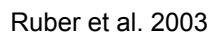
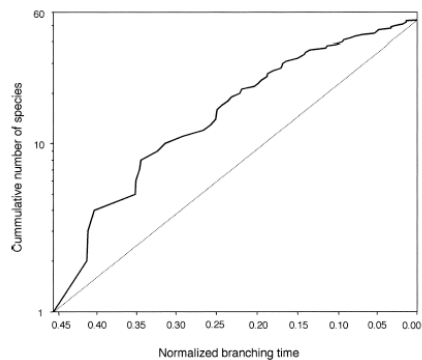


## This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- [illegible]

[illegible]

## Slowdown statistic

- Tests for longer branches near the tips of a phylogenetic tree
- Signature of an early, rapid radiation

$$\gamma = \frac{\left(\frac{1}{n-2} \sum_{i=2}^{n-1} \left(\sum_{k=2}^i k g_k\right)\right) - \left(\frac{T}{2}\right)}{T \sqrt{\frac{1}{12(n-2)}}}, T = \left(\sum_{j=2}^n j g_j\right).$$

## Slowdown statistic

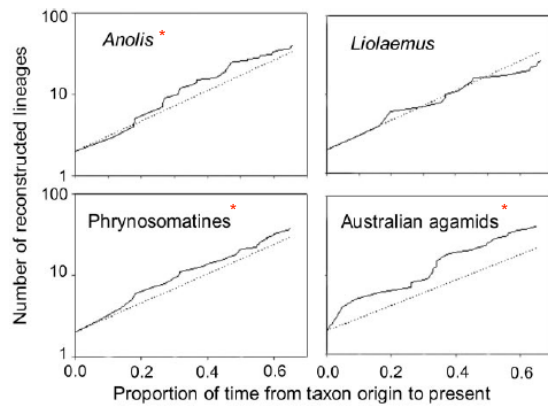
- Under pure-birth model,  $\gamma \sim N(0,1)$
- $\gamma < 0$  means diversification has slowed
- $\gamma > 0$  difficult to interpret

$$\gamma = \frac{\left(\frac{1}{n-2} \sum_{i=2}^{n-1} \left(\sum_{k=2}^i k g_k\right)\right) - \left(\frac{T}{2}\right)}{T \sqrt{\frac{1}{12(n-2)}}}, T = \left(\sum_{j=2}^n j g_j\right).$$

## Slowdown statistic

- Many (most?) clades show this signature of slowing rates of diversification through time
- Signature of adaptive radiation?

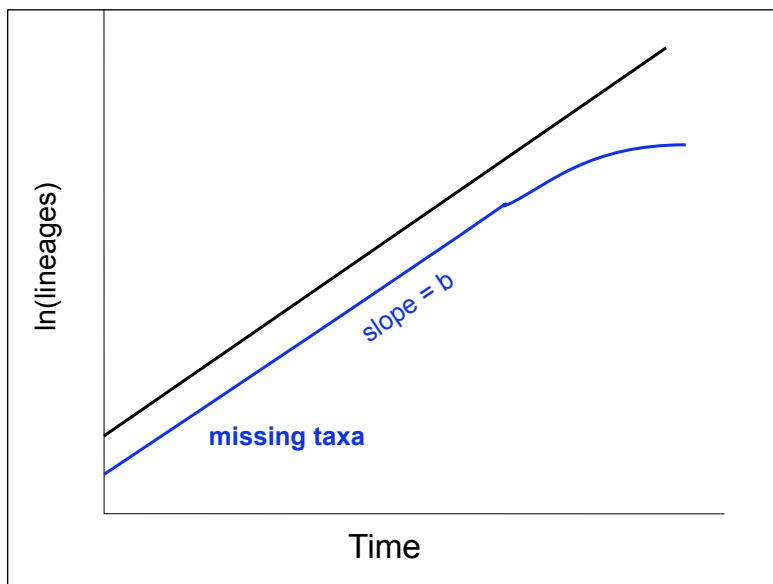
## Examples



(Harmon et al. 2005 Science)

## What about missing taxa?

- Missing taxa are more likely to connect to the tree near the tips
- You are missing taxa in your tree
- Randomly missing taxa looks like a slowdown



## Missing Taxa

- Have to account for missing taxa when interpreting a slowdown
- Can use simulations
- What if missing taxa are non-random?

