CH 4 - Mathematical Induction

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Notations

$$\sum_{i=m}^{n} x_i = x_m^2 + x_{m+1} = x_{m+2} + ... + x_{n-1} + x_n$$

$$\prod_{i=m}^{n} x_i = x_m \cdot x_{m+1} \cdot x_{m+2} \cdot ... \cdot x_{n-1} \cdot x_n$$

Properties

Constant multiplication

$$\sum_{i=m}^{k} cx_i = c \cdot \sum_{i=m}^{k} x_i$$

Addition/Subtraction

$$\sum_{i=m}^{k} x_i \pm \sum_{i=m}^{k} y_i = \sum_{i=m}^{k} x_i \pm \sum_{i=m}^{k} y_i$$

Index Shift

$$\sum_{i=m}^{k} x_i = \sum_{m \pm n}^{k \pm n} x_{i \mp n}$$

Breaking Sum

$$\sum_{i=m}^{k} x_i = \sum_{i=m}^{r} x_i + \sum_{i=r+1}^{k} x_i$$

Recurrence Relation

A sequence of values by giving one or more initial terms, together with an equation expressing each subsequent term in terms of earlier ones. (i.e. $s_1 = 1, s_n = s_{n-1} + n$ is the same as $\sum_{i=1}^{n} i$)

Proof by Induction

An **axiom** of a mathematical system is a statement that is assumed to be true. No proof is given. From axioms we derive proposition and theorems.

Info — Principle of Mathematical Induction
Let $P(n)$ be an open sentence that dependes on $n \in \mathbb{N}$
If statements 1 and 2 are both true:
1. $P(1)$
2. $\forall k \in \mathbb{N}$, if $P(k)$, then $P(k+1)$

Then statement 3 is true:
3. $\forall n \in \mathbb{N}, P(n)$