EC ENGR 111L

Experiment #4

Resonance Circuit

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Lab Section: 1E

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Part#1: Series Resonance Circuit

Objectives:

In the series resonance circuit, calculate the resonant frequency w_R and quality factor Q for each component, and compare to the theoretical value. Find the gain and phase graphs under the sinusoidal voltage input.

Theory:

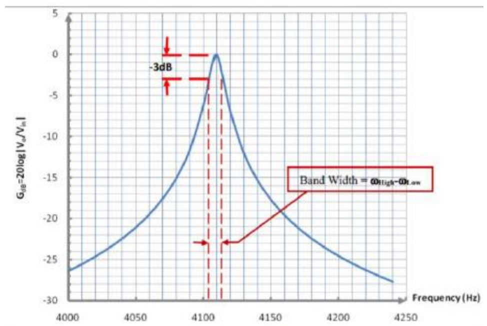
In series LRC circuit, maximum values for V_R, V_C, and V_L won't happen at the same frequency.

$(V_R)_{Max}$ at $\omega_0 = \frac{1}{\sqrt{LC}}$ 1-1

$(V_C)_{Max}$ at $\omega = \omega_0\sqrt{1 - \frac{R^2C}{2L}}$ 1-2

$(V_L)_{Max}$ at $\omega = \frac{\omega_0}{\sqrt{1-\frac{R^2C}{2L}}}$ 1-3

If $\frac{R^2C}{2L} \geq 1$ you don't see any resonance across C or across L



$$Q = \omega_{R/BW} \qquad 1\text{-}5$$

For a series RLC circuit, the Q factor may be expressed as a ratio of the reactance of the capacitor or inductor at resonance frequency to the resistance of the network. It may also be expressed as the ratio of the voltage across a reactive component at the resonance frequency to the input voltage that appears across the resistor.

$$Q = \frac{X_L}{R} = \frac{\omega_R L}{R} = \frac{L}{R\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad 1\text{-}6$$

Or alternatively,

$$Q = \frac{X_C}{R} = \frac{1}{\omega_R RC} = \frac{\sqrt{LC}}{RC} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad 1\text{-}7$$

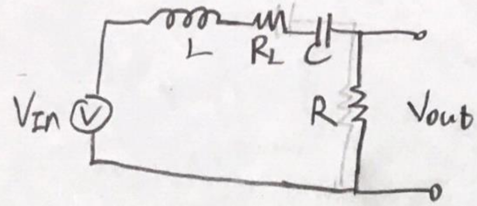
(If the inductor has internal resistance, R must be replaced by $R+R_L$).

$$Q = \frac{V_X}{V_{in}}$$

The transfer functions for each component.

Lab 4.

Series resonant



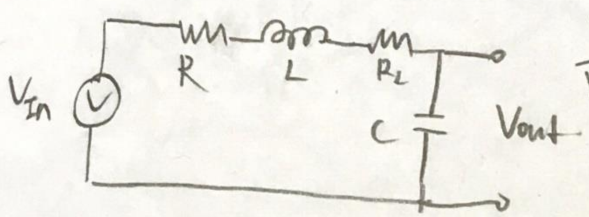
$$\frac{V_{out}}{V_{in}} = \frac{R}{R + Ls + R_L + \frac{1}{Cs}} = \frac{Rs}{(R+R_L)\cdot s + Ls^2 + \frac{1}{C}}$$

$$= \frac{R/L \cdot s}{s^2 + \left(\frac{R+R_L}{L}\right)\cdot s + \frac{1}{LC}}$$ Defined $\omega_0 = \frac{1}{\sqrt{LC}}$.

$$= \frac{\frac{R}{L}\cdot s}{s^2 + \frac{\omega_0}{\left(\frac{L}{R+R_L}\right)\omega_0}\cdot s + \omega_0^2}$$

$$\frac{1}{Q} = \frac{1}{\left(\frac{L}{R+R_L}\right)\cdot \omega_0} \qquad Q = \frac{L}{R+R_L}\cdot \omega_0$$

In resistor the $Q = \frac{L}{R+R_L}\cdot\frac{1}{\sqrt{LC}} = \frac{\sqrt{L}}{(R+R_L)\sqrt{C}}$.

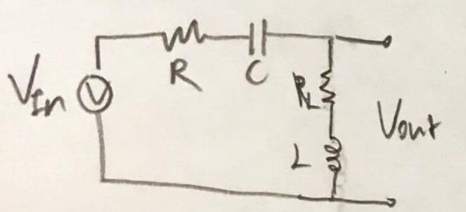


$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{Cs}}{\frac{1}{Cs} + R + R_L + Ls} = \frac{1}{1 + (R+R_L)Cs + LCs^2}$$

$$= \frac{1/LC}{s^2 + \left(\frac{R+R_L}{L}\right)s + \frac{1}{LC}}$$ define $\omega_0 = \frac{1}{\sqrt{LC}}$.

$$= \frac{\omega_0^2}{s^2 + \frac{1}{\frac{L}{(R+R_L)}\omega_0}\cdot\omega_0 s + \omega_0^2}$$

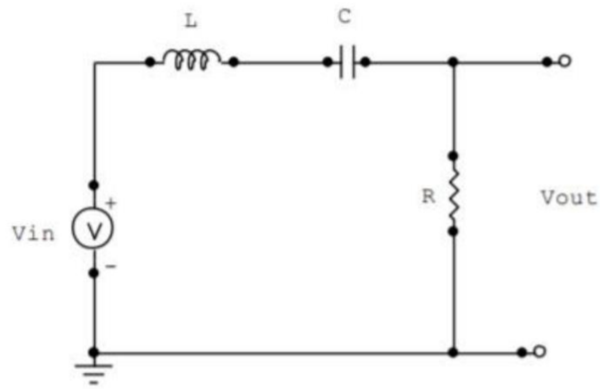
In capacitor, the $Q = \frac{L}{R+R_L}\cdot\omega_0 = \frac{\sqrt{L}}{(R+R_L)\sqrt{C}}$.



