An interactive introduction to topological data analysis

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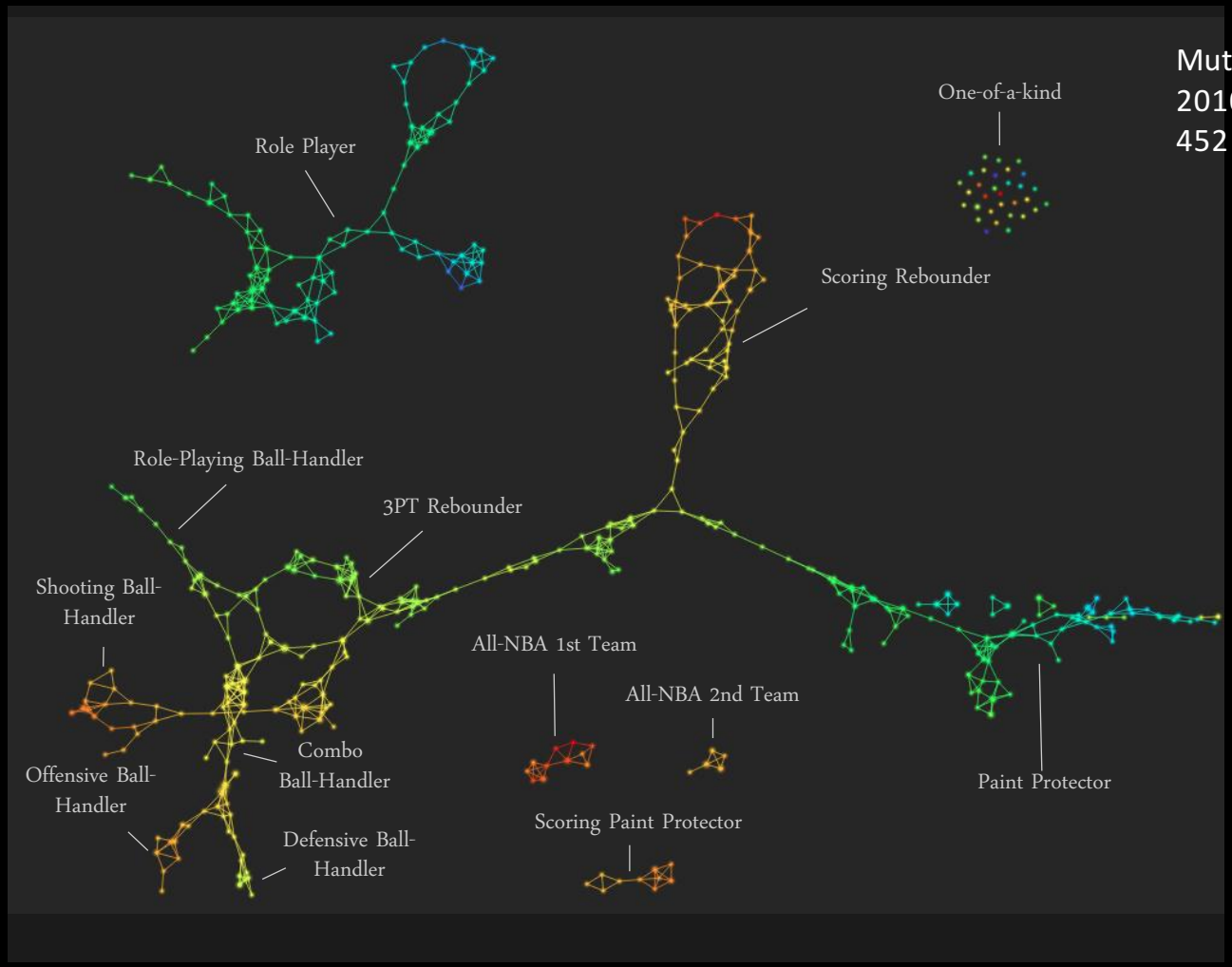