---

---

---

---

---

---

---

---

---

---

## The problem...



---

---

---

---

---

---

---

---

---

---

## Outline

- How do lineages accumulate under the birth-death model?
- Estimating speciation and extinction rates from trees
- Testing hypotheses about diversification

---

---

---

---

---

---

---

---

---

---

# Brownian Motion



Lecture 4

# Brownian Motion

- A model for the evolution of continuously-valued characters
- States change continuously through time
- After some time, expected character states follow a normal distribution

# Outline - BM

- What is Brownian motion?
- When might characters evolve in a Brownian-like way?
- Simulating Brownian motion on trees
- Independent contrasts

## Brownian Motion: The Model

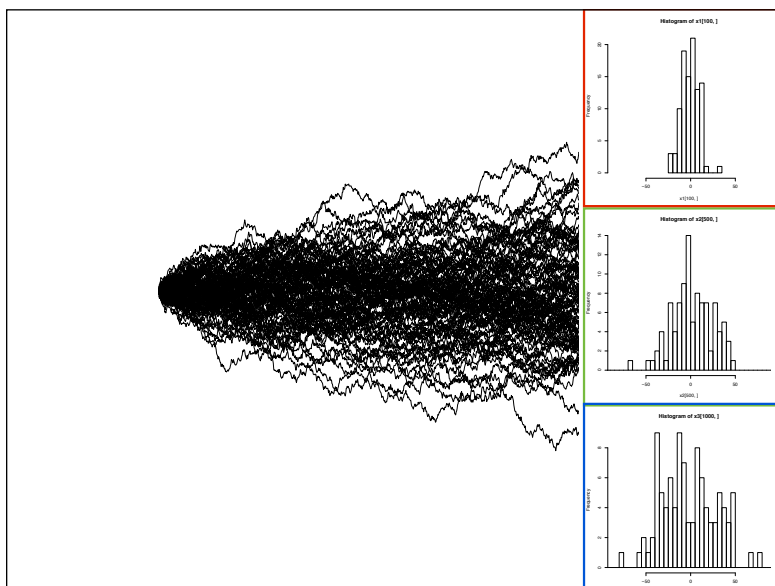
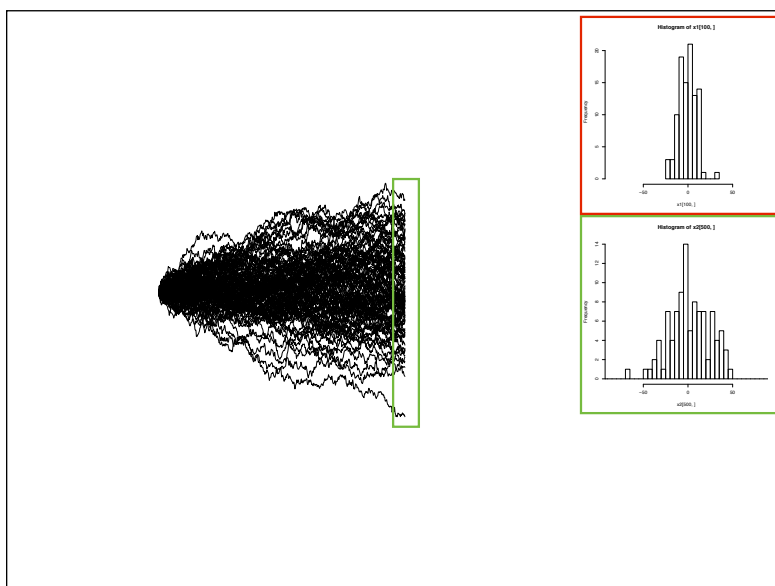
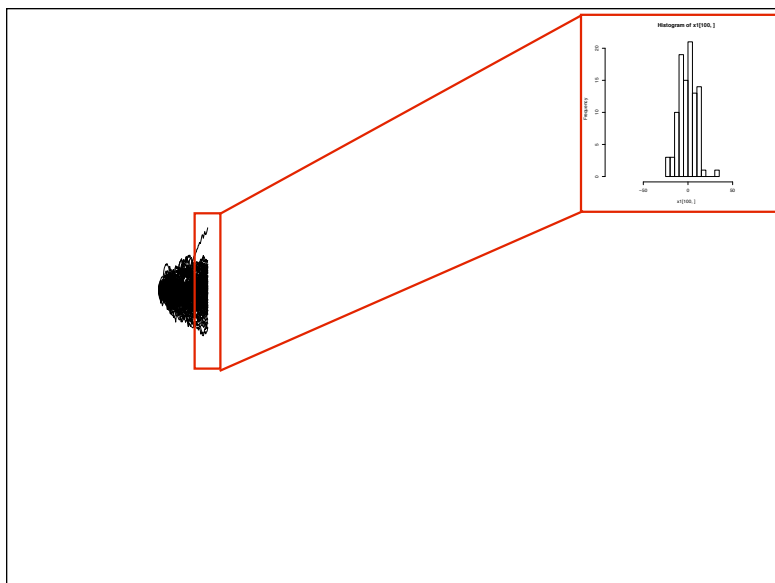
- Usually called the Wiener process
- A continuous-time stochastic process
- Describes a “random walk” of evolution for continuously-valued characters

