Electrical Engineering

Quals Questions

1996

From: mgbaker@plastique.Stanford.EDU
X-Authentication-Warning: plastique.Stanford.EDU: Host localhost.Stanford.EDU didn't use HELO protocol
X-Mailer: exmh version 1.6.4 10/10/95
To: shankle@ee.Stanford.EDU (Diane Shankle)
Subject: Re: Quals Questions 1996
Cc: mgbaker@plastique.Stanford.EDU, manning@ee.Stanford.EDU
Mime-Version: 1.0
Date: Thu, 01 Feb 96 23:41:28 PST

Here's a copy of my quals questions and answers. I asked different levels of these questions depending upon how much background the student has in the area.

1) Assume you have a file server that is able to broadcast at extremely high bandwidth (many gigabits per second). Assume it has various clients that are all able to listen to these broadcasts, but the clients have no back channel for communicating with the file server.

How would you set up this system so that the clients all get to hear the data for the files they need? (Don't worry about modifications to files from clients.)

> Answer: the file server can cycle through all its data broadcasting it. This may take several seconds or a minute, depending on the amount of data stored on the server, but the clients can listed until what they need is broadcast.

Now assume the clients do have a back channel, but it's a very weak one. They are able to send one bit of information after every file is broadcast. What would you do to improve the performance of the system?

> Answer: the clients can use their bit to vote about whether the last file was of interest to them. The server can collect these votes and use them to determine the frequency with which it should broadcast different files. More popular files should be broadcast more often so that clients don't need to wait as long for them on average.
>
> Extra points went to students who got through this quickly and had time to go on and consider how to handle modifications to files, caching mechanisms, and data consistency problems.

2) Assume you have a bunch of people wandering around the countryside, and they each have with them a portable computer with a battery-operating packet radio for communication. The radios have a limited range, so only some radios are within range of certain other radios. Also, their ability to hear each other is not transitive. This means radio A and radio B may both be able to talk with radio C, but A and B may not be able to communicate with each other directly.

Let's say one of the people on the ground encounters something interesting, such as a well or a source of electrical power. He wants to let certain others in his team know about this resource. How would you arrange for him to be able to send this information to them?

> Answer: the nodes in the network need to be able to accumulate routing information. It's not generally okay for a radio to broadcast to all its neighbors and allow them to broadcast to all theirs until the correct recipients get the message. This puts too much of a load on the network. If people came up with any sort of method

for building up route tables, they did well on this question. They also needed to understand that the route tables have to be dynamically configurable, since the people are wandering around, so they may move in and out of range with each other. Finally, people who came up with comparisons between totally distributed and somewhat centralized mechanisms did even better on this question.

Part 2

If you have many registers, the number of reachable states may be large. Explain a procedure that allows you to extract the reachable states without explicitly numbering the states.

You would represent the FSM by its state transition function δ, since both state diagrams and tables have exponential size. The procedure has to avoid representations that are surely of exponential size.

The states directly reachable from the reset state r_0 are the image of r_0 under δ and can be expressed implicitly as a function of the state variables. Similarly, the states directly reachable from any state set represented implicitly by r_k are the image of r_k under δ. By defining $r_{k+1}, k \geq 0$ to be the union of r_k and the image of r_k under δ, we specify an iterative method for implicit reachability computation. The iteration terminates when a fixed point is reached, i.e. when $r_k = r_{k+1}$ for some value of $k = k^*$. The expression for r_{k^*} encapsulates all reachable states, whereas the complement of r_{k^*} denotes unreachable states.

From: "Robert W. Dutton" <dutton@gloworm.Stanford.EDU>
X-Authentication-Warning: stjames.Stanford.EDU: Host dutton@localhost didn't use HELO protocol
To: shankle@ee.stanford.edu (Diane Shankle)
cc: group@stjames.Stanford.EDU
Subject: Quals Questions 1996
Date: Mon, 29 Jan 96 17:37:23 -0800

Hi Diane:
Here's my quals question as per your request.
It is my habit and tradition NOT to give an
answer to go with it...This is not intended
to be a "canned" study guide but to get the
students in the mood for the kind of question
that I ask.
Cheers,
Bob Dutton

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1) Given a 0.1um X 0.1um surface area, how much
capacitance can you achieve in an aggressive silicon
MOS (ie 50 Angstrom oxide) technology?
How much charge is transfered across this capacitor
for a 1V swing? How many electrons is that?

2) Assume that the oxide has electron traps in
it as shown below:

```
       / V (increasing positive)
      /
   Q /
_____________  Top electrode
              \
Traps - - - - -  } 50 Angstroms of oxide
_____________ /
(ground side in Si)
```

Sketch a plot of how the charge Q on the
top plate increases with positive voltage V
and discuss the effect and changes in this
curve with charge going into the traps.
(Hint: Assume that charge doesn't saturate with
voltage. In the discussion, talk about how the
charge actually moves onto the traps and
consider the limiting cases of no traps etc.)

3) Again, consider the minimum capacitor as
determined in part 1). Now assume you want
to make a memory system using it, as shown
below, where an MOS "pass transistor" is used
for access and the "black box" is for you to
innovate.

```
         (access voltage)
        |
    ______ (pass transistor)
      |  |
  -----  ------------- BLACK BOX CIRCUIT
  |                    (for read and write)
-----(minimum cap.)
-----
  |
(ground)
```

What are the problems/challenges with

this configuration? What are the effects
of the pass transistor and the elements
of the black box? (Hint: Consider the
size of the capacitors of all elements
and compare to that of the minimum cap.)

OFFICE MEMORANDUM ◇ STAR LABORATORY

January 26, 1996

To: Diane Shankle

From: Tony Fraser-Smith

Subject: Ph.D. Quals Question, 1996

Question: Suppose an earthquake occurs in the earth at a depth of $d = 10$ km (Figure 1). Although we have no idea what kind of electromagnetic signals, if any, are generated in the earth by earthquakes, let us assume that they do produce electromagnetic fields and that these fields have the same amplitude at all frequencies (Figure 2). Given the electrical conductivities (σ) shown in the figure, and given also that the skin depth δ at 1 Hz is 1.6 km for $\sigma = 0.1$ S/m, what frequencies are likely to be observed on the surface for earthquakes in California? Plot a figure equivalent to Figure 2 for the magnetic field measured on the surface. Are there any other ways in which the signal strength at the surface can be weakened in addition to absorption in the earth?

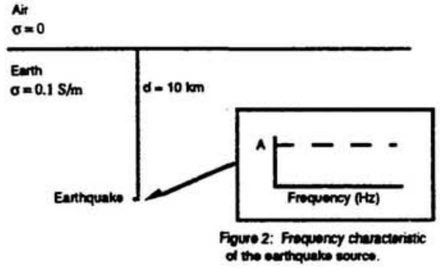


Figure 1. Earthquakes in California typically occur at depths (d, in km) of around 10 km. Lacking information about the frequencies of the electromagnetic fields they may generate, we will assume they generate white noise (Figure 2; A is the amplitude of the magnetic component).

Answer: After a brief discussion of good conductors and poor conductors, leading to the conclusion that the above problem must treat the earth as a good conductor ($\sigma/\omega\epsilon \gg 1$, where ω is the angular frequency and ϵ is the permittivity), the student must either remember or derive an expression for the attenuation of electromagnetic fields in a conducting medium in terms of the skin depth, i.e. $\boxed{\delta = \boxed{2/(\omega\mu\sigma)}^{1/2}}$, where μ is the permeability. In addition, the student should demonstrate some knowledge of how (1) the wave propagates with phase velocity $v = \omega/\beta = \omega\delta$, and (2) it is exponentially attenuated with attenuation constant $\alpha = 1/\delta$. phase velocity has $\omega^{1/2}$ frequency dependence

Using the above information, the spectrum shown in Figure 3 can be derived by considering the attenuation at just a few specific frequencies around 1 Hz – say 0.01 Hz (skin depth 16 km) and 1 Hz. Obviously, the attenuation declines to zero at frequencies lower than about 0.03 Hz, at which the skin depth is just equal to 10 km, and increases to very high values at higher frequencies.

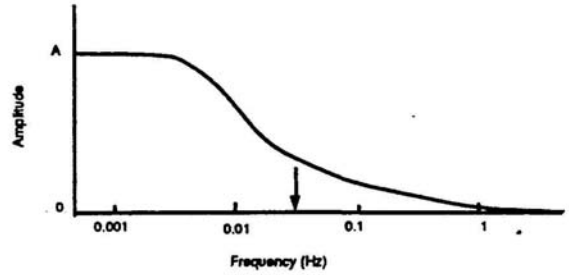


Figure 3. Spectrum (schematic) observed on the earth's surface. The arrow marks the frequency (0.03 Hz) at which the skin depth is just equal to 10 km, i.e., the depth to the source.

Students were asked if there could be any other factors in addition to absorption in the conducting medium that might reduce the strength of the earthquake signals as measured just above the earth's surface. The desired answer was some reference to reflection of the signals back down from the earth/air interface.

From: Hector Garcia-Molina <hector@DB.Stanford.EDU>
X-Authentication-Warning: Coke.Stanford.EDU: Host localhost [127.0.0.1] didn't use HELO protocol
To: shankle@ee.stanford.edu (Diane Shankle)
Subject: Re: Quals Questions 1996
Date: Sat, 27 Jan 96 09:29:24 -0800
X-Mts: smtp

> Send a copy of your Quals Question & Answer to
> Diane Shankle

Here is my question...
hector

EE QUALIFYING EXAM 1996
Hector Garcia-Molina

You have four computers, C1, C2, C3, C4, connected by a network.
Each computer Ci has a file Fi containing 1000 records.
Each record is 1000 bytes long, including an 8 byte key.
(Example: 1000 byte employee record, with 8 byte employee number as key.)
The keys range from 1 through 1,000,000.
Each computer has an unlimited amount of main memory.

We want to sort the records, so that we end up with four files,
G1, G2, G3, G4. File G1 is located at C1 and will contain the 1000
records with smallest keys. File G2 will be at C2 and will contain the
1000 records with the next 1000 smallest keys, and so on.

```
        ----------------------------------------------- network
          |           |           |           |
        ------      ------      ------      ------
        | C1 |      | C2 |      | C3 |      | C4 |
        | F1 |      | F2 |      | F3 |      | F4 |
        | G1 |      | G2 |      | G3 |      | G4 |
        ------      ------      ------      ------
```

(1) Suggest one or more efficient algorithms for sorting the files.

(a) A simple solution is to send all records to one computer, say C1,
and sort them all there. Then the records are distributed to the
appropriate computers.

(b) Another solution is to only send the 8 byte keys to C1.
C1 sorts the keys and determines the three breakpoints, b1, b2, b3.
(That is, the three records with keys less than b1 must go to C1;
the ones with keys between b1 and b2 go to C2, and so on.)
The breakpoints are sent to all computers, which then distribute the records
accordingly.

(c) A third solution is to determine the breakpoints without transmitting
all the keys. For instance, each computer can sort its records locally,
and determine three breakpoints that separate its own records into
four groups. All computers send their local breakpoints to say C1,
and C1 can determine "roughly" where the global b1, b2, b3 breakpoints are.
At this point C1 can either perform more iterations with the other computers
to refine the breakpoints, or it can go ahead and initiate the record
transfers. (If the breakpoints are not known exactly, some records may go to
the wrong computer, but when this is discovered, they can be routed
to the correct place.) Another variation is for computers to initially
send more than three breakpoints, e.g., they send a "histogram"
that more accurately reflects the key ranges it has. With these
histograms, C1 can more accurately determine the true global breakpoints.

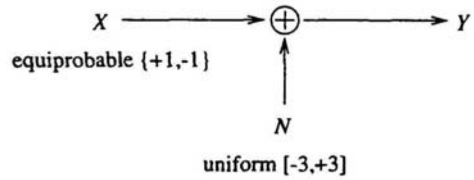
(2) What are the important metrics for evaluating the algorithms?
Can you roughly evaluate the performance of the algorithms?

Important metrics are total running time of the algorithm, number of bytes transmitted, number of messages, processor utilization. The evaluation of the algorithms depends on the exact variant that was proposed. During the exam, students we asked to focus only on the number of bytes transmitted.

1996 Qualifying Exam Questions

JOHN GILL

A communications system transmits binary data by sending one bit per unit time, representing binary values by the analog values $X = +1$ and $X = -1$.



The received signal is corrupted by additive noise; that is, $Y = X + N$. The noise N is uniformly distributed for $-3 \le N \le +3$. The two input values are equally probable.

1. Sketch the pdf (probability density function) of N.

2. What is the variance of N?

3. The noise power is defined to be its variance. What is the signal-to-noise ratio (SNR)?

4. Sketch the pdf of the received signal Y.

5. What is a good decision rule for estimating X given Y?

6. Suppose we use the simple decision rule:

$$\hat{X} = \begin{cases} +1 & \text{if } Y > 0 \\ -1 & \text{if } Y < 0 \end{cases}$$

   What is $\Pr(X = -1 \mid Y = +1)$, that is, the conditional error probability given $Y = +1$?

7. Find the overall error probability, $\Pr(\hat{X} \neq X)$.

8. To reduce the probability of error, we increase signal power by sending the same signal twice. The received signal Y is the sum of the two transmission:

$$Y = 2X + N_1 + N_2,$$

   where N_1 and N_2 are independent. What is the pdf of the received signal Y?

9. What is the optimum decision rule for estimating X given Y?

1996 PhD Quals Question
J. S. Harris

1. If I deposit a metal layer onto a semiconductor to form a metal-semiconductor junction:
   A. What occurs at the interface or junction?
   B. Draw an energy band diagram and explain what happens when a voltage is applied to the junction.

2. If I now insert a thin insulator between the metal and semiconductor:
   C. What does the insulator do?
   C. Draw an energy band diagram and explain what happens with bias.
   D. Since there is no current flow, an I-V characteristic is not meaningful. What measurement might be used to characterize an MIS structure and explain what you expect it to look like? CV measurem
   E What causes the inversion layer to form?

3. I can utilize this structure to form a transistor:
   F. What is the basis of forming such a transistor?
   G From where do the carriers originate that form the inversion layer of the transistor?
   H. Draw the Drain I-V characteristic for the transistor.
   I. What are V_{th} and V_{Dsat} and what is their origin?
   J. What does the potential look like along the channel for $V_D < V_{Dsat}$, $\boxed{V_D = V_{Dsat}}$ and $V_D > V_{Dsat}$?

when inversion layer start to form

Qualifying Exam Question

S. E. Harris
January 1996

I examined under the general heading of the basic physics of mechanics, heat, light, and sound. The student was shown a diagram of a (one-dimensional) mechanical system consisting of two masses coupled by three springs. The student was asked to come up with an electrical circuit that satisfies the same dynamic equations as do the coupled springs. We usually began by writing the differential equation for the motion of one of the masses. The student would then synthesize an appropriate coupled LC circuit. I was looking for the use of symmetry and combined mathematical-physical reasoning. For example, since the mechanical system was lossless, the inclusion of a resistor in the electrical system was a source of lost points.

The second part of the question was qualitative. I asked the student what the natural frequencies of the mechanical system are and, in particular, what is the natural frequency when the two masses move synchronously. We then examined the behavior of the mechanical system when one mass was pinned and the other was displaced.

Professor Mark Horowitz

Build a decoder for a 256 x 256 memory. The address lines A0 - A7 (and their complements, A0_b - A7_b) are available.

The only components that you can use in your design are inverters and 2 input NAND gates, and 2 input NOR gates.

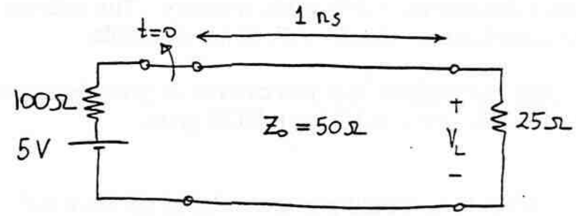
1. What logic would you use to build the decoder?
   If they have problems with that, suggest that they look at a single wordline first — WL0, and derive the logic function for this WL. If they build a full tree at each gate, need to ask, whether they can share logic ...

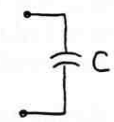
2. Having done the logic, how would you layout this design?
   Look for people worrying about the wires. Clearly the final NAND inverter should be next to the wordline, but so should the next level, unless you want to double the wire tracks

3. Assume that inverters would like to have a fanout of 4, and NAND and NOR gates would like to have a fanout of 3. Also assume that each memory cell contributes 10fF to the wordline load. Using these assumptions find the rough sizes of the gates in your decoder (you can measure the gate's size by its input capacitance).

Inan

Professor Inan/ PhD Quals Question#1 / January 1996





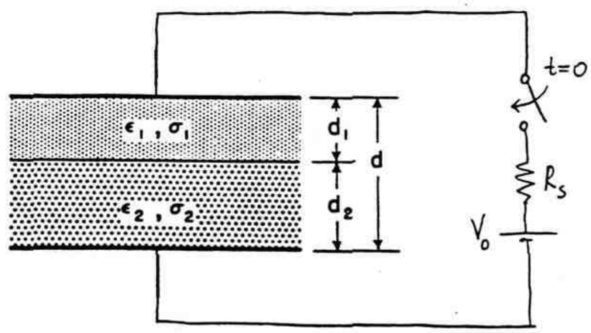
a) The students were told that the switch has been closed for a long time and that it was opened at $t = 0$. They were then told to sketch the load voltage $V_L(t)$ for $t > 0$.

In answering this question, it was important for the student to first assess the 'steady-state' situation in effect before the switch was opened. With the switch closed for a long time, the transmission line is fully charged and the load voltage is $V_L = \boxed{(5 \times 25)/(100+25) = 1\text{ V}}$. Once the switch is opened, V_L will remain unchanged until $t = 1$ ns, and then would change in discrete steps, eventually approaching zero as the line is discharged (thru the 25Ω load). To determine the manner in which V_L changes, it is better to first think in terms of the current on the line. Before the switch is opened, the current is $(1/25) = 0.04$ A. The opening of the switch requires that the current become zero. This new boundary condition requires that a new current of -0.04 A be launched from the source side. A current step of -0.04 A on a 50Ω line corresponds to a voltage step of -2 V. This voltage step of -2 V will now reflect at the load end at $t = 1$ ns, the reflected voltage will travel to the source end and reflect there at $t = 2$ ns, etc.. The reflection coefficients at both ends can be calculated and used for sketching $V_L(t)$.

load end $\Gamma_L = \frac{25-50}{75} = -\frac{1}{3}$

open end $\Gamma_L = \frac{\infty - 50}{\infty} = 1$

b) The students were then asked to comment on the same scenario (i.e., switch closed for a long time and then opened) for the case when the load consisted of a capacitance C (instead of the 25Ω resistor). In this case, the capacitor is fully charged to 5 V and the current is zero, and the opening of the switch does not introduce any new boundary conditions, hence nothing happens!

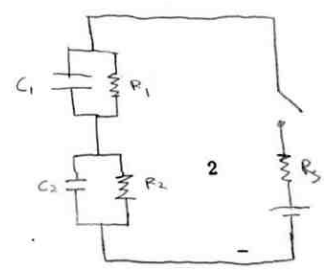


The students were told that the capacitor had two layers of $\boxed{\text{lossy}}$ dielectric material between its plates and that the switch was closed at $t = 0$. They were then asked to comment on the boundary conditions for the electric field at the interface between the two layers.

