EC ENGR 111L

Experiment #1

Steady-State Power Analysis

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Lab Section: 1E

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Objectives:

The purpose of this experiment part#1 and part#2 is to understand how source resistance of the voltage source affects the power transfer with a given load and how load resistance of the network affects the power transfer with a given load.

For part#3, the purpose is to find the power factor of an unknown load and to design the power factor adjustment component which is to make the power factor to be 1, no imaginary part.

Theory:

The concept of Thevenin equivalent theorem shows that the entire network can be replaced by an equivalent circuit that only contains an independent voltage source in series with a resistor. The Norton's theorem has a similar concept except that the equivalent circuit contains independent current source in parallel with a resistor. In this lab, we will use the RL to represents the equivalent load in the circuit, and Rs represents the voltage source resistor.

By using the phasor notation, the AC signal can be represented by the phasor from this formula

$x(t) = A\cos(\omega t + \theta) \rightarrow \mathrm{Re}[A\cdot\exp(j\omega t + j\theta)] \rightarrow \mathrm{Re}[A\cdot\exp(j\theta)\cdot\exp(j\omega t)] \rightarrow A\angle\theta°$

Calculate the power in steady-state signals:

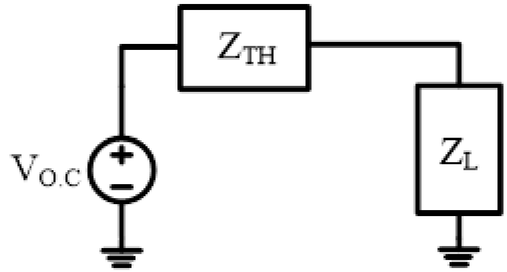
Average power:

$$
\begin{aligned}
P &= \frac{1}{T}\int_{t_0}^{t_0+T} p(t)dt = \frac{1}{T}\int_{t_0}^{t_0+T} V_M \cos(\omega t+\theta_v)\cdot I_M \cos(\omega t+\theta_i)dt \\
&= \frac{1}{T}\int_{t_0}^{t_0+T} \frac{V_M I_M}{2}[\cos(\theta_v-\theta_i)+\cos(2\omega t+\theta_v+\theta_i)dt \\
&= \frac{1}{2}V_M I_M \cos(\theta_v-\theta_i) \\
&= \frac{1}{2}\mathrm{Re}\left[\bar{V}\cdot\bar{I}^*\right]
\end{aligned}
$$

Maximum average power transfer:

$$P_L = \frac{1}{2}\cdot\frac{V_{OC}^2 R_L}{(R_{TH}+R_L)^2+(X_{TH}+X_L)^2}$$

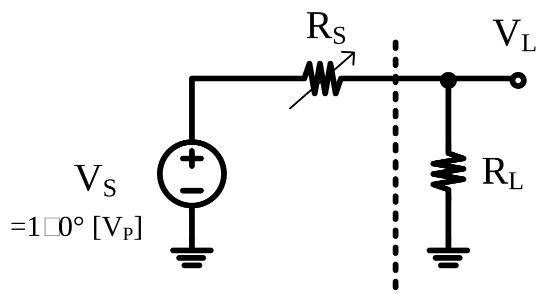
To obtain the maximum P_L, set the $X_{TH} = -X_L$, and $R_{TH} = R_L$. Thus, $Z_L = Z_{TH}^*$.



Part#1&2: Determine the source and load resistance

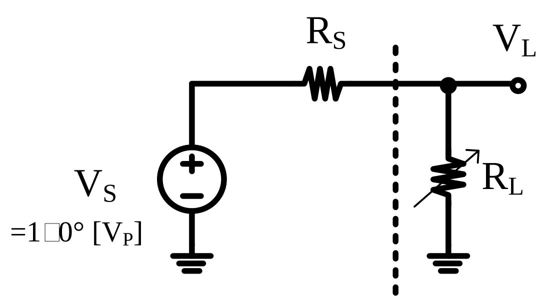
Procedure:

Part#1: Determine the source resistance, R_S



1. Build the given circuit that includes an ideal voltage source and equivalent resistors shown above.
2. Using function generator to generate the AC voltage source, and set the amplitude to be 2, the frequency to be 5kHz, DC offset to be 0.
3. Using R_L = 1 KΩ as a load resistor as well as a constant resistance. Placing Rs with resistance 100Ω,
4. Using oscilloscope to measure the voltage across the R_L and voltage source, record the Vrms for both.
5. Sweep the value of Rs resistance to 1kΩ and 10kΩ, repeat step 4.
6. Calculate the RMS power of R_L, P_L and voltage source, P_{supply} and the PE power efficiency P_L/P_{supply}

Part#2: Determine the load resistance, R_L



1. Similarly, following the same steps as part#1 but this part sweeps the R_L instead of sweeping the value of Rs. Rs is a constant resistance in this part. Calculate the RMS power of R_L, P_L and voltage source, P_{supply} and the power transfer efficiency P_L/P_{supply}

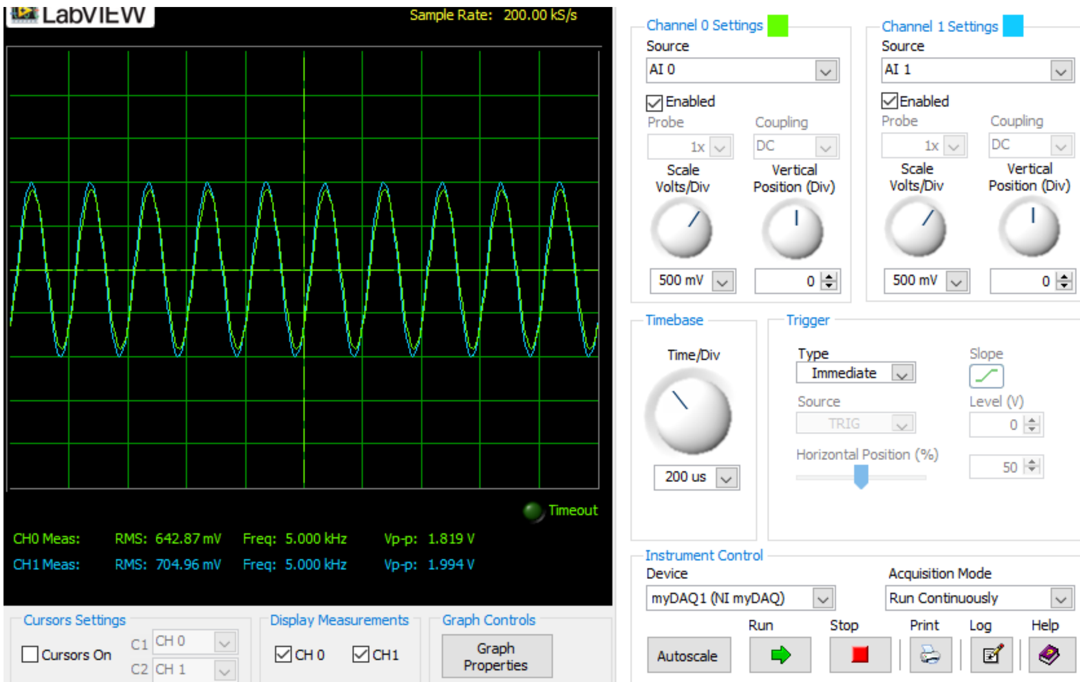
Data:

Resistor	**R_L**	**R_1**	**R_2**	**R_3**
Measured value (Ω)	0.990kΩ	98Ω	0.982kΩ	9.92kΩ

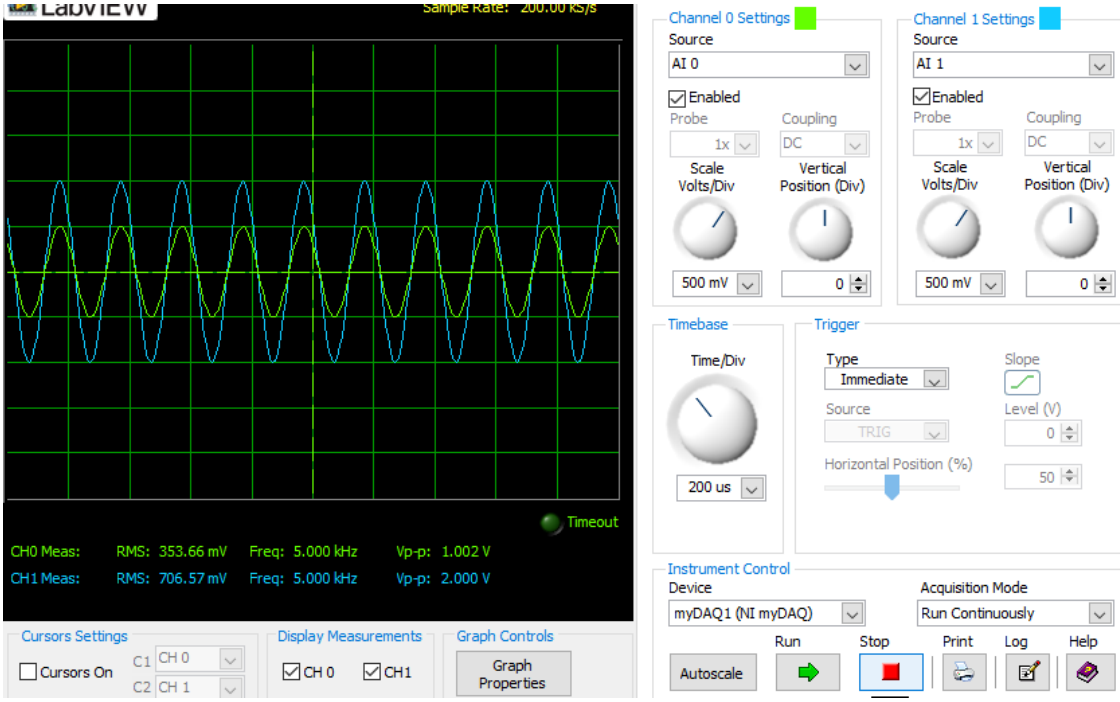
Part#2: Determine the source resistance, Rs

R_S [Ω]	R_L [Ω]	V_{supply}rms(V)	$V_L$rms(V)	P_L [W]	P_{supply} [W]	PE (P_L/P_{supply})
98Ω	0.990 KΩ	704.96mV	642.87mV	417.46μW	457.519μW	0.912
0.982kΩ	0.990 KΩ	706.57mV	353.66mV	125.63μW	251.54μW	0.499
9.92kΩ	0.990 KΩ	707.83mV	64.14mV	4.1555μW	45.87μW	0.091

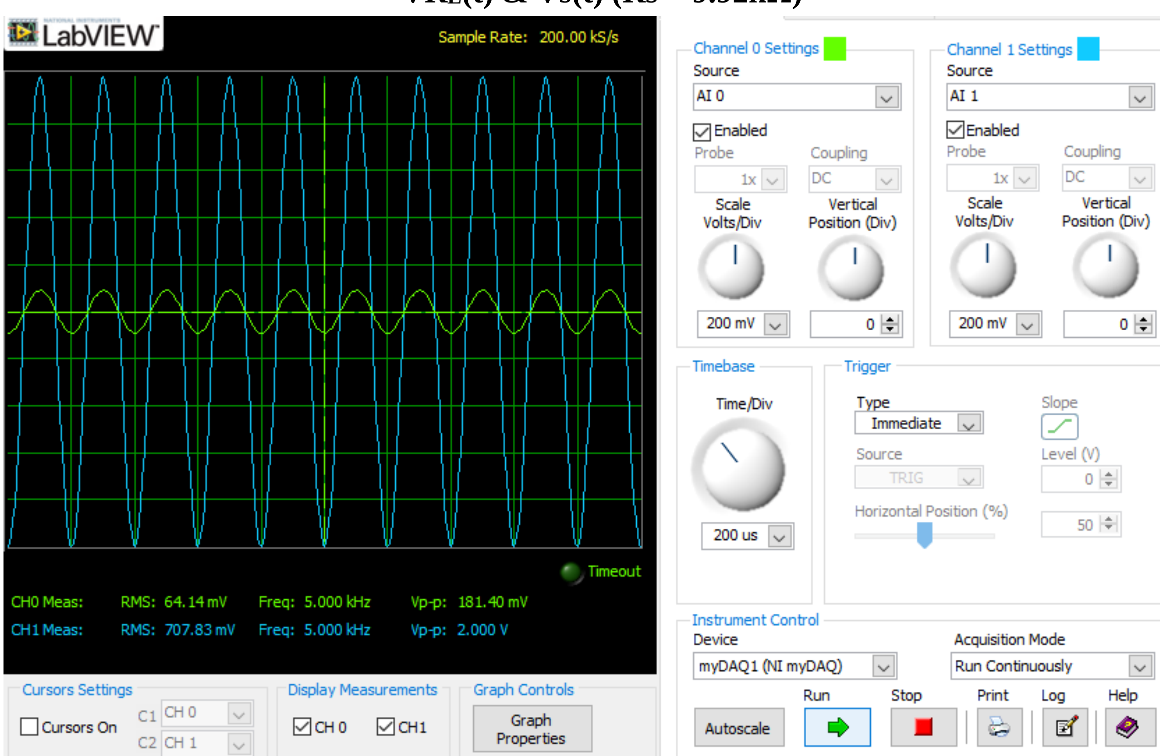
V_{RL}(t) & Vs(t) (Rs = 98Ω)



