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ECE102 Project

Project Report

Part I: Laplace analysis of closed-loop control system

Step 1:

a), b),

Part I:

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505-179-815
Project

$H_1(s)$: $h_1(t) \xrightarrow{\mathcal{L}_s} e^{-2s}$.

$H_2(s)$: $h_2(t) \xrightarrow{\mathcal{L}_s} \frac{e^{2s}}{s-2}$

$$H_{12}(s) = H_1(s) * H_2(s) = \frac{1}{s-2}.$$

$y(t) = h(t) * x(t)$

$y(t) = [x(t) - ky(t)] * h_{12}(t)$

$Y(s) = [X(s) - k \cdot Y(s)] \cdot H_{12}(s)$

$Y(s) + kH_{12}(s)\, Y(s) = X(s) \cdot H_{12}(s)$

① $Y(s)[1 + kH_{12}(s)] = X(s) \cdot H_{12}(s)$

② $Y(s) = H(s) \cdot X(s)$

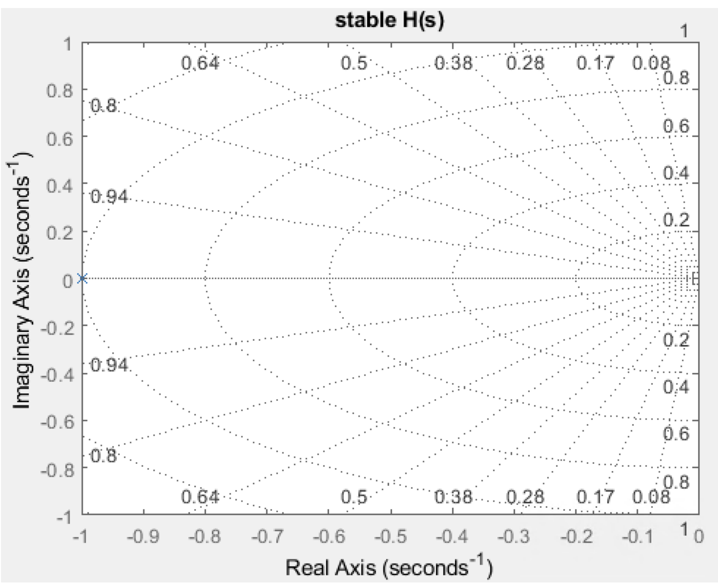
$$H(s) = \frac{H_{12}(s)}{1 + kH_{12}(s)} = \frac{\frac{1}{s-2}}{1 + \frac{k}{s-2}} = \frac{1}{s-2+k} \quad h(t) = e^{(2-k)t} \cdot u(t)$$

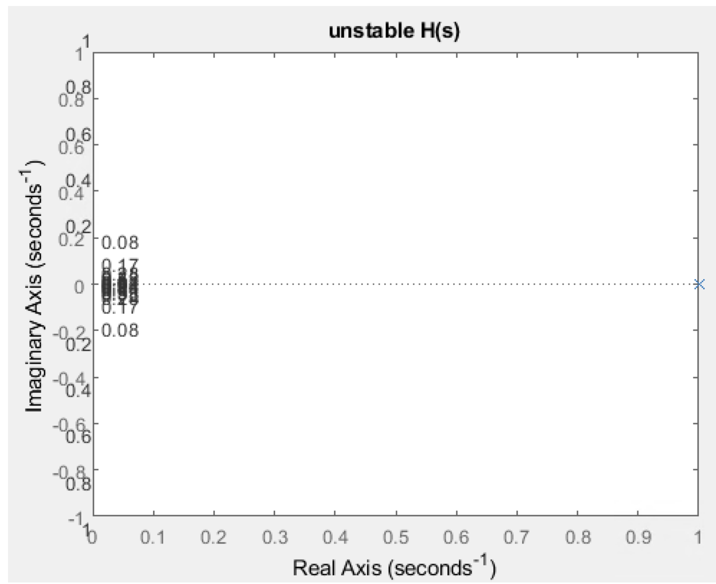
$k > 2$ is stable.

$k_1 = 1$; unstable
$k_2 = 3$; stable

c),

```
k1 = 1; %unstable;
k2 = 3; %stable


sys1 = tf(1,[1 -2+k1]);  %unstable
sys2 = tf(1,[1 -2+k2]); %stable
figure(1);
h1 = pzplot(sys1);
title("unstable H(s)");
grid on;

figure(2);
h2 = pzplot(sys2);
title("stable H(s)");
grid on
```