This problem is interesting because application of the most common form of the boundary conditions on the electric flux density and current (i.e., $\epsilon_1 E_1 = \epsilon_2 E_2$ and $\sigma_1 J_1 = \sigma_2 J_2$) leads to somewhat of a contradiction. The problem thus allows for a discussion of the continuity equation and the establishment of the surface charge at the interface. The proper answer is that *initially*, with no surface charge, the condition $\epsilon_1 E_1 = \epsilon_2 E_2$ is satisfied, and $\sigma_1 J_1 \neq \sigma_2 J_2$, since $\nabla \cdot \mathbf{J} = -\partial\rho/\partial t \neq 0$. Note that the initial current is determined simply by R_s, since the capacitor behaves as as a short circuit. We have $\partial\rho/\partial t \neq 0$, since current varies as the capacitor begins to accept charge. ***Eventually***, however, the system reaches a steady state so that $\partial\rho/\partial t = 0$ and thus $\nabla \cdot \mathbf{J} = 0$ and $\sigma_1 J_1 = \sigma_2 J_2$. At that point, a surface charge of ρ_s is established at the interface, so that $(\epsilon_1 E_1 - \epsilon_2 E_2) = \rho_s$. Note that a steady current $\mathbf{J}$ flows at the steady state, as determined by R_s and the series combination of the resistances of the two layers. The system can be modeled as a series combination of two paralel RC circuits; the surface charge varies from zero to ρ_s exponentially, with a time constant which can be determind from the model.

There are wrong. they should be $\sigma_1 E_1$ and $\sigma_2 E_2$

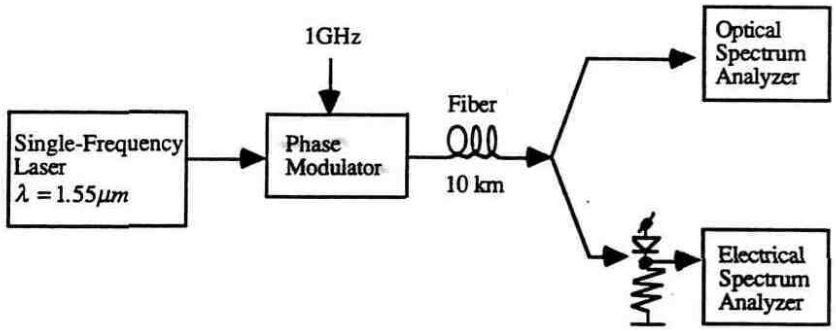
The students were simply asked to comment on the boundary conditions and were not asked to solve for $\rho_s(t)$.



Prof. Leonid Kazovsky

Ph.D. Quals. January 1996

Consider the experiment shown below:



Neglecting all noises, answer the following two questions:

1) The fiber has no dispersion: the dispersion coefficient at $\lambda = 1.55\mu m$ is $D \equiv 0$. Sketch the power spectral densities displayed by the two spectrum analyzers.

2) The fiber is strongly dispersive: the dispersion coefficient at $\lambda = 1.55\mu m$ is $D = 1.5ns/(nm \cdot km)$. Sketch the power spectral densities displayed by the two spectrum analyzers.

Mime-Version: 1.0
Date: Thu, 15 Feb 1996 12:30:29 -0800
To: shankle@ee.stanford.edu (Diane Shankle)
From: pierre@macro.stanford.edu (Pierre Khuri-Yakub)
Subject: Re: Quals Questions 1996

Diane:
My orals question was to find the inductance and capacitance of one of three wires such as in a three phase line. The answer can be found in textbooks.
Pierre Khuri-Yakub

Fabian pease

Pease's '96 QUals questions:

• What are commonest types of transistors and what are they used for?

•What are the desirable properties of a transistor used in digital logic?

•What are the limits of those properties as set by physics, and by technology?

•Suppose micromachining was developed so we could make electromechanical structures to 100nm, 10nm or even 1nm. Could we make a relay that would be as good, or even better, than a silicon transistor? In particular could we get a better on/off ratio for a given switching energy?

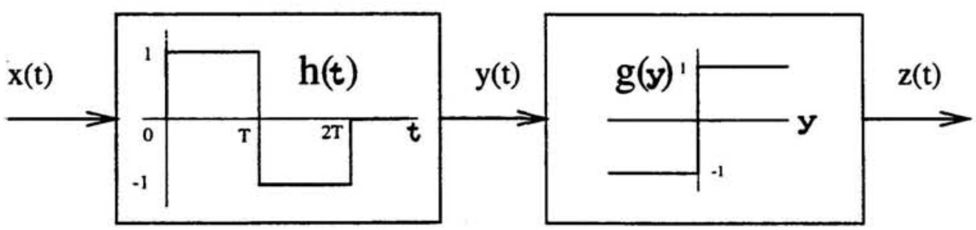
Paulraj

Paulraj: Quals Questions 1996

Let $x(t)$ be a zero mean, white, Gaussian, stationary process with a unit power spectral density.

Let $R_{xx}(\tau)$ and $R_{xy}(\tau)$ be the auto and cross correlation functions defined as usual.

Questions:



- If $x(t)$ is applied to linear time invariant system with an impulse response $h(t)$ shown and the output is $y(t)$
  - Is $y(t)$ Gaussian ?

    Answer : Yes
  - What is $R_{yy}(\tau)$

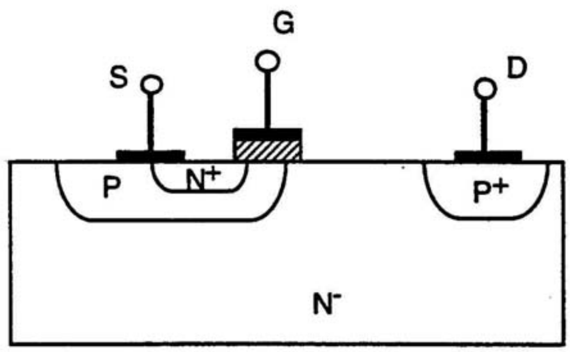
    Answer:

1996 EE QUALS

J. Plummer James

I asked a series of questions about the semiconductor device structure shown below. These questions focused first on the expected I-V characteristics of the device and why it would behave the way the student thought it would. While this looks initially like a simple structure, its behavior is actually quite complicated. Most students started by noting that it looks like an MOS transistor in series with a PN diode and therefore thought about MOS like I-V characteristics, offset from the origin by about 0.7 volts. This usually led to the observation (sometimes with some help), that actually there is a lateral PNP transistor which provides a parallel current path. So a better equivalent circuit is an MOS device driving a bipolar PNP. This gives MOS like I-V characteristics but with higher gain. Finally, some students noticed that the PNP collector current actually flows through the P region resistance on the left, generating an IR drop. This can result in latchup in the device because there is also a parasitic NPN bipolar transistor. Most students who got this far needed help to see this last possibility.

I also asked about the role that holes and electrons play in the current flow in the device. The scores I gave depended on the student's ability to reason through the operation of the device, and on how much help I needed to give during the exam.

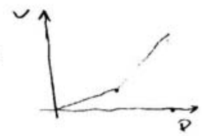


Date: Fri, 16 Feb 1996 01:45:32 PST
From: PIERO PIANETTA AT SSRL <pianetta@ssrl01.slac.stanford.edu>
To: shankle@ee.stanford.edu
Subject: RE: Quals Questions 1996

The question had three parts:

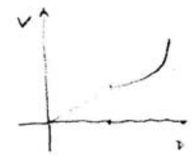
1. Assume you have two parallel plates separated by a distance, D, and filled to D/2 with a dielectric material. Assume that the remaining volume is vacuum. With a voltage V applied across the plates, plot the voltage as a function of position between the plates.

Answer: By using Maxwell's equations it is possible to calculate the voltage as a function of position between the plates in terms of the dielectric constant of the dielectric material, the distance, D, and the applied voltage. This shows that the slope of voltage versus position is smaller for the region within the dielectric versus free space. This could also be solved assuming that the system consists of two capacitors in series, one with dielectric and one without.



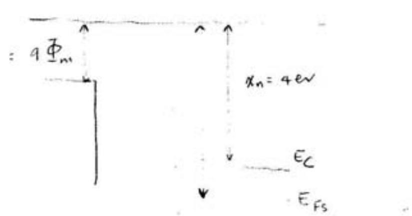
2. Replace the dielectric with a semiconductor such as silicon and determine what changes will be observed in the plot of voltage versus position.

Answer: An analogy could be made to an MOS capacitor if it was assumed that the air gap acted as the oxide realizing that the dielectric constant of vacuum is less than that of oxide. Then it is possible to determine the electric field and potential at zero applied voltage as well as consider the case with bias. The region in free space still has a linear voltage relationship while that in the semicondcutor is quadratic through the depletion region with the exact details being a function of the actual parameters assumed for the semiconductor and metal.



3. Assume that the semiconductor from part 2 has a work function of 5 eV and an electron affinity of 4 eV, and that the metal plate at the vacuum-semiconductor gap has a work function of 1 eV. Determine the band diagram and discuss what the implications the band diagram might have on the performance of such a structure.

Answer: The band diagram shows very P or n strong band bending and even possible accumulation if depending on the type of the semiconductor. There were no other expected answers to this part, primarily this part was to allow the students to give me their ideas on what might happen in such a structure which was similar to a familiar device but with very different physical constants.



1. In automata theory, what is a "language"?

2. What is a "regular language"? How many equivalent definitions do you know?

3. For every NFA, there is an equivalent DFA. However, the number of states of the DFA can be larger than the number of states of the NFA. How much larger?

4. Give an example of a language that is context free, but not a regular language. Prove each claim.

5. Let L1 and L2 be two regular languages. Is L1 union L2 a regular language? Is L1 intersect L2? Prove each claim.

Krishna Saraswat
1996 PhD Quals Question

Q1.
Fig A. shows an NMOS transistor fabricated in bulk Si wafer and Fig. B. shows an identical transistor fabricated in a quartz wafer. The Id - Vg characteristics for B show higher current than for A. How can you explain this discrepancy?

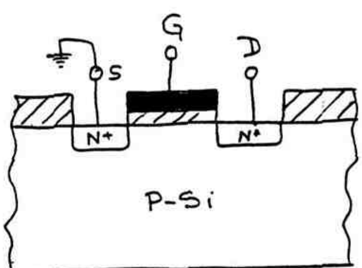
A.
The students were supposed to realize that the body of B is floating hence the source junction can get forward biased turning on a parasitic m-p-n bipolar transistor between source and drain, in addition to the normal MOS transistor.

Q2.
Will this phenomenon take place in a PMOS transistor?

