Contents

Real and Transcendental Functions

0. **Real-Valued Functions**
   Let X and Y be sets. A function f is a mapping that assigns to each $x \in X$ exactly one $y = f(x) \in Y$. We use the notation:

$$f : X \to Y, x \mapsto f(x)$$

0.1 Let $f : X \to Y$ be a function. We call the set of numbers for which the function f is well defined the **domain** of f.

$$D(f) = \{x : f(x) \text{ is well defined}\}$$

The **range** of a function $f : X \to Y$ is the set.

$$R(f) = \{f(x) : x \in X\}$$

Parity of Function: A function f is called **even** if $f(-x) = f(x)\ \forall x \in D$, generally symmetric on y axis graphically.
A function g is called **old** if $g(-x) = -g(x)\ \forall x \in D$, generally rotation of 180 degrees. 0.2
Examples of transcendental functions and its domain and range:

Function	**Domain**	**Range**
$y = x$	$\mathbb{R}$	$\mathbb{R}$
$y = x^2$	$\mathbb{R}$	$[0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \frac{1}{x}$	$\mathbb{R} \setminus \{0\}$	$\mathbb{R} \setminus \{0\}$
e^x	$\mathbb{R}$	$(0, \infty)$
$\ln(x)$	$(0, \infty)$	$\mathbb{R}$
$\sin(x)$	$\mathbb{R}$	$[-1, 1]$
$\cos(x)$	$\mathbb{R}$	$[-1, 1]$
$\tan(x)$	$\mathbb{R} \setminus \{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\}$	$\mathbb{R}$
$\arcsin(x)$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\arccos(x)$	$[-1, 1]$	$[0, \pi]$
$\arctan(x)$	$\mathbb{R}$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
$y = \|x\|$	$\mathbb{R}$	$[0, \infty)$

0.3.1 Composite functions
Let $f : X \to Y$ and $g : Y \to Z$. The composition of f and g, denoted $g \circ f$
$g \circ f : X \to Z$, $(g \circ f)(x) = g(f(x))$
0.3.2 Inverse function
Let $f : X \to Y$ be a function with domain X and range Y. Then f is invertible if $\exists f^-1 : Y \to X$ so that:

$$f^-1(f(x)) = x, \forall x \in X \text{ and } f^-1(f(x)) = x, \forall x \in Y$$

0.3.3 Piecewise function

$$f(x) = |x| = \begin{cases} x, & \text{if } x >= 0 \\ -x, & \text{if } x < 0 \end{cases}$$

The quantitative value $|a - b|$ represents the shortest distance between 2 points on a number line.

Examples:

1) $f(x) = \frac{x}{x^2-x} = \frac{x}{(x-1)(x)}; x \neq 0, 1$
   Therefore the domain should be $x \in \mathbb{R} \setminus \{0\}$, alternatively $x \in (\infty, 0) \cup (0, 1) \cup (1, \infty)$

2) The domain of $\cos(x)$ is R with range of $[-1, 1]$ Its inverse is $\arccos(x)$ will have domain $[-1, 1]$ and range of $[0, \pi]$
3) Parity questions:
   1) A fn that is odd that is not a power of x: $\sin(x)$

   2) A fn that is even that is not a power of x: $\cos(x)$.
   3) A fn that is neither even nor odd: $\sin(e^x)$ or e^x

   4) ==A fn that is even and odd: ==$f(x) = 0$

4) Factor $x^3 + 10x^3 + 13x - 24$
   Find a factor: $x = 1$, it will always be the factor of the constant term. So $x - 1$ is a factor of $x^3 + 10x^3 + 13x - 24$ $x^2 + 11x + 24$ is the quotient of the factorization of the polynomial. $x^2 + 11x + 24 = (x + 3)(x + 8)$ $x^3 + 10x^3 + 13x - 24 = (x - 1)(x + 3)(x + 8)$