

# CH 1 — Precalculus Review

Luke Lu • 2025-09-13

## Real and Transcendental Functions

### Real-Valued Functions

Let  $X$  and  $Y$  be sets. A function  $f$  assigns to each  $x$  in  $X$  exactly one  $y = f(x)$  in  $Y$ :

$$f : X \rightarrow Y; x \mapsto f(x).$$

### Domain and Range

For  $f : X \rightarrow Y$ :

- **Domain**  $D(f) = \{x : f(x) \text{ is well-defined}\}.$
- **Range**  $R(f) = \{f(x) : x \in X\}.$

### Parity

- $f$  is **even** if  $f(-x) = f(x)$  for all  $x$  in  $D(f)$  (y-axis symmetry).
- $g$  is **odd** if  $g(-x) = -g(x)$  for all  $x$  in  $D(g)$  ( $180^\circ$  rotation).

### Examples (domains & ranges)

Function	Domain	Range
$y = x$	$\mathbb{R}$	$\mathbb{R}$
$y = x^2$	$\mathbb{R}$	$[0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \frac{1}{x}$	$\mathbb{R} \setminus \{0\}$	$\mathbb{R} \setminus \{0\}$
$e^x$	$\mathbb{R}$	$(0, \infty)$
$\ln(x)$	$(0, \infty)$	$\mathbb{R}$
$\sin(x)$	$\mathbb{R}$	$[-1, 1]$
$\cos(x)$	$\mathbb{R}$	$[-1, 1]$
$\tan(x)$	$\mathbb{R} \setminus \{\frac{(2k+1)\pi}{2} : k \in \mathbb{Z}\}$	$\mathbb{R}$
$\arcsin(x)$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\arccos(x)$	$[-1, 1]$	$[0, \pi]$
$\arctan(x)$	$\mathbb{R}$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
$y =  x $	$\mathbb{R}$	$[0, \infty)$

## Function Operations

### Composition

Given  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$

$$(g \circ f) : X \rightarrow Z, (g \circ f)(x) = g(f(x))$$

**Inverse**

$f : X \rightarrow Y$  is invertible if there exists  $f^{\{-1\}} : Y \rightarrow X$  with

$$f^{-1}(f(x)) = x$$

for  $x \in X, f(f^{-1}(y)) = y$  (for)  $y \in Y$ .

**Piecewise (absolute value)**

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

**Examples:**

1. Find the domain of this function:

$$f(x) = \frac{x}{x^2 - x} = \frac{x}{(x-1)x}; x \neq 0, 1$$

$$\text{Domain} = \mathbb{R} \setminus \{0, 1\} = (-\infty, 0) \cup (0, 1) \cup (1, \infty).$$

2. The domain of  $\cos(x)$  is  $\mathbb{R}$  with range of  $[-1, 1]$  Its inverse is  $\arccos(x)$  will have domain  $[-1, 1]$  and range of  $[0, \pi]$

3. Parity questions:

- 1) A fn that is odd that is not a power of  $x$ :  $\sin(x)$
- 2) A fn that is even that is not a power of  $x$ :  $\cos(x)$
- 3) A fn that is neither even nor odd:  $\sin(e^x)$  or  $e^x$
- 4) A fn that is even and odd:  $f(x) = 0$

4) Factor  $x^3 + 10x^2 + 13x - 24$

Find a factor:  $x = 1$ , it will always be the factor of the constant term.

So  $x - 1$  is a factor of  $x^3 + 10x^2 + 13x - 24$

$x^2 + 11x + 24$  is the quotient of the factorization of the polynomial.

$$x^2 + 11x + 24 = (x + 3)(x + 8)$$

$$x^3 + 10x^2 + 13x - 24 = (x - 1)(x + 3)(x + 8)$$