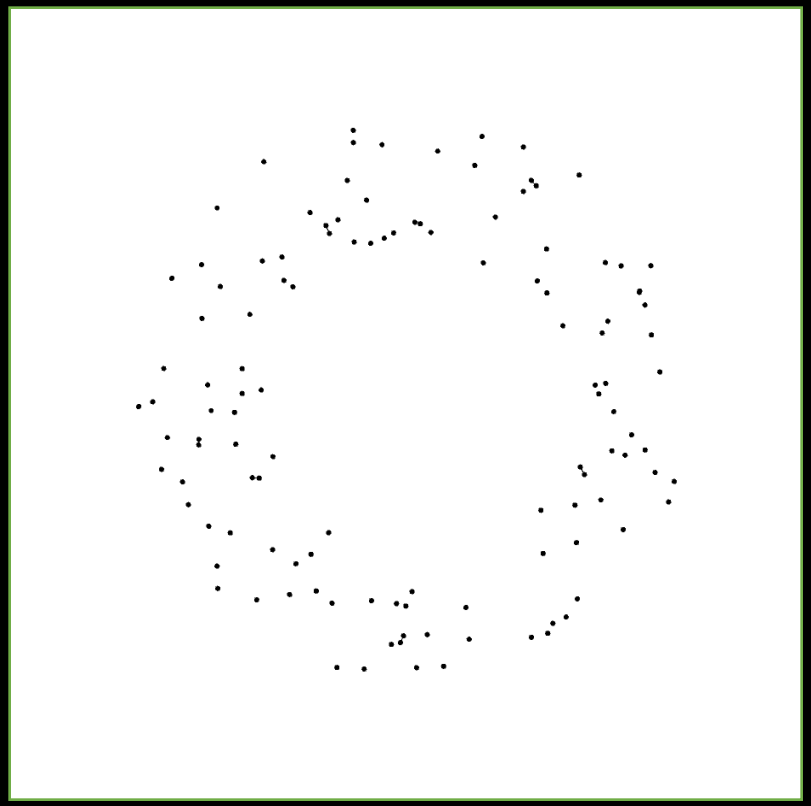
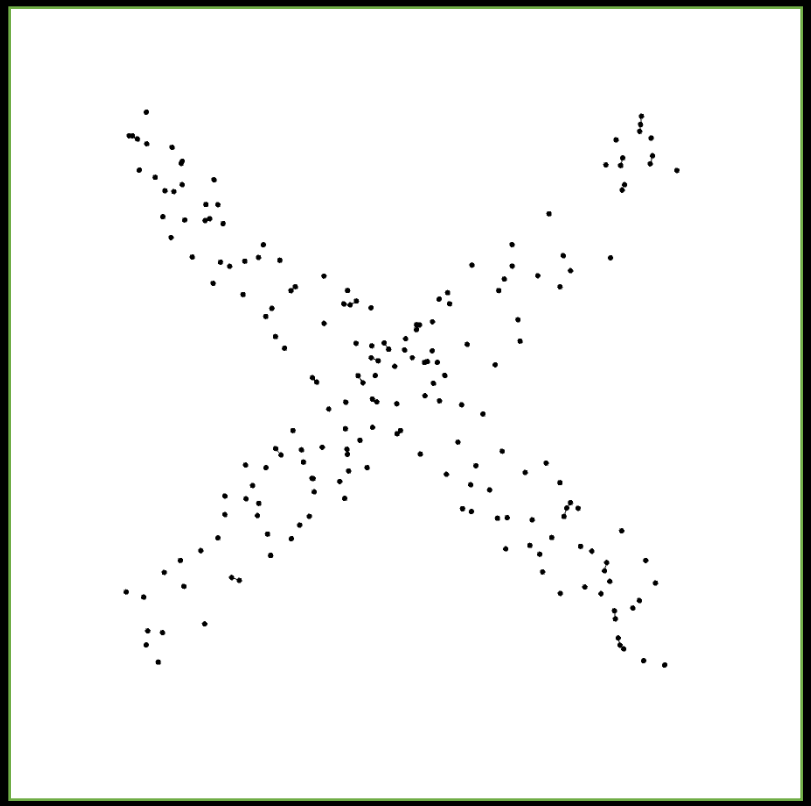
Bodies of Data Workshop
MAA MathFest, Portland 2014



