1 Show that the particular point topology is T_0 but not T_1

Consider the particular point topology of a set X cardinality ≥ 3 with particular point x, that is all sets containing x are open as is the empty set, but no other sets.

Consider two arbitrary "other" points x_1 and x_2.

To show X is T_0 consider the two cases:

Case 1: x and x_1. x has an open neighborhood $\{x\}$ that does not contain x_1.

Case 2: x_1 and x_2. x_1 has an open neighborhood $\{x, x_1\}$ that does not contain x_2.

This is sufficient to prove the statement for all pairs of points, since x_1 and x_2 were chosen arbitrarily, and every pair of points falls into one of these cases.

To show X is not T_1, consider the case x and x_1. Every open set containing x_1 will also contain x (since all non-empty open sets contain x)

2 Show that the particular point topology is not Hausdorf

Obviously if a space is not T_1 it can't be T_2.

A more illustrative proof starts with the fact limits in Hausdorf are unique. So we must construct a sequence that converges to more than one point.

Consider any sequence s_n that is eventually x. This sequence has x as a limit, but it also has every other point in the space. In the case of x_1, every neighborhood contains of $\{x, x_1\}$, which s_n eventually never leaves.