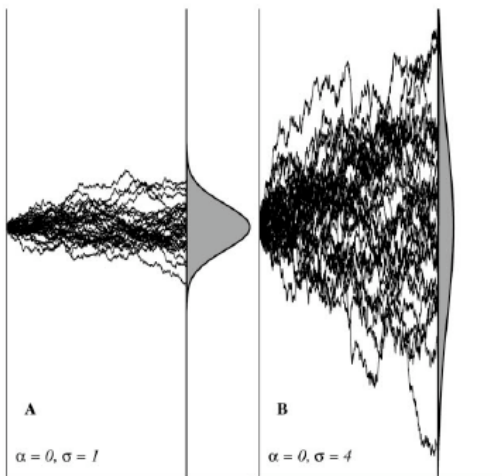
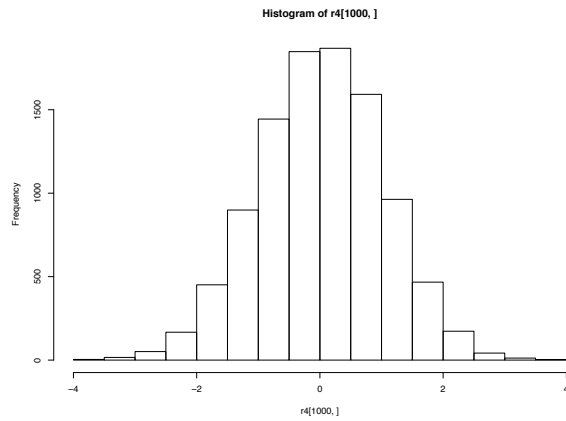
## Three Facts Describe Brownian Motion

- Let  $W(t)$  be the value of the character at time  $t$ . Then:
  - $E[W(t)] = W(0)$
  - Successive steps are independent
  - $W(t) \sim N(W(0), \sigma^2 t)$

## Parameters of BM

- Brownian motion models have two parameters:
  - $\Theta$ , the starting value;  $W(0) = \Theta$
  - $\sigma^2$ , the rate parameter





## Outline - BM

- What is Brownian motion?
- When might characters evolve in a Brownian-like way?
- Simulating Brownian motion on trees
- Independent contrasts



