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17337376 Assignment 1 CSU33081

Q1

E. None of these

It would be represented like this:

$[0 \quad 2 \quad 2 \quad -6] + [1 \quad 0 \quad 2 \quad -4]$

Q2

B =>
$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q3

C => 1 3

Q4

$f(x) = 3 - 17x^3$

$f(2.5) = 3 - 17(2.5)^3 \Rightarrow -262.625$

$f(x) = 3 - 17x^3$ $\quad f(x_0 = 2) = -133$

$f'(x) = -51x^2$ $\quad f(x_0 = 2) = -204$

$f''(x) = -102x$ $\quad f(x_0 = 2) = -204$

$$P_2 x = -133 + \frac{-204(x-2)}{1!} + \frac{-204(x-2)^2}{2!}$$

$$P_2(2.5) = -133 + \frac{-204(2.5-2)}{1!} + \frac{-204(2.5-2)^2}{2!}$$

$-133 - 102 - 25.5$

-260.5

Truncation error $f(x) - P_2(x)$

$-262.625 - -260.5 = -2.125$

Answer = E. None of these.

Q5

$$x_2 = x_1 - \frac{f(x_1)(x_0 - x_1)}{(f(x_0) - f(x_1))}$$

$$2.5 - \frac{(973.25)(3-2.5)}{(3098)(973.25)}$$

$$2.5 - \frac{486.625}{2124.75} =$$

$$2.5 - 0.229$$

$$= 2.271$$

$$x_3 = x_2 - \frac{f(x_2)(x_1 - x_2)}{f(x_1) - f(x_2)}$$

$$2.271 - \frac{(457.015)(2.5 - 2.271)}{(973.25) - (457.015)}$$

$$2.271 - \frac{104.656}{516.235}$$

$$2.271 - 0.202$$

$$= 2.069$$

$$x_4 = x_3 - \frac{f(x_3)(x_2 - x_3)}{f(x_2) - f(x_3)}$$

$$2.069 - \frac{32.549}{295.883}$$

$$2.069 - 0.109$$

$$= 1.96$$

$$x_5 = x_4 - \frac{f(x_4)\,(x_3 - x_4)}{f(x_3) - f(x_4)}$$

$$1.96 - \frac{(49.371)(0.109)}{(161.132) - (49.371)}$$

$$1.96 - 0.048$$

$$= 1.912$$

$$x_6 = x_5 - \frac{f(x_5)(x_4 - x_5)}{f(x_4) - f(x_5)}$$

$$1.912 - \frac{(8.976)(0.048)}{(49.371) - (8.976)}$$

$$1.912 - 0.011$$

$$= 1.901$$

$$x_7 = x_6 - \frac{f(x_6)\,(x_5 - x_6)}{f(x_5) - f(x_6)}$$

$$1.901 - \frac{(0.412)(0.011)}{(8.976)-(0.412)}$$

$$1.901 - \frac{0.005}{8.564}$$

$$1.901 - 0.00058$$

$$= \cancel{1.90042}$$

$$1.90042$$

$1.901 - 1.90042$

$= 0.00058$

which is < 0.001

Answer = C => 1.900475

Q6

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f'(x) = 6x^5 - 1$

$$x_1 = 1.5 - \frac{8.891}{44.565}$$

$1.5 - 0.199$

$= 1.301$

$$x_2 = 1.301 - \frac{2.548}{21.363}$$

$1.301 - 0.119$

$= 1.182$

$$x_3 = 1.182 - \frac{0.545}{12.843}$$

$1.182 - 0.042$

$= 1.14$

$$x_4 = 1.14 - \frac{0.055}{10.55}$$

$1.14 - 0.005$

$= 1.135$

$$x_5 = 1.135 - \frac{0.003}{10.301}$$

$$1.135 - 0.0003$$
$$= 1.1347$$

$x_5 - x_4 = 0.0003$
which is < 0.001

Answer $= A \Rightarrow 1.134778$

Q7

$$\frac{df_1}{dx} = 2x+y \quad \Big| \quad \frac{df_1}{dy} = x$$

$$\frac{df_2}{dx} = 3y^2 \quad \Big| \quad \frac{df_2}{dy} = 1+6xy$$

$$J = \begin{bmatrix} 2x+y & x \\ 3y^2 & 1+6xy \end{bmatrix}$$

with $x_0 = (1.5, 3.5]^T$ we get

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} x_n \\ y_n \end{bmatrix} - \begin{bmatrix} \frac{df_1}{dx} & \frac{df_1}{dy} \\ \frac{df_2}{dx} & \frac{df_2}{dy} \end{bmatrix}^{-1} \times f\left(\begin{matrix} x_n \\ y_n \end{matrix}\right)$$

$n=0 \qquad x=1.5 \quad y=3.5$

$$\begin{bmatrix} 1.5 \\ 3.5 \end{bmatrix} - \begin{bmatrix} 6.5 & 1.5 \\ 36.75 & 32.5 \end{bmatrix}^{-1} \times f\left(\begin{bmatrix} 1.5 \\ 3.5 \end{bmatrix}\right)$$

$$\begin{bmatrix} 1.5 \\ 3.5 \end{bmatrix} - \begin{bmatrix} 6.5 & 1.5 \\ 36.75 & 32.5 \end{bmatrix}^{-1} \times \begin{bmatrix} -2.5 \\ 1.625 \end{bmatrix}$$

$$= \begin{matrix} 2.036 \\ 2.844 \end{matrix}$$

$n=1 \qquad x=2.036 \quad y=2.844$

$$\begin{bmatrix} 2.036 \\ 2.844 \end{bmatrix} - \begin{bmatrix} 6.916 & 2.036 \\ 24.263 & 35.741 \end{bmatrix}^{-1} \times \begin{bmatrix} -0.064 \\ -4.756 \end{bmatrix}$$

$$= \begin{bmatrix} 1.999 \\ 3.002 \end{bmatrix}$$

Q7

$n = 2 \qquad x = 1.999 \qquad y = 3.002$

$$\begin{bmatrix} 1.999 \\ 3.002 \end{bmatrix} - \begin{bmatrix} 7 & 1.999 \\ 27.041 & 37.004 \end{bmatrix}^{-1} \times \begin{bmatrix} 0.005 \\ 0.05 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$x = 2 \qquad y = 3$

Answer = B, $x = 2$, $y = 3$

Q8

$$A = \begin{bmatrix} 0 & -3 & -2 \\ 1 & -4 & -2 \\ -3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R1 \leftrightarrow R2$

$$\begin{bmatrix} 1 & -4 & -2 \\ 0 & -3 & -2 \\ -3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R1 \times -3$

$$\begin{bmatrix} -3 & 12 & 6 \\ 0 & -3 & -2 \\ -3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 0 & -3 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 - R_1$

$$\begin{bmatrix} -3 & 12 & 6 \\ 0 & -3 & -2 \\ 0 & -8 & -5 \end{bmatrix} \begin{bmatrix} 0 & -3 & 0 \\ 1 & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$R_1 \times -\frac{1}{3}$

$$\begin{bmatrix} 1 & -4 & -2 \\ 0 & -3 & -2 \\ 0 & -8 & -5 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$R_2 \times -\frac{1}{3}$

$$\begin{bmatrix} 1 & -4 & -2 \\ 0 & 1 & \frac{2}{3} \\ 0 & -8 & -8 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{3} & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$R2 \times -4$

$$\begin{bmatrix} 1 & -4 & -2 \\ 0 & -4 & -\frac{8}{3} \\ 0 & -8 & -5 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ \frac{4}{3} & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$R1 - R2$

$$\begin{bmatrix} 1 & 0 & \frac{2}{3} \\ 0 & -4 & -\frac{8}{3} \\ 0 & -8 & -5 \end{bmatrix} \begin{bmatrix} -\frac{4}{3} & 1 & 0 \\ \frac{4}{3} & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$R_2 \times -\frac{1}{4}$ $\begin{bmatrix} 1 & 0 & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} \\ 0 & -8 & -5 \end{bmatrix} \begin{bmatrix} -\frac{4}{3} & 1 & 0 \\ -\frac{1}{3} & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix}$

$R_2 \times -8$ $\begin{bmatrix} 1 & 0 & \frac{2}{3} \\ 0 & -8 & -\frac{16}{3} \\ 0 & -8 & -5 \end{bmatrix} \begin{bmatrix} -\frac{4}{3} & 1 & 0 \\ \frac{8}{3} & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix}$

$R_3 - R_2$ $\begin{bmatrix} 1 & 0 & \frac{2}{3} \\ 0 & -8 & -\frac{16}{3} \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} -\frac{4}{3} & 1 & 0 \\ \frac{8}{3} & 0 & 0 \\ -\frac{8}{3} & 3 & 1 \end{bmatrix}$

$R_2 \times -\frac{1}{8}$ $\begin{bmatrix} 1 & 0 & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} -\frac{4}{3} & 1 & 0 \\ \frac{8}{3} & 0 & 0 \\ -\frac{8}{3} & 3 & 1 \end{bmatrix}$

$R_3 \times \cancel{3}2$ $\begin{bmatrix} 1 & 0 & \frac{2}{3} \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & \frac{2}{3} \end{bmatrix} \begin{bmatrix} -\frac{4}{3} & 1 & 0 \\ \frac{8}{3} & 0 & 0 \\ -\frac{16}{3} & 6 & 2 \end{bmatrix}$

$R_2 - R_3$ $\begin{bmatrix} 1 & 0 & \frac{2}{3} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{2}{3} \end{bmatrix} \begin{bmatrix} -\frac{4}{3} & 1 & 0 \\ 5 & -6 & -2 \\ -\frac{16}{3} & 6 & 2 \end{bmatrix}$

$R_1 - R_3$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 4 & 5 & -2 \\ 5 & -6 & -2 \\ -\frac{16}{3} & 6 & 2 \end{bmatrix}$

$R_3 \times \frac{3}{2}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 4 & -5 & -2 \\ 5 & -6 & -2 \\ -8 & 9 & 3 \end{bmatrix}}$

Answer = D

Q9

From the given equations

$x_{k+1} = \frac{1}{12}(2 - 7y_k - 3z_k)$

$y_{k+1} = \frac{1}{5}(-5 - x_{k+1} - z_k)$

$z_{k+1} = \frac{1}{-11}(6 - 2x_{k+1} - 7y_{k+1})$

Initial guess $x = 1$ $y = 3$ $z = 5$

1st Approximation

$x_1 = \frac{1}{12}(2 - 7(3) - 3(5)) = \frac{1}{12}(-34) = -2.833$

$y_1 = \frac{1}{5}(-5 - (-2.833) - (5)) = \frac{1}{5}(-7.167) = -1.433$

$z_1 = \frac{1}{-11}(6 - 2(-2.833) - 7(-1.433)) = \frac{1}{-11}(27) = -1.973$

2nd Approximation

$x_2 = \frac{1}{12}(2 - 7(-1.433) - 3(-1.973)) = \frac{1}{12}(17.952) = 1.496$

$y_2 = \frac{1}{5}(-5 - (1.496) - (-1.973)) = \frac{1}{5}(-4.523) = -0.905$

$z_2 = \frac{1}{-11}(6 - 2(1.496) - 7(-0.905)) = \frac{1}{-11}(9.341) = -0.849$

3rd Approximation

$x_3 = \frac{1}{12}(2 - 7(-0.905) - 3(-0.849)) = \frac{1}{12}(10.88) = 0.907$

$y_3 = \frac{1}{5}(-5 - (0.907) - (-0.849)) = \frac{1}{5}(-5.058) = -1.012$

$z_3 = \frac{1}{-11}(6 - 2(0.907) - 7(-1.012)) = \frac{1}{-11}(11.267) = -1.02$

After 3 iterations, we have

$x = 0.907$

$y = -1.012$

$z = -1.024$

Answer = C

Q10

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 3 \times R_1$

$$= \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 2 & 6 & 13 \end{bmatrix}$$

$R_3 \leftarrow R_3 - 2 \times R_1$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 2 & 5 \end{bmatrix}$$

$R_3 \leftarrow R_3 - 1 \times R_2$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\therefore U = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

L is just the multipliers we used in Gaussian elimination with 1s on the diagonal

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$

$Ly = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{pmatrix} 3 \\ 13 \\ 4 \end{pmatrix}$$

$y_1 = 3$
$3y_1 + y_2 = 13$
$2y_1 + y_2 + y_3 = 4$

$3(3) + y_2 = 13$
$y_2 + 9 = 13$
$y_2 = 13 - 9$
$y_2 = 4$

$2y_1 + y_2 + y_3 = 4$
$2(3) + (4) + y_3 = 4$
$y_3 + 6 + 4 = 4$
$y_3 + 10 = 4$
$y_3 = 4 - 10$
$y_3 = -6$

$Ux = y$

$$\begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ -6 \end{bmatrix}$$

$x_1 + 2x_2 + 4x_3 = 3$
$2x_2 + 2x_3 = 4$
$3x_3 = -6$

$\Rightarrow$

$3x_3 = -6$
$x_3 = -2$

$2x_2 + 2(-2) = 4$
$2x_2 - 4 = 4$
$2x_2 = 4 + 4 = 8$
$x_2 = 4$

$\Rightarrow$

$x_1 + 2x_2 + 4x_3 = 3$
$x_1 + 2(4) + 4(-2) = 3$
$x_1 + 8 - 8 = 3$
$x_1 = 3$

Answer $=$ $x = \begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix}$

Answer $=$ B