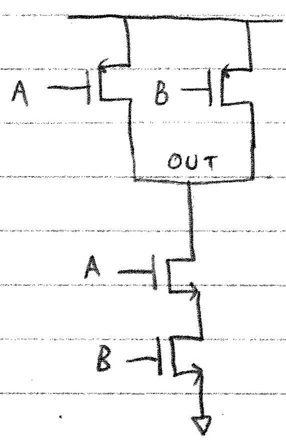


Note that $I_{ds} \neq 0$ $\forall V_g$.
⇒ can't turn transistor OFF.

8. No. These ambipolar transistors will not work well in digital logic, where we'd like to be able to use MOSFETs as switches and thus be able to turn them OFF completely.

(Wow it's says useless since you can't turn it off (it's ambipolar)

9.

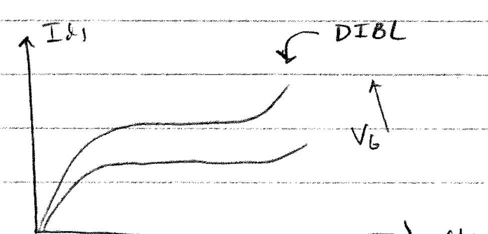


A NAND B

A	B	OUT
1	1	0
1	0	1
0	1	1
0	0	1

No, this will not work if the transistor is made of the ambipolar transistor above.

5.



looks like a MOSFET.

redid answer: see last page of devices.

↓

see EE311 L10-1 slide 50.

Landauer's principle.

2004 - Saraswat

- Ultimate limit to device scaling?

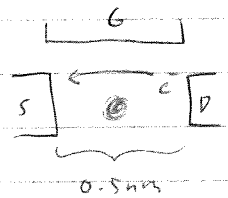
atomic size, because each device element needs at least one atom to exist, E.g. channel, source, drain, gate.

Unit cell length of Si is $\sim 5\text{Å} \approx \boxed{0.5nm}$

- What is the minimum time a device will need to switch its state.

Assume OFF → ON is limited by transit time.

⇒



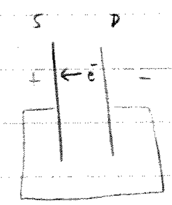
$KE = U \qquad \frac{1}{2}mv^2 = qV$

$\Rightarrow v = \sqrt{\frac{2q}{m} V}$ assume max v is $c = 3 \cdot 10^8 m/s$

$$\Rightarrow \boxed{t_{min} \approx \frac{0.5 \cdot 10^{-9}}{3 \cdot 10^8} = 1.6 \cdot 10^{-18} s}$$

- What will be the corresponding energy requirement.

if we model it as a parallel plate:



$W = \int_S^D F \cdot ds$

$W = q_e \int_S^D \vec{E}\, ds = q_e V$

entropy: S units of J/K joules per kelvin.

$S_i = k \log(\# \text{ of states})$ (k: boltzmann constant) ⇒ assume we have 10 bits,

$\Rightarrow S_i = k \log(2^{10})$ if we switch to a specific desired combo of bits,

$\Rightarrow S_f = k \log(1) = 0 \Rightarrow \Delta S = S_i - S_f = k \log(2^{10}) = \frac{\Delta Q}{T}$ ← change in Energy, ← Temperature

$\Rightarrow \Delta Q = kT \log(2^{10})$

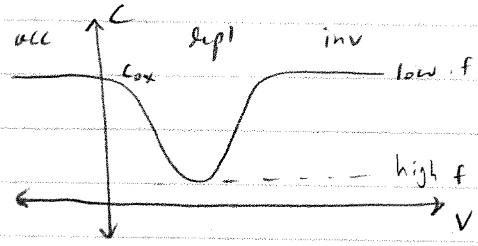
$\Rightarrow$ for 300K $\Delta Q = 3000\, k \log(2)$ (boltzmann.)

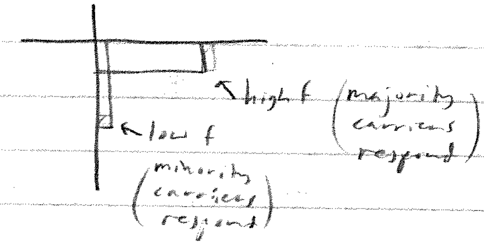
⇒ for a single bit, the fundamental limit is $\boxed{\Delta Q = kT \ln(2)}$

Simon

Pre-1993 - Wong

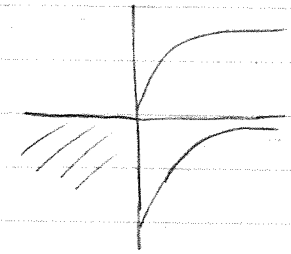
1.



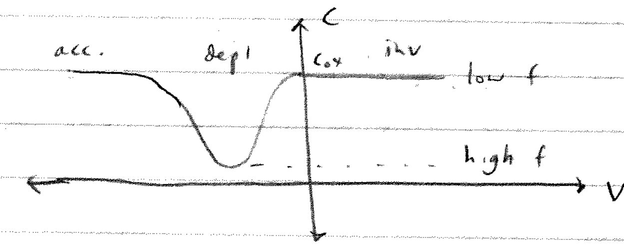


minority carriers come from recombination / generation which are slow processes ⇒ can't respond at high freq.

2.



⇒



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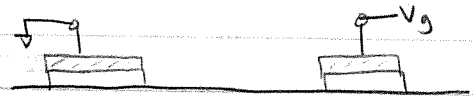
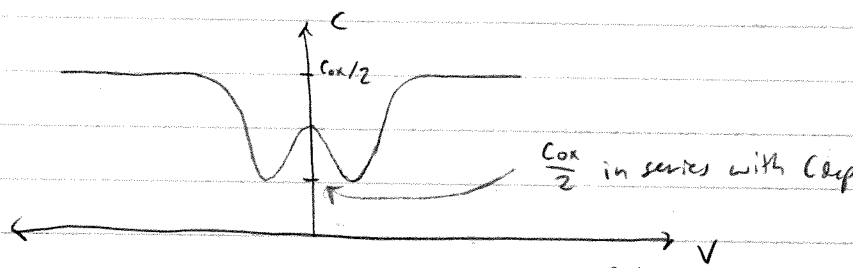
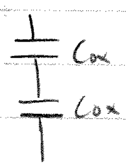
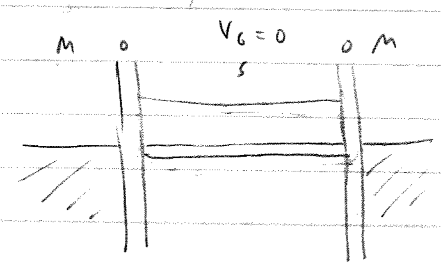
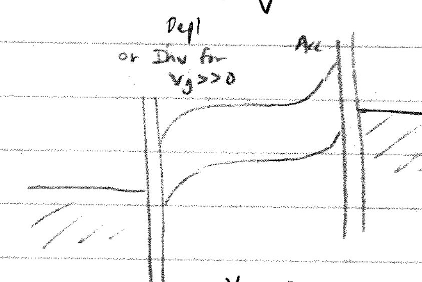


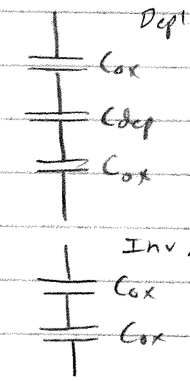




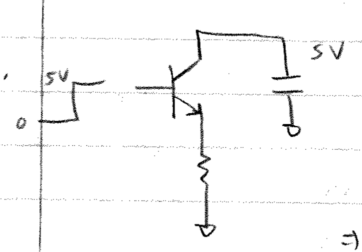




$V_G > 0$



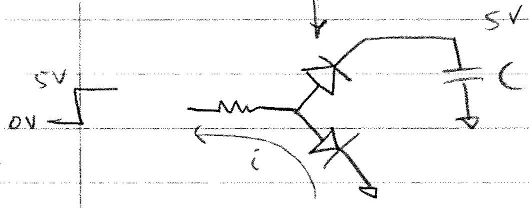
4.



final value of C is 0V.

initially, BJT is on the brink of forward active ⇒ $V_{be} > 0$ $V_{ce} = 0$ ⇒ since $V_{be} > 0.7$, BJT turns on, starts drawing current & discharging C. ⇒ $V_{ce} > 0$ ⇒ BJT enters saturation ⇒ looks resistive. So C will completely discharge.

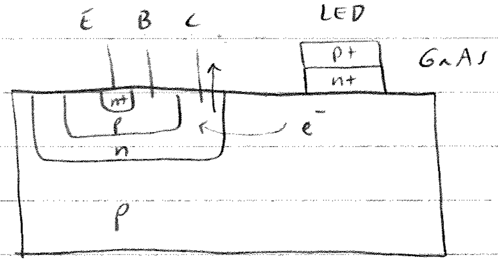
voltage across remains 0.
⇒ stays off. ⇒ doesn't discharge C.



⇒ final value of C is 5V.

Explain why the two diodes don't make a transistor.
In a BJT, the base is very narrow, so e's entering the base from the forward biased E-B diode get swept across and into the collector even though the V_{BC} diode is reverse biased. In the above case, the diodes are completely independent, so the electrons simply leave through the resistor.

4.



- e's simply contribute to collector current (leave through C), since $V_{bc} < 0$ ⇒ b-c diode is reverse biased.

- Assume LED light is incident on the base. What happens in the base? How are the emitter, base and collector currents affected?

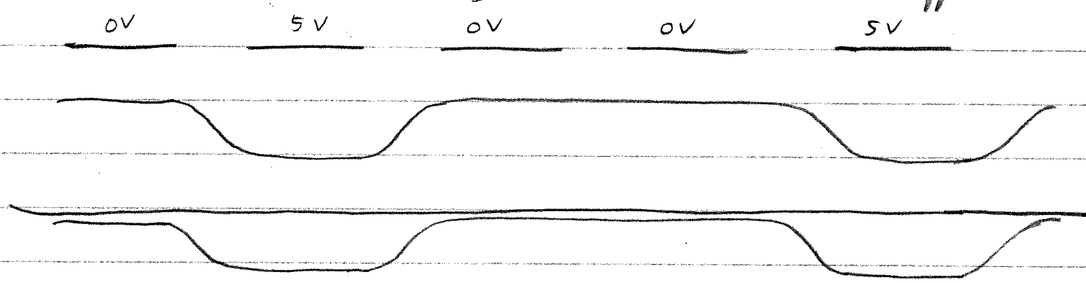
$e^- - h^+$ pairs are generated in the base. e^-'s flow into the collector, and holes flow out of the base.
$\Rightarrow i_b \downarrow \Rightarrow i_c \downarrow$

$$i_b' = i_b - \Delta i_b \qquad i_c' = i_c - \beta \Delta i_b$$

2010 - Wong

1. Do you know how a CCD works?

2. Draw the energy band diagram along the SiO_2/Si interface for the following device structure and applied bias.



3. Assume some $\bar{e}s$ are collected under G2, now we want to move the $\bar{e}s$ to the right under G3, what biases would you apply to the gates G1 to G5?

	G1	G2	G3	G4	G5
ϕ_1	0	5	0	0	5
ϕ_2	0	5	5	0	5
ϕ_3	0	0	5	0	5

4. What are the forces acting on the electrons that move the charges from G2 to G3?

ϕ_2 : Diffusion

ϕ_3 : Drift

5. If we want to move the electrons faster, what would you do? You can change anything you want, including (but not limited to) applied biases, device structure, doping, and the materials of the device.

$v_d = \mu_n \vec{E} \Rightarrow$ increase $\vec{E}$ and μ_n.

$D_n = \mu_n \frac{kT}{q} \Rightarrow$ increase μ_n.

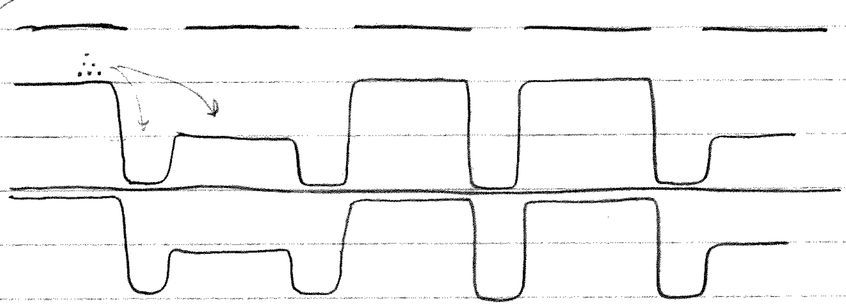
To Increase $\vec{\mathcal{E}}$: Increase applied bias, decrease separation of gates.

To Increase μ_n: Decrease p-type doping ($\mu \propto \frac{T^{3/2}}{N_I}$ for low T).
Use higher mobility material like GaAs.

however, if we do this then $V_T \downarrow$ $\Rightarrow$ we cannot increase applied bias, else we would get inversion.

?, check this.

6. If there is n^+ doping in the p-Si in between the gates, how does the band diagram look like and how does it effect the charge transfer?

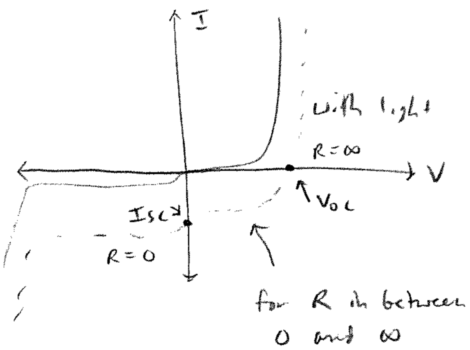


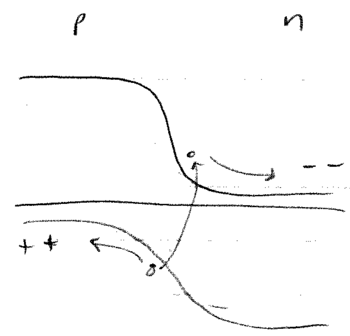
as the charges transfer, some are trapped in the n^+ regions.
$\Rightarrow$ this reduces the transfer efficiency + introduces some noise in the signal.

2008 - Wong

1. P-N junction, has V_{bi}. generated e^- h^+ pairs in depl. region or around depletion region can be separated by V_{bi}.

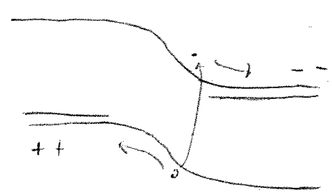
2.





