# CH 1 — Precalculus Review

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## Real and Transcendental Functions

### Real-Valued Functions

Let $X$ and $Y$ be sets. A function $f$ assigns to each $xinX$ exactly one $y = f(x)inY$:

$$f : XtoY,; x \mapsto f(x).$$

### Domain and Range

For $f : XtoY$:

- **Domain** $D(f) = \{x : f(x) \text{ is well-defined}\}$.
- **Range** $R(f) = \{f(x) : xinX\}$.

### Parity

- $f$ is **even** if $f(-x) = f(x)$ for all $xinD(f)$ (y-axis symmetry).
- $g$ is **odd** if $g(-x) = -g(x)$ for all $xinD(g)$ (180° rotation).

### Examples (domains & ranges)

| Function | Domain | Range |
|---|---|---|
| $y = x$ | $\mathbb{R}$ | $\mathbb{R}$ |
| $y = x^2$ | $\mathbb{R}$ | $[0, \infty)$ |
| $y = \sqrt{x}$ | $[0, \infty)$ | $[0, \infty)$ |
| $y = \frac{1}{x}$ | $\mathbb{R} \setminus \{0\}$ | $\mathbb{R} \setminus \{0\}$ |
| $e^x$ | $\mathbb{R}$ | $(0, \infty)$ |
| $ln(x)$ | $(0, \infty)$ | $\mathbb{R}$ |
| $\sin(x)$ | $\mathbb{R}$ | $[-1, 1]$ |
| $\cos(x)$ | $\mathbb{R}$ | $[-1, 1]$ |
| $\tan(x)$ | $\mathbb{R} \setminus \{\frac{(2k+1)\pi}{2} : k \in \mathbb{Z}\}$ | $\mathbb{R}$ |
| $\arcsin(x)$ | $[-1, 1]$ | $[-\frac{\pi}{2}, \frac{\pi}{2}]$ |
| $\arccos(x)$ | $[-1, 1]$ | $[0, \pi]$ |
| $\arctan(x)$ | $\mathbb{R}$ | $(-\frac{\pi}{2}, \frac{\pi}{2})$ |
| $y = \lvert x \rvert$ | $\mathbb{R}$ | $[0, \infty)$ |

### Function Operations

### Composition

Given $f : X \to Y$ and $g : Y \to Z$
$(g \circ f) : X \to Z, (g \circ f)(x) = g(f(x))$

**Inverse**

$f : X \to Y$ is invertible if there exists $f^{\{-1\}} : Y \to X$ with

$$f^{-1}(f(x)) = x$$
$$\text{for } x \in X, f(f^{-1}(y)) = y (\text{for}) y \in Y.$$

**Piecewise (absolute value)**

$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

**Examples:**

1. Find the domain of this function:

$f(x) = \frac{x}{x^2 - x} = \frac{x}{(x-1)x}; x \neq 0, 1$

Domain $= \mathbb{R} \setminus \{0, 1\} = (-\infty, 0) \cup (0, 1) \cup (1, \infty)$.

2. The domain of $\cos(x)$ is $\mathbb{R}$ with range of $[-1, 1]$ Its inverse is $\arccos(x)$ will have domain $[-1, 1]$ and range of $[0, \pi]$
3. Parity questions:
   1) A fn that is odd that is not a power of $x$: $\sin(x)$
   2) A fn that is even that is not a power of $x$: $\cos(x)$
   3) A fn that is neither even nor odd: $\sin(e^x)$ or $e^x$
   4) A fn that is even and odd: $f(x) = 0$

4) Factor $x^3 + 10x^3 + 13x - 24$

Find a factor: $x = 1$, it will always be the factor of the constant term.

So $x - 1$ is a factor of $x^3 + 10x^3 + 13x - 24$

$x^2 + 11x + 24$ is the quotient of the factorization of the polynomial.

$x^2 + 11x + 24 = (x + 3)(x + 8)$

$x^3 + 10x^3 + 13x - 24 = (x - 1)(x + 3)(x + 8)$