$V_{RL}(t)$ & $V_S(t)$ (R_S = 0.982 kΩ)



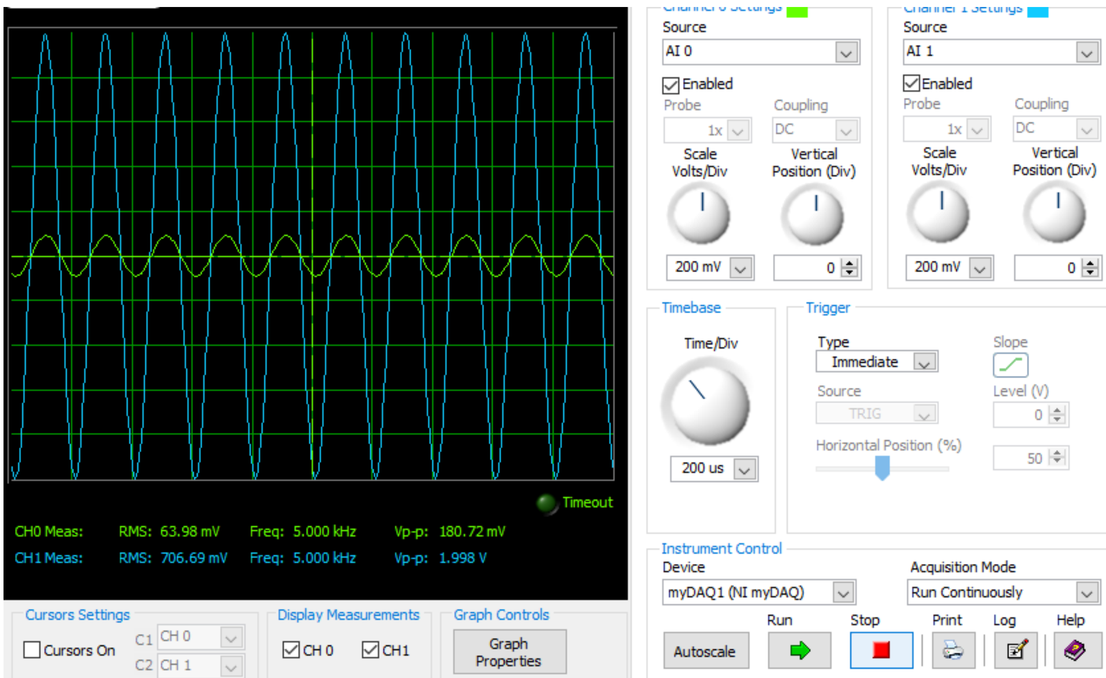
$V_{RL}(t)$ & $V_S(t)$ (R_S = 9.92kΩ)



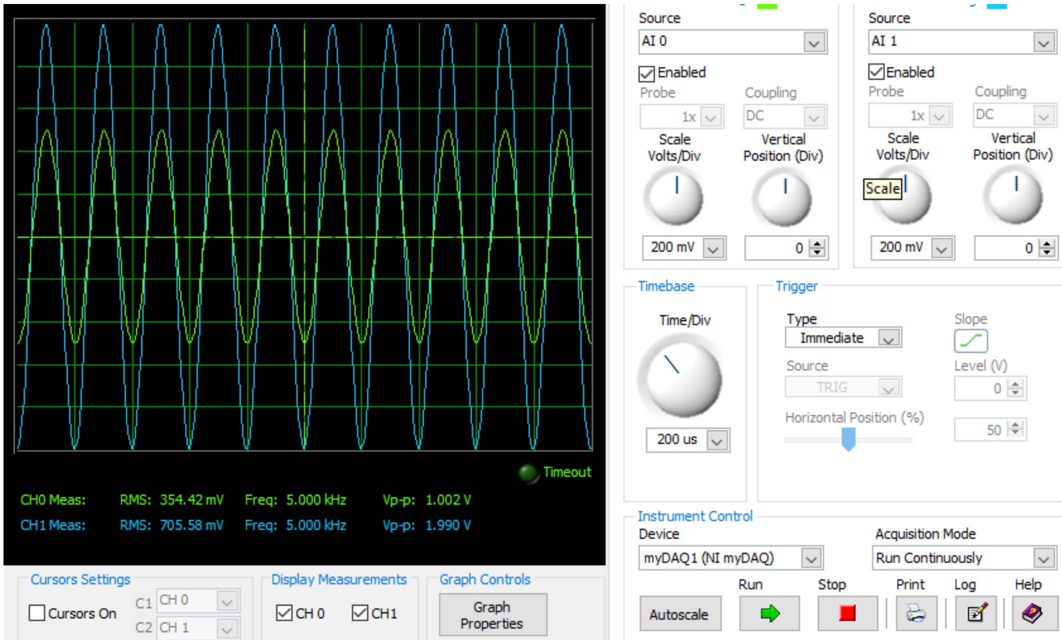
Part#2: Determine the load resistance, R_L

R_S [Ω]	R_L [Ω]	$V_s rms$(V)	$V_L rms$(V)	P_L [W]	P_{supply} [W]	PE (P_L/P_{supply})
0.984 kΩ	**98Ω**	706.69mV	63.98mV	41.77µW	461.47µW	0.091
0.984 kΩ	**0.982kΩ**	705.58mV	354.42mV	127.92µW	254.71µW	0.502
0.984 kΩ	**9.92kΩ**	707.56mV	642.10mV	41.56µW	45.78µW	0.908

