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Week 2 Questions

Question 1

a) A six sided die is rolled three times. Each of the three rolls that occur has six possible outcomes. Therefore there are $6^3 = 216$ possible outcomes. This means that there are 216 elements in the sample space.

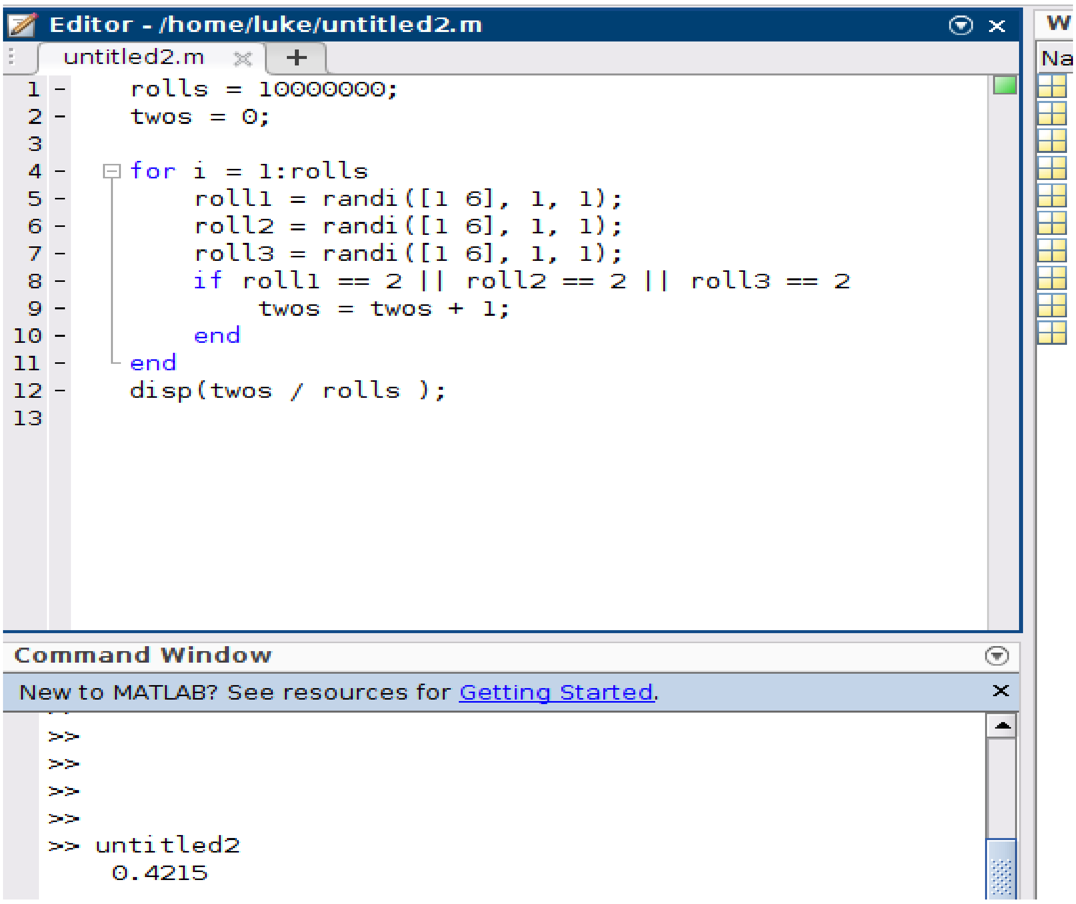
b) To calculate the rolls where there is a single 2, there are 5 options for the remaining two rolls and $\binom{3}{2}$ ways to order them. $5 \cdot 5 \cdot \binom{3}{2} = 75$.

To calculate the number of rolls where exactly two 2's are rolled, there are 5 options for the remaining rolls and $\binom{3}{1}$ ways of ordering the rolls. $5 \cdot \binom{3}{1} = 15$.

