1 Let $e_0, e_1, ...$ be an orthonormal basis for $\mathscr{H}$ and let $\alpha_0, \alpha_1, ...$ be complex numbers. Define $\mathscr{D} = \{h \in \mathscr{H} : \sum_0^\infty |\alpha_n \langle h, e_n\rangle|^2 < \infty\}$ and let $Ah = \sum_0^\infty \alpha_n \langle h, e_n\rangle e_n$ for h in $\mathscr{D}$. Then $A \in \mathscr{C}(\mathscr{H})$ with dom $A = \mathscr{D}$. Also, dom $A^* = \mathscr{D}$ and $A^*h = \sum_0^\infty \overline{\alpha_n}\langle h, e_n\rangle e_n$ for all h in $\mathscr{D}$

Exercise X.1.2: Prove claims in X.1.9

Since Hilbert Spaces in the chapter are assumed to be separable, to show that $\mathscr{D}$ is dense we must show that for every $h \in \mathscr{H}$, there exists a sequence $h_\bullet$ in $\mathscr{D}$ that converges to h. Let $h = \sum_0^\infty \beta_n e_n$ where all but finite number of β_n are 0. ???

Next we have to show that A is closed, that is $\{h \oplus Ah : h \in \mathscr{D}\}$ is closed in $\mathscr{H} \oplus \mathscr{H}$. Again since $\mathscr{H}$ is separable, this amounts to showing that the limit of an arbitrary sequence $h_\bullet$ of $\mathscr{D}$ is still in $h \oplus Ah$. Again not sure where to go from here.

To show that $A^*h = \sum_0^\infty \overline{\alpha_n}\langle h, e_n\rangle e_n$, start with $\langle Ah, k\rangle = \langle h, A^*k\rangle$.

$\langle \sum_0^\infty \alpha_n \langle h, e_n\rangle e_n, k\rangle$