# A physical model for BM



## Why Normal?

- BM can be used to describe motion that results from the combination of a large number of independent weak forces
- Adding many small independent variables result in normal distributions, no matter the original distribution (Central limit theorem)

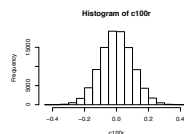
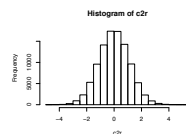
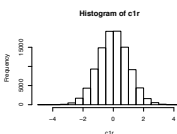
### Mean of

$n=1$

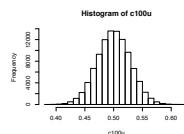
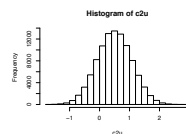
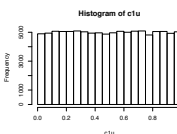
$n=2$

$n=100$

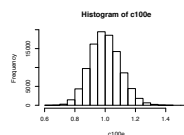
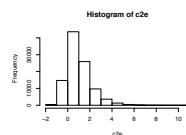
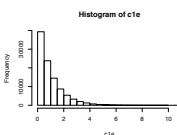
Normal



Uniform



Exponential



## Evolution might approximate BM...

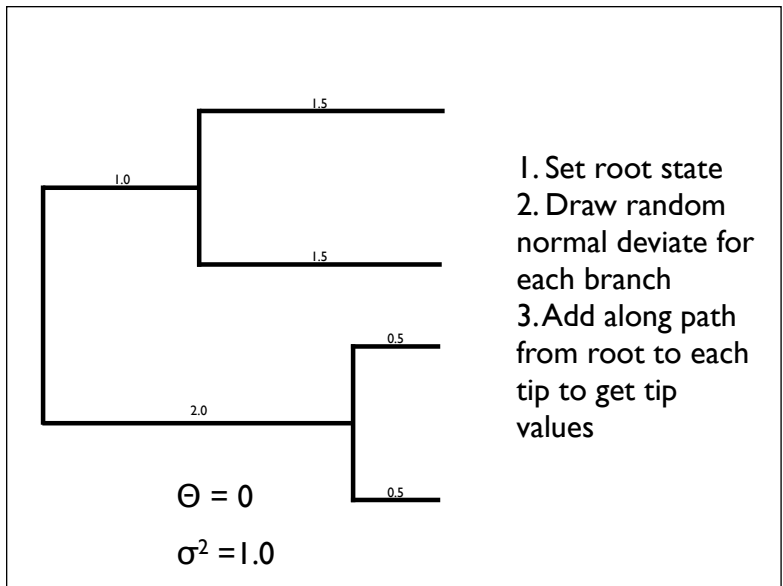
- Genetic drift
- Random punctuated change
- Selection that is weak relative to the time interval considered
- Selection that changes randomly through time

## Outline - BM

- What is Brownian motion?
- When might characters evolve in a Brownian-like way?
- Simulating Brownian motion on trees
- Independent contrasts

## Simulating BM

- Simulating Brownian motion involves drawing values from normal distributions
- Variance of the distribution depends on  $\sigma^2$  and  $t$
- Values along adjacent branches are added from the root to the tips of the tree




