1 If p is a seminorm on $\mathscr{X}$, $\mathscr{M}$ is a linear manifold in $\mathscr{X}$, and $\bar{p} : \mathscr{X}/\mathscr{M} \mapsto [0, \infty]$ is defined by $\bar{p}(x + \mathscr{M}) = \inf\{p(x + y) : y \in \mathscr{M}\}$, then $\bar{p}$ is a seminorm on $\mathscr{X}/\mathscr{M}$

.

Triangle Inequality: Must show that $\bar{p}(x_1+x_2+M) \leq \bar{p}(x_1+\mathscr{M})+\bar{p}(x_2+\mathscr{M})$

$\inf\{p(x_1 + x_2 + y) : y \in \mathscr{M}\} \leq \inf\{p(x_1 + y) : y \in \mathscr{M}\} + \inf\{p(x_2 + y) : y \in \mathscr{M}\}$.

Apply the triangle inequality somehow inside the infimums to expand into a larger expression that is still "less than or equal to" all the way through???

Absolute homogeneity: Must show that $\bar{p}(\alpha x + \mathscr{M}) = |\alpha|\bar{p}(x + \mathscr{M})$

$\inf\{p(\alpha x + y) : y \in \mathscr{M}\} = \inf\{p(\alpha x + \alpha y) : y \in \mathscr{M}\} = \inf\{|\alpha|p(x + y) : y \in \mathscr{M}\} = |\alpha| \inf\{p(x + y) : y \in \mathscr{M}\}$

The first equality is justified since both y and αy are in $\mathscr{M}$, and the second is since p is itself a seminorm.

2 Show that if $\mathscr{M} \leq \mathscr{X}$ and $\mathscr{M}$ is topologically complimented in $\mathscr{X}$, then $\mathscr{M}^{\perp}$ is topologically complimented in $\mathscr{X}^*$ and that its complement is weak-star and linearly homeomorphic to $\mathscr{X}^*/\mathscr{M}^{\perp}$