EC ENGR 111L

Experiment #3

Transfer Function Synthesis

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Lab Section: 1E

Date:5/23/2019

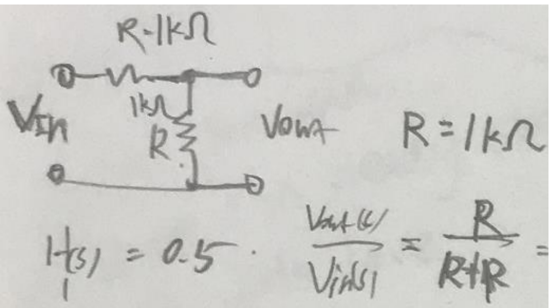
Part#1: Zero-order transfer function synthesis

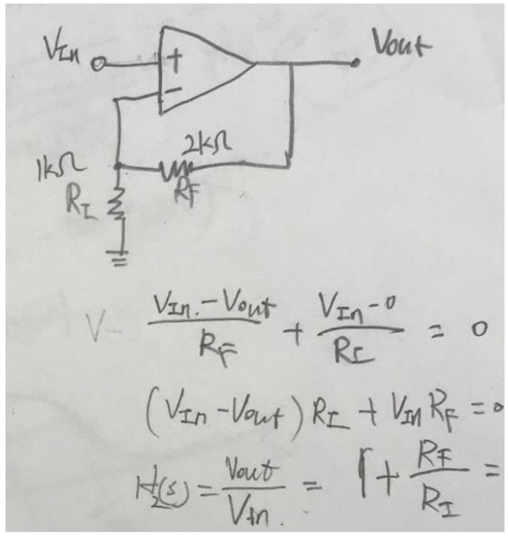
Objectives:

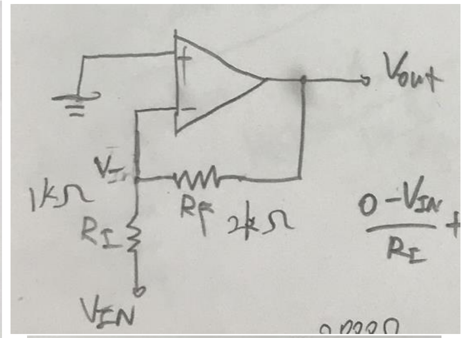
The purpose for part one is to design a circuit with corresponding constant transfer functions.

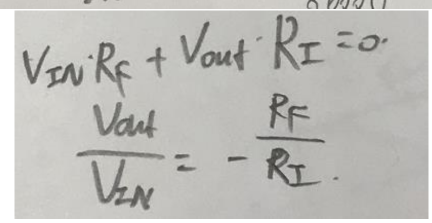
Theory:

In order to design a circuit with the constant transfer function, we can implement resistors and Op-Amp to get the corresponding transfer functions. Since there is no frequency in the zero-order transfer function, the transfer functions are below.









Procedure:

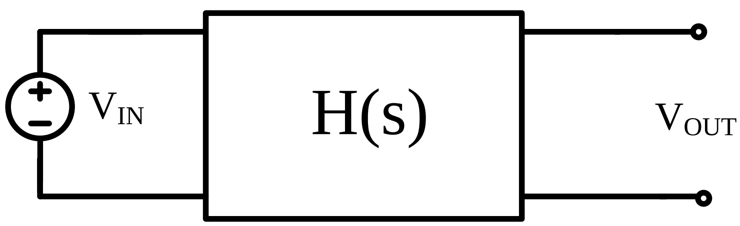
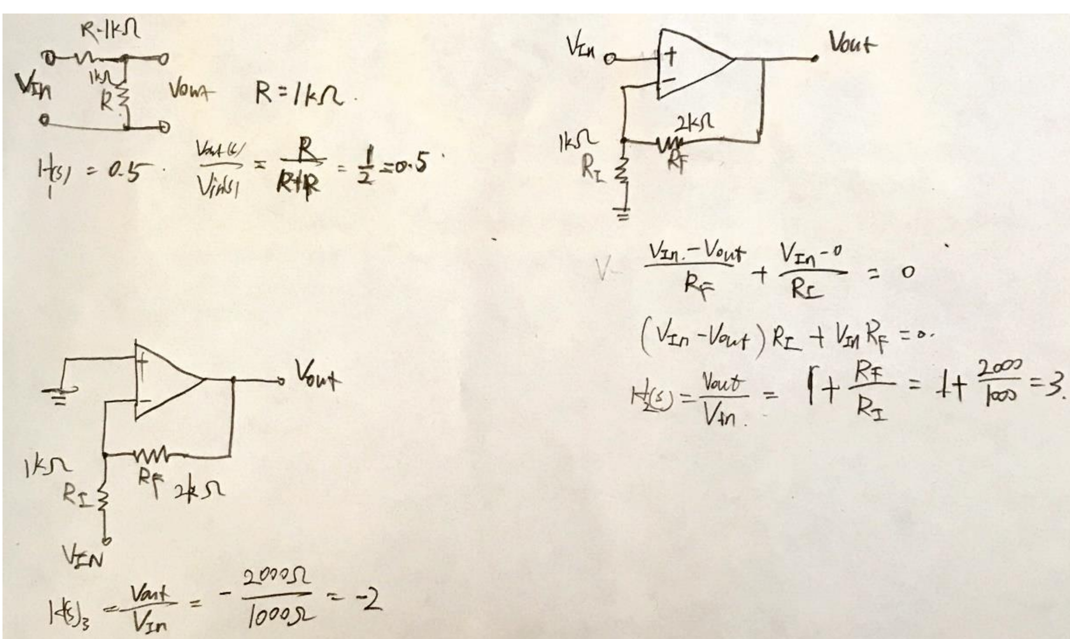
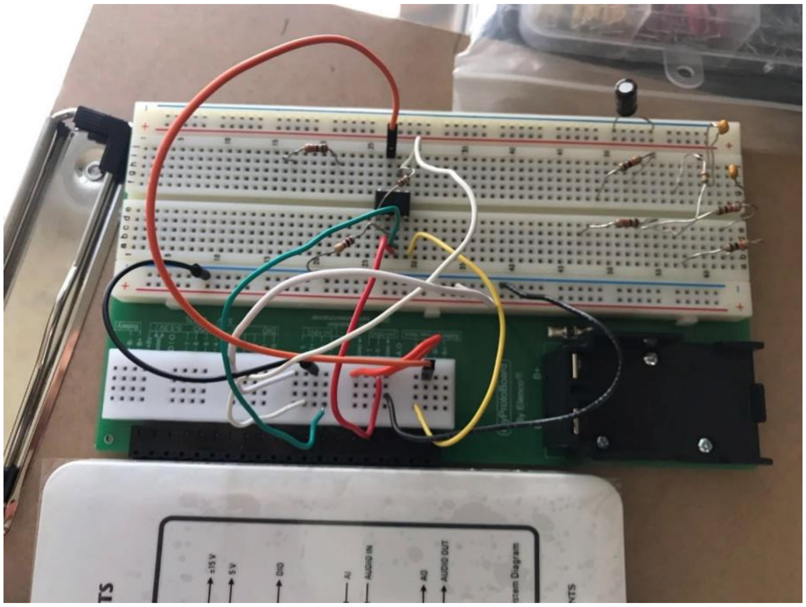


Fig. 1. Test bench with an arbitrary transfer function.

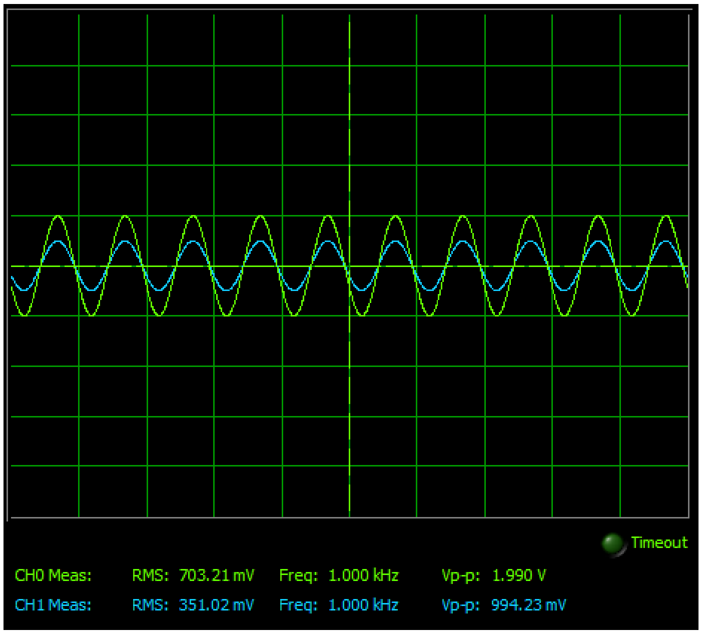


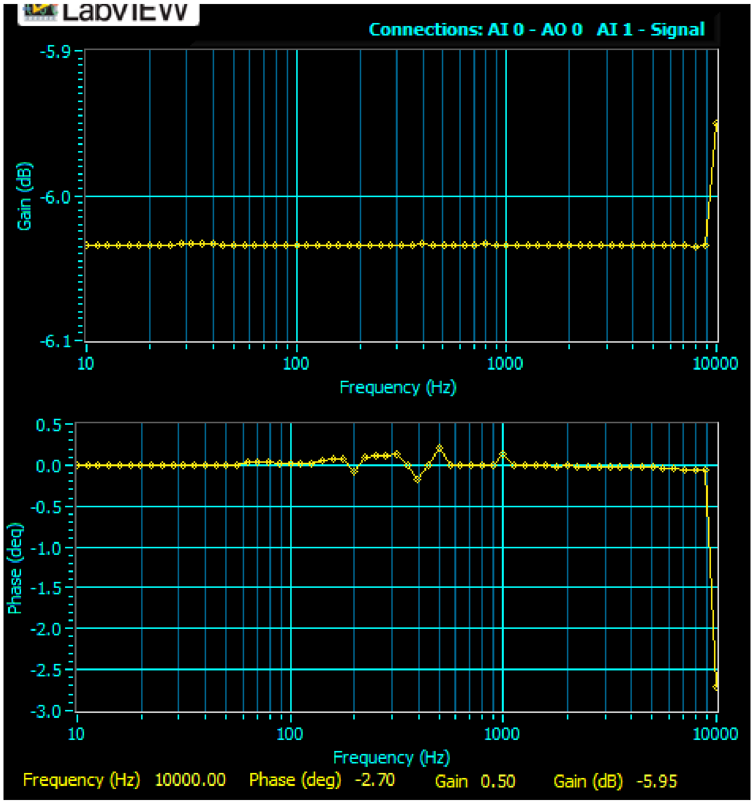
1. Implement the circuits with corresponding transfer functions as below.
2. Using function generator to generator an input voltage with Vp is 1V, 1kHz.
3. Using the oscilloscope to measure the output voltage to verify the transfer functions.
4. Using myDAQ to draw the bode plots and analyze observations.



