

# Contents

## Real and Transcendental Functions

### 0. Real-Valued Functions

Let  $X$  and  $Y$  be sets. A function  $f$  is a mapping that assigns to each  $x \in X$  exactly one  $y = f(x) \in Y$ . We use the notation:

$$f : X \rightarrow Y, x \mapsto f(x)$$

0.1 Let  $f : X \rightarrow Y$  be a function. We call the set of numbers for which the function  $f$  is well defined the **domain** of  $f$ .

$$D(f) = \{x : f(x) \text{ is well defined}\}$$

The **range** of a function  $f : X \rightarrow Y$  is the set.

$$R(f) = \{f(x) : x \in X\}$$

==**Parity of Function**==: A function  $f$  is called **even** if  $f(-x) = f(x) \forall x \in D$ , generally symmetric on  $y$  axis graphically. A function  $g$  is called **odd** if  $g(-x) = -g(x) \forall x \in D$ , generally rotation of 180 degrees

0.2 Examples of transcendental functions and its domain and range:

Function	Domain	Range
$y = x$	$\mathbb{R}$	$\mathbb{R}$
$y = x^2$	$\mathbb{R}$	$[0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \frac{1}{x}$	$\mathbb{R} \setminus \{0\}$	$\mathbb{R} \setminus \{0\}$
$e^x$	$\mathbb{R}$	$(0, \infty)$
$\ln(x)$	$(0, \infty)$	$\mathbb{R}$
$\sin(x)$	$\mathbb{R}$	$[-1, 1]$
$\cos(x)$	$\mathbb{R}$	$[-1, 1]$
$\tan(x)$	$\mathbb{R} \setminus \{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\}$	$\mathbb{R}$
$\arcsin(x)$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\arccos(x)$	$[-1, 1]$	$[0, \pi]$
$\arctan(x)$	$\mathbb{R}$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
$y =  x $	$\mathbb{R}$	$[0, \infty)$

#### 0.3.1 Composite functions

Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  The composition of  $f$  and  $g$ , denoted  $g \circ f : X \rightarrow Z$ ,  $(g \circ f)(x) = g(f(x))$

#### 0.3.2 Inverse function

Let  $f : X \rightarrow Y$  be a function with domain  $X$  and range  $Y$ . Then  $f$  is invertible if  $\exists f^{-1} : Y \rightarrow X$  so that:

$$f^{-1}(f(x)) = x, \forall x \in X \text{ and } f^{-1}(f(x)) = x, \forall x \in Y$$

#### 0.3.3 Piecewise function

$$f(x) = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

The quantitative value  $|a - b|$  represents the shortest distance between 2 points on a number line.

Examples:

1)  $f(x) = \frac{x}{x^2 - x} = \frac{x}{(x-1)(x)}$ ;  $x \neq 0, 1$

Therefore the domain should be  $x \in \mathbb{R} \setminus \{0\}$ , alternatively  $x \in (\infty, 0) \cup (0, 1) \cup (1, \infty)$

- 2) The domain of  $\cos(x)$  is  $\mathbb{R}$  with range of  $[-1, 1]$  Its inverse is  $\arccos(x)$  will have domain  $[-1, 1]$  and range of  $[0, \pi]$
- 3) Parity questions:
  - 1) A fn that is odd that is not a power of  $x$ :  $\sin(x)$
  - 2) A fn that is even that is not a power of  $x$ :  $\cos(x)$
  - 3) A fn that is neither even nor odd:  $\sin(e^x)$  or  $e^x$
  - 4) ==A fn that is even and odd: == $f(x) = 0$  4). Factor  $x^3 + 10x^3 + 13x - 24$  Find a factor:  $x = 1$ , it will always be the factor of the constant term. So  $x - 1$  is a factor of  $x^3 + 10x^3 + 13x - 24$   
 $x^2 + 11x + 24$  is the quotient of the factorization of the polynomial  $x^2 + 11x + 24 = (x + 3)(x + 8)$   
 $x^3 + 10x^3 + 13x - 24 = (x - 1)(x + 3)(x + 8)$