CH 2 — Logical Analysis

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Truth Tables and Negation

A is a statement with an assigned truth value and can be manipulated as a logical expression using the operators not, or, and, =>

Negation table

A	not A	not(not A)
T	F	T
F	T	F

Compound Statements

A compound statement is built from simpler statements using or and and

- $\vee$ is disjunction
- $\wedge$ is conjunction

Tables for

A	B	A or B	A and B
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

Logical Laws

1. De Morgan's Laws for statements A and B
    1. $\neg(A \wedge B) \equiv \neg A \vee \neg B$
    2. $\neg(A \vee B) \equiv \neg A \wedge \neg B$
2. Commutative, Associative, and Distributive Laws
    1. Commutative
        - $A \vee B \equiv B \vee A$
        - $A \wedge B \equiv B \wedge A$
    2. Associative
        - $A \wedge (B \wedge C) \equiv (A \wedge B) \wedge C$
        - $A \vee (B \vee C) \equiv (A \vee B) \vee C$
    3. Distributive
        - $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$
        - $A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$

Truth-table proof of De Morgan Law

A	B	not A	not B	A and B	not(A and B)	not A or not B
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Since the columns for $\neg(A \wedge B)$ and $\neg A \vee \neg B$ are identical in the table, they are logically equivalent.

Example:
Truth table proof of Distributive $\wedge$ over $\vee$

Show $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$

A	B	C	B or C	A and (B or C)	A and B	A and C	(A and B) or (A and C)
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Implications

$\Rightarrow$ is implication

B if A

B when A

B whenever A

A is a sufficient condition for B

Key equivalence
$(A \Rightarrow B) \equiv (\neg A \vee B)$

A	B	A => B	not A	(not A) or B
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

So $\neg(A \Rightarrow B) \equiv \neg(\neg A \vee B) \equiv A \wedge \neg B$

Practice

$((A \vee B) \Rightarrow C) \equiv (A \Rightarrow C) \wedge (B \Rightarrow C)$

A	B	C	A or B	(A or B) => C	A => C	B => C	(A => C) and (B => C)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

Converse

$B \Rightarrow A$ is the converse of $A \Rightarrow B$

Contrapositive

$(\neg B) \Rightarrow (\neg A)$ is the contrapositive of $A \Rightarrow B$

A	B	A => B	B => A	not A	not B	(not B) => (not A)
T	T	T	T	F	F	T
T	F	F	T	F	T	F
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Examples

1. If $x > y$ then $x \geq y$
2. Converse If $x \geq y$ then $x > y$
3. Contrapositive If $x < y$ then $x \leq y$

If and Only If

$\Leftrightarrow$ means if and only if

A	B	A => B	B => A	A <=> B	(A => B) and (B => A)
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

What is $\neg(A \Leftrightarrow B)$?

Solution:

$\equiv \neg((A \Rightarrow B) \wedge (B \Rightarrow A))$

$\equiv \neg(A \Rightarrow B) \vee \neg(B \Rightarrow A)$

$$\equiv \neg(\neg A \vee B) \vee \neg(A \vee \neg B)$$

$$\equiv (A \wedge \neg B) \vee (\neg A \wedge B)$$