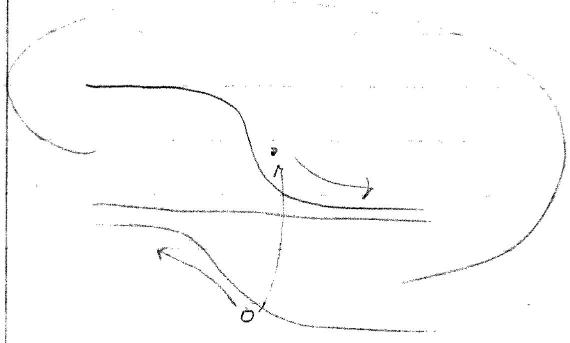
for open circuit

⇒



⇒ PN junction becomes slightly forward biased, fermi levels split into quasi-fermi levels.

for short circuit



$I_{sc} = I_L$

Fermi level is at normal level (from before any light is shone on it)

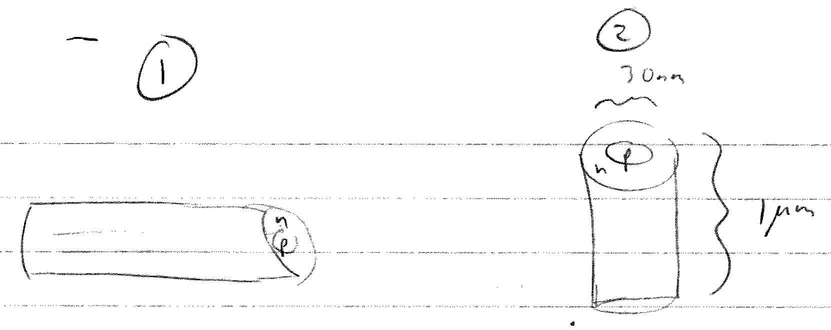
3.

$$(E_F - E_i) = kT \ln\left(\frac{N_D}{n_i}\right) \quad \text{for } n \text{ side}$$

$$(E_i - E_F) = kT \ln\left(\frac{N_A}{n_i}\right) \quad \text{for } p \text{ side.}$$

4. Area of junction → determines area of depletion region. want it to be large ⇒ don't want 3 and 4.

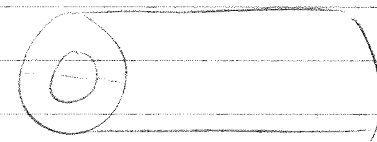
Also 3 + 4 will have more surface recombination as carriers diffuse into junction.



if α is large, would want to use (1), otherwise, should use (2).

(Note: optimal bandgap for solar is ~1.4eV & GaAs is good, but one problem is it has higher absorption coefficient, and has higher surface recombination than Si, since Si grows SiO_2 which passivates the surface very well, whereas GaAs does not).

5. Use PIN structure. OR



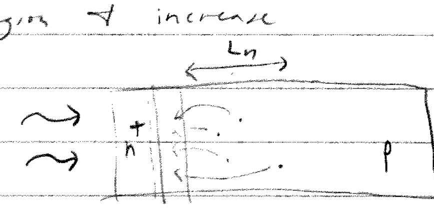
Antireflection coating on top.
Reflection coating on bottom.

also decreases contact resistance. & increases depletion width on other side.

Make outer shell heavily doped and thin to allow light to go through without being absorbed. Make inner shell lightly doped to increase size of depletion region & increase diffusion length.

$\tau = \frac{1}{cN_T}$

trap coefficient



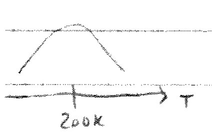
$$L_n = \sqrt{D_n \tau} = \sqrt{\mu_n \frac{kT}{q} \tau}$$

carrier lifetime

$\sigma = q(n\mu_n + p\mu_p)$

Make inner shell p and outer shell n^+, since n^+ is more conductive and $L_n > L_p$ since $\mu_n > \mu_p$ so we want the large region to be p.

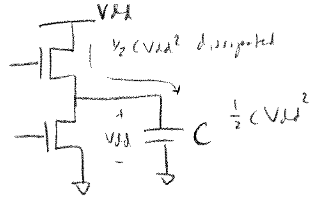
Make the semiconductor very pure (no traps, etc), that way the $L_n \uparrow$.



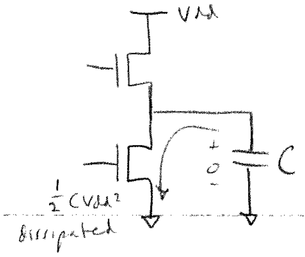
Use a good heat sink since at high T, the semiconductors look more intrinsic, and also $\mu \downarrow \Rightarrow L_n \downarrow \Rightarrow$ efficiency degrades.

doesn't look like a p-n junction anymore. just looks like a photoresistor.

CV_{dd}^2 / cycle



⇒



2005 - Saraswat

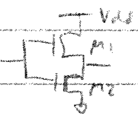
Scaling ⇒ power density ↑

1. a) Static Power Dissipation (I_{sub})

b) Dynamic Power Dissipation (Capacitor on/off) $P = CV_{DD}^2 f$

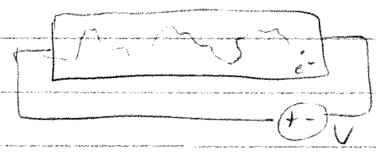
c) oscillator + relay

d) Dynamic Switching when both M1 + M2 are on



If 'constant $\vec{E}$' scaling was done, active power density should remain constant. But this is not what is done. Vdd is not scaled at the same rate the dimensions in order to get higher performance) active power $CV_{dd}^2 f$ increases.

2. Resistive elements will convert electrical energy to thermal kinetic energy.



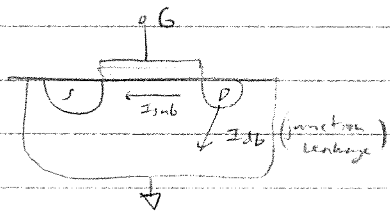
e's accelerated by $\vec{E}$, + collide with lattice.

⇒ phonon scattering. ⇒ heat.

3. Transistors (channel resistance, junction leakage (drain to bulk), gate leakage, I_{sub}), Resistors, Interconnects (Resistors).

Also V_t ↓ ⇒ subthreshold current increases.

4.



dynamic { In the channel resistance, (+ contact resistance) during SAT + Triode.

static { From I_{db}, I_{sub}, and $I_{Gate\ leakage}$ for static case.

$\frac{I_{ON}}{I_{OFF}}$ ↓

and t_{ox} ↓ ⇒ I_{gate} ↑ (90% of I_{OFF} for 65nm)

⇒ static power increases.

I_{gate} can be from tunneling (direct, trap assisted, FN) or hot carrier injection.

5. 1) Increase t_{ox} while using high K, high bandgap dielectric.

⇒ C_{ox} is the same, but t_{ox} is higher ⇒ less gate leakage. but same active performance.

2) μ ↑, L ↓ (decrease channel resistance)

3) grow clean oxide with no traps (decreases I_{gate} from trap assisted tunneling)

4) use SOI to reduce junction leakage (I_{db}) and punch through. ↓ improves isolation.

5) use a good heat sink.

6) decrease contact resistance

7) ☆ Use trigate or finfet. !!

For constant $\vec{E}$ field scaling,

$$\underset{\frac{1}{\alpha}}{C}\ \underset{\frac{1}{\alpha^2}}{V_{dd}^2}\ \underset{\alpha}{f} = P$$

$I \to \frac{1}{\alpha}$

$f = \frac{I}{CV} \to \alpha$

$P \to \frac{1}{\alpha^2}$

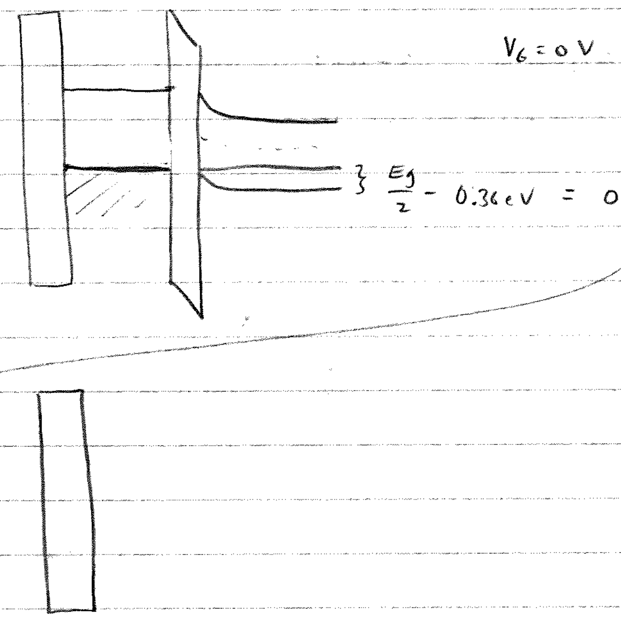
$A \to \frac{1}{\alpha^2}$

⇒ $\frac{P}{A}$ remains constant

↑ power density.

2005 - Wong

1.



$\frac{E_g}{2} - 0.36eV = 0.2eV \Rightarrow V_{FB} = 0.2V$

$E_i - E_F = kT \ln\left(\frac{N_A}{n_i}\right) = 0.026 \ln\left(\frac{10^{16}}{10^{10}}\right) = 0.36eV$ $\quad \phi_F = q(E_i - E_f) = 0.36V$

$V_{th} = 2\phi_F + \frac{k_s}{k_o} t_{ox} \sqrt{\frac{2 q N_A}{k_s \epsilon_0} 2\phi_f}$

$= 2 \cdot 0.36 + \frac{3.9}{11.8} \cdot 10^{-6} cm \sqrt{\frac{2 \cdot 1.6 \cdot 10^{-19} \cdot 10^{16} (0.72)}{3.9 \cdot 8.85 \cdot 10^{-14}}} = 0.92V$

$V_{inv} = V_{FB} + V_{th} = 0.2 + 0.92 = 1.02V$

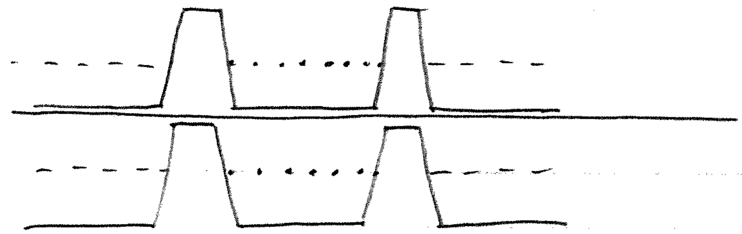
~~rule of thumb: $V_{inv} \approx qE_b \approx 1V$ for Si.~~

Wrong, consider just making a really thick oxide.

(But Φ_s should be less than qE_b for inversion.)

Case A: —
Case B: - - -
Case C: • • • •

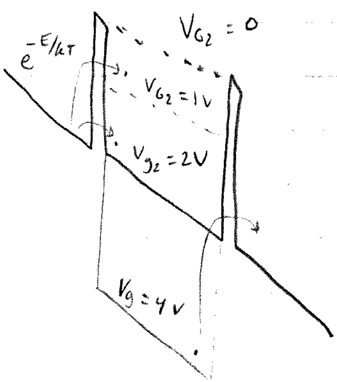
2.



3. - Reduce gate spacing ⇒ fringing field will reduce bump.

- reduce substrate doping
- use n or n+ regions between gates. X not a good idea.
- use heterostructure with lower bandgap material between gates. ← infeasible for fabrication.
- use lower bandgap material (e.g. Ge ~ 0.6 eV)

4.



Wong: There's no barrier as we've drawn. The fringing field from the gates removes the bump.

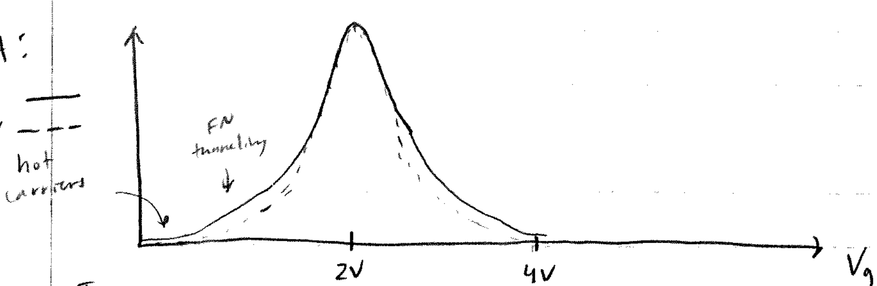
⇒ it should just look like a MOS.

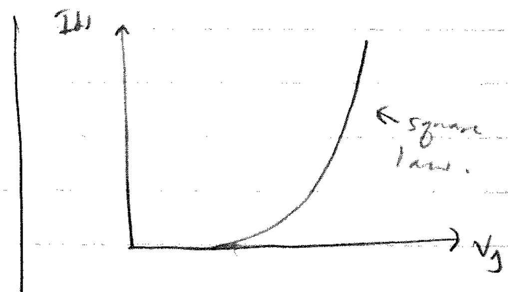
(always ask if you should consider fringing fields)

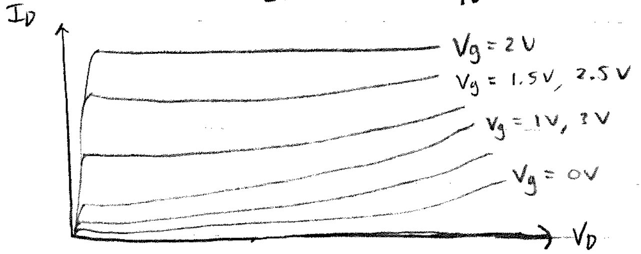
for $V_{G_2} = 0\,V$ - no tunnelling, almost no diffusion.
$V_{G_2} = 2\,V$ - tunnelling. (direct)

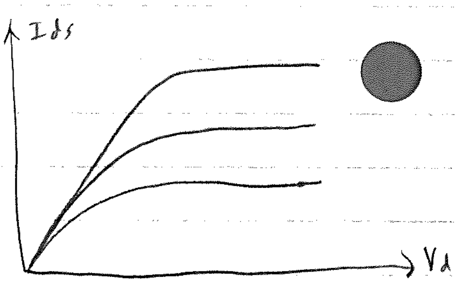
in between, we should see an exponential increase in current since only the carriers with sufficient energy can tunnel + carriers have a boltzmann energy distribution.

CASE A:
$V_D = 2V$ —
$V_D = 50mV$ - - -





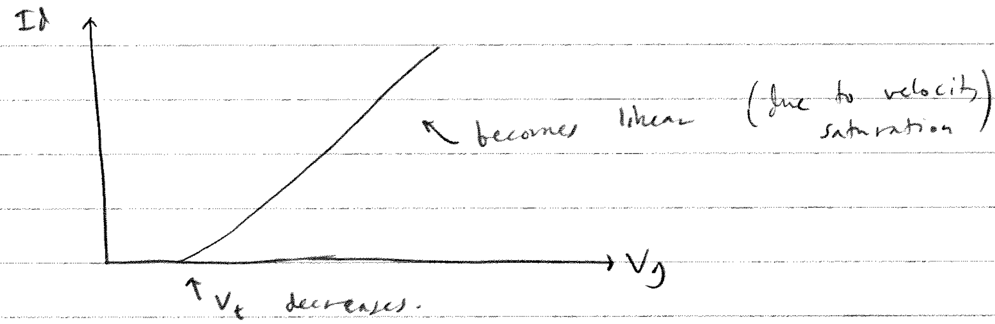


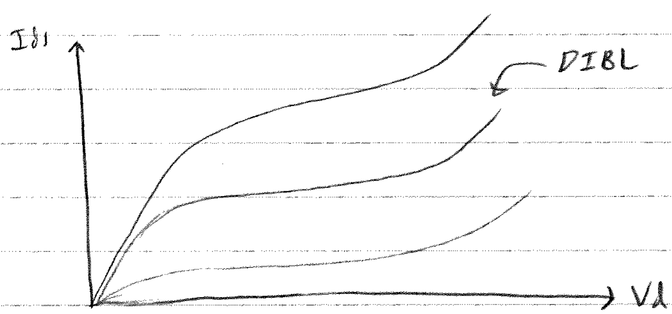


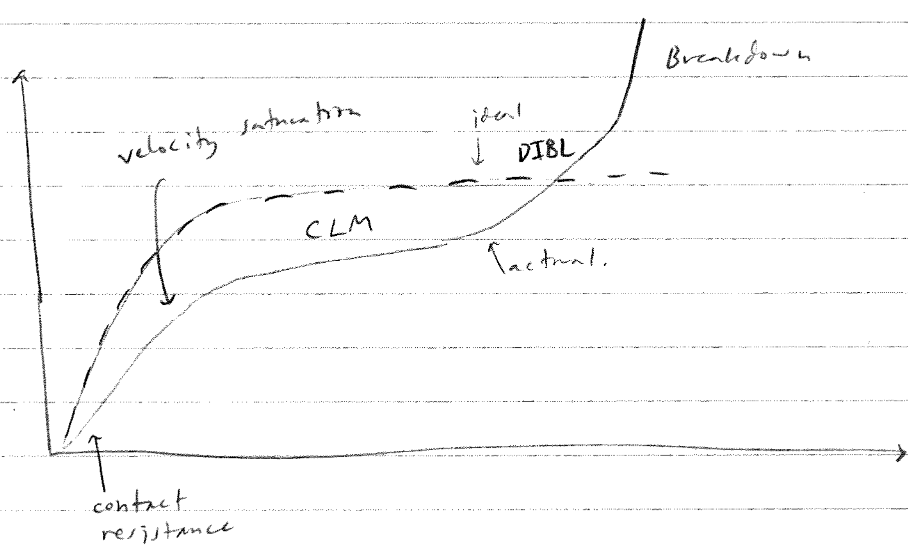
~~?? 5. L↓ ⇒ steeper bending for a given V_D ⇒ more FN tunneling.~~

~~ask Wong.~~

5. Now you should just consider all short channel effects.



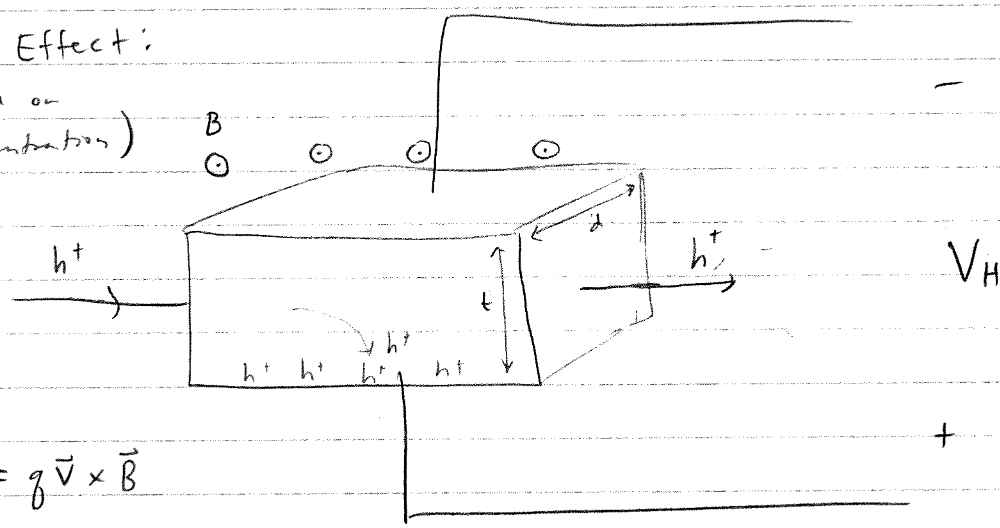




1995 - Wang

Hall Effect:

(can measure μ or carrier concentration)



$F = q\vec{E} = q\vec{v} \times \vec{B}$

$$E_y = \frac{V_H}{t} = |v| \cdot |B| = \frac{B \cdot I}{p\,q\,d \cdot t}$$

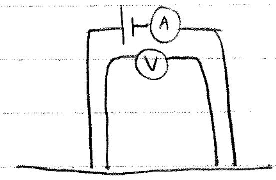
$$\Rightarrow p = \frac{d}{V_H} \cdot \frac{B \cdot I}{q \cdot d \cdot t} = \frac{IB}{qV_H d}$$

$$\Rightarrow p = \frac{IB}{qV_H d} \qquad V_H = \frac{IB}{qpd}$$

$$n = -\frac{IB}{qV_H d} \qquad V_H = -\frac{IB}{qnd}$$

Four Point Probe Measurement: Measures sheet resistance

can determine σ or ρ



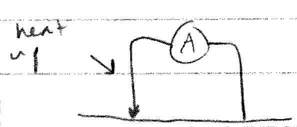
$V = IR$

$R = \frac{V}{I}$

$(\sigma = q\mu_n n + q\mu_p p)$

No current flows through voltmeter.

Hot Point Probe: Measures n-type or p-type.



if p-type, current flows away from hot probe.
if n-type, current flows into hot probe.

carriers always diffuse away from hot probe.

2004 - Saraswat

- Thermodynamic Limit:

$E_b \geq kT \ln 2$

- Quantum Mechanics

$\Delta x \, \Delta p \geq h \Rightarrow x_{min}$ (integration density)

$\Delta E \, \Delta t \geq h \Rightarrow \tau$ (switching speed)

- Ultimate Limit:

after this ¿

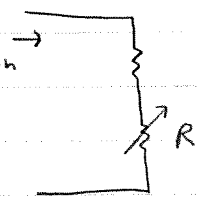
$x_{min} \approx 0.5\,nm \Rightarrow 5 \cdot 10^3$ transistors / cm^3
Switching $t_{min} \sim 0.04\,ps \Rightarrow 25\,THz$
Switching Energy $E_{bit} = 17\,meV$
Power $= 55\,nW/bit$

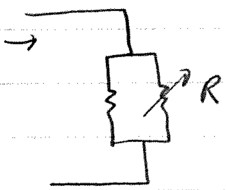
- Total Power Density $\approx 370\,W/cm^2$

Systems

1995 - Boyd

(1)



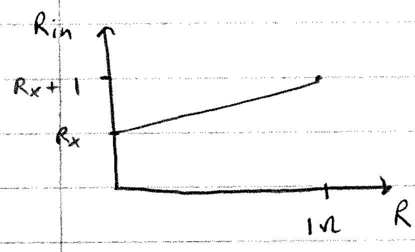


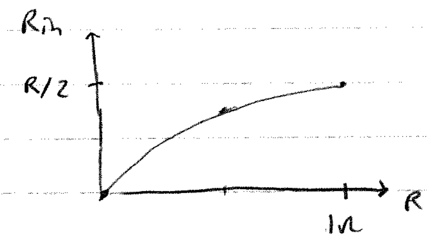
Sketch R_{in} vs R

$0 \le R \le 1\Omega$

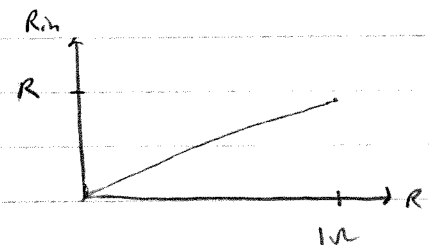
$R_{in} = R_x + R$

$$R_{in} = \frac{R_x \cdot R}{R_x + R}$$

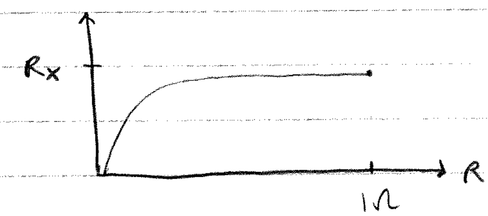




if $R_x = R$

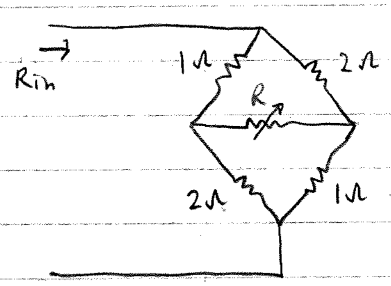


if $R_x >> R$

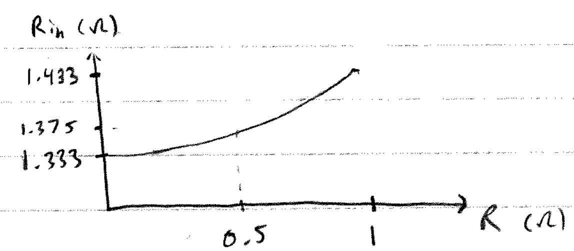


if $R_x << R$

(2)

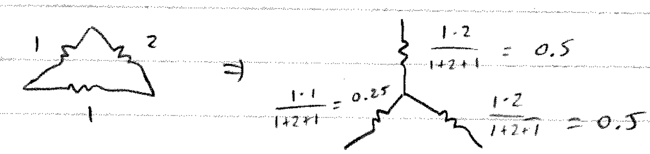


$0 < R < 1\Omega$

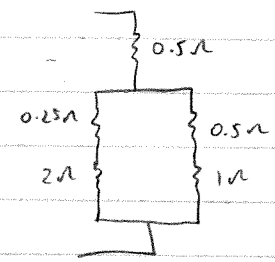


For $R=0$: $\quad R_{in} = \frac{2\cdot 1}{2+1} + \frac{2\cdot 1}{2+1} = \frac{4}{3} = 1.333$

For $R=1$:



$\Rightarrow$

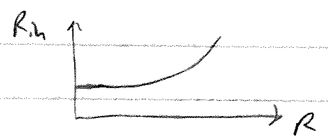


$$\Rightarrow R_{in} = 0.5 + \left(\frac{\frac{9}{4}\cdot\frac{3}{2}}{\frac{9}{4}+\frac{3}{2}}\right) = 0.5 + \frac{27}{8}\cdot\frac{4}{15} = \frac{43}{30} = 1.433\Omega$$

Similarly for $R=0.5$: $\quad R_{in} = 1.375\Omega$

Since $$\frac{\left(R_{in}\big|_{R=0} + R_{in}\big|_{R=1}\right)}{2} = 1.383\Omega > 1.375\Omega$$

The R_{in} vs. R graph has positive second derivative.



as desired.

?! ! related to positive definiteness?

1999 - Boyd

Composite ranking (A) = $1^T M_A w$

where $M_{A_{ij}} = \begin{cases} 1 & \text{if ranked } j\text{th by evaluator } i \\ 0 & \text{if not ranked } j\text{th by evaluator } i \end{cases}$

$\boxed{\text{Yes}}$ - This could happen if A, B, C, and D all recieve the same score from every evaluator.

i.e. A receives 10 #1 rankings.

$$M_A = \begin{bmatrix} 1&0&0&0\\ 1&0&0&0\\ 1&0&0&0\\ 1&0&0&0\\ 1&0&0&0\\ 1&0&0&0\\ 1&0&0&0\\ 1&0&0&0\\ 1&0&0&0\\ 1&0&0&0 \end{bmatrix}$$ etc. for B, C, D.

This assumes weight (1) < weight (2) < weight (3) < weight (4)

$\Rightarrow CR_A < CR_B < CR_C < CR_D$ No matter what.

OR, this could also happen if 2 (e.g. A + B) both recieve 5 1sts and 5 2nds and the other 2 (e.g. C + D) both receive 5 3rds and 5 4ths.

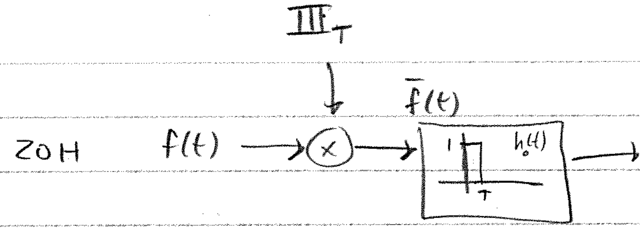
$\Rightarrow$ No matter what $CR_A = CR_B < CR_C = CR_D$

Now solve more mathematically.

Hint: $\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ R_{21} & R_{22} & R_{23} & R_{24} \\ R_{31} & R_{32} & R_{33} & R_{34} \\ R_{41} & R_{42} & R_{43} & R_{44} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}$ $0 < w_1 < w_2 < w_3 < w_4$

when R_{11} = # of times A ranked 1st
R_{12} = # of times A ranked 2nd
R_{21} = # of times B ranked 1st.

$\frac{1}{T} = 2B$



$$Ш_p(x) = \sum_{k=-\infty}^{\infty} \delta(x - kp)$$

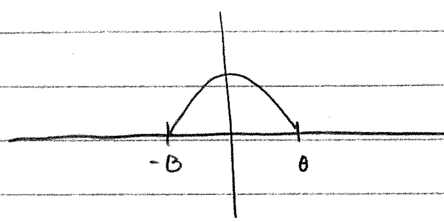
$$f(x)\, Ш_p(x) = \sum_{k=-\infty}^{\infty} f(x)\, \delta(x - kp) = \sum_{k=-\infty}^{\infty} f(kp)\, \delta(x - kp)$$

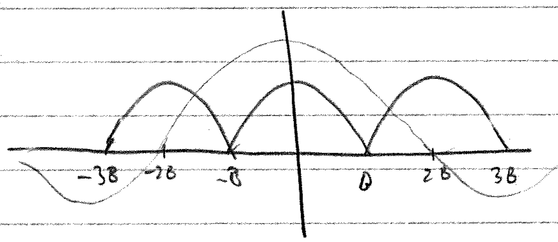
$\Rightarrow$ p is spacing of samples.

$$\mathcal{F}g = \Pi_p (\mathcal{F}g * Ш_p) \qquad p = 2B$$

$$g(t) = \sum_{k=-\infty}^{\infty} g\left(\frac{k}{p}\right) \operatorname{sinc}\left(p\left(x - \frac{k}{p}\right)\right)$$

$$h_0(t) = \Pi_T(t - T/2) \Leftrightarrow H(s) = e^{-2\pi i s T/2}\, T \operatorname{sinc}(Ts)$$

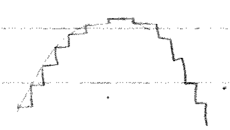




1993 - Nishimura

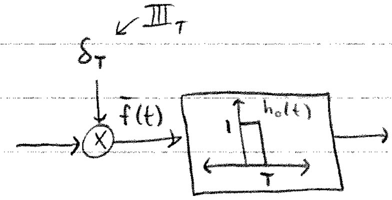
1. Increase the frequency of the sinewave from the function generator. You should hear the pitch increasing. Eventually, the pitch will suddenly drop due to aliasing. This should be roughly the Nyquist frequency $\Rightarrow f_s = 2f_w$ where f_w is the frequency of the waveform generator at this point.

2. 10 Hz is very low. But if this tone is sampled, we should also get a replica around f_s $\Rightarrow$ we probably hear this higher tone.



* Dominant tone is at sampling frequency.

