EC ENGR 111L

Experiment #2

Operational Amplifiers

Name: Ma, Yunhui

UID: 505-179-815

Lab Section: 1E

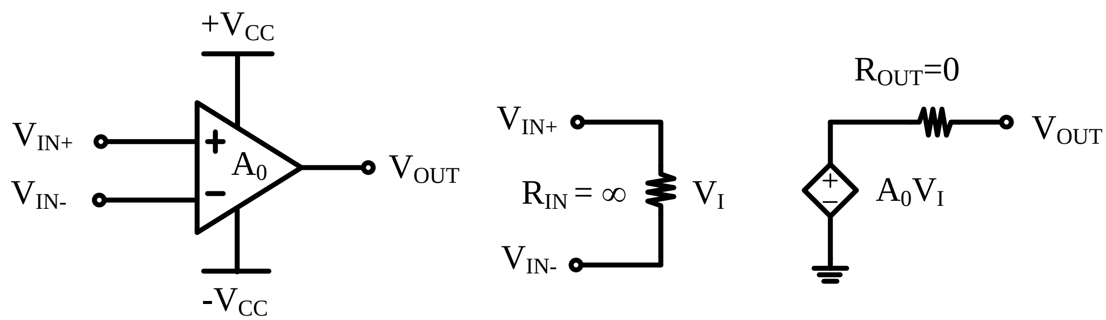
Date:5/3/2019

Objectives:

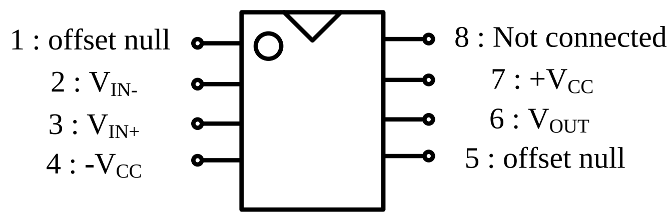
The purpose of this lab is to understand and use the operational amplifier.

Theory:

The ideal Op-Amp should have a very high gain as well as a very small output impedance.



The Op-amp we use in this experiment is the 741 op-amp. Its description is below.



$$\% \, Error = \left| \frac{Y_{experiment} - Y_{theory}}{Y_{theory}} \right|$$

Part 1: Unity-Gain buffer design – I

Objectives:

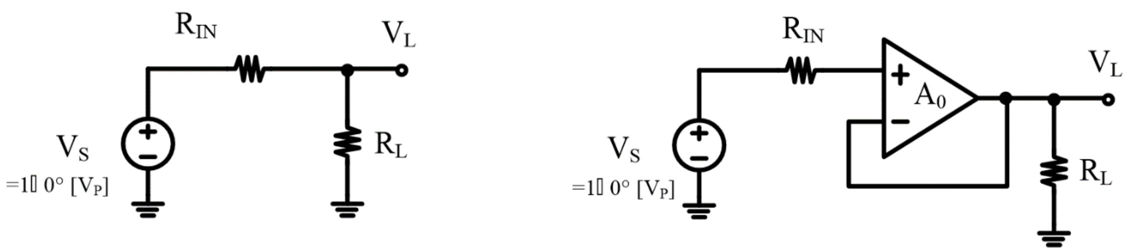
The purpose of part 1 is to design a voltage divider using an operational amplifier as a unity-gain buffer.

Theory:

Part a circuit is a voltage divider. $V_L = Vs*(R_L/(R_L+R_{IN}))$.

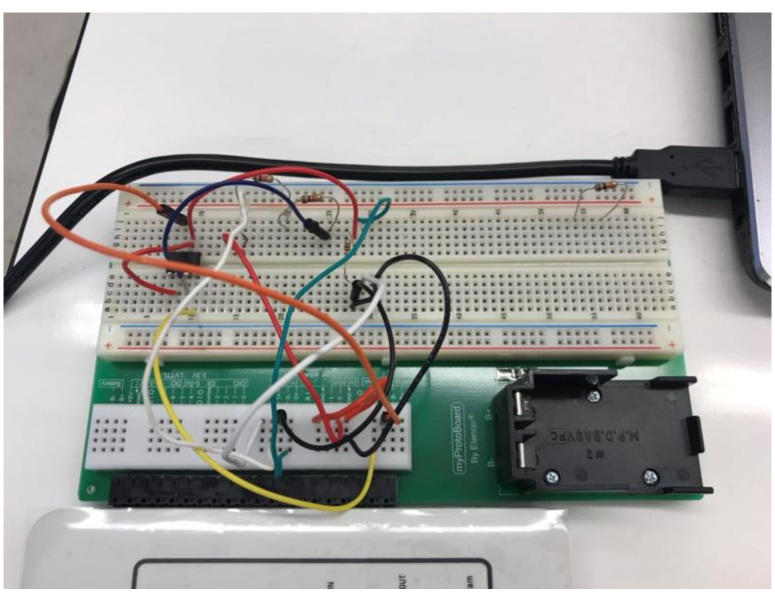
Part b uses an Op-Amp as a unity-gain buffer. The op-amp is a voltage-controlled voltage source (amplifier) with a very high gain. It has a very large input impedance (ideally infinite) and very small output impedance (ideally zero). (ECE 111L lab manual). The voltage at the V- and V+ input is approximately equal. Due to the large input impedance, the current entering both inputs are about 0A. Therefore, the Vout (V_L) is the same as the Vs. $V_L/Vs = 1$.

Procedure:



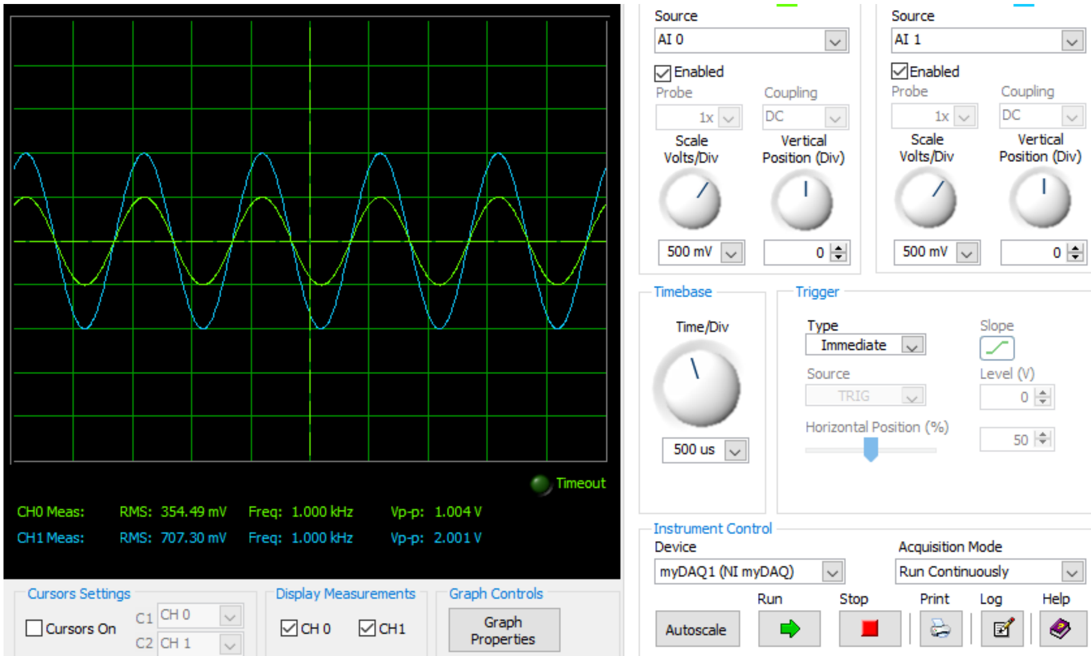
Voltage dividers: (a) passive alone, (b) with unity-gain buffer.

1. Implement the passive along circuit shown as above (a). (Rin = 2K, RL = 2K)
2. Using function generator to generate the AC voltage source, and set the amplitude to be 2, the frequency to be 1kHz, DC offset to be 0.
3. Using oscilloscope to output waveform $V_L(t)$ and Vs(t);
4. Implement the unity-gain buffer circuit shown as above (b). (Rin = 2K, RL = 2K)
5. Repeat step 2 and step 3.

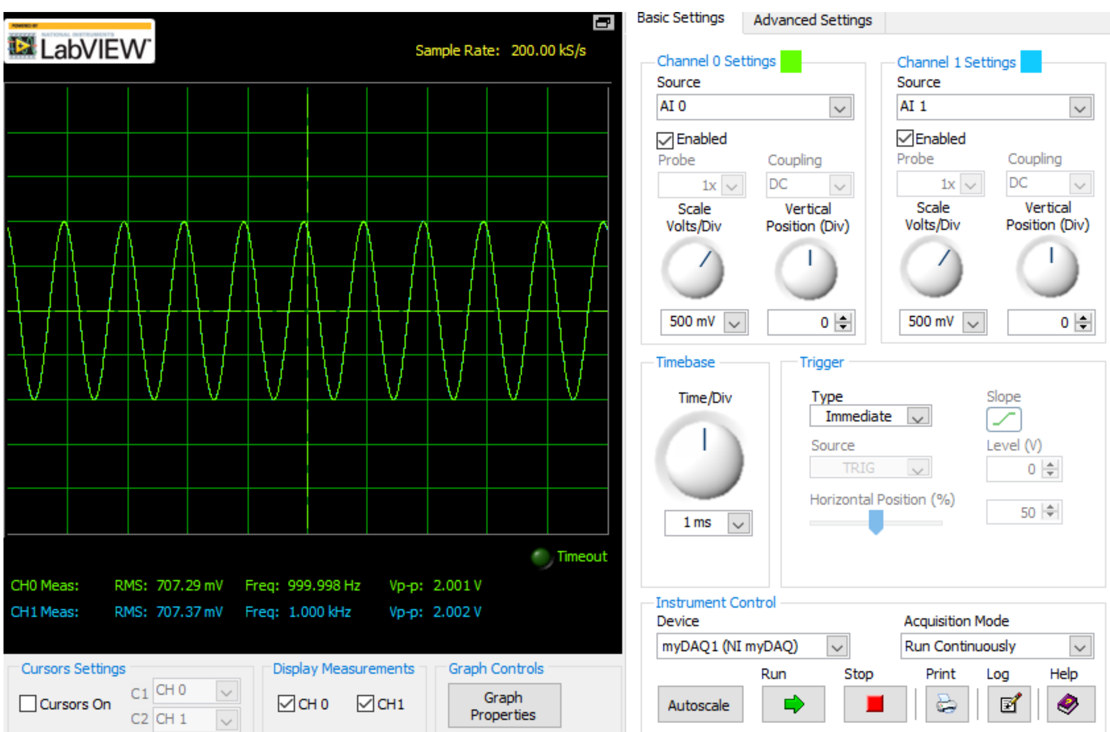


Data:

Part a:



Part b:



Data Analysis:

	V_L	V_S
Part a	354.49mV	707.30mV
Part b	707.29mV	707.37mV

	Theoretical V_L/V_S	Experimental V_L/V_S	Error %
Part a	0.499	0.50	0.20
Part b	1	0.9999	0.01

Part a calculation:

Theoretical: $V_L = Vs*(R_L/(R_L+R_{IN}))$, $V_L/Vs = 0.5$, $V_L = 707.29mV * 0.5 = 353.645mV$

$V_L/V_S = 353.645mV/707.30mV = 0.499$

Part b calculation:

Theoretical: $V_L = Vs = 707.37mV$, $V_L/Vs = 1$.

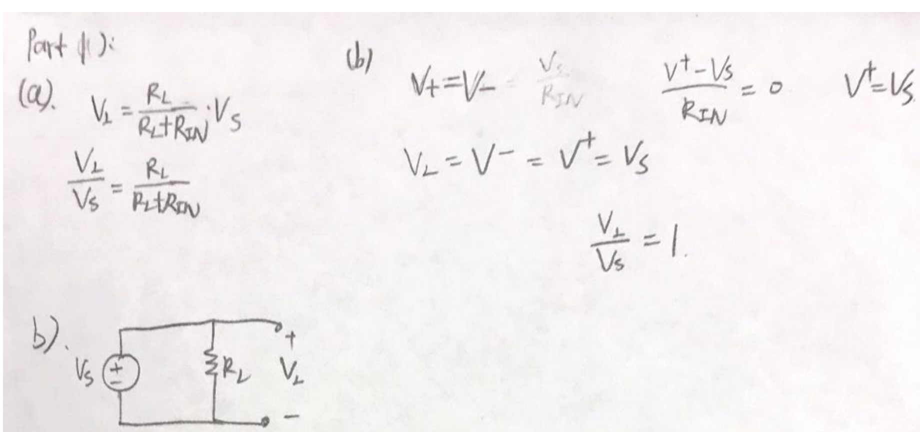
Error %:

Error % = 100*(0.50-0.499)/0.50 = 0.20%

Discussion:

From the data analysis, the percent error is very small, the experimental value for part a and part b verifies the theoretical value. In the part a, the voltage is divider into a half of the Voltage source by the two same resistors in series in circuit. The transfer function $V_L/Vs = 0.5$. In the part b, the V_L/V_S is about equal to 1 due to the Op-Amp. The Op-Amp acts as a unity-gain buffer, and it prevents the current flows from the source to the load. Therefore, it prevents the voltage divider, the gain is 1.

a), b):



Part 2: Unity-Gain buffer design – II

Objectives:

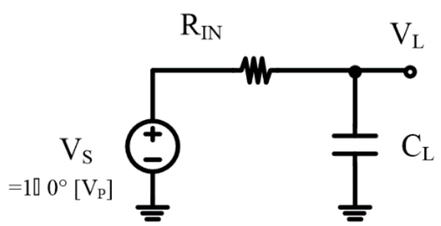
The purpose of part 2 is to design a circuit using an operational amplifier as a unity-gain buffer that matches the behavior of a RC series network which is a low pass filter.

