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Week 3 Questions

Question 1

a) The probability of each roll is $\frac{1}{6}$, so the probability is:

$$\left(\frac{1}{6}\right)^6 = \frac{1}{46656} = 0.000021433$$

b) The probability of each 3 that is rolled is $\frac{1}{6}$ and there is $\binom{6}{4}$ ways to order them. The probability of not rolling one is $\frac{5}{6}$. Therefore:

$$\left(\frac{1}{6}\right)^4 \cdot \binom{6}{4} \cdot \left(\frac{5}{6}\right)^2 = 0.00803$$

c) The probability of the 1 being rolled is $\frac{1}{6}$ and the probability of the other rolls is $\frac{5}{6}$. There are $\binom{6}{1}$ ways to order the rolls. Therefore the probability is:

$$\left(\frac{1}{6}\right) \cdot \binom{6}{1} \cdot \left(\frac{5}{6}\right)^5 = 0.40187$$

d) The probability of this happening is the inverse of the probability that we don't roll any 1s at all.

$$1 - \left(\frac{5}{6}\right)^6 = 0.665102$$

Question 2

P(A) is $\frac{1}{6}$. The only way that the sum of two rolls could be 2 is if the second roll is also a 1. P(B) is therefore $\frac{1}{6} \cdot \frac{1}{20} = \frac{1}{120}$.

Therefore the probability of A and B happening is the probability of a 1 being rolled on the first roll and a 1 being rolled on the second roll. P(A n B) = $\frac{1}{120}$.

Two events X and Y are said to be independent if P(X n Y) = P(X) · P(Y)
This is not true for A and B because P(A n B) is $\frac{1}{120}$ and P(A) · P(B) = $\frac{1}{6} \cdot \frac{1}{120} = \frac{1}{720}$.
It is clear that these two events are not independent.

Question 3

a) On the hacker's first try the probability that she is unsuccessful is $\frac{n-1}{n}$, on her second attempt (because she remoed the incorrect password from her last attempt) the probability of being unsuccessful is $\frac{n-2}{n-1}$, and so on. So we can say that on the k-th try, the probability of her being successful is -> $\frac{1}{n-k+1}$

b) Substituting the values into the formula we get $\frac{1}{4}$

c) For the first k-1 attempts, the probability of her being unsuccessful is $\frac{n-1}{n}$.
On the k-th attempt, the probability of her being successful is $\frac{1}{n}$.
Therefore the probability is given by

$$\left(\frac{n-1}{n}\right)^{(k-1)} \cdot \frac{1}{n}$$

d) Substituing the values into the formula we get

$$\left(\frac{5}{6}\right)^{2} \cdot \left(\frac{1}{6}\right) = 0.11574074$$

Question 4

a) The probability that they fail at least one test is the inverse of the probability that they don't fail any tests.

$$1 - \left(\frac{1}{3}\right)^{3} = 0.973$$

b) The probability of the person failing at least one test is the inverse of the probability that they don't fail any of the tests.

$$1 - \left(\frac{95}{100}\right)^{3} = 0.142625$$

c) Let R be the event that the user is a robot, and F be the event that the user gets flagged.

Given that a user is a robot, the probability that they are flagged is:

$$P(F \mid R) = 0.973$$

The probability that a user is a robot, prior to receiving any new evidence:

$$P(R) = 0.1$$

Using marginalisation we calculate:

$$P(F) = P(F \mid R) \cdot P(R) + P(F \mid R') \cdot P(R')$$

$$= 0.973 \cdot 0.1 + 0.1426 \cdot (1 - 0.1) = 0.22564$$

Therefore the probability that the user is a robot, given that they are flagged is:

$$P(R \mid F) = \frac{P(F|R) \cdot P(R)}{P(F)} = \frac{0.973 \cdot 0.1}{0.22564} = 0.43121$$