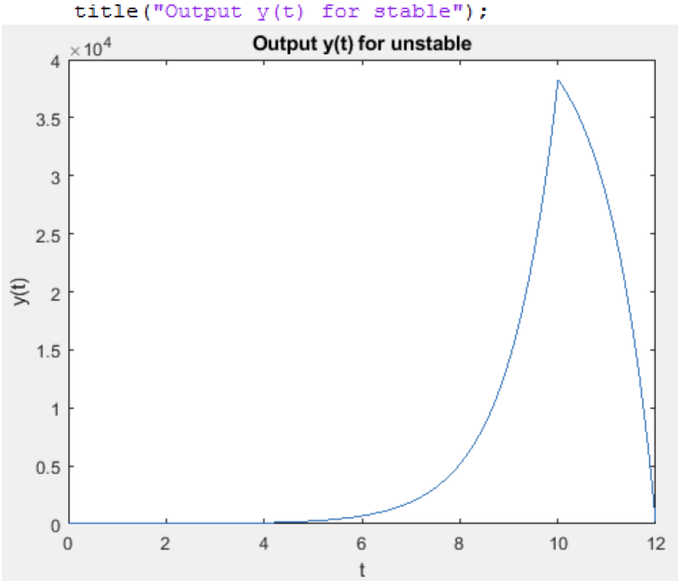


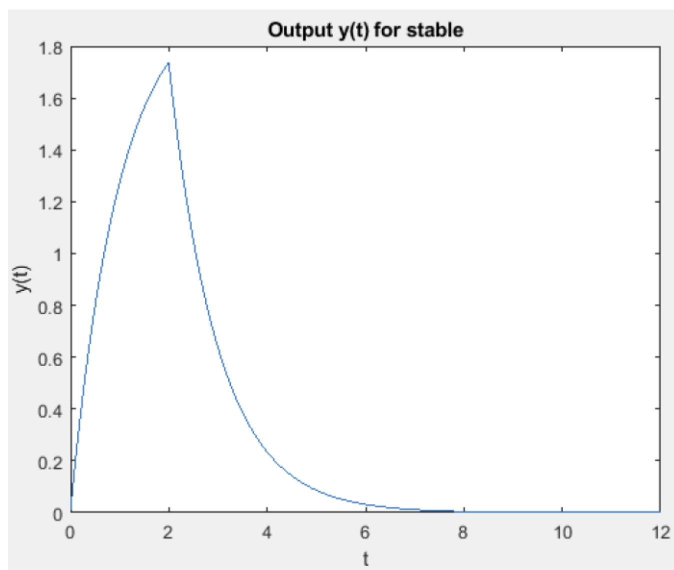


Step 2:

a),

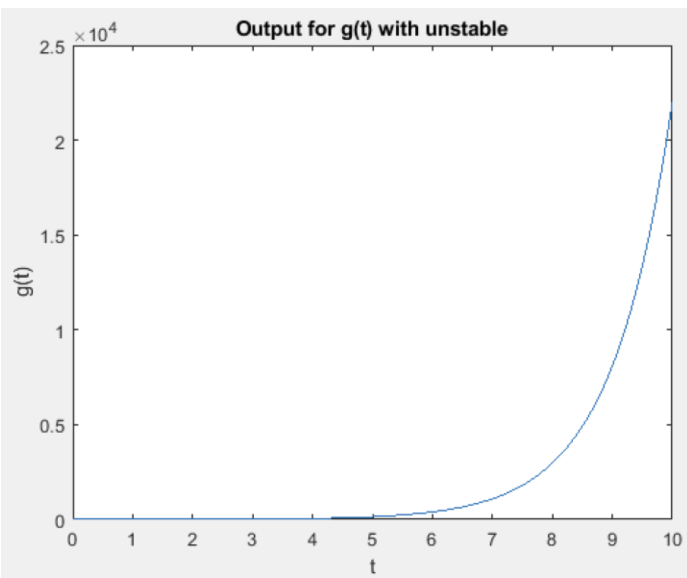
```
%Step 2 a)
syms s
Ts = 0.01;
xtime = 0:Ts:2;
htime = 0:Ts:10;

xt = 2*(heaviside(xtime)-heaviside(xtime-2));

h1 = exp((2-k1)*htime);%unstable
h2 = exp((2-k2)*htime);%stable
y1 = conv(xt, h1)*Ts; %unstable
y2 = conv(xt, h2)*Ts; %stable

ytime1 = (0:length(y1)-1)*Ts;
ytime2 = (0:length(y2)-1)*Ts;
figure(3);
plot(ytime1, y1);
xlabel("t");
ylabel("y(t)");
title("Output y(t) for unstable");

figure(4);
plot(ytime2, y2);
xlabel("t");
ylabel("y(t)");
title("Output y(t) for stable");
```

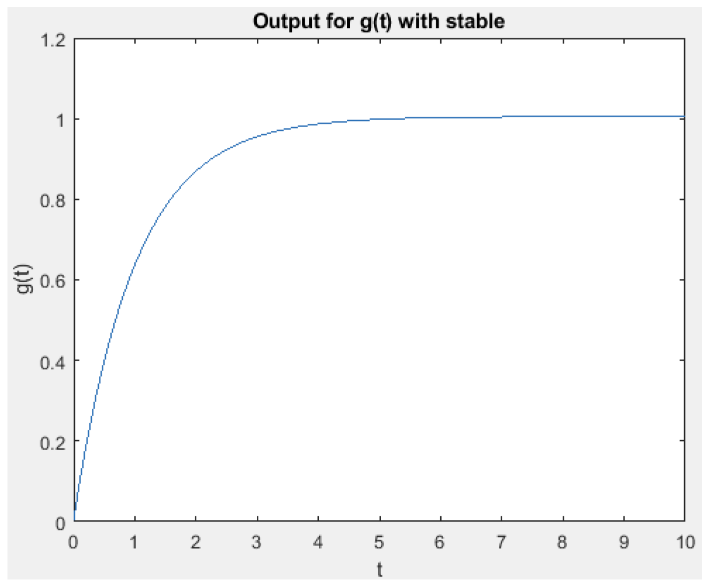




b),

```
%Step 2 b)
x1time = 0:Ts:10;
unit_x1 = heaviside(x1time);
gt = conv(unit_x1, h1)*Ts;
gtime = (0:length(gt)-1)*Ts;
figure(5);
plot(gtime, gt);
xlim([0 10]);
xlabel("t");
ylabel("g(t)");
title("Output for g(t) with unstabel");

gt2 = conv(unit_x1, h2)*Ts;
gtime2 = (0:length(gt2)-1)*Ts;
figure(6);
plot(gtime2, gt2);
xlim([0 10]);
xlabel("t");
ylabel("g(t)");
title("Output for g(t) with stabel");
```





Part II: Fourier series and transform of audio signals

Step 3)

a), b).