Muthu Alagappan, Stanford
2010-2011
452 NBA Players, 7 normalized statistics

If your data form a high-dimensional point cloud, what is the shape?





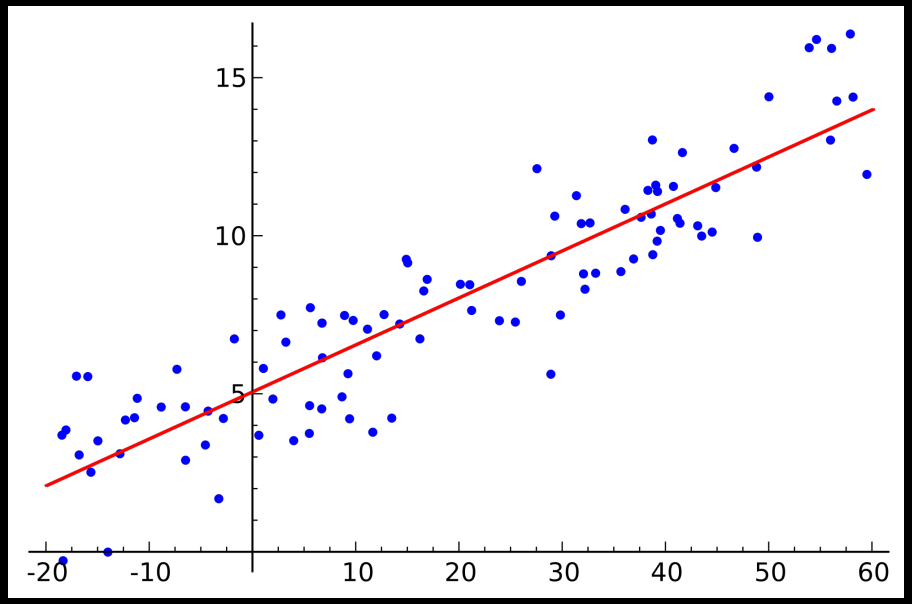
How is TDA different?

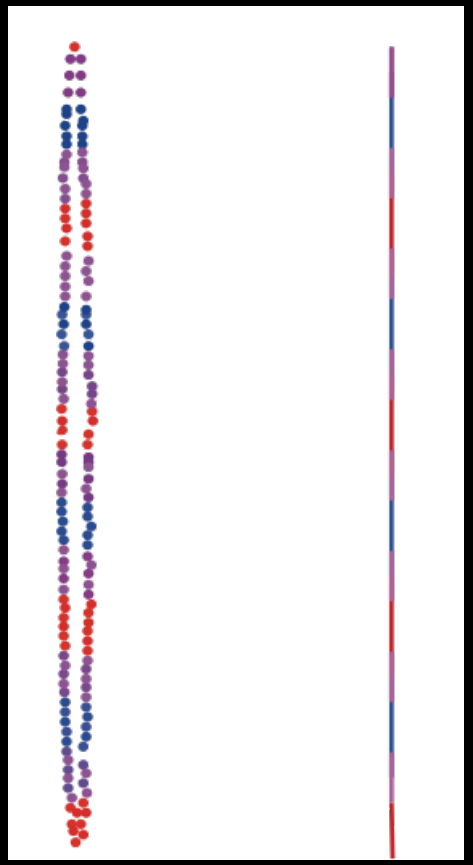
-- holistic/global: don't have to select for features beforehand

