CH 2 — Logical Analysis

Luke Lu • 2025-09-29

Truth Tables and Negation

A is a statement with an assigned truth value and can be manipulated as a logical expression using the operators not, or, and, =>

Negation table

A	not A	not(not A)
T	F	T
F	T	F

Compound Statements

A compound statement is built from simpler statements using or and and

- $\lor$ is disjunction
- $\land$ is conjunction

Tables for

A	B	A or B	A and B
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

Logical Laws

1. De Morgan's Laws for statements A and B
    1. $\neg(A \land B) \equiv \neg A \lor \neg B$
    2. $\neg(A \lor B) \equiv \neg A \land \neg B$
2. Commutative, Associative, and Distributive Laws
    1. Commutative
        - $A \lor B \equiv B \lor A$
        - $A \land B \equiv B \land A$
    2. Associative
        - $A \land (B \land C) \equiv (A \land B) \land C$
        - $A \lor (B \lor C) \equiv (A \lor B) \lor C$
    3. Distributive
        - $A \land (B \lor C) \equiv (A \land B) \lor (A \land C)$
        - $A \lor (B \land C) \equiv (A \lor B) \land (A \lor C)$

Truth-table proof of De Morgan Law

A	B	not A	not B	A and B	not(A and B)	not A or not B
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Since the columns for $\neg(A \land B)$ and $\neg A \lor \neg B$ are identical in the table, they are logically equivalent.

Example:
Truth table proof of Distributive $\land$ over $\lor$

Show $A \land (B \lor C) \equiv (A \land B) \lor (A \land C)$

A	B	C	B or C	A and (B or C)	A and B	A and C	(A and B) or (A and C)
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Implications
$\Rightarrow$ is implication

B if A

B when A

B whenever A

A is a sufficient condition for B

Key equivalence
$(A \Rightarrow B) \equiv (\neg A \lor B)$

A	B	A => B	not A	(not A) or B
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

So $\neg(A \Rightarrow B) \equiv \neg(\neg A \lor B) \equiv A \land \neg B$

Practice

$((A \vee B) \Rightarrow C) \equiv (A \Rightarrow C) \wedge (B \Rightarrow C)$

A	B	C	A or B	(A or B) ⇒ C	A ⇒ C	B ⇒ C	(A ⇒ C) and (B ⇒ C)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

Converse

$B \Rightarrow A$ is the converse of $A \Rightarrow B$

Contrapositive

$(\neg B) \Rightarrow (\neg A)$ is the contrapositive of $A \Rightarrow B$

A	B	A ⇒ B	B ⇒ A	not A	not B	(not B) ⇒ (not A)
T	T	T	T	F	F	T
T	F	F	T	F	T	F
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Examples

1. If $x > y$ then $x \geq y$
2. Converse If $x \geq y$ then $x > y$
3. Contrapositive If $x < y$ then $x \leq y$

If and Only If

$\Leftrightarrow$ means if and only if

A	B	A ⇒ B	B ⇒ A	A ⇔ B	(A ⇒ B) and (B ⇒ A)
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

What is $\neg(A \Leftrightarrow B)$?

Solution:

$\equiv \neg((A \Rightarrow B) \wedge (B \Rightarrow A))$

$\equiv \neg(A \Rightarrow B) \vee \neg(B \Rightarrow A)$

$$\equiv \neg(\neg A \vee B) \vee \neg(A \vee \neg B)$$

$$\equiv (A \wedge \neg B) \vee (\neg A \wedge B)$$