Part II: Fourier series and transform of audio signals

a). $H(s) = \frac{1}{1+Bs}$ $B = \frac{1.27}{8000}$ $x(t) = \cos(\Omega_0 t) + \cos(\Omega_1 t)$

$\Omega_0 = 2\pi \times 500$, $\Omega_1 = 2\pi \times 3000$.

$$y(t) = |H(jk\Omega)| \cos(k\Omega t + \theta_k + \angle H(jk\Omega))$$

$$H(j\Omega_0) = \frac{1}{1+Bj\Omega_0} = \frac{1 - B\Omega_0 j}{1^2 + (B\Omega_0)^2} = \frac{1}{1^2+(B\Omega_0)^2} - \frac{B\Omega_0}{1^2+(B\Omega_0)^2} j = 0.8 - 0.39939j.$$

$$H(j\Omega_1) = \frac{1}{1+Bj\Omega_1} = \frac{1}{1+2.992j} = \frac{1-2.992j}{9.954} = 0.1 - 0.3j$$

$$y(t) = |H(j\Omega_0)| \cdot \cos(\Omega_0 t + \angle H(j\Omega_0)) + |H(j\Omega_1)| \cos(\Omega_1 t + \angle H(j\Omega_1))$$

$$= 0.894 \cos(1000\pi t - 0.463) + 0.316 \cos(6000\pi t - 1.249)$$

b). Find X_k and Y_k.

The period of $x(t)$.

$T_0 = \frac{2\pi}{2\pi \times 500} = 0.002$ $T_1 = \frac{2\pi}{2\pi \times 3000} = \frac{1}{3000}$.

$\frac{T_0}{T_1} = \frac{0.002}{1/3000} = 6$

$T = T_0 = 6 \times T_1 = 0.002$

Fundamental frequency

$\Omega = \frac{2\pi}{T} = 1000\pi$

Input: $x(t) = \cos(\Omega t) + \cos(6\Omega t)$

$= \frac{1}{2}[e^{j\Omega t} + e^{-j\Omega t} + e^{j6\Omega t} + e^{-j6\Omega t}]$

$= X_1 = X_{-1} = X_6 = X_{-6} = \frac{1}{2}$ O.W. $X_k = 0$.

Output: $Y_k = X_k H(jk\Omega)$ $k = -1, 1, 6, -6$. O.W. $Y_k = 0$

$Y_{-1} = \frac{1}{2}(H(j(-1)\Omega)) = \frac{1}{2} \frac{1}{1 - \frac{1.27}{8000} \times j \times 1000\pi} = 0.4 + 0.1997j$

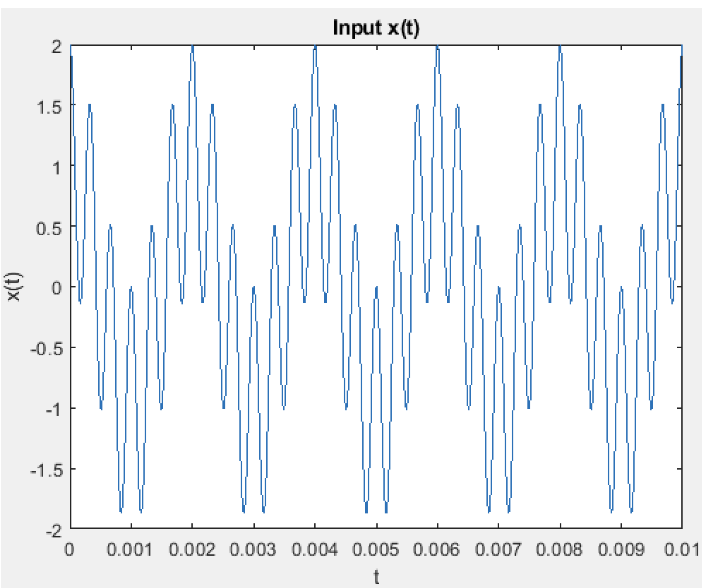
$Y_1 = \frac{1}{2}(H(j\Omega)) = 0.4 - 0.1997j$.

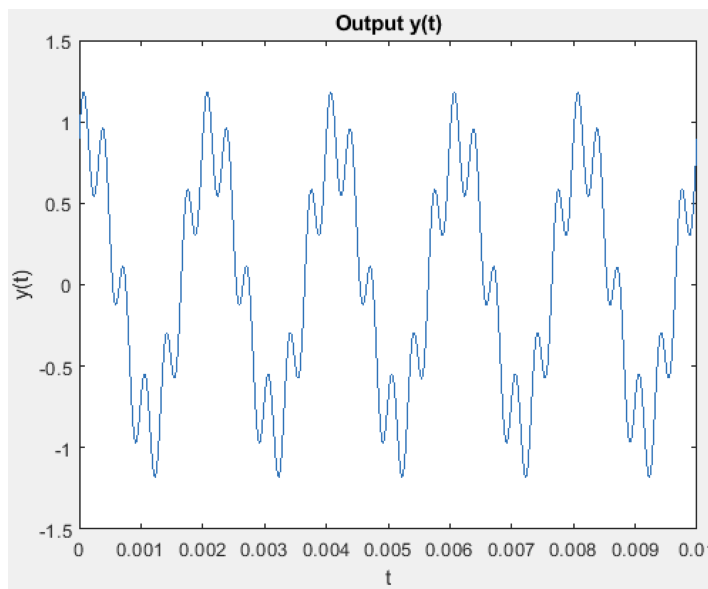
$Y_6 = \frac{1}{2}[H(j6\Omega)] = 0.05 - 0.15j$.