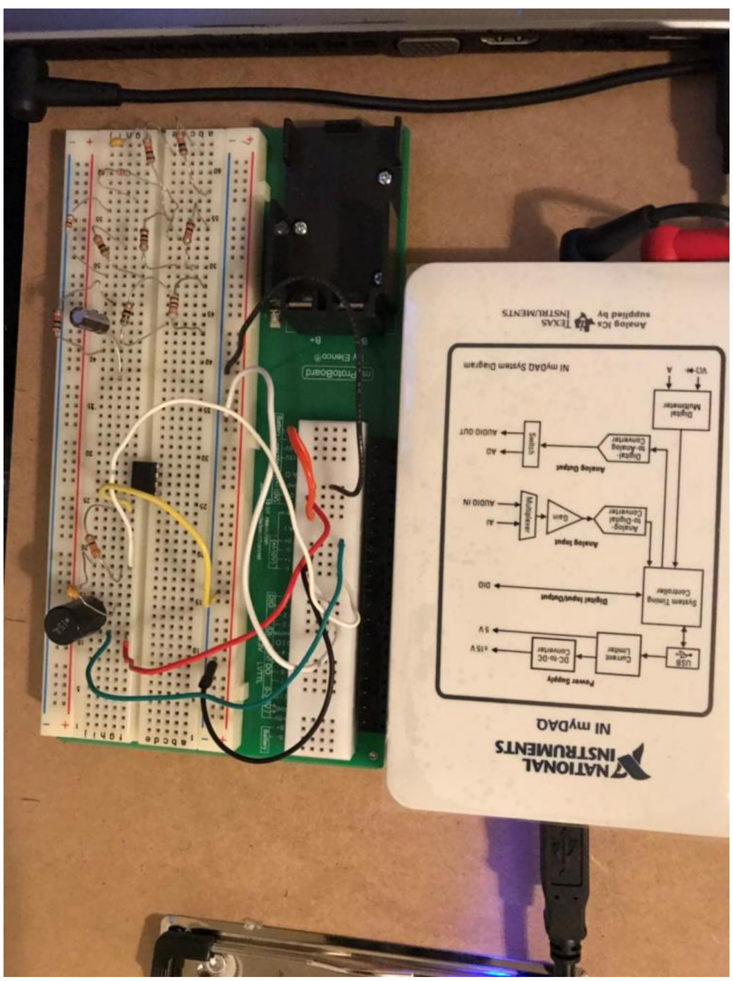
$$\frac{V_{out}}{V_{in}} = \frac{R_L + Ls}{R_L + Ls + R + \frac{1}{Cs}}$$ similarly,

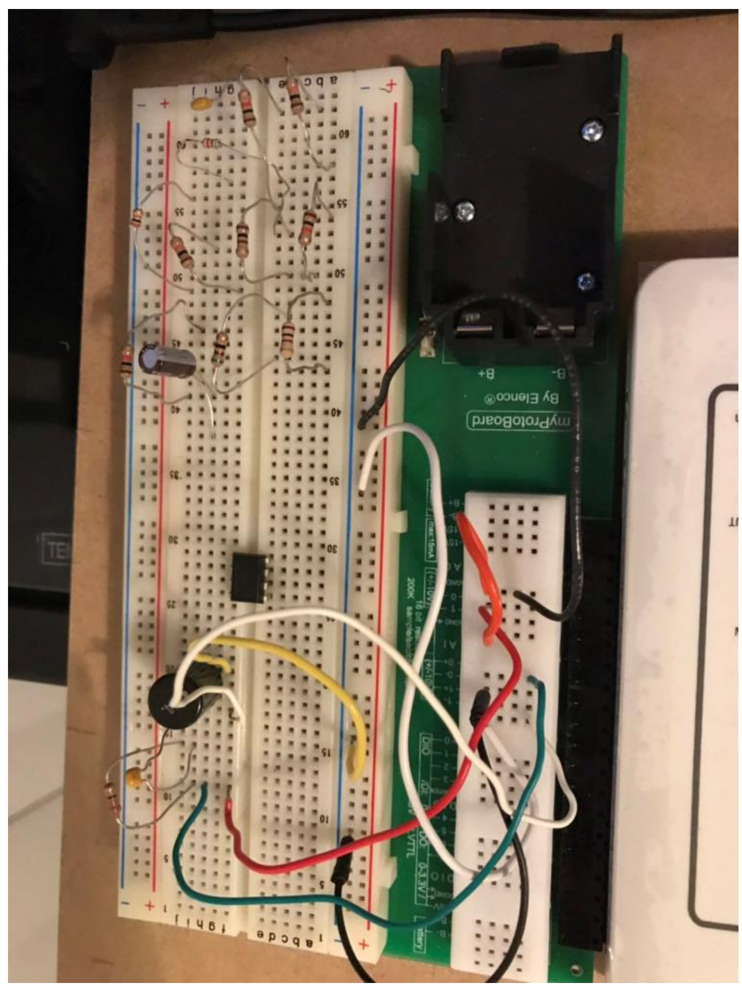
the Q for the inductor is $\frac{1}{R+R_L}\sqrt{\frac{L}{C}}$.

Procedure:



1. Implement the circuit as above.
2. Using Bode analysis to get the Bode plot for Resistor.
3. Repeat 1 to 3 for inductor and capacitor.
4. Find the resonant frequency when the magnitude bode plot for voltage is maximum. Compare it with experimental value and find the percent error.
5. From the magnitude bode plot, decrease the maximum voltage by 3-dB and find the frequency difference which is the bandwidth.
6. Using the resonant frequency w_R for L and C to divide by their bandwidth to the experimental value of Q_L, Q_C.
7. Calculate the thermotical value Q_L and Q_C by using the equation 1-6, 1-7. Compare it with experimental value and find the percent error.

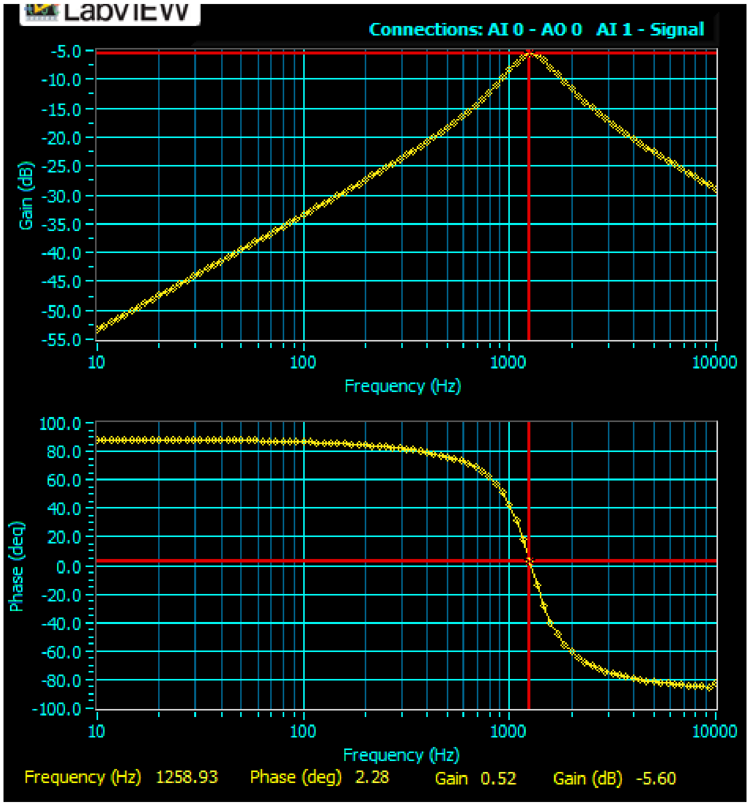


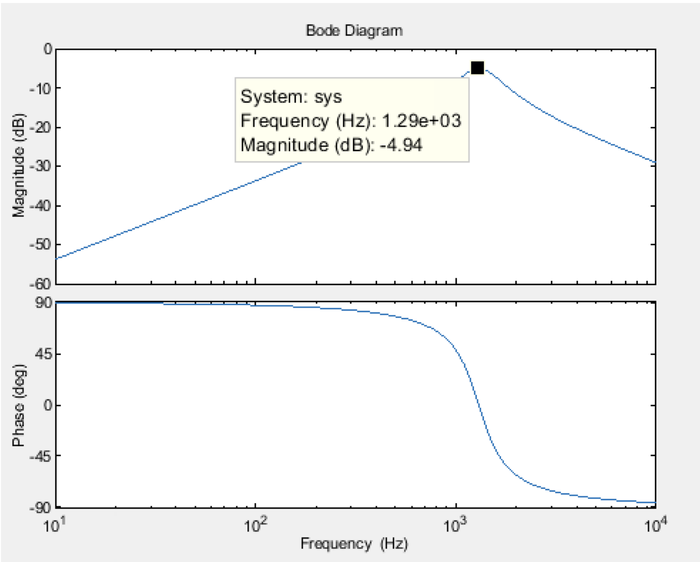


Data:

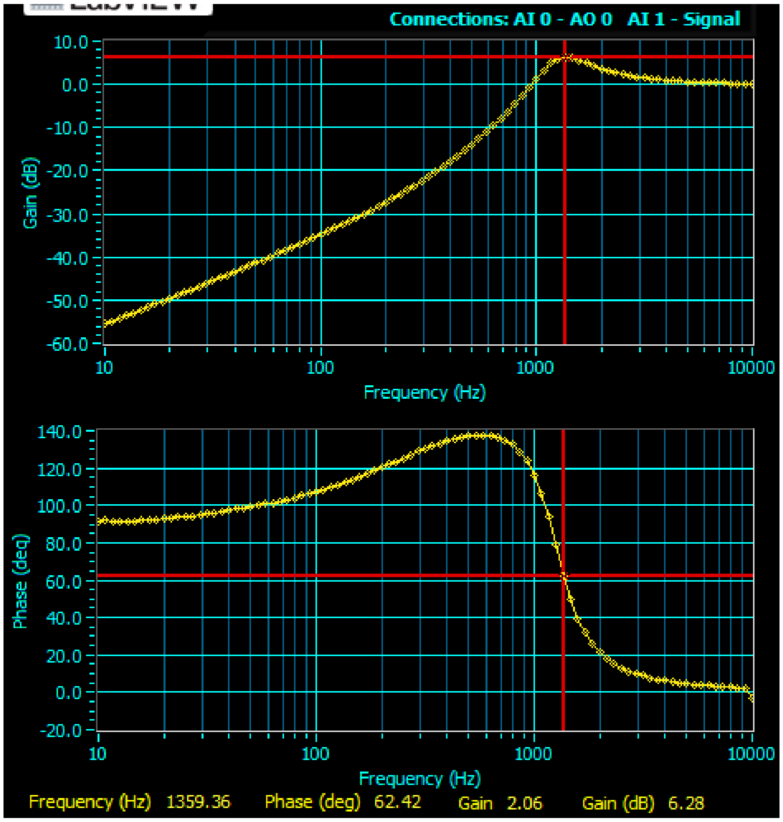
	Inductor	R_L	Capacitor	Resistor
Measured value	150mH	249Ω	100nF	325Ω