$\frac{1}{T} = 2B$



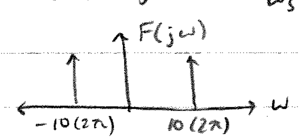
ZOH: $f(t)$

$$H(j\omega) = e^{-j\pi(\omega/\omega_s)} \operatorname{sinc}\left(\frac{\omega}{\omega_s}\right)$$

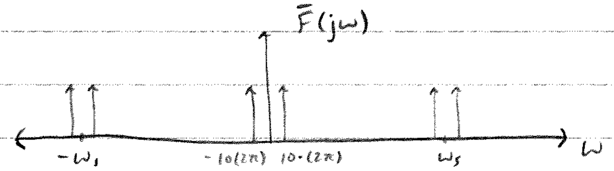
where $\omega_s = \frac{2\pi}{T}$ sampling frequency.

$$\Rightarrow \bar{F}(j\omega) = \frac{1}{2\pi} F(j\omega) * \frac{2\pi}{T} \delta_{\frac{2\pi}{T}}(\omega) = \frac{\omega_s}{2\pi} F(j\omega) * \delta_{\omega_s}(\omega)$$

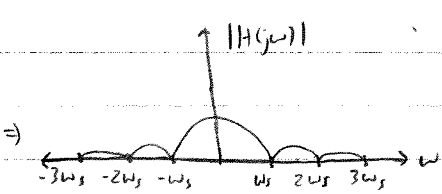
$\Rightarrow$ let $f(t) = \cos(2\pi(10)t)$



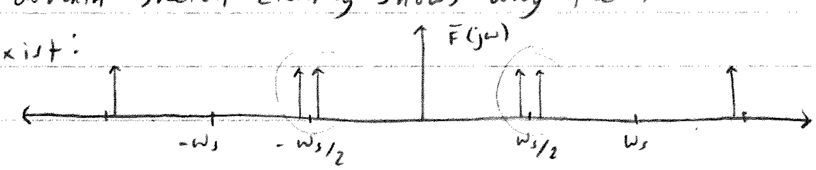
$\Rightarrow$



3. $$|H(j\omega)| = |e^{-j\pi(\omega/\omega_s)}|\,\left|\operatorname{sinc}\left(\frac{\omega}{\omega_s}\right)\right| = \left|\operatorname{sinc}\left(\frac{\omega}{\omega_s}\right)\right| \Rightarrow$$



4. The wavering nature of the output is due to the beating of two frequencies. Frequency domain sketch clearly shows why the two frequency components exist:



1996 - Nishimura

~~Since the frame rate of the Video camera or TV is below nyquist for the fan rotation frequency, we will see aliasing.~~

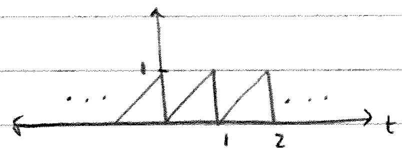
1. Given a TV frame rate of 30 HZ + the fact that you see 3 markers appearing stationary, the fan is rotating at 10 Hz, one third of the sampling rate.

2. Yes, there are other rotational speeds that would give a video identical to what were just seen.

   $(10+30n)$ Hz rotations would give the same video, frame by frame, $n \in \mathbb{Z}$. Note too that a 20 Hz rotation would appear to be the same, but not on a frame by frame basis (i.e. we assume that you can tell the individual fan blades apart).

3. The fan appears to be nearly stationary since there are 3 fan blades and the rotation rate is one-third the sampling rate. If there were 2 fan blades and the rotation rate was one half the sampling rate, it would also appear stationary.

1999 - Osgood



F.S. $D_n = \frac{1}{T}\int_{\tau}^{\tau+T} f(t)\, e^{-j\omega_0 n t}\,dt = \int_0^1 t\, e^{-j2\pi n t}\,dt$

Integrate by parts: $u = t \quad dv = e^{-j2\pi n t}\,dt \quad v = \frac{j}{2\pi n} e^{-j2\pi n t}$

$$\Rightarrow \frac{j}{2\pi n} t e^{-j2\pi n t}\Big]_0^1 - \frac{j}{2\pi n}\int_0^1 e^{-j2\pi n t}\,dt = \frac{j}{2\pi n} e^{-j2\pi n} + \frac{1}{4\pi^2 n^2}\left(e^{-j2\pi n t}\Big]_0^1\right)$$

$$= \frac{j}{2\pi n} + \frac{1}{4\pi^2 n^2}\left(\cancel{e^{-j2\pi n}}^{\,1} - 1\right) = \frac{j}{2\pi n}$$

Parsevals Theorem: $\lim_{N\to\infty} \sum_{n=-N}^{N} |D_n|^2 = \frac{1}{T}\int_{t_0}^{t_0+T} |f(t)|^2\,dt$

$D_0 = \int_0^1 f(t)\,dt = \frac{1}{2} \quad \Rightarrow \quad D_n = \begin{cases} \frac{1}{2}, & n = 0 \\ \frac{j}{2\pi n}, & \text{otherwise} \end{cases}$

$$\Rightarrow \sum_{n=-\infty}^{\infty} |D_n|^2 = D_0^2 + 2\sum_{n=1}^{\infty} |D_n|^2 = \int_0^1 t^2\,dt = \frac{1}{3}t^3\Big]_0^1 = \frac{1}{3}$$

$$\Rightarrow 2\sum_{n=1}^{\infty}\left(\frac{1}{2\pi n}\right)^2 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{4\pi^2}{24}$$

$$\Rightarrow \boxed{\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}} \quad ✓$$

Fourier Transform exists for any energy signal and many power signals.

$E_x = \lim_{t\to\infty} \int_{-T}^{T} |x(t)|^2\,dt$ Energy Signal: $0 < E_x < \infty$, $P_x = 0$

$P_x = \lim_{t\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2\,dt$ Power Signal: $0 < P_x < \infty$, $E_x = \infty$

Sawtooth is a power signal. Doesn't have a F.T. But it is periodic, so it definitely has a F.S.

1994 - Nishimura

1. $\cos(\omega t) \Leftrightarrow \pi\left(\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\right) \qquad \angle F(j\omega) = 0$

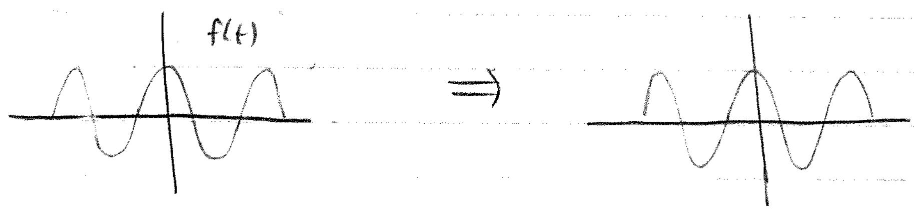
$\Rightarrow \quad g(t) = f(t)$

$\angle G(j\omega) = -\angle F(j\omega) \Rightarrow \tan\left(\frac{\mathrm{Im}(G)}{\mathrm{Re}(G)}\right) = -\tan\left(\frac{\mathrm{Im}(F)}{\mathrm{Re}(F)}\right)$

$\Rightarrow \mathrm{Im}(G) = -\mathrm{Im}(F) \Rightarrow G(j\omega) = F^*(j\omega) \Rightarrow g(t) = f^*(-t)$

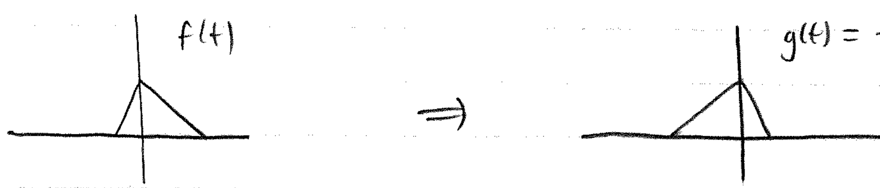
If the input signal is real, it Fourier transform has hermitian symmetry.

a)



$g(t) = f^*(-t) = f(t)$

b)



$g(t) = f^*(-t) = f(-t)$

A system is <u>causal</u> if the output depends only on past or present inputs. $\Rightarrow$ $\boxed{\text{non-causal}}$

$f^*(-(t-\tau)) = f^*(-t+\tau) = g(t+\tau) \Rightarrow \boxed{\text{time variant}}$

$(\alpha f)^*(-t) \neq \alpha g(t)$ if $\alpha \in \mathbb{C}$ $\Rightarrow \boxed{\text{nonlinear}}$

If the input is constrained to be real valued, the system behaves linearly.

2. a) Yes, one can throw away every other sample and still exceed the Nyquist frequency.

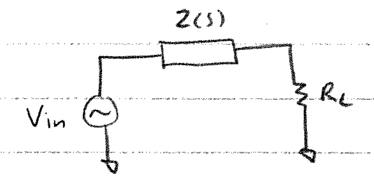
b) Yes, model the "defective" sampling by $f(t)[\delta_T(t) - \delta(t)]$. The transform of this quantity is the original replicated spectrum shifted vertically based on the value of $f(0)$. Therefore, $f(0)$ can be determined by evaluating the spectrum in the gaps between replication islands since the values should be zero,

$$f(t)[\delta_T(t) - \delta(t)] = f(t)\delta_T(t) - f(0)\delta(t) \iff \overbrace{\frac{1}{T} F(j\omega) * \delta_{\omega_0}(\omega)}^{\text{ideal sampled spectrum}} - f(0)$$

c) No, not possible because of aliasing.

1994 - Boyd

1. Assume :

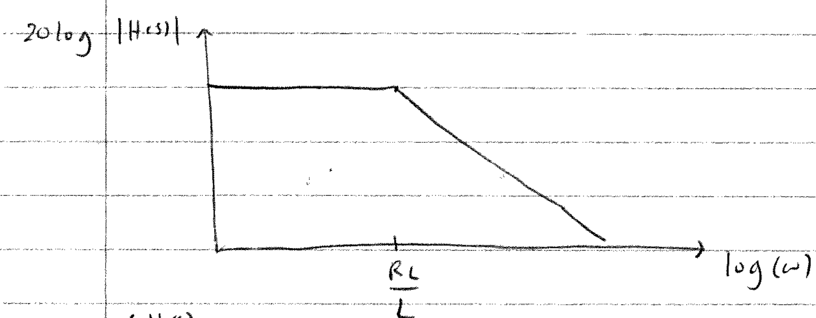


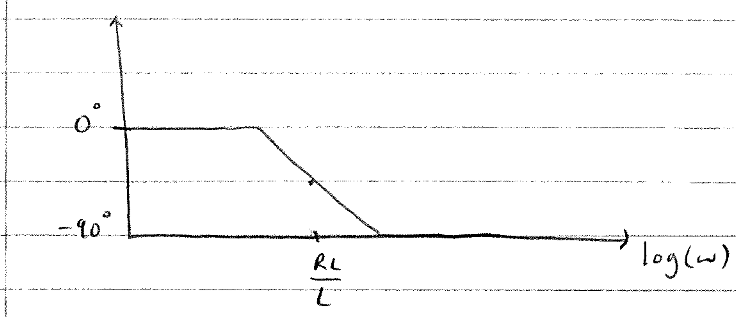
$V_L(s) = \frac{R_L}{R_L + Z(s)} V_{in}(s)$ model $Z(s)$ as L

$\Rightarrow Z(s) = sL \Rightarrow H(s) = \frac{R_L}{R_L + sL} = \frac{1}{1 + s\frac{L}{R_L}}$

pole at $\omega_p = -\frac{R_L}{L}$

Ignore this part. It is wrong.



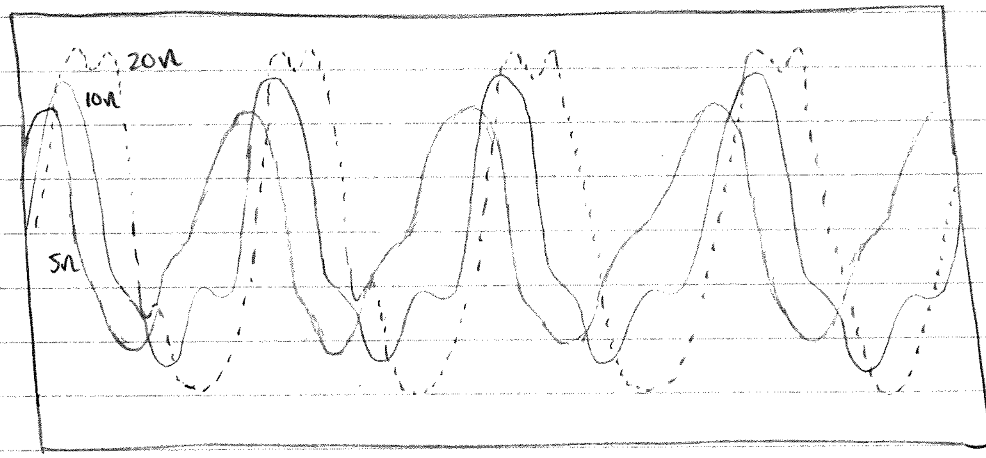


$\Rightarrow V_L(s)$ will always lag $V_{in}(s)$ by between $0°$ and $90°$.

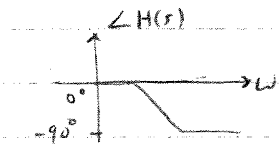
V_{in} looks like it's composed of 2 sinusoidal components; $\cos(\omega t)$ and $\cos(2\omega t)$. Changing R_L from 10Ω to 20Ω effectively doubles the bandwidth of the LPF. $\Rightarrow$ we expect $|V_L(s)|$ to increase and $\angle V_L(s)$ to increase as well. ~~This matches the waveforms given.~~ $\Rightarrow$ for $R_L = 5\Omega$, we would expect the bandwidth to shrink. $\Rightarrow |V_L(s)|$ should decrease and $\angle V_L(s)$ should decrease. Furthermore, since $\omega_{p_{20\Omega}} = 2 \cdot \omega_{p_{10\Omega}}$ and $\omega_{p_{10\Omega}} = 2 \cdot \omega_{p_{5\Omega}}$ we can expect the phase shift and magnitude decrease factor from 10Ω to 5Ω to be roughly the same as the decrease from 20Ω to 10Ω assuming the input frequencies are within ~2 decades of $\omega_{p_{10\Omega}}$. We might also expect the amplitude of the high frequency component to be further suppressed. So the waveform becomes smoother.

No. Wrong. Actually, 10Ω waveform leads 20Ω waveform.

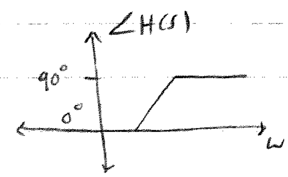
⇒ We expect something like:

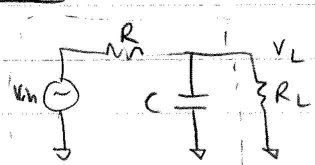


Notes: For a first order LPF, V_{out} will always lag V_{in} by a value between 0° and 90°



For a first order HPF, V_{out} will always lead V_{in} by a value between 0° and 90°





$$\Rightarrow \frac{V_L(s)}{V_{in}(s)} = \frac{\frac{R_L}{1+sR_LC}}{R + \frac{R_L}{1+sR_LC}} = \frac{R_L}{R_L + R(1+sR_LC)}$$

$$= \frac{R_L}{R+R_L}\left(\frac{1}{1+s(R\|R_L)C}\right)$$

⇒ Assume we have a signal around ω_s

⇒ $|H(j\omega_s)|_{R_L=10\Omega} < |H(j\omega_s)|_{R_L=20\Omega}$ since $\frac{R_L}{R+R_L}$ increases for $R_L = 20\Omega$

$\angle H(j\omega_s)_{R_L=10\Omega} > \angle H(j\omega_s)_{R_L=20\Omega}$ since pole decreases for $R_L = 20\Omega$

This is consistent with the observed waveforms. ⇒ for 5Ω, we expect the waveform to have smaller amplitude & greater phase as pictured above.

1997 - Boyd

1. $\int_{-\infty}^{\infty} |u(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |U(j\omega)|^2 d\omega$ $\quad Y(j\omega) = H(j\omega)\, U(j\omega)$

$\int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(j\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(j\omega)\, U(j\omega)|^2 d\omega$

$\Rightarrow$ $\boxed{\text{No}}$ $\boxed{\text{False}}$. For example. Consider the case when the system is a LPF with $H(j\omega) = \text{rect}\left(\frac{\omega}{4\pi}\right)$

let $u(t) = 2\,\text{rect}(t)\cos(10\pi t)$ $\quad v(t) = \text{rect}(t)$

$\int_{-\infty}^{\infty} u(t)^2 dt \approx 2 \cdot \frac{1}{2\pi}\int_{-\infty}^{\infty} \text{sinc}\left(\frac{\omega}{2\pi}\right) d\omega > \int_{-\infty}^{\infty} v(t)^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} \text{sinc}\left(\frac{\omega}{2\pi}\right) d\omega$

but $\int_{-\infty}^{\infty} y(t)^2 dt \ngeq \int_{-\infty}^{\infty} z(t)^2 dt$,

since $U(j\omega) = \text{sinc}\left(\frac{\omega - 10\pi}{2\pi}\right) + \text{sinc}\left(\frac{\omega + 10\pi}{2\pi}\right)$ $\quad V(j\omega) = \text{sinc}\left(\frac{\omega}{2\pi}\right)$

and $H(j\omega) = \text{rect}\left(\frac{\omega}{4\pi}\right)$

$\Rightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} (H(j\omega)\, U(j\omega))^2 d\omega < \frac{1}{2\pi}\int_{-\infty}^{\infty} (H(j\omega)\, V(j\omega))^2 d\omega$

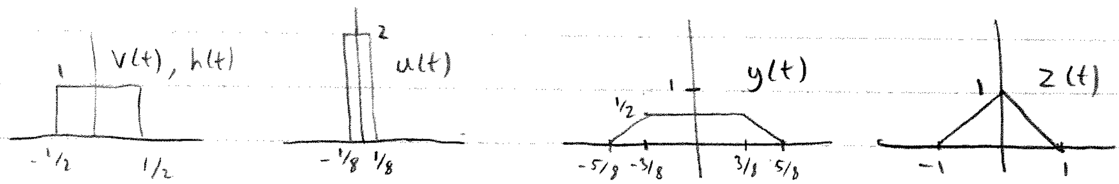
since most of $U(j\omega)$ is filtered out.

2. $\boxed{\text{No}}$. $\quad y(t) = (u * h)(t) \quad z(t) = (v * h)(t)$

let $h(t) = \text{rect}(t) \quad u(t) = 2\,\text{rect}(4t) \quad v(t) = \text{rect}(t)$

$\Rightarrow \max_t |u(t)| = 2 \geq \max_t |v(t)| = 1$

but $\max_t |y(t)| = \frac{1}{2} \ngeq \max_t |z(t)| = 1$



1997 - Nishimura

1. $\alpha f(x) \Leftrightarrow \alpha F(s)$ $\quad |\alpha F(s)|^2 = \alpha^2 |F(s)| \neq \alpha G(s)$ $\Rightarrow$ nonlinear

$f(x-\tau) \Leftrightarrow e^{js\tau} F(s)$ $\quad |e^{js\tau} F(s)|^2 = |F(s)|^2$ $\Rightarrow$ time variant

$\frac{G(s)}{F(s)} = \frac{|F(s)|^2}{F(s)}$ $\quad g(x) =$ autocorrelation of $f(x)$

2. $g(x) = f(-x)$ $\quad \alpha f_1(-x) + \beta f_2(-x) = \alpha g_1(x) + \beta g_2(x)$ $\Rightarrow$ linear

$f(-(x-\tau)) = f(-x+\tau) \neq g(x-\tau)$ $\Rightarrow$ time variant.

3. $H(s) = e^{-j2\pi s}\, 2\pi j s$ Linear, time invariant.

$\frac{d}{dx}(\alpha f_1(x-1) + \beta f_2(x-1)) = \alpha \frac{d f_1(x-1)}{dx} + \beta \frac{d f_2(x-1)}{dx} = \alpha g_1(x) + \beta g_2(x)$ ✓

$\frac{d}{dx} f((x-\tau)-1) = \frac{d}{dx} f(x-\tau-1) = g(x-\tau)$ ✓

4. $\sum_{n=-\infty}^{\infty} \alpha f_1(x-n) + \beta f_2(x-n) = \alpha \sum_{n=-\infty}^{\infty} f_1(x-n) + \beta \sum_{n=-\infty}^{\infty} f_2(x-n) = \alpha g_1(x) + \beta g_2(x)$ ✓

$\sum_{n=-\infty}^{\infty} f((x-\tau)-n) = \sum_{n=-\infty}^{\infty} f(x-\tau-n) = g(x-\tau)$ ✓

$G(s) = F(s)\cdot \delta_1(s) \Rightarrow H(s) = \delta_1(s) \leftarrow$ comb(s)

5. $H(s) = \text{rect}(s)\, \delta_1(s)\, \text{sinc}^2(s)$ Linear (all elements linear)

Time variant (If input shifted, linear phase in s-domain that should result does not because of replication islands that are not filtered out. Also, interpolation filter does linear interpolation. So consider a slightly shifted version of the input. The piecewise linear output cannot be a shifted version of the original piecewise linear input. Note that if the interpolation filter is a sinc interpolator of appropriate bandwidth, the system is LTI.

6. $\int_{-\infty}^{x} \alpha f_1(u) + \beta f_2(u)\, du = \alpha \int_{-\infty}^{x} f_1(u)\, du + \beta \int_{-\infty}^{x} f_2(u)\, du = \alpha g_1(x) + \beta g_2(x)$ ✓

$\int_{-\infty}^{x} f(u-\tau)\, du = \int_{-\infty}^{x-\tau} f(u)\, du = g(x-\tau)$ $\Rightarrow$ Linear, Time-invariant

$H(s) = \frac{1}{2}\delta(s) + \frac{1}{j2\pi s}$

2001 - Nishimura

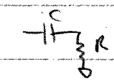
$f(t) \rightarrow \boxed{\quad} \rightarrow g(t)$

1) Yes

2) No.

3) No. Output must go negative at some points

4) No.

5) Yes 

$\frac{R}{R+\frac{1}{sC}} = \frac{sCR}{1+sCR}$

6) No. Time-Variant

7) Yes. Narrow Bandpass filter.

2) let T be the period of $f(t)$. Clearly $g(t)$ does not have period T.

$\Rightarrow \quad f(t) \rightarrow g(t) \quad$ and $\quad f(t+T) = f(t) \rightarrow g(t) \neq g(t+T)$

4) $f(t) \rightarrow g(t) \quad \Rightarrow \quad g(t) = w(t)\,f(t)$ where $w(t)$ is a windowing function.

$\Rightarrow \quad f(t-\tau) \rightarrow w(t)\,f(t-\tau) \neq g(t-\tau) = w(t-\tau)\,f(t-\tau)$

$X_{in} \qquad X_{out} = z^{-100}(X_{in} + \alpha X_{out})$

$$X_{out}(1-\alpha) = z^{-100} X_{in}$$

$$\frac{X_{out}}{X_{in}} = \frac{z^{-100}}{1-\alpha z^{-100}}$$

$1-\alpha z^{-100} = 0 \quad \Rightarrow \quad z^{-100} = \frac{1}{\alpha}$

$z^{100} - \alpha = 0 \qquad z^{100} = \alpha \qquad \alpha^{1/100}$

2004 - Boyd

a) $u \to \boxed{z^{-1}} \to \cdots \to \boxed{z^{-1}} \to y$

$x(t+1) = Ax(t) + Bu(t) \qquad y(t) = Cx(t) + Du(t)$

$$\Rightarrow \quad x(t) = \begin{bmatrix} u(t-1) \\ \vdots \\ \\ u(t-100) \end{bmatrix} \qquad A = \underbrace{\begin{bmatrix} 0 & \cdots & & & 0 \\ 1 & 0 & \cdots & & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & & \cdots & 1 & 0 \end{bmatrix}}_{100 \times 100} \qquad B = \underbrace{\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{100 \times 1}$$

$$C = \underbrace{[0\ 0 \cdots 0\ 1]}_{1 \times 100} \qquad D = 0$$

b) Eigenvalues of A are all 0.

The diagonal entries of a triangular matrix are its eigenvalues.

- The determinant of a triangular matrix equals the product of its diagonal entries.
- Since for any triangular matrix A, the matrix $sI - A$ (whose determinant is the characteristic polynomial of A) is also triangular.

c) $x(t)$, B, C and D remain the same.

$$A = \begin{bmatrix} 0 & & \cdots & 0 & \alpha \\ 1 & 0 & & & 0 \\ & 1 & 0 & & 0 \\ & & \ddots & \ddots & \vdots \\ 0 & & \cdots & 1 & 0 \end{bmatrix}$$

$\alpha = 10^{-5}$

d) Find the eigenvalues of A.

Eigenvalues of A are the poles of the transfer function:

$$Y = z^{-100}(U + \alpha Y) \Rightarrow Y(1 - \alpha z^{-100}) = z^{-100} U$$

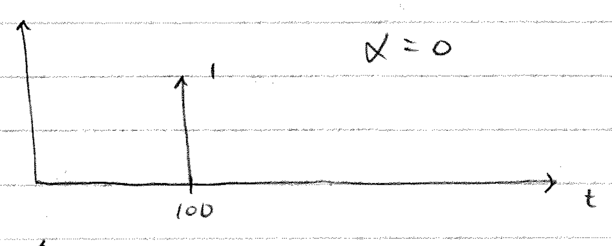
$$\Rightarrow \frac{Y}{U} = \frac{z^{-100}}{1 - \alpha z^{-100}} = \frac{1}{z^{100} - \alpha}$$

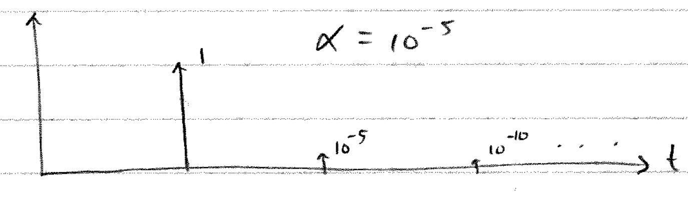
$$\Rightarrow z^{100} - \alpha = 0 \Rightarrow z^{100} - 10^{-5} = 0 \Rightarrow z^{100} = 10^{-5}$$

$$\Rightarrow z = 10^{-5/100} e^{2\pi k/100} \qquad k = 0, \ldots, 99$$

These are points spaced 3.6° apart on a circle of radius $10^{-5/100} = 0.8913$. These eigenvalues are distinct, so A is diagonalizable.

e) How different is the impulse response of the system with feedback $(\alpha = 10^{-5})$ and without feedback $(\alpha = 0)$

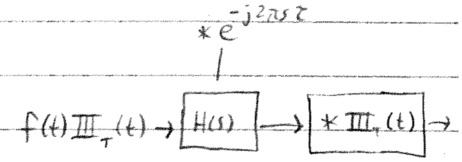




$\Rightarrow$ There is little noticeable difference in the impulse response. One might expect a more dramatic change, since the transfer function with $\alpha = 10^{-5}$ has many poles in the right half plane.

2005 - Pauly

1) $f(t)\,\text{III}_T(t)$ $\qquad \frac{1}{T} = 2B \Rightarrow T = \frac{1}{2B}$



$$f(t)\,\text{III}_T(t) \Longleftrightarrow F(s) * 2B\,\text{III}_{2B}(s)$$

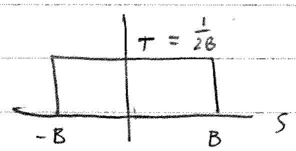
$$f(t-\tau)\,\text{III}_T(t) \Longleftrightarrow e^{-j2\pi s\tau} F(s) * 2B\,\text{III}_{2B}(s)$$

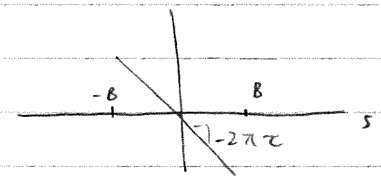
$\Rightarrow$ can implement a phase shift $e^{-j2\pi s\tau}$ before resampling.

$w = 2\pi s$

WRONG. This would be for IIR

2) $|H(s)|$ $\qquad \angle H(s)$ $\qquad \boxed{H(s) = \frac{1}{2B}\,\Pi_{2B}\, e^{-j2\pi s\tau}}$

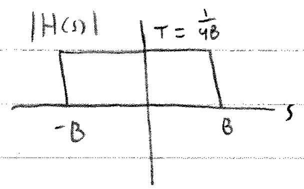


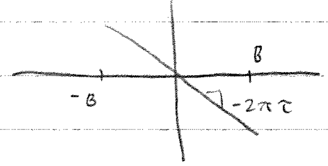


$$e^{-j2\pi s\tau} = \cos(2\pi s\tau) - j\sin(2\pi s\tau) \Rightarrow \angle e^{-j2\pi s\tau} = \tan^{-1}\left(\frac{-\sin(2\pi s\tau)}{\cos(2\pi s\tau)}\right)$$

$$= \tan^{-1}(-\tan(2\pi s\tau)) = -2\pi s\tau$$

3) $|H(s)|$ $\qquad \angle H(s)$





$$\boxed{H(s) = \frac{1}{4B}\,\Pi_{2B}\, e^{-j2\pi s\tau}}$$

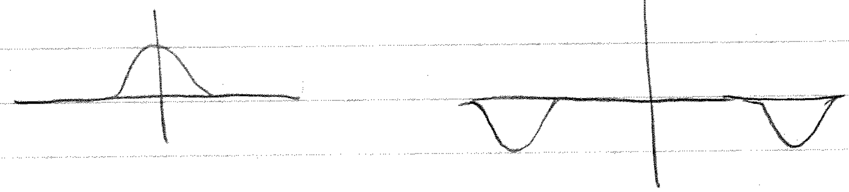
just decrease the amplitude of the FIR filter by a factor of 2.

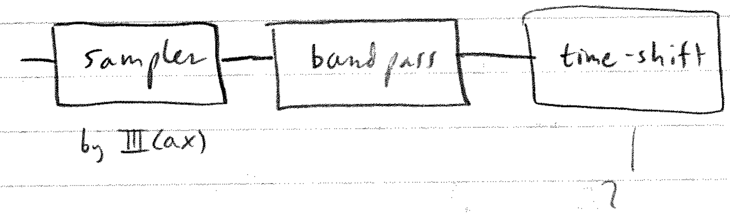
2) for FIR, we must approximate this IIR filter. The IIR is a shifted sinc. Thus, the FIR should be a truncated shifted sinc.

$$\Rightarrow h(t) = \text{rect}\left(\frac{t-\tau}{W}\right)\text{sinc}(2B(t-\tau)) \Rightarrow H(s) = \left(\frac{1}{2B}\,\Pi_{2B} * W\text{sinc}\left(\frac{s}{W}\right)\right)\cdot e^{-j2\pi s\tau}$$

for $W \gg T$, this approximates a shifted rect (with some smoothed corners). But the phase remains linear in the passband.

2010 - Nishimura

1. 



$Ш(ax) \Leftrightarrow \frac{1}{|a|} Ш\left(\frac{s}{a}\right)$

$\delta(t-\tau) \Leftrightarrow e^{-2\pi i s \tau}$ we want phase shift of $-\pi$ for all s. $\Rightarrow e^{-\pi i}$

To approximate this, we could choose a τ s.t. $e^{-2\pi i a \tau} = e^{-\pi i}$

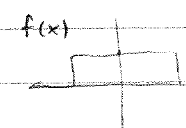
$\Rightarrow \tau = \frac{1}{2a}$

2. Yes. We know $f(x) \Leftrightarrow F(s)$ we want to find $\int_{-\infty}^{\infty} x f(x)\, dx$

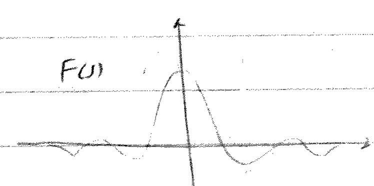
and we know $\int_{-\infty}^{\infty} x^n f(x)\, dx = \left(\frac{j}{2\pi}\right)^n F^{(n)}(0)$

$\Rightarrow \int_{-\infty}^{\infty} x f(x)\, dx = \boxed{\frac{j}{2\pi} F'(0)}$

3. It would be best to agree and disagree with Dr. T. Dr. T is partially correct, but additional constraints on s_0 and $F(s_0)$ are needed to avoid potential ambiguities in determining the center of mass.

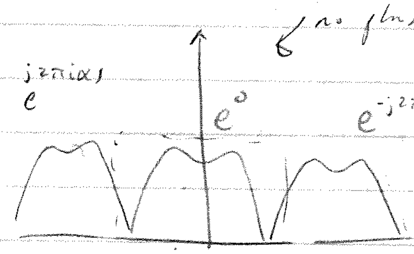
e.g. 

center of mass is $x=0$



Can't readily tell what center of mass is from a single point $F(s_0)$ (unless of course, we somehow know that $F(s_0) = F'(0)$)

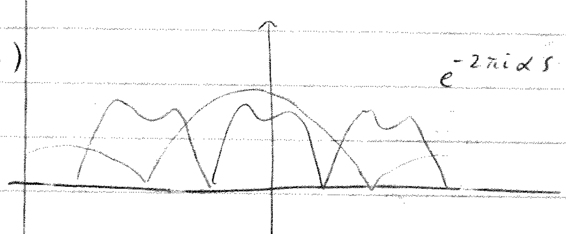
1998 - Nishimura



$\Rightarrow \quad g_r(x) = g(x)$

1. a) no

b)



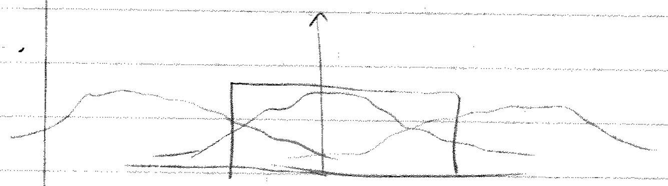
$$\epsilon = \int_{-\infty}^{\infty} |g_r(x) - g(x)|^2 dx = \int_{-\infty}^{\infty} |G_r(s) - G(s)|^2 ds$$

$$= \int_{-1/2}^{1/2} |\mathrm{sinc}^2 s\, G(s) - G(s)|^2 ds + \int_{-\infty}^{-1/2} |e^{?}\,\mathrm{sinc}^2 G(s)|^2 ds$$

$$+ \int_{1/2}^{\infty} |e^{?}\,\mathrm{sinc}^2 G(s)|^2 ds$$

2. $\Rightarrow$ doesn't depend on α.

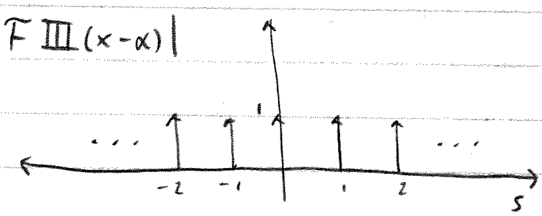
3.



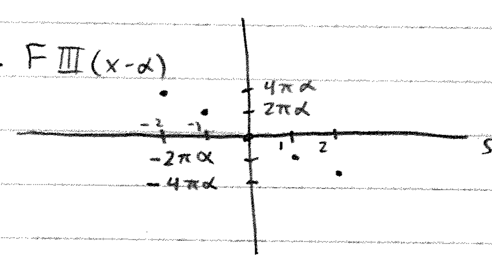
$\Rightarrow$ yes, since phase shifted copies alias into the part that's filtered.

1. $$Ш(x-\alpha) \Leftrightarrow e^{-2\pi i s\alpha}\, Ш(s) = \sum_{n=-\infty}^{\infty} e^{-2\pi i n\alpha}\,\delta(s-n)$$

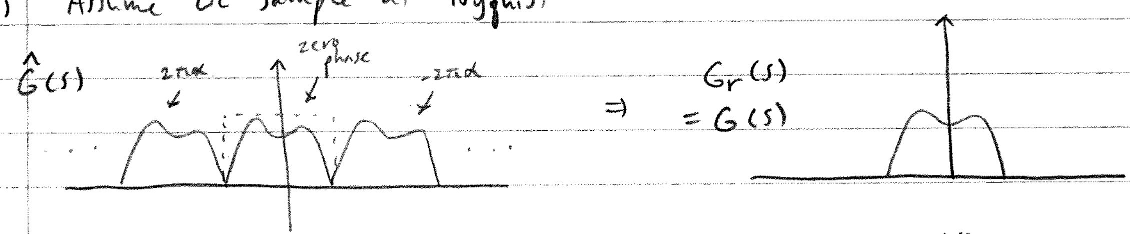
shifted Shah: $|\mathcal{F}\, Ш(x-\alpha)|$



$\angle\, \mathcal{F}\, Ш(x-\alpha)$

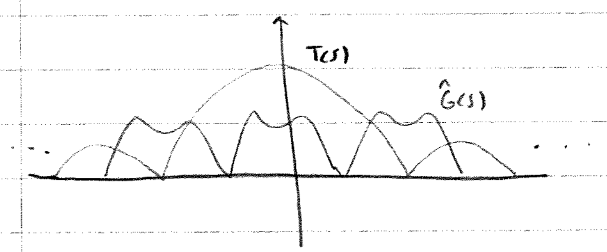


a) Assume we sample at Nyquist



by parseval's: $\epsilon = \int_{-\infty}^{\infty} |g_r(x) - g(x)|^2 dx = \int_{-\infty}^{\infty} |G_r(s) - G(s)|^2 ds = \int_{-\infty}^{\infty} 0 \, ds = 0$

b) $t(x) = \Lambda(x) \Rightarrow T(s) = \text{sinc}^2(s)$



zero phase $\Rightarrow$
doesn't depend on α

$$\epsilon = \int_{-\infty}^{\infty} |g_r(x) - g(x)|^2 dx = \int_{-\infty}^{\infty} |G_r(s) - G(s)|^2 ds = \int_{-1/2}^{1/2} |G_r(s) - G(s)|^2 ds$$

$$+ \int_{-\infty}^{-1/2} |G_r(s)|^2 ds + \int_{1/2}^{\infty} |G_r(s)|^2 ds \Rightarrow \boxed{\text{No}}, \ \epsilon \text{ won't depend on } \alpha$$

magnitude doesn't depend on phase, and thus α.

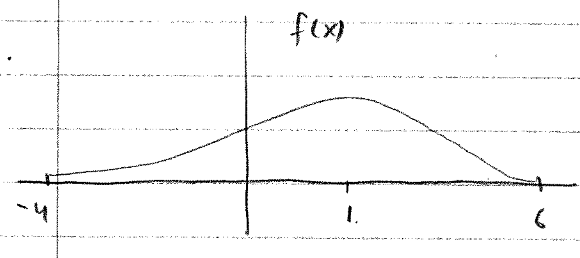
2. $\boxed{\text{No}}$ for $t(x) = \text{sinc}(x)$
$\boxed{\text{Yes}}$ for $t(x) = \Lambda(x)$

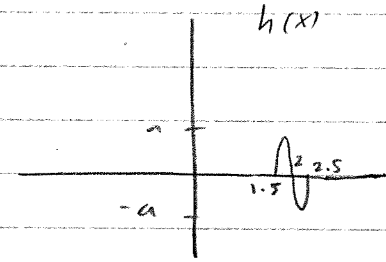
3. Yes. Now islands with nonzero phase will alias into the passband (for both $t(x) = \text{sinc}(x)$ and $t(x) = \Lambda(x)$)

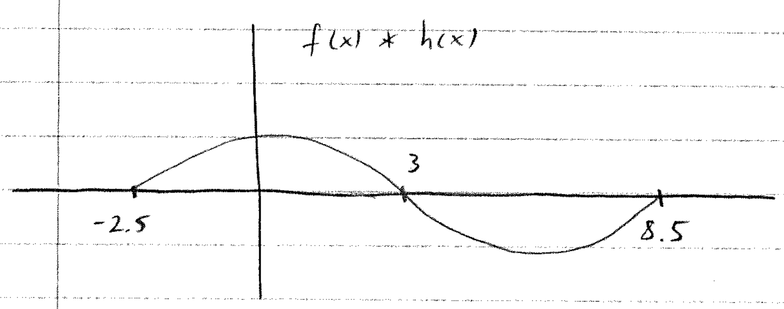
$\Rightarrow$ $G_r(s) - G(s)$ will depend on α $\Rightarrow$ ϵ will depend on α.

2009 - Nishimura

1.

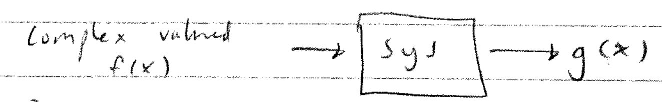


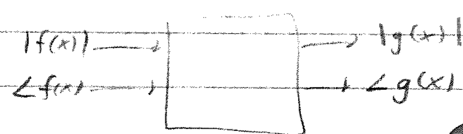




approximately takes the derivative of $f(x)$.

2. a)





linear?

Sys: $|f(x)| = |g(x)|$

phase of $f(x)$ passes through LTI system.

$$f(x) = re^{i\theta} \qquad g(x) = re^{i\phi}$$

$f_1(x) = r_1 e^{i\theta_1}$

$f_2(x) = r_2 e^{i\theta_2}$ $\Rightarrow$ $\boxed{\text{nonlinear}}$. sum of magnitude $\neq$ magnitude of sum.

e.g. consider $f_1(x) = -f_2(x) \Rightarrow |f_1(x) + f_2(x)| = 0 \neq |f_1(x)| + |f_2(x)| = 2|f_1(x)|$

b) time invariant? Yes, magnitude operation is time invariant, and phase operation is time invariant since it passes through an LTI system.

$f(x) = re^{i\theta}$

$f(x-\alpha) = r(x-\alpha)\, e^{i\theta(x-\alpha)}$ $\qquad g(x-\alpha) = r(x-\alpha)\, e^{i\phi(x-\alpha)}$

$\Rightarrow$ $\boxed{\text{time invariant}}$

c) magnitude operation is causal

but causality of overall system depends on causality of LTI system that phase passes through.

2007 - Nishimura

1. $\boxed{\text{Yes.}}$ This looks like simple time delay. $\Rightarrow$ $H(s) = e^{-2\pi i s \alpha}$

2. BIBO stable: $\|h\|_1 = \int_{-\infty}^{\infty} |h(t)|\,dt < \infty$ $\Rightarrow$ $h(t) = \cos(t)$ is not BIBO stable.

↖ for continuous case.

L^1 norm → $\|h\|_1 = \sum_{n=-\infty}^{\infty} |h[n]| < \infty$ ← for discrete case.

$h(t)$ must fall off faster than $\frac{1}{x}$ $\Rightarrow$ $\boxed{\text{No}}$

(aside $\sum_{x=-\infty}^{\infty} \frac{1}{x^p}$ only converges for $p > 1$)

3. $\boxed{\text{No}}$ this is time reversal which is not LTI (it's time variant)

also, for an LTI system, you can't add new frequencies. (can only remove frequencies) since we multiply by $H(s)$, from the graphs, we can see a point on the right where $|F(s)| = 0$ and $G(s) \neq 0$, which violates this principle.

for time reversal: $\mathcal{F}(f(-t)) = F(-s) = R(-s)\, e^{j\phi(-s)}$

?. 4. $\boxed{\text{Yes}}$

5. $\boxed{\text{No}}$ $f(x) = \int_{-\infty}^{\infty} F(s)\, e^{j2\pi s x}\, dx$

$\Rightarrow$ $f(0) = \int_{-\infty}^{\infty} F(s)\, ds$

from the graph we see that $\int_{-\infty}^{\infty} G(s)\, ds \neq 0$

and we are given that $g(0) = 0$ $\Rightarrow$ False

6. $\boxed{\text{No}}$ Fourier transform of autocorrelation always has zero phase.

$F(\bar{g} * g) = |G(s)|^2$ (aside: if $f(x)$ is real, $|F(s)|$ is even) due to Hermitian symmetry. $F(-s) = F^*(s) \Rightarrow |F(-s)| = |F(s)|$.

2005 - Osgood

Find the fourier transform of $f(t) = |t|$

Existence of the Fourier Transform:

Conditions for the existence of the Fourier transform are complicated to state in general, but it is sufficient for $x(t)$ to be absolutely integrable (i.e. $x \in L_1$, meaning x has finite L_1 norm, $\|x\|_1 < \infty$)

$$\|x\|_1 = \int_{-\infty}^{\infty} |x(t)|\, dt < \infty .$$

It is also sufficient for $x(t)$ to be square integrable ($x \in L_2$)

$$\|x\|_2 = \int_{-\infty}^{\infty} |x(t)|^2\, dt < \infty$$

$f(t) = |t| \notin L_1$ or L_2

but $f'(t) = \mathrm{sgn}(t) \Rightarrow \mathcal{F}f'(t) = \frac{1}{\pi i s} = 2\pi i s\, \mathcal{F}f(s)$

$$\Rightarrow \boxed{\mathcal{F}f(s) = -\frac{1}{2\pi^2 s^2}}$$

2003 - Osgood

1. Express $f(t)$ as a finite sum of weighted sinusoids:

$$f(t) = \sum_{k=-N}^{N} c_k e^{2\pi i k t/T} \qquad c_{-k} = \overline{c_k}$$

$f(t)$ is periodic of period T.

There are $2N+1$ terms in the sum, so it takes $2N+1$ samples per period to determine $f(t)$ completely.

$$\mathcal{F}f(s) = \sum_{k=-N}^{N} c_k\, \delta\left(s - \frac{k}{T}\right)$$

spectrum goes from $-\frac{N}{T}$ to $\frac{N}{T}$.

Applying the sampling formula:

$$f(t) = \sum_{k=-\infty}^{\infty} f(t_k)\, \mathrm{sinc}\, p\,(t - t_k) \qquad \text{where } t_k = k\frac{T}{M}$$

where $M = 2N+1$ defines the needed sample spacing (samples per period)

and $p = \frac{2N+1}{T} = \frac{M}{T}$ such that all δ's in freq. domain are evenly spaced.

$$t_k = \frac{k}{p} = \frac{kT}{M}$$

$\Rightarrow$ the sample points are spaced a fraction of a period apart, $\frac{T}{M}$, and after $f(t_0), f(t_1), \dots, f(t_{M-1})$ the sample values repeat (i.e. $f(t_M) = f(t_0)$, $f(t_{M+1}) = f(t_1)$)

$$\Rightarrow f(t_{k+k'M}) = f(t_k) \qquad \text{for } k, k' \in \mathbb{Z}$$

Using this periodicity of coefficients in the sampling formula, the single sampling sum splits into M sums as:

$$\sum_{k=-\infty}^{\infty} f(t_k)\, \mathrm{sinc}\, p(t-t_k) = f(t_0) \sum_{m=-\infty}^{\infty} \mathrm{sinc}(pt - mM) + \dots + f(t_{M-1}) \sum_{m=-\infty}^{\infty} \mathrm{sinc}(pt - (M-1+mM))$$

The sums on the right have a simple closed form expression:

$$\sum_{m=-\infty}^{\infty} \operatorname{sinc}(pt-k-mM) = \frac{\operatorname{sinc}(p(t-t_k))}{\operatorname{sinc}(\frac{1}{T}(t-t_k))}$$

$$\Rightarrow f(t) = \sum_{k=-N}^{N} c_k e^{2\pi i k t/T} = \sum_{k=0}^{2N} f(t_k) \frac{\operatorname{sinc}(p(t-t_k))}{\operatorname{sinc}(\frac{1}{T}(t-t_k))}$$

Finite sampling Theorem →

$$= \sum_{k=0}^{2N} f(t_k) \frac{\operatorname{sinc}((2\nu_{max}+\nu_{min})(t-t_k))}{\operatorname{sinc}(\nu_{min}(t-t_k))}$$

where $p = \frac{2N+1}{T}$ $\quad t_k = \frac{k}{p} = \frac{kT}{2N+1}$

---

e.g. for $f(t) = \sin(2\pi t)$ $\quad T=1 \quad N=1$

$\Rightarrow$ applying the sampling theorem:

$$f(t) = \sum_{k=-\infty}^{\infty} f(t_k) \operatorname{sinc} p(t-t_k) \qquad t_k = \frac{kT}{2N+1} = \frac{k}{3} \qquad p = \frac{2N+1}{T} = 3$$

applying the finite sampling theorem:

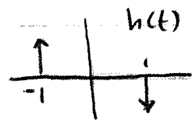
$$f(t) = \sum_{k=0}^{2N} f(t_k) \frac{\operatorname{sinc}(p(t-t_k))}{\operatorname{sinc}(\frac{1}{T}(t-t_k))}$$

$$= \sum_{k=0}^{2} f(\tfrac{k}{3}) \frac{\operatorname{sinc}(3(t-t_k))}{\operatorname{sinc}(t-t_k)}$$

$$= 0 + \sin(2\pi \tfrac{1}{3}) \frac{\operatorname{sinc}(3(t-\frac{1}{3}))}{\operatorname{sinc}(t-\frac{1}{3})} + \sin(2\pi \tfrac{2}{3}) \frac{\operatorname{sinc}(3(t-\frac{2}{3}))}{\operatorname{sinc}(t-\frac{2}{3})}$$

$$= \sin\left(\frac{2\pi}{3}\right) \frac{\operatorname{sinc}(3t-1)}{\operatorname{sinc}(t-\frac{1}{3})} + \sin\left(\frac{4\pi}{3}\right) \frac{\operatorname{sinc}(3t-2)}{\operatorname{sinc}(t-\frac{2}{3})}$$

2002 - Nishimura

1. $\boxed{\text{Yes.}}$ 

$y(t) = x(t) * h(t)$

2. $\boxed{\text{No.}}$ LSI but not BIBO stable.

$h(t) = u(t) \Rightarrow \int_{-\infty}^{\infty} |h(t)|\,dt \not< \infty$

Also, the system is an integrator with transfer function (laplace) $\frac{1}{s}$

$\Rightarrow$ pole is <u>on</u> $j\omega$ axis. $\Rightarrow$ not BIBO stable.

BIBO stability requires all poles to be in left half plane.

3. $\boxed{\text{No.}}$ Can't compress in time. This is the same as adding new frequencies. $a > 1$, $f(at) \Leftrightarrow \frac{1}{|a|} F\left(\frac{s}{a}\right)$

Since $f * h \Leftrightarrow F(s)H(s)$, multiplication by $H(s)$ can only remove frequency content from $F(s)$.

LTI means you put a sine wave in, you get the same sine wave out, possibly with different amplitude + phase, but with the same frequency.

4. regular gaussian (centered at zero) has zero phase. $\Rightarrow$ $\boxed{\text{Yes}}$ $\mathcal{F}(e^{-\pi t^2}) = e^{-\pi s^2}$

Gaussian $\Leftrightarrow$ Gaussian

5. No. $V_{in}(0) = 0$ but $V_{out}(0) \neq 0$

$\Rightarrow \alpha V_{in}(0) = 0 \neq \alpha V_{out}(0) = V_{out}(0) \Rightarrow$ nonlinear

2001 - Osgood

- Existence of Fourier Transform:

if $\int_{-\infty}^{\infty} |f(t)|^2 dt < \infty$ or $\int_{-\infty}^{\infty} |f(t)| dt < \infty$,

then $\mathcal{F}f$ exists.

This condition is not satisfied for polynomials.

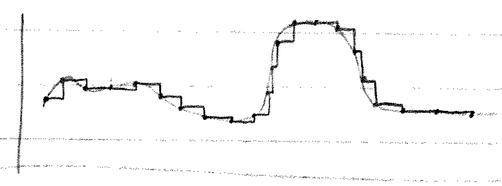
$$t\, f(t) \Leftrightarrow \left(\frac{j}{2\pi}\right) F'(s)$$

$$t^n f(t) \Leftrightarrow \left(\frac{j}{2\pi}\right)^n F^{(n)}(s)$$

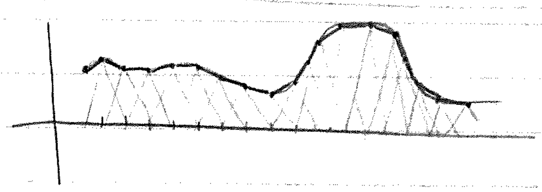
let $f(t) = 1$

$\Rightarrow$ $t^n \Leftrightarrow \left(\frac{j}{2\pi}\right)^n \delta^{(n)}(s)$ $\Rightarrow$ polynomials don't yield meaningful Fourier transforms.

A better strategy would be interpolation using either zero-order hold or linear interpolation to approximate $f(t)$



ZOH



Linear Interpolation

expand on this: Ch 7 of Oppenheim

2000 - Nishimura

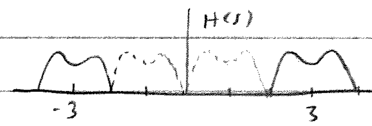
1. $f_s = 2 \qquad R(s) = \Pi\left(\frac{s}{2}\right)$

2. $H(s) = 2\pi i s\, G(s) \qquad f_s = 2 \qquad R(s) = \Pi\left(\frac{s}{2}\right)$

3. $H(s) = (G * G * G)(s) \qquad f_s = 6 \qquad R(s) = \Pi\left(\frac{s}{6}\right)$

4. $H(s) = G(s+1) \qquad f_s = 2 \qquad R(s) = \Pi\left(\frac{s+1}{2}\right)$

5. $H(s) = \frac{1}{2}\left(G(s-3) + G(s+3)\right)$



$f_s = 2 \qquad R(s) = \frac{1}{2}\left(\Pi\left(\frac{s-3}{2}\right) + \Pi\left(\frac{s+3}{2}\right)\right)$

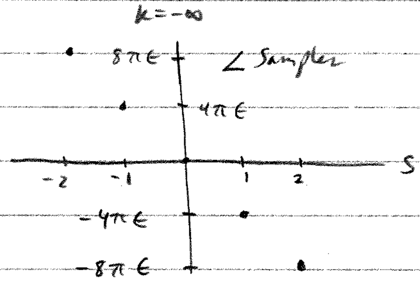
---

2. $R(s) = \frac{1}{2\pi i s}\Pi\left(\frac{s}{2}\right)$

5. $\boxed{\text{No}}$. $\frac{1}{2}\,\text{III}_{\frac{1}{2}}(t-\epsilon) \iff e^{-2\pi i s \epsilon}\,\text{III}_2(s)$

$T = \frac{1}{f_s}$

$$= e^{-2\pi i s \epsilon}\sum_{k=-\infty}^{\infty}\delta(s-2k) = \sum_{k=-\infty}^{\infty} e^{-2\pi i 2k\epsilon}\,\delta(s-2k)$$



$\Rightarrow$ at 3, $\hat{H}(s)$ has a $G(s)\left(1 + e^{-i12\pi\epsilon}\right)$
at -3, $\hat{H}(s)$ has a $G(s)\left(1 + e^{i12\pi\epsilon}\right)$

$\Rightarrow$ unless ϵ satisfies a special property (i.e. $6\epsilon \in \mathbb{Z}$ in which case we could reconstruct as in Q5.) we cannot recover $h(t)$.

2006 - Osgood

$$(f * g)[k] = \sum_{n=-\frac{N}{2}+1}^{\frac{N}{2}} f[n]\, g[n+k] = \sum_{n=0}^{N-1} f[n]\, g[n+k]$$

a) Show that $\mathcal{F}(f*g) = \overline{\mathcal{F}f}\,\mathcal{F}g$

$$\mathcal{F}f[m] = \sum_{n=0}^{N-1} f[n]\, e^{-2\pi i n m/N} \Rightarrow \overline{\mathcal{F}f} = \sum_{n=0}^{N-1} f[n]\, e^{2\pi i n m/N}$$

$$\mathcal{F}g[m] = \sum_{n=0}^{N-1} g[n]\, e^{-2\pi i n m/N}$$

$$\overline{\mathcal{F}f}\,\mathcal{F}g[m] = \left(\sum_{n=0}^{N-1} f[n]\, e^{2\pi i n m/N}\right)\left(\sum_{l=0}^{N-1} g[l]\, e^{-2\pi i l m/N}\right)$$

$$\mathcal{F}(f*g)[m] = \sum_{k=0}^{N-1}\left(\sum_{n=0}^{N-1} f[n]\, g[n+k]\right) e^{-2\pi i k m/N}$$

$$= \sum_{l=0}^{N-1}\sum_{n=0}^{N-1} g[l]\, f[n]\, e^{-2\pi i (l-n)m/N}$$

??? $e^{2\pi i n m/N}\, e^{-2\pi i (k+n)m/N}$ "

$$\mathcal{F}(f*g)[m] = \sum_{k=0}^{N-1}\left(\sum_{n=0}^{N-1} f[n]\, g[n+k]\right) e^{-2\pi i k m/N}$$

$$= \sum_{k=0}^{N-1}\left(\sum_{n=0}^{N-1} f[n]\, e^{2\pi i n m/N}\right) g[n+k]\, e^{-2\pi i (k+n) m/N}$$

$$= \sum_{k=0}^{N-1}$$

2006 - Osgood

1. $(f \star g)[k] = \sum_{n=-\frac{N}{2}+1}^{\frac{N}{2}} f[n]\, g[n+k] = \sum_{n=0}^{N-1} f[n]\, g[n+k]$

a) show that $\mathcal{F}(f \star g) = \overline{\mathcal{F}f}\, \mathcal{F}g$

$$\cancel{\overline{\mathcal{F}f}\, \mathcal{F}g = \sum_{p=0}^{N-1} \overline{f[p]}\, \omega^{p} \cdot \sum_{q=0}^{N-1} g[q]\, \omega^{-q}}$$

$$\cancel{= \sum_{p=0}^{N-1} \sum_{q=0}^{N-1} f[p]\, g[q]\, \omega^{-(q-p)}}$$

$$\cancel{= N \sum_{p=0}^{N-1} f[p]\, g[-q]}$$

$$\overline{F[m]} = \sum_{n=0}^{N-1} f[n]\, e^{2\pi i n m/N} \qquad G[m] = \sum_{n=0}^{N-1} g[n]\, e^{-2\pi i n m/N}$$

$$\ni \overline{F[m]}\, G[m] = \sum_{p=0}^{N-1} f[p]\, e^{2\pi i p m/N} \sum_{n=0}^{N-1} g[n]\, e^{-2\pi i n m/N}$$

$$= \sum_{p=0}^{N-1} \sum_{n=0}^{N-1} f[p]\, g[n]\, e^{-2\pi i m(n-p)/N} \qquad k = n-p$$

$$= \sum_{k=0}^{N-1} \sum_{p=0}^{N-1} f[p]\, g[p+k]\, e^{-2\pi i m k/N}$$

b) $(f \star g)(0)$ max value is $\sum_{n=0}^{N-1} f[n]^2$, if $f[n] = g[n]$

2007 - Osgood

$$f(t) = \int_{-\infty}^{\infty} F(s)\, e^{j2\pi st}\, ds$$

$$y(t) = \int_{-\infty}^{\infty} F(s)\, H(s)\, e^{j2\pi st}\, ds$$

$$= \int_{-\infty}^{\infty} F(s) \int_{-\infty}^{\infty} h(\tau)\, e^{-j2\pi s\tau}\, d\tau\, e^{j2\pi st}\, ds$$

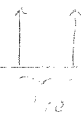
$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(s)\, h(\tau)\, e^{j2\pi s(t-\tau)}\, d\tau\, ds$$

$$= \int_{-\infty}^{\infty} h(\tau) \int_{-\infty}^{\infty} F(s)\, e^{j2\pi s(t-\tau)}\, ds\, d\tau$$

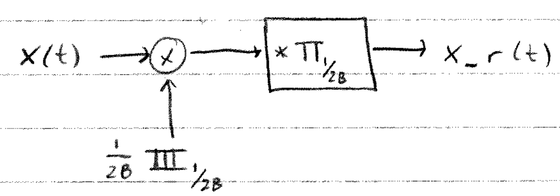
$$= \int_{-\infty}^{\infty} h(\tau)\, f(t-\tau)\, d\tau$$

$$= (h * f)(t) \Rightarrow LTI$$

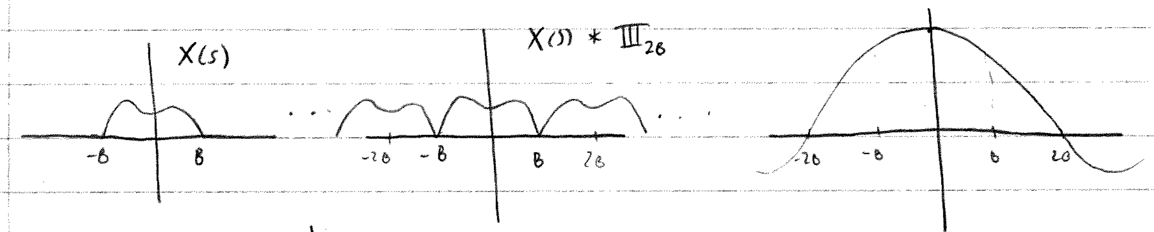
2009 - Pauly



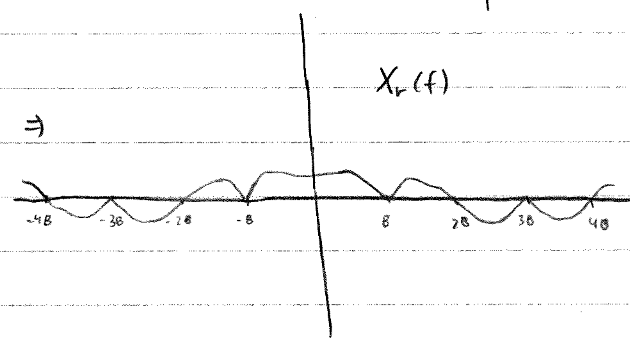
1.



$Z(s) = \frac{1}{2B}\,\mathrm{sinc}\left(\frac{s}{2B}\right)$



⇒



2. LPF x_r(t). $H(s) = \frac{2B}{\mathrm{sinc}\left(\frac{s}{2B}\right)}\,\Pi_{2B}$

(or you could predistort the signal)

1995 - Nishimura

1. No.

2. Yes.

3. $\operatorname{sgn}(t) \Longleftrightarrow \frac{1}{\pi i s} \Rightarrow \frac{1}{2}(1+\operatorname{sgn}(t)) \Rightarrow \frac{1}{2}\left(\delta(s) + \frac{1}{\pi i s}\right)$

$\Rightarrow \frac{1}{2}\left(\delta(-t) - \frac{1}{\pi i t}\right) \Longleftrightarrow \frac{1}{2}(1+\operatorname{sgn}(s)) = H(s)$

$\Rightarrow h(t) = \frac{1}{2}\delta(t) - \frac{1}{2\pi i t}$

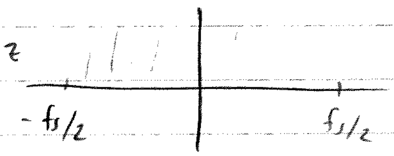
$\Rightarrow x(t) * h(t) = \frac{1}{2}x(t) - \frac{1}{2\pi i t} * x(t)$

$= \frac{1}{2}x(t) + \underbrace{\frac{i}{2\pi}\left(t * x(t)\right)}_{\text{imaginary.}}$

$\Rightarrow y(t) = \frac{1}{2}x(t)$

64 samples.

2. $T = 1s$ $\Rightarrow$ freq. resolution is $\frac{1}{64}$ Hz



2007 - Pauly

Bandlimited signal $x(t)$, Sample at seemingly random but known times. Can you reconstruct $x(t)$? What conditions do you need?

Maybe if average sampling rate is above Nyquist.

??

To the first order, you could do linear interpolation.

1998 - Osgood

let $R(t)$ denote the population of robins
$W(t)$ denote the population of worms

If left completely alone, each population would ideally grow exponentially.

$$P(t) = P_0 e^{\alpha t} \qquad \alpha \in \mathbb{R}, \ \alpha > 0$$

Can write this as $\frac{\partial P}{\partial t} = \alpha P(t)$

Interaction between the populations (Robins eating worms) should have a ~ linear effect on each population.

$$\Rightarrow \boxed{\begin{aligned} \frac{\partial R}{\partial t} &= A \cdot R(t) + B \cdot W(t) \\ \frac{\partial W}{\partial t} &= C \cdot W(t) - D \cdot R(t) \end{aligned}}$$

??

2004 - Osgood

if $f(t)$ is real, $|Ff|$ is even + $\angle Ff$ is odd due to hermitian symmetry properties.

Since $f[n]$ is real + even, we expect $F[m]$ to be real and even (zero imaginary component).

Student entered [11100... 01111] when he should have entered [111100... 0111]. Thus, the $f[n]$ he entered was not even. It was a shifted version of $f[n]$. Accordingly, $F[m]$ ended up with an imaginary component (nonzero phase).

, 2006 - Nishimura

$$\frac{f_{in}}{f_s} = \frac{16}{128} = \frac{1}{8} \Rightarrow f_s = 8 \cdot f_{in}$$

max point the FFT can represent is $\frac{f_s}{2}$

1. Check power connections.

2. f_{in} was reduced or f_{samp} was increased.

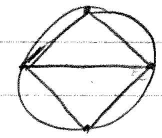
3. Signal being clipped $\Rightarrow$ nonlinearity from ADC.

4. Spike noise in time signal (white noise)

5. Spectral leakage $\Rightarrow$ $\frac{f_{in}}{f_s}$ is not rational.

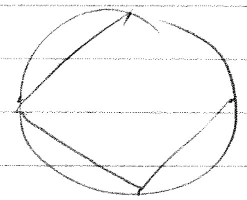
6. time signal rectified

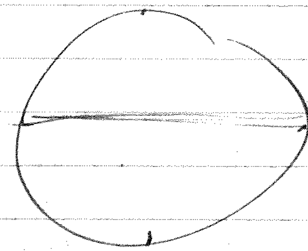
2000 - Osgood

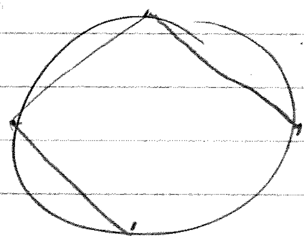


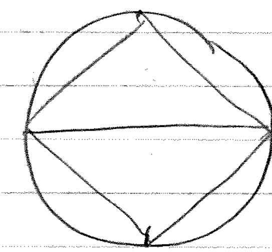
$$F[m] = \sum_{k=0}^{3} f[n]\, e^{-2\pi i k m/4}$$

$$F[m] = f[0]\, e^{0} + f[1]\, e^{-\pi i m/2} + f[2]\, e^{-\pi i m} + f[3]\, e^{-3\pi i m/2}$$









2000 - Spielman

1. * Question of sampling rate. Is the function bandlimited?

treat rainy day on 1/3 as an impulse.

$\Rightarrow$ impulse response decays as $e^{-t/\tau}$ $\quad 5(1-\frac{1}{e}) \simeq (0.63)(5) = 3.15$

$\Rightarrow \tau \approx 1$ day $\qquad BW \simeq \frac{1}{2\pi\tau} = f_{3dB}$

$\approx \frac{1}{6 \text{ days}} = \frac{1}{6}$ cycles/day

$\Rightarrow$ Nyquist is $\frac{1}{3}$ cycles/day $\Rightarrow$ 2 measurements per day is above Nyquist's

Is this an LTI system? To first order, it should be.
Linear since more rain $\Rightarrow$ more water $\Rightarrow$ faster flow rate.
linear if we assume river width is relatively constant.
Time invariant since delaying rain causes a delay in water level rising.

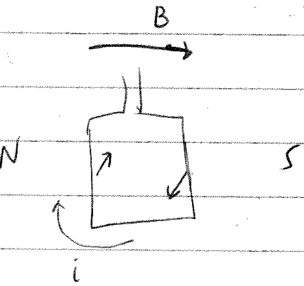
2. 2 cycles/day by Nyquist.

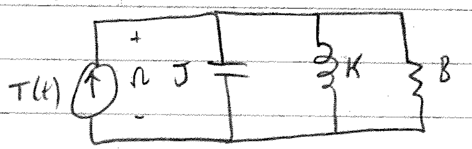
3. $r'(t) \Leftrightarrow 2\pi i s\, R(s)$

I would say no. We can see that based on 1/3, this strategy would cause a significant error.

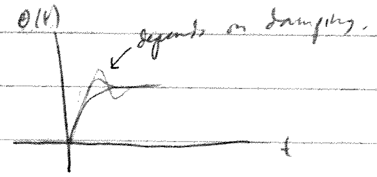
2001 - Spielman

1. $F = q\vec{v} \times \vec{B} = \vec{i} \times \vec{B}$





2. a)

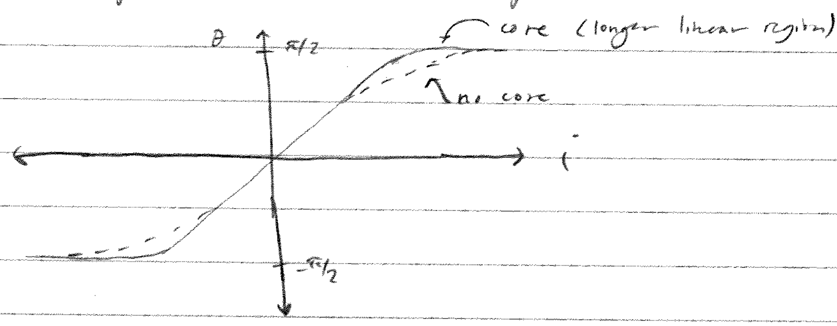


will also have noise. which will be strongest near resonant frequency.

b) critical damping. increase air resistance around needle.

3. nonlinear but still time invariant

- Resonant frequency unchanged.
- For small θ, system is LTI as before.
- Can compute steady state response

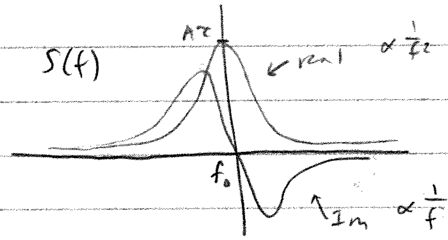


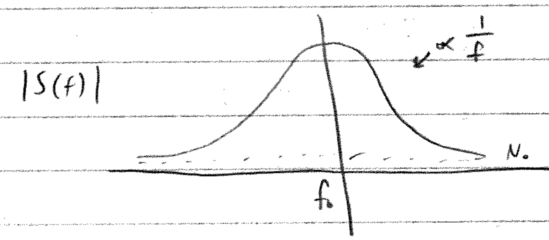
2003 – Spielman

1. $s(t) = A e^{-t/\tau} e^{j2\pi f_0 t} + n(t)$

$$e^{-at} \Leftrightarrow \frac{1}{a + 2\pi i s}$$

$$\Rightarrow S(f) = A\tau \left(\frac{1}{1 + (2\pi\tau(f - f_0))^2} - j \frac{2\pi\tau(f-f_0)}{1 + (2\pi\tau(f-f_0))^2} \right) + N_0$$





compressed by τ

FWHM $\propto 1/\tau$

FWHM $\propto 1/\tau$

2 Additive noise term in $S(f)$ is zero mean, complex WGN. While noise in $|S(f)|$ is white, it is in general not Gaussian. The noise is well approximated zero mean WGN where the signal component is large + Rayleigh distributed where the signal is small, and obeys Rician statistics for intermediate values.

2004 - Spielman

1. No

Yes

$h(t) = 3\delta(t) - 5$

2. Yes

No

$h(t, t_0) = e^{-i2\pi t t_0}$ $\delta(t - t_0) \to e^{-i2\pi t t_0}$

3. Yes

Yes

$\delta(t)$

4. Yes

Yes

$\Pi_{2W} \leftrightarrow 2W \operatorname{sinc}(2Wt) = h(t)$

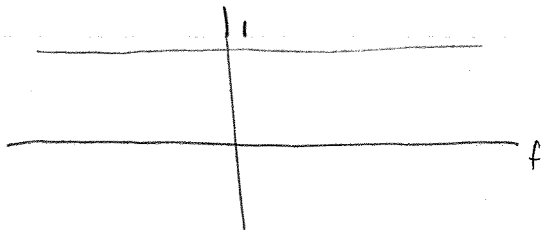
5. Yes, No.

$$h(t) = \begin{cases} \delta(t - t_0), & -T < t_0 < T \\ 0, & |\tau| > T \end{cases}$$

2010 - Spielman

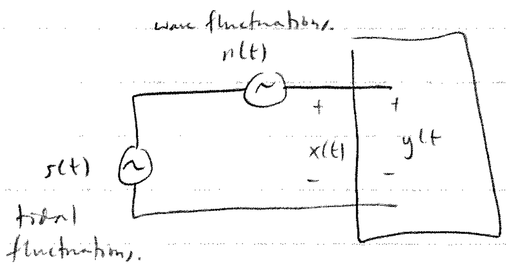
1. Yes. $y(t) = x(t)$

$\frac{Y(f)}{X(f)} = H(f)$



2. Advantages: inexpensive, easy to build, durable.

Disadvantages: need a boat to make the measurements.
More importantly, data will be noisy. The measurements are subject to unwanted variations from surface wave fluctuations. An improved device would filter out this unwanted high frequency noise.

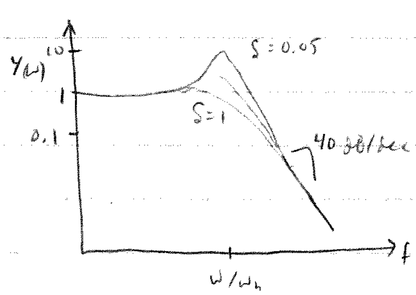


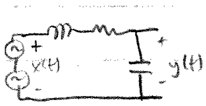
damping factor ↓
$\zeta = \frac{1}{2Q}$

3. There are lots of choices for better measurement systems which can incorporate the desired low-pass filtering.

critical damping when $\zeta = 1$ $(Q = \frac{1}{2})$

4. Device 2 is a low pass filter (equivalent to an LRC circuit). The frequency response would look something like (depending on the choices of A, L, + D):





In choosing A, L, and D, we want to avoid an underdamped system with a natural frequency in the range of ocean surface waves, typically on the order of 0.1 Hz.
$\omega_n = \frac{1}{\sqrt{LC}}$ where L_f is the inductance (fluid analog of electrical inductance) and C_f is fluid capacitance.
The damping ratio equals $\frac{1}{2Q} = \zeta = \frac{R}{2\sqrt{L/C}}$ where R_f is fluid resistance.
We want surface wave fluctuations $\gg \omega_n$ and tidal variations $\ll \omega_n$

Damped Harmonic Oscillators in General:

$$\frac{\partial^2 x}{\partial t} + 2 \zeta \omega_0 \frac{dx}{dt} + \omega_0^2 x = 0 \qquad \omega_0 = \sqrt{\frac{k}{m}} = \frac{1}{\sqrt{LC}}$$

$$\zeta = \frac{c}{2m\omega_0} = \frac{1}{2Q}$$

$$Q = \frac{1}{2\zeta}$$

Overdamped $\zeta > 1$
Critically damped $\zeta = 1$
Underdamped $\zeta < 1$

5. a) Increasing A increases the fluid capacitance, thereby decreasing the natural frequency ω_n.

b) Increasing L increases the fluid inertance, thereby decreasing the natural frequency ω_n, and increases the fluid resistance (thereby increasing the damping).

c) Increasing D decreases the fluid inertance, thereby increasing the natural frequency ω_n, and decreases the fluid resistance (thereby decreasing damping).