1 Prove that for a finite topological spaces, discrete is equivalent to Hausdorf

Assume a finite topological space X is discrete. Then given two points x_1 and x_2 there are the open sets $\{x_1\}$ and $\{x_2\}$ that are non-intersecting, thus X is Hausdorf. Note the assumption that the space was finite was not needed.

Assume a finite topological space X is Hausdorf. Pick an arbitrary point x. For every $x_i \in X$ where $1 \leq i \leq n = |X|$, there exists an open set containing x that does not contain x_i, call it U_i. Take $U = \bigcap_{i=0}^{n} U_i$. $U = \{x\}$ since it is in every set U_i but every other point in X is not in at least one U_i. U is open, since it is the finite intersection of open sets.

Since x was chosen arbitrarily, the singleton set around every point is open. Thus since the union of open sets is open, any subset of X is open, thus X is discrete.