$Y_{-6} = \frac{1}{2}[H(-j6\Omega)] = 0.05 + 0.15j$

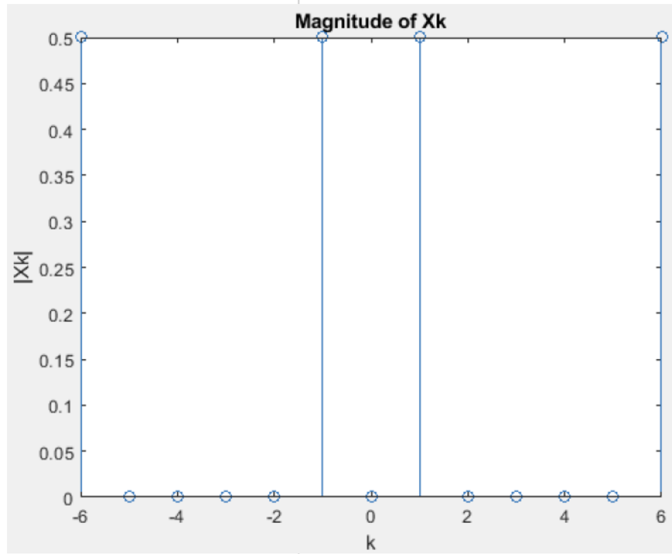
c),

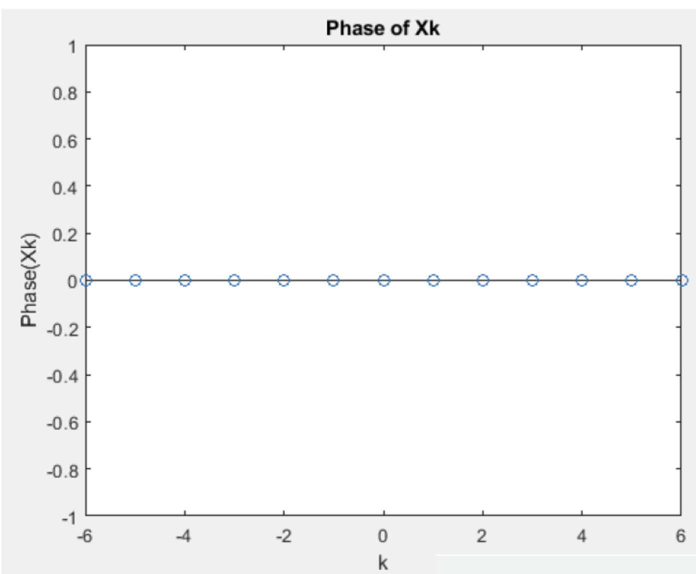
```
%project part2
%Step 3
%part c)
B=1.27/8000;
w0=2*pi*500;
w1=2*pi*3000;
w=1000*pi;
j=sqrt(-1);
ts=1/100000;
t=0:ts:0.01;

x = cos(w0*t) + cos(w1*t);
y = 0.894*cos(1000*pi*t-0.463)+0.316*cos(6000*pi*t-1.249);
figure(1);
plot(t, x);
xlabel("t");
ylabel("x(t)");
title("Input x(t)");

figure(2);
plot(t, y);
xlabel("t");
ylabel("y(t)");
title("Output y(t)");
```

