CH 4 - Derivatives

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Velocity

Info — Average Velocity and Instantaneous Velocity

$$v_{avg} = \frac{s(t_1) - s(t_0)}{t_1 - t_0}$$

$$v_{inst} = \lim_{t \to t_0} \frac{s(t) - s(t_0)}{t - t_0} = \lim_{h \to 0} \frac{s(t_0 + h) - s(t_0)}{h}$$

Definition of Derivatives

Info — Average Rate of Change and Instantaneous Rate of Change (Derivative)

$$f_{avg} = \frac{f(b) - f(a)}{b - a}$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

If $f'(x)$ exists at $x = a$, then $f(x)$ is **differentiable** at $x = a$

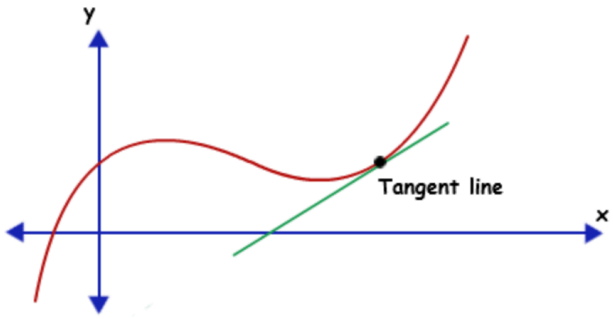
Info — Tangent Line

If $f(x)$ is differentiable at $x = a$, then the **tangent line** to $f(x)$ at $x = a$ is the line passing through $(a, f(a))$ with slope $f'(a)$

The equation of the tangent line

$$y = f'(a)(x - a) + f(a)$$

$(a, f(a))$ is the **point of tangency**



Examples:

Find the tangent line to $f(x) = \frac{1}{x+5}$ at $x = 2$
$f(3) = \frac{1}{8}$
$f'(3) = f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{\frac{1}{a+h+5}-\frac{1}{a+5}}{h} =$
$\lim_{h\to 0} \frac{1}{h}\frac{a+5-(a+h+5)}{(a+5)(a+h+5)} = \lim_{h\to 0} -\frac{1}{(a+5)(a+h+5)} = -\frac{1}{(a+5)^2} = -\frac{1}{64}$

$y = -\frac{1}{64}(x-3) + \frac{1}{8}$