

1 If $S : l_p \rightarrow l_p$ is defined by $S(\alpha_1, \alpha_2, \dots) = (0, \alpha_1, \alpha_2, \dots)$ describe the lattice of invariant subspaces of S

$\mathcal{M}_n = \{x \in l^p : x(k) = 0 \text{ for } 1 \leq k \leq n\}$, then $\mathcal{M}_n \in \text{Lat} S$, if $x \in \mathcal{M}_n$, so is $S(x)$, since they both start with at least n 0s.

$\mathcal{M}_{n+1} \in \mathcal{M}_n$ since if a series begins with $n+1$ 0s it will also begin with n zeros.

Claim: These subspaces, together with the zero element and all of l^p represent all of $\text{Lat } T$ (which is thus a totally ordered set).

Let X be an arbitrary set in l_p , and n be the number such every element of X has at 0s in at least the first n slots. The claim is $X = M_n$.

That $X \subset M_n$ is obvious. To show $M_n \subset X$, pick an arbitrary element that starts with n 0s, say $x_0 = (0_1, 0_2, \dots, 0_n, a_{n+1}, a_{n+2}, \dots)$. Then X must also include $x_1 = (0_1, 0_2, \dots, 0_n, 0_{n+1}, a_{n+1}, a_{n+2}, \dots)$ and so on. The linear combination of all x is (almost) all of M_n . (TODO: Finish. Take the closure maybe?)