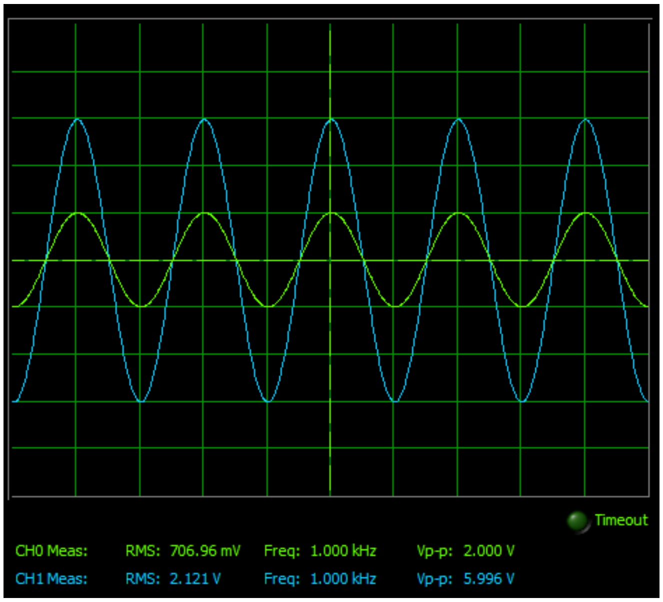
Data:

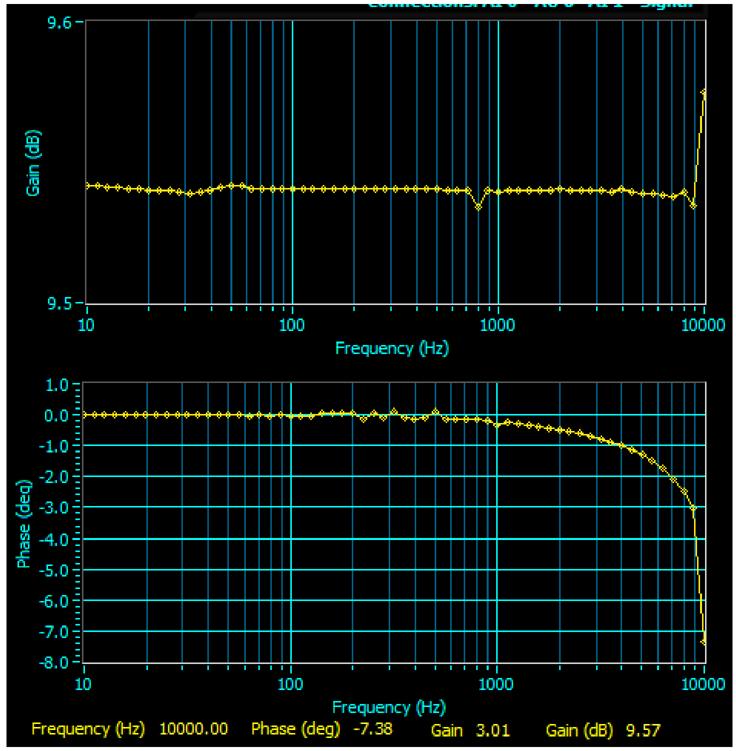
(a) $H_1(s) = 0.5$



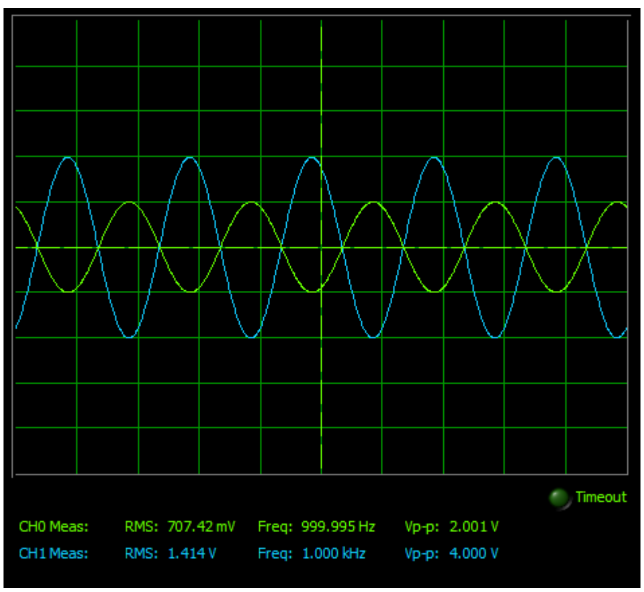


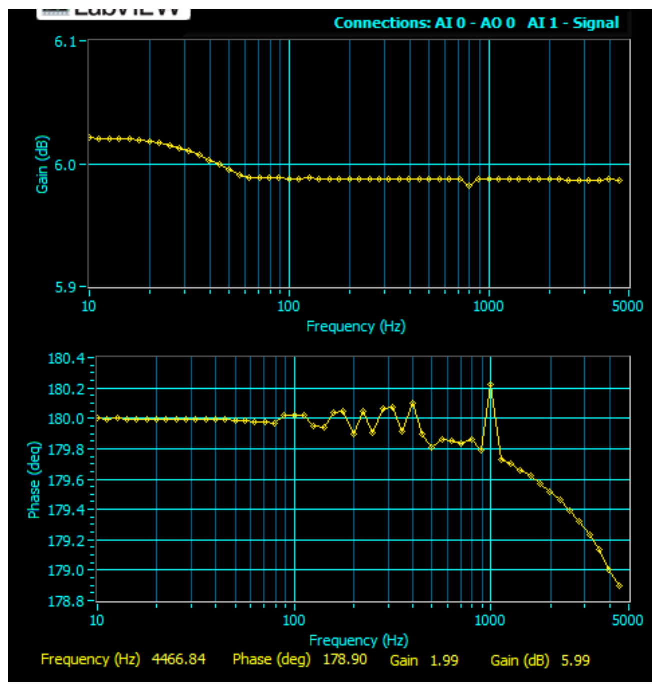
(b) $H_2(s) = 3$


Timeout
CH0 Meas: RMS: 706.96 mV Freq: 1.000 kHz Vp-p: 2.000 V
CH1 Meas: RMS: 2.121 V Freq: 1.000 kHz Vp-p: 5.996 V


Gain (dB)
9.6
9.5
10
100
1000
10000
Frequency (Hz)
Phase (deg)
1.0
0.0
-1.0
-2.0
-3.0
-4.0
-5.0
-6.0
-7.0
-8.0
Frequency (Hz) 10000.00 Phase (deg) -7.38 Gain 3.01 Gain (dB) 9.57

(c) $H_3(s) = -2$


Timeout
CH0 Meas: RMS: 707.42 mV Freq: 999.995 Hz Vp-p: 2.001 V
CH1 Meas: RMS: 1.414 V Freq: 1.000 kHz Vp-p: 4.000 V


Connections: AI 0 - AO 0 AI 1 - Signal
Gain (dB)
6.1
6.0
5.9
10
100
1000
5000
Frequency (Hz)
Phase (deg)
180.4
180.2
180.0
179.8
179.6
179.4
179.2
179.0
178.8
Frequency (Hz) 4466.84 Phase (deg) 178.90 Gain 1.99 Gain (dB) 5.99

Data Analysis:

	Vin	Vout	Magnitude (Vout/Vin)	Phase (degree)
$H_1(s) = 0.5$	1.990V	0.994V	0.4996	0
$H_2(s) = 3$	2.000V	5.996V	2.998	0
$H_3(s) = -2$	2.001V	4.000V	1.999	180

	Error%
$H_1(s) = 0.5$	0.08%
$H_2(s) = 3$	0.067%
$H_3(s) = -2$	0.05%

The theoretical values calculations are in the **theory**.

One example of the experimental value calculation:

$$\frac{V_{out}}{V_{in}} = \frac{0.994V}{1.990V} = 0.4996.$$

$$Error\% = \frac{0.4996 - 0.5}{0.5} \times 100\% = 0.08\%.$$

Discussion:

In this part, the experimental values of designed network are accurate, the percent errors are very small. For the H_1 and H_2, their phase difference is 0. For the H_3 the phase difference is 180 because of the inverting gain. Since there is no frequency dependence in this circuit, the constant number of transfer function H(s) is equal to the gain.

a. Because it's a zero-order network, there are no poles and zeros in the transfer function.
b. The gain factor K is just equal to the transfer function.
   Such as $K_1 = H_1(s) = 0.5$, $K_2 = H_2(s) = 3$, $K_3 = H_3(s) = -2$

Part#2: First-order transfer function synthesis

Objectives:

The purpose for part two is to design a circuit with first-order transfer function. In this part, the transfer function would have one real pole, no real zero and the gain factor 1, -5.

Theory:

Part 2.

(a) One real pole at -10^3 Hz, no real zero, gain factor $=1$ $\lim_{s\to 0} H(s) = 1$

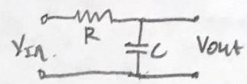
$H(s) = \frac{A}{s+\omega}$ $H(s) = \frac{A/10^3}{1+s/10^3}$. $H(s) = \frac{1}{\frac{1}{10^3}s+1}$.

$H(s) = \frac{A}{s+10^3}$ In order to get gain factor $=1$ $A/10^3 = 1$, $A = 10^3$.

Therefore, $H(s) = \frac{10^3}{s+10^3}$.

$\frac{1}{10^3} = 10^{-3} = RC$ $\boxed{R = 1k\Omega,\ C = 10^{-6}F = 1\mu F}$

To determine the component values to match this transfer function, From professor video, we found that the RC circuit in series matches this requirement.



(b). One real pole at -10^3 Hz, no real zero, gain factor $= -5$. $\lim_{s\to 0} H(s) = -5$

$H(s) = \frac{A}{s+\omega} = \frac{A}{s+10^3}$ $\lim_{s\to 0} H(s) = \frac{A}{10^3} = -5$ $A = -5\times10^3$.

$H(s) = \frac{-5\times10^3}{s+10^3}$

$H(s) = -\frac{R_2}{R_1}\left(\frac{1}{1+sR_2C_2}\right)$

$= -\frac{R_2}{R_1+sR_1R_2C_2}$

$= -\frac{1/R_1C_2}{1/R_2C_2 + s}$.

$\frac{1}{R_1C_2} = 5\times10^3$.

$\frac{1}{R_2C_2} = 10^3$.

$\boxed{C_2 = 1\mu F.\ \ R_1 = 200\Omega \\ R_2 = 1000\Omega}$

In order to satisfy the transfer function $H(s) = \frac{-5\times10^3}{s+10^3}$

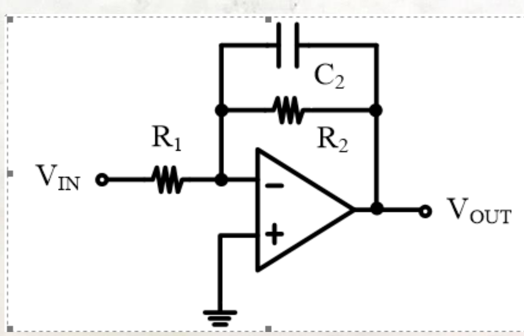
We design this circuit.

$\frac{0-V_{IN}}{R_1} + \frac{0-V_{out}}{R_2} + \frac{0-V_{out}}{\frac{1}{C_2s}} = 0$.

$-V_{IN}\cdot\frac{R_2}{C_2s} - V_{out}\frac{R_1}{C_2\cdot s} - V_{out}\cdot R_1R_2 = 0$

$-\left(\frac{R_1+R_1R_2\cdot C_2\cdot s}{C_2s}\right)V_{out} = V_{IN}\cdot\frac{R_2}{C_2s}$

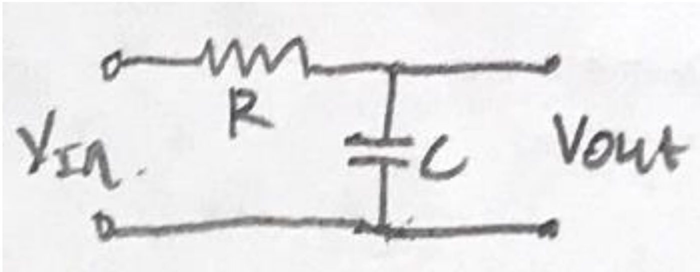
$\frac{V_{out}}{V_{IN}} = -\frac{R_2}{R_1}\left(\frac{1}{1+R_2C_2\cdot s}\right)$



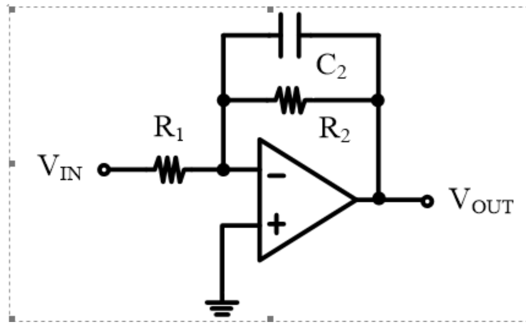
$$H(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + b_{m-2}s^{m-2} + \cdots + b_1 s^1 + b_0}{a_n s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_1 s^1 + a_0}$$

$$H(s) = \frac{N(s)}{D(s)} = K\cdot\frac{(s-z_1)(s-z_2)\cdots(s-z_{m-1})(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_{n-1})(s-p_n)}$$

Procedure:

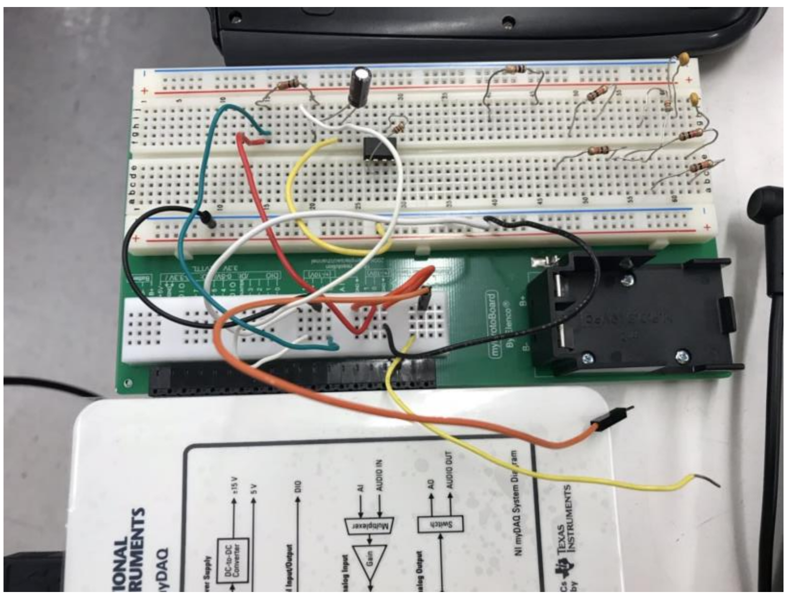


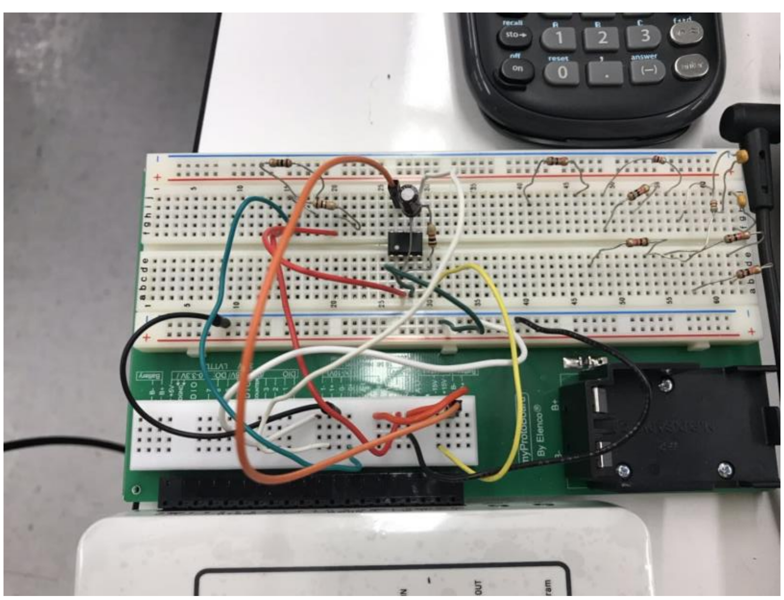
(a)



(b)

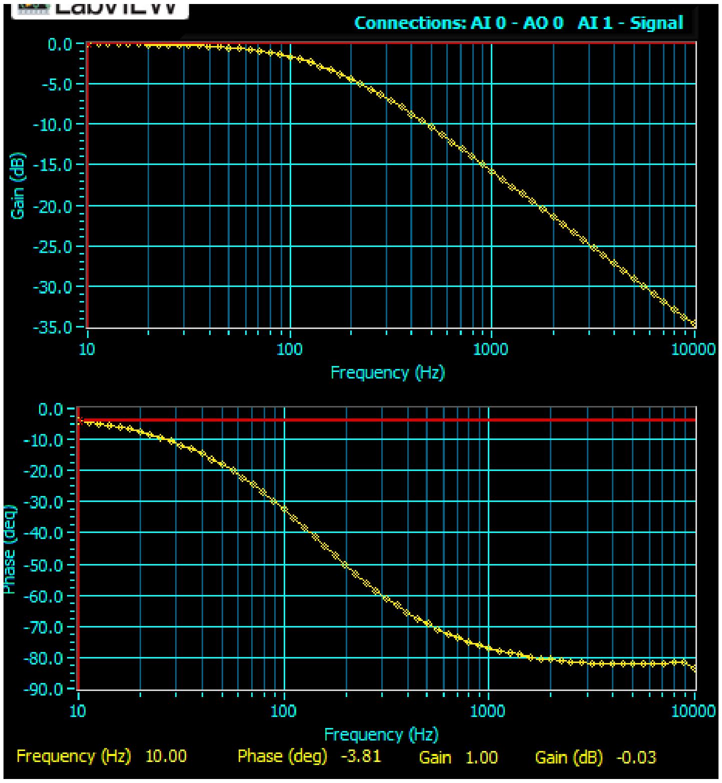
1. Implement the circuit as above (a) and (b). For the (a) circuit, R is equal 1kΩ, C is 1uF. For the (b) circuit, R_1 is 200Ω, R_2 = 1000Ω, C_2 = 1uF
2. Using myDAQ to draw the bode plots and analyze frequency response and confirm with the bode plot.

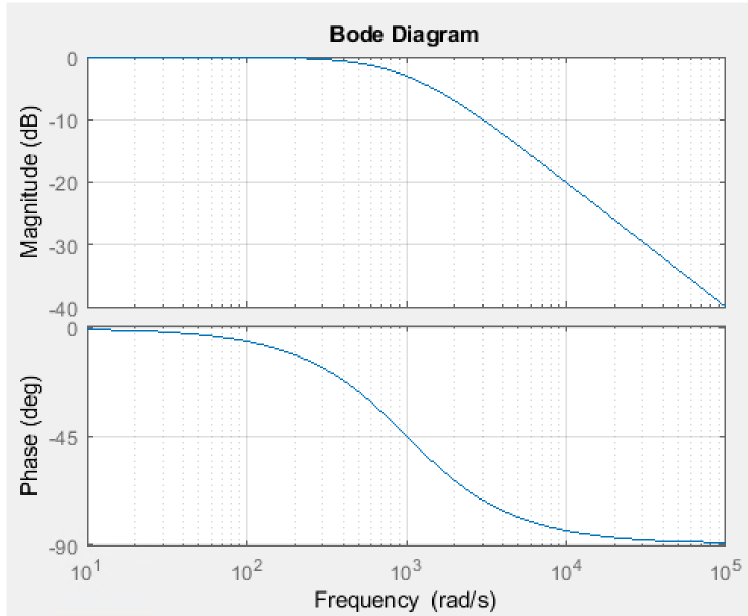




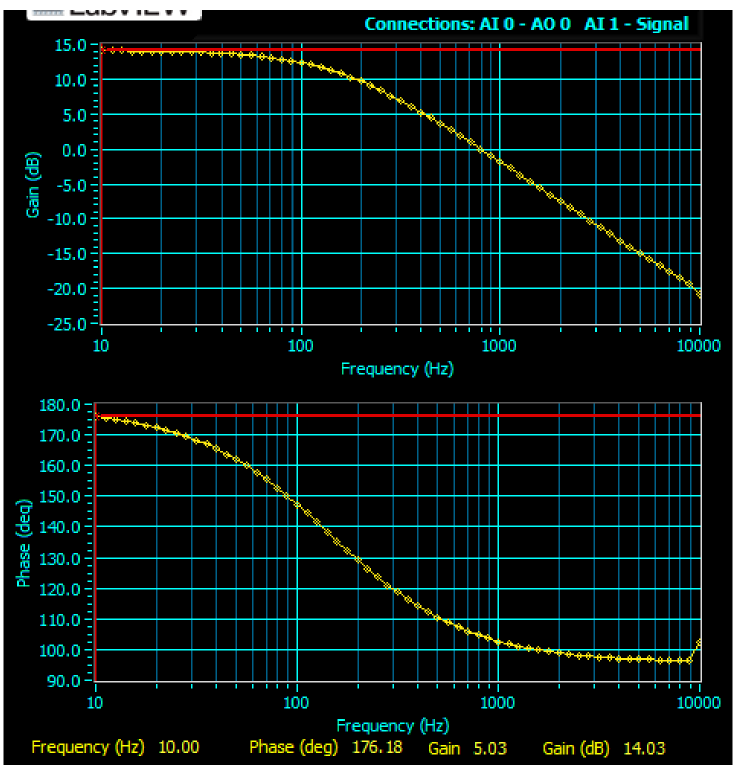
Data:

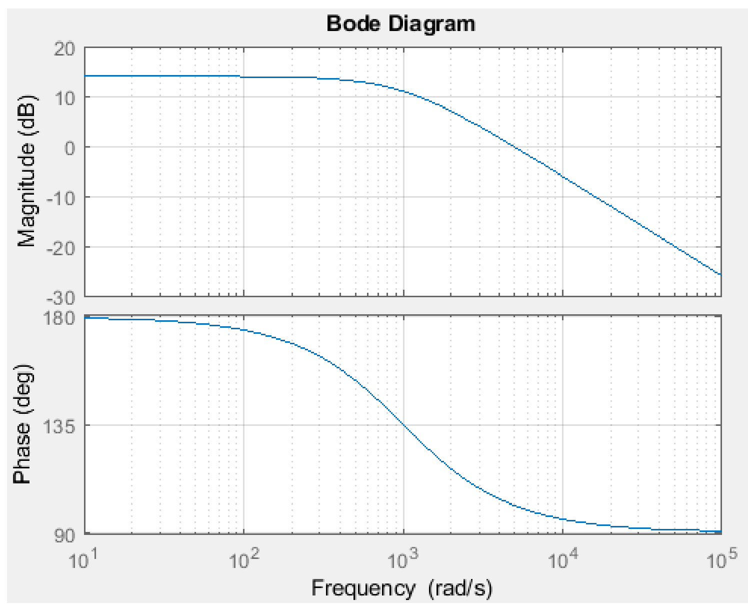
(a) Bode plot





(b) Bode plot





Data analysis:

(a)

	Gain (experimental)	Gain (theoretical)	Percent Error%	Phase (degree) (experimental)	Phase (degree) (theoretical)	Percent Error%
10Hz	1	0.998	0.2%	-3.81	-3.595	5.98%
100Hz	0.83	0.847	2.01%	-32.42	-32.14	0.87%
500Hz	0.30	0.30	0%	-69.11	-72.34	4.47%
1000Hz	0.16	0.157	1.91%	-77.00	-80.96	4.89%

(b)

	Gain (experimental)	Gain (theoretical)	Percent Error%	Phase (degree) (experimental)	Phase (degree) (theoretical)	Percent Error%
10Hz	5.03	4.99	0.8%	176.18	176.4	0.12%
100Hz	4.16	4.23	1.65%	147.42	147.86	0.5%
500Hz	1.51	1.52	0.66%	110.53	107.66	2.67%
1000Hz	0.81	0.79	2.53%	102.55	99.04	3.54%

Calculation:

Part 2.