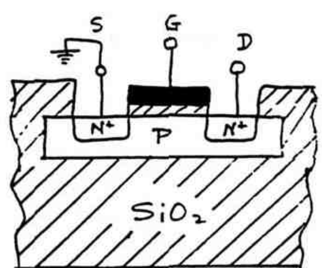
A.
Yes, but the extent will be smaller, because h_{FE} of a p-n-p bipolar is smaller, since holes have lower mobility.

Q3.
If some point defects are introduced in B as illustrated in Fig. C the excess current disappears. Why?

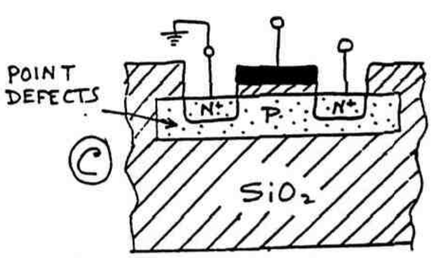
A.
The point defects act as generation - recombination centers, thus reducing the lifetime of the minority carriers. This reduces the h_{FE} of the parasitic bipolar transistor and hence the excess current disappears.



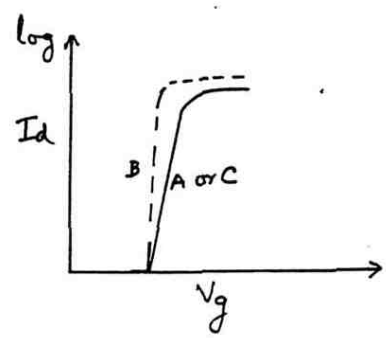
A



B



C



To: shankle@ee.stanford.edu (Diane Shankle)
Subject: Re: Quals Questions 1996
Date: Fri, 26 Jan 1996 17:31:27 -0800
From: Jennifer Widom <widom@DB.Stanford.EDU>

1996 qual questions
Jennifer Widom

Consider two relations stored in files on disk:

STUD(ID, Dnum)
DEPT(Dnum, Bldg)

The STUD relation has 20,000 records and the DEPT relation has 100 records. The files are divided into pages (blocks), and for both relations 20 records fit on each page. We have a small main-memory buffer that can store two pages of records.

We wish to compute the "natural join" of the two relations:

STUD-DEPT(ID, Dnum, Bldg)

Question 1: If neither relation is sorted, there are no indexes, and we use a standard "nested block" algorithm with STUD as the outer relation, approximately how many page fetches will it take to compute the join?

[Answer: 1000 + (1000 * 5) = 6000]
[Optimized: 1000 + 5 + (999 * 4) = 5001]

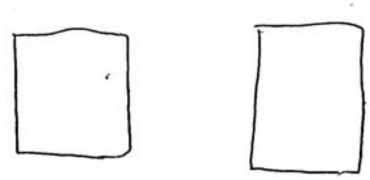
1 1
1 2
1 3
1 4
1 5
2 4
2 3
2 2
2 1
3 1

Question 2: Now suppose both relations are sorted on attribute Dnum. Approximately how many page fetches will it take to compute the join?

[Answer: 1000 + 5 = 1005]

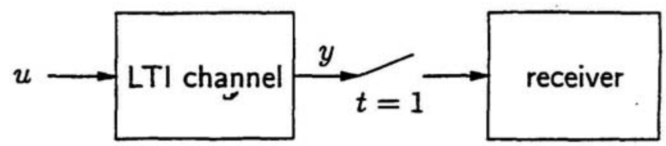
Question 3: Now suppose relation STUD is sorted on attribute Dnum and relation DEPT has a dense index that guarantees exactly 2 page accesses per lookup. Approximately how many page fetches will it take to compute the join?

[Answer: 1000 + (100 * 2) = 1200]



Please ask for further clarification.

Signal design for a simple one-bit communication system



input signal $u(t)$ is either $u_1(t)$ or $u_0(t)$
($u_i(t) = 0$ for $t < 0$)

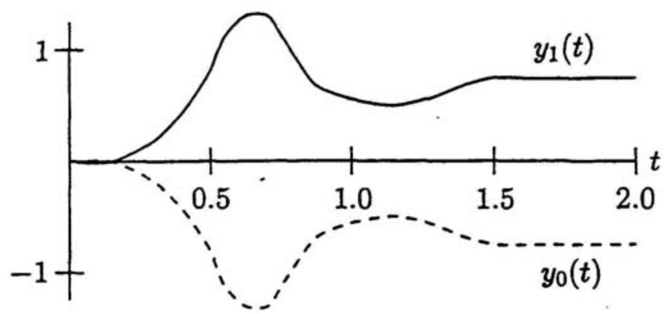
input signals are amplitude limited: $|u_i(t)| \leq 1$ for all t

receiver samples output y at $t = 1$ to determine which input signal was sent

goal: find input signals u_1 and u_0 that are maximally distinguishable to the receiver an engineer suggests the simple choice:

$$u_1(t) = 1, \quad u_0(t) = -1$$

which results in outputs y_1, y_0:



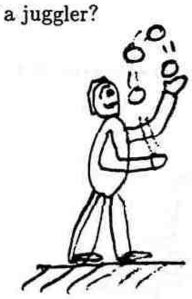
can you do better?

1996 Quals (T. Cover)

Three Questions

Juggling

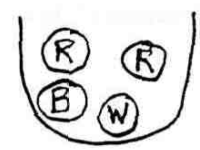
What is the average weight of a juggler?



Answer: The juggler's weight plus the weight of the balls. Otherwise the system would fly away. *why? Even if total weight is less than juggler's weight plus the weight of the ball, the system still would not fly away.*

****

Urn



Place a (nonzero) number of red, white and blue balls in an urn. Now draw n balls with replacement from the urn.

Can you make the probability that all n balls are red equal to the probability that either all are white or all are blue?

T. Cover

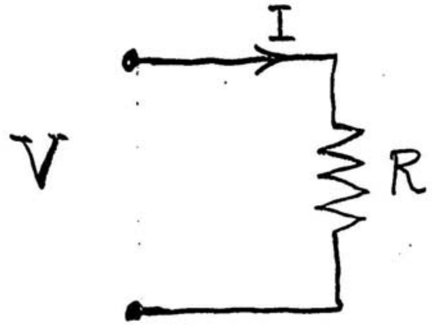
Answer: Choose r, b, w so that

$$\left(\frac{r}{r+b+w}\right)^n = \left(\frac{b}{r+b+w}\right)^n + \left(\frac{w}{r+b+w}\right)^n$$

or $r^n = b^n + w^n$. Not possible, thanks to Fermat, for $n \geq 3$. (Recognizing Fermat's theorem was not required for full credit.)

****

Current



Let V and R be the true voltage and resistance.

Let $\hat{V} = V + Z_1$ be the measured voltage and let $\hat{R} = R + Z_2$ be the measured resistance. Let Z_1 and Z_2 be independent and have mean zero and variance σ^2. Let $\hat{I} = \frac{\hat{V}}{\hat{R}}$ be the estimated current. Estimate $E(\hat{I})$ and compare to I.

Answer: A sophisticated answer would be

$$E\hat{I} = E\frac{\hat{V}}{\hat{R}} = E\hat{V}E\frac{1}{\hat{R}} \geq \frac{E\hat{V}}{E\hat{R}} = \frac{V}{R},$$

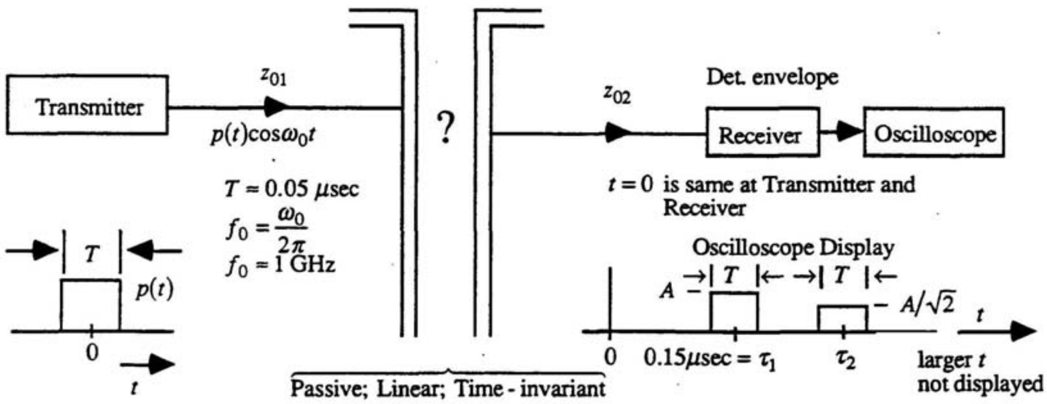
by independence and Jensen's inequality. Thus the average value of $\hat{I}$ is too high.

A perfectly adequate answer (full credit) would be to expand the denominator and look at the first order terms

$$\begin{aligned} E\hat{I} &= E\tfrac{\hat{V}}{\hat{R}} \\ &= E\tfrac{V+Z_1}{R+Z_2} \\ &= E\tfrac{V}{R(1+\frac{Z_2}{R})} \\ &= \tfrac{V}{R}E(1 - \tfrac{Z_2}{R} + \left(\tfrac{Z_2}{R}\right)^2 + \ldots) \\ &= \tfrac{V}{R}(1 + \tfrac{\sigma^2}{R^2}) + \ldots \end{aligned}$$

Ph.D. Oral Qual Question
Donald C. Cox
January 1996

The exam started with the sketch shown below on a white board on a wall. The problem was described with reference to the sketch as follows.



You go into a lab as indicated on the upper left. In the lab is a transmitter connected to a transmission line that has a characteristic impedance z_{01}. The transmitter is matched to z_{01}. The transmission line disappears through a wall as indicated. The transmitted signal is represented as $p(t)\cos\omega_0 t$ and $p(t)$ is the single rectangular pulse with period, T. The period, T, is about 0.05 μ sec and the carrier frequency, f_0, is about 1 GHz. These parameters are not too important, but they establish a framework for the problem.