-- scale-independent

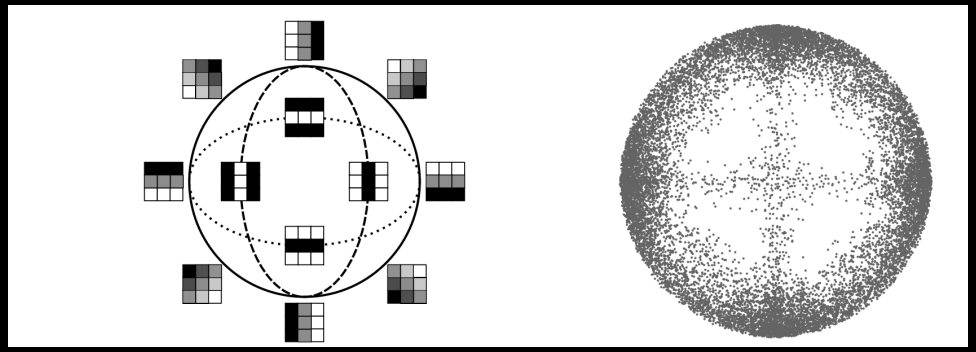
-- robust to noise and deformation

-- unique insights

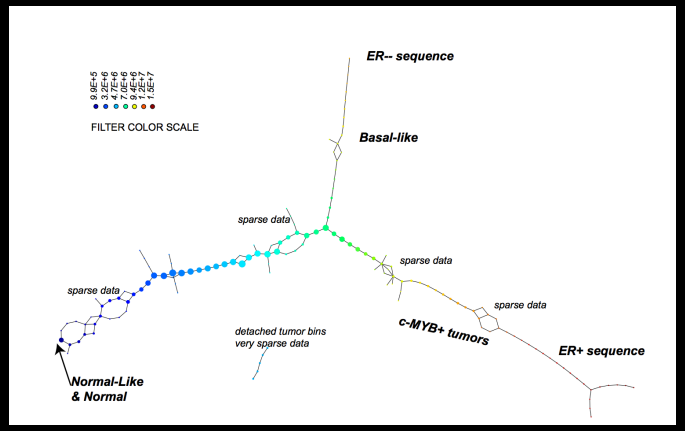




Examples of successes:



G Carlsson et al. International Journal of Computer Vision, 2008.



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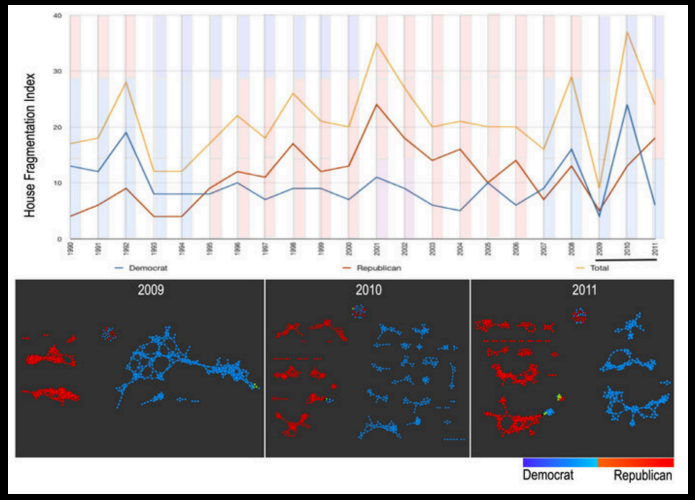

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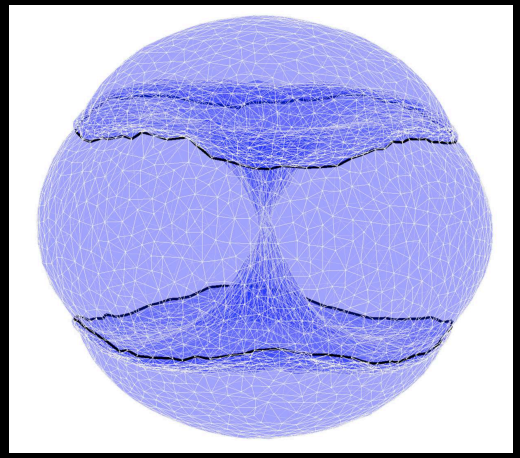
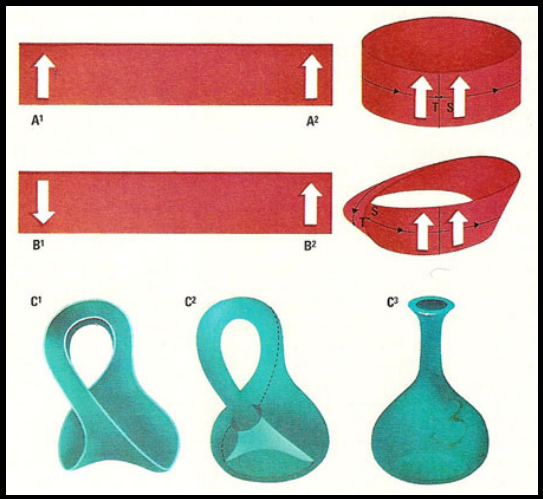
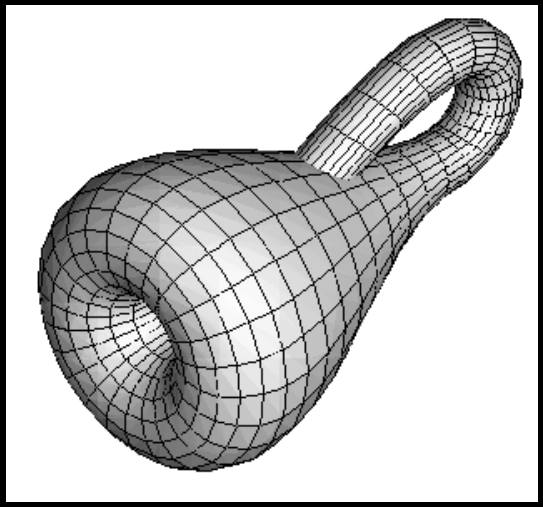


S Martin, JP Watson. Computational Geometry, 2011.