(a). $|H(2\pi fj)| = \frac{10^3}{\sqrt{(2\pi f)^2 + (10^3)^2}}$ $\quad \angle H(2\pi fj) = -\tan^{-1}(2\pi f \times \frac{1}{10^3})$

$|H(2\pi \times 10 \cdot j)| = \frac{10^3}{\sqrt{(2\pi \times 10)^2 + 10^6}} = 0.998$

$\angle H(2\pi \times 10 \times j) = -\tan^{-1}(2\pi \times 10 \frac{1}{10^3}) = -3.595°$

$\text{Error\%}(\text{Gain}) = \frac{|0.998 - 1|}{0.998} \times 100\% = 0.2\%$

$\text{Error\%}(\text{phase}) = \frac{|-3.595 - (-3.81)|}{3.595} = 5.98\%$

(b). $|H(2\pi fj)| = \frac{\frac{1000}{200}}{\sqrt{(2\pi \times 1000 \times 10^{-6} f)^2 + 1}}$

$|H(2\pi \times 10 j)| = \frac{5}{\sqrt{(2\pi \times 1000 \times 10^{-6} \times 10)^2 + 1}} = 4.99$

$\angle H(2\pi fj) = \tan^{-1}(-R_2 C_2 \cdot 2\pi f)$

$\angle H(2\pi \times 10 j) = \tan^{-1}(-1000 \times 10^{-6} \times 2\pi \times 10)$

$= -3.595° \quad 180° - 3.595° = 176.4°$

$\text{Error\%}(\text{Gain}) = \frac{|4.99 - 5.03|}{4.99} \times 100\% = 0.8\%$

$\text{Error\%}(\text{phase}) = \frac{|176.4 - 176.18|}{176.4} \times 100\% = 0.12\%$

$H(s) = -\frac{R_2}{R_1} \frac{1}{1 + R_2 C_2 j 2\pi f}$

$-\frac{R_2}{R_1} \frac{1 - R_2 C_2 j 2\pi f}{1 + R_2^2 C_2^2 (2\pi f)^2} = \frac{-\frac{R_2}{R_1} + \frac{R_2^2}{R_1} C_2 \cdot 2\pi f j}{1 + R_2^2 C_2^2 (2\pi f)^2}$

$\angle H(s) = \tan^{-1} \frac{\frac{R_2^2}{R_1} \cdot C_2 \cdot 2\pi f}{-\frac{R_2}{R_1}} = \tan^{-1}(-R_2 C_2 2\pi f)$

Discussion:

From the measurement data, when the frequency is small, the gain factors are accurate. As the frequency increases, the gain decreases, and the phase would approach to 90 degree. The percent error % with the high frequency is high because the myDAQ cannot accurately measure the data when the frequency is very high. However, the Bode plots are almost the same as the theoretical bode plots (MATLAB). Both circuits are LPF.

a) The transfer functions are derived in the theory section.
b) Time-constant: a) 1/RC = 1/(1000*10^-6) = 1000   b) 1/(R_2C) = 1/(1000*10^-6) = 1000;

Part#3: Second-order transfer function synthesis

Objectives:

The purpose of this part is to design second-order circuit by only a single value for resistor and capacitor, and to satisfy the transfer function requirements.

Theory:

The second order circuit can be created by two cascading first order circuits. For this part, it only allows to use only a single value for R and C. Therefore, we can cascade two first order RC circuit to build the second order circuit of RC.

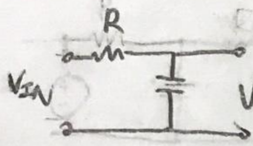
Part 3

(a) Two real poles $s_1 = -\left(\frac{3+\sqrt{5}}{2}\right)\times10^3$, $s_2 = -\left(\frac{3-\sqrt{5}}{2}\right)\times10^3$, no real zeros, $\lim_{s\to0} H(s) = 1$

$$H(s) = \frac{A}{(s+s_1)(s+s_2)} \qquad \lim_{s\to0} H(s) = \frac{A}{s_1\cdot s_2} = 1 \qquad \frac{A}{\left(\frac{3+\sqrt{5}}{2}\right)\times10^3\times\left(\frac{3-\sqrt{5}}{2}\right)\times10^3} = 1.$$

$$A = \left(\frac{9}{4}-\frac{5}{4}\right)\times10^6 = 10^6$$

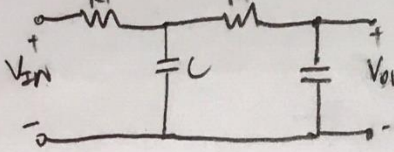
$$H(s) = \frac{10^6}{s^2+(s_1+s_2)\cdot s+s_1\cdot s_2} = \frac{10^6}{s^2+(3\times10^3)s+10^6}$$



the transfer function for the RC first order network is

$$H_1(s) = \frac{\frac{1}{C\cdot s}}{\frac{1}{C s}+R} = \frac{1}{1+RCs}.$$

For the 2nd-order transfer function, it has two different real poles. It doesn't need an unity-gain buffer. Therefore, to satisfy $H(s) = \frac{10^6}{s^2+(3+10^3)s+10^6}$, we can cascade two RC first order network.



the transfer function for this circuit is

$$\frac{V_{out}}{V_{IN}} = \frac{1}{R_1^2C^2s^2+3R_1Cs+1}$$

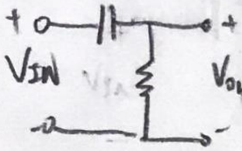
$$H(s) = \frac{1}{\frac{1}{10^6}s^2+(3\times10^{-3})s+1} \qquad R_1^2C^2 = \frac{1}{10^6} \qquad R_1C = 10^{-3}. \qquad \boxed{R_1 = 1k\Omega,\ C = 1\mu F}$$

b). For part b, we have two real poles, and two zeros at DC, $\lim_{s\to\infty} H(s)=1$

it's similar to part a. except there are two zeros at DC,

Base on the data, the transfer function is below

$$H(s)=\frac{s^2}{s^2+(3\times10^3)s+10^6}.$$



the transfer function for this RC network is $H_1(s)=\frac{R}{\frac{1}{Cs}+R}=\frac{RCs}{1+RCs}.$

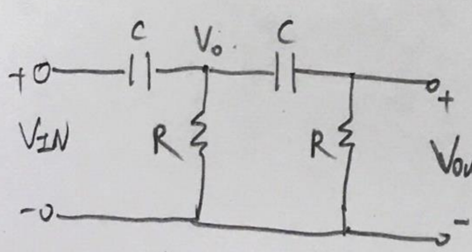
Similar to part a). since we have two zeros at DC, we can cascade two of this RC circuit to satisfy this second order transfer.

$$\frac{V_{out}}{V_{IN}}=\frac{s^2}{s^2+\frac{3}{RC}s+\frac{1}{R^2C^2}}.$$

$RC=10^{-3}$ $R^2C^2=10^6.$

$\boxed{R=1k\Omega,\ C=1\mu F}$.

$\Downarrow$



KCL At V_o.

$$\frac{V_o-V_{IN}}{\frac{1}{Cs}}+\frac{V_o}{R}+\frac{V_o-V_{out}}{1/Cs}=0$$

$$\frac{V_{out}-V_o}{1/Cs}+\frac{V_{out}}{R}=0.$$

$Cs\cdot V_{out}-Cs\cdot V_o+\frac{1}{R}V_{out}=0.$

$V_{out}(Cs+\frac{1}{R})=Cs\cdot V_o.$

$V_o=(1+\frac{1}{RCs})V_{out}.$

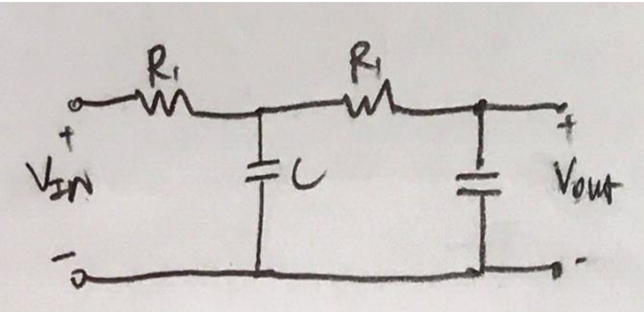
$$Cs(1+\frac{1}{RCs})V_{out}-CsV_{IN}+(\frac{1}{R}+\frac{1}{R^2Cs})V_{out}+Cs(1+\frac{1}{RCs})V_{out}-CsV_{out}=0.$$

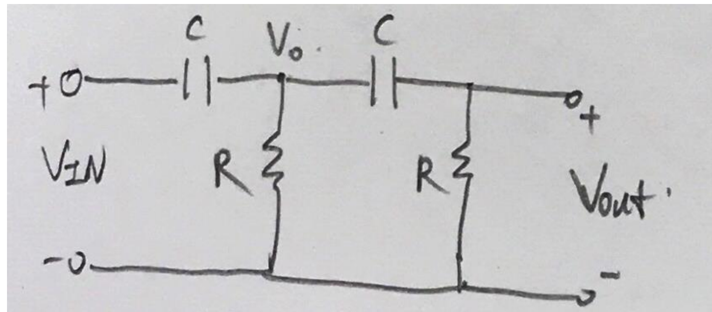
$$[\cancel{Cs}+\frac{1}{R}+\frac{1}{R}+\frac{1}{R^2Cs}+Cs+\frac{1}{R}-\cancel{Cs}]V_{out}=Cs\cdot V_{IN}$$

$$[\frac{3}{R}+\frac{1}{R^2Cs}+Cs]V_{out}=CsV_{IN}$$

$$\frac{V_{out}}{V_{IN}}=\frac{Cs}{\frac{3}{R}+\frac{1}{R^2Cs}+Cs}=\frac{s^2}{s^2+\frac{3}{RC}s+\frac{1}{R^2C^2}}$$

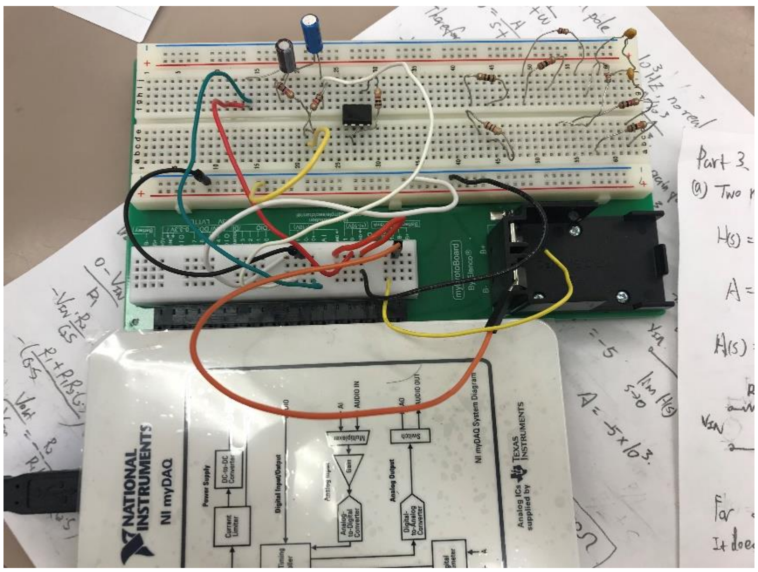
Procedure:

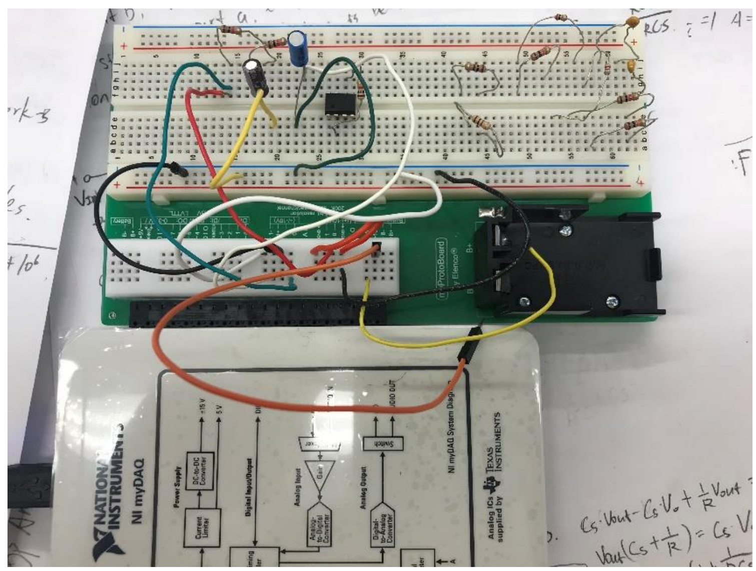




(a) (b)

