① I ran the code in MATLAB

Answer = C. 2.430

② I ran the code in MATLAB.

Answer = A. $\begin{bmatrix} 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & -1 & 3 & 1 \\ -1 & -1 & 1 & 3 \end{bmatrix}$

③ I ran all of the codes in MATLAB to see which was most appropriate.

Answer = A. plot (x, y1, 'r', x, y2, 'g', x, y3, 'b');

(4) $\begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$ $\det(A - \lambda I) = 0$

$$A - \lambda I = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

$$= \begin{pmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 6 & -6 & 4-\lambda \end{pmatrix}$$

$$1-\lambda \begin{vmatrix} -5-\lambda & 3 \\ -6 & 4-\lambda \end{vmatrix} - -3 \begin{vmatrix} 3 & 3 \\ 6 & 4-\lambda \end{vmatrix} + 3 \begin{vmatrix} 3 & -5-\lambda \\ 6 & -6 \end{vmatrix}$$

$$(1-\lambda)\left((-5-\lambda)(4-\lambda) - (3)(-6)\right) - (-3)\left((3)(4-\lambda) - (6)(3)\right)$$
$$+3\left((3)(-6) - (-5-\lambda)(6)\right)$$

$$= (1-\lambda)\left(-20 + 5\lambda - 4\lambda + \lambda^2 + 18\right) - (-3)(12 - 3\lambda - 18)$$
$$+3(-18 + 30 + 6\lambda)$$

$$= (1-\lambda)(\lambda^2 + \lambda + 2) - (-3)(-3\lambda - 6) + 3(-18 + 30 + 6\lambda)$$

$$= \lambda^2 + \lambda - 2 - \lambda^3 - \lambda^2 + 2\lambda - 9\lambda - 18 - 54 + 90 + 18\lambda$$

$$= -\lambda^3 + 12\lambda + 16 = 0$$

$$= -(\lambda + 2)(\lambda^2 - 2\lambda - 8)$$

$$= -(\lambda+2)(\lambda+2)(\lambda-4)$$

eigenvalues

$x_1 = 4$

$x_2 = -2$

Answer = C. 4, -2

(5) $A = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 4 & -3 \\ 0 & -1 & 1 \end{bmatrix}$ $x_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

1st iteration

$$Ax_0 = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 4 & -3 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

Scaling approximation

$$\Rightarrow x_1 = \frac{1}{2} \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -0.5 \\ 0 \end{bmatrix}$$

2nd iteration

$$Ax_1 = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 4 & -3 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 3.5 \\ -4 \\ 0.5 \end{bmatrix}$$

Scaling approximation

$$x_2 = \frac{1}{4} \begin{bmatrix} 3.5 \\ -4 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 0.875 \\ -1 \\ 0.125 \end{bmatrix}$$

3rd iteration

$$Ax_2 = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 4 & -3 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0.875 \\ -1 \\ 0.125 \end{bmatrix} = \begin{bmatrix} 3.625 \\ -6.125 \\ 1.125 \end{bmatrix}$$

(5) Scaling approximation

$$\Rightarrow x_3 = \frac{1}{6.125}\begin{bmatrix} 3.625 \\ -6.125 \\ 1.125 \end{bmatrix} = \begin{bmatrix} 0.59183673 \\ -1 \\ 0.18367347 \end{bmatrix}$$

4th iteration

$$Ax_3 = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 4 & -3 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 0.59183673 \\ -1 \\ 0.18367347 \end{bmatrix} = \begin{bmatrix} 2.7755102 \\ -5.73469388 \\ 1.18367347 \end{bmatrix}$$

Scaling approximation

$$x_4 = \frac{1}{5.73469388}\begin{bmatrix} 2.7755102 \\ -5.73469388 \\ 1.18367347 \end{bmatrix} = \begin{bmatrix} 0.48398577 \\ -1 \\ 0.20640569 \end{bmatrix}$$

5th iteration

$$Ax_4 = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 4 & -3 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 0.48398577 \\ -1 \\ 0.20640569 \end{bmatrix} = \begin{bmatrix} 2.4519573 \\ -5.58718861 \\ 1.20640569 \end{bmatrix}$$

Scaling approximation

$$x_5 = \frac{1}{5.58718861}\begin{bmatrix} 2.4519573 \\ -5.58718861 \\ 1.20640569 \end{bmatrix} = \begin{bmatrix} 0.4388535 \\ -1 \\ 0.21592357 \end{bmatrix}$$