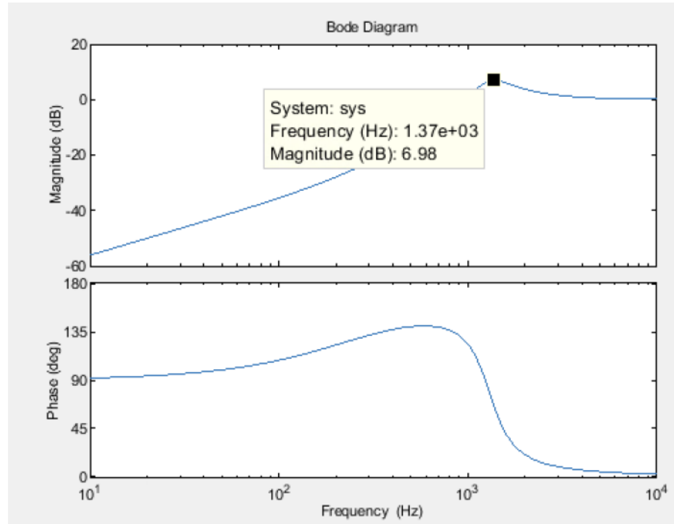
1. **Voltage across the resistor**



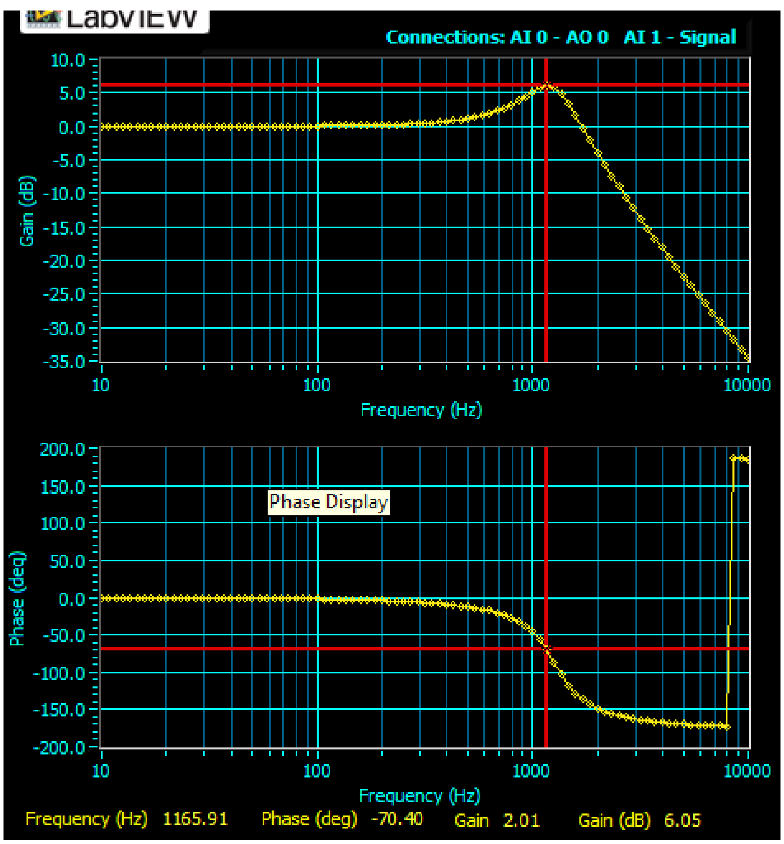


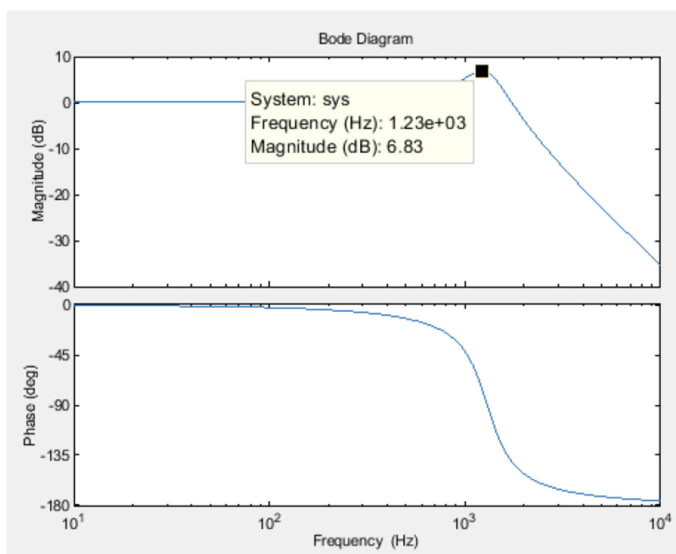
2. Voltage across the inductor

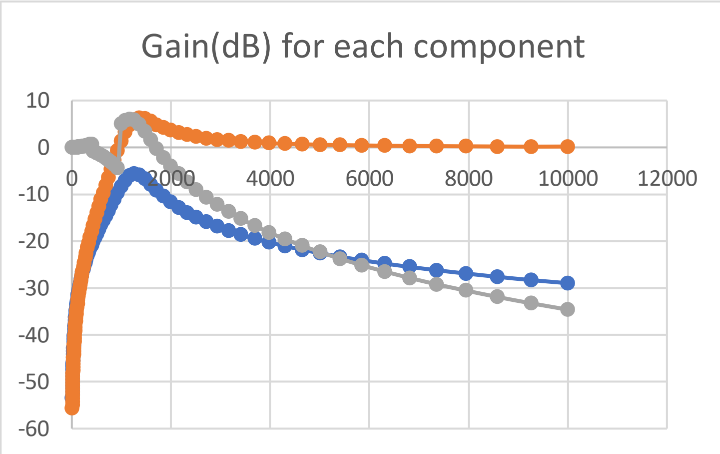


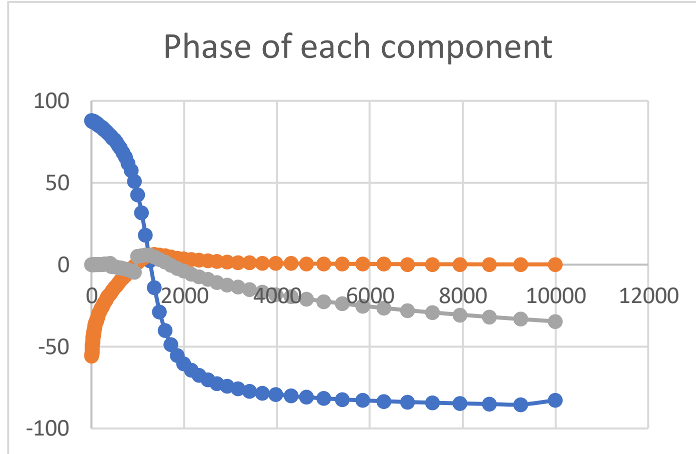


3. Voltage across the capacitor









Data Analysis:

Output	**Experimental Resonant frequency (Hz)**	**Theoretical Resonant frequency (Hz)**	**Percent error%**	**Experimental Q factor**	**Theoretical Q factor**	**Percent error%**
Resistor (325Ω)	1258.93	1299.50	3.12%	1.77	2.13	16.9%
Inductor(150mH)	1359.36	1322.999	2.75%	1.27	2.13	40.38%
Capacitor (100hF)	1165.91	1276.42	8.66%	1.73	2.13	18.78%

The bandwidth from V_R, those data is from the myDAQ bode plot log. Repeat the same thing for V_L, V_C to get their bandwidth.

