CH 1 — Precalculus Review

Luke Lu • 2025-12-17

Real and Transcendental Functions

Real-Valued Functions

Let X and Y be sets. A function f assigns to each $x \in X$ exactly one $y = f(x) \in Y$:

$$f : X \to Y, x \to f(x).$$

Domain and Range

For $f : X \to Y$:
- **Domain** $D(f) = \{x : f(x) \text{ is well-defined}\}$.
- **Range** $R(f) = \{f(x) : x \in X\}$.

Parity

- f is **even** if $f(-x) = f(x)$ for all $x \in D(f)$ (y-axis symmetry).
- g is **odd** if $g(-x) = -g(x)$ for all $x \in D(g)$ (180° rotation).

Examples (domains & ranges)

Function	Domain	Range
$y = x$	$\mathbb{R}$	$\mathbb{R}$
$y = x^2$	$\mathbb{R}$	$[0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \frac{1}{x}$	$\mathbb{R} \setminus \{0\}$	$\mathbb{R} \setminus \{0\}$
e^x	$\mathbb{R}$	$(0, \infty)$
$\ln(x)$	$(0, \infty)$	$\mathbb{R}$
$\sin(x)$	$\mathbb{R}$	$[-1, 1]$
$\cos(x)$	$\mathbb{R}$	$[-1, 1]$
$\tan(x)$	$\mathbb{R} \setminus \{\frac{(2k+1)\pi}{2} : k \in \mathbb{Z}\}$	$\mathbb{R}$
$\arcsin(x)$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\arccos(x)$	$[-1, 1]$	$[0, \pi]$
$\arctan(x)$	$\mathbb{R}$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
$y = \lvert x \rvert$	$\mathbb{R}$	$[0, \infty)$

Function Operations

Composition

Given $f : X \to Y$ and $g : Y \to Z$
$(g \circ f) : X \to Z, (g \circ f)(x) = g(f(x))$

Inverse

$f : X \to Y$ is invertible if there exists $f^{-1} : Y \to X$ with

$$f^{-1}(f(x)) = x \text{ for } x \in X, f(f^{-1}(y)) = y \text{ for } y \in Y.$$

Piecewise (absolute value)

$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

Examples:

1. Find the domain of this function:

$f(x) = \frac{x}{x^2-x} = \frac{x}{(x-1)x}; x \neq 0, 1$

Domain $= \mathbb{R} \setminus \{0, 1\} = (-\infty, 0) \cup (0, 1) \cup (1, \infty)$.

2. The domain of $\cos(x)$ is $\mathbb{R}$ with range of $[-1, 1]$ Its inverse is $\arccos(x)$ will have domain $[-1, 1]$ and range of $[0, \pi]$
3. Parity questions:
   1) A fn that is odd that is not a power of x: $\sin(x)$
   2) A fn that is even that is not a power of x: $\cos(x)$
   3) A fn that is neither even nor odd: $\sin(e^x)$ or e^x
   4) A fn that is even and odd: $f(x) = 0$

4) Factor $x^3 + 10x^3 + 13x - 24$
Find a factor: $x = 1$, it will always be the factor of the constant term.
So $x - 1$ is a factor of $x^3 + 10x^3 + 13x - 24$
$x^2 + 11x + 24$ is the quotient of the factorization of the polynomial.
$x^2 + 11x + 24 = (x + 3)(x + 8)$
$x^3 + 10x^3 + 13x - 24 = (x - 1)(x + 3)(x + 8)$

CHAPTER ENDS