

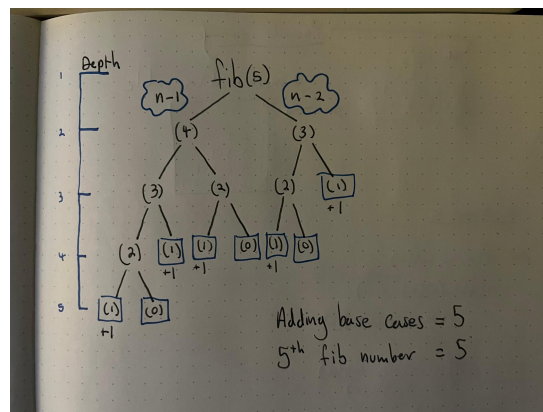
1. Explain how the larger problem of solving for the  $n$ th Fibonacci number, can be solved by finding the solutions to its smaller, subproblems.

The base cases are important because they provide a “stopping point” that tells the program when to add values. Adding each base case results in calculating the  $n$ th fibonacci number.

2. What are the base case (s)? In this problem, why do they represent the smallest subproblem(s) we want to recurse down too?

The base cases for this problem are 0 and 1. These represent the lowest possible values of  $n$ . In the recursive “naive” solution, the tree is split at each stage until  $n$  is either 0 or 1. In either case, each split is added together and returned up the recursive call.

3. Draw the recursive tree for Solution #1 (recursive, non-dynamic), up to a depth of 5. Identify the overlapping subproblems.

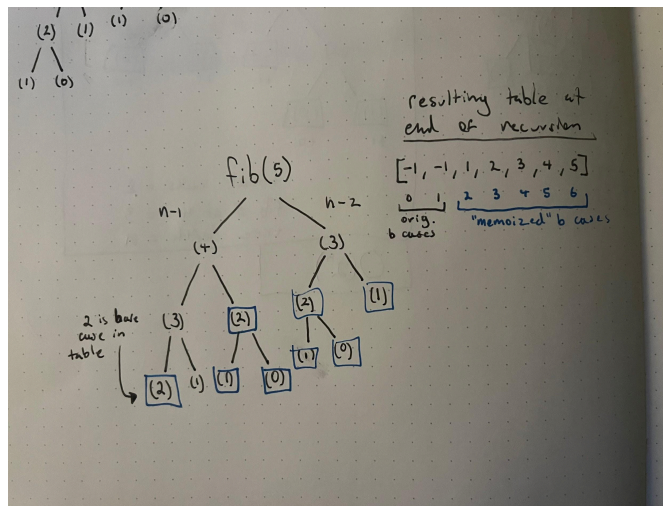


Notice the trees for 2 and 3 repeat. The values for these trees can be recorded and used as base cases for future recursive calls. Note that the base cases are NOT saved for the non-dynamic solution.

4. Why is Solution #1, the non-dynamic recursive solution, so slow in terms of runtime efficiency?

The naive approach is so slow because the tree splits  $2^n$  times. Until  $n \leq 1$ , the tree continues to split. As  $n$  increases, the runtime becomes extremely slow. For example, when I tried  $n = 50$ , it took so long I lost patience and quit the program before it could finish.

5. Draw one possible recursive tree for Solution #2 (recursive, Memoized solution), up to a depth of 5.



(Sorry! I forgot to add one more level! There should be another tree splitting from the last 2 on the left.)

6. With Solution #2, the dynamic programming recursive implementation, how does Memoization work to improve the runtime efficiency?

Memoization works to improve the runtime efficiency by recording new base cases as recursive calls are made. I made a note of the final table, which represents the value of each case corresponding to the  $n$  value by index. So, when  $n$  is 2, its value of 1 is stored in the table at index 2.