How does it work? What is topology?





Topology studies space and shape, allowing for continuous deformation.
Algebraic topology uses algebra to understand topology.

$$H_k(S^1 \times D^2) \cong H_k(S^1) \cong \begin{cases} \mathbb{Z} & \text{if } k = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

Most common method in TDA: persistent homology

Point cloud X

↓

Fatten X via r

↓

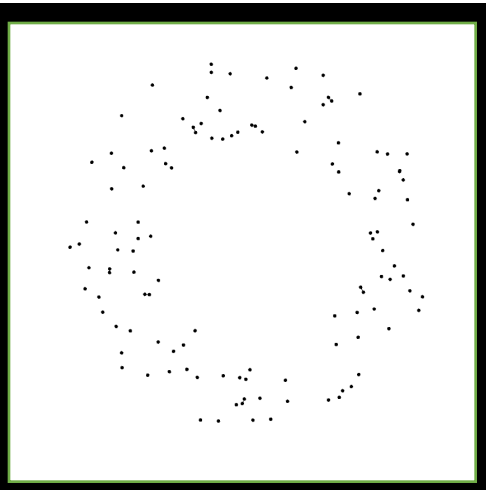
Vietoris-Rips complex VR(X,r)

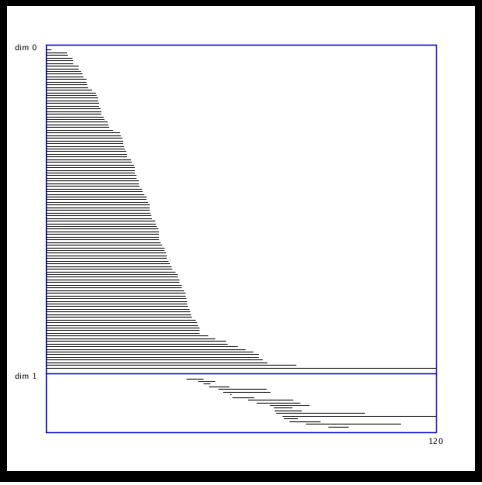
↓

Take homology

↓

Visualize in barcode





Demo using Processing…

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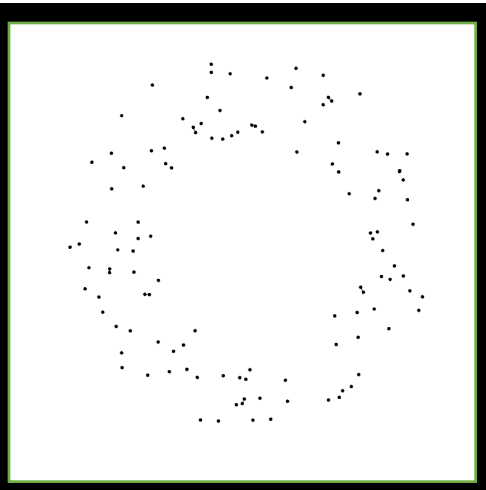
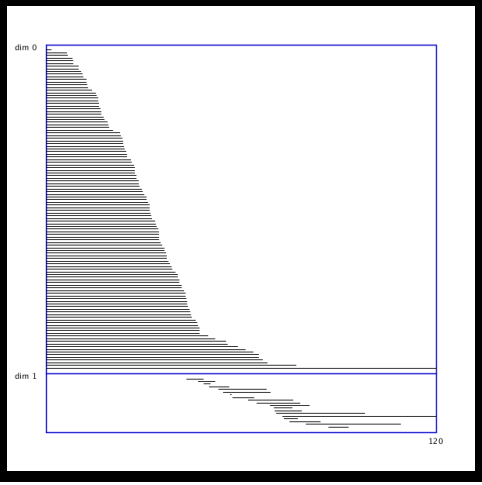
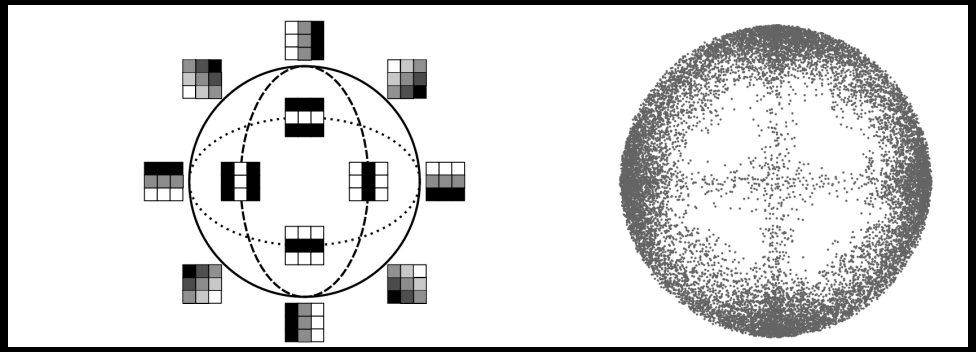