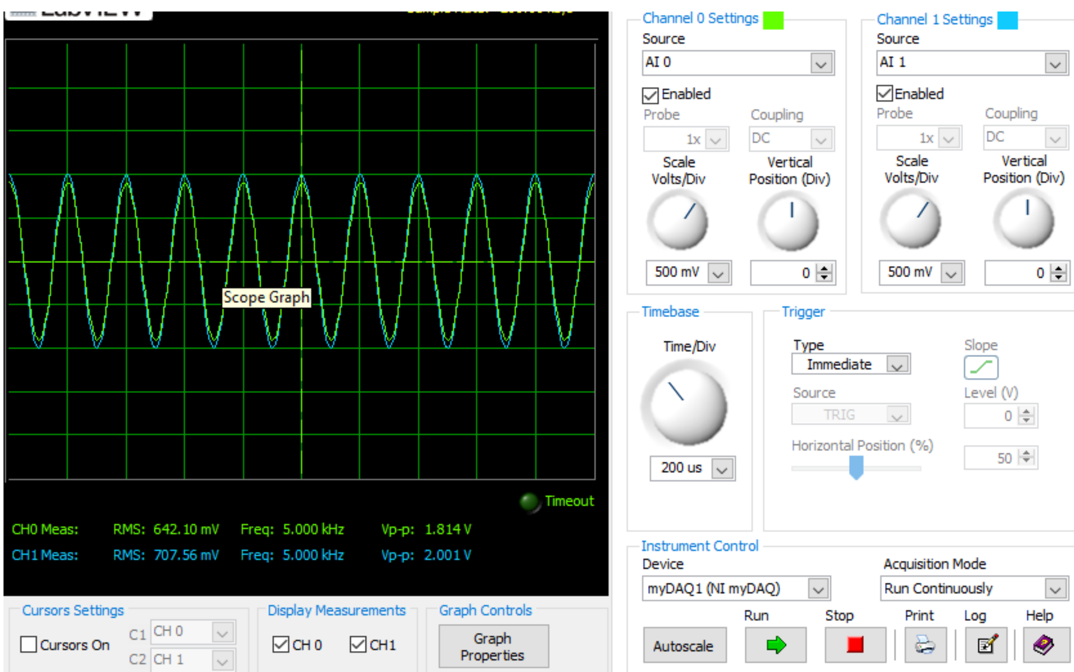
$V_{RL}(t)$ & $V_s(t)$ (R_L = 98Ω)



VRL(t) & Vs(t) (RL = 0.982kΩ)



VRL(t) & Vs(t) (RL = 9.92kΩ)



Data Analysis:

Calculation for the P_{supply} and P_L,

The formulas using for theoretical calculation are below: V = 1V.

$$Vrms = \frac{V}{\sqrt{2}} \qquad P_L = \frac{Vrms^2 \times RL}{(RL+Rs)^2} \qquad Psupply = \frac{Vrms^2}{(RL+Rs)}$$

To calculate the power of the load resistor, using the oscilloscope to measure the voltage for the load resistor. And use the rms voltage for calculation. After getting the power of the load resistance, use V/R to get the current I. Using this current to calculate the power supply by the equation P_{supply} = VI.

$$P_L = \frac{\mathrm{VLrms}^2}{RL} \qquad I = \frac{VLrms}{RL} \qquad P_{supply} = \mathrm{Vsupply} * \mathrm{I} \qquad PTE = \frac{PL}{Psupply}$$

$$\%\,Error = \left|\frac{Y_{experiment} - Y_{theory}}{Y_{theory}}\right|$$

Part#1: RL = 0.990 KΩ (1kΩ)

	Measured	**Theoretical**	**%Error**
P_L (Rs = 98Ω)	417.46 µW	414.73 µW	0.69%
P_{supply} (Rs = 98Ω)	457.519 µW	455.37 µW	0.47%
P_L (Rs = 0.982kΩ)	125.63 µW	125 µW	0.50%
P_{supply} (Rs = 0.982kΩ)	251.54 µW	250 µW	0.61%
P_L (Rs = 9.92kΩ)	4.156 µW	4.132 µW	0.58%
P_{supply} (Rs = 9.92kΩ)	45.87 µW	45.45 µW	0.92%

Part#2: Rs = 0.984kΩ (1kΩ)

	Measured	**Theoretical**	**%Error**
P_L (R_L = 98Ω)	41.77 µW	41.32 µW	1.09%
P_{supply} (R_L = 98Ω)	461.47 µW	454.55 µW	1.52%
P_L (R_L = 0.982kΩ)	127.92 µW	125 µW	2.34%
P_{supply} (R_L = 0.982kΩ)	254.71 µW	250 µW	1.88%
P_L (R_L = 9.92kΩ)	41.56 µW	41.32 µW	0.58%
P_{supply} (R_L = 9.92kΩ)	45.78 µW	45.45 µW	0.73%

The example of calculation for Part#1 and a theoretical value:

Calculation For Part # 1 & Part #2.

$R_s = 98\Omega$

$$P_L = \frac{V_{L\,rms}^2}{R_L} = \frac{(642.87 \times 10^{-3})^2 V}{990\Omega} = 417.46\,uW$$

$$I = \frac{V_{L\,rms}}{R_L} = \frac{642.87\,mV}{990\Omega} = 0.649\,mA$$

Theoretical:

$$P_L = \frac{\left(\frac{1}{\sqrt{2}}\right)^2 \times 1000}{(1000+98)^2} = 414.73\,uW$$

$$P_{supply} = \frac{\left(\frac{1}{\sqrt{2}}\right)^2}{(1000+98)} = 455.37\,uW.$$

$$P_{supply} = V_{supply} \cdot I = 704.96\,mV \times 0.649\,mA = 457.519\,uW$$

$$PTE: P_L / P_{supply} = \frac{417.46}{457.519} = 0.912$$

$R_s = 0.982\,k\Omega$

$$P_L = \frac{V_{L\,rms}^2}{R_L} = \frac{(352.66\,mV)^2}{990\Omega} = 125.63\,uW$$

$$I = \frac{V_{L\,rms}}{R_L} = \frac{352.66\,mV}{990\Omega} = 0.356\,mA$$

$$P_{supply} = V_{supply} \cdot I = 706.57\,mV \times 0.356\,mA = 251.54\,uW$$

$$PTE: P_L / P_{supply} = \frac{125.63\,uW}{251.54\,uW} = 0.499$$

$R_s = 9.92\,k\Omega$

$$P_L = \frac{V_{L\,rms}^2}{R_L} = \frac{(64.14\,mV)^2}{990\Omega} = 4.1555\,uW$$

$$I = \frac{V_{L\,rms}}{R_L} = \frac{64.14\,mV}{990\Omega} = 0.0648\,mA$$

$$P_{supply} = V_{supply} \cdot I = 707.83\,mV \times 0.0648\,mA = 45.87\,uW$$

$$PTE: P_L / P_{supply} = \frac{4.1555\,uW}{45.87\,uW} = 0.091$$

Discussion:

The measured and theoretical values of the load and source power are less than 5% error. The experiment indeed shows the power transfer for the load resistor and the voltage source. However, the %error still exist between 1% for the measured values because of the tolerance of resistors and the uncertainty of measured device such as the MyDAQ. The uncertainty of oscilloscope is 0.5% which also produces the error when measuring the voltage from the circuit.

