Muon Physics

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Muons are a type of elementary particle similar to electrons. The purpose of this experiment was to measure and characterize the behavior of Muons related to time and energy. In particular, our goal was to test and determine the lifetime of a Muon, as well as the effects of time dilation and the force of the weak interaction. This experiment was performed using a plastic scintillator that can measure the amount of time it takes for a muon of sufficiently low energy to enter the scintillator, stop, and decay inside. By properly fitting the decay distributions with an exponential function, we are able to calculate a value for how long a muon can propagate in terms of its energy. And with that relationship modeled, it becomes possible to extrapolate muon count rates for any point in the range of their travel in earth's atmosphere. This is to be demonstrated, once we correct for time dilation and energy/intensity variations.

I. Introduction to Muons

A muon is a kind of elementary particle that is "electron-like" in terms of its atomic interactions. Specifically, Muons have half-integer spin, and do not interact by the strong force. Particles that exhibit these properties are known as leptons. Electrons and muons are both charged leptons, however muons are not characteristically found amongst common matter like electrons are.

Muons are generated in Earth's upper atmosphere as a result of high energy cosmic rays that disintegrate into numerous particles upon contact with atmospheric nuclei. Pions are one such particle to result from this, and it is from them that the Earth's diffusive population of muons originates. A charged pion can spontaneously decay by the weak force into a muon and a neutrino. Not all Pions do this; Some will continue to travel, losing energy to air nuclei via the strong interaction.

On their own, Muons will simply travel through the atmosphere until they lose enough kinetic energy and decay by the weak interaction into an electron and a neutrino. It is this decay that will be used to measure the muon lifetime. Where and when this decay occurs is related to the energy of the muon, and a distribution for the muon's decay time can be constructed to demonstrate this.

Muon decay is represented in much the same way as radioactive decay. The equation for radioactivity is:

$$dN/dt = -\lambda N \tag{1}$$

Where N is the total number of atoms, t is time, and λ is the "decay constant" which characterizes the rate at which the material will decay radioactively. This relationship can also be used to describe the decay of particles not by radioactivity, but by the weak interaction.

For our experiment the number of muons present in a system is considered to be a decreasing function of time, N(t), and from the radioactivity model we have:

$$dN/N(t) = -\lambda dt \tag{2}$$

Then by integration, the number of muons left after time t, is:

$$N(t) = N_0 e^{-\lambda t} \tag{3}$$

where N_0 is the initial number of muons. So The muon population is of a simple exponential form. The "lifetime" of a muon, is simply the reciprocal of the decay constant λ:

$$\tau = \frac{1}{\lambda} \tag{4}$$

By plugging 3 into 2, and then differentiating, an expression is found for the decay time distribution D(t), of a muon:

$$D(t) = -dN/N_0 = \lambda e^{-\lambda t} \quad (5)$$

This is the decay probability of a muon as a function of time. Since it has an exponential distribution, and is unchanged by the value of N_0, it is the case that at any time, any muon can be described by this distribution. It is through averaging the decay probability across different time intervals that the Muon lifetime τ, can be determined. To clarify, it does not matter that we are unable to observe a muon's entire life cycle start to end. All muons are subject to the same decay probability at any given time. The muon lifetime is dependent on its *energy.* The amount of energy lost by a muon as it traverses matter is pleasantly constant. About (2 MeV/c^2). So as long as the energy of the muon is known, then it is trivial to determine when it will lose enough energy to decay. The muon lifetime tells us the relationship between energy and time for muons and that is what's important for understanding particle behavior.

II. Equipment & Procedure

A scintillator was used to measure the decay time interval of muons. A scintillator is a material that fluoresces when excited by ions. Common scintillation materials are typically made of a mixture of fluor molecules that saturate a large volume of polymers. Such is the case for the "plastic scintillator" used in the lab.