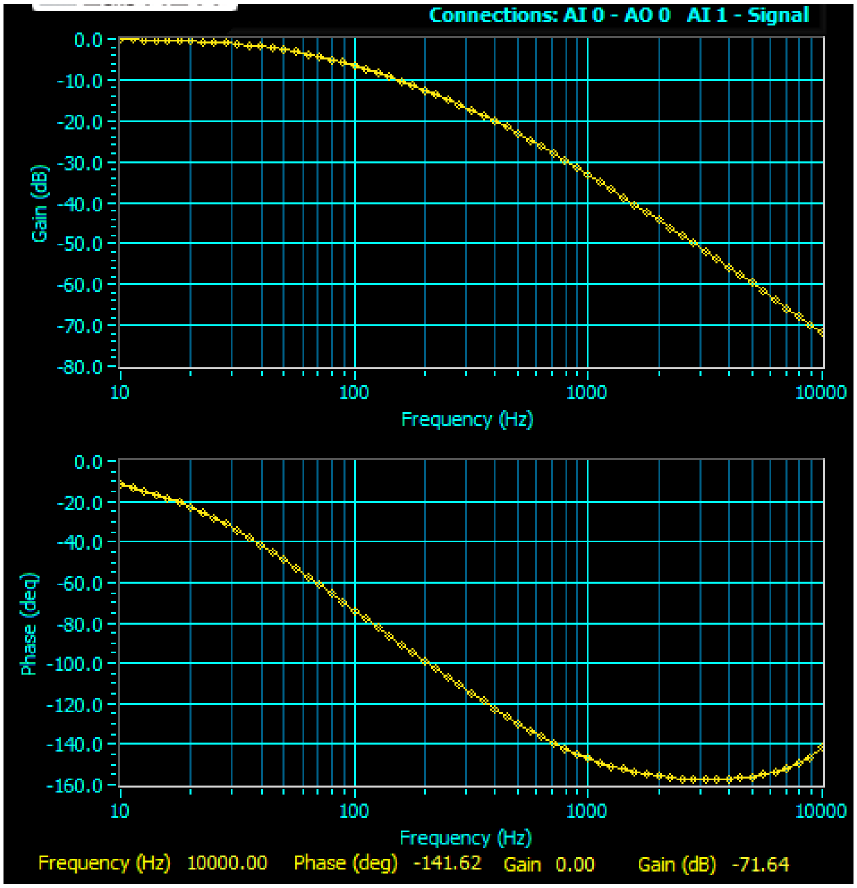
1. Implement the circuit as above. For (a), R = 1kΩ, C=1uF. For (b), R = 1kΩ, C=1uF.
2. Using the Bode Analyzer from myDAQ to find the Bode plots of both circuits.
3. Using MATLAB to confirm with the measured Bode plot.

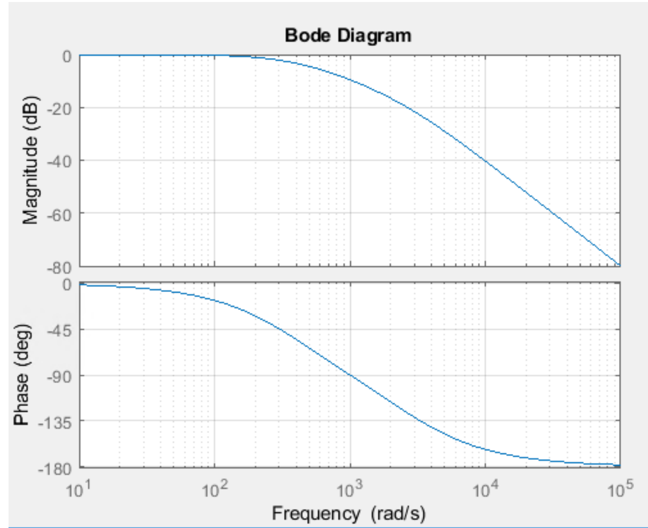




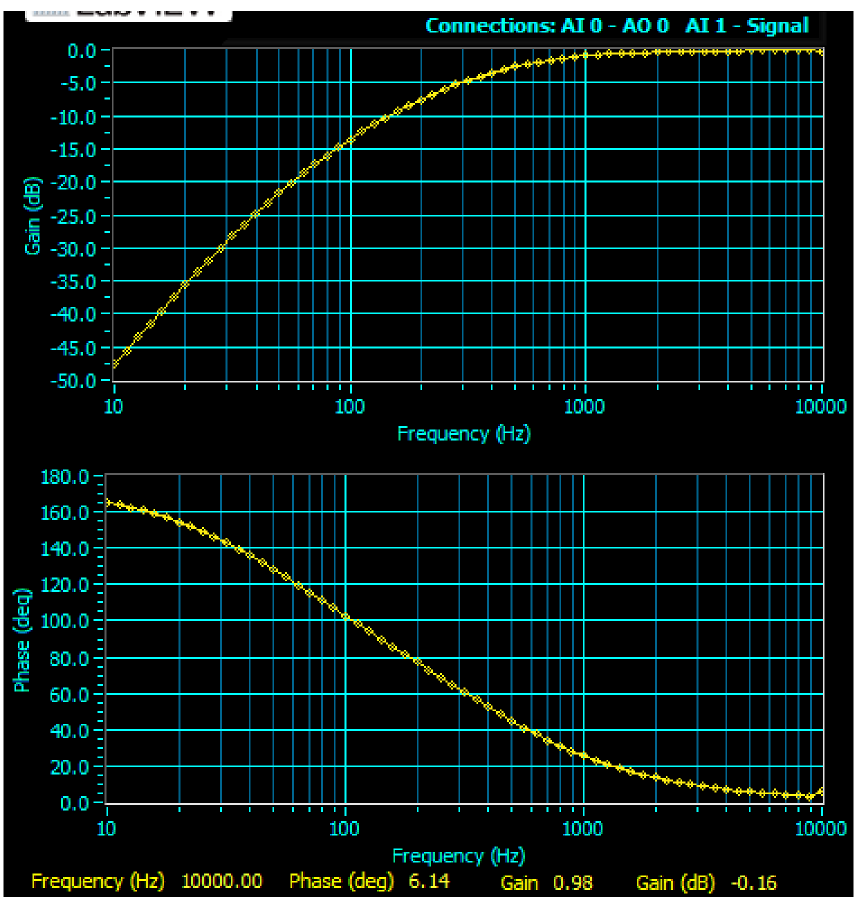
Data:

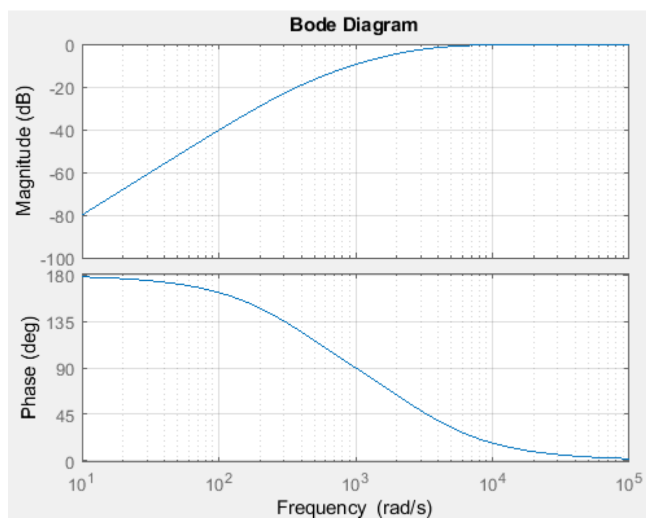
(a)


Connections: AI 0 - AO 0 AI 1 - Signal
Gain (dB)
0.0
-10.0
-20.0
-30.0
-40.0
-50.0
-60.0
-70.0
-80.0
10
100
1000
10000
Frequency (Hz)
Phase (deg)
0.0
-20.0
-40.0
-60.0
-80.0
-100.0
-120.0
-140.0
-160.0
10
100
1000
10000
Frequency (Hz)
Frequency (Hz) 10000.00 Phase (deg) -141.62 Gain 0.00 Gain (dB) -71.64


Bode Diagram
Magnitude (dB)
0
-20
-40
-60
-80
Phase (deg)
0
-45
-90
-135
-180
10^1
10^2
10^3
10^4
10^5
Frequency (rad/s)

(b)


Connections: AI 0 - AO 0 AI 1 - Signal
Gain (dB)
0.0
-5.0
-10.0
-15.0
-20.0
-25.0
-30.0
-35.0
-40.0
-45.0
-50.0
10
100
1000
10000
Frequency (Hz)
Phase (deg)
180.0
160.0
140.0
120.0
100.0
80.0
60.0
40.0
20.0
0.0
10
100
1000
10000
Frequency (Hz)
Frequency (Hz) 10000.00 Phase (deg) 6.14 Gain 0.98 Gain (dB) -0.16


Bode Diagram
Magnitude (dB)
0
-20
-40
-60
-80
-100
Phase (deg)
180
135
90
45
0
10^1
10^2
10^3
10^4
10^5
Frequency (rad/s)

Data analysis:

(a)

	Gain (experimental)	Gain (theoretical)	Percent Error%	Phase (degree) (experimental)	Phase (degree) (theoretical)	Percent Error%
10Hz	0.981	0.987	0.61%	-11.833	-10.72	10.38%
100Hz	0.466	0.505	7.72%	-73.968	-72.20	2.45%
500Hz	0.07	0.077	9.09%	-129.816	-133.26	2.58%
1000Hz	0.022	0.0233	5.58%	-146.788	-153.9	4.62%

(b)

	Gain (experimental)	Gain (theoretical)	Percent Error%	Phase (degree) (experimental)	Phase (degree) (theoretical)	Percent Error%
10Hz	0.004	0.00389	2.83%	165.433	169.28	2.27%
100Hz	0.210	0.199	5.53%	102.710	107.8	4.72%
500Hz	0.748	0.763	1.97%	44.685	46.74	4.40%
1000Hz	0.899	0.921	2.39%	25.725	26.099	1.43%

Calculation:

(a). $|H_a(s)| = \left|\frac{10^6}{(j2\pi f)^2 + (3\times10^3)(j2\pi f) + 10^6}\right|$

$= 0.987\angle -10.72^\circ$ for frequency = 10 Hz

Gain 0.987

phase -10.72°

percent Error%: $\frac{|0.987-0.981|}{0.987}\times 100\% = 0.61\%$

$\frac{|-10.72^\circ+11.833^\circ|}{10.72^\circ} = 10.38\%$

(b) $|H_b(s)| = \frac{(j2\pi f)^2}{(j2\pi f)^2+(3\times10^3)(j2\pi f)+10^6}$

$0.00389\angle 169.28^\circ$ for frequency = 10Hz

Gain = 0.00389

phase 169.28°

percent Error%: $\frac{|0.00389-0.004|}{0.00389}\times 100\% = 2.83\%$

$\frac{|169.28^\circ-165.43^\circ|}{169.28^\circ} = 2.27\%$

Discussion:

The general graph of measured Bode plots match up with the bode plots which are from MATLAB. The error percentages are generally small except some were greater than 10%. The limitation of myDAQ produces higher errors especially when it measured the high frequency. However, the desired second-order transfer function can be obtained by cascading two first-order network.

a). The derived transfer functions are in the theory section.

b). Since the transfer functions have two real poles, their response is overdamped.

c). Part a is LPF. Part b is HPF. The rest of answer is in the theory section.

d). The answers are in the theory section.

Part#4: 2nd-order transfer function synthesis – II

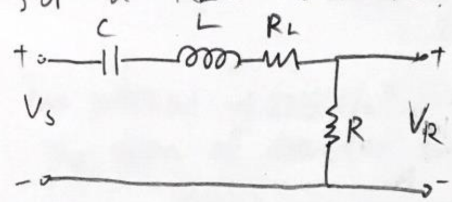
Objectives:

The purpose of this part is to design RLC network circuit to satisfy the transfer function requirements.

Theory:

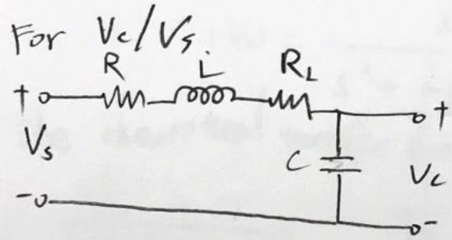
First, we derive the transfer function for each voltage output of resistor, inductor and capacitor.

a) For a RLC network. $R_L = 150\Omega$.