“Choose your own adventure” time:

1. Stability theorems: How do we know barcodes are telling us anything?
2. Revisit some of the applications: How has TDA been used?
3. The category of barcodes: What additional structure can we add to barcodes?
4. Where do high-dimensional bubbles come from?
5. Big picture: What does this mean for me and my life?

Examples of successes:



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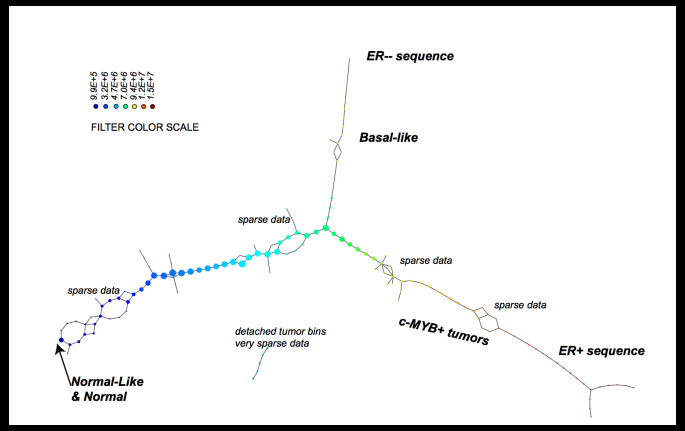

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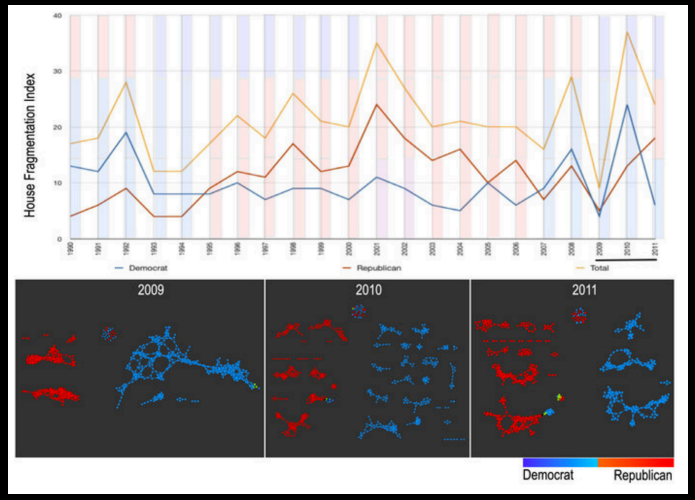

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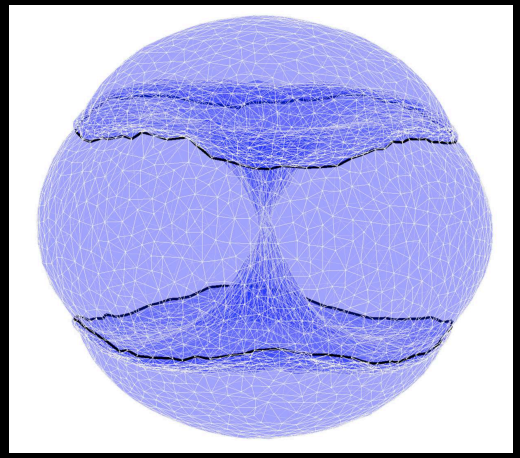

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The End.
Thank you!