The dominant eigenvalue = 5.59

The dominant eigenvector is $\begin{bmatrix} 0.44 \\ -1 \\ 0.22 \end{bmatrix}$

Answer = E. None of these

(6)

x	y	x^2	xy
30	70	900	2100
40	90	1600	3600
40	100	1600	4000
50	120	2500	6000
50	130	2500	6500
50	150	2500	7500
60	160	3600	9600
70	190	4900	13300
70	200	4900	14000
80	200	6400	16000
80	220	6400	17600
80	230	6400	18400
$\Sigma = 700$	$\Sigma = 1860$	$\Sigma = 44,200$	$\Sigma = 118,600$

find the best slope (m) and y-intercept (b) that suits $y = mx + b$

Step 1, for each (x, y) calculate x^2 and xy

$$m = \frac{N\Sigma(xy) - \Sigma x \Sigma y}{N\Sigma(x^2) - (\Sigma x)^2} = \frac{12 \times 118,600 - (700)(1860)}{12 \times 44,200 - (700)^2}$$

$$= \frac{121,200}{40,400} = 3$$

calculate intercept b

(6) $b = \dfrac{\Sigma y - m\Sigma x}{N}$

$b = \dfrac{1860 - 3(700)}{12}$

$b = -20$

$y = mx + b$

$y = 3x - 20$

(i) $x = 35 \Rightarrow y = 3(35) - 20$

$y = 85m$

(ii) $x = 85 \Rightarrow y = 3(85) - 20$

$y = 235m$

(iii) $x = 100 \Rightarrow y = 3(100) - 20$

$y = 280m$

Answer = B. 85m, 235m, 280m

(7) $p = k_1 e^{-0.1315h}$

$P_{sea\text{-}level} = k_1 e^{-0.1315(0)}$

$= k_1$

$P_{atmosphere} = k_1 e^{-0.1315(atmosphere)}$

$$\frac{P_{sea\text{-}level}}{P_{atmosphere}} = \frac{k_1}{k_1 e^{-0.1315(atmosphere)}}$$

$$\frac{P_{sea\text{-}level}}{\frac{1}{1000} P_{sea\text{-}level}} = \frac{1}{e^{-0.1315(atmosphere)}}$$

$$h_{atmosphere} = \frac{\ln \frac{1}{1000}}{-0.1315}$$

$h_{atmospher} = 52.53km$

Answer = D. 52.5 km

(8) First order polynomial, velocity is

$$v(t) = a_0 + a_1 t$$

to find v at $t=16$, we choose the two data points nearest to $t=16$

$t_0 = 15$ $\quad$ $t_1 = 18$

$t_0 = 15, \; v(t_0) = 24$
$t_1 = 18, \; v(t_1) = 37$

gives

$$v(15) = a_0 + a_1(15) = 24$$
$$v(18) = a_0 + a_1(18) = 37$$

writing them in matrix form

$$\begin{bmatrix} 1 & 15 \\ 1 & 18 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 24 \\ 37 \end{bmatrix}$$

Solving these equations leaves us with

$$a_0 = -41$$
$$a_1 = 4.33333$$

hence $v(t) = a_0 + a_1 t$

$= -41 + 4.33333\, t \quad , \quad 15 \leq t \leq 18$

$v(16) = -41 + 4.33333(16)$
$= 28.3333 \text{ ms}$

Answer = B $\quad$ 28.33ms

(9) we want the Lagrange polynomials at $u = 2.5$, so

$$L_1(2.5) = \frac{(2.5-u_2)(2.5-u_3)(2.5-u_4)(2.5-u_5)}{(u_1-u_2)(u_1-u_3)(u_1-u_4)(u_1-u_5)}$$

$$= \frac{(2.5-0.5)(2.5-1)(2.5-1.5)(2.5-2)}{(0-0.5)(0-1)(0-1.5)(0-2)}$$

similar calculations for the other Lagrange polynomials leave us with

$L_2(2.5) = -5$
$L_3(2.5) = 10$
$L_4(2.5) = -10$
$L_5(2.5) = 5$

So our interpolated polynomial at $x = 2.5$ is

$$p(2.5) = f_1 L_1(2.5) + f_2 L_2(2.5) + f_3 L_3(2.5) + f_4 L_4(2.5) + f_5 L_5(2.5)$$

$$= 0 \times 1 + 19.32 \times -5 + 90.62 \times 10 + 175.71 \times -10 + 407.11 \times 5$$

$$= 1088.05$$

Answer = C. 1088N

(10)

x	2	3	7
y	$\sqrt{2}$	$\sqrt{3}$	$\sqrt{7}$

$$f(x) = y = a_1 + a_2(x - x_1) + a_3(x - x_1)(x - x_2)$$

x_i	y_i		
a_1 2	$\sqrt{2}$		
		a_2 $\frac{\sqrt{3}-\sqrt{2}}{3-2} = \boxed{0.3178}$	
3	$\sqrt{3}$		a_3 $\frac{0.2284 - 0.3178}{7-2} = \boxed{-0.01788}$
		$\frac{\sqrt{7}-\sqrt{3}}{7-3} = \boxed{0.2284}$	
7	$\sqrt{7}$		

$$f(x) = y = \sqrt{2} + 0.3178(x-2) + -0.01788(x-2)(x-3)$$

$$= \sqrt{2} + 0.3178x - 0.6356 - 0.01788(x^2 - 5x + 6)$$

$$0.3178x + 0.7786 - 0.01788x^2 + 0.0894x - 0.10728$$

$$= -0.01788x^2 + 0.4072x + 0.67132$$

$$f(2.5) = -0.01788(2.5)^2 + 0.4072(2.5) + 0.67132$$

$$= 1.57757$$

Answer = E. None of these.