Freq (Hz)	Gain (dB)	Phase (deg)
926.119	-9.734	50.886
1000.000	-8.348	42.700
1079.775	-7.074	31.845
1165.914	-6.076	18.293
1258.925	-5.605	2.281
1359.356	-5.833	-13.752
1467.799	-6.696	-28.649
1584.893	-7.887	-40.060
1711.328	-9.169	-48.607
1847.850	-10.436	-55.250
1995.262	-11.654	-60.362
2154.435	-12.791	-64.309
2326.305	-13.893	-67.554
2511.886	-14.936	-70.192
2712.273	-15.924	-72.371
2928.645	-16.863	-74.195
3162.278	-17.756	-75.728
3414.549	-18.620	-77.061
3686.945	-19.461	-78.228
3981.072	-20.272	-79.240
4298.662	-21.047	-80.116

BW

Theoretical Analysis

$\omega_0 = 2\pi f_R$

$V_{R(max)}$ $f_R = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.15\times 100\times 10^{-9}}} = 1299.50\,Hz$

$Q_R = \frac{1}{R+R_L}\sqrt{\frac{L}{C}} = \frac{1}{325+249}\sqrt{\frac{0.15}{100\times 10^{-9}}} = 2.1337$ $= 1276.42\,Hz$

$V_{C(max)}$ $f_C = \frac{\omega}{2\pi} = \frac{\omega_0 1}{2\pi\sqrt{LC}}\sqrt{1-\frac{R^2C}{2L}} = 1299.50\times\sqrt{1-\frac{325^2\times 100\times 10^{-9}}{2\times 0.15}}$ $= 1276.42\,Hz$

$Q_C = \frac{1}{R+R_L}\sqrt{\frac{L}{C}} = \frac{1}{325+249}\sqrt{\frac{0.15}{100\times 10^{-9}}} = 2.1337$

$V_{L(max)}$ $f_L = \frac{\omega}{2\pi} = \frac{\frac{1}{2\pi\sqrt{LC}}}{\sqrt{1-\frac{R^2C}{2L}}} = \frac{1299.50}{\sqrt{1-\frac{325^2\times 100\times 10^{-9}}{2\times 0.15}}} = 1322.999\,Hz$

$Q_L = \frac{1}{R+R_L}\sqrt{\frac{L}{C}} = 2.1337$

calculation for experimental value of Q

Example For Q_R inductor

$\omega_R = 2\pi f_R = 2\pi\times 1258.93 = 7910.09\,rad/s$

$BW = 1711.328\,Hz - 1000\,Hz = 711.328\,Hz$

$Q_R = \frac{\omega_R}{BW} = \frac{7910.09}{711.328\times 2\pi} = 1.77$

BW for Inductor: $|2154.35 - 1079.775| = 1074.575\,Hz$

BW for Capacitor: $|1467.799 - 794.328| = 673.471\,Hz$

Discussion:

The percent error of resonant frequency is less than 10%. They are closed to the theoretical resonant frequency. However, the percent error of quality factor is very large especially the from $V_{Inductor}$ Bode plot which is about 40% error. When we count down -3dB from the peak in the magnitude bode plot, it can produce a huge difference between the theoretical and experimental value for the resonant frequency. In addition, the myDAQ has limitation to measure the value when the frequency is high or small. It's difficult to find accurate value of bandwidth. Therefore, the percent error of Quality factor can be large due to vary effects. Moreover, the bode plots for each component are matched with the theoretical bode plots from the MATLAB. From the resistor bode plots, it shows that it's a bandpass filter circuit. It makes sense because at low frequency, the capacitor acts as open circuit. At the high frequency, the inductor acts as open circuit. In the capacitor bode plot, at the low frequency, the gain (dB) is 0, At high frequency, the gain(dB) decreases significantly, which it filters high frequency. In the inductor bode plot, at high frequency, the gain(dB) is 0. At low frequency, the gain(dB) decreases significantly, which it filters low frequency.

Discussion question:

The equations 1-1, 1-2, and 1-3 for calculating the resonant frequency are confirmed.

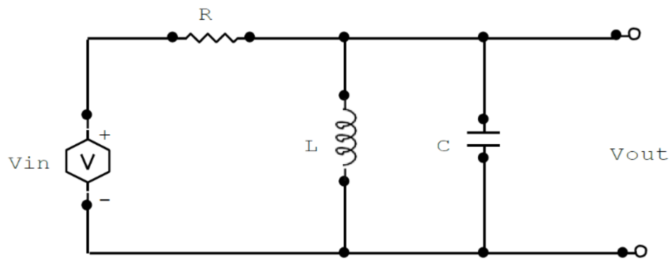
The percent error for resonant frequency and Quality factor are in the data analysis.

Part#2: Parallel Resonance Circuit

Objectives:

The purpose of this part is to learn the characteristics of parallel resonance circuit. Find the value of components with the specific resonance frequency.

Theory:



$$Q = R\sqrt{\frac{C}{L}}$$

Figure 7: Parallel circuit using voltage

For above circuit resonance frequency is $\omega_0 = \sqrt{\frac{1}{LC} - \left(\frac{R_L}{L}\right)^2}$ where R_L is internal resistance of the inductor. We can show that maximum of $Vout$ not appears at $\omega = \omega_0$. The resonant frequency should be calculated by the equation below.

$$\omega = \omega_m = (x - y)^{1/2}$$

$$x = (a + b)^{\frac{1}{2}}$$

$$a = \frac{1}{(LC)^2}\left(1 + \frac{2R_l}{R}\right)$$

$$b = \left(\frac{R_L}{L}\right)^2\left(\frac{2}{LC}\right)$$

$$y = \left(\frac{R_L}{L}\right)^2$$

The resonance frequency for this circuit is

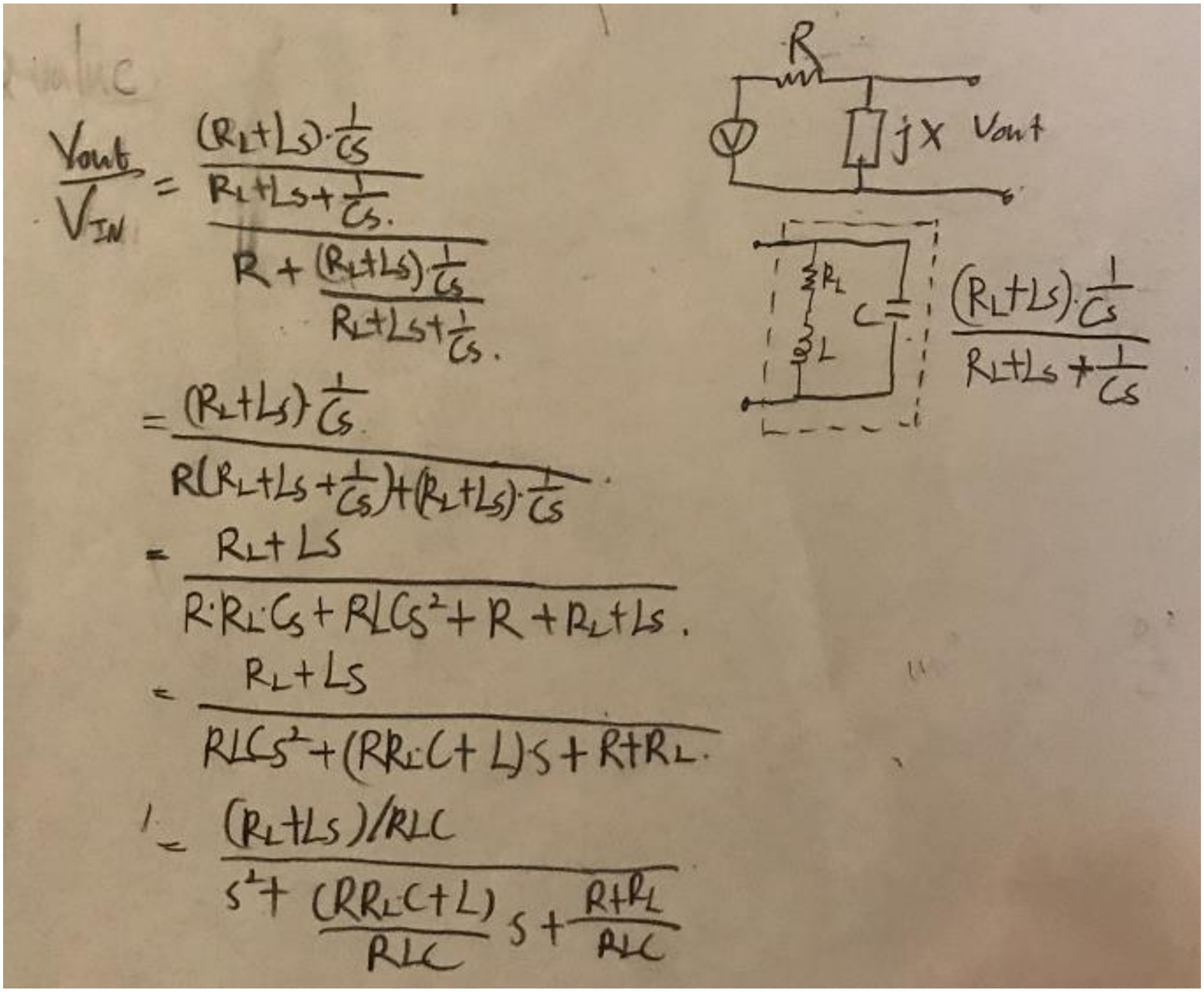
$$W_o = \sqrt{\frac{1}{LC} - \left(\frac{R_L}{L}\right)^2}$$

$$2\pi \times 4100 = \sqrt{\frac{1}{150mH \times C} - \left(\frac{250}{150mH}\right)^2}$$

$$C = 10nF.$$

The capacitance is 10nF.

The transfer function for this circuit is:



$$\frac{V_{out}}{V_{IN}} = \frac{\dfrac{(R_L+Ls)\cdot\frac{1}{Cs}}{R_L+Ls+\frac{1}{Cs}.}}{R+\dfrac{(R_L+Ls)\cdot\frac{1}{Cs}}{R_L+Ls+\frac{1}{Cs}.}}$$

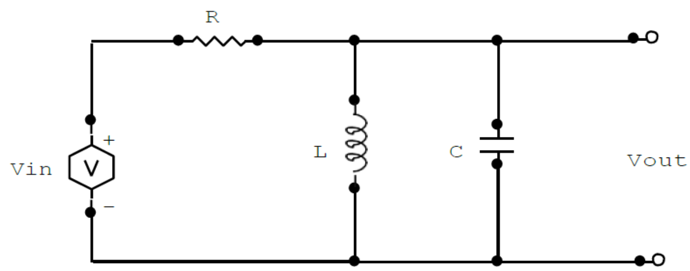
$$= \frac{(R_L+Ls)\cdot\frac{1}{Cs}}{R(R_L+Ls+\frac{1}{Cs})+(R_L+Ls)\cdot\frac{1}{Cs}}.$$

