Algorithm Testing

Algorithm Statistics

In this paper, we compare the effectiveness of multiple algorithms on their ability to estimate location based on signal strengths to nearby beacons.

Variables

Let's define the following variables:

- n is the number of beacons used in the location computation
- (x_{true}, y_{true}, $floor_{true}$) is the true position of the testing device (phone)
- (x_i, y_i, $floor_i$) is the position of a nearby beacon
    - $1 \leq i \leq n$
- (x_0, y_0, $floor_0$) is the estimated position of the testing device (phone) using one of the algorithms
- R_i is the estimated distance from a beacon based on rssi

Algorithms

In this section, we will define the algorithms used to compute position.

Algorithm 1

This is the algorithm that was first used on the BOSSA platform to compute position. It minimizes the following expression:

$$\sum_{i=1}^{n} \frac{(x_i - x_0)^2 + (y_i - y_0)^2 - R_i}{R_i}$$

Algoirthm 2

This is the algorithm currently being used on the BOSSA platform. It minimizes the following expression:

$$\sum_{i=1}^{n} \frac{\sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} - R_i}{R_i}$$

Algorithm 3

Minimize the following expression:

$$\sum_{i=1}^{n} \frac{\sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}}{R_i^2}$$

Algorithm 4

Minimize the following expression:

$$\sum_{i=1}^{n} \frac{(x_i - x_0)^2 + (y_i - y_0)^2}{R_i^2}$$

Calculating R_i

Recall that R_i is the estimated distance the testing device is from a beacon. This is calculated using the RSSIs a phone gets from a beacon. More specifically, we use a set of four bins to map RSSI to distance.

For algorithms 1, 3, and 4, we use the following table to convert RSSI to distance:

0 dBm	-50 dBm	-60 dBm	-70 dBm
2 meters	3 meters	7 meters	15 meters

For algorithm 2, we use the following table to convert RSSI to distance

0 dBm	-50 dBm	-60 dBm	-70 dBm
1 meter	2 meters	5 meters	9 meter

If your average signal strength to a beacon over a period of time is in the range [0, -50), you would be mapped to a distance of two meters for algorithms 1, 3, and 4 and one meter for algorithm 2.

Data Collection

In order to test the effectiveness of the algorithms, we gathered test data from buildings where beacons were deployed. In order to do this, we marked testing positions on a map of the building, used an app to automatically gather signals strengths of nearby beacons for a period of time on a phone, and then run the algorithms over the data collected by the phone.

In this paper, we use two data sets to analyze the algorithms: one data set was collected a few years ago using a different type of beacon while the other was collected this year using an assortment of AXA beacons.

Comparison

In order to compare the effectiveness of the algorithms, we will use paried t-tests to show if the error of the location estimates using one algorithm is significantly different from the estimates of another. Note, the paired t-test requires that the difference between the samples follows a roughly normal distribution. In other words, we need to show that the difference between the errors of the two algorithms being compared follows a normal distribution.

The error of a location algorithm is calculated by finding the distance between the calculated position and the true position of the tester. In other words:

$$Error = \sqrt{(x_{true} - x_0)^2 + (y_{true} - y_0)^2}$$

We will also choose n = 3. In other words, we will select the three beacons with the highest signal strengths when computing position.

Showing Normality

In this section, we show that the data is adequately normal using a table of histograms on the test data collected from a few years ago. These histograms are shown on the next page.

Floor 0 Error Differences

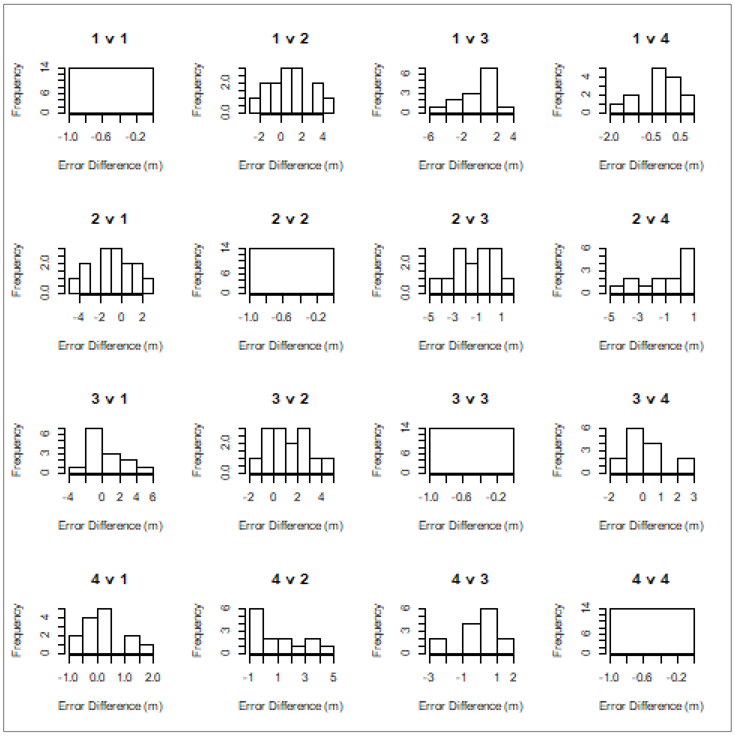

Figure 1 – This figure shows the histograms of error between the algorithms on data collected on floor 0.

Floor 2 Error Differences

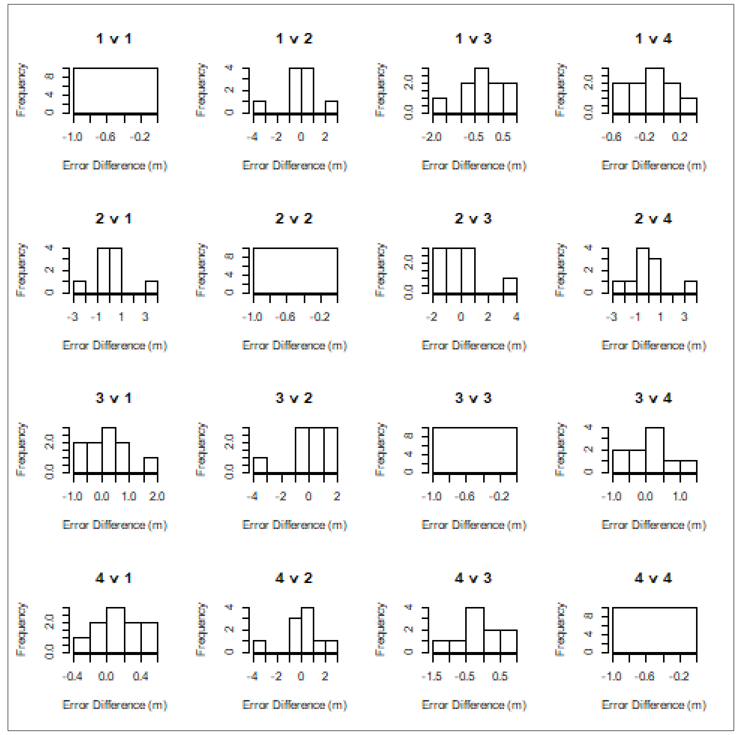

Figure 2 – This figure shows the histograms of error between the algorithms on data collected on floor 2.

NOTE: the titles of each histogram mean which algorithms are being compared. For example, 1 v 2 means we are showing the difference between the errors of algorithms 1 and 2.

We see that the diagonal of the histogram tables all show that the difference between the errors are 0. This is because we are showing the difference between an algorithm and itself.

For the most part, these graphs seem approximately normal. The only exception to this is with the graph 4v2 (and 2v4) in figure 1. Half of the data is located in the range -1 to 0, and the other half is located in the range 1 to 5. This means that the useage of a t-test to compare between algorithms 4 and 2 should be taken with a grain of salt.

T-Tests

In this section we will compute the mean and standard deviation of error for each algorithm and run a paired t-test between each pair of algorithms.

Here we show the mean and standard deviation of the error of the different algorithms on the data collected from floors 0 and 2 in the data set from a few years ago.

	Algorithm 1	Algorithm 2	Algorithm 3	Algorithm 4
Mean floor 0	4.104	4.024	3.926	3.967
Standard deviation floor 0	3.055	2.490	2.766	3.185
Mean floor 2	5.056	5.917	4.597	4.826
Standard deviation floor 2	3.140	4.129	3.099	3.073

Table 1 – This shows the mean and standard deviations of the error each algorithm has on approximating location using the beacons collected from a few years ago.

From table 1, we see that algorithms 3 and 4 seem to perform better than algorithms 1 and 2. In the tables 3 and 4, we show the probability that the difference between the usage of one algorithm versus another occurs just by chance.

	Algorithm 1	Algorithm 2	Algorithm 3	Algorithm 4
Algorithm 1	1	0.879	0.516	0.145
Algorithm 2	0.879	1	0.838	0.913
Algorithm 3	0.516	0.838	1	0.842
Algorithm 4	0.145	0.913	0.842	1

Table 2 – the probabilities that the difference in location approximation on floor 0 between two algorithms occurs by chance

	Algorithm 1	Algorithm 2	Algorithm 3	Algorithm 4
Algorithm 1	1	0.126	0.380	0.279
Algorithm 2	0.126	1	0.010	0.022
Algorithm 3	0.380	0.010	1	0.481
Algorithm 4	0.279	0.022	0.481	1

Table 3 – the probabilities that the difference in location approximation on floor 2 between two algorithms occurs by chance

For example, on floor 0, when comparing the difference between algorithms 1 and 2, we see that the difference in the error of algorithms 1 and 2 on approximating position occur by chance with a probability of 87.9%. This is a very high chance, and thus we can conclude that there isn't sufficient evidence to suggest that using algorithm 1 is better or worse than using algorithm 2. We will assume that a probability of .1 is sufficient evidence to suggest that there is a significant difference between two algorithms.

For the most part, most of the differences between the algorithms are not statistically significant. On floor 0, there isn't any evidence to suggest that using any one of the algorithms is superior to the others. However, on floor 2, we see that the difference between algorithms 2 and 3 and 2 and 4 are significantly different. Since algorithms 3 and 4 have a smaller mean error than algorithm 2 on floor 2, algorithm 2 performs significantly worse than these two algorithms.

Changing Bin Sizes

In this section, we will tune the bin sizes to fit a particular beacon type. From experimentation, we know that AXA beacons will produce a signal strength of roughly -60 dBm when you are roughly 1 meter away from the beacon with no obstructions. It’s also not likely that a person would be standing within a meter from a beacon.