1 Why does X not contradict Y

Let $\mathscr{A} \subseteq \mathscr{B}$ be Banach Algebras.

Take $X \equiv \{a - \lambda I : \lambda \in \mathbb{C}\}$. By XXXX, $\sigma_{\mathscr{B}}(a) \subseteq \sigma_{\mathscr{A}}(a)$.

This can be interpreted as saying in general that there are elements of X that are not invertible in $\mathscr{A}$ but are invertible in $\mathscr{B}$ since there simply more elements in $\mathscr{B}$, that might be the inverse of an element in X.

However, if both $\mathscr{A}$ and $\mathscr{B}$ are C^*, then by XXX, going from $\mathscr{A}$ to $\mathscr{B}$ does not add any inverses.

Subsequently the elements of X that are invertible is the same when taken as a subset of $\mathscr{A}$ or $\mathscr{B}$, thus $\sigma_{\mathscr{B}}(a) = \sigma_{\mathscr{A}}(a)$.