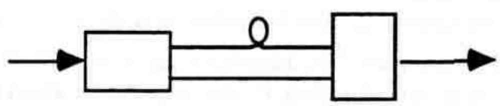
You then go to another lab as indicated at the upper right. You do not know where the second lab is located with respect to the first one. It could be next door or it could be down the street somewhere. In the second lab you see a transmission line coming out of a wall. The characteristic impedance, z_{02}, of this line may or may not be the same as z_{01}. A receiver is connected to the second transmission line and is matched to z_{02}. There is an envelope detector in the receiver that detects the envelope of the signal on the transmission line. The output of the envelope detector is displayed on an oscilloscope. There is a time reference available to mark when t=0 occurs at the transmitter, i.e. time is synchronized at the receiver and transmitter. The oscilloscope display is as indicated at the lower right. At a time τ_1 after the pulse is transmitted, a replica of the pulse envelope is received and displayed on the oscilloscope. The amplitude of the received pulse envelope is A and the time delay $\tau_1 = 0.15\ \mu$ sec. A short time later at τ_2 a second replica of the transmitted pulse envelope is received having an amplitude of $A/\sqrt{2}$. You don't know what happens after τ_2, there may or may not be other replicas of the transmitted pulse envelope.

You do not know what is providing the connection (transmission) between the two labs, but you are told that it is passive, linear, and time invariant. There are many possibilities for the transmission. We want to discuss a few. I will change the problem as we discuss it.

The first thing I want you to do is to describe one or two of the possibilities that could produce the observed waveform.

Note: passive and linear rules out detecting the original transmitted pulse and retransmitting two pulses, but this note was not stated to the examinee.

Some students observed that the pulse was not distorted, thus no dispersion. The most common reaction was to note correctly that there was a reflection. In some cases it took some discussion to elicit a reflection "in or from what." One common reaction was to connect the transmission lines and have $z_{01} \neq z_{02}$. When asked to trace the signal paths, some noted that the reflection would be absorbed by the matched transmitter, and immediately put in another transmission line in between with characteristics impedance $z_{03} \neq z_{01} \neq z_{02}$. Other suggestions noted once or twice each included: a) putting an impedance across a transmission line to produce one or both reflections, b) placing two couplers in the line and connecting with different length lines



c) connecting a shorted or open transmission line to a line connecting the labs

and, d) connecting both lines to antennas and placing a reflecting surface somewhere. Some students did not succeed in constructing any possible configuration, even with discussion and several hints.

Either, after a possible configuration was proposed (and I would say that looks like a possibility, and if we had time, perhaps you could calculate values and see if you could produce the amplitudes and delays seen, and propose we alter the question), or it became obvious that a solution was not likely, I would say that now we would add more information to the problem before we proceed. You now look out the window and see two poles or towers, one at the transmitter lab and one at the receiver lab. Transmission lines run up the poles to antennas on top. The antennas can be any kind you want, but they are the same height above the GROUND. There is nothing between the tower and antennas but air. The question now is, can the two antennas produce the display on the oscilloscope.

L: x

A few students proceeded to work their way through to a solution without help, most needed some discussion and hints, and a few made little or no progress, even with discussion and hints. Some started correctly noting a reflection was involved. When asked from what, some suggested the possibility of a building (one suggested an airplane) - to which I said these were possibilities, but looking out the window you note that there are no buildings around; the antennas and towers are isolated by themselves. Sometimes, after no progress was evident, I would say again that the antennas are equal height above the ground, or after that hint and no progress would ask what is holding the towers up. If a reflection from the ground were noted, they would be told to assume the ground was flat and perfectly conducting (aluminum).

Then I would ask how far apart the antennas were, assuming all the delay was in the air between the antennas, and would then tell them that the velocity of radio propagation in air was about 1000 ft/μ sec. Most readily made the trivial distance calculation, but a few even had trouble with that!

I would then ask if they could tell how high the antennas were from the information available assuming the antennas radiated equally in all directions. Some quickly noted either that power decreased as $1/d^2$ because of the expanding spherical wavefront or that E field in the far field went as $1/d$. Some needed to be asked what decreased as $1/d^2$ for them to realize it was (amplitude)^2. Very few (a disappointment to me) recognized that the $\sqrt{2}$ distance increased on the reflected path indicated 45° angles, but quite a few who made it this far succeeded in doing the geometry one way or another.

They would then be asked what τ_2 was; many did not make it this far.

For the very few that completed to this point, I would go back and ask for other transmission line configurations that could possibly produce the observed waveform. A very few suggested another possibility before time ran out.

There was a large range of reactions to this question ranging from not being able to proceed in any direction without hints, to working through the questions with no help.

Qualifying Examination 1996

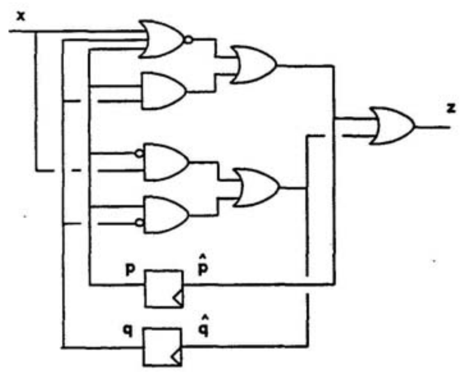
Giovanni De Micheli

January 1996

This exam has two parts. The first checks basic knowledge while the second tests intuition and ability to do research. Top scores are reserved to those who get to part 2, which requires finishing part 1 relatively quickly. (e.g., 8 min))

Part 1

The figure shows a simple finite-state machine. Assume it has a reset state $p = 0$, $q = 0$.



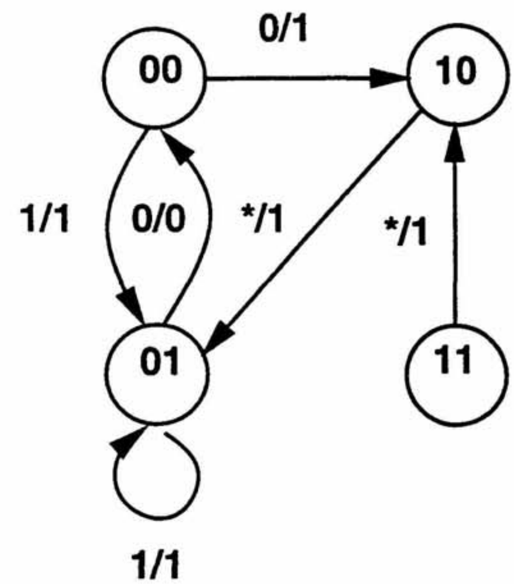
Can you derive its state diagram?

Solution

The state transition function is: $\widehat{p} = x'p'q' + pq$ *and* $\widehat{q} = xp' + pq'$. *When* $(p = 0; q = 0)$ *the function reduces to* $\widehat{p} = x'$ *and* $\widehat{q} = x$. *Equivalently, the states reachable from the reset state are encoded as:* $(p = 1; q = 0)$ *and* $(p = 0; q = 1)$.

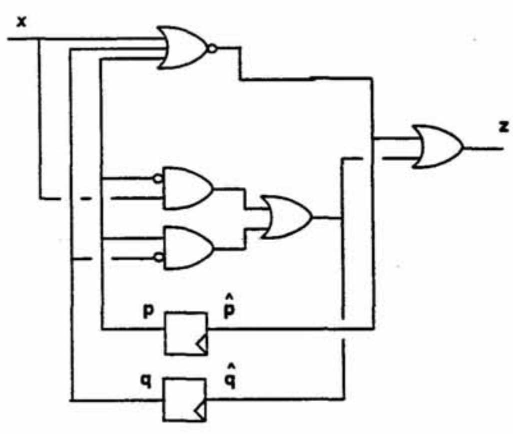
The state reachable from $(p = 1; q = 0)$ *is* $(p = 0; q = 1)$ *for any input. The states reachable from* $(p = 0; q = 1)$ *are* $(p = 0; q = 0)$ *with* $x = 0$ *and* $(p = 0; q = 1)$ *with* $x = 1$ *(self-loop). State* $(p = 1; q = 1)$ *has a transition into* $(p = 1; q = 0)$ *for any input. The state diagram is shown next.*

De Micheli



Can the encoding of the unreachable state be used as *don't care* condition to reduce the size of the combinational logic? How?

Solution Yes. *Since* $(p = 1; q = 1)$ *can never happen,* $pq = 0$ *always and an AND gate can be removed. Consequently and OR gate can be replaced by a wire.*

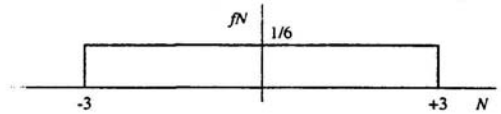


1996 Qualifying Exam Answers

John Gill

The first three questions are warmup questions. The last two questions are bonus questions; about 15% of the examinees reached the last two questions.

1. The pdf of N has the constant value $1/6$ between -3 and $+3$, zero elsewhere.



2. The variance of a random variable uniformly distributed on the interval $[a, b]$ is $(b-a)^2/12$. So the variance of the noise N is $6^2/12 = 3$. The variance can also be calculated from the definition:

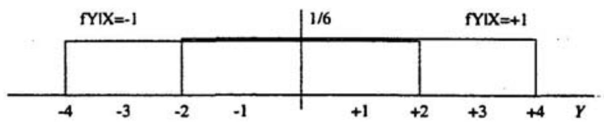
$$\sigma_N^2 = E[N^2] - E[N]^2 = \int_{-3}^{+3} \frac{1}{6} x^2 \, dx = \frac{x^3}{18} \Big[_{-3}^{+3} = \frac{(+3)^3 - (-3)^3}{18} = 3 \,.$$

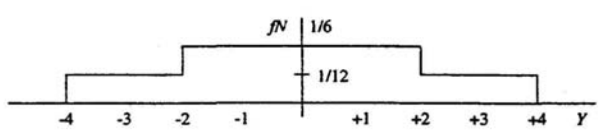
3. The power of the signal X is 1 since the magnitude of X is always 1. The variance of X is also 1 because

$$\sigma_X^2 = E[X^2] - E[X]^2 = \frac{1}{2}(+1)^2 + \frac{1}{2}(-1)^2 = 1 \,.$$

Therefore the signal-to-noise ratio is $1/3$.

