

1 Show that wk is the smallest topology on \mathcal{X} such that each x^* in \mathcal{X}^*

Must show that an arbitrary open set in wk can be generated by some collection of sets of the form $x^{*-1}(V)$.

Start with an arbitrary open set U . Since X with wk is a locally convex set, $\bigcap_{i=1}^n \{x \in \mathcal{X} : p_j(x - x_0) < \varepsilon_j\} \subseteq U$ for some finite list of ε and p , where $p_{x^*} = |\langle x, x^* \rangle|$. Say it's $x_1^*, x_2^* \dots x_n^*$.

Since U is generated by the subbase of preimages of the collection of x^* and U is arbitrary, all open sets of wk are generated in this way, thus is the smallest possible topology.