---

---

---

---

---

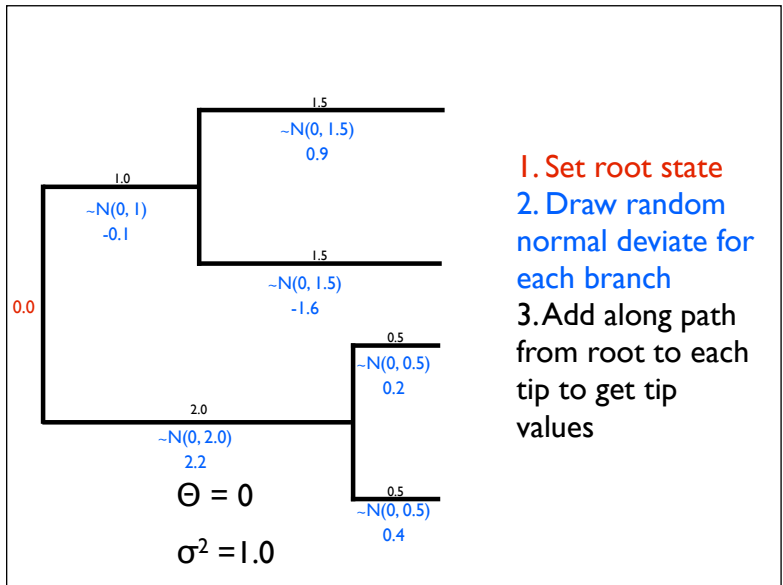
---

---

---

---

---




---

---

---

---

---

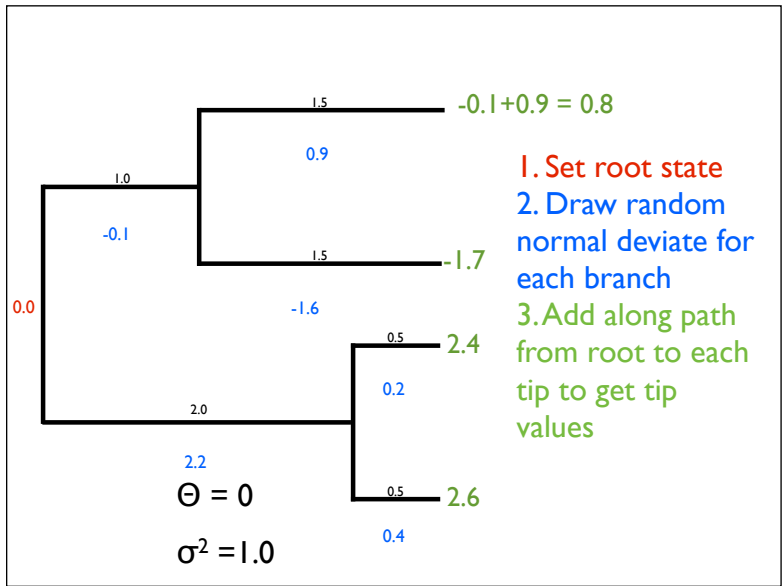
---

---

---

---

---




---

---

---

---

---

---

---

---

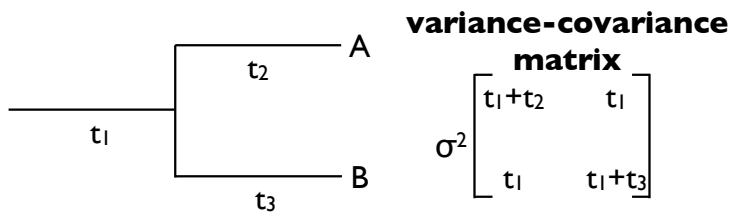
---

---

# Properties of BM on trees

- Variance increases with both  $\sigma^2$  and  $t$
- Expected (mean) value of any tip is always  $\Theta$
- Closely related species tend to be similar (they covary)

## How do they covary?

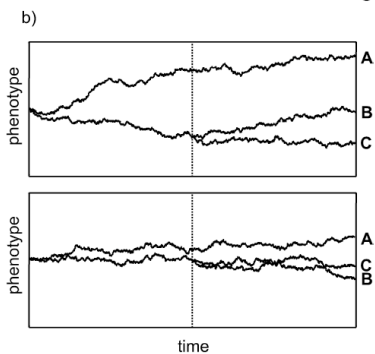
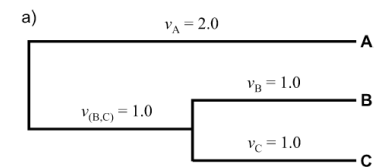


$$\text{var}(A) = \sigma^2(t_1+t_2) \quad \text{cov}(A,B) = \sigma^2(t_1)$$

$$\text{var}(B) = \sigma^2(t_1+t_3)$$

## General form

- Tip data follow a multivariate normal distribution with mean vector  $\Theta$  and variance-covariance matrix where
- $\text{var}(i) = \sigma^2(d_i)$ ;  $d_i$  = distance from root to tip  $i$
- $\text{cov}(i,j) = \sigma^2(c_{i,j})$ ;  $c_{i,j}$  = shared path of tip  $i$  and  $j$



c)