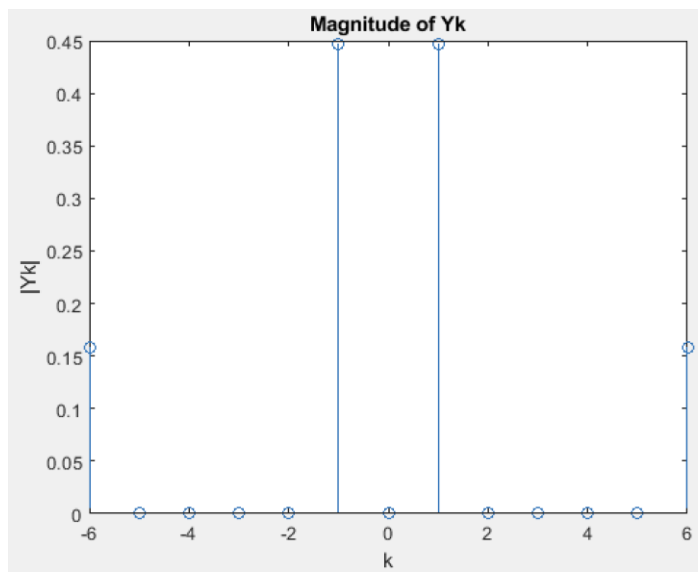


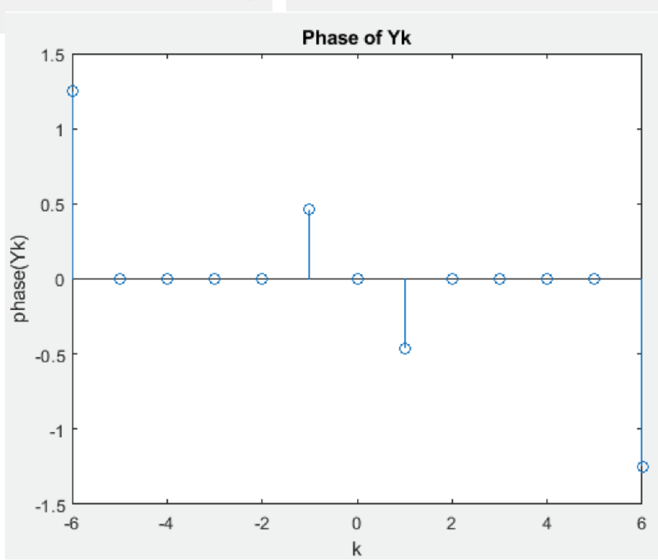


```
k=-6:1:6;
Xk=1/2.*(k==1) + 1/2.*(k==-1) + 1/2.*(k==6) + 1/2.*(k==-6);
Yk=(0.4+0.1997*j).*(k==-1) + (0.4-0.1997*j).*(k==1) + (0.05-0.15*j).*(k==6) + (0.05+0.15*j).*(k==-6);
MX = abs(Xk);
PX = angle(Xk);
figure(3);
stem(k, MX);
xlabel("k");
ylabel("|Xk|");
title("Magnitude of Xk");

figure(4);
stem(k, PX);
xlabel("k");
ylabel("Phase(Xk)");
title("Phase of Xk");

figure(5);
stem(k, abs(Yk));
xlabel("k");
ylabel("|Yk|");
title("Magnitude of Yk");

figure(6);
stem(k, angle(Yk));
xlabel("k");
ylabel("phase(Yk)");
title("Phase of Yk");
```



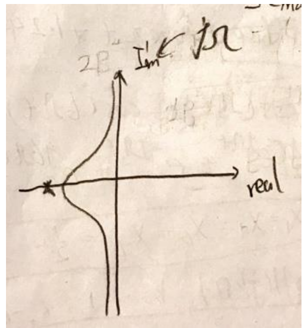






d).

1+Bs = 0;
S = -1/B = -6299;
Poles: -6299
Pole is real number.



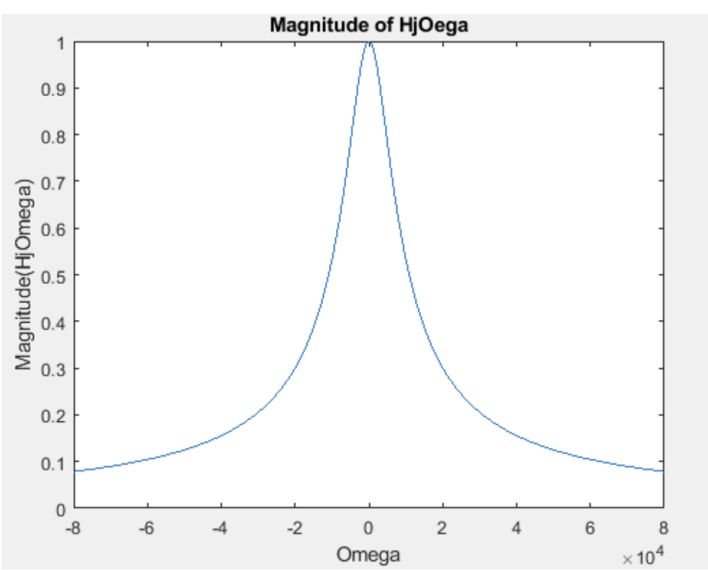
The highest magnitude of the Output is when the frequency closes to the poles (imaginary axis is 0). The imaginary axis can be assumed to be jOmega axis, which is the j frequency axis, the output magnitude is highest when the frequency is equal to the 0 which is the real axis (poles).

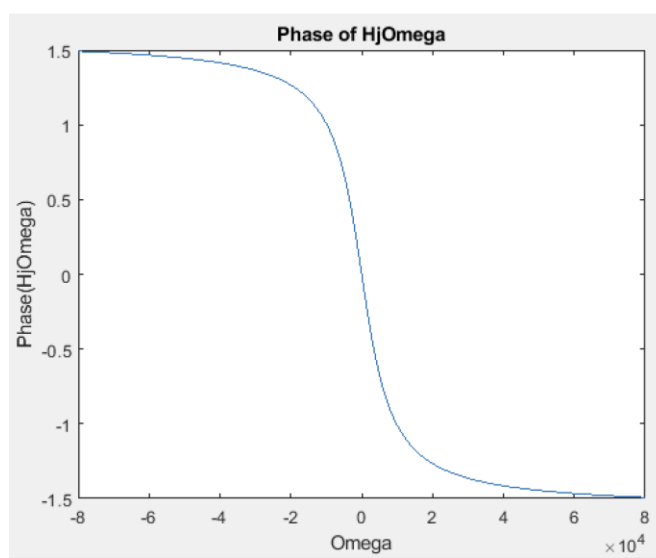
Step 4)

a).

