CH 1 — Integration

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Definite Integrals

🐳 **Info — Riemann Sums**

Given $f(x)$ that is defined over $[a, b]$ with $a < b$, the area under function $f(x)$ can be found by

1. **Left-Endpoint Riemann Sum**

$$L_n = \sum_{i=0}^{n-1} f(x_i^*)\Delta x$$

2. **Right-Endpoint Riemann Sum**

$$R_n = \sum_{i=1}^{n} f(x_i^*)\Delta x$$

where

- $\Delta x = \frac{b-a}{n}$ under regular partition
- $x_i^* = a + \Delta x i = a + \frac{b-a}{n} i$
- $\lim_{n\to\infty} R_n = \lim_{n\to\infty} L_n$ for $f(x)$ on interval $[a, b]$
- Regular Parition means interval $[a, b]$ is equally partitioned into n rectangles with identical width
- A Riemann Sum equal 0 does not always mean that there is no area.

Example:

Estimate area under the curve for $f(x) = x^2$ on $x \in [0, 1]$

$R_n = \sum_{i=1}^{n} \frac{1}{n} f\left(\frac{i}{n}\right) = \sum_{i=1}^{n} \frac{1}{n}\left(\frac{i}{n}\right)^2 = \frac{1}{n^3}\sum_{i=1}^{n} i^2 = \frac{(n+1)(2n+1)}{6n}$
$\lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n} = \frac{1}{3}$

🐳 **Info — Definite Integral**

$f(x)$ defined on $x \in [a, b]$ with regular partition with n subintervals

The definite integral of $f(x)$ on $[a, b]$ is denoted

$$\int_a^b f(x) \,\mathrm{d}x = \lim_{x\to\infty} \sum_{i=1}^{n} \Delta x f(x_i^*)$$

A function is integrable on $x \in [a, b]$ provided that the limit of Riemann Sum exists and has the same value regardless of the choice of x_i^*

In simpler words: integrability equals continuity