4. For each value of X, the conditional probability density of Y is uniformly distributed about that value of X. The conditional densities $f_Y(y \mid X = \pm 1)$ and the unconditional density $f_Y(y)$ are shown below.





Combining the two conditional densities is the same as convolving the pdf of X, which is two impulses of height $1/2$ located at ± 1 with the pdf of N.

5. The obvious decision rule is to estimate that $X = +1$ when $Y > 0$ and that $X = -1$ when $Y < 0$. This decision rule is optimal, as shown below.

6. The conditional probability of error given any particular received value y is

$$\Pr(X \neq \hat{X} \mid Y = y) = \frac{\Pr(X \neq \hat{X} \text{ and } Y = y)}{\Pr(Y = y)} .$$

When $Y = +1$ the estimate of X is $\hat{X} = +1$, so the conditional error probability is

$$\frac{\Pr(X \neq -1 \text{ and } Y = +1)}{\Pr(Y = +1)} = \frac{\Pr(X = -1)\Pr(Y = +1 \mid X = -1)}{\Pr(Y = +1)} = \frac{(1/2)\cdot(1/6)}{(1/6)} = \frac{1}{2} .$$

Both $\Pr(Y = +1)$ and $\Pr(Y = +1 \mid X = -1)$ are values of probability density functions, so their quotient is meaningful.

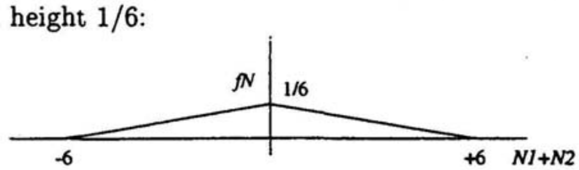
The same conditional error probability of $1/2$ is obtained for all values of Y between -2 and $+2$. For these values of Y, the receiver knows no more about X after receiving Y than before X was transmitted. Therefore $\hat{X} = +1$ and $\hat{X} = -1$ are equally good estimates, so the simple decision rule based on the sign of Y is optimal. (Or $\hat{X}$ could be decided by tossing a coin when $-2 < Y < +2$.)

7. When $Y < -2$ it is certain that $X = -1$, and when $Y > +2$ it is certain that $X = +1$. For Y in these two intervals, the conditional error probability is 0. When $-2 < Y < +2$, the conditional error probability is $1/2$. The overall error probability is

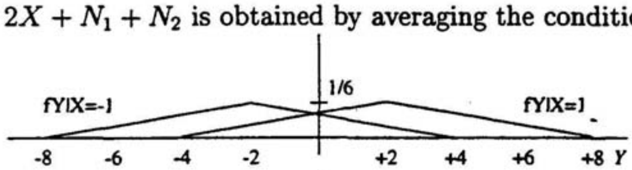
$$\Pr(|Y| > 2)\cdot 0 \; + \; \Pr(|Y| < 2)\cdot \tfrac{1}{2} \; = \; \tfrac{2}{3}\cdot\tfrac{1}{2} \; = \; \tfrac{1}{3} .$$

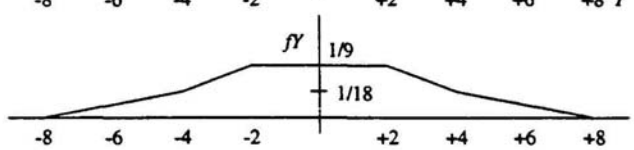
Another way to evaluate the error probability is to condition on X. When $X = -1$, an error occurs when $Y = X + N > 0$, that is, when $N > 1$. From the pdf for N, we see that $\Pr(N > 1) = 1/3$, so $\Pr(\hat{X} \neq X \mid X = -1) = 1/3$. By a similar calculation, $\Pr(\hat{X} \neq X \mid X = +1) = 1/3$. The overall error probability, which is the average of these two conditional probabilities, is $1/3$.

8. The combined noise is the sum of two independent uniformly distributed random variables. The pdf of the sum is the convolution of two rectangle functions, which is a triangle with range $[-6, +6]$ and height $1/6$:



The pdf for $Y = 2X + N_1 + N_2$ is obtained by averaging the conditional pdfs:





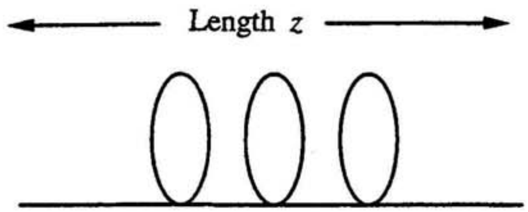
9. The optimum decision rule is the obvious decision rule:

$$\hat{X} = \begin{cases} +1 & \text{if } Y > 0 \\ -1 & \text{if } Y < 0 \end{cases}$$

Since the values of X are equiprobable, minimum error decoding is the same as maximum-likelihood decoding. Because the pdfs of Y given X are triangular, $\Pr(Y = y \,|\, X = -1) > \Pr(Y = y \mid X = +1)$ when $y < 0$, whereas $\Pr(Y = y \mid X = -1) < \Pr(Y = y \mid X = +1)$ when $y > 0$.

The overall error probability is the conditional error probability given either value of X. For $X = -1$ this conditional probability is the area under the left triangle to the right of the vertical axis. Thus $\Pr(\hat{X} \neq X) = \Pr(N_1 + N_2 > 2) = 2/9$. As expected, this error probability is smaller than $1/3$, the error probability for a single transmission of X.

Qualifying Exam Questions
J.W. Goodman



$s_1(t) = A\cos(\omega t + \varphi)$ $\qquad\qquad$ $s_1(t) = A\cos(\omega t - \beta(\omega)z + \varphi)$

$$\beta(\omega) = \beta_1\omega + \beta_3\omega^3$$

A signal $s_1(t)$ is applied to a length of optical fiber z meters long. The response at the fiber output is $s_2(t)$ with the form shown above, valid for all A, ω, and φ.

1. Is this system linear? How do you know?

   There are some subtleties to this question. No direct information about additivity has been given. We do know that the complex exponentials are the eigenfunctions of linear, time-invariant (LTI) systems. Is this equation telling us that a complex exponential excitation emerges from the system as a complex exponential of the same frequency? Almost. If $(Ae^{j\varphi})e^{j\omega t}$ emerges from the system as $(Ae^{j\varphi})e^{j[\omega t-\beta(\omega)z]}$, then the system must be LTI. It is always mathematically possible that the $e^{j\omega t}$ component of the input emerges from the system as an $e^{-j\omega t}$ output, but this would be rather bizarre behavior. We know that we are dealing with real optical fiber here, so we can rule out pathological behavior on physical grounds. Almost no one recognized this subtlety. Most said the system was linear and then tried to find some way to prove additivity.

2. Is it time-invariant? How do you know?

   Simple substitution of $t-\tau$ for t in the input shows that the output suffers a simple time delay by τ, so this system is obviously time invariant.

3. Does it have a transfer function? If so, what is it? If not, why not?

   Accepting that it is linear, and knowing that it is time invariant, the system must have a transfer function. By considering a complex exponential input, it is easily shown to be $e^{-j\beta(\omega)z} = e^{-j(\beta_1\omega+\beta_3\omega^3)z}$.

4. Interpret the various phase terms in the transfer function. What are their effects individually?

   The linear phase term causes a simple delay by time $\beta_1 z$. The cubic phase term causes dispersion of the signal, that is a blurring or smearing of the signal at the output.

Goodman

5. An arbitrary periodic excitation $p(t)$ is applied to the input of the fiber. It is claimed that for certain lengths z of the fiber, the output will suffer no dispersion. What lengths are these?

First, we are interested only in a periodic signal $p(t)$ with period we will call T. Since $p(t)$ can be expanded in a complex Fourier series, $p(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi n}{T}t}$, making the dispersion vanish at the discrete set of frequencies $\frac{n}{T}$ will suffice. The linear phase term introduces only a time delay, so it can be ignored. For the cubic phase term, evaluated at these discrete frequencies, we will have no dispersion if $\beta_3\left(2\pi\frac{n}{T}\right)^3 z = k2\pi$, where k is an integer. Solving for z we see that there is an infinite set of lengths z where dispersion vanishes. They are $z = \frac{kT^3}{\beta_3(2\pi)^2 n^3}$. The shortest distance for which dispersion vanishes is $z = \frac{T^3}{\beta_3(2\pi)^2}$. Note that each harmonic suffers a phase change of a different multiple of 2π radians. Other distances where dispersion vanishes are integer multiples of this distance.

1996 Quals question of R.M. Gray

The quals question is presented with a variety of solutions.

A system takes as input an N-dimensional vector $x = (x_0, x_1, \ldots, x_{N-1})^t$ (t denotes transpose) and converts it into an N-dimensional vector $y = (y_0, y_1, \ldots, y_{N-1})^t$ using the formula $y = Hx$, where H is an $N \times N$ matrix. Let I denote the $N \times N$ identity matrix and define the $N \times N$ matrix S by

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \cdot & & \cdot & & & & & \cdot \\ \cdot & & & & \cdot & & & \cdot \\ \cdot & & & & & \cdot & & \cdot \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

1. Suppose that $H = S$. If you are given the output vector y, how do you compute the input vector x?

   Solution and Comment:

   In general the answer is to find the inverse matrix H^{-1} and find x as $x = H^{-1}y$, where $H^{-1}H = I$. The meat of the question, however, is to actually evaluate the inverse. Cookbook formulas are not of immediate help here and are hard to apply, so the idea is to find a way that works for arbitrary N. The easiest way is to consider what

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S does to a vector and figure out how to undo it. A little thought shows that

$$Sx = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \cdot & & \cdot & & & & & \cdot \\ \cdot & & & & \cdot & & & \cdot \\ \cdot & & & & & & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_{N-1} \end{pmatrix} = \begin{pmatrix} x_{N-1} \\ x_0 \\ x_1 \\ \vdots \\ x_{N-2} \end{pmatrix},$$

i.e., the matrix S just cyclically rotates the elements of x. This is a *cyclic shift* operation in digital signal processing. This is undone by reversing the shift direction and rotating in the other direction. Again a little thought shows that this is accomplished by the matrix

$$S^t = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ \cdot & & & & \cdot & & & & \cdot \\ \cdot & & & & & \cdot & & & \cdot \\ \cdot & & & & & & & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

and hence $S^{-1} = S^t$.