H(jOmega) = 1/(1+B*j*Omega)

```
%Step 4)
%a), b)
dOmega = 10;
Omega = -80000:dOmega:80000;
HjOmega = 1./(j*B*Omega+1);
figure(7);
plot(Omega,abs(HjOmega));hold on
xlabel("Omega");
ylabel("Magnitude(HjOmega)");
title("Magnitude of HjOega");

figure(8);
plot(Omega,phase(HjOmega));hold on
xlabel("Omega");
ylabel("Phase(HjOmega)");
title("Phase of HjOmega");
```





Step 4

a). $H(j\Omega) = \frac{1}{1+Bj\Omega}$

c). $H(j\Omega) = \frac{1-Bj\Omega}{1+B^2\Omega^2} = \frac{1}{1+B^2\Omega^2} - \frac{B\Omega}{1+B^2\Omega^2} j$

$$|H(j\Omega)| = \sqrt{\left(\frac{1}{1+B^2\Omega^2}\right)^2 + \left(\frac{B\Omega}{1+B^2\Omega^2}\right)^2}$$

$$= \frac{\sqrt{1+B^2\Omega^2}}{\sqrt{(1+B^2\Omega^2)^2}} = (1+B^2\Omega^2)^{-\frac{1}{2}}$$

$$\frac{d|H(j\Omega)|}{d\Omega} = -\frac{1}{2} \cdot 2B^2\Omega(1+B^2\Omega^2)^{-\frac{3}{2}}$$

$\Omega_{max} = 0.$ $|H(j\Omega)| = $ maximum

c).
From the Matlab result, when Omega is 0, the magnitude response also is maximum. Same as the calculation.

d).
The system H(s) decreases the higher frequency's magnitude while the frequency which closes to zero would have higher magnitude. As the frequency increases, its magnitude decreases. H(s) is the high frequency filter.

Step 5)
a). Calculation by hand.
b). The input signal of this step is x(t) = cos(Omega$_0$t)u(t) + cos(Omega$_1$t)u(t) which is a causal signal. While the signal in step 3 is a non-causal signal. When a causal signal convolves with h(t) which is also a causal signal, it would get a different output than with one causal and one non causal signal.

Part II.

Step 5). $x(t) = \cos(\Omega_0 t)u(t) + \cos(\Omega_1 t)u(t)$

a) From fourier table

$$\cos(\Omega_0 t)u(t) \xrightarrow{F} \frac{\pi}{2}[\delta(\Omega-\Omega_0)+\delta(\Omega+\Omega_0)] + \frac{j\Omega}{\Omega_0^2-\Omega^2}$$

$$X(\Omega) = F(x(t)) = \frac{\pi}{2}[\delta(\Omega-\Omega_0)+\delta(\Omega+\Omega_0)] + \frac{j\Omega}{\Omega_0^2-\Omega^2} + \frac{\pi}{2}[\delta(\Omega-\Omega_1)+\delta(\Omega+\Omega_1)] + \frac{j\Omega}{\Omega_1^2-\Omega^2}$$

$$H(\Omega) = \frac{1}{1+jB\Omega}$$

$$\boxed{Y(\Omega) = X(\Omega) \cdot H(\Omega)}$$

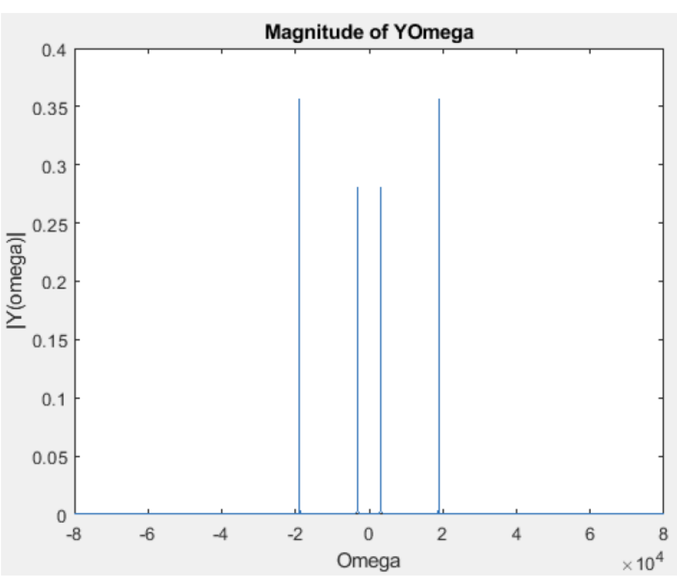
b). $x(t) = \cos(\Omega_0 t)u(t) + \cos(\Omega_1 t)u(t)$

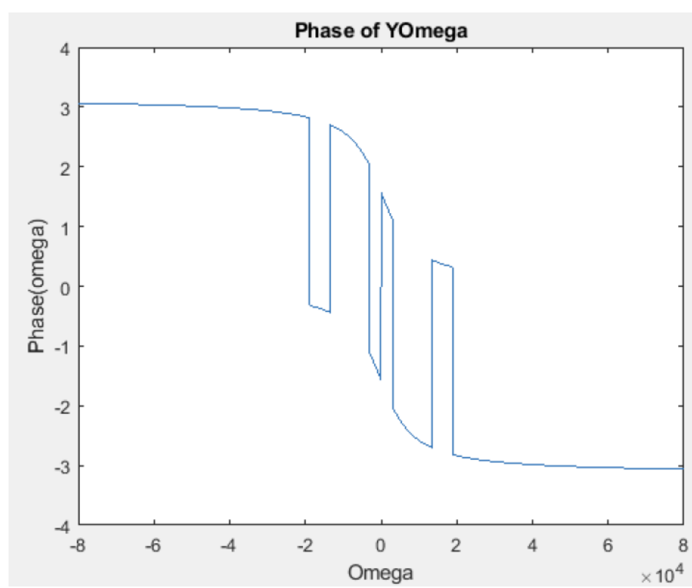
$$H(s) = \frac{1}{1+Bs} \qquad H(s) = \frac{\frac{1}{B}}{\frac{1}{B}+s} = \frac{1}{B} \cdot \frac{1}{s+\frac{1}{B}}$$