Theory:

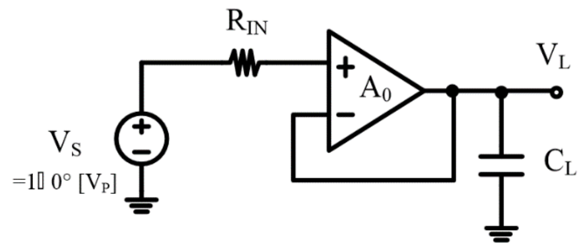
The transfer function for part a is:

$$\frac{V_{L(s)}}{V_{IN(s)}} = \frac{\frac{1}{C_L s}}{R_{IN} + \frac{1}{C_L s}}$$

Procedure:

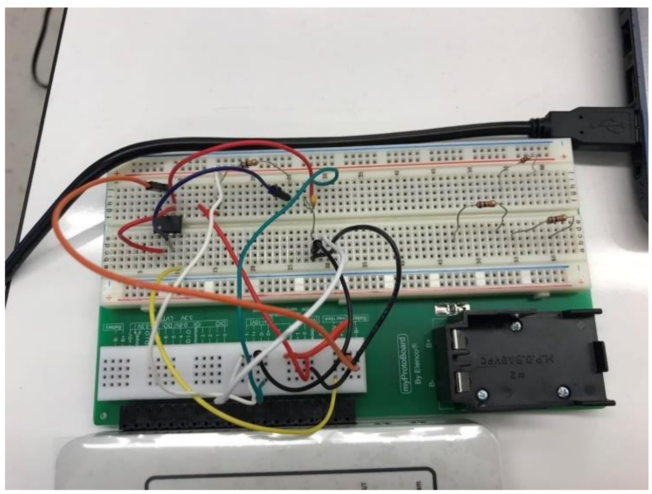


(a) passive alone,



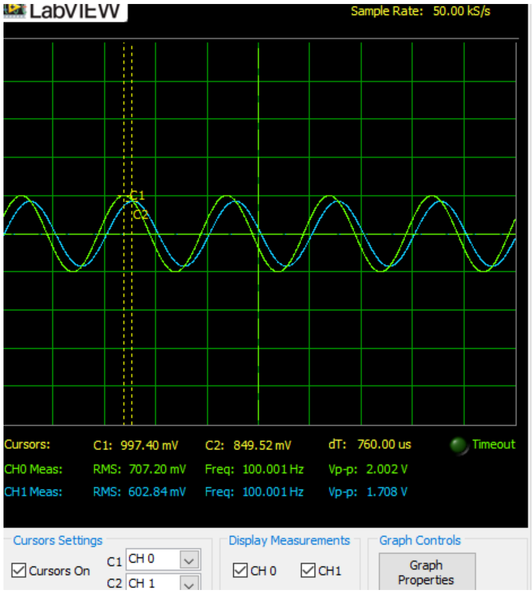
(b) with unity-gain buffer.

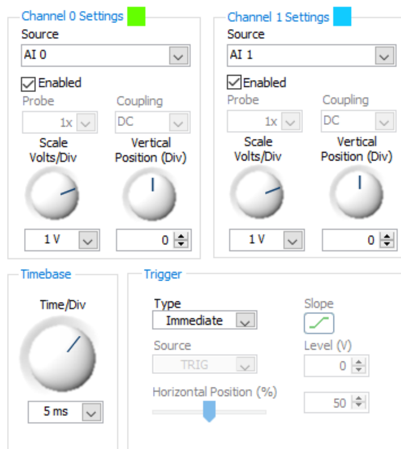
1. Implement the passive along circuit shown as above (a). (Rin = 10Kohm, CL = 100nF)
2. Using function generator to generate the AC voltage source, and set the amplitude to be 2, the frequency to be 100Hz, DC offset to be 0.
3. Using oscilloscope to output waveform $V_L(t)$ and Vs(t);
4. Implement the unity-gain buffer circuit shown as above (b). (Rin = 10Kohm, CL = 100nF)
5. Repeat step 2 and step 3
6. Draw the Bode plot for both cases part a and part b.



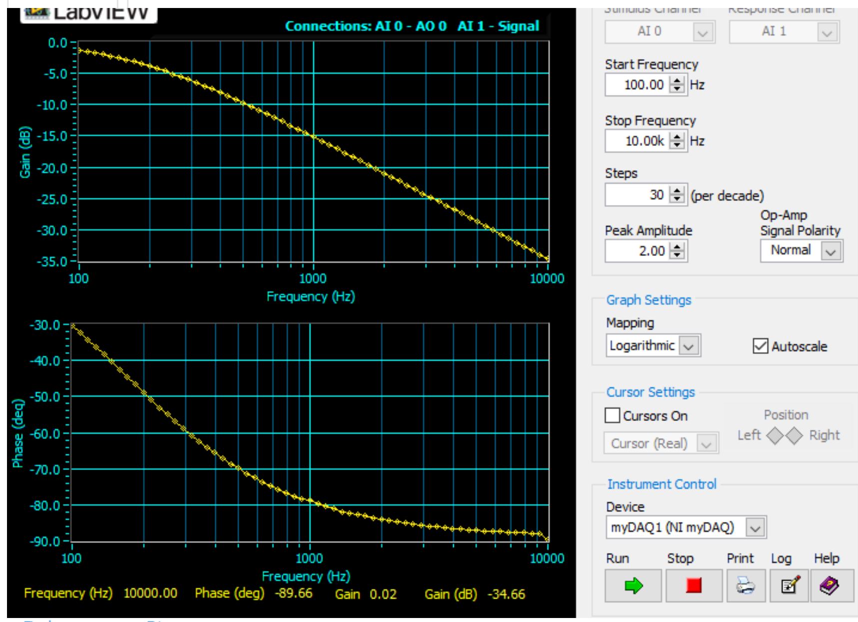
Data:

Case a: 100Hz: output waveform

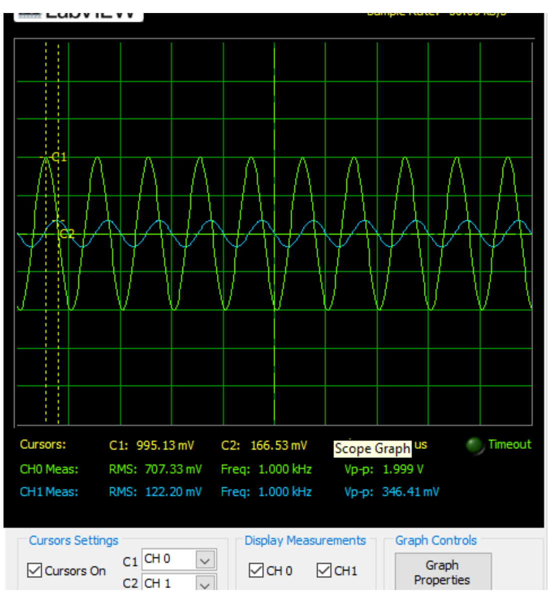


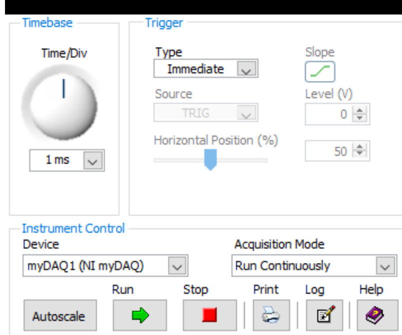


Bode plot part a:

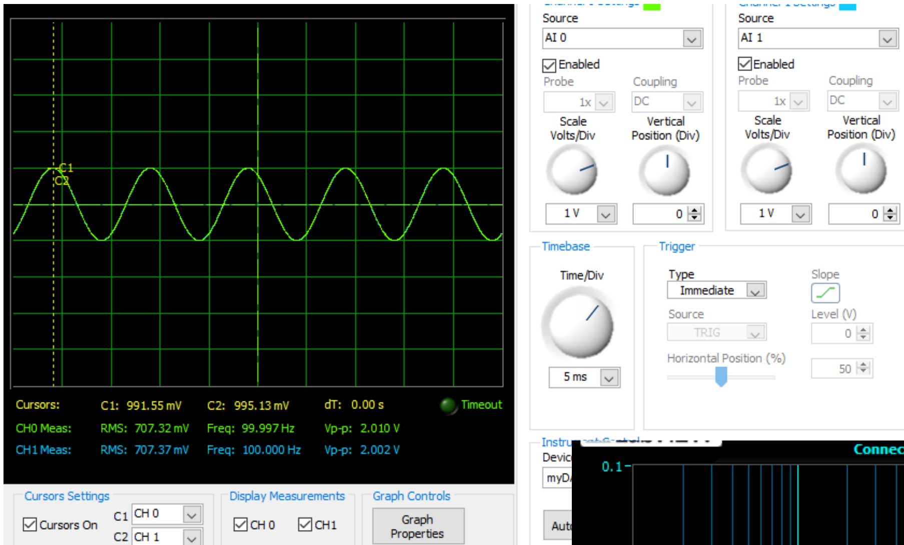


1000Hz: output waveform

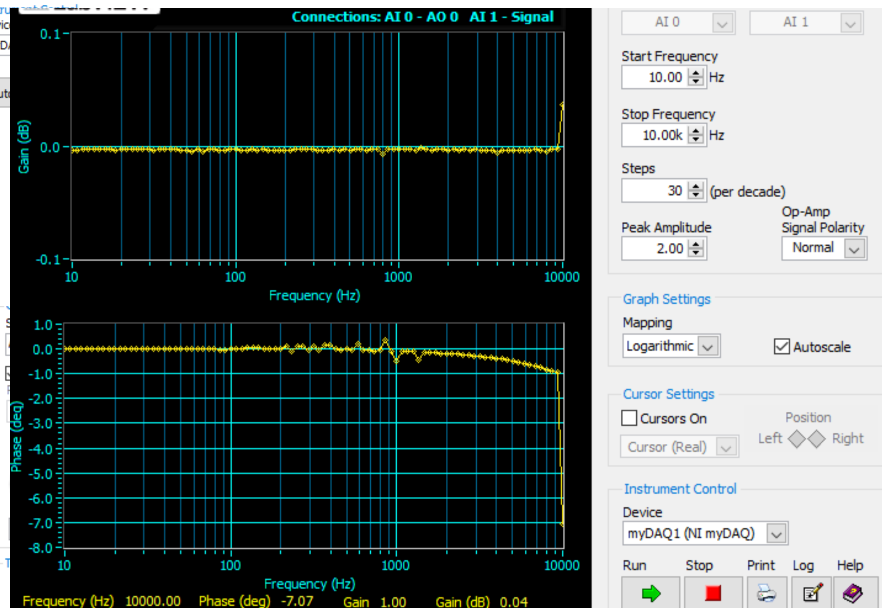




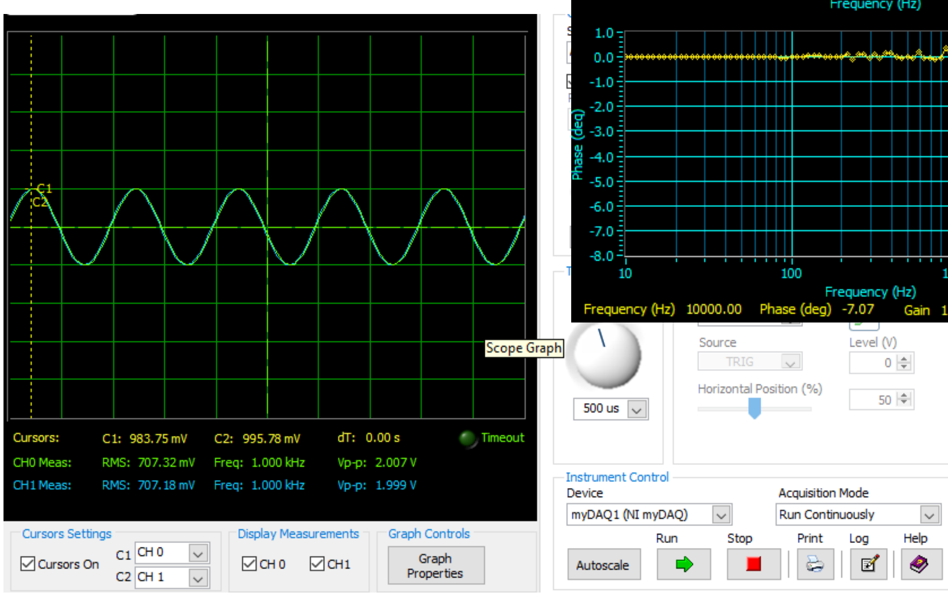
Case b: 100Hz: output waveform



Bode plot part b:



1000Hz: output waveform



Data Analysis:

	V_L	Vs
Part a (100Hz)	602.84mV	707.20mV
Part a (1k Hz)	122.20mV	707.33mV
Part b (100Hz)	707.37mV	707.32mV
Part b (1k Hz)	707.18mV	707.32mV

	Theoretical V_L/V_S	Experimental V_L/V_S	Error %
Part a (100 Hz)	0.8467	0.8524	0.673%
Part a (1k Hz)	0.157	0.1728	10.06%
Part b (100 Hz)	1	0.9999	0.01%
Part b (1 kHz)	1	0.9998	0.02%

Calculation:

Theoretical V_L/V_s.

$$\frac{V_L}{V_{in}} = \frac{\frac{1}{Cj\omega}}{\frac{1}{Cj\omega}+R} = \frac{1}{1+j\omega RC}$$

$$= \frac{1}{1+j(2\pi\times100\times10000\times10^{-7})} = \frac{1}{1+j(0.2\pi)}$$

$$= 0.8467\angle{-32^\circ}$$

$$\boxed{\left|\frac{V_L}{V_{in}}\right|_{100Hz} = 0.8467.} \qquad \boxed{\left|\frac{V_L}{V_{in}}\right|_{1000Hz} = 0.157.}$$

Experimental

Part a (100Hz) $\frac{V_L}{V_s} = \frac{602.84mV}{707.20mV} = 0.8524.$

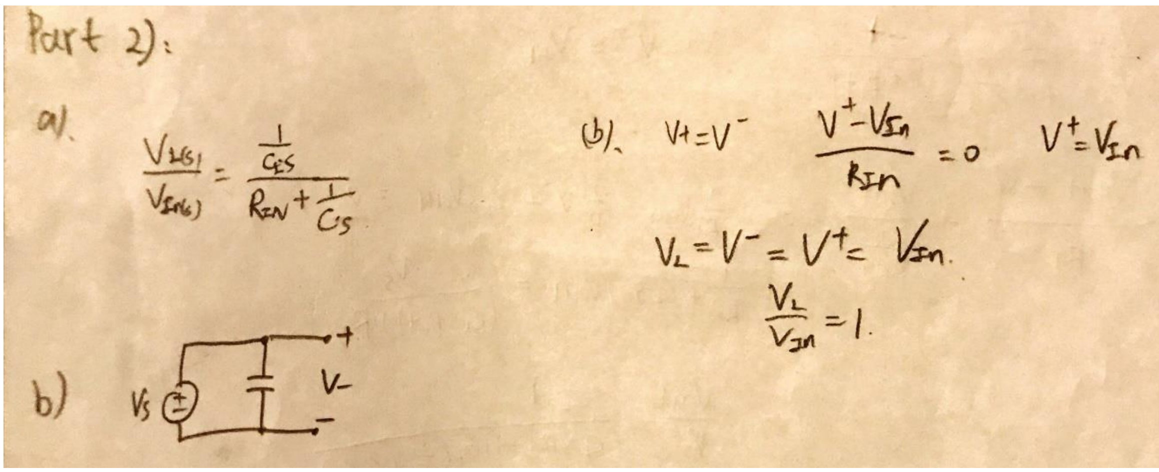
$$\text{Error\%} = \frac{0.8524-0.8467}{0.8467}\times100\% = 0.673\%$$