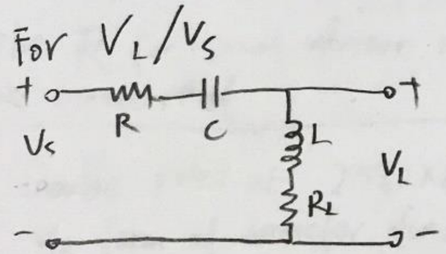
$$\frac{V_R}{V_s} = \frac{R}{\frac{1}{Cs} + Ls + R_L + R} = \boxed{\frac{\frac{R}{L}\cdot s}{s^2 + \left(\frac{R_L+R}{L}\right)s + \frac{1}{LC}}}$$

For V_c/V_s.



$$\frac{V_c}{V_s} = \frac{\frac{1}{C\cdot s}}{\frac{1}{Cs} + R + R_L + Ls} = \frac{\frac{1}{LC}}{s^2 + \left(\frac{R+R_L}{L}\right)s + \frac{1}{LC}}$$

For V_L/V_s



$$\frac{V_L}{V_s} = \frac{R_L + Ls}{R + R_L + \frac{1}{Cs} + Ls} = \frac{s(R_L + Ls)}{(R+R_L)s + \frac{1}{C} + Ls^2} = \frac{s\left(s + \frac{R_L}{L}\right)}{s^2 + \left(\frac{R+R_L}{L}\right)\cdot s + \frac{1}{LC}}$$

Part 4). 2nd-transfer function

$$H(s) = \frac{s^2 + B_1 s + B_2}{s^2 + A_1 s + A_2}$$

a). Two poles at -1.225×10^3, -5.442×10^3, no zeros, DC gain is 1

The form of transfer funistion is

$$H(s) = \frac{A}{(s+1.225\times10^3)(s+5.442\times10^3)}. \qquad H(0) = \frac{A}{6.67\times10^6} = 1 \quad A = 6.67\times10^6.$$

$$H(s) = \frac{6.67\times10^6}{s^2 + 6.67\times10^3 s + 6.67\times10^6}$$

The theoretical transfer function $\boxed{V_c/V_s}$ can satisfy this $H(s)$.

$$\frac{V_c}{V_{IN}} = \frac{\frac{1}{LC}}{s^2 + \left(\frac{R+R_L}{L}\right)s + \frac{1}{LC}}. \qquad \frac{1}{LC} = 6.67\times10^6.$$

$$R + 250 = 150\times10^{-3}\times6.67\times10^3$$

$$\boxed{\begin{gathered} L = 150\text{mH}, \quad C = 1\mu F \\ R_L = 250\Omega. \\ R = 750\Omega \end{gathered}}$$

The R_L for 150mH inductor is 250Ω as we measured.

---

b). Double poles at -2.582×10^3, no zeros, DC gain is 1.

The form of transfer function is.

$$H(s) = \frac{A}{(s+2.582\times10^3)(s+2.582\times10^3)} \qquad H(0) = \frac{A}{6.67\times10^6} = 1 \quad A = 6.67\times10^6.$$

$$H(s) = \frac{6.67\times10^6}{s^2 + 5.164\times10^3 s + 6.67\times10^6}.$$

The theoretical transfer function $\boxed{V_c/V_s}$ can satisfy this $H(s)$.

$$\frac{V_c}{V_s} = \frac{\frac{1}{LC}}{s^2 + \left(\frac{R+R_L}{L}\right)s + \frac{1}{LC}} \qquad \frac{1}{LC} = 6.67\times10^6$$

$$\boxed{\begin{gathered} L = 150\text{mH} \quad C = 1\mu F \\ R_L = 250\Omega. \end{gathered}}$$

$$R + 250 = 150\times10^{-3}\times5.164\times10^3 \qquad \boxed{R = 525\Omega}.$$

c). Complex poles at $-1.67\times10^3 \pm j1.972\times10^3$, no zeros, DC gain is 1.

$$H(s) = \frac{A}{(s+1.67\times10^3+j1.972\times10^3)(s+1.67\times10^3-j1.972\times10^3)}$$

$$= \frac{A}{s^2+(3.34\times10^3)s+6.67\times10^6}$$

$H(0)=1$ $A=6.67\times10^6$

$$= \frac{6.67\times10^6}{s^2+(3.34\times10^3)s+6.67\times10^6}$$

V_c/V_s can satisfy this $H(s)$. $\frac{1}{LC}=6.67\times10^6$

$R+250\Omega = 150\times10^{-3}\times3.34\times10^3$

$= 250\Omega$

$\boxed{L=150mH,\ C=1\mu F \quad R_L=250\Omega \quad R=250\Omega}$

---

d) Two poles at -1.225×10^3, -5.442×10^3, two zeros at DC $\lim_{s\to\infty} H(s)=1$.

$$H(s) = \frac{s(s+A)}{(s+1.225\times10^3)(s+5.442\times10^3)}$$

$A=\frac{R_L}{L}=\frac{250}{150\times10^{-3}}=1667$

$$H(s) = \frac{s^2+As}{s^2+6.667\times10^3 s+6.67\times10^6}$$

$\frac{1}{LC}=6.67\times10^6.$

The V_L/V_s can satisfy this $H(s)$.

$$\frac{V_L}{V_s} = \frac{s(s+\frac{R_L}{L})}{s^2+(\frac{R+R_L}{L})s+\frac{1}{LC}}$$

$R+250\Omega = 150\times10^{-3}\times6.67\times10^{-3}$

$\boxed{L=150mH,\ C=1\mu F \quad R_L=250\Omega \quad R=750\Omega}$

e). Double poles at -2.582×10^3 Hz, two zeros at DC, $\lim_{s\to\infty} H(s)=1$.

$$H(s) = \frac{s(s+A)}{s^2+5.164\times10^3 s+6.67\times10^6}$$

$\frac{1}{LC}=6.67\times10^6$

V_L/V_s can satisfy this $H(s)$.

$R+250\Omega = 150\times10^{-3}\times5.164\times10^3$

$\boxed{L=150mH,\ C=1\mu F \quad R_L=250\Omega, \quad R=525\Omega}$

---

f) Complex poles at $-1.67\times10^{-3}\pm j1.972\times10^3$ Hz, two zeros at DC, $\lim_{s\to\infty} H(s)=1$

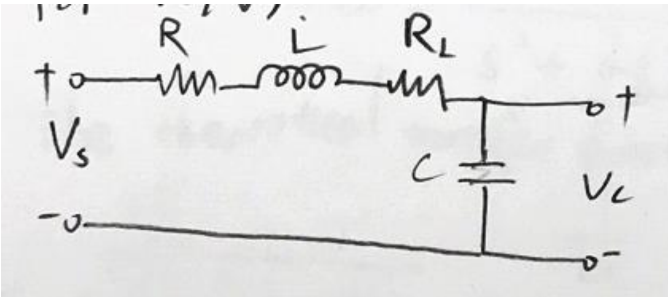
$$H(s) = \frac{s(s+A)}{s^2+(3.34\times10^3)s+6.67\times10^6}$$

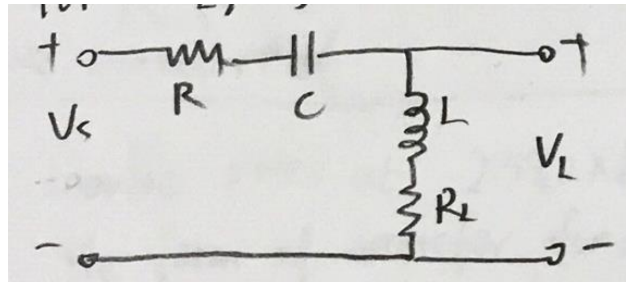
$\boxed{L=150mH,\ C=1\mu F \quad R_L=250\Omega \quad R=250\Omega}$

V_L/V_s can satisfy this $H(s)$.

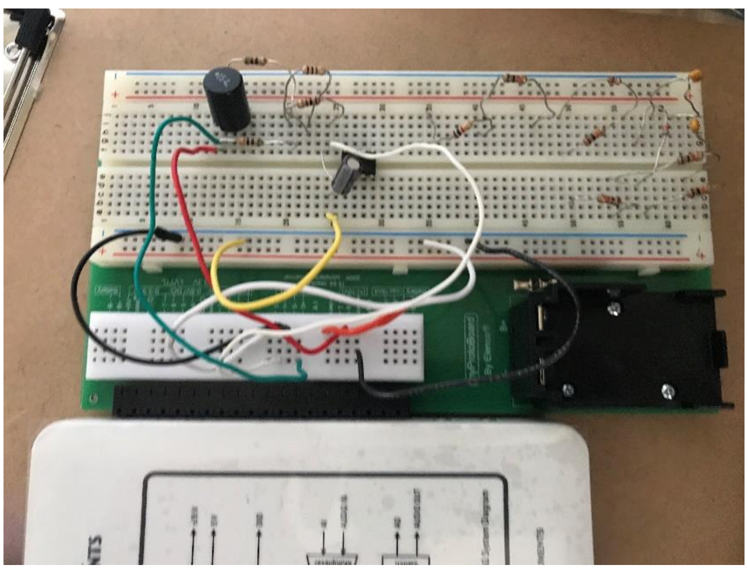
$R+250\Omega = 150\times10^{-3}\times3.34\times10^3$

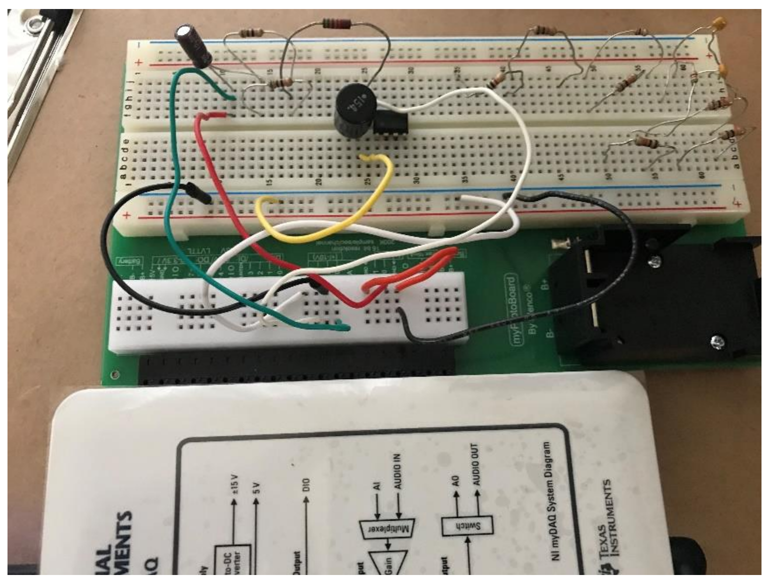
Procedure:





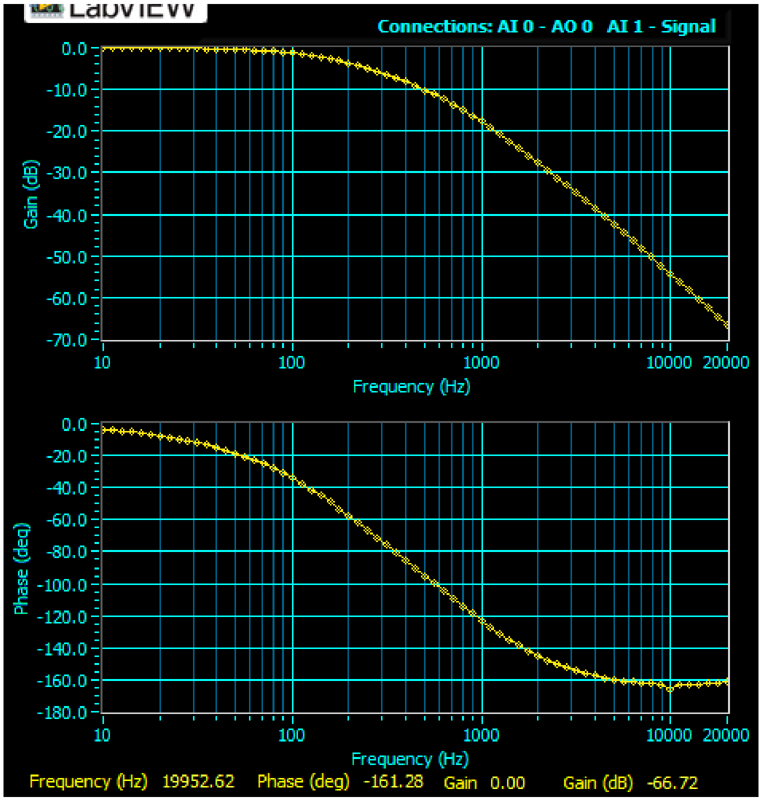
1. Implement the circuit above. (R_L=250Ω).
    a. For part a), using Vc/Vs circuit. R = 750Ω, C = 1uF, L=150mH. Output voltage at the capacitor.
    b. For part b), using Vc/Vs circuit. R=525Ω, C=1uF, L=150mH. Output voltage at the capacitor.
    c. For part c), using Vc/Vs circuit. R=250Ω, C=1uF, L=150mH. Output voltage at the capacitor.
    d. For part d), using V_L/Vs circuit. R=750Ω, C=1uF, L=150mH. Output voltage at the inductor.
    e. For part e), using V_L/Vs circuit. R=525, C=1uF, L=150mH. Output voltage at the inductor.
    f. For part f), using V_L/Vs circuit. R=250, C=1uF, L=150mH. Output voltage at the inductor.
2. Using the Bode Analyzer from myDAQ to find the Bode plots of both circuits.
3. Using MATLAB to confirm with the measured Bode plot.



