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1 If $x \in X$ and $\delta_x(f) = f(x)$ for all f in $C_b(X)$, show that $||\delta_x|| = 1$

$$
\begin{aligned}
||\delta_x|| &= \sup\{||\delta_x(f)|| : f \in C_b(X), ||f|| \leq 1\} && (1)\\
&= \sup\{||f(x)|| : f \in C_b(X), ||f|| \leq 1\} && (2)\\
&= 1 && (3)
\end{aligned}
$$

Since $||f|| \leq 1$ are exactly those functions where $f(x) \leq 1$ for all x.