Discussion:

The percent error is very small except for part a) circuit at 1kHz frequency because the value of this transfer function (V_L/Vs) is very small which is about 0.157. The Experimental value is 0.1728 which is also very close to the theoretical value. Also, the myDAQ has limitations to measure the small-scale value. Therefore, it makes this percent error bigger. The low pass filter circuit network would filter out the high frequency voltage. We can see that the V_L for part a) at 1kHz is much smaller than part a) 100Hz. After adding the unity-gain buffer, the behavior of low pass filter is broken. The V_L is the same as Vs. Therefore, adding the unity-gain buffer would not have the same behavior of low pass filter. The transfer functions V_L/V_{IN} are different.

a), They are not the same.

b),



Part 3: 2nd-Order LPF design

Objectives:

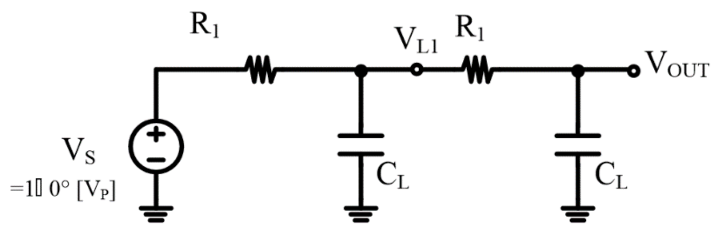
The purpose of part 3 is to design a second order low-pass filter using two cascaded first order passive filter (resistors and capacitors)

Theory:

The transfer function for two cascaded 1st-order passive filter is:

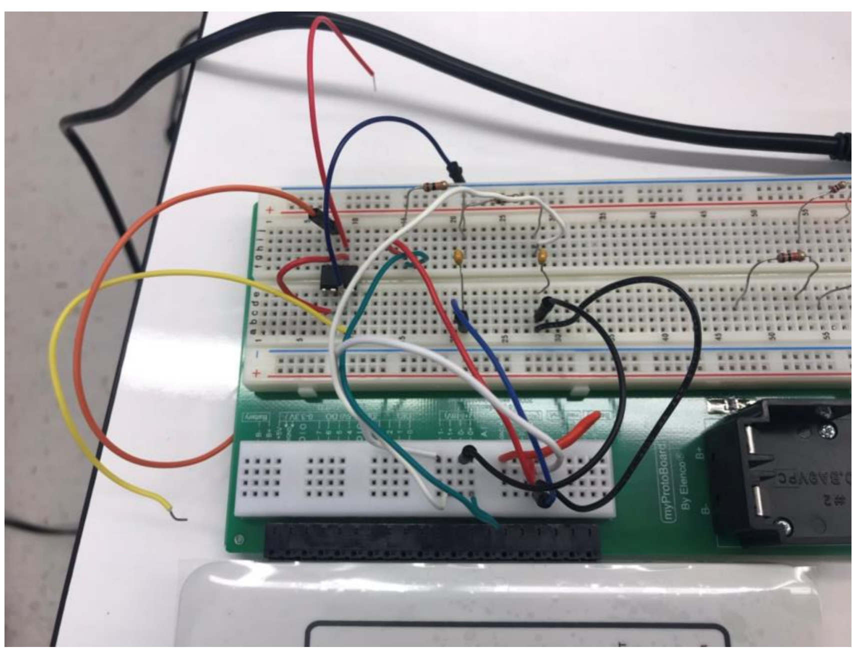
$$\frac{V_{OUT}}{V_S}(s) = H(s)^2 = \left[\frac{1}{1 + sR_1C_L}\right]^2$$

Procedure:



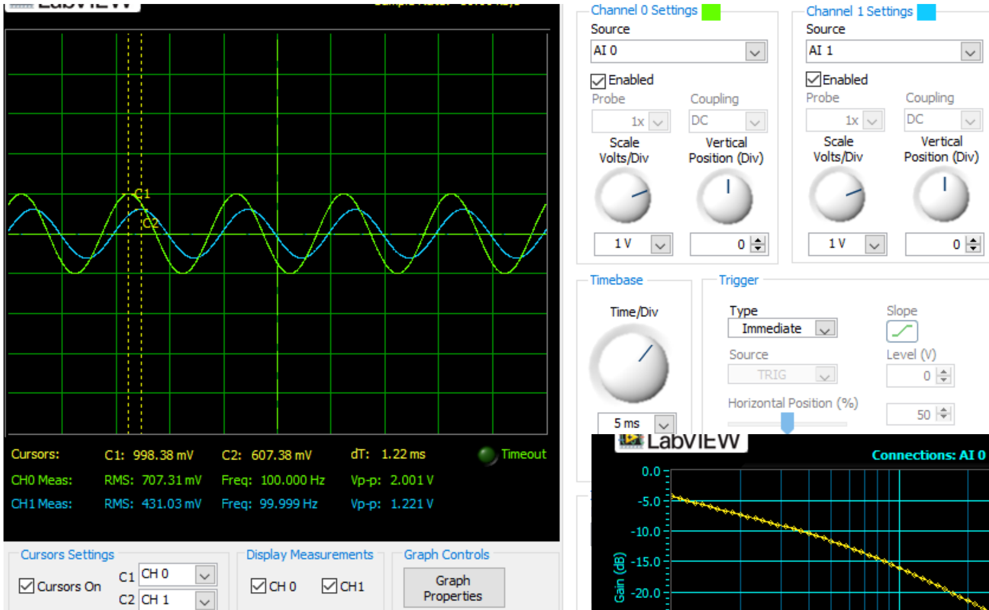
Passive 2nd-Order filter

1. Implement 2nd -order RC LPF (R1 = 10K, CL = 100n)
2. Using function generator to generate the AC voltage source, and set the amplitude to be 2, the frequency to be 100Hz and 1kHz, DC offset to be 0.
3. Using oscilloscope to output waveform $V_{L1}(t)$, Vout(t) and Vs(t);
4. Draw Bode plot for V_{L1} and V_{OUT} using ‘Bode’ in myDAQ.

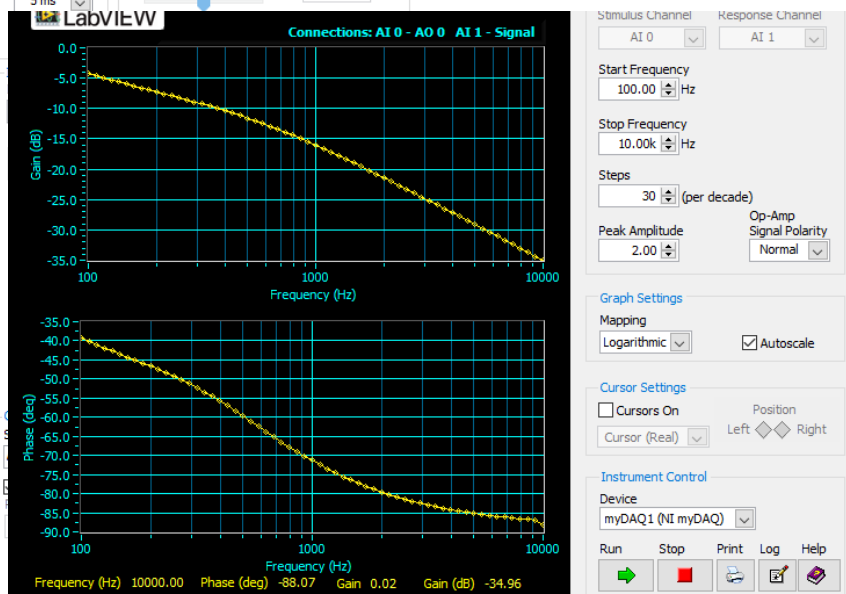


Data:

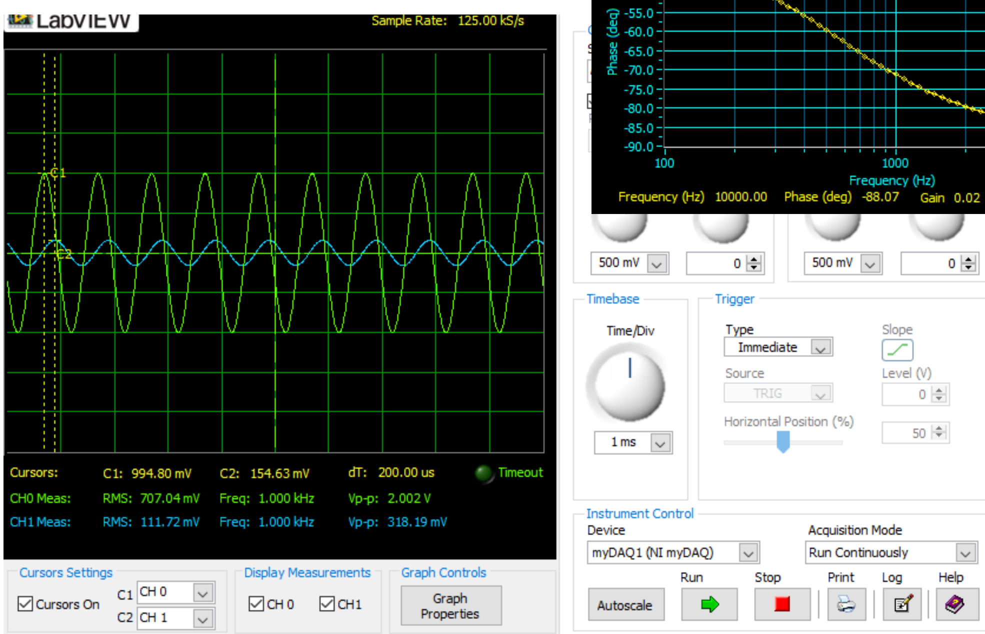
100Hz: VL1 vs Vs



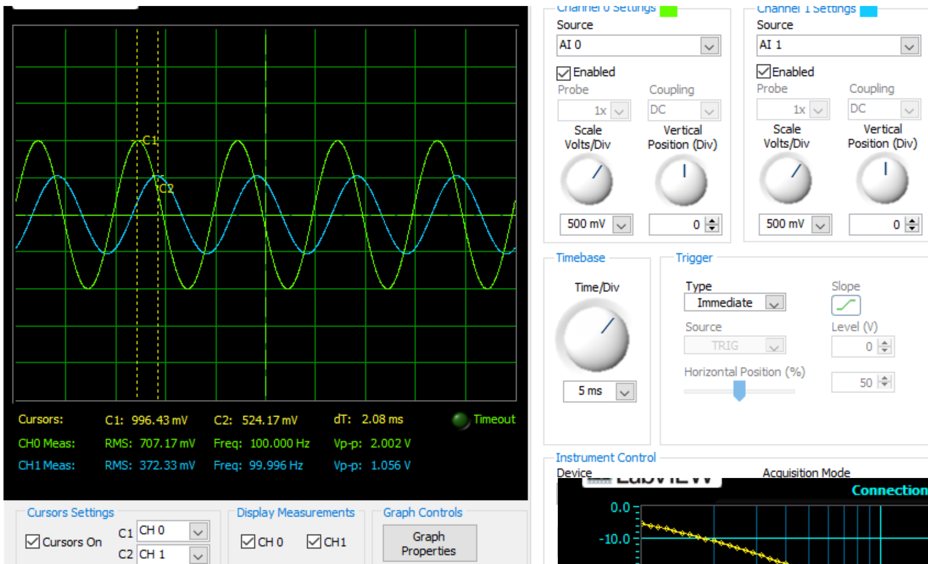
Bode plot for VL1



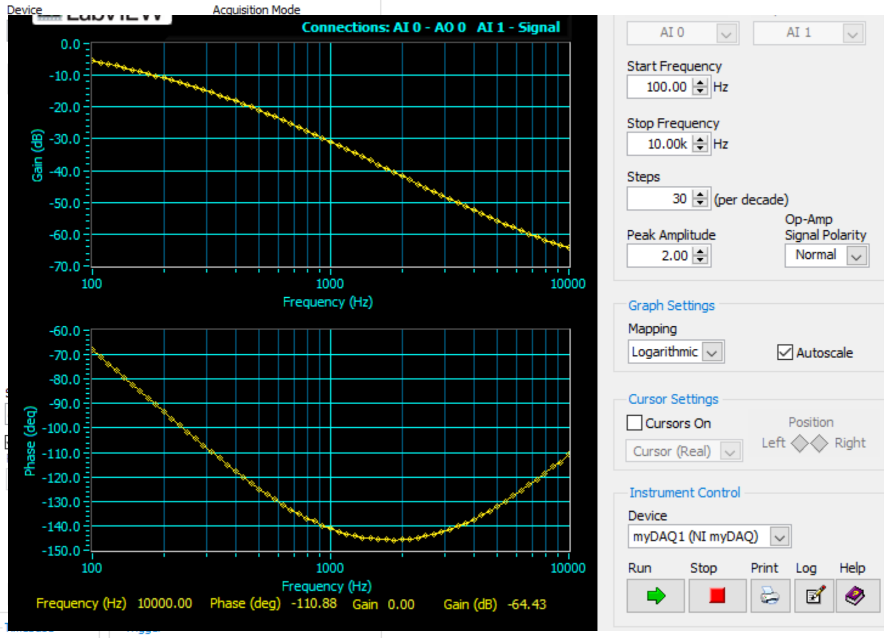
1000Hz: VL1 vs Vs



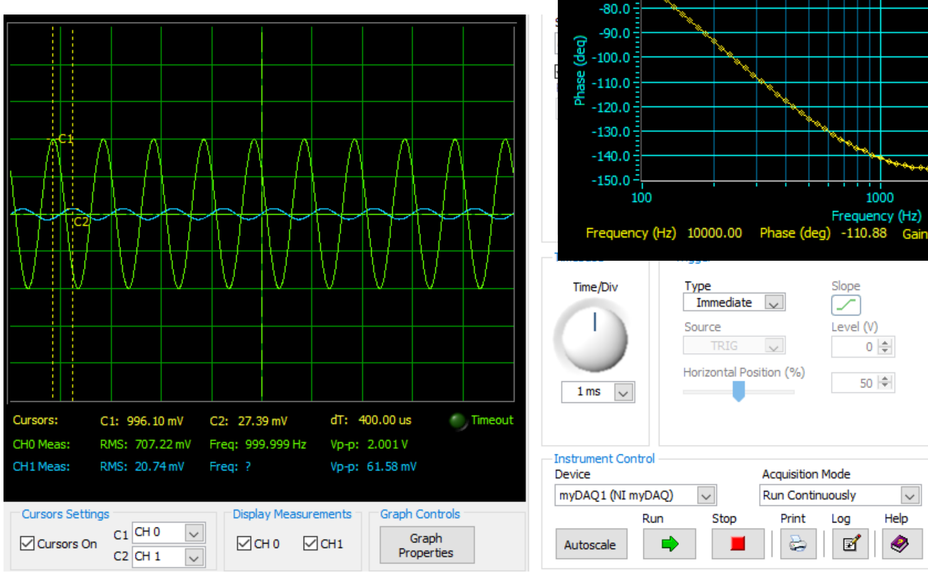
100Hz: Vout vs Vs



Bode plot Vout



1000Hz: Vout vs Vs



Data Analysis:

	Vout	**Vs**
100Hz	372.33mV	707.17mV
1k Hz	20.74mV	707.22mV

	V_L	**Vs**
100Hz	431.03mV	707.31mV
1k Hz	111.72mV	707.04mV

	Theoretical	**Experimental**	**Error %**
V_{out}/V_S, 100 Hz	0.505	0.5265	4.25%
V_{out}/V_S, 1 kHz	0.023	0.0293	27.39%

Calculation:

$$\frac{V_{out}(s)}{V_S(s)} = \frac{1}{R_1^2 C_1^2 (j\omega)^2 + 3R_1 C_1 (j\omega) + 1}$$

Theoretical: $$\frac{V_{out}}{V_S} = \frac{1}{10000^2 \times (10^{-7})^2 (j2\pi 100)^2 + 3 \times 10000 \times 10^{-7} \times j \times 2\pi \times 100 + 1}$$

$$\left|\frac{V_{out}}{V_S}\right|_{100\,Hz} = \frac{1}{-0.3948 + 1.8850j + 1} = 0.505 \angle -72.20^\circ$$

$$\left|\frac{V_{out}}{V_S}\right|_{1000\,Hz} = \frac{1}{-39.478 + j6\pi + 1} = 0.0233 \angle -15^\circ$$

Experimental: **V_{out}/V_S =** 372.33mV/707.17mV = 0.5265

Discussion:

The percent error of transfer function at frequency 1kHz for Vout is 27.39%. It might be caused through the myDAQ limitations and a non-ideal capacitor. The measurement for a higher frequency also reduces the accuracy of myDAQ.

$$\frac{V_{OUT}}{V_S}(s) = H(s)^2 = \left[\frac{1}{1+sR_1C_L}\right]^2$$ Due to this transfer function, we know that the gain (dB) and phasor of Vout is two times of V_{L1}'s gain (dB) and phasor. However, from the Bode plot of Vout and V_{L1}, we can find out that the output voltage transfer function does not match this requirement. In other words, it doesn't match the square of the first-order transfer function. In addition, the higher frequency of the phasor of the output voltage transfer function goes up, which doesn't match this requirement again. The first-order transfer function of phasor should keep going down (Bode plot V_{L1}).

a),

Part 3):

a) $\frac{V_{L1}-V_s}{R_1} + \frac{V_{L1}}{\frac{1}{sC_L}} + \frac{V_{L1}-V_{out}}{R_1} = 0$. $\frac{V_{out}-V_{L1}}{R_1} + \frac{V_{out}}{\frac{1}{C_L s}} = 0$. $\left(\frac{1}{R_1}+C_L s\right)V_{out} = \frac{V_{L1}}{R_1}$

$V_{L1} = V_{out}\left(\frac{1}{R_1}+C_L s\right)R_1 = V_{out}(1+R_1C_L s)$

$\frac{V_{out}(1+R_1C_L s)}{R_1} - \frac{V_s}{R_1} + sC_L V_{out}(1+R_1C_L s) + \frac{(1+R_1C_L s)}{R_1}V_{out} - \frac{1}{R_1}V_{out} = 0$.

$$\frac{V_{out}}{V_s} = \frac{1}{R_1^2C_L^2s^2+3R_1C_Ls+1} \qquad s = \frac{-3R_1C_L \pm\sqrt{(3R_1C_L)^2-4R_1^2C_L^2}}{2R_1^2C_L^2}$$

s has $\boxed{2 \text{ poles}}$

b),

$$s = \frac{-3R_1C_L \pm \sqrt{9R_1^2C_L^2-4R_1^2C_L^2}}{2R_1^2C_L^2}$$

$$= \frac{-3R_1C_L \pm \sqrt{5R_1^2C_L^2}}{2R_1^2C_L^2}$$

$$= \frac{-3R_1C_L \pm \sqrt{5}R_1C_L}{2R_1^2C_L^2}$$

$$= \frac{-3\pm\sqrt{5}}{2R_1C_L}$$

$$s_1 = \frac{-3+\sqrt{5}}{2\times 10000\times 10^{-7}} = \boxed{-381.966}$$

$$s_2 = \frac{-3-\sqrt{5}}{32\times 10000\times 10^{-7}} = \boxed{-2618.03}$$

Part 4: 2nd-Order LPF design with a unity-gain buffer

Objectives:

The purpose of part 3 is to design a second order low-pass filter by using a unity-gain buffer (Op-Amp) in between the two first order passive filter (resistors and capacitors)

Theory:

The transfer function for this part is:

Part 4:

$$\frac{V_{L1}}{V_s} = \frac{\frac{1}{C_1 s}}{R_1 + \frac{1}{C_1 s}} = \frac{1}{C_1 s \cdot R_1 + 1} \qquad V^+ = V^- = V_{L1}$$

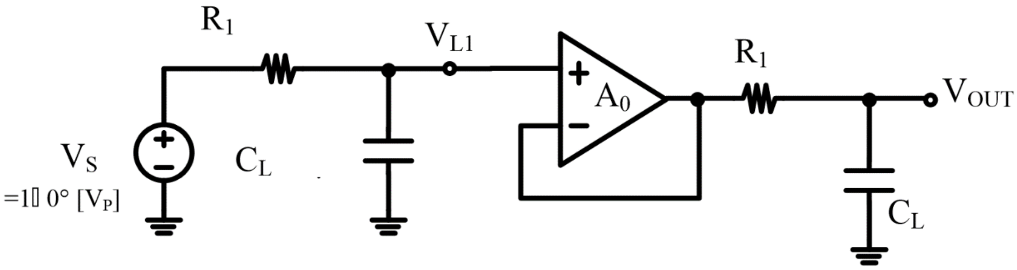
$$\frac{V_{out} - V^-}{R_1} + \frac{V_{out}}{\frac{1}{C_1 s}} = 0 \qquad \frac{1}{R_1} V_{out} - \frac{1}{R_1} V^- + C_1 s V_{out} = 0$$

$$\left(\frac{1}{R_1} + C_1 s \right) V_{out} = \frac{V_s}{(C_1 s R_1 + 1) \cdot R_1}$$

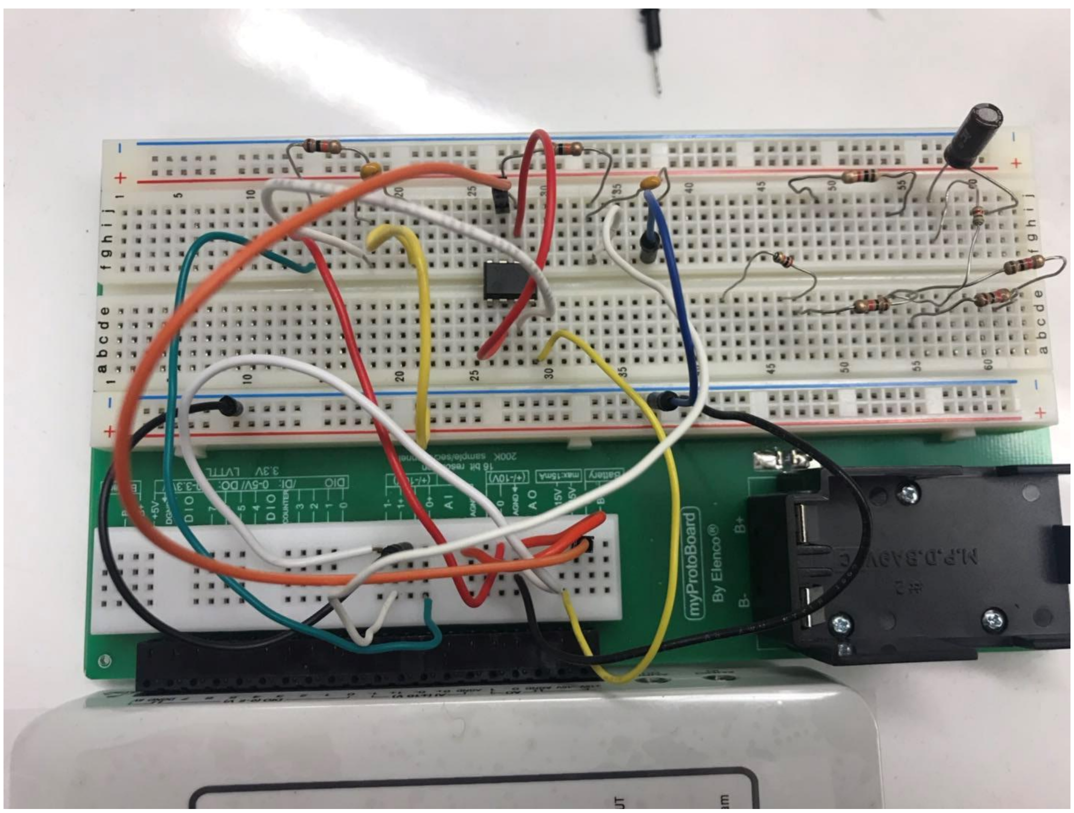
$$\frac{V_{out}}{V_s} = \frac{1}{(C_1 s R_1 + 1) R_1 \left(\frac{1}{R_1} + C_1 s \right)}$$

$$= \frac{1}{(1 + C_1 s R_1)^2}$$

Procedure:

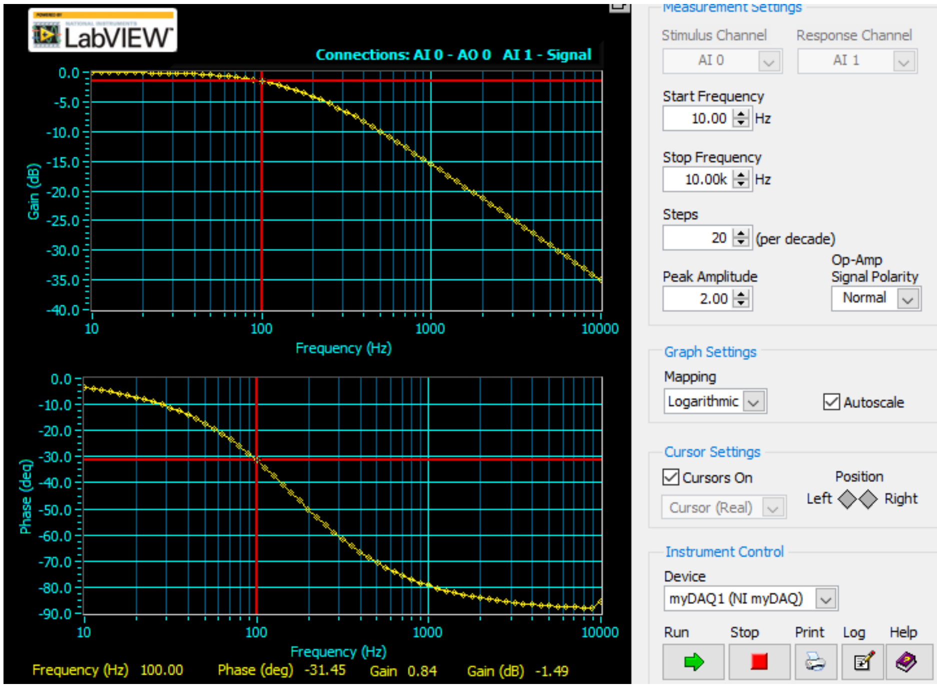


1. Implement 2nd -order RC LPF with a unit-gain buffer circuit as above (R1 = 10K, CL = 100n)
2. Using function generator to generate the AC voltage source, and set the amplitude to be 2, the frequency to be 100Hz and 1kHz, DC offset to be 0.
3. Using oscilloscope to output waveform $V_{L1}(t)$, Vout(t) and Vs(t);
4. Draw Bode plot for V_{L1} and V_{OUT} using ‘Bode’ in myDAQ.

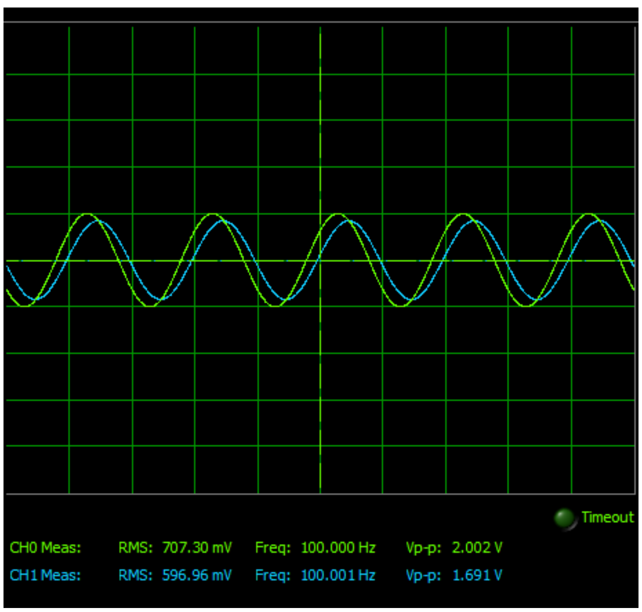


Data:

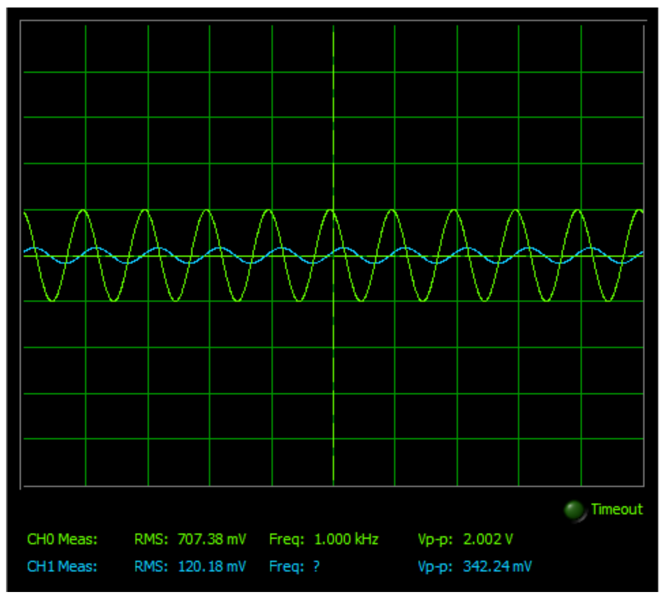
Bode plot at V_{L1}



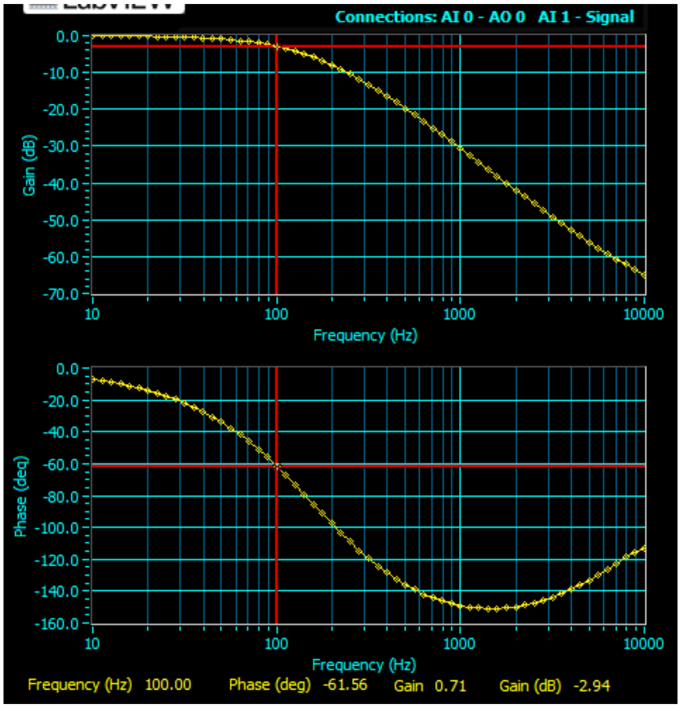
waveforms of V_{L1} and Vs at 100Hz



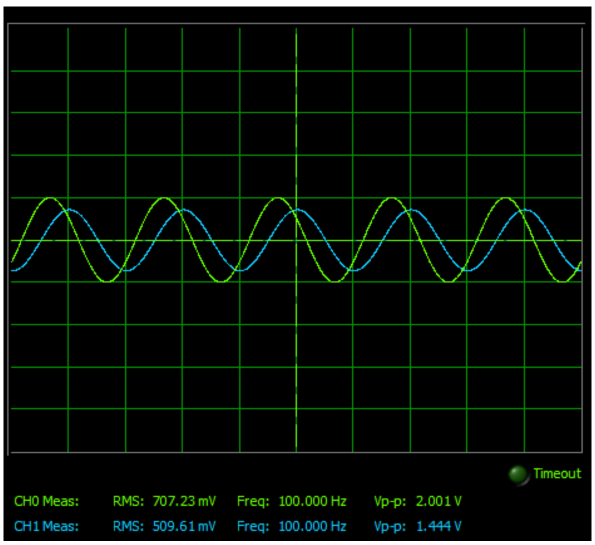
at 1kHz

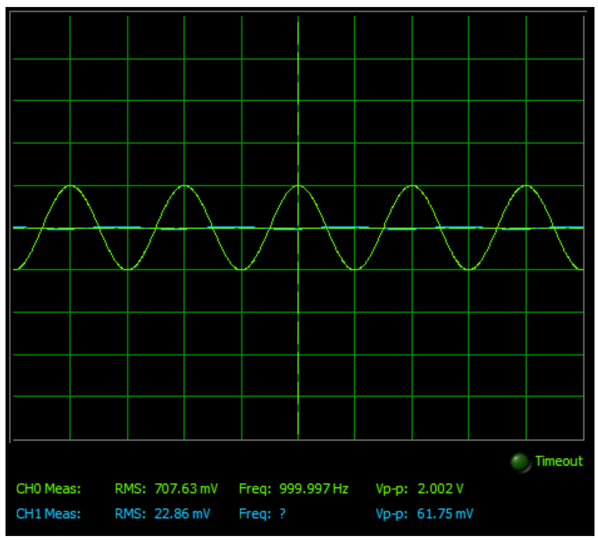


Bode plot at V_{out}



waveforms of V_{out} and Vs at 100Hz at 1kHz





Data Analysis:

	V_{L1}	Vs
100Hz	596.96mV	707.30mV
1k Hz	121.18mV	707.38mV

	V_{out}	Vs
100Hz	509.61mV	707.23mV
1k Hz	22.86mV	707.63mV

	Theoretical	Experimental	Error %
V_{out}/V_S, 100 Hz	0.717	0.721	0.558%
V_{out}/V_S, 1 kHz	0.0247	0.0323	30.77%

Calculations:

Theoretical:

$$\frac{V_{out}}{V_S}\bigg|_{100Hz} = \frac{1}{(1+10^{-7} \times j \cdot 2\pi \times 100 \times 10000)^2} = 0.717 \angle -64^\circ$$

$$\frac{V_{out}}{V_S}\bigg|_{1000Hz} = 0.0247 \angle -16^\circ$$

Experimental

$$\frac{V_{out}}{V_S}\bigg|_{100Hz} = \frac{509.61}{707.23} = 0.721$$

Discussion:

Due to the myDAQ limitation and the non-ideal capacitors, the experimental value of transfer function has higher error compared to the theoretical value for the high frequency measurement. In this part, with the unity-gain buffer (Op-Amp) which is implemented in between two first order passive filter, we successfully design a second order low-pass filter by using two cascaded first order passive filter. From the V_L's Bode plot, the gain value is 0.84 at 100Hz. From the Vout's Bode plot, the gain value is 0.71 at 100Hz. The square of 0.84 is equal to about 0.71 which is the same as Vout's gain. Therefore, the transfer function of this second order filter is square of the transfer function of the first order filter.

a). In theory section.

b). From the bode plot, we can see that the Gain(dB) of the 2nd-order LPF response at V_{OUT} is two times of the Gain(dB) of the 1st-order LPF response at V_{L1} the first order. The phase of V_{out} is also two times of the phase of V_{L1}

c). The 3dB bandwidth of the filter is 2*pi*100 = 628.32rad/s

d). There are two poles, and they are equal to -10^3

e). The LPF response in part 3 has two different poles, while the LPF in this part has two same poles. The unity-gain buffer in between two first order filter makes the transfer function of network become the square of the transfer function of a first order filter.

Part5: Inverting / Non-Inverting gain amplifiers.

Objectives:

The purpose of this part is to build an Inverting / Non-inverting gain amplifier to verify the superposition theorem.

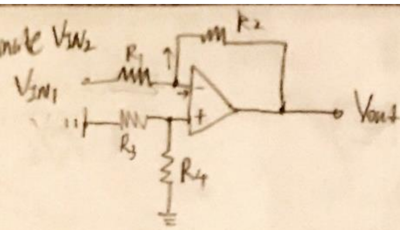
Theory:

The superposition theory shows that the response in any element of LTI linear circuit network which has more than one sources is the sum of the responses produced by the sources each acting independently.

The functions used to verify superposition theory in this part are:

Part 5) First, terminate V_{IN_2}

a) $\frac{V_{out}}{V_{IN1}}$



$$\boxed{\frac{V_{out}}{V_{IN1}} = -\frac{R_2}{R_1}}$$

$V_+ = V_-$

$V^+ = 0$

$V^- = 0$

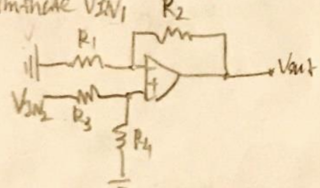
$$\frac{V^+}{R_3} + \frac{V^+}{R_4} = 0$$

$$\frac{V^- - V_{IN1}}{R_1} + \frac{V^- - V_{out}}{R_2} = 0 \qquad \frac{V_{IN1}}{R_1} = -\frac{V_{out}}{R_2}$$

$$\boxed{\frac{V_{out}}{V_{IN1}} = -\frac{R_2}{R_1}}$$

Terminate V_{IN_1}

(b).



$V^+ = V^-$

$$\frac{V^-}{R_1} + \frac{V^- - V_{out}}{R_2} = 0$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) V^- = \frac{1}{R_2} V_{out}$$

$$\frac{V^+ - V_{IN_2}}{R_3} + \frac{V^+}{R_4} = 0$$

$$\frac{1}{R_3}V^+ - \frac{1}{R_3}V_{IN_2} + \frac{V^+}{R_4} = 0.$$

$$\left(\frac{1}{R_3} + \frac{1}{R_4}\right) V^+ = \frac{1}{R_3} V_{IN_2}.$$

$$V^+ = \frac{\frac{1}{R_3}}{\frac{1}{R_3} + \frac{1}{R_4}} V_{IN_2}.$$

$$\frac{\frac{1}{R_3}}{\frac{1}{R_3} + \frac{1}{R_4}} \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_{IN_2} = \frac{1}{R_2} V_{out}$$

$R_1 = R_3$, $R_2 = R_4$.

$$\frac{1}{R_1} V_{IN2} = \frac{1}{R_2} V_{out}$$

$$\boxed{\frac{V_{out}}{V_{IN_2}} = \frac{R_2}{R_1}}$$

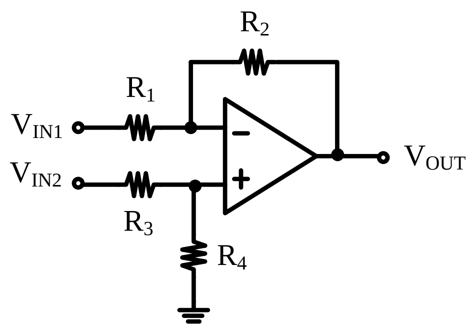
b) $V_{out} = -\frac{R_2}{R_1} \cdot V_{IN_1} + \frac{R_2}{R_1} V_{IN_2}$.

c). $R_1 = R_3$, $R_2 = R_4$.

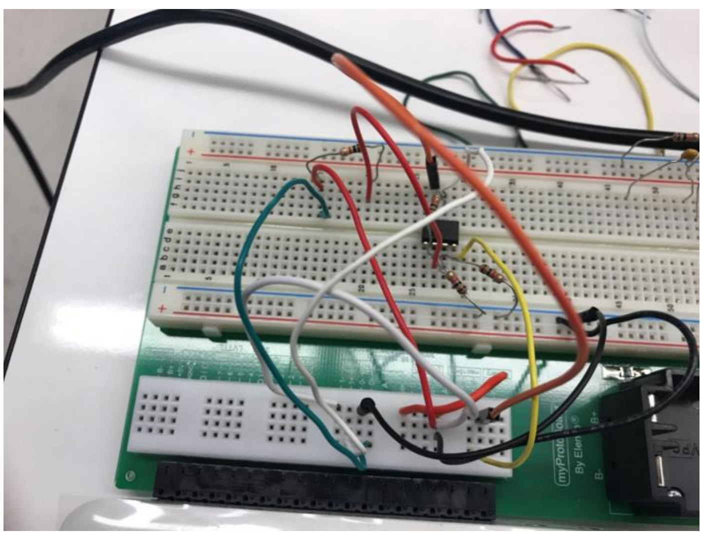
$$V_{out} = -\frac{R_2}{R_1} V_{IN_1} + \frac{R_2}{R_1} V_{IN_2}$$

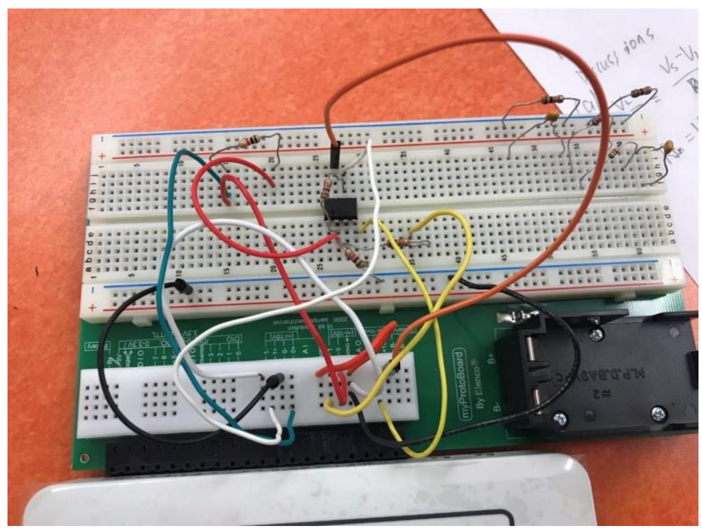
$$\boxed{V_{out} = \frac{R_2}{R_1}(V_{IN_2} - V_{IN_1})}$$

Procedure:



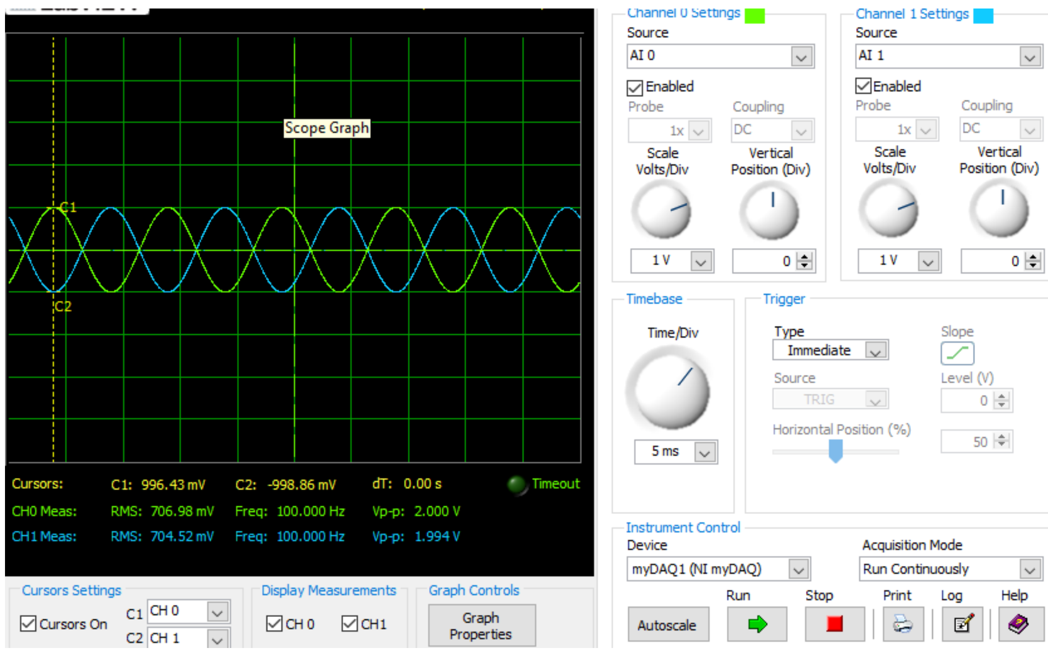
1. Implement the circuit as above. Set up the R1~R4 as 1kOhm, f_{IN} = 100 Hz in this test.
2. Terminating V_{IN2}, Draw $V_{IN1}(t)$, and $V_{OUT}(t)$
3. Terminating V_{IN1}, Draw $V_{IN2}(t)$, and $V_{OUT}(t)$
4. Set $V_{IN1}(t) = \sin(2\pi\times 100Hz\times t)$, $V_{IN2}(t) = \sin(2\pi\times 100Hz\times t)$ with ARB, and draw the output.
5. Set $V_{IN1}(t) = \sin(2\pi\times 100Hz\times t)$, $V_{IN2}(t) = -\sin(2\pi\times 100Hz\times t)$, and draw the output.
6. Set $V_{IN1}(t) = \sin(2\pi\times 100Hz\times t)$, $V_{IN2}(t) = -\sin(2\pi\times 100Hz\times t)$. Draw the output.
7. Set $V_{IN1}(t) = \sin(2\pi\times 100Hz\times t)$, $V_{IN2}(t) = -\sin(2\pi\times 1000Hz\times t)$. Draw the output and its spectrum.



