

- 1 Let  $A$  be a commutative ring. Let  $M$  be a module, and  $N$  a submodule. Let  $N = Q_1 \cap \dots \cap Q_r$  be a primary decomposition of  $N$ . Let  $\bar{Q}_i = Q_i/N$ . Show that  $0 = \bar{Q}_1 \cap \dots \cap \bar{Q}_r$  is a primary decomposition of  $0$  in  $M/N$ . State and prove the converse.

Firstly, let's take a concrete example,  $A = \mathbb{Z}$ ,  $M = \mathbb{Z}_{24}$ ,  $N = \mathbb{Z}_{12}$