

# CS135 L04 — Recursion

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## From “...” to Recursion

**Goal:** Formalize computations that math often writes with ellipses (“...”).

**Example — Sum to n** Let  $f(n)$  be the sum of natural numbers up to  $n$ . Observe:

- $f(n) = f(n - 1) + n$
- Base:  $f(0) = 0$

**Direct Racket translation (Nat recursion with zero?/sub1):**

```
;; sum-to: Nat -> Nat
(define (sum-to n)
  (cond [(zero? n) 0]
        [else (+ n (sum-to (sub1 n)))]))

(require rackunit)
(check-expect (sum-to 4) 10)
(check-expect (sum-to 8) 36)
(check-expect (sum-to 13) 91)
(check-expect (sum-to 0) 0)
```

**Key ideas:**

- A **recursive function** calls itself with a “smaller” argument until a **base case**.
- For Naturals in this course: “smaller” = `sub1`, base test = `zero?`.

**The “Tripod” of Computation (Turing completeness in our model):** 1) Arbitrarily large numbers • 2) Conditionals • 3) Recursion.

## Rules for Recursion (version 1)

1) **Change exactly one argument** closer to termination on the recursive call. 2) For Naturals: **use** `sub1` and test termination with `zero?`.

**Template for Natural-number recursion:**

```
(define (natural-template n)
  (cond
    [(zero? n) ...] ; base case result
    [else (... n ... (natural-template (sub1 n)))] ; use n and recur on (sub1 n)
  )
```

 **Tip — Three properties to watch:**

- **Termination:** Guaranteed by shrinking the Nat with `sub1` until `zero?`.
- **Correctness:** In CS135 we lean on tests (you could also prove by induction).
- **Efficiency:** Measured by **stepper** (count substitution steps).

## Names & Scope

- **Scope** = where an identifier (function/parameter/constant) has effect.

- Two kinds for now: **global** and **function** scope.
- The **smallest enclosing scope** takes priority.
- **Duplicate identifiers in same scope → error:**

```
(define f 3)
(define (f x) (sqr x)) ; error: f already defined as a value
```

- DrRacket has **scope tools** to visualize where names refer.

## Our 5-Step Design Pattern

1) **Purpose:** Write what the function produces/consumes. 2) **Contract & Header:** Types (kinds) and parameter names. 3) **Examples/Tests:** Work by hand; include boundary cases. 4) **Body from Template:** Start with the recursion template, fill in specifics. 5) **More Tests:** Add any missing edge/coverage tests.

## Worked Example — Factorial

**Math idea:**  $n! = n \cdot (n - 1)!$  with base  $0! = 1$  (by convention).

**Definition (Nat recursion):**

```
;; factorial: Nat -> Nat
;; Produce n!
(define (factorial n)
  (cond
    [(zero? n) 1]
    [else (* n (factorial (sub1 n)))]))
```

```
(require rackunit)
(check-expect (factorial 3) 6)
(check-expect (factorial 5) 120)
(check-expect (factorial 7) 5040)
(check-expect (factorial 0) 1)
(check-expect (factorial 1) 1)
```

**Variant with a shorter name:**

```
;; !: Nat -> Nat
(define (! n)
  (cond
    [(zero? n) 1]
    [else (* n (! (sub1 n)))]))
```

### ⚠ Warning — Common mistakes:

- Changing **multiple** arguments on the recursive call (breaks our v1 rules).
- Missing or incorrect **base case** (non-terminating or wrong result).
- Forgetting to **use n in the step** (e.g., summing/multiplying the wrong thing).