# CH 5 - Applications of Derivatives

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## Related Rates

💡 **Tip** — Steps for Related Rates Questions

1. Draw diagram
2. Identitfy **changing** quantities
3. Find **constant** quantities (if possible)
4. Derive equations relating the quantities that are changing
5. **Implicitly differentiate** the key equations
6. Solve for the desired rate of change, substituting in known quantities.
7. **Concluding statement** (and also check units)

Example:

1. Laindon is taking a hot air ballon ride. A giant fan is blowing hot air into the ballon in a rate of $200\frac{\mathrm{m}^3}{\mathrm{min}}$. Assuming that at any given point in time the balloon sphere, find the rate at which the radius of the balloon is changing when the diameter is 12 m.

   ANS:

   1. Picture: The problem is trivial so the graph is omitted
   2. Changing variable: Volume($\mathrm{m}^3$), Radius(m), time(t)
   3. Constant quantities: $\frac{\mathrm{d}V}{\mathrm{d}t} = 200\frac{\mathrm{m}^3}{\mathrm{min}}$
   4. Key Equation: $V = \frac{4}{3}\pi r^3(t)$
   5. Implicit Differentiation: $\frac{\mathrm{d}V}{\mathrm{d}t} = 4\pi r^2(t) \cdot \frac{\mathrm{d}r}{\mathrm{d}t}$
   6. $\frac{\mathrm{d}r}{\mathrm{d}t}\big|_{r=6} = \frac{1}{4\pi(6)^2} \cdot 200 = \frac{200}{144\pi} = \frac{25}{18\pi}\frac{\mathrm{m}}{\mathrm{min}}$
   7. Concluding statement: When the diameter of the balloon is 12m, the rate of change of the radius is expanding by $\frac{25}{18\pi}\frac{\mathrm{m}}{\mathrm{min}}$

2. The construction workers building M4 accidentally left a 20 foot ladder propped up against a concrete wall that is 80 feet in height. The base of the ladder begins to slide away from the wall at a rate of 2ft/sec, and the top begins to move down as a result. When the base of the ladder is 14 ft from the wall, how fast is the top of the ladder sliding down the wall?

   ANS:

   1. Picture is omitted and left as an exercise for the reader
   2. Changing variable: Distance from wall of base of ladder (m), Heing where ladder touches the wall (m)
   3. Constant quantities : $\frac{\mathrm{d}x}{\mathrm{d}t} = 2$
   4. Key Equation: $x^2 + y^2 = 20^2$

5. Implicit Differentiation: $2x\frac{\mathrm{d}x}{\mathrm{d}t} + 2y\frac{\mathrm{d}y}{\mathrm{d}t} = 0 \Longrightarrow \frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{x}{y}\frac{\mathrm{d}x}{\mathrm{d}t}$
6. $\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{14}{\sqrt{400-14^2}} \cdot 2 = -\frac{14}{\sqrt{51}}\frac{\mathrm{ft}}{\mathrm{sec}}$
7. Concluding statement: When the base of ladder is 14cm, the top of the ladder is falling at a speed of $\frac{14}{\sqrt{51}}\frac{\mathrm{ft}}{\mathrm{min}}$