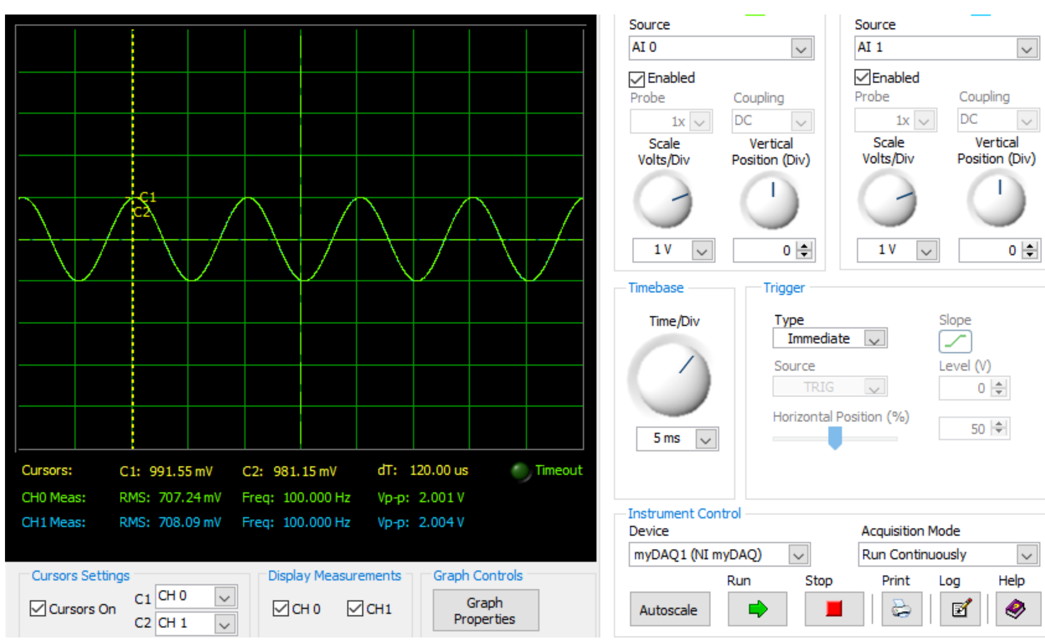


Data:

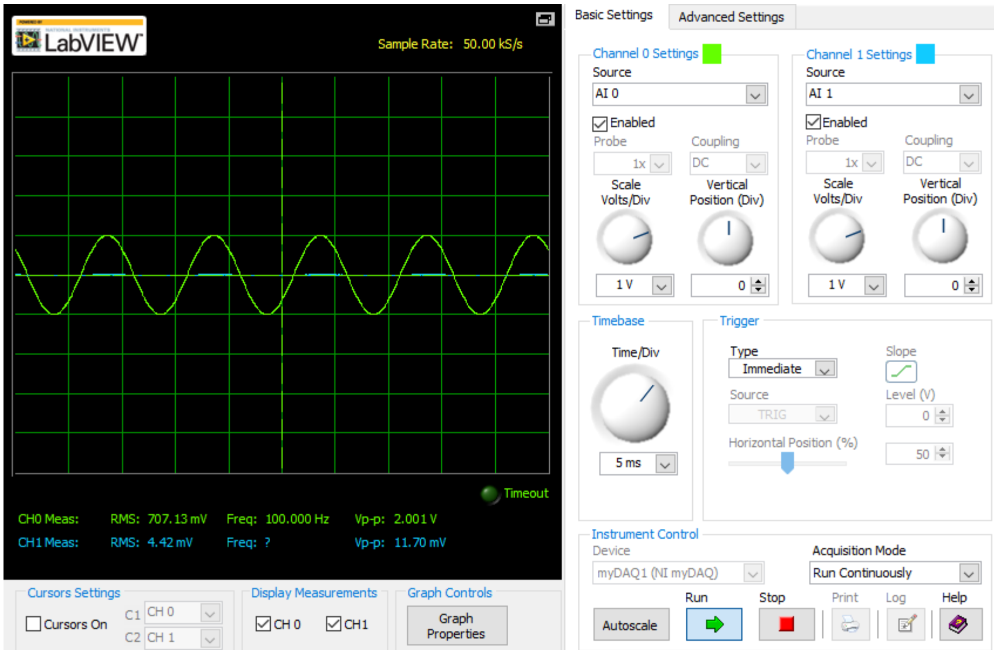
Terminating V_{IN2}: V_{out} and V_{IN1} f_{in} = 100Hz (inverting Amp)



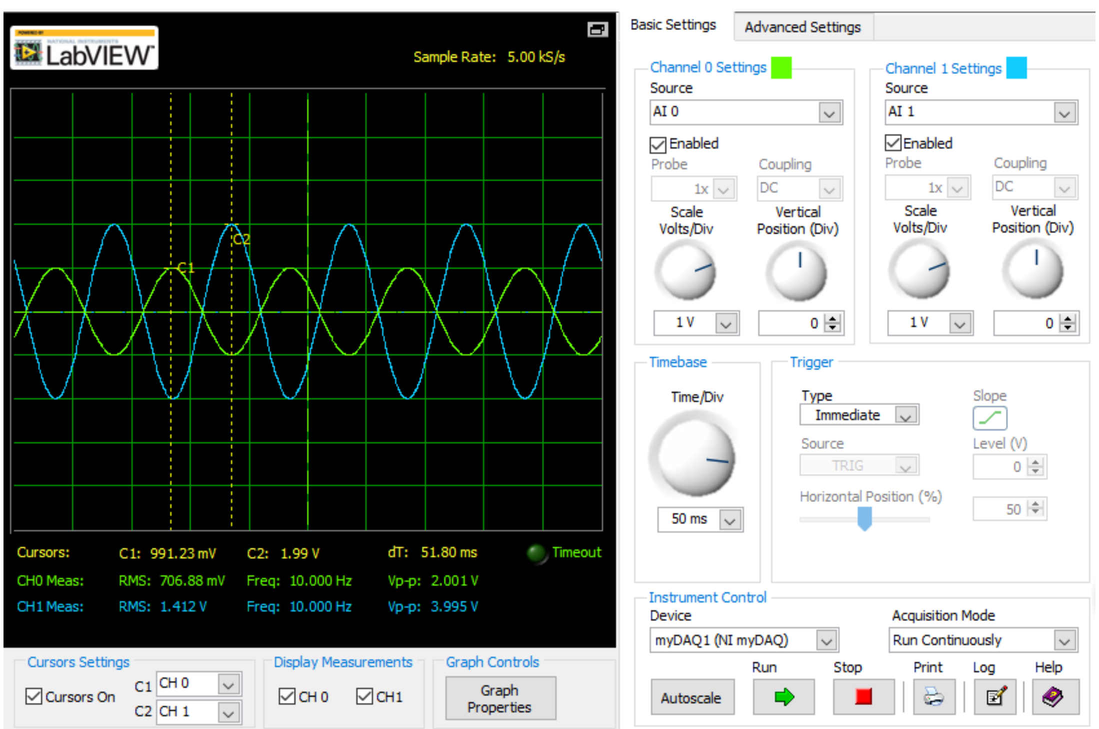
Terminating V_{IN1}: V_{out} and V_{IN2} f_{in} = 100H (non-inverting Amp)



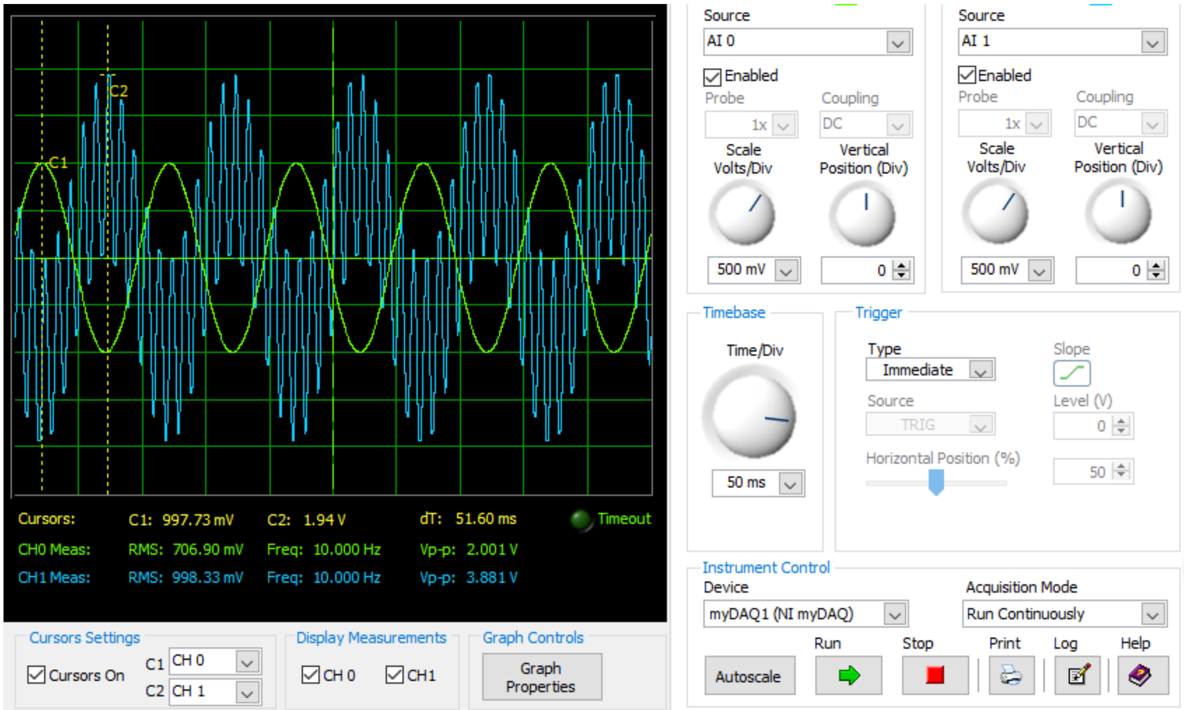
Step 4: $V_{IN1}(t) = \sin(2\pi\times100Hz\times t)$, $V_{IN2}(t) = \sin(2\pi\times100Hz\times t)$



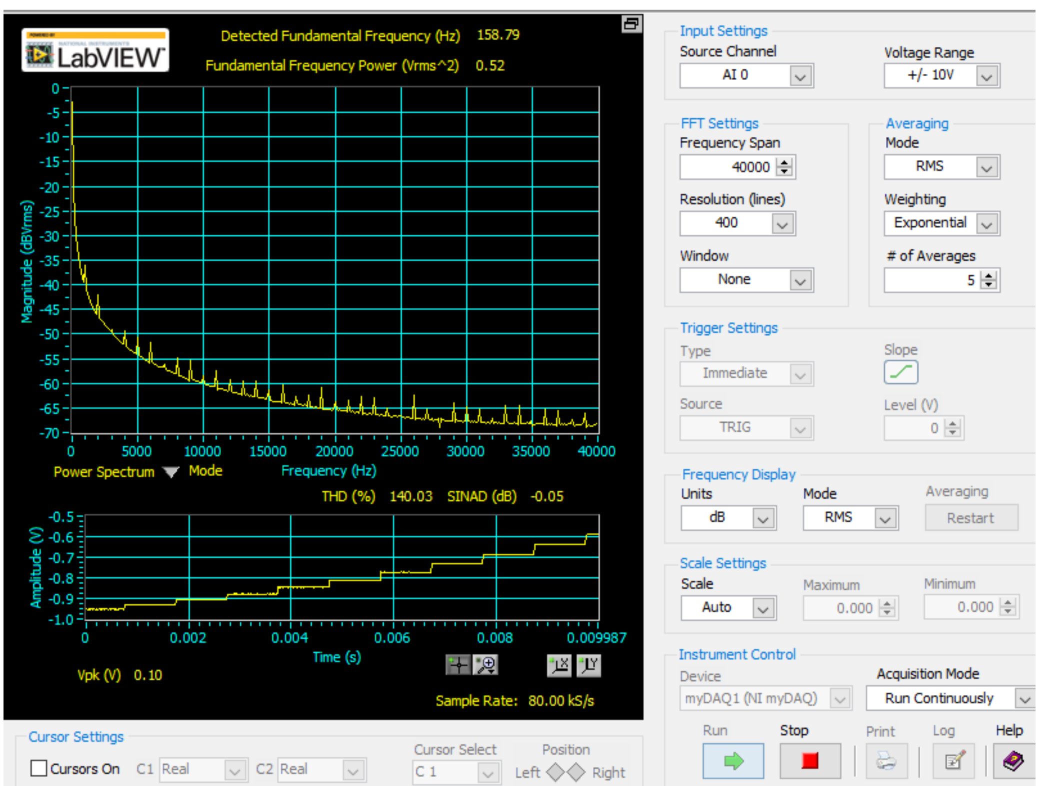
Step 5: $V_{IN1}(t) = \sin(2\pi\times100Hz\times t)$, $V_{IN2}(t) = -\sin(2\pi\times100Hz\times t)$



Step 6: $V_{IN1}(t) = \sin(2\pi\times100Hz\times t)$, $V_{IN2}(t) = -\sin(2\pi\times1000Hz\times t)$



The spectrum:



Data Analysis:

	V_{out}	Vs
Inverting (V_{out}/V_{in1})	-704.52mV	706.98mV
non-Inverting (V_{out}/V_{in2})	708.09mV	707.24mV

	Theoretical	Experimental	Error %
Step 4 V_{out} (V_p)	0.0V	4.42mV	None
Step 5 V_{out} (V_p)	-2V	-1.99V	0.5%
Step 6 V_{out} (V_p)	-2V	-1.94V	3.0%

	Theoretical	Experimental	Error %
Inverting (V_{out}/V_{in1})	-1.0	-0.9965	0.35%
non-Inverting(V_{out}/V_{in2})	1.0	1.001	0.1%

Calculation:

$$V_{out} = \frac{R_2}{R_1}(V_{IN_2} - V_{IN_1}) = (V_{IN_2} - V_{IN_1}).$$

Inverting $\left(\frac{V_{out}}{V_{IN_1}}\right) = 0 - 1 = -1$

non-Inverting $\left(\frac{V_{out}}{V_{IN_2}}\right) = 1 - 0 = 1.$

Step 4: $V_{out} = \sin(2\pi \times 1000Hz\, t) - \sin(2\pi \times 1000Hz\, t) = 0\,V$

step 5: $V_{out} = -\sin(2\pi \times 1000Hz\, t) - \sin(2\pi 1000Hz \times t) = -2\sin(2\pi \times 1000Hz \times t).$

step 6: $V_{out} = -\sin(2\pi \times 1000 \times t) - \sin(2\pi \times 100t)$

Discussion:

In this part, the experimental values match the theoretical values, and the percent error is very small. Therefore, the implement of Op-Amp in this circuit accurately works the inverting and non-inverting amplifier. To verify the superposition theorem, we terminate V_{IN2} and measure the output voltage. Then, we terminate V_{IN1} and measure the output voltage. The sum of these two output voltages is about equal to 0. The superposition theorem is verified. Throughout the step 4 to step 6, we verify the transfer function $\frac{V_{OUT}}{V_S} = \frac{R_2}{R_1}\left(V_{IN2} - V_{IN1}\right)$. Step 6 is the subtraction of two different frequency sinusoid inputs.

a), b), c) answers are in theory section.

