1 If $S : l_p \to l_p$ is defined by $S(\alpha_1, \alpha_2, ...) = (0, \alpha_1, \alpha_2, ...)$ describe the lattice of invariant closed subspaces of S

$\mathscr{M}_n = \{x \in l^p : x(k) = 0 \text{ for } 1 \leq k \leq n\}$, then $\mathscr{M}_n \in \text{Lat}S$, if $x \in \mathscr{M}_n$, so is $S(x)$, since they both start with at least n 0s.

$\mathscr{M}_{n+1} \in \mathscr{M}_n$ since if a series begins w n+1 0s it will also begin with n zeros.

Claim: These subspaces, together with the zero element and all of l^p, represent all of Lat T (which is thus a totally ordered set).

Start with the element $x_1 = (1, 0, 0, ...)$. It will be shown that the smallest closed invariant subspace X that contains x_1 is fact the whole space.

If $x_1 \in X$, so is any element of the form $(\alpha_1, 0, 0, ...)$, since X is a subspace so should be closed under scalar multiplication. Also, if $x_1 \in X$, so is $S(x_1) = (0, 1, 0, 0, ...) = x_2$, as well as $(0, \alpha, 0, 0)$. Since a subspace is closed under vector addition, all elements of the form $(\alpha_1, \alpha_2, \alpha_3, ..., 0, 0, ...)$ are thus in X. Call the set of all such points $Y \subset X$.

Now we must show that Y is dense in l_p, hence that $X = l_p$. Given an arbitrary element of l_p and ϵ, since $l_p -> 0$, we can produce an element $y \in Y$ such that $||l_p - y|| < \epsilon$

By the same argument, any element whose first n entries are 0 will be identical to $\mathscr{M}_n$, thus proving the claim.