Part#1:

a. The 98Ω source resistor gives the maximum power with P_L = 421.07µW.
b. The 98Ω source resistor gives the maximum power transfer efficiency with PE=0.912
c. The result is that as the source resistor is getting smaller, the power transfer is bigger.
d. Base on the data in the experiment, when the source resistor is zero, the power efficiency would be 100%. But in reality, it's impossible to allow the source resistor to be zero Ohm because the voltage source needs a source resistor to protect itself. Therefore, the source resistor would decrease the power transfer to the network. Also, some power would be lost and becomes the heat through the resistor

Part#2:

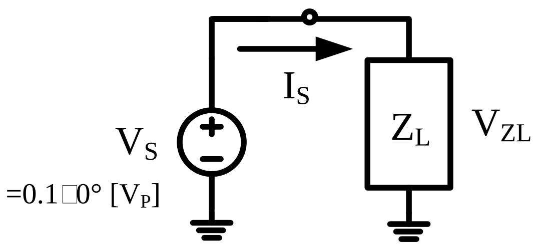
a. The 0.982kΩ load resistor, R_L gives the maximum power with P_L = 127.92µW.
b. The 9.92kΩ load resistor, R_L gives the maximum power efficiency with PE = 0.908
c. When the R_L is equal to Rs, the voltage source provides the maximum power to the load. The power transfer efficiency is 50%.
d. In order to get the maximum power transfer to the load, it's better to determine the load resistance is equal to the source resistance. But in reality, every device has its own load resistance. The best way to do is try to make the PE is equal or close to 50%, which means that the load resistance equals to the source resistance, and the power of load would be max.

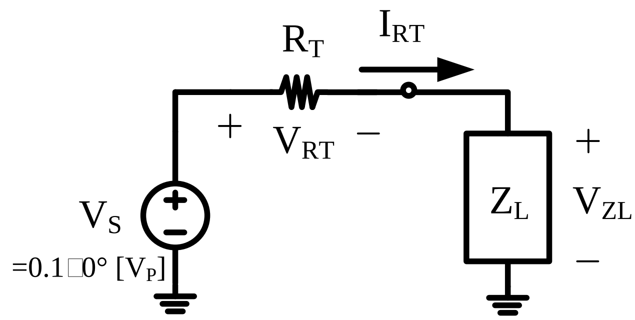
Conclusion:

In this experiment, if the load resistance is constant, the circuit network would get the maximum power transfer if the source resistance is smallest. However, if the source resistance is constant, in order to get the maximum power transfer to the network, the load resistance is equal to the source resistance. The power efficiency is about 50%. It proved that in reality, the power efficiency should 50% instead of approaching to 100%. Every voltage source has its own source resistance, it cannot transfer 100% power to the circuit network.

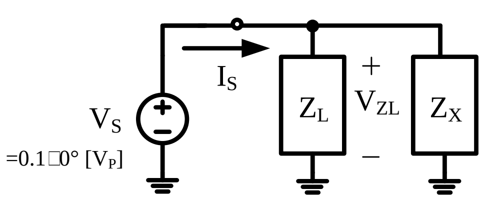
Part#3: Power Factor adjustment circuit design with an unknown load

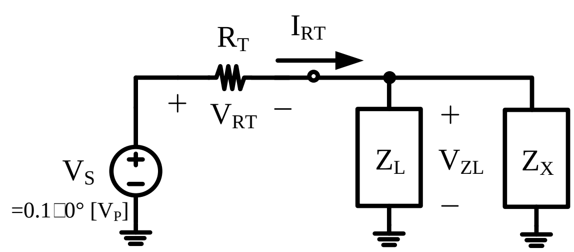
Procedure:





1. Build a circuit which is including a voltage source, a resistor R_T with resistance 10kΩ and a black box which includes a resistor 1000Ω and an inductor.
2. Using function generator to generate a voltage source. Set the frequency to be 5kHz, the amplitude to be 0.2V and the DC offset to be 0.
3. Using oscilloscope to measure the voltage of source and R_T, record the Vp-p and phased difference between Vs and V_{RT}. Vp-p/2 is the amplitude of $V_{RT}(t)$ and Vs(t).
4. Calculate the current I_{RT}.
5. Use (Vs - V_{RT}) to get V_L.
6. Calculate the Z_L (V_L/I_{RT}).
7. Calculate the current Is. (Is = Vs(t)/Z_L)
8. Calculate the power factor of ZL.





9. Add a power factor adjustment component (such as 10nF, 5nF capacitor) to the Z_L in parallel. The 5nF capacitor could be combine with two 10nF capacitor in series.
10. Repeat step 3 to step 7 to get the new Is, compare the new Is to the previous Is. Check the phased difference.

Data:

Power Factor adjustment circuit design with an unknown load

Frequency: 5kHz

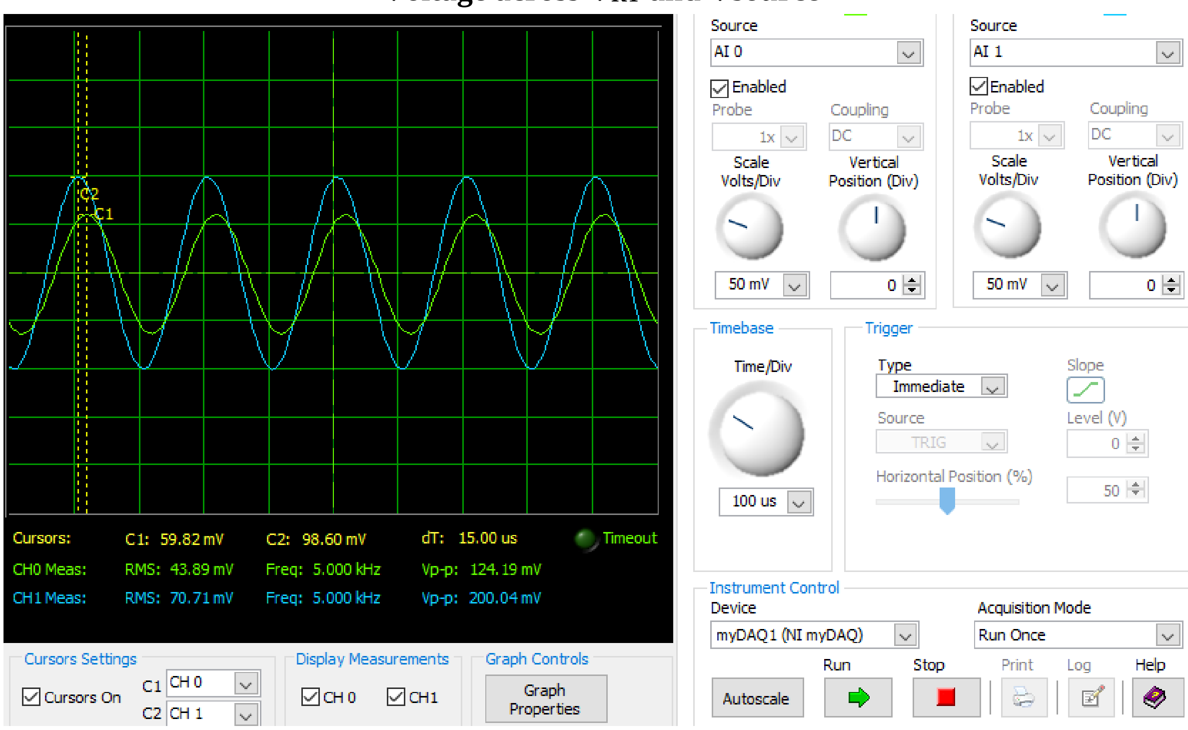
The measured values for the resistor (R_T) and black box elements (5000Ω resistor in series with 150mH inductor):