Part 6: Active filter design

Objectives:

The purpose of part 6 is to design an active LPF that meets the required specifications.

Theory:

The transfer function for this part is below:

Part 6)

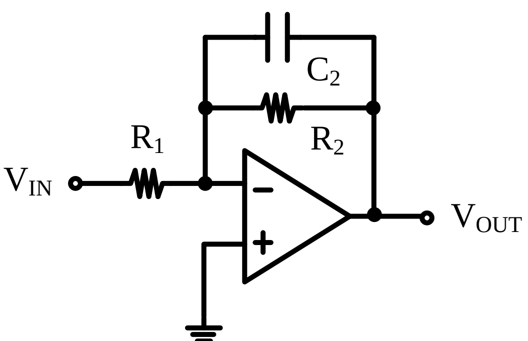
$$V^- = V^+ = 0$$

$$\frac{V^- - V_{IN}}{R_1} + \frac{V^- - V_{out}}{R_2} + \frac{V^- - V_{out}}{\frac{1}{C_2 \cdot s}} = 0$$

$$\frac{-V_{IN}}{R_1} - \frac{V_{out}}{R_2} - \frac{V_{out}}{\frac{1}{C_2 \cdot s}} = 0 \qquad \frac{V_{out}}{R_2} + C_2 \cdot s\, V_{out} = -\frac{V_{IN}}{R_1}$$

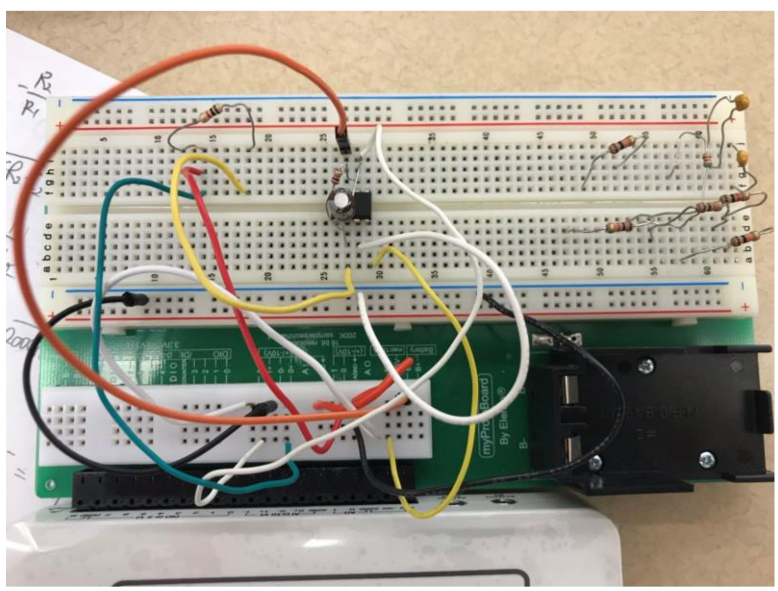
$$\frac{V_{out}}{V_{IN}} = \frac{-\frac{1}{R_1}}{\frac{1}{R_2} + C_2 \cdot s} = \frac{-\frac{1}{R_1}}{\frac{1 + C_2 R_2 \cdot s}{R_2}} = -\frac{R_2}{R_1}\,\frac{1}{1 + sR_2C_2}$$

Procedure:



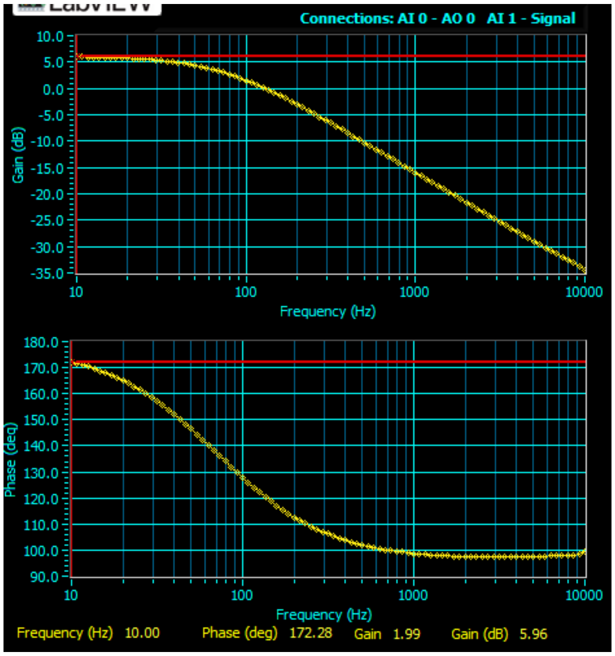
Active LPF (R_1 = 1kohm)

1. Implement the active LPF circuit as above.
2. Draw the Bode plot using the transfer function by MATLAB and compared to the BODE function module result.

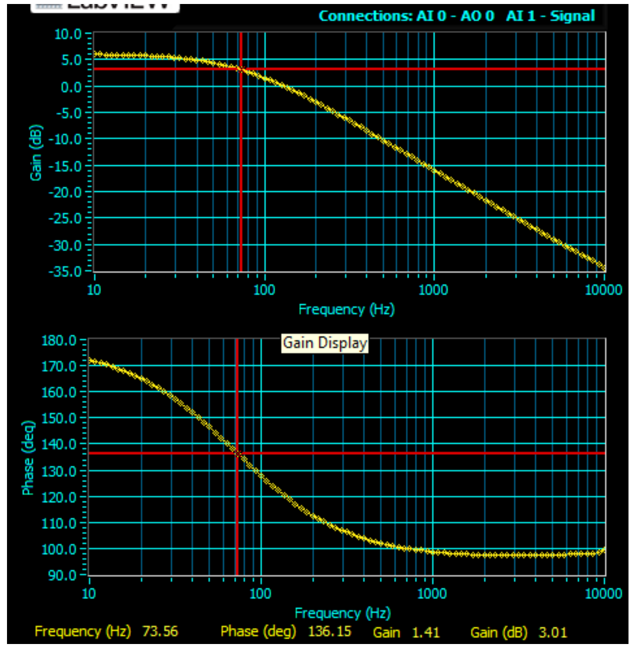


Data:

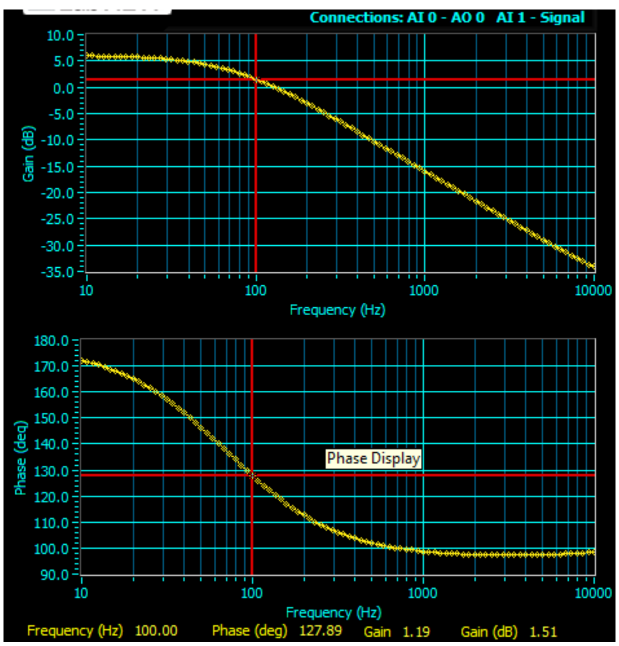
Bode plot at frequency 10Hz



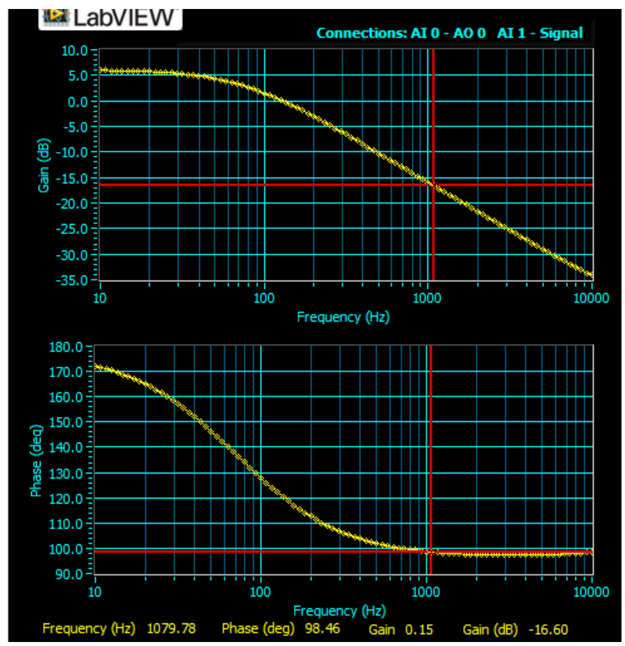
Bode plot at Gain(dB) 3.01



Bode plot at frequency 100Hz



Bode plot at frequency 1000Hz

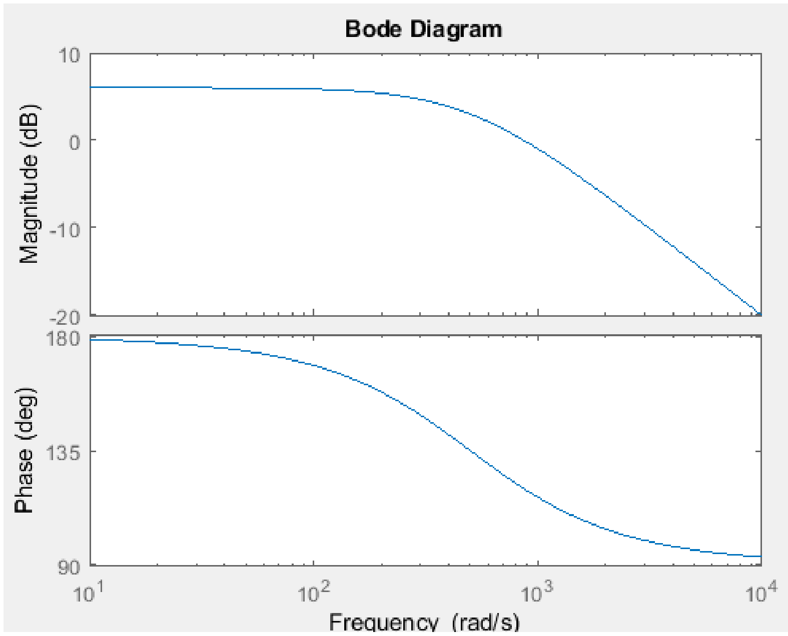


Data Analysis:

From the Bode plot at Gain(dB) 3.01, the frequency is 73.56Hz. The theoretical frequency for 3-dB bandwidth is 80Hz. The percent error is (80-73.56)/80*100 = 8.05%.

	Gain	**Gain(dB)**
10Hz	1.99	5.96
100Hz	1.19	1.51
1000Hz	0.15	-16.60

Bode plot by MATLAB:



Discussion:

The myDAQ measurement limitations and non-ideal capacitor would produce the percent error. The bode plot generated by myDAQ approximately matches the bode plot by using MATLAB. We can see that as the frequency increases, the gain of this circuit is decreases. Also, when the frequency is close to 0, the gain is equal to 2. The low pass filter is designed successfully with the gain of 2 at lowest frequency.

a), In the theory section.

b).

$H(j\omega) = \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}\left(\frac{1}{1+j\omega R_2 C_2}\right)$. $\boxed{R_2 = 2k\Omega}$. if $C \to 0$ $\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1} = 2$

$R_2 = 2k\Omega$.

$H(s) = -\frac{R_2}{R_1}\left(\frac{1}{1+sR_2C_2}\right)$. pole: $s = \frac{1}{R_2C_2}$

(b) $\omega_{3dB} = \frac{1}{R_2C_2}$

$2\pi \times 80Hz = \frac{1}{2000\Omega \times C_2}$ $\boxed{C_2 = 1\mu F}$

Part 7: Integrator design

Objectives:

The purpose of this part is to design and analyze an integrator.

