1 Show that wk is the smallest topology on $\mathscr{X}$ such that each x^* in $\mathscr{X}^*$

Must show that an arbitrary open set in wk can be generated by some collection of sets of the form $x^{*-1}(V)$.

Start with an arbitrary open set U. Since X with wk is a locally convex set, $\bigcap_{i=1}^{n}\{x\epsilon\mathscr{X} : p_j(x-x_0) < \varepsilon_j\} \subseteq U$ for some finite list of ε and p, where $p_{x^*} = |\langle x, x^*\rangle|$.

$\bigcap_{i=1}^{n}\{x \in \mathscr{X} : p_j(x-x_0) < \varepsilon_j\}$ is open and the intersection of pre-images of open sets from $\mathbb{F}$ (why?).

Since U is generated by the subbase of preimages of the collection of x^* and U is arbitrary, all open sets of wk are generated in this way, thus is the smallest possible topology.

2 Show that wk* is the smallest topology on $\mathscr{X}^*$ such that for each x in $\mathscr{X}$, $x^* \mapsto \langle x, x^*\rangle$

This purported topology σ is generated by open sets that are preimages of open sets of functions of the form $x^* \mapsto \langle x, x^*\rangle$. By continuity of composition, they are also the preimages of open sets of functions of the form $x^* \mapsto |\langle x, x^*\rangle|$. Open sets of $\mathbb{R}$ are generated by open balls, so the pre-images are generated by sets of the form $|\langle x, x^*\rangle| < \varepsilon$.

A set U of $\mathscr{X}^*$ is weakly open if and only if for every x_0^* in U there is an ε and there are $x_1, ...x_n$ in $\mathscr{X}$ such that

$$\bigcap_{i=1}^{n}\{x^* \in \mathscr{X}^* : |\langle x_k, x^* - x_0^*\rangle| < \varepsilon_j\} \subseteq U$$

Every such set U can be generated as part of σ and therefore is the smallest topology

3 If $A \subseteq \mathscr{X}$, show that A is weakly bounded if and only if A° is absorbing in $\mathscr{X}^*$

Assume A is weakly bounded.

Assume A° is absorbing in $\mathscr{X}^*$