R_T	R_b(5000Ω)	R_L (inductor)	R_L(black box) R_b+R_L	X (black box) X = jwL	L
9.95kΩ	5020Ω	250Ω	5270Ω	4712Ω	150mH

Measured voltage from oscilloscope reading before adding a power factor adjustment

Δ t (second)	15us
Phased difference (V_{RT} and Vs)	-27° (V_{RT} is shifted to the left)
Vp-p(source)	200.04mV
Vp-p(R_T)	124.19mV

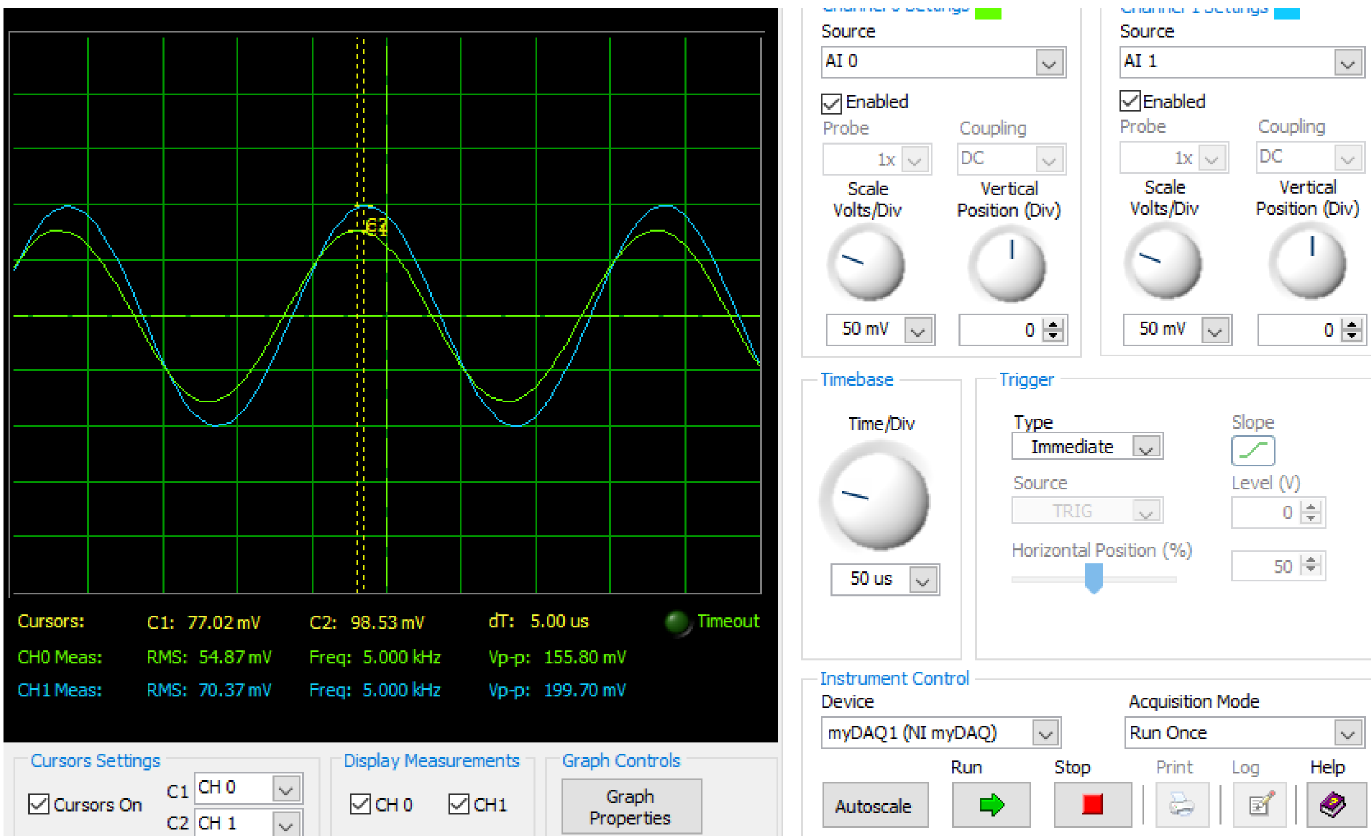
Voltage across V_{RT} and Vsource



Measured voltage from oscilloscope reading after adding a power factor adjustment (10nF)

Δ t (second)	5us
Phased difference (V_{RT} and Vs)	9° (V_{RT} is shifted to the left)
Vp-p(source)	199.70mV
Vp-p(R_T)	155.80mV