Theory:

The transfer function for this part is below:

$$V^- = V^+ = 0.$$

b) $\left|\frac{V_{out}}{V_{IN}}\right|$

$$\frac{V^- - V_s}{R_1} + \frac{V^- - V_{out}}{\frac{1}{C_1 s}} = 0$$

$$\frac{V_s}{R_1} = -\frac{V_{out}}{\frac{1}{C_1 s}}$$

$$\frac{V_{out}}{V_{IN}} = -\frac{1}{C_1 R_1 s} = -\frac{1}{j\omega R_1 C_1} = \frac{j}{\omega R_1 C_1}$$

Procedure:

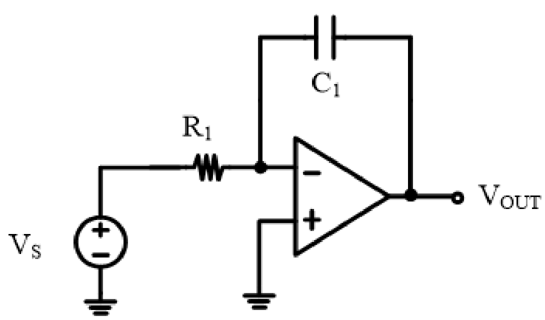
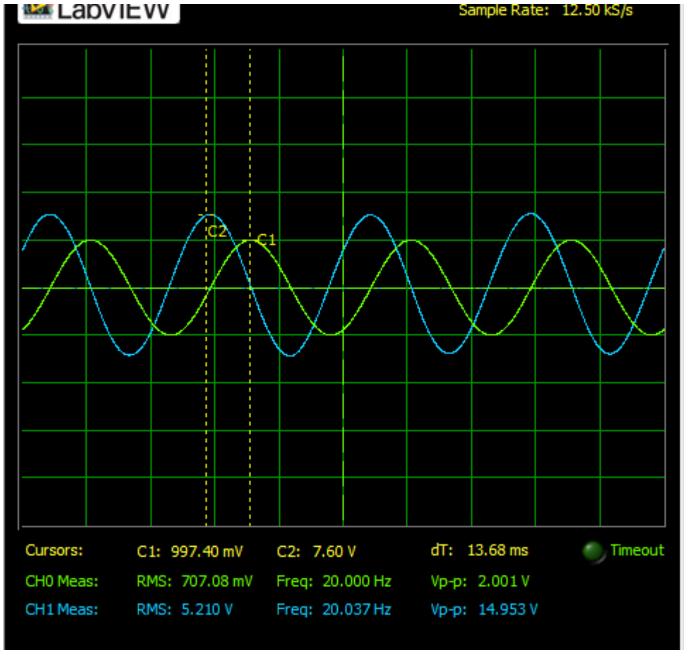


Fig.9. Integrator ($C_1 = 1\ \mu F$)

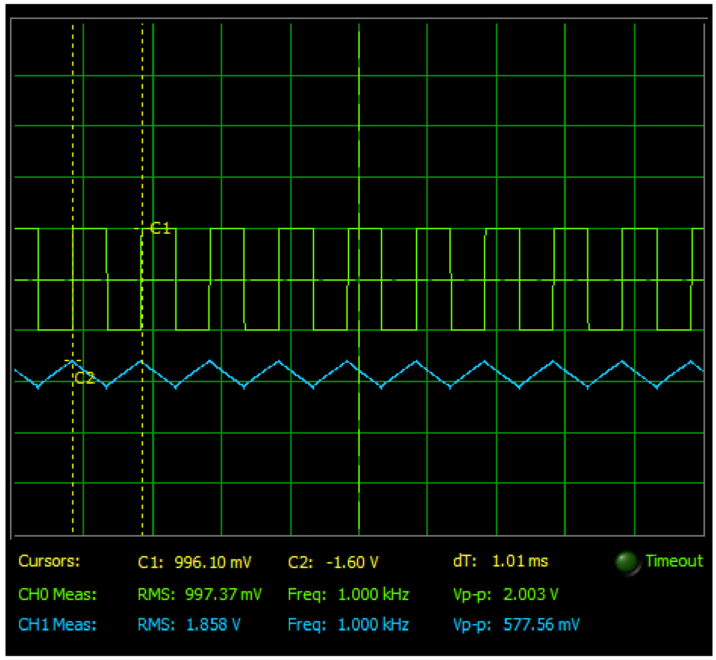
1. Implement the integrator circuit as above (R_1 = 1000Ohm)
2. Using Arbitrary waveform generator to generate $V_{IN}(t) = \cos(2\pi\times 1kHz\times t)$ [V_P] and measure the Vout.
3. Using function generator to generate rectangular signal with an amplitude of 1 V_P and 1 kHz input frequency and measure the Vout.

Data:

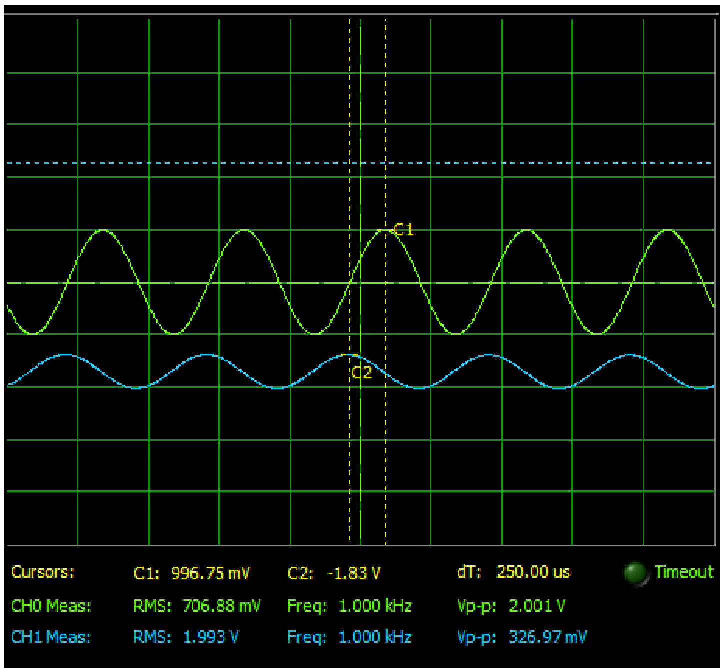
Vout(t) when $V_{IN}(t) = \cos(2\pi \times 1kHz \times t)$



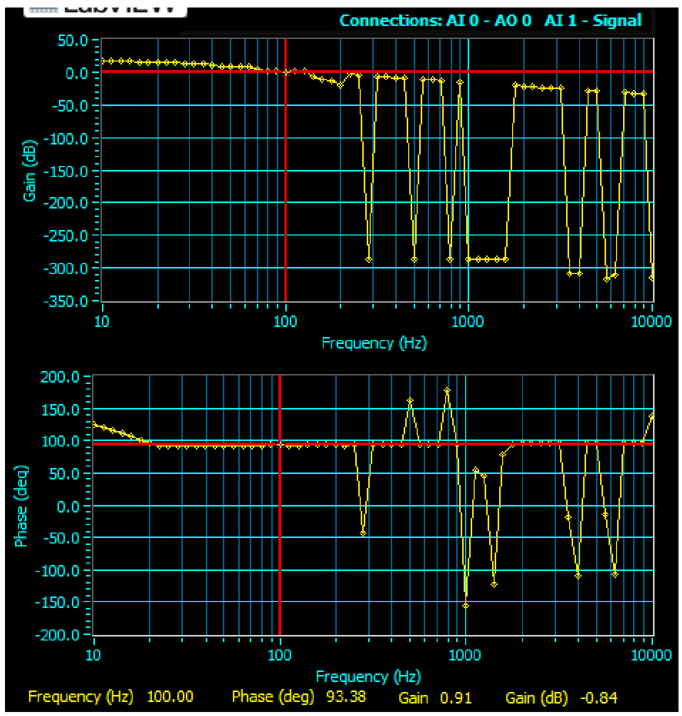
Vout(t) when rectangular signal



Vout for function generator (1kHz)

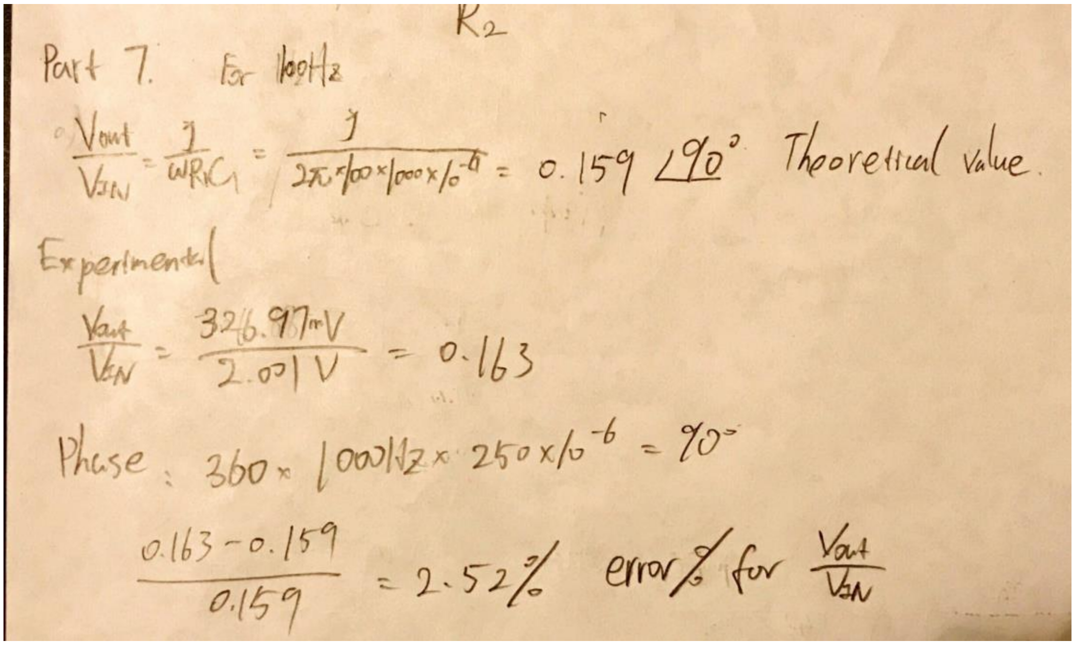


Bode plot:



Data analysis:

Due to the Arbitrary waveform generator which cannot accurately generate cos(2π×1kHz×t) stable voltage, I use function generator to generate a sin(2π×1kHz×t) voltage to measure the data.



R_2

Part 7. For 1000Hz

$$\frac{V_{out}}{V_{IN}} = \frac{j}{\omega R_1 C_1} = \frac{j}{2\pi\times1000\times1000\times10^{-6}} = 0.159\angle 90^\circ$$ Theoretical value.

Experimental

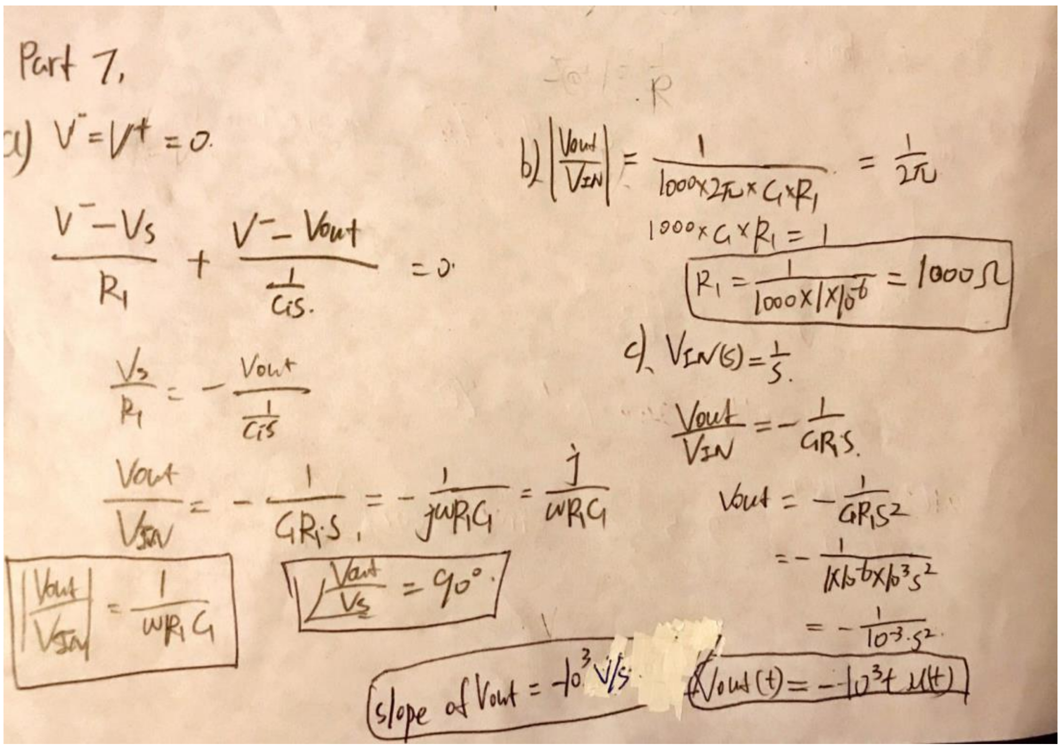
$$\frac{V_{out}}{V_{IN}} = \frac{326.97\,mV}{2.001\,V} = 0.163$$

Phase: $360\times 1000Hz\times 250\times10^{-6} = 90^\circ$

$$\frac{0.163-0.159}{0.159} = 2.52\%$$ error% for $\frac{V_{out}}{V_{IN}}$

Discussion:

From the data analysis, we can see that the experimental phase is 90 degree, and the gain is 0.163 at 1kHz. They are theoretically predicted. From the Bode plot, before the graph messes up, we can see that the gain(dB) is a horizontal line. It means that the slope of gain is a constant. The phase is also equal to 90 degree. From the rectangular signal, we can see that the output voltage is triangle signal. The Integrator is designed successfully. In addition, the reason for the bode plot messes up at higher frequency is that the gain is almost equal to zero after 100Hz. Therefore, the myDAQ cannot accurately measure the rest of gain(dB) and phase.



Part 7.

a) $V^- = V^+ = 0$.

$$\frac{V^- - V_s}{R_1} + \frac{V^- - V_{out}}{\frac{1}{C_1 s}} = 0$$

$$\frac{V_s}{R_1} = -\frac{V_{out}}{\frac{1}{C_1 s}}$$

$$\frac{V_{out}}{V_{IN}} = -\frac{1}{C_1R_1 s} = -\frac{1}{j\omega R_1 C_1} = \frac{j}{\omega R_1 C_1}$$

$$\boxed{\left|\frac{V_{out}}{V_{IN}}\right| = \frac{1}{\omega R_1 C_1}} \qquad \boxed{\angle\frac{V_{out}}{V_s} = 90^\circ}$$

b) $$\left|\frac{V_{out}}{V_{IN}}\right| = \frac{1}{1000\times2\pi\times C_1\times R_1} = \frac{1}{2\pi}$$

$1000\times C_1\times R_1 = 1$

$$\boxed{R_1 = \frac{1}{1000\times1\times10^{-6}} = 1000\Omega}$$

c) $V_{IN}(s) = \frac{1}{s}$.

$$\frac{V_{out}}{V_{IN}} = -\frac{1}{C_1R_1 s}$$

$$V_{out} = -\frac{1}{C_1R_1 s^2} = -\frac{1}{1\times10^{-6}\times10^3 s^2} = -\frac{1}{10^{-3}\cdot s^2}$$

$\boxed{\text{slope of } V_{out} = -10^3\, V/s}$ $\boxed{V_{out}(t) = -10^3 t\, u(t)}$

Conclusion:

Through all of the experiments, the operational amplifier plays an important rule. We can design an unity-gain buffer using Op-Amp. The unity-gain buffer can be able to provide a voltage is the same as input voltage, the it can prevent voltage divider. With the Op-Amp, we can design a second order low pass filter using two cascaded first order passive filter. In addition, the Op-Amp can invert or not invert the output voltage by setting the specification. The active LPF can prevent higher frequency voltage pass to the output. The Integrator can also increate a phase difference 90 degree in between the input and output voltage. Most of the experimental values match with the theoretical calculations excepted some high frequency measurement.