Astronomer's Spectral Line Project

Luke Johnston
Biola University

Numerical Analysis

November 10, 2022

Abstract

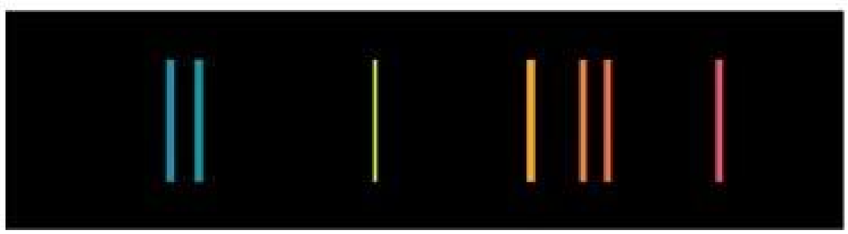
This paper demonstrates the application and results of the Astronomer's Spectral Line project for Math 321 - Numerical Analysis. This project applies methods to approximate the integral of a function given only data points (x,y). Six spectral lines were isolated from the data and further analyzed to find the integral corresponding to each line.

1 Spectral Line Introduction

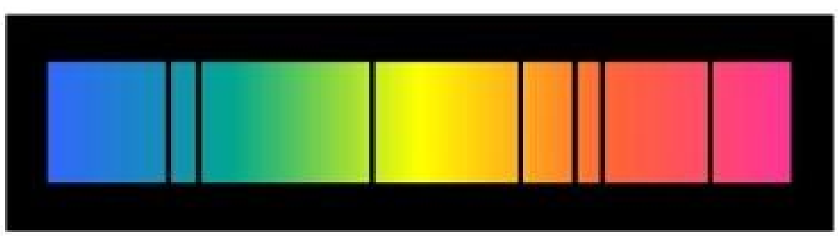
An astronomer has given us spectral data, which is the measurements of light intensity at different frequencies for a star. The astronomer wants us to analyze the spectral lines found in the data, especially the line strength, and return our findings to them. Before we can begin we must understand spectral lines.

The light from stars can be analyzed using spectral grating, to split the light up into frequencies. These frequencies correspond to colors, and we can measure the intensity of the light at each frequency. A spectral line can be used to identify the elements present in a cloud of interstellar gas, galaxy or star. Each element creates specific spectral lines at certain frequencies, so by identifying the frequencies of the lines, we can find out what atoms are present in a star.

If we separate the light using a prism, we will see a spectrum of colors crossed with lines. There are two kinds of spectral lines: emission and absorption lines. If the gas is cool then it will absorb light, but if the gas is hot then it will emit light at the wavelengths.



Emission lines are seen as coloured lines on a black background.



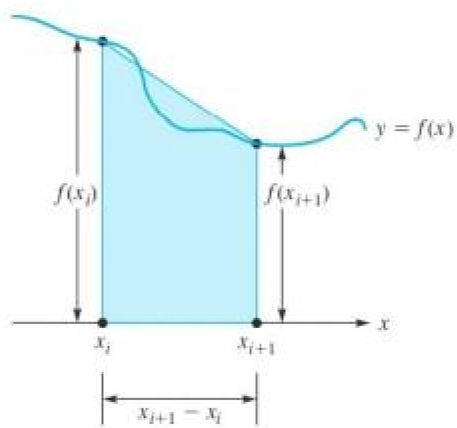
Absorption lines are seen as black lines on a coloured background.

(Cosmos.com)

2 Trapezoidal Method Application

2.1 Theory

The Trapezoid method uses trapezoids to estimate the area under a curve. The estimation of the integral from a to b of the function, is found by first dividing the interval [a, b] into subintervals according to: $\mathbf{a = x_0 < x_1 < x_2 < x_3 < \ldots < x_n = b}$. The points x_i do not have to be evenly spaced.

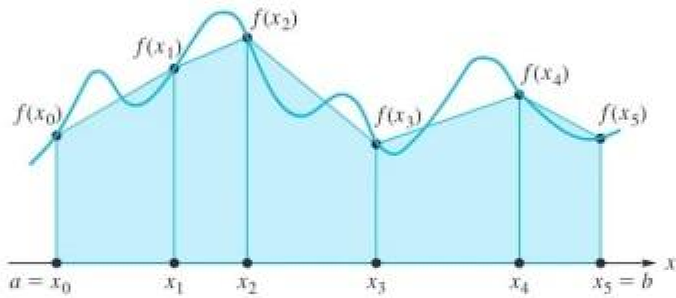


(Cheney & Kincaid, p.229)

The image above shows a trapezoid created using the subinterval [x_i , x_{i+1}] as the base and $f(x_i)$ and $f(x_{i+1})$ as the sides. The area of the trapezoid is equal to the base times the average height, which is the basic trapezoid rule.

$$\int_{x_i}^{x_{i+1}} f(x)\,dx \approx \frac{1}{2}(x_{i+1} - x_i)[f(x_i) + f(x_{i+1})]$$

This basic trapezoid rule can be used to find the total area of all the trapezoids from x_1 to x_n which is [a, b]. We can find the trapezoid from each subinterval, and summing them up will give us the estimation for the interval [a, b]. This is called the composite trapezoid rule. Below is an image showing all the trapezoids needed to estimate the area under the function.

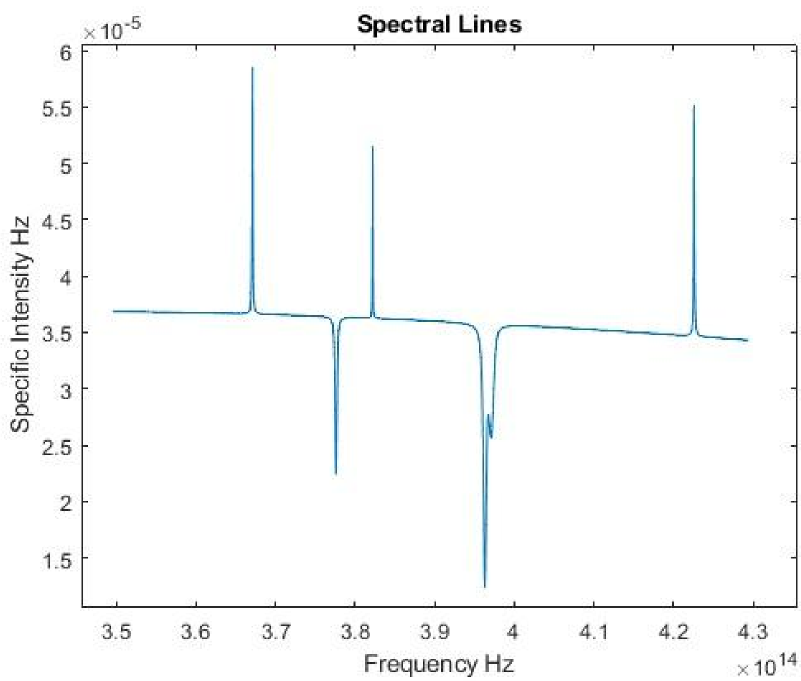


(Cheney & Kincaid, p.229)

The Trapezoid method is not the only way to estimate the integral of a function, but it is what we have decided to use for this project. The results may have been more or less accurate using a different method to find estimates.

2.2 Applying the Trapezoidal Method to Spectral Lines

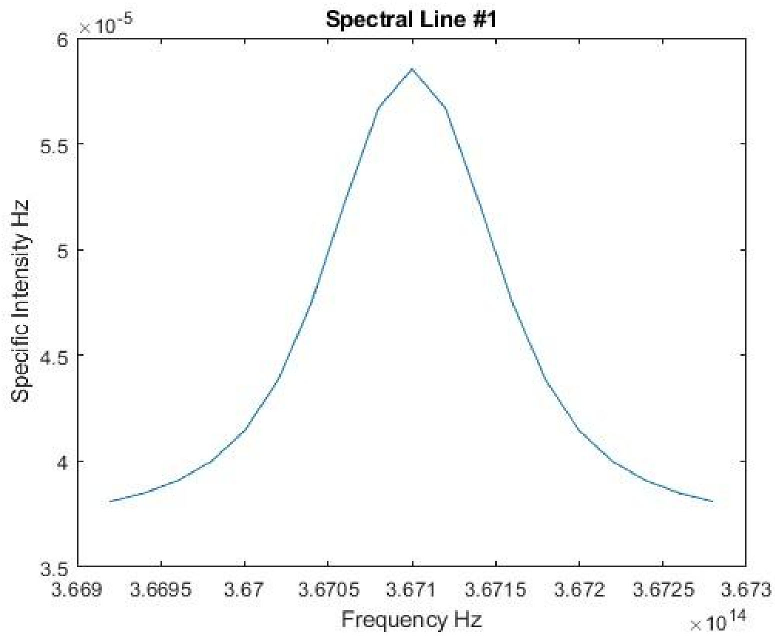
The data given by the astronomer has 4,000 frequency measurements. Each frequency data point has a corresponding light intensity. In this data set there are three emission lines and three absorption lines (six spectral lines in total). As shown in the graph below.



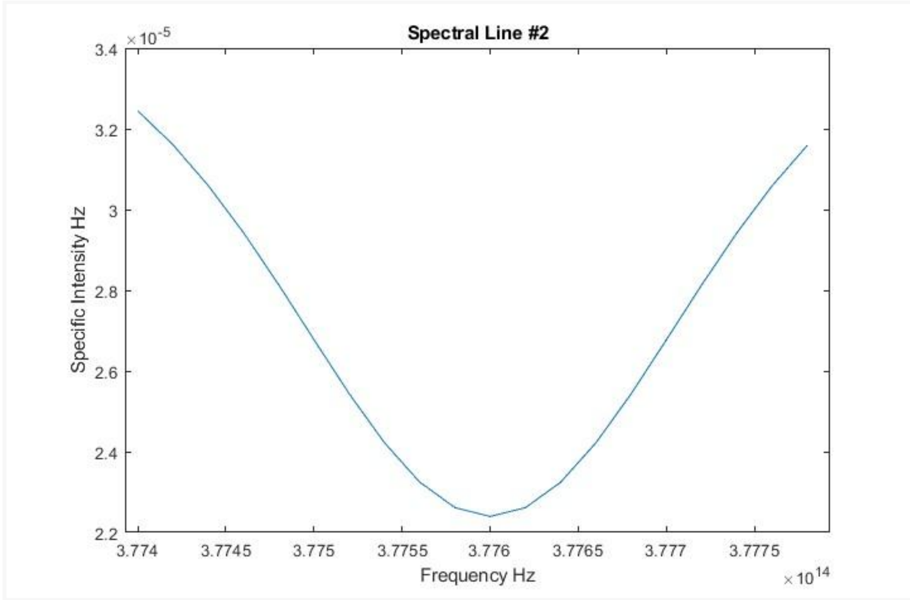
Two of the emission lines appear to be very close together in this graph but they are distinct from each other. We isolated each spectral line in the graph and gave them all the same width of 0.004

frequency. This was done so that the width will not affect the comparisons of spectral line intensity. The lines were numbered going from left to right about the frequency axis (x). So Spectral Line #1 is the leftmost spike and Spectral Line #6 is the rightmost spike. Using the composite trapezoidal method to integrate the spike interval, gives us the strength of the spectral line. The line strength is the intensity of each line integrated over frequency (in/m^2). We will now discuss each line separately.