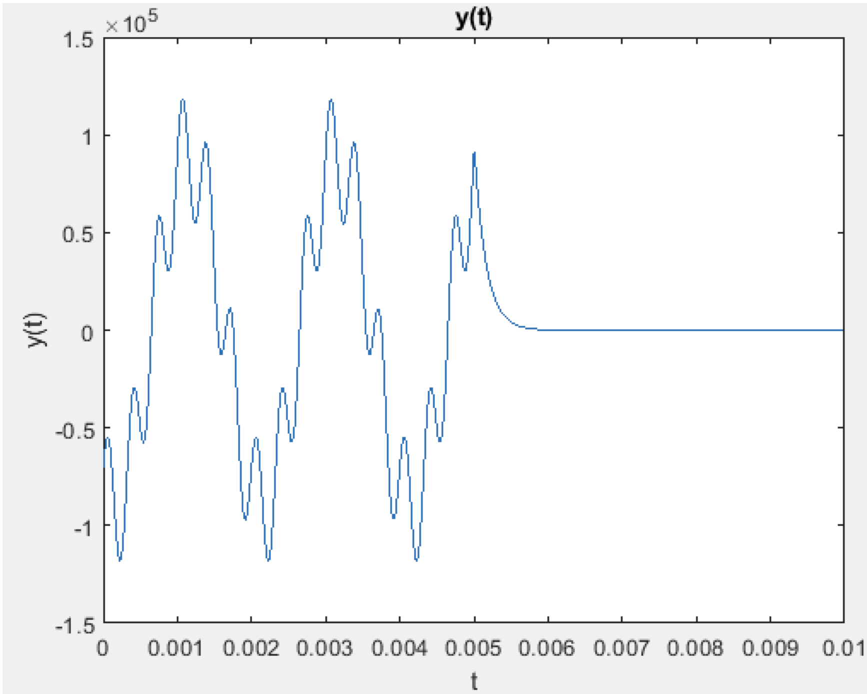
$$\boxed{h(t) = \frac{1}{B} e^{-\frac{1}{B}t} u(t)}$$

$$y(t) = h(t) * x(t)$$

```
%Step 5)
%(a)
B=1.27/8000;
Omega0=2*pi*500;
Omega1=2*pi*3000;
j=sqrt(-1);

dOmega = 10;
Omega = -80000:dOmega:80000;

HjOmega = 1./(j*B*Omega+1);
XOmega = (pi/2).*(dirac(Omega-Omega0)+dirac(Omega+Omega0))...
    + (j.*Omega)./(Omega0.^2-Omega.^2) + (pi/2)*(dirac(Omega-Omega1)...
    + dirac(Omega+Omega1)) + (j*Omega)./(Omega1.^2-Omega.^2);
YOmega = XOmega.*HjOmega;

figure(1);
plot(Omega,abs(YOmega));hold on
xlabel("Omega");
ylabel("|Y(omega)|");
title("Magnitude of YOmega");

figure(2);
plot(Omega,angle(YOmega));hold on
xlabel("Omega");
ylabel("Phase(omega)");
title("Phase of YOmega");

%Step 5,(b)
ts=1/100000;
t=0:ts:0.01;
ht=(1/B).*exp(-(1/B)*t).*heaviside(t);
xt=cos(Omega0*t).*heaviside(t)+cos(Omega1*t).*heaviside(t);
yt=conv(xt,ht, 'same');
figure(3);
plot(t,yt);
xlabel("t");
ylabel("y(t)");
title("y(t)");
```





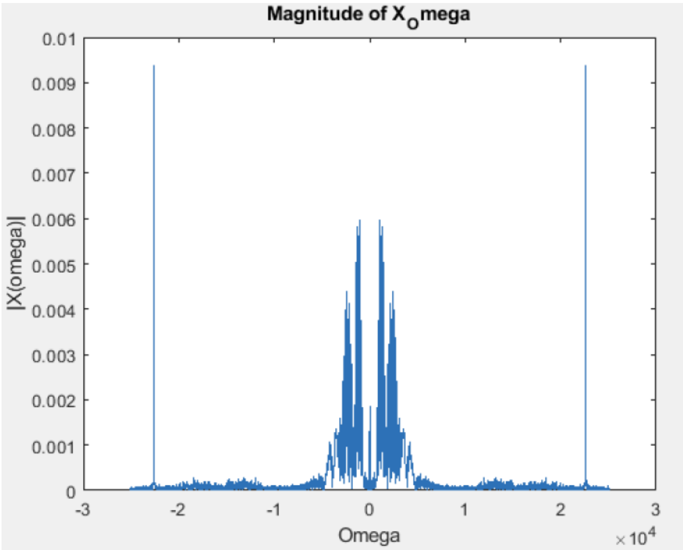

y(t)
×10^5
1.5
1
0.5
0
-0.5
-1
-1.5
y(t)
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01
t

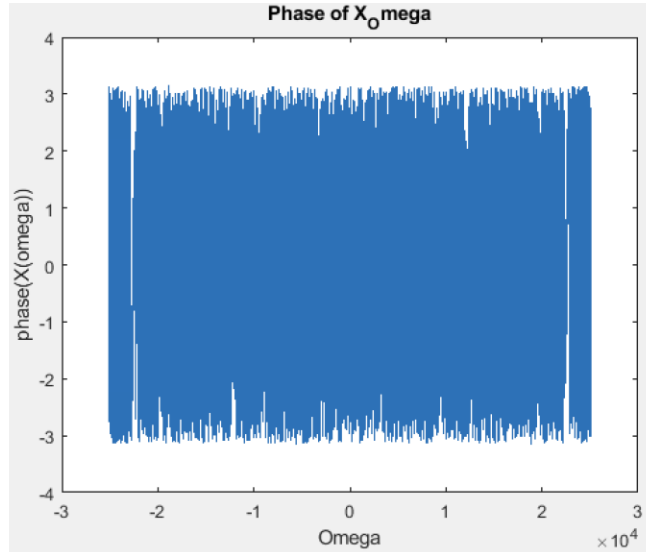
Step 6)

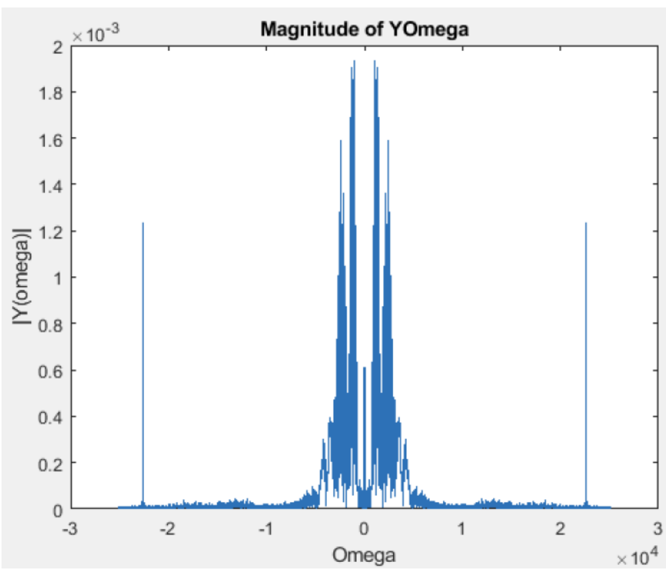
6a). There is single tone of sound like ‘B……’ which frequency is higher than human, and it is louder than the human sound and makes the distortion. Therefore, the distortion of this file is the single tone.

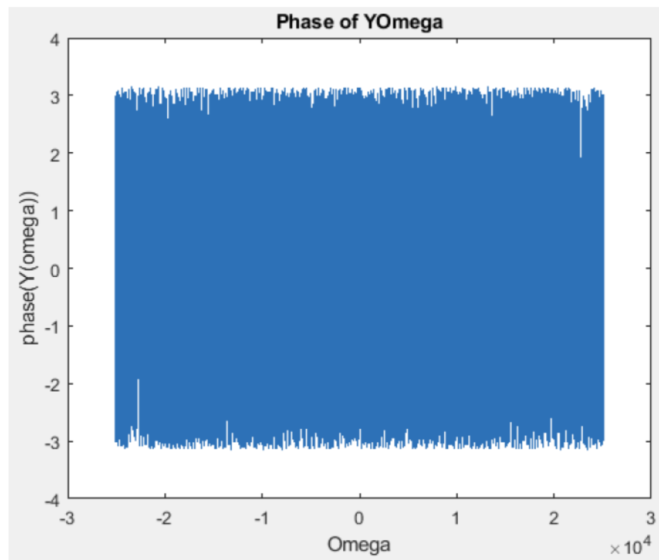
6b)

```
[x,fs]=audioread('Z:\Assignment3\distorted.wav');

B=1.27/8000;
j=sqrt(-1);
t=1/fs:1/fs:length(x)/fs;

ht=(1/B).*exp(-(1/B)*t).*heaviside(t);
[Omega,X_Omega]=calc_dft(x,fs);
HjOmega = 1./(j*B*Omega+1);

y=conv(x, ht).*(1/fs);
[yOmega,YOmega]=calc_dft(y,fs);

figure(1);
plot(Omega,abs(X_Omega));hold on
xlabel("Omega");
ylabel("|X(omega)|");
title("Magnitude of X_Omega");

figure(2);
plot(Omega,angle(X_Omega));hold on
xlabel("Omega");
ylabel("phase(X(omega))");
title("Phase of X_Omega");

figure(3);
plot(yOmega,abs(YOmega));hold on
xlabel("Omega");
ylabel("|Y(omega)|");
title("Magnitude of YOmega");

figure(4);
plot(yOmega,angle(YOmega));hold on
xlabel("Omega");
ylabel("phase(Y(omega))");
title("Phase of YOmega");

sound(y,fs);
audiowrite('Z:\Assignment3\recovered.wav',y,fs);

function [Omega,X_Omega] =calc_dft(x,fs)
x=x(:);
f = linspace(-fs/2,fs/2,length(x));
Omega=f(:)*2*pi;
X_Omega= fftshift(fft(x))/length(x);
end
```









6c). From the magnitude spectrum, H(s) decreases the frequencies magnitude especially the higher frequency. The magnitude of high frequency (noise) decreases from 0.009 to 0.0012. The human voice's (low frequency) magnitude is higher than the noise's (high frequency) magnitude after pass this this system. Therefore, the distortion is affected and becomes lower. H(s) is the high frequency filter, and it filters out the higher frequency signal.