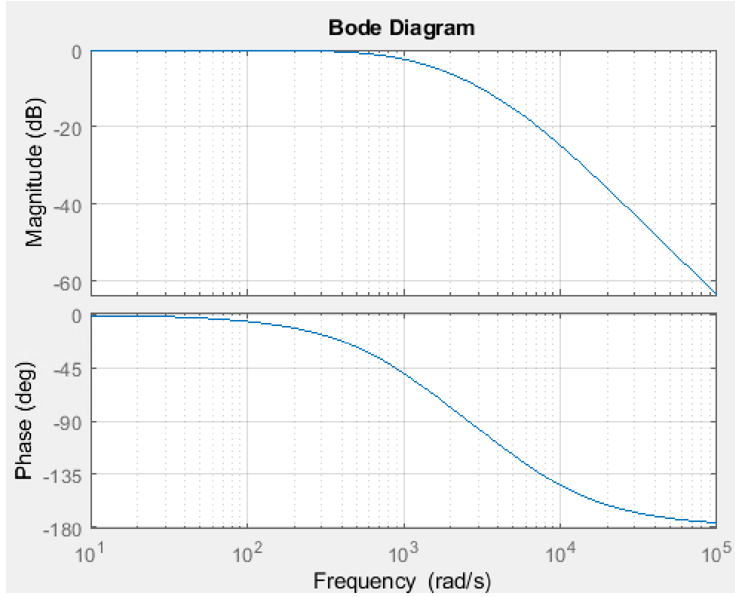


Data:

(a)

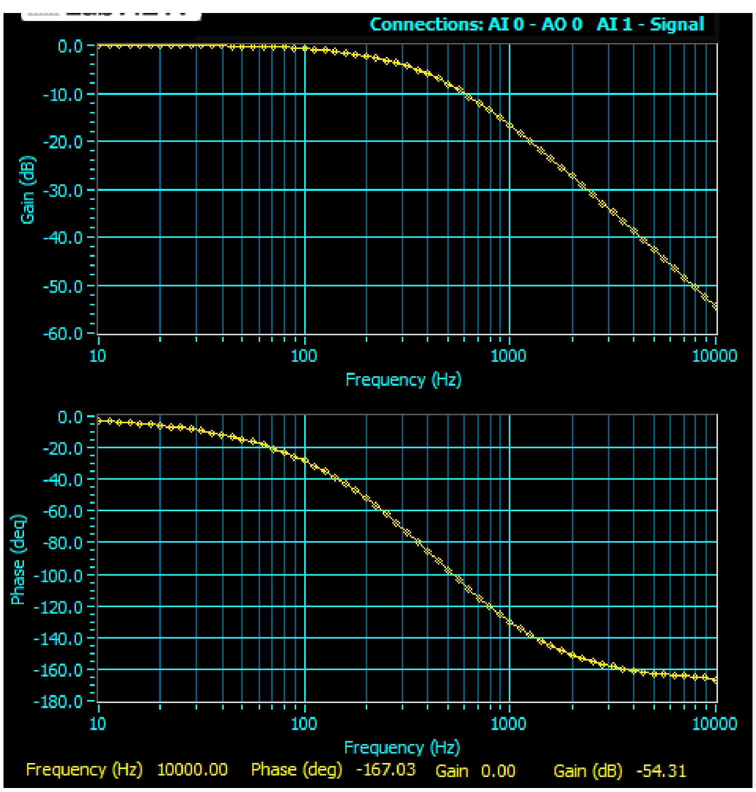


Connections: AI 0 - AO 0 AI 1 - Signal
Gain (dB)
Frequency (Hz)
Phase (deg)
Frequency (Hz) 19952.62 Phase (deg) -161.28 Gain 0.00 Gain (dB) -66.72

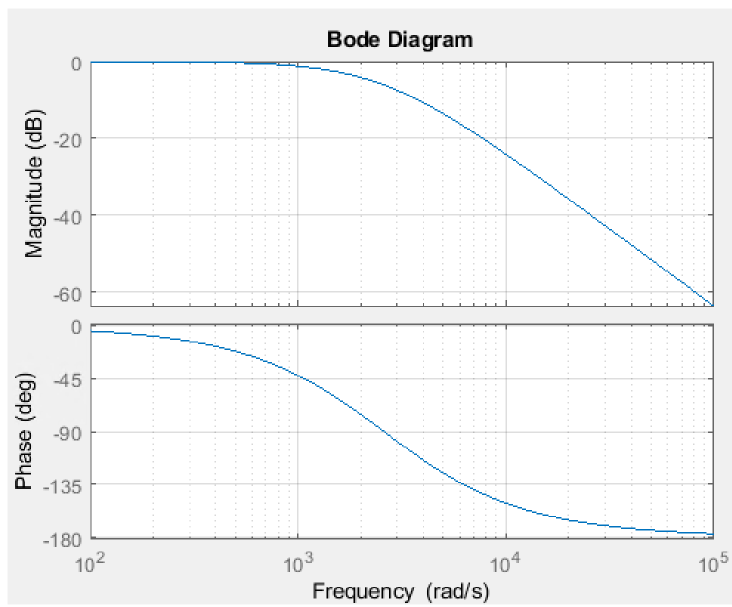


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

(b).

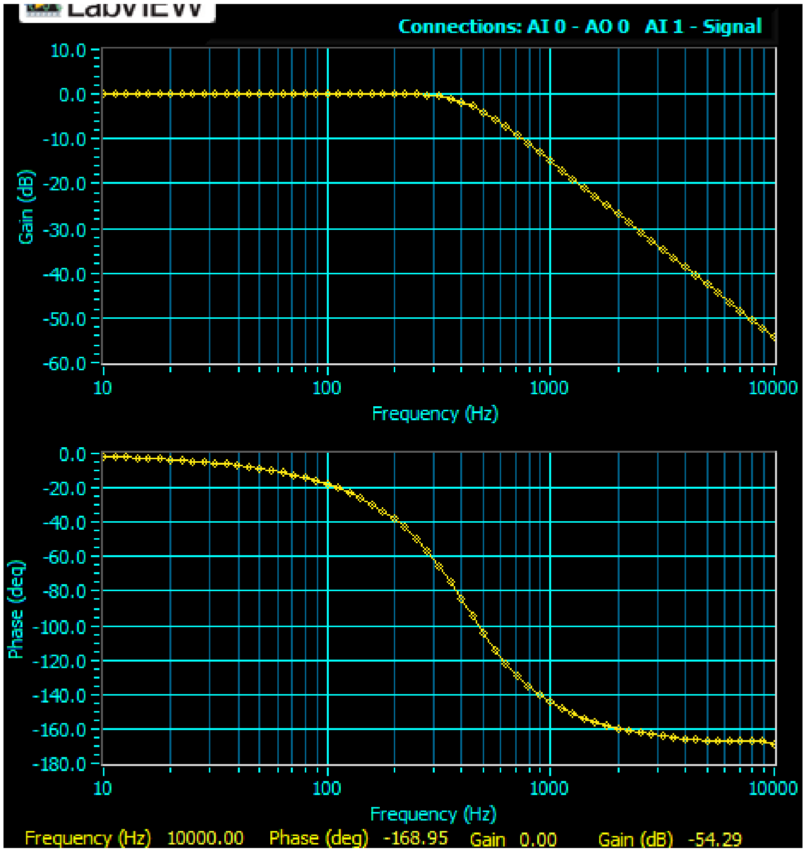


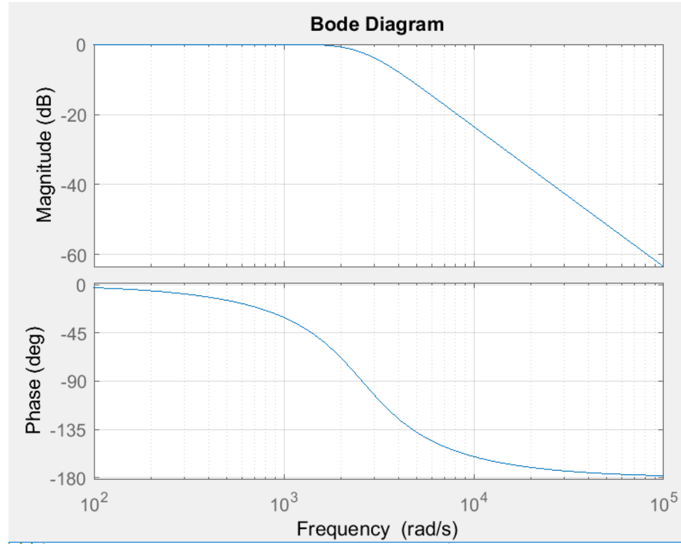
Connections: AI 0 - AO 0 AI 1 - Signal
Gain (dB)
Frequency (Hz)
Phase (deg)
Frequency (Hz) 10000.00 Phase (deg) -167.03 Gain 0.00 Gain (dB) -54.31



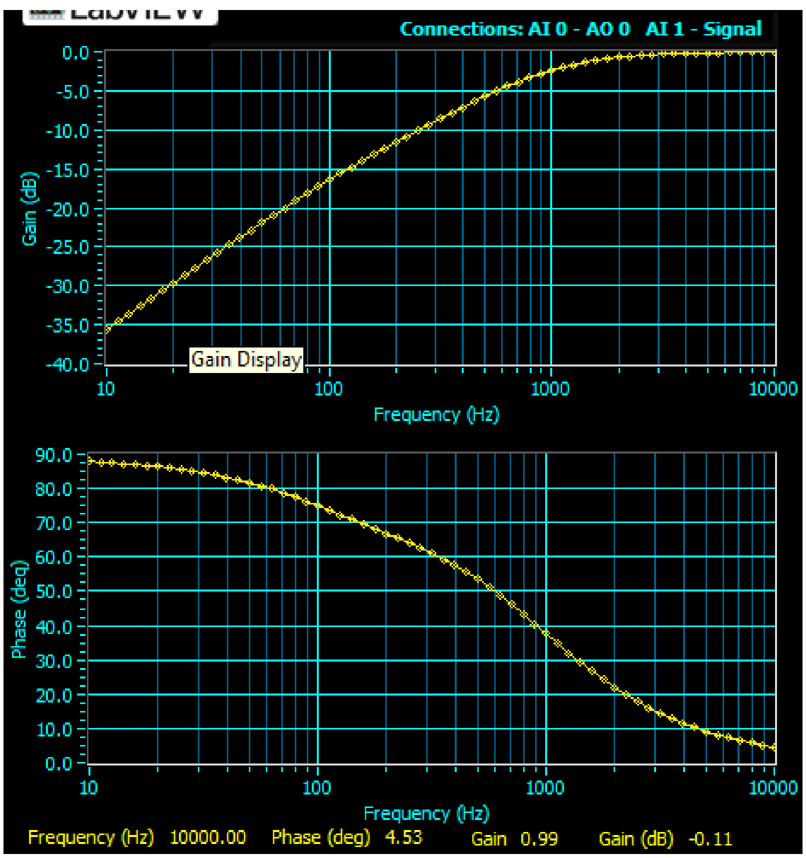
Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

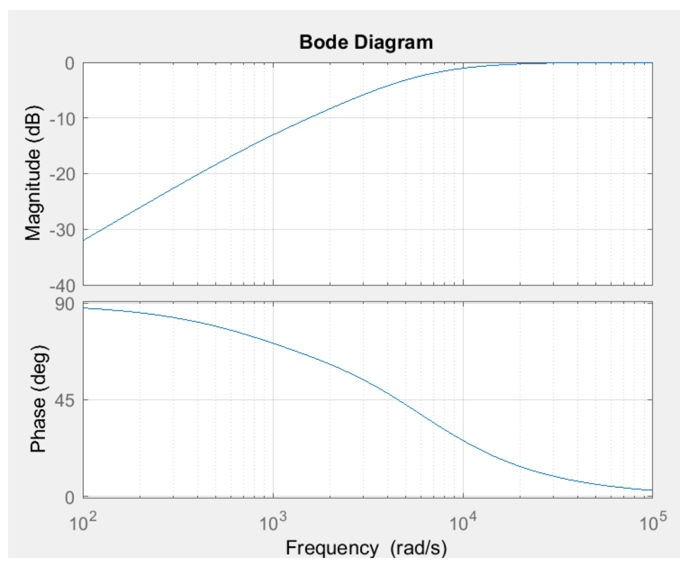
(c).