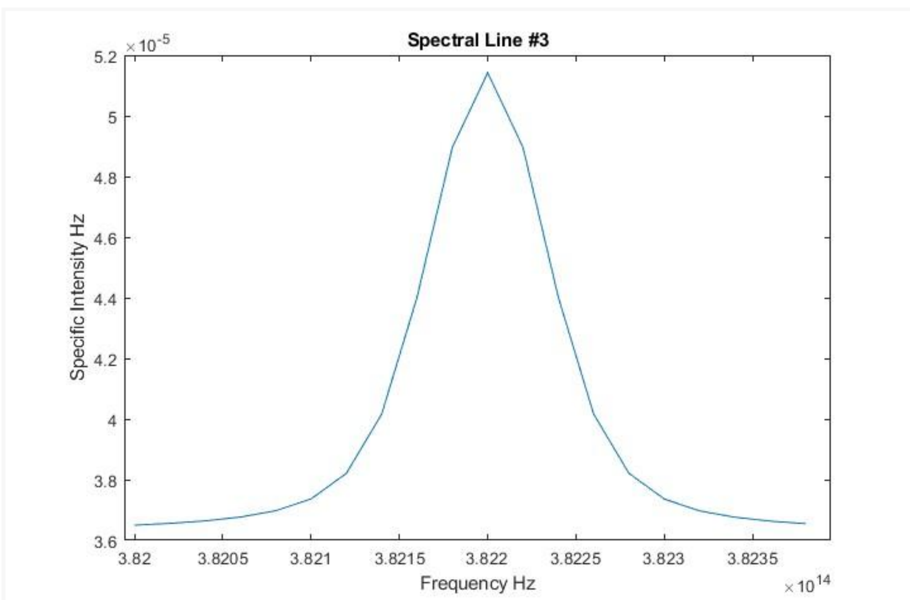
Spectral Line #1:
Centered at frequency 3.671e+14 Hz.
Total interval is [3.669e+14, 3.673e+14].
Line strength is 1.6301e+07.



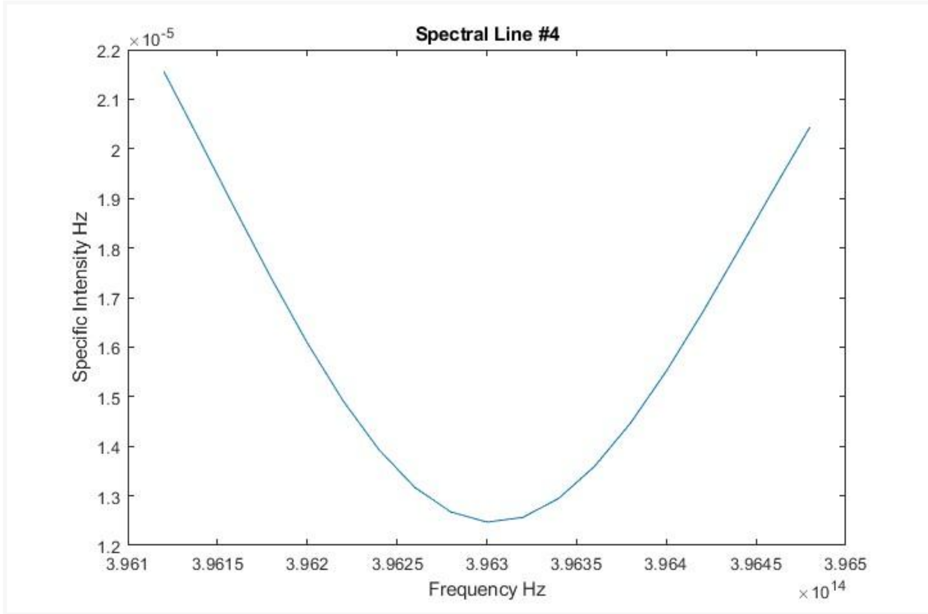
Spectral Line #2:
Centered at frequency 3.776e+14 Hz.
Total interval is [3.774e+14, 3.778e+14].
Line strength is 1.0128e+07.



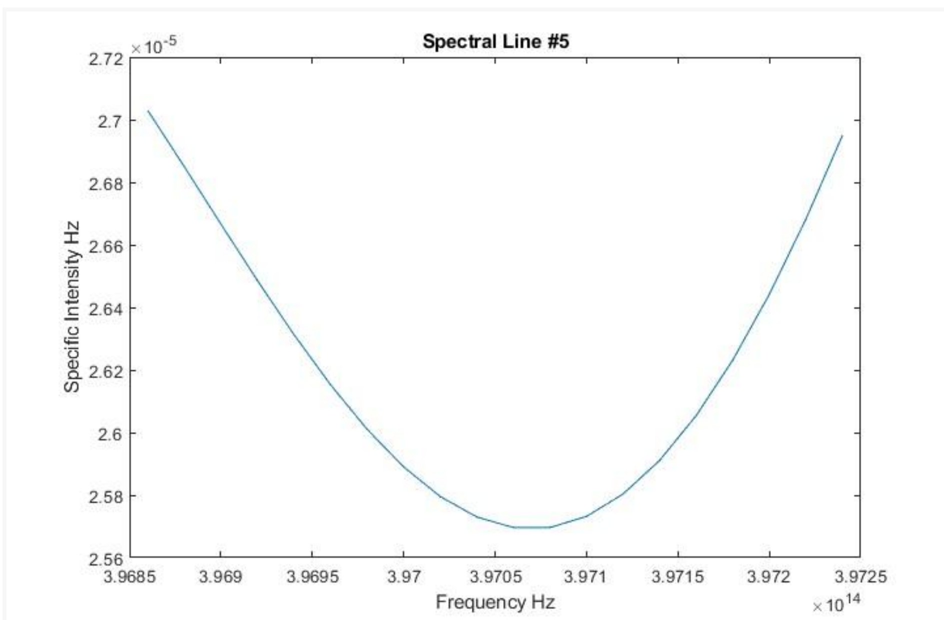
Spectral Line #3:
Centered at frequency 3.882e+14 Hz.
Total interval is [3.880e+14, 3.824e+14].
Line strength is 1.5252e+07.



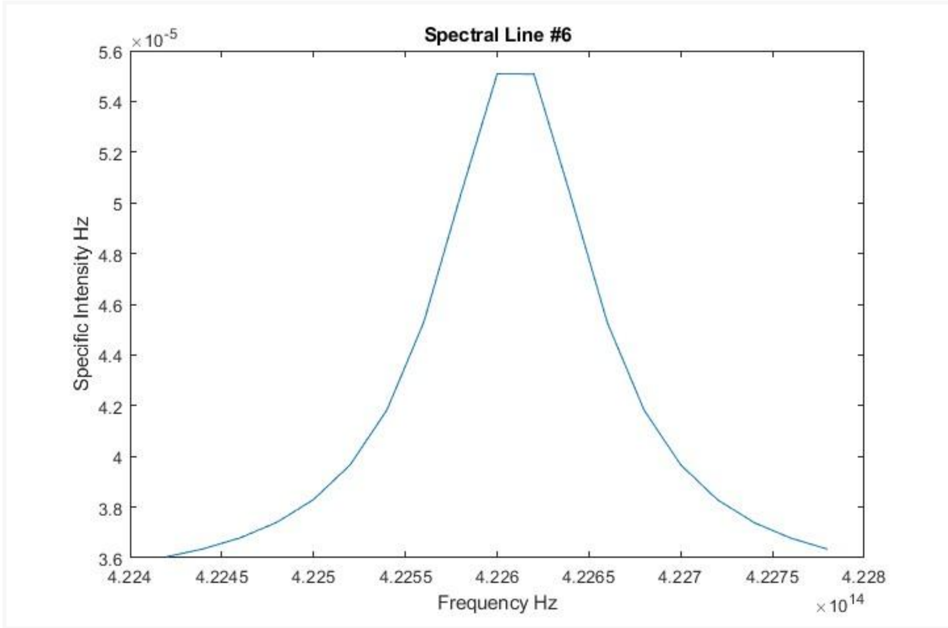
Spectral Line #4:
Centered at frequency 3.963e+14 Hz.
Total interval is [3.961e+14, 3.965e+14].
Line strength is 5.6694e+06.



Spectral Line #5:
Centered at frequency 3.9706e+14 Hz.
Total interval is [3.9686e+14, 3.9726e+14].
Line strength is 9.9429e+06.



Spectral Line #6:
Centered at frequency 4.226e+14 Hz.
Total interval is [4.224e+14, 4.228e+14].
Line strength is 1.3154e+07.



This data on each spectral line will be used to help the astronomer identify what elements are present in the star. The line strength was found for each line, and they fit with what is expected based on the overall graph.

3 Appendix: Matlab Script

```
%uiimport('spectrum.xls');
x=spectrum{:,1};
y=spectrum{:,2};
plot(x,y);

[s1, s2, s3, s4, s5, s6] = deal(4000);   %start variables
[f1, f2, f3, f4, f5, f6] = deal(0);      %finish variables

for i=1:4000
   if ((x(i) > 3.669e+14) && (x(i) < 3.673e+14))       %line #1
       if (i<s1)
           s1=i;
       end
       if (i>f1)
           f1=i;
       end
   end

   if ((x(i) > 3.774e+14) && (x(i) < 3.778e+14))       %line #2
       if (i<s2)
           s2=i;
       end
       if (i>f2)
           f2=i;
       end
   end

   if ((x(i) > 3.82e+14) && (x(i) < 3.824e+14))       %line #3
       if (i<s3)
           s3=i;
       end
       if (i>f3)
           f3=i;
       end
   end

   if ((x(i) > 3.961e+14) && (x(i) < 3.965e+14))       %line #4
       if (i<s4)
           s4=i;
       end
       if (i>f4)
           f4=i;
       end
   end

   if ((x(i) > 3.9686e+14) && (x(i) < 3.9726e+14))       %line #5
       if (i<s5)
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            s5=i;
        end
        if (i>f5)
            f5=i;
        end
    end

    if ((x(i) > 4.224e+14) && (x(i) < 4.228e+14))        %line #6
        if (i<s6)
            s6=i;
        end
        if (i>f6)
            f6=i;
        end
    end
end

Qsum1 = 0;          %calculate the sum of trapezoids for the spectral line #1
for i=s1:f1-1
    Q = 1/2*(x(i+1)-x(i))*(y(i)+y(i+1));
    Qsum1 = Qsum1 + Q;
end
%plot(x(s1:f1),y(s1:f1));
disp('strength of spectral line #1:');
disp(Qsum1);

Qsum2 = 0;          %calculate the sum of trapezoids for the spectral line #2
for i=s2:f2-1
    Q = 1/2*(x(i+1)-x(i))*(y(i)+y(i+1));
    Qsum2 = Qsum2 + Q;
end
%plot(x(s2:f2),y(s2:f2));
disp('strength of spectral line #2:');
disp(Qsum2);

Qsum3 = 0;          %calculate the sum of trapezoids for the spectral line #3
for i=s3:f3-1
    Q = 1/2*(x(i+1)-x(i))*(y(i)+y(i+1));
    Qsum3 = Qsum3 + Q;
end
%plot(x(s3:f3),y(s3:f3));
disp('strength of spectral line #3:');
disp(Qsum3);

Qsum4 = 0;          %calculate the sum of trapezoids for the spectral line #4
for i=s4:f4-1
    Q = 1/2*(x(i+1)-x(i))*(y(i)+y(i+1));
    Qsum4 = Qsum4 + Q;
end
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```
%plot(x(s4:f4),y(s4:f4));
disp('strength of spectral line #4:');
disp(Qsum4);

Qsum5 = 0;          %calculate the sum of trapezoids for the spectral line #5
for i=s5:f5-1
   Q = 1/2*(x(i+1)-x(i))*(y(i)+y(i+1));
   Qsum5 = Qsum5 + Q;
end
%plot(x(s5:f5),y(s5:f5));
disp('strength of spectral line #5:');
disp(Qsum5);

Qsum6 = 0;          %calculate the sum of trapezoids for the spectral line #5
for i=s6:f6-1
   Q = 1/2*(x(i+1)-x(i))*(y(i)+y(i+1));
   Qsum6 = Qsum6 + Q6;
end
%plot(x(s6:f6),y(s6:f6));
disp('strength of spectral line #6:');
disp(Qsum6);

title('Spectral Lines')
xlabel('Frequency Hz')
ylabel('Specific Intensity Hz')
```

References

Cheney, W., Kincaid, D., (2013). Numerical Mathematics and Computing (7th ed.). Cengage Learning.

Spectral Line. COSMOS. astronomy.swin.edu.au/cosmos/S/Spectral+Line.