$$= \frac{R_L+Ls}{R\cdot R_L\cdot Cs+RLCs^2+R+R_L+Ls}.$$

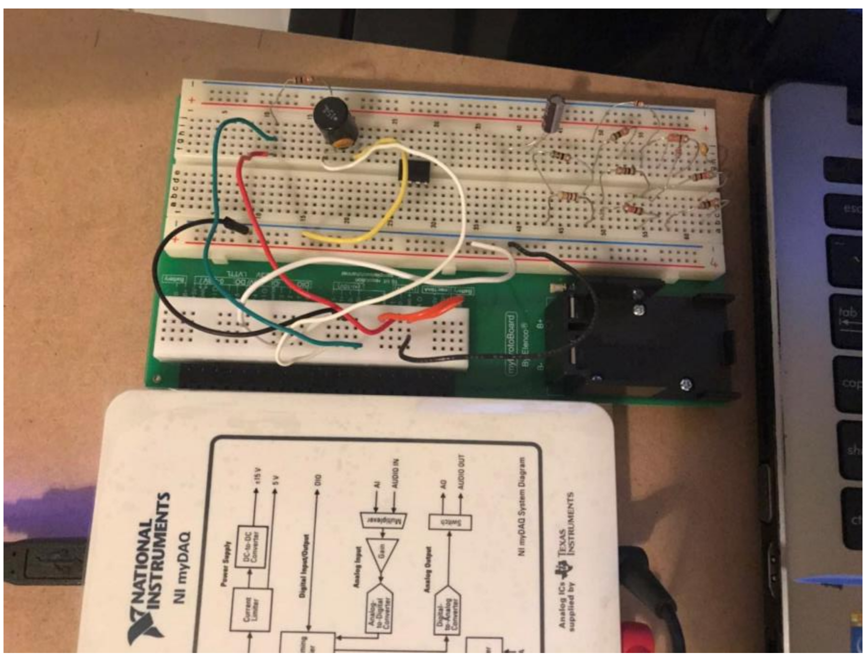
$$= \frac{R_L+Ls}{RLCs^2+(RR_LC+L)s+R+R_L}.$$

$$= \frac{(R_L+Ls)/RLC}{s^2+\frac{(RR_LC+L)}{RLC}s+\frac{R+R_L}{RLC}}$$

Procedure:



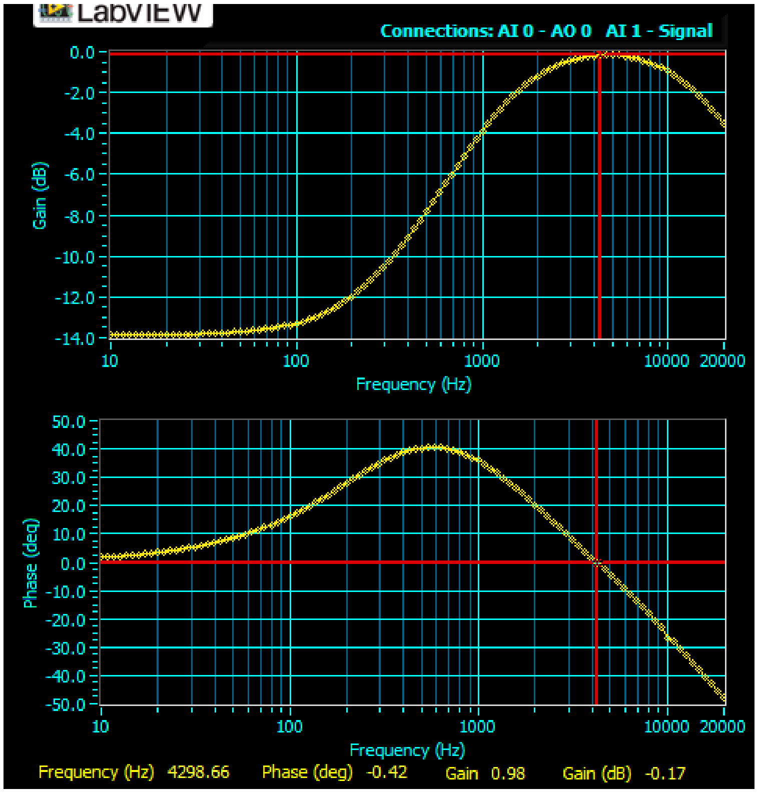
1. Calculate the capacitor to make resonance frequency equal to 4.1 kHz.
2. Implement the circuit as above. L=150mH, R=1kΩ, C=10nF.
3. Using Bode analyzer to generator the Bode plot.
4. Find the resonant frequency when the magnitude bode plot for voltage is maximum. Compare it with experimental value and find the percent error.
5. From the magnitude bode plot, decrease the maximum voltage by 3-dB and find the frequency difference which is the bandwidth.
6. Find the Quality factor and compare it with theoretical value.



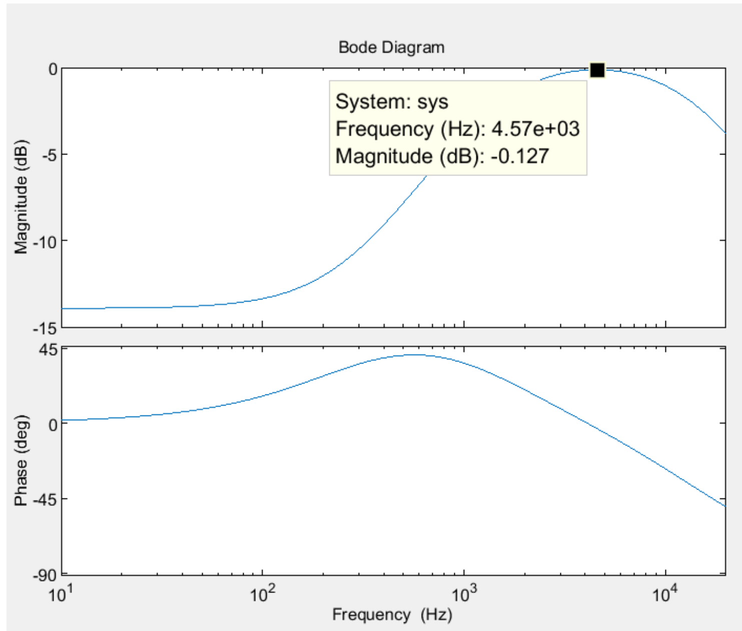
Data:

	Inductor	**R_L**	**Capacitor**	**Resistor**
Measured value	150mH	249Ω	10nF	988Ω

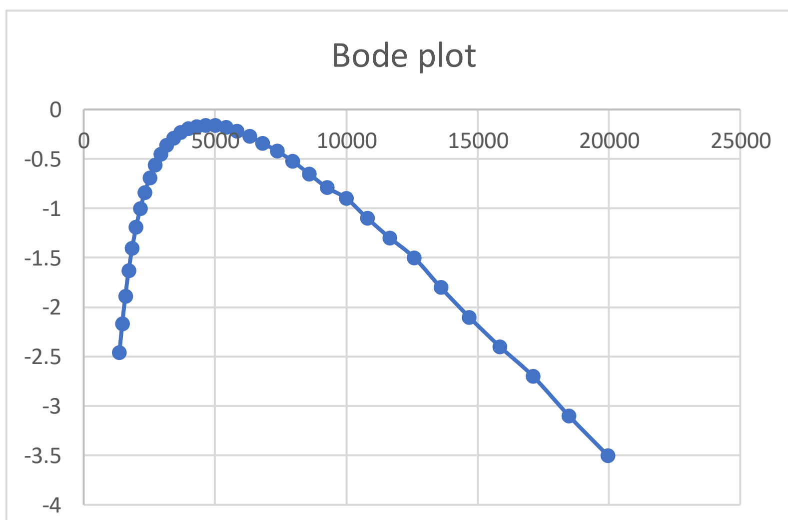
Bode plot for Vout



MATLAB:



Freq (Hz)	Gain (dB)	Phase (deg)
1711.328	-1.638	24.355
1847.850	-1.403	22.382
1995.262	-1.191	20.382
2154.435	-1.008	18.391
2326.305	-0.840	16.332
2511.886	-0.692	14.257
2712.273	-0.566	12.172
2928.645	-0.458	10.086
3162.278	-0.368	8.007
3414.549	-0.295	5.911
3686.945	-0.238	3.784
3981.072	-0.196	1.650
4298.662	-0.172	-0.420
4641.589	-0.162	-2.579
5011.872	-0.166	-4.713
5411.695	-0.186	-6.939
5843.414	-0.222	-9.149
6309.573	-0.274	-11.383
6812.921	-0.342	-13.644
7356.423	-0.428	-15.973
7943.282	-0.526	-18.229
8576.959	-0.653	-20.651
9261.187	-0.798	-23.084
10000.000	-0.928	-26.888
10797.752	-1.147	-27.944
11659.144	-1.358	-30.441
12589.254	-1.594	-32.955
13593.564	-1.860	-35.548
14677.993	-2.144	-38.028
15848.932	-2.454	-40.503
17113.283	-2.796	-43.013
18478.498	-3.160	-45.446
19952.623	-3.549	-47.841



Data Analysis:

	Experimental value	Theoretical value	Percent error%
Frequency at Vout maximum	4641.589Hz	4549.28Hz	2.029%
Quality factor	0.2483	0.2555	2.82%

$w = w_m = (x-y)^{1/2}$

$x = (a+b)^{1/2}$

$a = \frac{1}{(LC)^2}\left(1 + \frac{2R_L}{R}\right)$

$b = \left(\frac{R_L}{L}\right)^2\left(\frac{2}{LC}\right)$

$y = \left(\frac{R_L}{L}\right)^2$

$a = \frac{1}{(0.15\times10\times10^{-9})^2}\left(1+\frac{2\times249}{988}\right) = 6.68466\times10^{17}$

$b = \left(\frac{249}{0.15}\right)^2\left(\frac{2}{0.15\times10\times10^{-9}}\right) = 3.6741\times10^{15}$

$x = (a+b)^{\frac{1}{2}} = (6.68466\times10^{17} + 3.6741\times10^{15})^{1/2}$

$= 8.198\times10^{8}$

$y = \left(\frac{249}{0.15}\right)^2 = 2.7556\times10^{6}$

$w_m = (x-y)^{1/2} = (8.198\times10^{8} - 2.7556\times10^{6})^{1/2}$

$= 28583.989$ rad/s

Theoretical value: $\boxed{f_m = 4549.28Hz}$

at maximum Voltage

percent error% for Resonant frequency

$\frac{4641.589 - 4549.28}{4549.28}\times100\% = 2.029\%$

theoretical Q

$Q = R\sqrt{\frac{C}{L}} = 988\times\sqrt{\frac{10\times10^{-9}}{0.15}} = 0.2555$

From the Bode plot, the peak to -3dB.

The Bandwidth: $18478.498 - 1165.914 = 17312.584$ Hz

Experimental $Q = \frac{f_R}{BW} = \frac{4298.66Hz}{17312.584Hz} = 0.2483$

Error%: $\frac{0.2483 - 0.2555}{0.2555}\times100\% = 2.82\%$

Discussion:

The percent error for this part is much small which is lower than 5%. The experimental bode plot is similar to the theoretical bode plot. The errors are still large even though they are smaller than part 1. The actual value of capacitor and inductor might be different from the labeled value. Also, due to the limitation of myDAQ for measuring the data, it can produce uncertainty for measurement. From the bode plot, we can see that the parallel RLC circuit is a bandpass filter. It makes sense because capacitor and inductor filter out high and low frequency signals.

Discussion question:

The equation for finding the Quality factor is confirmed.

The percent error for resonant frequency and Quality factor are in the data analysis.

Conclusion:

Throughout this experiment, the measured bode plots are matched with theoretical bode plots which are generated by the MATLAB.

We can see that a bandpass filter circuit would filter out high and low frequency. For example, when we measure the output voltage for a resistor in RLC series circuit. When the input voltage has low frequency, the capacitor acts as an open circuit. No current would cross the circuit. On the other hand, when the input voltage has high frequency, the inductor acts as open circuit, and no current crosses the circuit. At the certain frequency, it would have maximum current goes through the circuit. Therefore, we can see there is a peak at the bode plot. If we measure the output voltage for another component such as capacitor. At the high frequency, the capacitor acts as a wire, and it would not drop any voltage. Therefore, it filters out high frequency voltage.

When we calculate the Quality factor for the RLC network, we need consider the internal resistance of inductor. Otherwise, it would produce a largely different result. We can find the resonant frequency at the maximum voltage in the bode plot. However, in the part2, the resonant frequency is not the frequency at the maximum voltage. By decreasing 3dB from the peak point, we can also find the different frequency which is the bandwidth. And using the formula to calculate the quality factor.

Most of the equations are confirmed based on the experimental measurement. Most data result in a less than 10% error except the quality factor in the part 1. It's difficult to pick a point to find the bandwidth. In addition, the quality factor itself is very small. Those many reasons produce the large percent error.