LabVIEW
Connections: AI 0 - AO 0 AI 1 - Signal
Gain (dB)
10.0
0.0
-10.0
-20.0
-30.0
-40.0
-50.0
-60.0
10
100
1000
10000
Frequency (Hz)
Phase (deg)
0.0
-20.0
-40.0
-60.0
-80.0
-100.0
-120.0
-140.0
-160.0
-180.0
10
100
1000
10000
Frequency (Hz)
Frequency (Hz) 10000.00 Phase (deg) -168.95 Gain 0.00 Gain (dB) -54.29

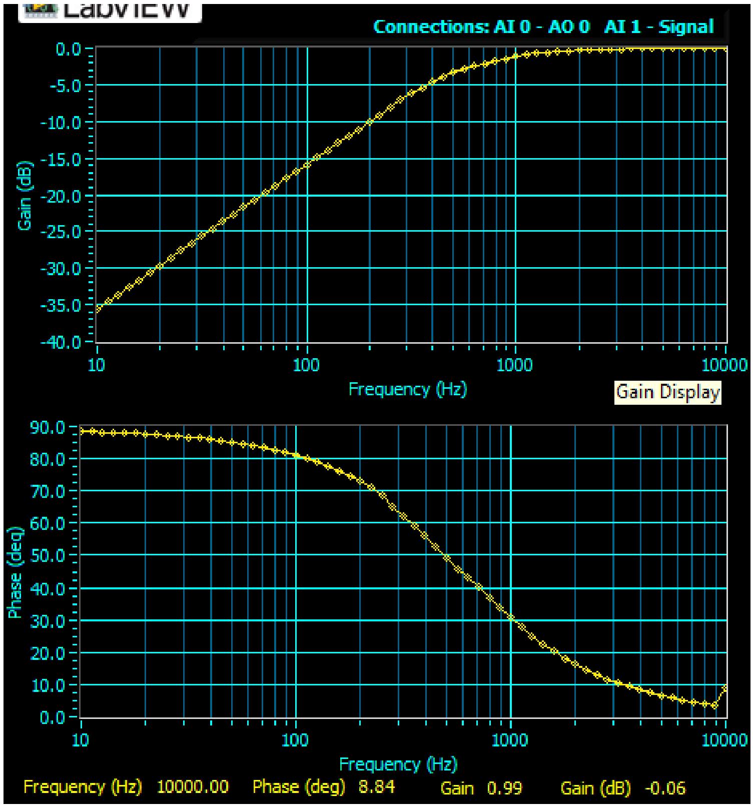

Bode Diagram
Magnitude (dB)
0
-20
-40
-60
Phase (deg)
0
-45
-90
-135
-180
10^2
10^3
10^4
10^5
Frequency (rad/s)

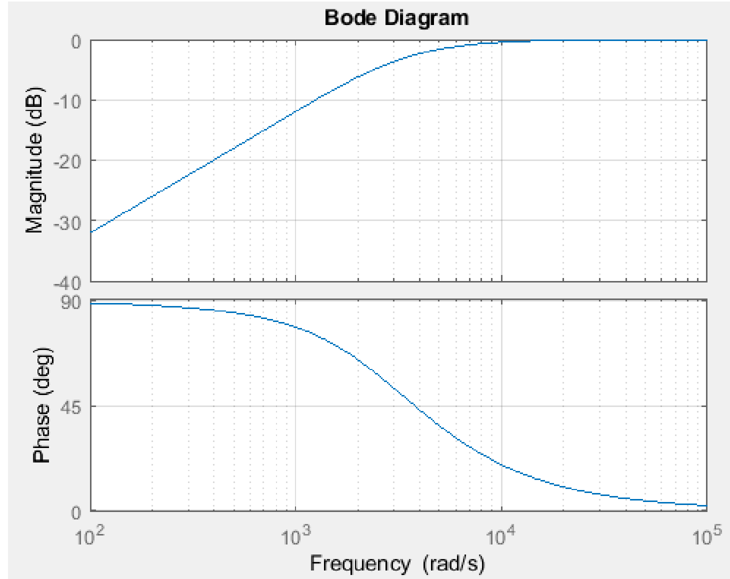
(d).


LabVIEW
Connections: AI 0 - AO 0 AI 1 - Signal
Gain (dB)
0.0
-5.0
-10.0
-15.0
-20.0
-25.0
-30.0
-35.0
-40.0
Gain Display
10
100
1000
10000
Frequency (Hz)
Phase (deg)
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
10
100
1000
10000
Frequency (Hz)
Frequency (Hz) 10000.00 Phase (deg) 4.53 Gain 0.99 Gain (dB) -0.11

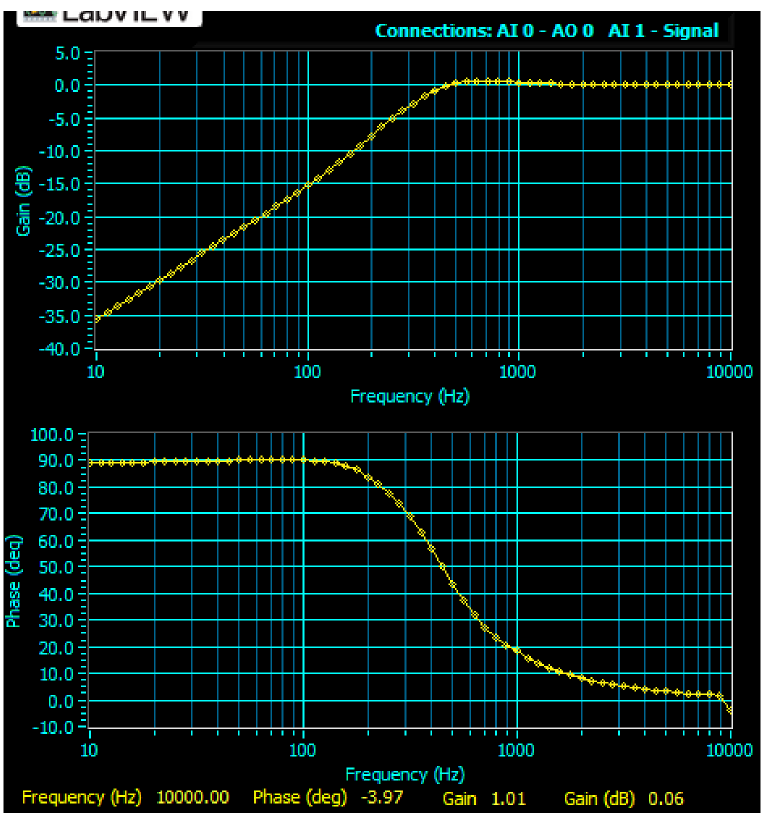

Bode Diagram
Magnitude (dB)
0
-10
-20
-30
-40
Phase (deg)
90
45
0
10^2
10^3
10^4
10^5
Frequency (rad/s)

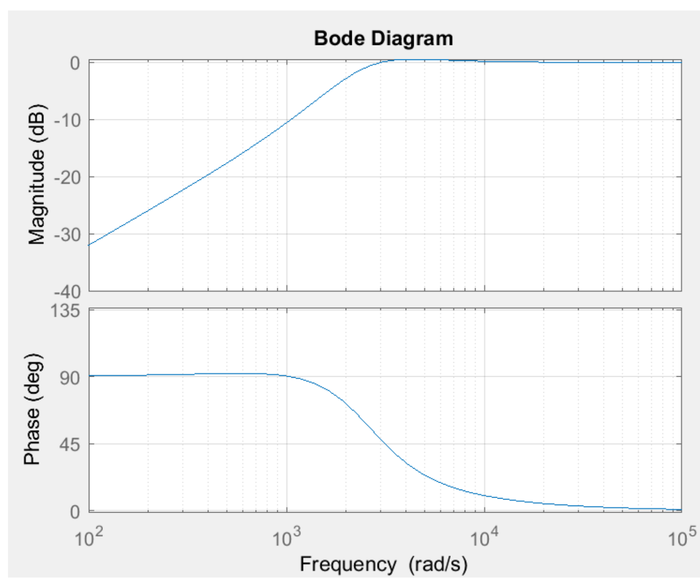
(e).


Connections: AI 0 - AO 0 AI 1 - Signal
Gain (dB)
Frequency (Hz)
Gain Display
Phase (deg)
Frequency (Hz)
Frequency (Hz) 10000.00
Phase (deg) 8.84
Gain 0.99
Gain (dB) -0.06


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

(f).


Connections: AI 0 - AO 0 AI 1 - Signal
Gain (dB)
Frequency (Hz)
Phase (deg)
Frequency (Hz)
Frequency (Hz) 10000.00
Phase (deg) -3.97
Gain 1.01
Gain (dB) 0.06


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

Data analysis:

	Gain (experimental)	Gain (theoretical)	Percent Error%	Phase (degree) (experimental)	Phase (degree) (theoretical)	Percent Error%
(a) at 100Hz	0.860	0.884	2.71%	-34.240	-33.74	1.48%
(b) at 100Hz	0.923	0.944	2.22%	-28.260	-27.32	3.44%
(c) at 100Hz	1.003	1.008	0.50%	-18.064	-18.49	2.30%
(d) at 100Hz	0.150	0.148	1.35%	74.955	76.92	2.55%
(e) at 100Hz	0.161	0.158	1.27%	80.964	83.33	2.84%
(f) at 100Hz	0.174	0.160	8.75%	89.988	93.18	3.43%

Calculation:

- Example for (a) at 100Hz

$$|H(s)| = \frac{6.67\times10^6}{(j\times2\pi\times100)^2+6.67\times10^3(j2\pi\times100)+6.67\times10^6}$$

$|H(s)| = 0.884$

$\angle H(s) = -33.74°$

Percent Error% for gain $\frac{|0.884-0.860|}{0.884}\times100\% = 2.71\%$

Percent Error% for phase $\frac{|-33.74° - (-34.240)|}{33.74°}\times100\% = 1.48\%$

Discussion:

For all parts of this experiment, the measured Bode plots match up with the theoretical bode plots which are obtained from MATLAB. We pick a frequency at 100Hz and compare their gain and phase value. From part a to part f, the percent errors are relatively small. Throughout this experiment, in the RLC circuit, if we measure the output voltage of capacitor, the transfer function would not have zeros. If we measure the output voltage of inductor, the transfer function would have two zeros. Measuring the output voltage of resistor has one zero in the transfer function.

All discussion question answers are in the theory section.

Conclusion:

Throughout this experiment, a desired transfer function can be obtained by the specific circuit network. For the Bode plot, the zeros would increase the slope, while the poles would decrease the slope. When the transfer function is equal to a constant number, it would not have any poles and zeros. Because there is no phase shift, the circuit doesn't need capacitor and inductor. From the transfer function, it can also be able to determine the circuit is LPF or HPF, phase and gain. In addition, all measured Bode plots are closed to the theoretical Bode plots, which we expected.