Expected covariance among species ( $\sigma^2 = 1.0$ ):

$$C = \begin{bmatrix} v_A & 0.0 & 0.0 \\ 0.0 & v_B + v_{(B,C)} & v_{(B,C)} \\ 0.0 & v_{(B,C)} & v_C + v_{(B,C)} \end{bmatrix} = \begin{bmatrix} 2.0 & 0.0 & 0.0 \\ 0.0 & 2.0 & 1.0 \\ 0.0 & 1.0 & 2.0 \end{bmatrix}$$

Observed covariance among species  
(N=100 simulations):

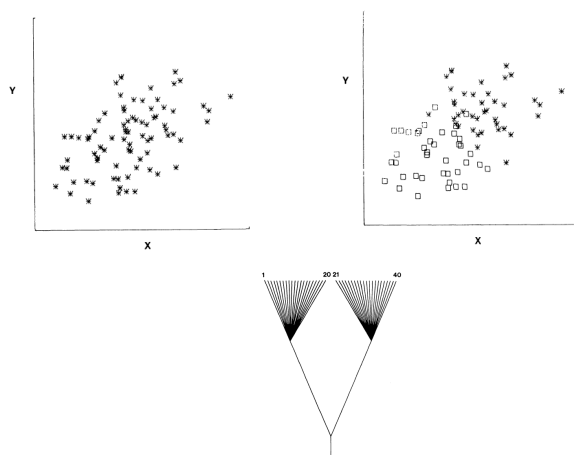
$$\bar{C} = \begin{bmatrix} 1.85 & 0.11 & -0.24 \\ 0.11 & 2.08 & 0.84 \\ -0.24 & 0.84 & 1.94 \end{bmatrix}$$

from Revell et al. 2008

## Outline - BM

- What is Brownian motion?
- When might characters evolve in a Brownian-like way?
- Simulating Brownian motion on trees
- Independent contrasts

## Felsenstein's "Worst Case"



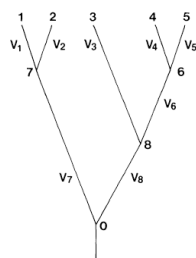


TABLE 1  
THE FOUR CONTRASTS EXTRACTED FROM THE PHYLOGENY  
SHOWN IN FIGURE 9, EACH WITH ITS VARIANCE, ALL  
COMPUTED USING STEPS 1-4 IN THE TEXT

CONTRAST	VARIANCE
$X_1 - X_2$	$v_1 + v_2$
$X_4 - X_5$	$v_4 + v_5$
$X_3 - X_6$	$v_3 + v'_6$
$X_7 - X_8$	$v'_7 + v'_8$

where

$$X_6 = \frac{v_4 X_5 + v_5 X_4}{v_4 + v_5}$$

$$v'_6 = v_6 + v_4 v_5 / (v_4 + v_5)$$

$$X_7 = \frac{v_2 X_1 + v_1 X_2}{v_1 + v_2}$$

$$v'_7 = v_7 + v_1 v_2 / (v_1 + v_2)$$

$$X_8 = \frac{v'_6 X_3 + v_3 X_6}{v_3 + v'_6}$$

$$v'_8 = v'_7 + v_3 v'_6 / (v_3 + v'_6)$$

(Felsenstein 1981)

## What are contrasts?

- Each standardized contrast is telling us something about the RATE of evolution
- The contrasts have a close relationship with  $\sigma^2$ , the rate parameter from BM
- The sum of the squared contrasts divided by n gives the ML estimate of  $\sigma^2$