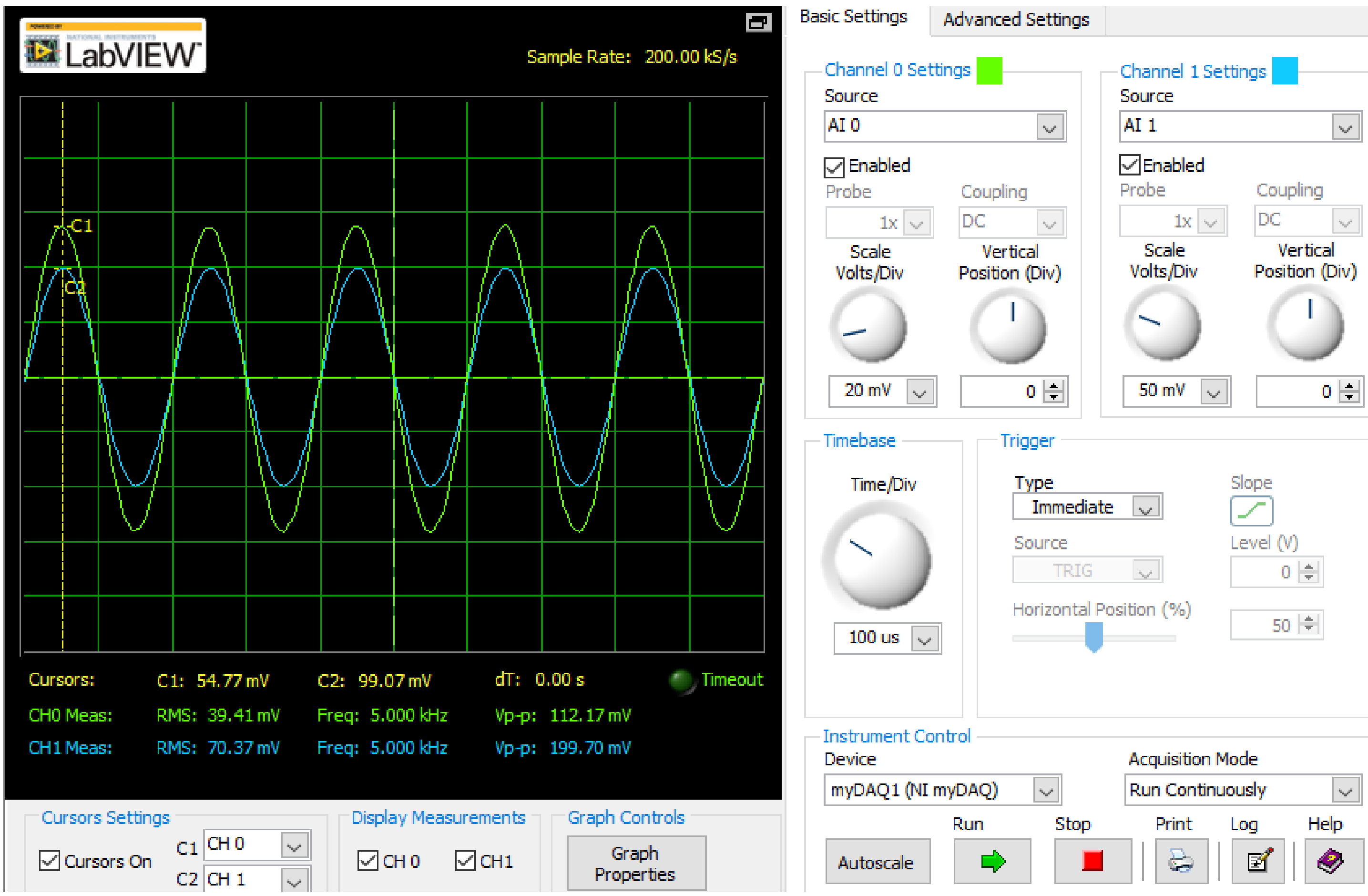
Voltage across V_{RT} and Vsource



Measured voltage from oscilloscope reading after adding a power factor adjustment (5nF)

Δ t (second)	0us
Phased difference (V_{RT} and Vs)	0° (V_{RT} is shifted to the left)
Vp-p(source)	199.70mV
Vp-p(R_T)	112.17mV

Voltage across V_{RT} and Vsource



Data Analysis:

The formulas used to analyze the data are below:

$V = Vp\text{-}p/2\angle\alpha$ α (phase shift) $= 360°\cdot\Delta t\cdot f$ $I_{RT} = \frac{V}{RT}$

$V_L = V_{source} - V_{RT}$ $Z_L = \frac{VL}{IRT}$ $Is = \frac{Vs}{ZL}$ $Zt = \frac{Z1\cdot Z2}{Z1+Z2}$

Power factor = $\cos \Omega_{ZL}$

$$\%\ Error = \left|\frac{Y_{experiment} - Y_{theory}}{Y_{theory}}\right|$$

In order to get the phase shift, using the cursors to measure the dt with V_{RT} and V_{source} from oscilloscope. Then get the voltage of R_T with the phase shift and use this voltage to calculate the current I_{RT}. The voltage source minus V_{RT} to get the voltage across the black box. Finally, with the V_L and I_{RT}, it can get the load impedance for the black box which is the Z_L. The current Is can also be found through the formula ($Z_L = \frac{VL}{IRT}$) after getting the value of Z_L.

After the calculation, the result for the power factor adjustment component is a capacitor with 4.38nF capacitance. Therefore, using two 10nF capacitor in series can produce a about 5nF capacitor. Redo the calculation and get the ZL as well as Is. The calculation for the power factor adjustment component is in discussion b).

Before adding a component:

	RL	**XL**	**Power factor**
Theoretical value	5270 Ω	4712Ω	0.745
Measured value	4327.14 Ω	7274.32 Ω	0.511
% Error	17.89%	54.39%	31.41%

After adding a component: (adding a 10nF capacitor in parallel)

	RL	**XL**	**Power factor**
Theoretical value	1779	-3701.19	0.433
Measured value	2646.65	-1995.11	0.799
% Error	48.77%	46.10%	84.53%

After adding a component: (adding a 5nF capacitor in parallel (two 10nF capacitor in series))

	RL	**XL**	**Power factor**
Theoretical value	1779	-3701.19	0.861
Measured value	7787.2	0	1
% Error	48.77%	46.10%	16.14%

An example of calculation for ZL, Is before and after adding a 10nF capacitor.

The step to calculate the ZL after adding the 5nF capacitor and the thermotical ZL is similar.

$R_b = 5020\Omega$ $\Delta t = 15us.$ $\theta_{RT} = 360° \times 15\times10^{-6} \times 5000 = 27°$

$V_{RT} = 0.062095\angle -27°$ $V_s = 0.1\angle 0°$

$I_{RT} = \frac{V_{RT}}{R_T} = \frac{0.062095\angle -27°}{9950} = 0.006241\ mA\ \angle -27°$

$V_L = V_s - V_{RT} = 0.1\angle 0° - 0.062095\angle -27°$

$V_L = 0.0528\angle 32.25°$

$Z_L = \frac{V_L}{I_{RT}} = \frac{0.0528\angle 32.25°}{0.006241\times10^{-3}\angle -27°} = 8464.03\angle 59.25°$

$Z_L = 4327.14 + 7274.32j$ $R_L = 4327.14$

$X_L = 7274.32$

$I_s = \frac{V_s}{Z_L} = \frac{0.1}{8464.03\angle 59.25}$

$= 0.011815mA\angle -59.25°$ Power factor $|\cos 59.25°| = \boxed{0.511}$

After adding a capacitor

$\Delta t = 5\ us$ $\theta = 360° \times 5\times10^{-6} \times 5000 = 9°$

$V_{RT} = 0.0779\angle 9°$ $V_s = 0.09985\angle 0°$

$I_{RT} = \frac{V_{RT}}{R_T} = 0.007829\ mA\angle 9°$ $I_s = \frac{V_s}{Z_L} = \frac{0.09985\angle 0°}{3314.4\angle -37.01°}$

$V_L = V_s - V_{RT} = 0.0259\angle -28°$ $= 0.0301mA\angle 37.01°$

$Z_L = \frac{V_L}{I_{RT}} = \frac{0.0259\angle -28°}{0.007829\times10^{-3}\angle 9°} = 3314.4\angle -37.01°$ The power factor is getting bigger

$Z_L = 2646.65 - 1995.11j$ $|\cos(-37.01°)| = \boxed{0.799}$

Calculation for the theoretical data for RL and XL after adding the 10nF and 5Nf capacitor

Theoretical value.