There is only one outcome where all rolls result in a 2.

$$\frac{75+15+1}{216} = 0.4212963$$

c) The code I wrote for my simulation:




```
rolls = 10000000;
twos = 0;

for i = 1:rolls
    roll1 = randi([1 6], 1, 1);
    roll2 = randi([1 6], 1, 1);
    roll3 = randi([1 6], 1, 1);
    if roll1 == 2 || roll2 == 2 || roll3 == 2
        twos = twos + 1;
    end
end
disp(twos / rolls );
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    0.4215
```

d) There is only one combination of numbers that sums up to 17, and that is 6, 6, 5. There is $\binom{3}{2}$ ways that these numbers could be rolled.

$$\left(\frac{1}{6}\right)^3 \cdot \binom{3}{2} = 0.01388888$$

e) Because the first roll was a 1 and the sum is 12 altogether, this means that the remaining two rolls must sum up to 11. The only way we can achieve this is by some ordering of 5 and 6. There are $\binom{2}{1}$ orderings of these rolls.

$$\left(\frac{1}{6}\right)^2 \cdot \binom{2}{1} = 0.555555555$$

Question 2

a) The probability of the first roll being a one is $\frac{1}{6}$, and so the probability of it not being a one is $\frac{5}{6}$. Given that the first roll is a one, a six sided dice is rolled so the probability of a five being rolled is $\frac{1}{6}$. If it is not a one, a twenty sided dice is rolled, so the probability of a five being rolled is $\frac{1}{20}$.

$$\frac{1}{6} \cdot \frac{1}{6} + \frac{5}{6} \cdot \frac{1}{20} = 0.069444444$$

b) The second roll can only be a 15 if a one is not rolled in the first roll. The probability of a one not being rolled is $\frac{5}{6}$. The probability of a 15 being rolled on the twenty sided dice is $\frac{1}{20}$.

$$\frac{5}{6} \cdot \frac{1}{20} = 0.0416666666$$

Question 3

Let G be the event that someone is guilty, and let C be the event that someone possesses a certain characteristic.

Given that a person is guilty, the probability that they have a certain characteristic is P(C | G) = 1

The probability of someone being guilty, before this new evidence is given as P(G) = 0.6

Using marginalisation we calculate:

$$P(C) = P(C \mid G) \cdot P(G) + P(C \mid G') \cdot P(G') = 1 \cdot 0.6 + 0.2 \cdot (1 - 0.6) = 0.68$$

Using Bayes Rule to calculate the probability that a suspect is guilty given that they possess the certain characteristic :

$$P(G \mid C) = \frac{P(C|G) \cdot P(G)}{P(C)} = \frac{(1) \cdot (0.6)}{(0.68)} = 0.88235$$

Question 4

Let L be the event that a person is in a particular location.
Let O be the event that a person is observed to be in a particular location.

For each cell in the grid, given the two observed bars, the probability of the phone being in the location is:

$$P(L \mid O) = \frac{P(O|L) \cdot P(L)}{P(O|L) \cdot P(L) + P(O|L') \cdot P(L')}$$

I wrote a program that calculates the probability of each cell's location given the observation.

My code stores the probability of the known location, and the observed location in two 4x4 arrays. I then initialised an array of zeros to store the results. Finally for each value I used the formula above to calculate the new probabilities and store them in the new array.

The code and the results can be seen below.

Editor - /home/luke/untitled2.m

untitled2.m

```
p =      [ 0.05 0.10 0.05 0.05 ;
           0.05 0.10 0.05 0.05 ;
           0.05 0.05 0.10 0.05 ;
           0.05 0.05 0.10 0.05 ];

p_obs =  [ 0.75 0.95 0.75 0.05 ;
           0.05 0.75 0.95 0.75 ;
           0.01 0.05 0.75 0.95 ;
           0.01 0.01 0.05 0.75 ];

pLocObs = zeros ( 4 , 4 ) ;

for i = 1:numel(pLocObs)
    num = p_obs(i) * p(i);
    den_a = (p_obs(i) * p(i));
    den_b = ((1 - p_obs(i)) * (1 - p(i)));
    pLocObs(i) = num / (den_a + den_b );
end

disp(pLocObs);
```

ommand Window

ew to MATLAB? See resources for Getting Started.

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>> untitled2
    0.1364    0.6786    0.1364    0.0028
    0.0028    0.2500    0.5000    0.1364
    0.0005    0.0028    0.2500    0.5000
    0.0005    0.0005    0.0058    0.1364
```