

1 If p is a seminorm on \mathcal{X} , \mathcal{M} is a linear manifold in \mathcal{X} , and $\bar{p} : \mathcal{X}/\mathcal{M} \mapsto [0, \infty]$ is defined by $\bar{p}(x + \mathcal{M}) = \inf\{p(x + y) : y \in \mathcal{M}\}$, then \bar{p} is a seminorm on \mathcal{X}/\mathcal{M}

Triangle Inequality: Must show that $\bar{p}(x_1 + x_2 + \mathcal{M}) \leq \bar{p}(x_1 + \mathcal{M}) + \bar{p}(x_2 + \mathcal{M})$
 $\inf\{p(x_1 + x_2 + y) : y \in \mathcal{M}\} \leq \inf\{p(x_1 + y) : y \in \mathcal{M}\} + \inf\{p(x_2 + y) : y \in \mathcal{M}\}$.

Apply the triangle inequality somehow inside the infimums to expand into a larger expression that is still "less than or equal to" all the way through???

Absolute homogeneity: Must show that $\bar{p}(\alpha x + \mathcal{M}) = |\alpha| \bar{p}(x + \mathcal{M})$
 $\inf\{p(\alpha x + y) : y \in \mathcal{M}\} = \inf\{p(\alpha x + \alpha y) : y \in \mathcal{M}\} = \inf\{|\alpha| p(x + y) : y \in \mathcal{M}\} = |\alpha| \inf\{p(x + y) : y \in \mathcal{M}\}$

The first equality is justified since both y and αy are in \mathcal{M} , and the second is since p is itself a seminorm.