The way scintillation is used to detect muons involves integration with an electronic circuit that can use voltage signals to operate a special timer. The first component for this circuit is a photomultiplier tube (or PMT). The PMT converts incident photons from the scintillator into an electrical signal.

The signal produced by the PMT consists of an alternating current that is subsequently sent through a two-stage amplifier. An amplifier uses externally supplied voltage to increase the amplitude of the signal that is supplied to it. This supplies the discriminator with a signal of higher voltage.

From there, the discriminator, or "voltage comparator", is used to 'decide' whether or not to activate the timer. It does this by comparing the voltage input given to it with a reference voltage.If the signal voltage is higher than the reference voltage, then the discriminator outputs a logic signal that will activate the timer. If the reference voltage is greater than the signal produced by the amplifier, then the output will not trigger the timer.

The final component, the device that actually takes the data, is an FPGA timer. An FPGA, or "Field-Programmable Gate Array" is an integrated circuit that is customizable for a range of different uses. An FPGA is intended to be configured by the end user in a way that allows for a logical circuit or processor to be designed separately, and then programmed into a "blank" chip, the gate array.

The experimental procedure involved the use of this circuit to measure muon activity at the elevation of Athens,GA over a large time period. 4 separate trials were run and the threshold, or 'reference voltage' was increased for each successive run. The way that the apparatus detects and measures a muon decay is when two consecutive electrical signals are picked up by the PMT within a short enough time interval. This indicates that a muon entered the scintillator, at which point a signal is generated and if the voltage is sufficiently high, the FPGA timer will start. Then the muon must lose enough kinetic energy to stop, and decay inside the scintillator. The muon decays into a neutrino and an electron. The electron in particular is much less massive than the muon, and as such has significantly more energy. Thus, the electron will again cause the scintillator to fluoresce, sending another signal through to the FPGA timer. This second signal stops the timer, and the measured time interval is what gets recorded in the data.

III. Analysis & Results

The initial data set was collected with the discriminator value set at 148 mV. Of the 1.1 million scintillation signals produced, 13082 were instances of decay. The time intervals for each of these decays were assembled in a histogram by frequency, and then normalized to give the distribution below:

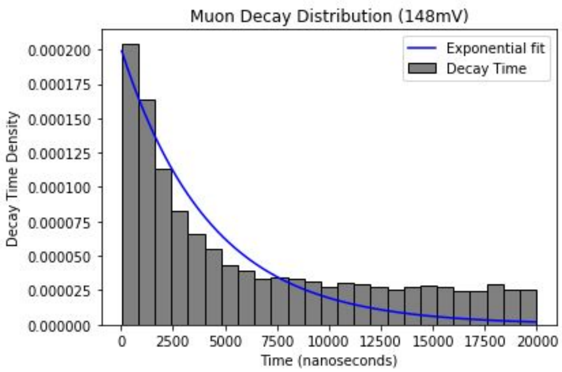

Figure 1

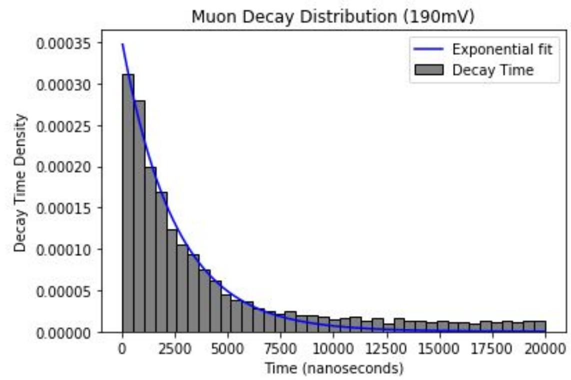

Figure 2

The curve fitted to the histogram, (and that will be fitted to each of the data sets to follow,) is of the exponential decay form for D(t) in equation (5). By performing a curve fit to the histogram, the values for the muon lifetime τ, and the decay constant λ, can be determined. The function used for the fit is given in terms of τ:

$$D(t) = Ae^{-\frac{t}{\tau}}$$

Here the coefficient A includes a factor of λ, by equation (5). For the first data set, the calculated values were: $A = 2.00789 \times 10^{-4}$, $\tau = 4.28039 \times 10^{3}$. From equation (4) we have that $\lambda = 1 / 4.28039 \times 10^{3}$ The accepted value for the muon lifetime is 2,2 microseconds. This means the percent error in τ for this first instance is alarmingly high. 94.54%. This is not particularly surprising though, given that our observed value is almost twice what the accepted value is. Fortunately however, by inspection of the histogram it seems likely that the egregiousness of the error is a result of the data containing a lot of excess time intervals that likely aren't true muon decays. Most likely this is due to the discriminator voltage being too low, thereby allowing too much noise in the data, and making an accurate curve fit impossible as a result.

The remaining three data sets were analyzed following the same procedure: The decay intervals were sorted out, and a curve was fitted to the resulting normalized histogram, The second set of data was collected with the discriminator set to 190 mV. Figure 2 gives the plot of the results.

And the coefficients for the curve were as follows:

$$D(t) = 3.52804x10^{-4} e^{-\frac{t}{2.52769x10^{3}}}$$

This gives 2.52769×10^{3} ns for the muon lifetime τ, and then decay constant $\lambda = 1 / 2.52769 \times 10^{3}$. The error in our observed value for the second set is 14.86%.While this is undeniably an improvement, it is still distastefully high. Looking at the histogram, it seems to still contain some background detection, as there are many signals still being detected beyond the expected muon lifetime. As the timer goes past 2.2 microseconds, we expect that the frequency of lengthier intervals will eventually decrease to zero.

For the third set of data, the discriminator was set to 260 mV, and the plot is shown below.

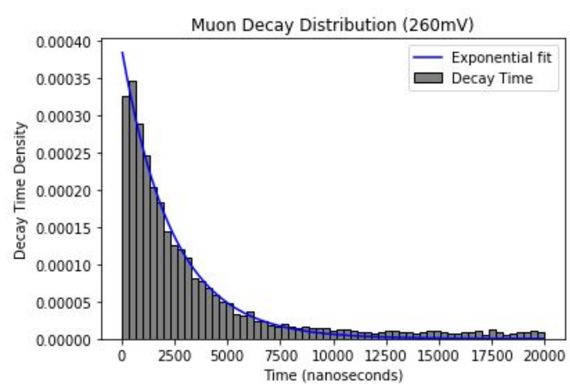

Figure 3

The coefficients were calculated:

$$D(t) = 3.91107x10^{-4}e^{-\frac{t}{2.39119x10^3}}$$

And so we have $2.39119x10^3$ ns for the muon lifetime. This value presents an error of 8.68%, which means we've gotten τ to fall within the same order of magnitude as the accepted value.. Looking at figure 3, the lengthy decay intervals, ones that are almost certainly not proper muon decays, do seem to be disappearing. This more or less confirms that the discriminator voltage was far too low in the first two runs, and as the discriminator filters out more low voltage signals the data fits true muon behavior much more precisely.

The last set of data was gathered with the discriminator set to 550 mV. Figure 4 shows the plot.

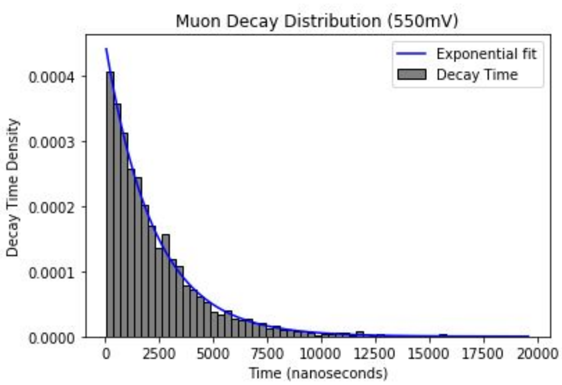

Figure 4

With the following curve:

$$D(t) = 4.49619x10^{-4}e^{-\frac{t}{2.25455x10^3}}$$

Here $\tau = 2.25455x10^3$ ns. This gives the final data set a percent error of 2.45%. Though this is still not an ideal amount of error, it does strongly suggest that the function D(t) that we have tried to fit to the distribution is in fact, the correct form for density. Having the correct model means that the error at this point is primarily statistical; Further minimization would require the collecting of better, larger data sets, rather than optimization of the apparatus. In point of fact, the plot in figure 4 shows almost zero detection of decay past the expected intervals for muons, suggesting that the best voltage for the discriminator to detect muon decays exclusively is about 550mV. Hence, there is little more adjustment needed with the setup.

IV. Conclusions

The value for the muon lifetime is given by:

$$\tau = (192\pi^3\hbar^7)/(G_f^2 m^5 c^4)$$

Where m is the muon mass, and G_f is the Fermi Coupling constant. Using the observed $\tau = 2.25455x10^3$ ns from the last set of data, and the muon mass m = 105.65838 MeV / c^2, the Fermi coupling constant can be calculated. As such, our experimental value is $G_f = 1.25698x10^{-5}$ (GeV$^{-2.}$) The accepted value of Fermi's Coupling constant is $G_f = 1.1663787x10^{-5}$. This puts our error at 7.76% which is higher than expected, given that the error in the muon lifetime was less than half of this. However, since τ is merely an *average*, and its observed value is heavily dependent on the data being used to determine it, G_f will certainly be subject to the same error as τ, or in some cases more so.

τ_{obs} denotes the average expected lifetime for a muon given that differently charged muons are subject to differing decay rates for λ. Positively charged muons will live slightly longer in the scintillator as they do not interact with nuclei in the material. Negatively charged muons have a tendency to lose kinetic energy faster to nuclei in the scintillator via the weak interaction. Unfortunately this means that any experimentally derived value for τ will actually be a result of the mean of two seperate lifetime values. Turning to our most accurate measurement for τ = 2.2545 microseconds, we can use $\tau_{obs} = \tau$ to help determine a value for the ratio of positive and negative charged muons in the scintillator.

This ratio is given by ρ:

$$\rho = -\frac{\tau^+}{\tau^-}\left(\frac{\tau^- - \tau_{obs}}{\tau^- - \tau_{obs}}\right) = \frac{N^+}{N^-}$$

Where τ^+, and τ^- represent the true muon lifetimes for positive and negative charged particles, and N^+, N^- both give the respective number of muons present within the system. We can then use $\tau = \tau_{obs} = 2.25455$ from our forth data set, and ascertain whether or not the average is skewed towards muons of a particular charge.

As it turns out, the measured p = N^+/N^-, was 3.9. This is 3 times the expected value of 1,3. Knowing this, it is now clear that our determined value for τ might be less than ideal. Even though the error for the muon lifetime was small, all that really means is that during the collection of the data, there was around 3 times as many positively charged muons entering the scintillator as there were negatively charged ones. If we wished to improve this measurement we should consider focusing on the collecting of better data, as this error seems fairly non-systematic and random.

The stopping rate for Muons in Athens is .02/sec. Dividing this by the correction factor for variation of energy spectrum: 1,5 +- .2, gives (.013+-.00234)/sec. Accounting for the energy lost by the particle as it travels vertically a distance of H=14806m, we have that a muon will lose ΔE = 36.9641680295 MeV of energy in transit. The transit time of the particle is t' = (2.37x10^-10) seconds, and from this we can calculate the stopping rate ratio between Athens and 15000m.This is given by R = e^(- t'/τ) = .999894565. Now the stopping rate for the upper altitude can be predicted by solving : (.02/sec) / y = .999894565. And we have that the stopping rate of muons detected 14806 m above Athens is 49.99/sec.