Another way to do the problem is to write out the definition of an inverse and solve the resulting system of linear equations. Suppose

$S^{-1} = \{\sigma_{k,j};\ k = 0, \ldots, N-1;\ j = 0, \ldots, N-1\}$ and $S = \{s_{k,j};\ k = 0, \ldots, N-1;\ j = 0, \ldots, N-1\}$. Then the formula for the inverse becomes

$$\sum_k \sigma_{l,k} s_{k,m} = \delta_{l-m},$$

where δ_i is the Kronecker delta (1 if $i = 0$ and 0 otherwise). Since $s_{k,m}$ is 1 if and only if the row index is one greater than the column index (modulo N), $s_{k,m} = \delta_{k-1-m}$, and the formula becomes

$$\sum_k \sigma_{l,k} \delta_{k-1-m} = \sigma_{l,m+1} = \delta_{l-m}$$

which means that $\sigma_{l,k} = \delta_{l-(k-1)} = \delta_{l-k+1)}$ is 1 if and only if $l = k-1$, i.e., the row index is one less than the column index, as found before.

A third way to solve the problem would be to find the eigenvalues and eigenvectors and to use these to construct an inverse. This is harder and requires the next part of the question.

The point was look at the structure of the matrix and not just try to plug into likely forgotten complicated formulas.

2. What are the eigenvalues and eigenvectors of H?

Solution and Comment:

Eigenvalues λ and eigenvectors u are solutions to the equation

$$Hu = \lambda u$$

or, equivalently, $(H - \lambda I)u = 0$, where I is the identity matrix. In the case of $H = S$, this is a system of equations

$$u_{N-1} = \lambda u_0$$

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$$u_{n-1} = \lambda u_n; \; n = 1, 2, \ldots, N-1. \qquad (1)$$

This means that the eigenvector must satisfy $u_n = \lambda^{-n} u_0$ for $n = 1, 2, \ldots, N-1$ and that $u_0 = \lambda^{-1} u_{N-1} = \lambda^{-N} u_0$. Thus λ must satisfy $\lambda^N = 1$, that is, it must be a root of unity, e.g., have the form $\lambda = e^{\frac{j2\pi k}{N}}$ for some integer k. (For those rusty with the idea of roots of unity, if follows most easily from noting that $1 = e^{j2\pi k}$ for any integer k, hence $e^{\frac{j2\pi k}{N}}$ are Nth roots of 1. These are distinct for $k = 0, 1, \ldots, N-1$.) Thus we have N distinct eigenvalues. For each eigenvalue λ, (1) implies that the corresponding eigenvalue must have the form $(1, \lambda^{-1}, \lambda^{-2}, \ldots, \lambda^{-(N-1)})$ (where I have chosen $u_0 = 1$, any other constant will do). Thus 1 is an eigenvalue with eigenvector the all ones vector. $e^{\frac{j2\pi}{N}}$ is an eigenvalue with eigenvector $(1, e^{\frac{-j2\pi}{N}}, \ldots,)$, etc.

In terms of discrete Fourier transforms, these eigenvalues correspond to the discrete Fourier transforms of shifted Kronecker deltas, which are the rows of the S matrix.

Another way to tackle the eigenvalues is to consider the solutions of the equation

$$\det|H - \lambda I| = 0.$$

For those adept at manipulating determinants this gives the eigenvalues, but you still need to find the eigenvectors.

3. Now suppose that $H = I - aS$, where $0 < a < 1$. Again find the inverse, eigenvalues, and eigenvectors of H.

Solution and Comment:

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Now we are interested in the inverse, eigenvalues, and eigenvectors of the more complicated matrix

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 & -a \\ -a & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -a & 1 & 0 & 0 & \cdots & 0 & 0 \\ \cdot & & \cdot & & & & & \cdot \\ \cdot & & & & \cdot & & & \cdot \\ \cdot & & & & & \cdot & & \cdot \\ 0 & 0 & 0 & 0 & 0 & \cdots & -a & 1 \end{bmatrix}$$

The easiest part is to find the eigenvalues and eigenvectors, so do this first. The easiest way to do this is to note that the eigenvalue/eigenvector equation is now

$$Hu = (I - aS)u = \alpha u$$

where u is the eigenvector and α the eigenvalue. This is just

$$((1-\alpha)I - aS)u = 0$$

or

$$(S - (\frac{1-\alpha}{a})I)u = 0,$$

which looks *exactly* like the previously solved problem of $(S - \lambda I)u = 0$. Thus the new eigenvalues α relate to the old eigenvalues λ as

$$\frac{1-\alpha}{a} = \lambda$$

and the eigenvectors are exactly what they were before!

Of course the problem can also be solved in the direct manner, by just writing out the equations: for each k

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$$\sum_m h_{k,m} u_m = \lambda u_k$$

and

$$\sum_m (\delta_{k-m} - a\delta_{k-m-1}) u_m = u_k - a u_{k-1}$$

so that

$$u_k = \frac{a}{1-\lambda} u_{k-1}$$

provides a recursion for the eigenvector elements. Again using the modulo property, the eigenvalues must satisfy

$$\left(\frac{a}{1-\lambda}\right)^N = 1.$$

Note that this means that now $a/(1-\lambda)$ must be a root of unity, and hence the eigenvectors will be *exactly as in the previous simple case!*

What is going on here is that both of these matrices are *circulant matrices*, formed by taking all cyclic shifts of one row. All such matrices have the same set of eigenvectors, formed by complex exponentials. These manipulations are, in fact, just another way of looking at the discrete Fourier transform. Diagonalizing such a matrix using the eigenvalues and eigenvectors is in fact just writing down the matrix form of the inverse DFT. The problem, however, was aimed at testing skills at inverting matrices and solving equations that arise in systems analysis (especially when done digitally) when the cookbook formulas fail, but the underlying structure still allows for solutions.

The inverse is the only tricky part and only a few people solved it. Unlike the previous case this is not obvious intuitively, although it is possible to construct the inverse by intuitive arguments. Here the straightforward way

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of writing out the equations is best. Suppose that
$H^{-1} = \{b_{k,j};\ k = 0, \ldots, N-1;\ j = 0, \ldots, N-1\}$ and let
$H = \{h_{k,j};\ k = 0, \ldots, N-1;\ j = 0, \ldots, N-1\}$. Since $H = I - aS$ we know that $h_{k,m} = \delta_{k-m} - a\delta_{k-m-1}$ and hence the formula for the inverse becomes

$$\sum_k b_{l,k} h_{k,m} = \delta_{l-m}$$

or

$$\sum_k b_{l,k}[\delta_{k-m} - a\delta_{k-m-1}] = b_{l,m} - ab_{l,m+1} = \delta_{l-m},$$

where as before we evaluate the subscripts as modulo N. To see what this means, fix the row index l. If the column index m is the same as l, then we must have

$$b_{l,l} - ab_{l,l+1} = 1,$$

that is, the diagonal entry $b_{l,l}$ must be $1 + a$ times the entry to the right in the same row. If $l \neq m$ so that we are looking at a nondiagonal entry, then

$$b_{l,m} = ab_{l,m+1};\ l \neq m,$$

that is, the entry is a times the entry in the same row on the right. Analogous to the eigenvalue computation previously done, iteratively applying this relation and using the previous relation we have that

$$b_{l,l+1} = a^{N-1} b_{l,l}$$

and

$$b_{l,l+1} = \frac{b_{l,l} - 1}{a}$$

and hence

$$a^{N-1} b_{l,l} = \frac{b_{l,l} - 1}{a}$$

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or $b_{l,l} = \frac{1}{1-a^N}$. Thus the matrix is

$$H^{-1} = \frac{1}{1-a^N} \begin{bmatrix} 1 & a^{N-1} & a^{N-2} & a^{N-3} & a^{N-4} & \cdots & a^2 & a \\ a & 1 & a^{N-1} & a^{N-2} & a^{N-3} & \cdots & a^3 & a^2 \\ a^2 & a & 1 & a^{N-1} & a^{N-2} & \cdots & a^4 & a^3 \\ \cdot & & \cdot & & & & & \cdot \\ \cdot & & & & \cdot & & & \cdot \\ \cdot & & & & & \cdot & & \cdot \\ a^{N-1} & a^{N-2} & a^{N-3} & a^{N-4} & a^{N-5} & \cdots & a^2 & 1 \end{bmatrix}$$

There is another way to do this which is more sneaky or elegant, depending on your point of view. The matrix $(I - aS)^{-1}$ can be expanded in a way analogous to the geometric series sum $(1 - a)^{-1}$, i.e.,

$$(I - aS)^{-1} = \sum_{k=0}^{\infty} a^n S^n$$

is the matrix analog of

$$\frac{1}{1-a} = \sum_{k=0}^{\infty} a^n.$$

But the matrix S^n is just a cyclic shift of n, and this is periodic with period N, that is, $S^k = S^{k+MN}$ for any integer M. Thus this sum becomes

$$\sum_{k=0}^{N-1} \left(\sum_{l=0}^{\infty} a^{k+lN} \right) S^k = \frac{1}{1-a^N} \sum_{k=0}^{N-1} a^k S^k,$$

which is just what was found above.

From: ag@pepper.Stanford.EDU
To: shankle@ee.stanford.edu (Diane Shankle)
Cc: ag@pepper.Stanford.EDU
Subject: Re: Quals Questions 1996
Date: Tue, 20 Feb 96 11:34:55 PST

Here are the questions. -- Anoop.

P.S> I hope this email is fine and that you don't need a hardcopy.

Here are the questions for EE quals. I am not including the answers, because I accepted a wide range of answers. Also, many students attempted only a subset of the questions.

1. After all this talk about RISC and providing only for primitives and not solutions in ISA, why are ISAs growing again? Can you give examples of instructions that are being added? Do you think this is a good long-term trend, or are we goint to be regretting it?

2. In the MIPS architecture, branches and loads have delay slots associated with them. DEC Alpha architecture does not have delay slots. What are the tradeoffs? Which do you think is a better architecture?

3. What is meant by the term "working set" of a program? Does a program have only one, or may it have a hierarchy of working sets? What is the importance of understanding working sets in computer system design?

4. Consider scientific codes (flt. pt. intensive and array based). What are the relative tradeoffs, from a cost-performance perspective, between a superscalar CPU and vector CPU?

5. What is meant by the term memory consistency model for a microprocessor? How is it different from cache coherence? Give an example of a relaxed consistency model?

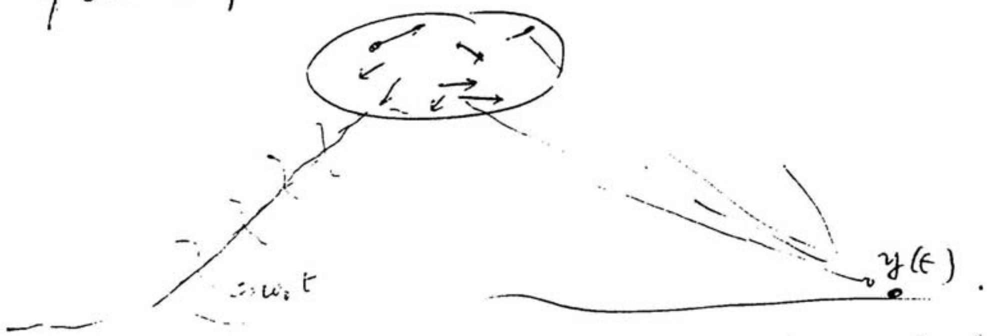
6. With processor clock rates going up much faster than memory speeds, memory stalls are becoming a major performance bottleneck. What are the various ways to reduce the latency problem? Give examples of ways to reduce latency and to hide latency.

7. Microprocessor chips in the future may have a 100 million transistors? As an employee of intel, who still wants to dominate the market, suggest some ways in which you would use these 100 million transistors?

1996 Ph.D. Quals. — T. Kailath. (1)

Prob: A model for scattered signals.

A sinusoidally-varying plane wave is scattered in various directions by a cloud of small particles, moving around randomly.



Make various "reasonable assumptions to develop a model for the signal $y(\cdot)$.

Solution: Skipping a lot of the words:

$y(t) = \sum_{i}^{N} \rho_i \cos(\omega_0 t + \theta_i)$, θ_i and ρ_i assumed to be independent & ~~iid~~ the $\{\rho_i\}, \{\theta_i\}$ to be i.i.d.

We assume the θ_i to be uniformly distributed over $(-\pi, \pi$

Then ~~write~~:

$$y(t) = A \cos \omega_0 t - B \sin \omega_0 t, \quad A = \sum_{1}^{N} \rho_i \cos \theta_i, \; B = \sum_{1}^{N} \rho_i \sin$$

We see that $E[A] = 0 = E[B]$, $E[AB] = 0$.

~~Though~~ Since A and B are uncorrelated, we can use a multidimensional version of the Central Limit

Theorem to conclude that for large N, $\{A, B\}$ are independent normal random variables. Then we can write

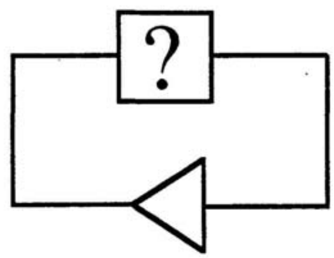
$$y(t) = \sqrt{A^2 + B^2} \cos(\omega_c t + \phi),$$

where $\sqrt{A^2+B^2}$ will have a Rayleigh distribution while ϕ will be uniformly dist. over $(-\pi, \pi)$.

G. KOVACS' QUALS QUESTION JAN 1996

This question involved a physical/electronic feedback circuit and was intended to help provide insight into the examinee's thinking processes.

The examinee was seated in front of an assemblage of electronic equipment, and their attention was specifically directed toward a small region containing a (full) can of beer with some objects glued onto opposite sides. The student was told that the system was a feedback loop. An oscilloscope in the apparatus displayed a flat line, representing a sample of the signal somewhere in the loop. The loop looked like (question mark representing the region the student was asked to look at):



The examinee was shown that when the amplifier gain was increased, oscillations began in the loop. The question posed was to explain, physically, how the oscillations occurred, paying particular attention to the region of the apparatus around the beer can.

The examinee was free to physically investigate the region of the apparatus in question, but was instructed to ask before touching anything.

The apparatus consisted of an ordinary metal nut glued to one side of the beer can and positioned close to the pole piece of an electromagnet. On the opposite side of the beer can was an accelerometer, also glued to the beer can, and providing feedback for the loop. The output of the amplifier was applied to the electromagnet, which could pull on the nut, thus slightly deforming the beer can. The mechanism of oscillation was that, with sufficient amplifier gain, the beer can could be driven into resonance.

By physically interacting with the apparatus and asking questions, the examinee was potentially able to deduce the nature of the oscillator.

The student's *approach* to the problem was most important.

Nishimura
[illegible]

Videotape Problem

A fan will start and eventually reach its fastest rotational speed. Consider what is happening when it reaches its fastest rotational speed. A marker (piece of tape) on 1 of the 3 fan blades will help you watch it.

Now Watch the Videotape

- At what speed is the fan rotating? Explain.

- Are there other rotational speeds that would give a video identical to what you have just seen (when the fan is rotating at its fastest speed)?

- Why does the fan appear to be nearly stationary in the video?

Nishimura

Answers: Videotape Problem

- At what speed is the fan rotating? Explain.

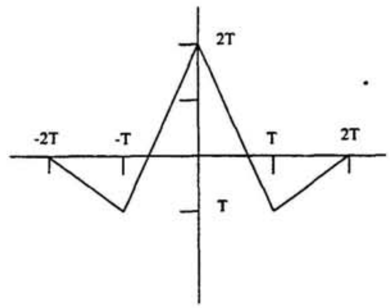
Given a TV frame rate of 30 Hz and the fact that you see 3 markers appearing stationary, the fan is rotating at 10 Hz, one-third the sampling rate.

- Are there other rotational speeds that would give a video identical to what you have just seen (when the fan is rotating at its fastest speed)?

Yes, $(10 + 30n)$ Hz rotations would give the same video, frame by frame. Note that n could be a positive or negative integer. Note too that a 20 Hz rotation would appear to be the same *perceptually*, but not on a frame-by-frame basis.

- Why does the fan appear to be nearly stationary in the video?

Because there are 3 fan blades and the rotation rate is one-third the sampling rate. If there were 2 fan blades and the rotation rate was one-half the sampling rate, it would also appear stationary.



– What is $R_{xy}(\tau)$

Answer: $h(\tau)$

- If $y(t)$ is applied to a memoryless device $g(y) = sgn(y)$ as shown and the output is $z(t)$

  – Is $z(t)$ Gaussian ?

  Answer: No

  – What is $R_{zz}(\tau)$

  Answer:

$$R_{zz}(\tau) = \frac{2}{\pi} \arcsin\left(\frac{R_{yy}(\tau)}{R_{yy}(0)}\right)$$

  – What is $R_{yz}(\tau)$

  Answer:

$$R_{yz}(\tau) = \sqrt{\frac{2}{\pi R_{yy}(0)}}\, R_{yy}(\tau)$$

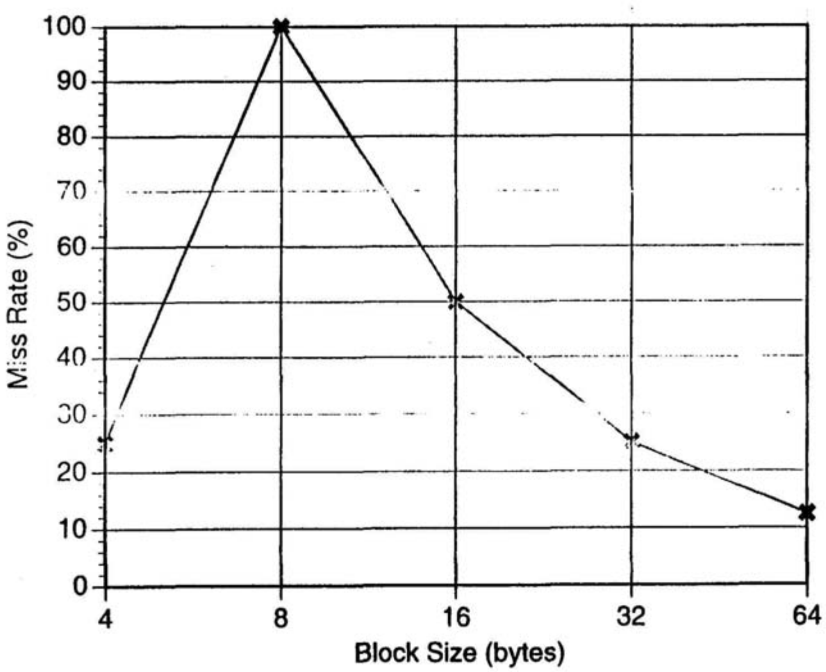
<u>Cache</u>

16384 bytes
Fully associative
LRU replacement

<u>Application</u>

```
int x[8192], y, i, j; /* 4 byte integers */

for (i = 0; i < 4; i++)
   for (j = 0; j < 8192; j += 2)
       y = x[j];
```



Q: Graph the miss rate vs. block size for the cache and application.
A: Shown on graph

Q: What types of locality are being exploited at the various block sizes?
A: 4B-temporal, 8B-none, 16B–64B-spatial

Q: What types of misses occur at the at the various block sizes? Use 3-Cs model
A: 4B-compulsory, 8B–64B-compulsory, capacity. No conflict misses.

Q: Which block size provides the best performance? Assume 4B wide refill bus.
A: Depends on latency (LA) and bandwidth (BW) of cache refill. To achieve higher performance with a 64B block than with a 4B block: LA > 60B/BW.