$V_s = 0.1\angle 0^\circ$

$Z_L = 5250 + 4712j$

$I_s = \frac{V_s}{Z_L} = \frac{0.1}{5250+4712j} = 0.0142\angle -41.90$

power factor $|\cos(-41.90^\circ)| = 0.744$.

$V_s = 0.1\angle 0^\circ$ $\quad \boxed{C = 10nF} \quad Z_c = \frac{1}{j\omega C} = -\frac{j}{\omega \cdot C} = -\frac{j}{2\pi \times 5000 \times 10\times10^{-9}}$

$= -3183.1j$

$Z_{LC} = Z_L \| Z_C = \frac{Z_L \cdot Z_L}{Z_L + Z_C} = 4106.56\angle -64.33^\circ$

$I_s = \frac{V_s}{Z_{LC}} = \frac{0.1\angle 0^\circ}{4106.56\angle -64.33^\circ} = \boxed{0.0244\angle 64.33^\circ}$

power factor $\cos(64.33^\circ)$

$= 0.433$

If C is 5nF.

$Z_c = -\frac{j}{\omega C} = -\frac{j}{2\pi \times 5000 \times 5\times10^{-9}} = -6366.2j$

$Z_{LC} = Z_L \| Z_C = 8185.89\angle -30.6^\circ$

power factor $\cos(30.6^\circ)$

$= 0.861$

$I_s = \frac{V_s}{Z_{LC}} = \boxed{0.0122\angle 30.6^\circ}$

Discussion:

Base on the measured data, the RL and XL have a big percent error compared to the theoretical data with 17.58% and 54.39%. One of the reasons is when measuring the phase shift from the oscilloscope, the minimum distance that the cursors can move is 5us. For example, from the data, the phase shift from V_{RT} and Vs is 15us. In fact, the more accurate phase shift might be 11 us or 12us. Even though the difference between 15 and 12 us is very small, 1us could make the calculation for the Z_L a huge difference compared to the theoretical data. The MyDAQ oscilloscope could not provide an accurate phase shift measurement, although some other reasons such as the tolerance of resistors uncertainty of oscilloscope also produce the error, the phase shift measurement plays a big rule. Because of the error for ZL, the power factor also has a significant error with 31.31%.

After adding the capacitor components, the phase shift is getting small and the power factor approaches to 1. It matches the goal of this experiment. From the discussion b), as the ZL adding the capacitor components, it could be able to eliminate the imaginary part. Therefore, the ZL becomes purely resistive, and its power factor is 1. For the theoretical data, since it uses the 10nF and 5nF capacitor to calculate the new ZL with the theoretical RL and XL, the theoretical power factor would have value different from the experiment value.

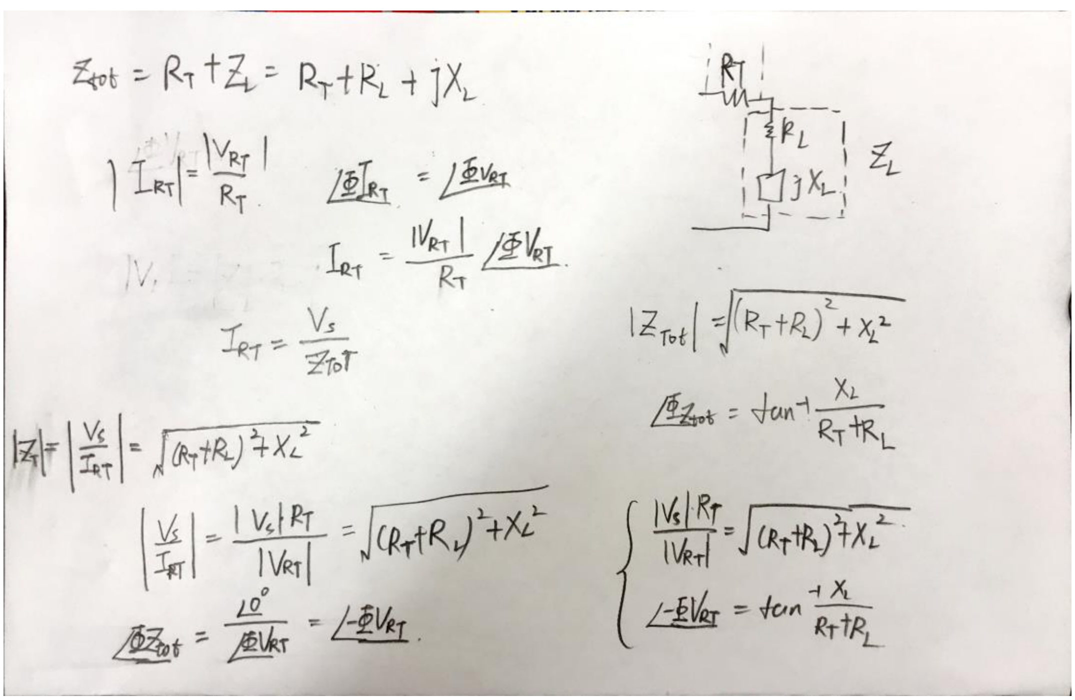
A trick thing happened to the theoretical data is using the discussion b) result to calculate the power factor adjustment component. Xx = $-X_L$. The imaginary part still exists.

$$Z_{CL}= (Zc*Z_L)/(Zc+Z_L) = (-X_Lj)*(R_L+X_Lj)/(-X_Lj + R_L + X_Lj)$$

$$Z_{CL}=\frac{Zc*ZL}{Zc+ZL}=\frac{(-XLj)*(RL+XLj)}{(-XLj + RL + XLj)}=\frac{-XLRLj+XL^2}{RL}$$

Discussion question:

a),



b),

$$\mathrm{Im}(Y_L + Y_x) = 0$$

$$Y_L + Y_x = \frac{1}{R_L + jX_L} + \frac{1}{R_x + jX_x} \qquad \frac{1}{R_L + jX_L} + \frac{1}{jX_x}$$

$$= \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{R_x - jX_x}{R_x^2 + X_x^2}$$

$$= \frac{R_L}{R_L^2 + X_L^2} - \frac{jX_L}{R_L^2 + X_L^2} + \frac{R_x}{R_x^2 + X_x^2} - \frac{jX_x}{R_x^2 + X_x^2}$$

$-X_L - X_x = 0$ Therefore the X_x component needs to be a capacitor

$-X_L = X_x$. $X_L = -X_x$

X_x could be a capacitor from my data, X_L is 7274.32.

$$X_x = \frac{1}{\omega C} = 7274.32 \qquad \boxed{Z_x = jX_x = -jX_L}$$

$$\boxed{C = 4.38 \times 10^{-9}\ F}$$

Therefore Z_x is a capacitor.

Conclusions: