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Week 4 Questions

Question 1

(a) There is only one way that two dice rolls can sum to 2, and that is if both of the rolls are 1.

$$\{(1, 1)\}$$

(b) The only ways that the sum can be 3 is if a r1 is rolled along with a 2. The two numbers can be rolled in any order of 1 and 2 or 2 and 1.

$$\{(2, 1), (1, 2)\}$$

(c) The sum can be 4 if: a 3 and a 1 are rolled, or a 2 and a 2 are rolled. The 3 and 1 can be rolled in any order of 1 and 3 or 3 and 1.

$$\{(3, 1), (1, 3), (2, 2)\}$$

(d) The event contains 3 possible outcomes and there are a total of 6^2 elements in the sample space.

$$P(X = 1) = \left(\frac{3}{6^2}\right) = 0.083333333$$

Question 2

(a) These are the possible outcomes of heads and tails, along with the relevant values for X

$$(3H, 0T) = 3 - 0, \quad X = +3$$

$$(2H, 1T) = 2 - 1, \quad X = +1$$

$$(1H, 2T) = 1 - 2, \quad X = -1$$

$$(0H, 3T) = 0 - 3, \quad X = -3$$

(b) In this case we have 3 heads and 0 tails rolled. There are $\binom{3}{0}$ ways to order the rolls and the total number of outcomes is 2^3 .

$$\frac{\binom{3}{0}}{2^3} = 0.125$$

(c) In this case we have rolled 1 head and 2 tails. There are $\binom{3}{1}$ ways to order the rolls and the total number of outcomes is 2^3 .

$$\frac{\binom{3}{1}}{2^3} = 0.375$$

(d) The probabilities are as follows:

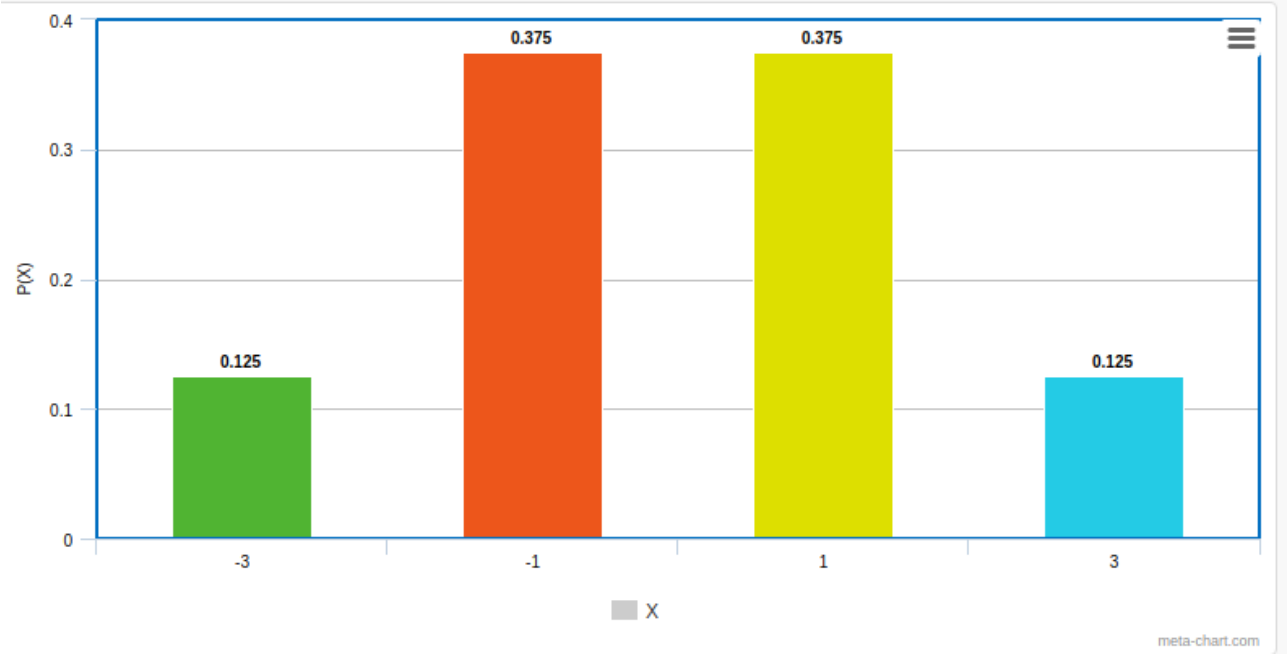
X = -3, P(X) = 0.125

X = -1, P(X) = 0.375

X = 1, P(X) = 0.375

X = 3, P(X) = 0.125

They can be graphed like so:



The cumulative probabilities are:

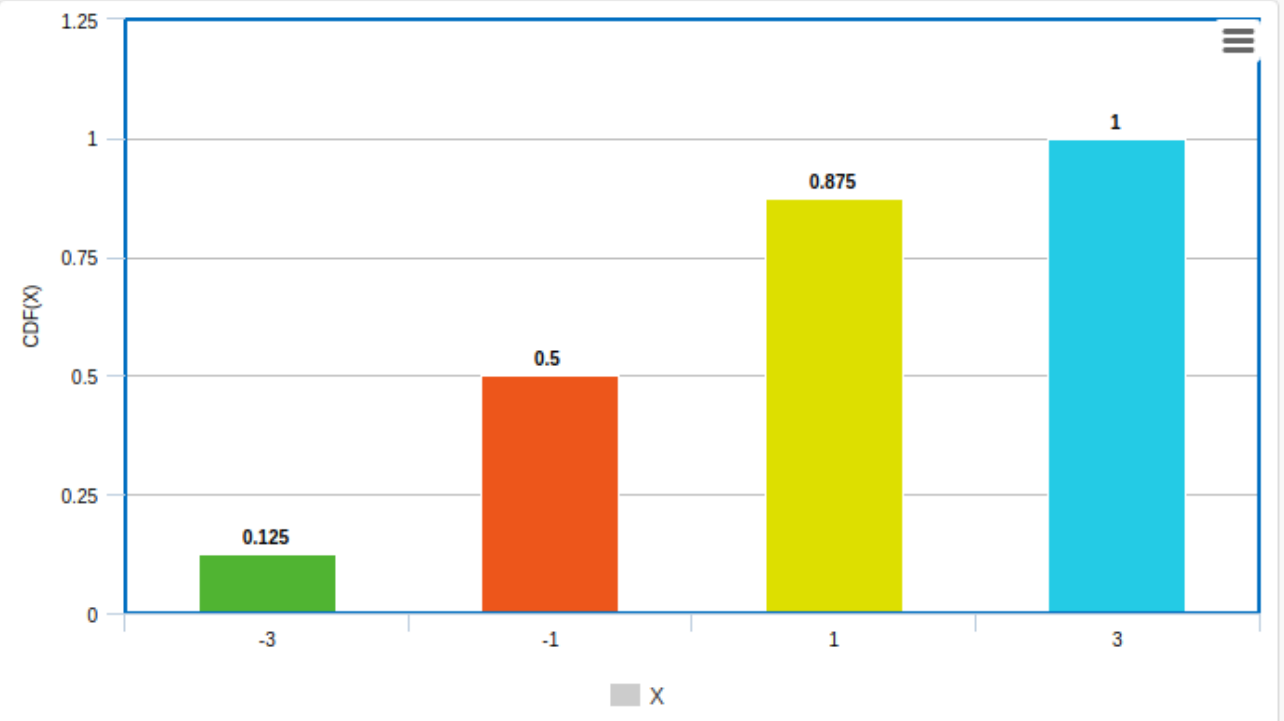
X = -3, CDF(X) = 0.125

X = -1, CDF(X) = 0.5

X = 1, CDF(X) = 0.875

X = 3, CDF(X) = 1

They can be graphed like so:



Question 3

(a) It's not possible for any dice roll to be less than 1. So the probability is:

$$P(X \geq 1) = 1.0$$

(b) The probability of every roll being greater or equal to 2 is the same as the probability that none of the rolls are equal to 1. For each roll the probability is $\frac{5}{6}$.

$$P(X \geq 2) = \left(\frac{5}{6}\right)^4 = 0.48225$$

(c) The probability that $X \leq 1$ is the inverse of the probability that none of the dice rolls are equal to 1.

$$P(X \leq 1) = 1 - \left(\frac{5}{6}\right)^4 = 0.51774$$

The probability that $X \leq 2$ is the inverse of the probability that none of the dice rolls are equal to 1 or 2. This is the cumulative probabilty of $P(X \leq 1) + P(X \leq 2)$.

$$P(X \leq 2) = 1 - \left(\frac{4}{6}\right)^4 = 0.80246$$

From viewing these 2 calculations we can derive a formula:

$$P(X \leq k) = 1 - \left(\frac{6-k}{6}\right)^4$$

So we just replace k with the remaining values 3, 4, 5, 6 to get the remaining probabilities.

$$P(X \leq 3) = 1 - \left(\frac{3}{6}\right)^4 = 0.9365$$

$$P(X \leq 4) = 1 - \left(\frac{2}{6}\right)^4 = 0.98765$$

$$P(X \leq 5) = 1 - \left(\frac{1}{6}\right)^4 = 0.99922$$

$$P(X \leq 6) = 1 - \left(\frac{0}{6}\right)^4 = 1$$

And we can graph these probabilties like so: