Brain Modelling Workshop

Session 3:

Bifurcations in Neural Dynamics and Cognition

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Housekeeping

Slides + practical files:

github.com/lukewtait/intro_to_modelling

Questions: brainmodelworkshop.freeforums.net

Please unmute your mic at any point to ask questions, or write on chat. There are tutors monitoring it and ready to help you!

Plan

Talk:

1) Recap & Stability analysis
2) Types of bifurcations
3) Example from cognitive neuroscience
4) Intro to exercises

Exercises:

- Decision modeling with Wong&Wang model


$\frac{dV}{dt} = f(V)$
Stability analysis

Recap

Dynamical system - system that evolves according to a set of rules.

$$\dot{x} = f_r(x)$$

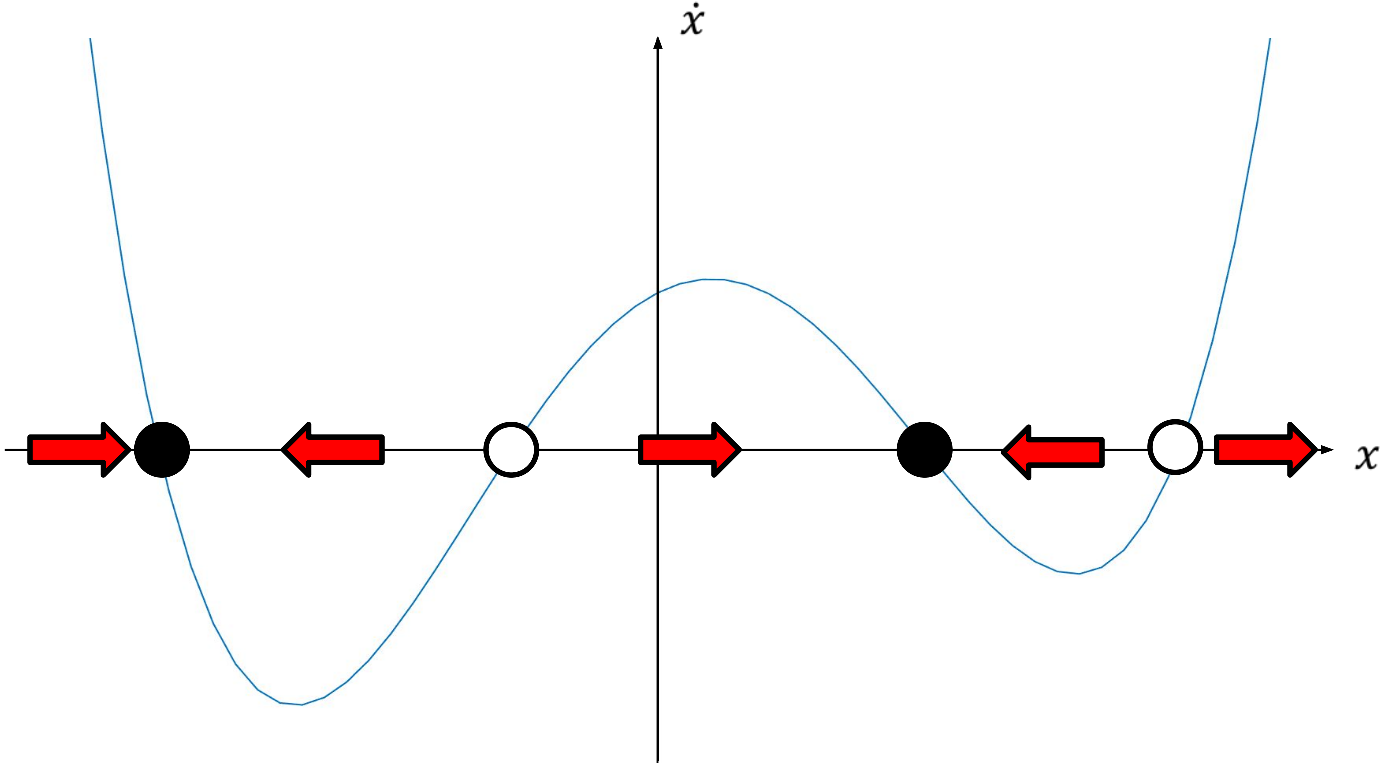
Parameters   state variables

$$f = r_0 \exp(x^2) + r_1$$

Recap

- ➢ We describe (continuous) **dynamical systems** using ordinary differential equations (ODEs)
- ➢ A one dimensional system (one equation) has 2 types of asymptotic dynamics:
  - $X \to \infty$
  - $X \to X_s$
- ➢ X_s is called a steady state, and can be stable or unstable

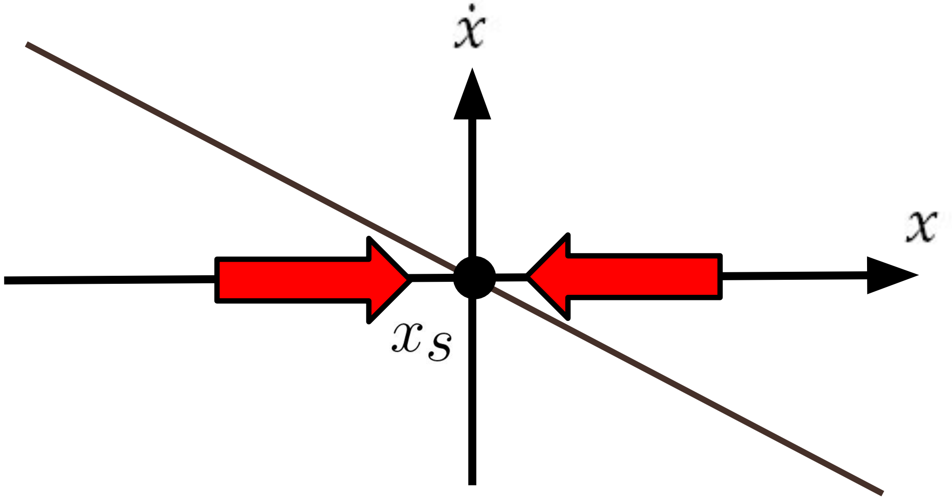
Stability analysis



Stability analysis

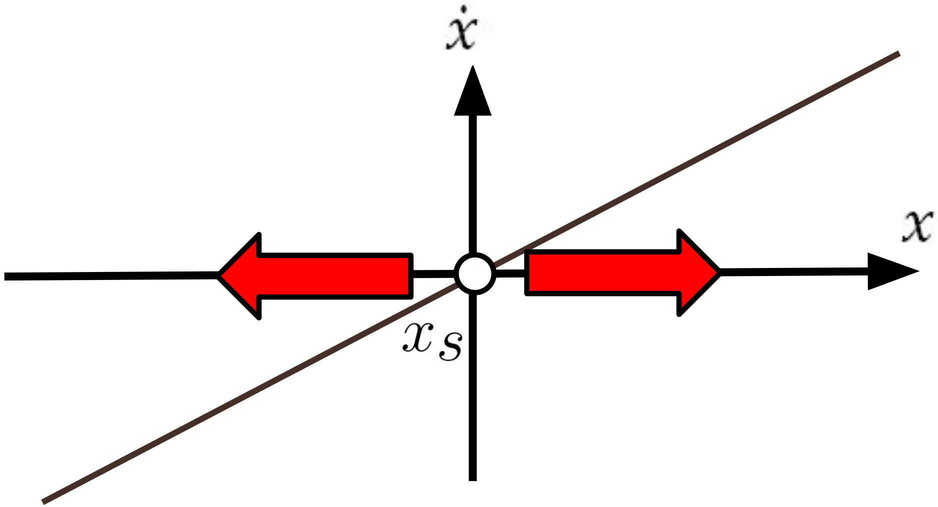
Stability analysis

Stable



Gradient = $\frac{df}{dx}(x_s) < 0$

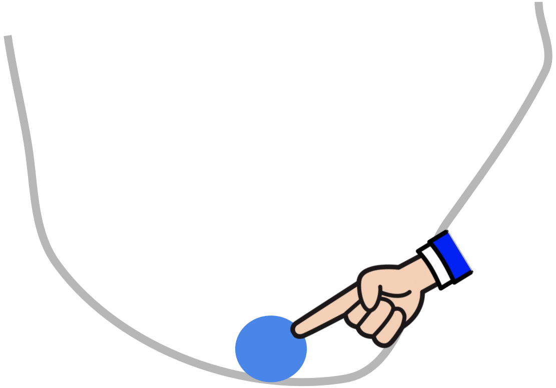
Unstable



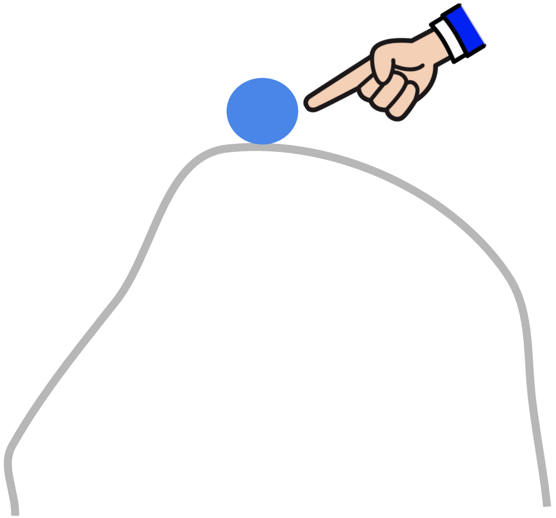
Gradient = $\frac{df}{dx}(x_s) > 0$

Stability analysis

Gradient = $\frac{df}{dx}(x_s) < 0$



Gradient = $\frac{df}{dx}(x_s) > 0$



Stability analysis

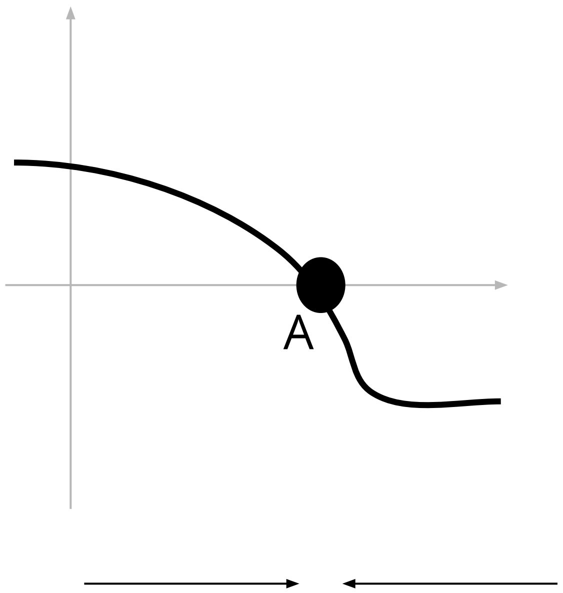
$$\frac{df(x_s)}{dx} < 0$$

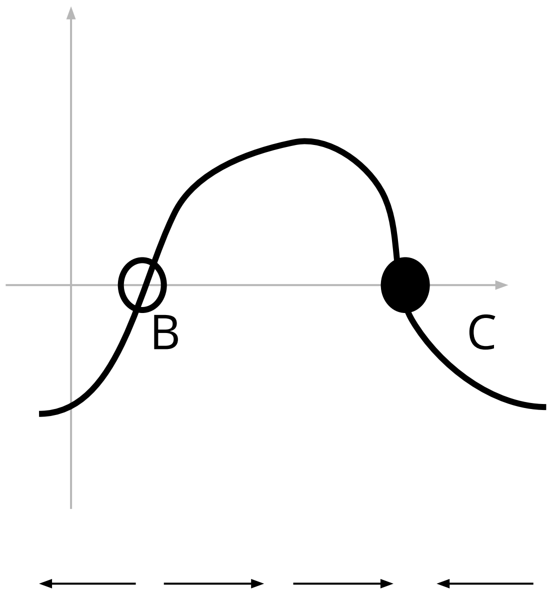
$$\frac{df(x_s)}{dx} > 0$$

stable

unstable

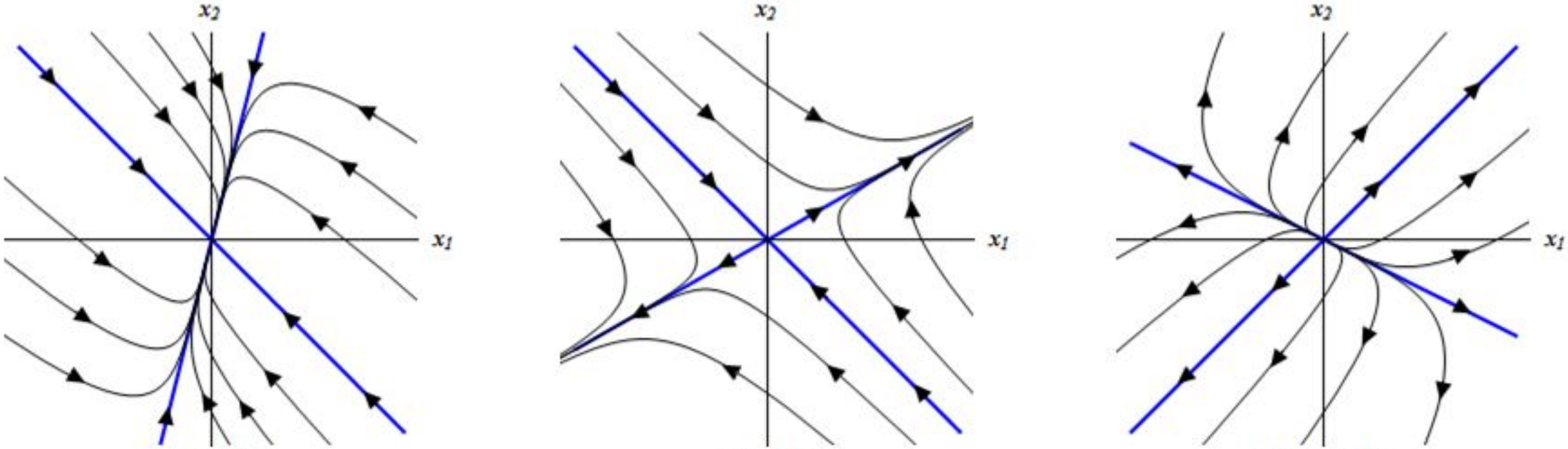
Stability analysis - exercise





Stability analysis in higher dimensions

In higher dimensions, we can calculate the gradient in each of the 'blue' directions below.
These 'gradients' are called **eigenvalues***



Stable
$\lambda_1, \lambda_2 < 0$

Saddle
$\lambda_1 < 0, \lambda_2 > 0$

Unstable
$\lambda_1, \lambda_2 > 0$

*Technically, they are eigenvalues of the *Jacobian matrix*, which quantifies the gradient in each variable with respect to all other variables.

Stability analysis in higher dimensions

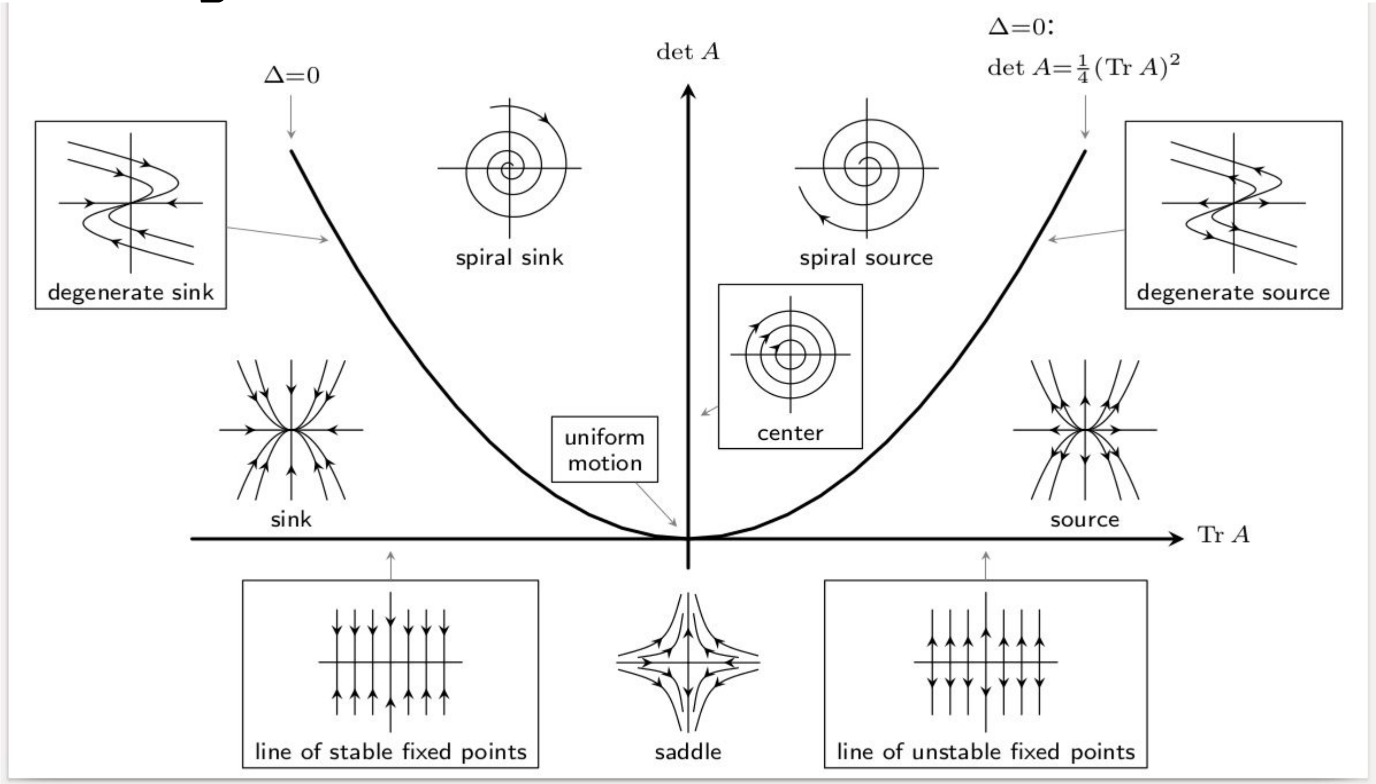
$\lambda < 0$

$\lambda > 0$

stable

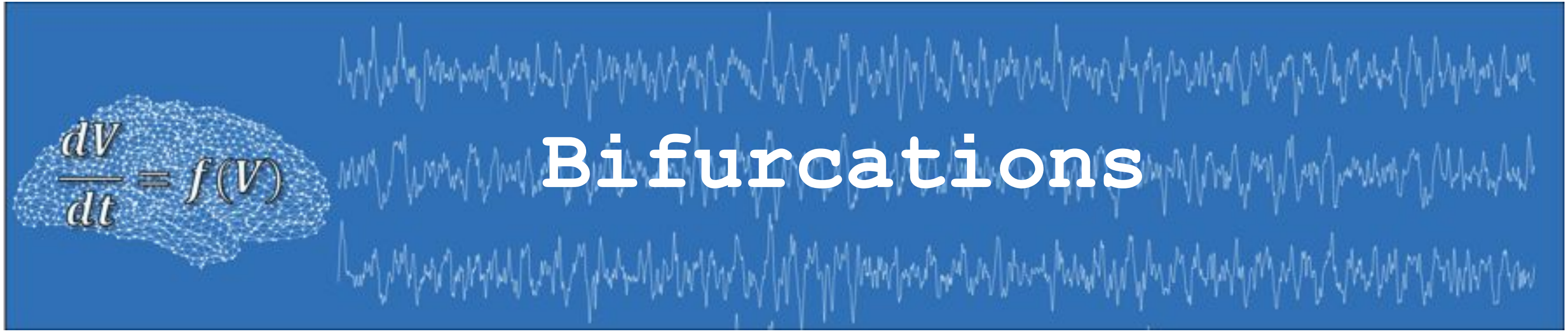
unstable

Poincare Diagram



https://en.wikipedia.org/wiki/Stability_theory

Bifurcations



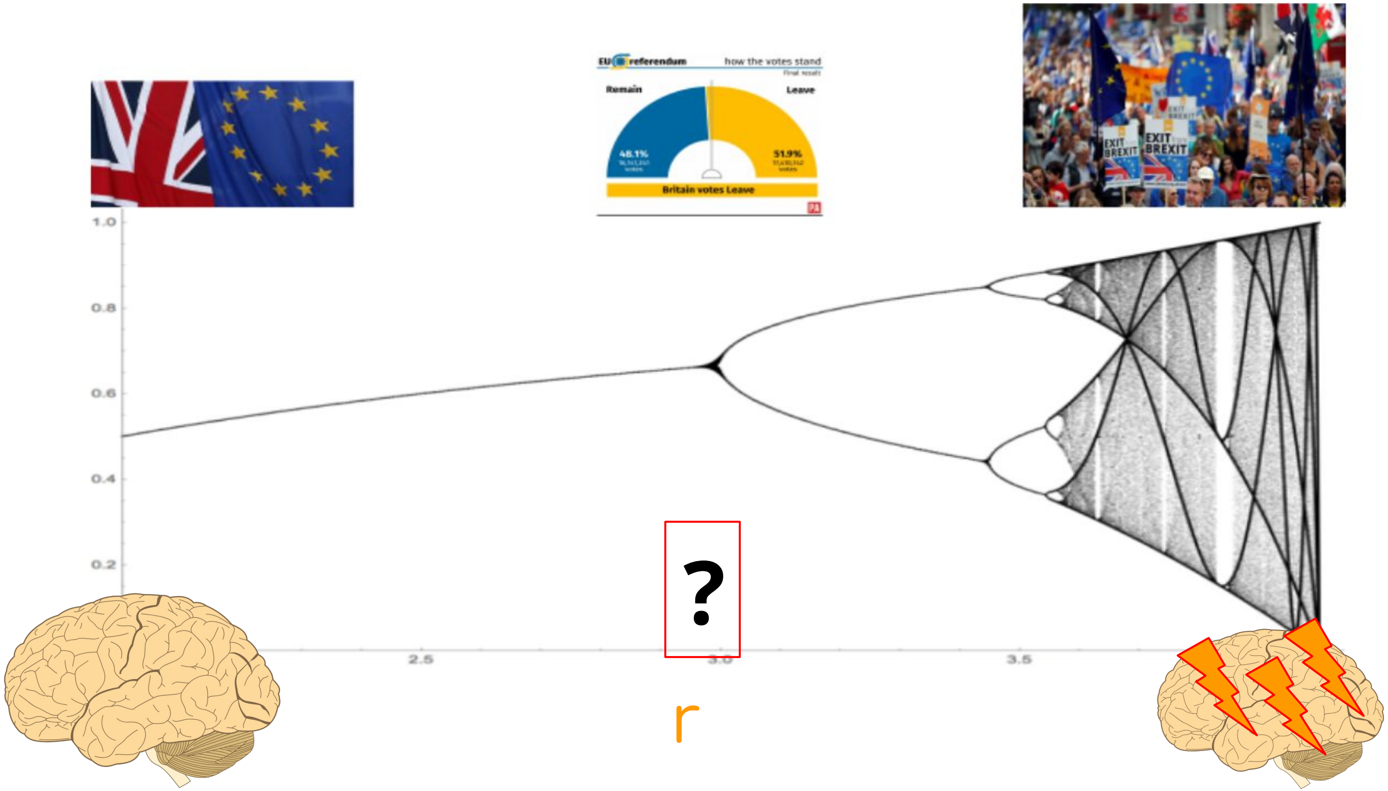
What is bifurcation?

Bifurcation occurs when a small smooth change made to the parameter values (the bifurcation parameters) of a system causes a sudden 'qualitative' change in its behavior. For example, a sudden change in stability of a steady state or oscillation.

$$f_r(x)$$

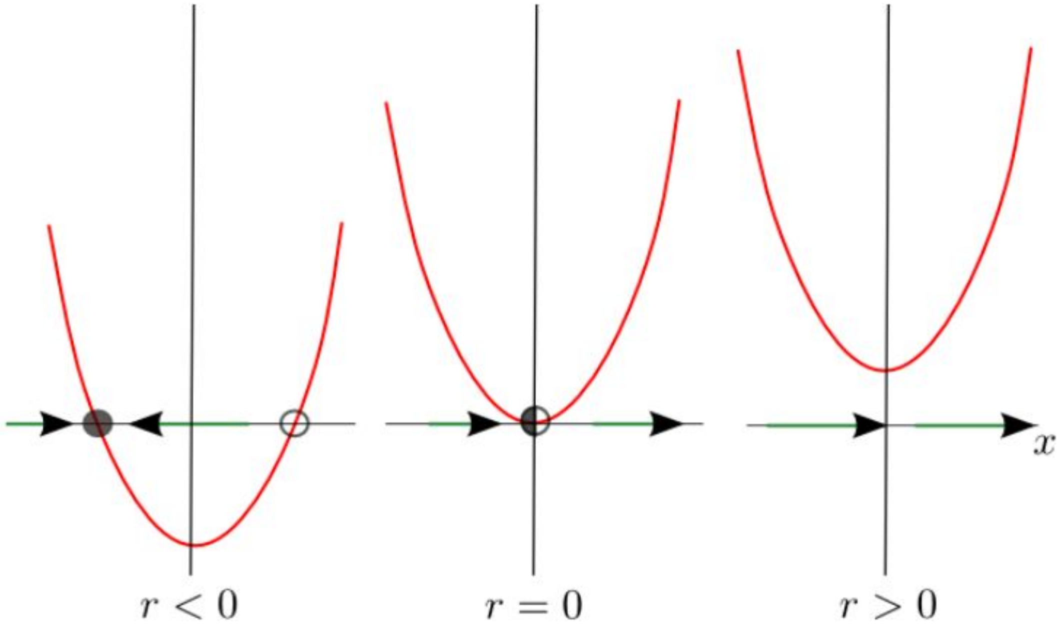
!!!

What is bifurcation?



Bifurcations

$$\dot{x} = r + x^2$$

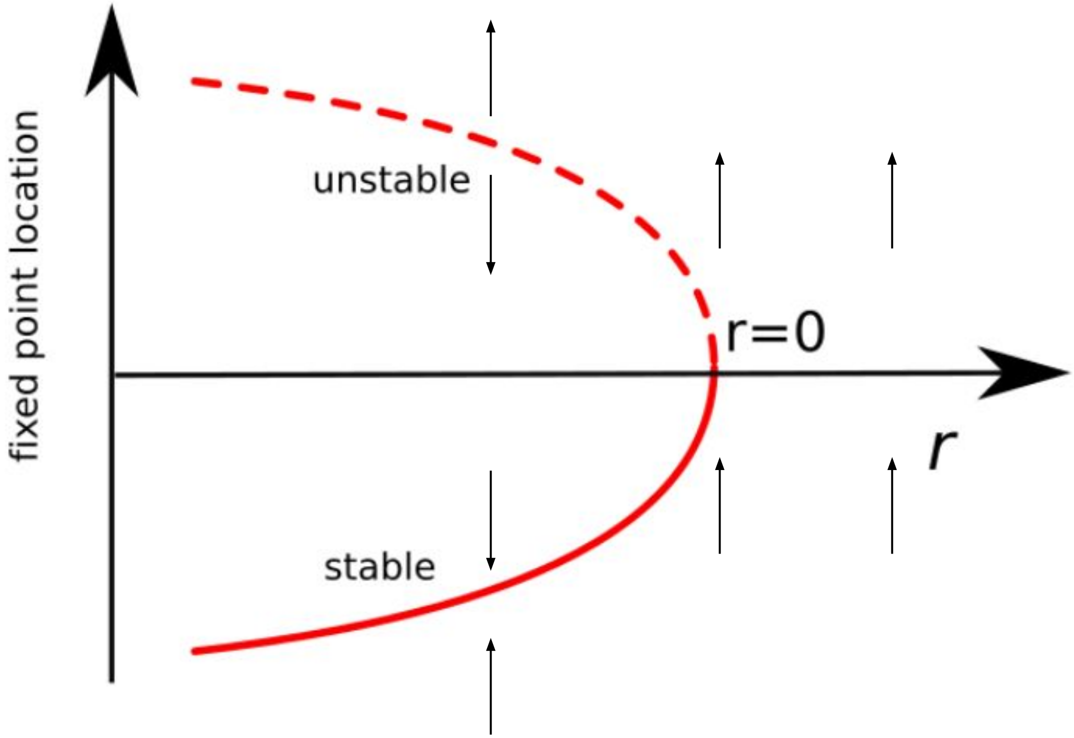


Some plots come from PHY411 Lecture notes Part 4 by Alice Quillen

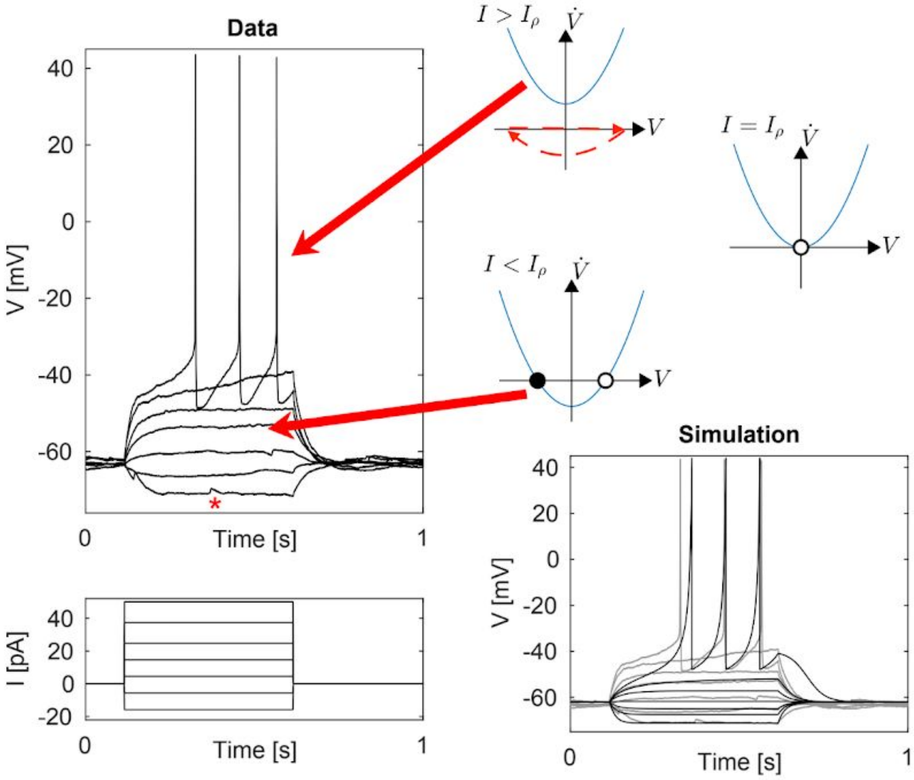
Bifurcation plot

Saddle-node bifurcation

$$\dot{x} = r + x^2$$



Quadratic neuron model



$$\dot{x} = r + x^2$$

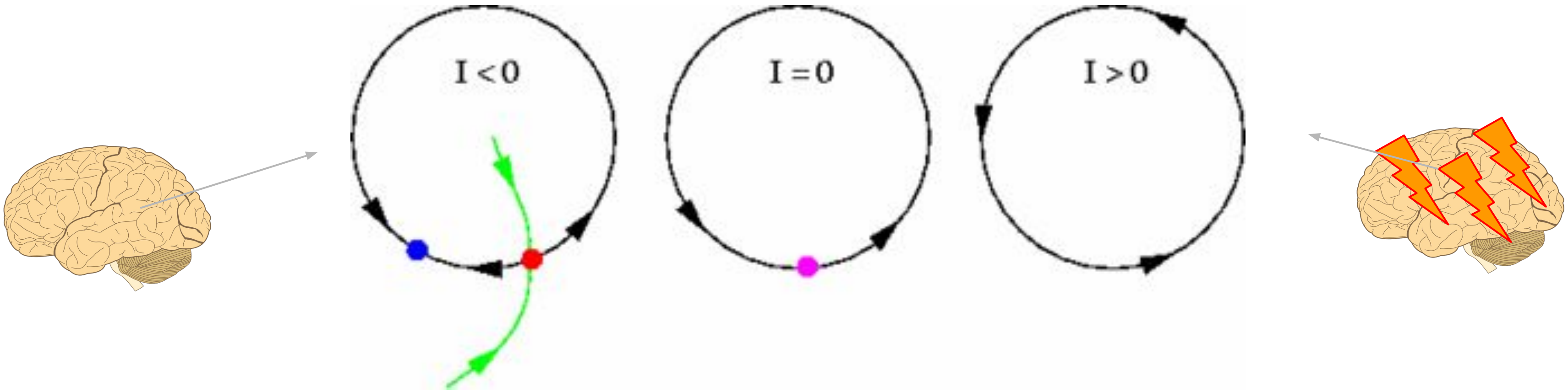
$$\dot{V} = V^2 + I$$

(with reset mechanism)

Handy model for mathematical analysis of **type I neurons**

Theta model

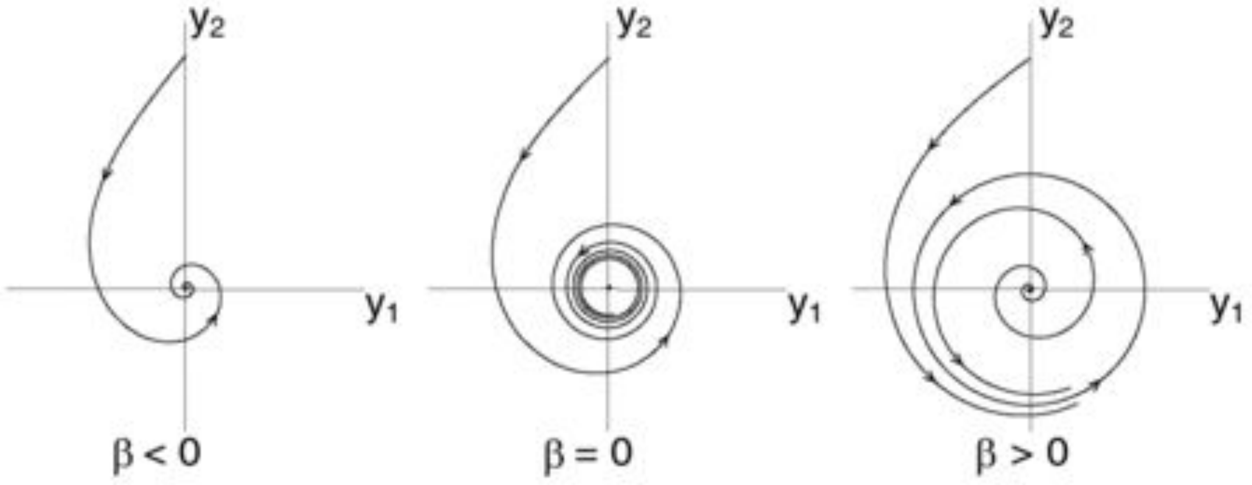
$$\frac{d\theta}{dt} = \dot{\theta} = 1 - \cos\theta + (1 + \cos\theta)I$$



Saddle node on limit cycle.

http://www.scholarpedia.org

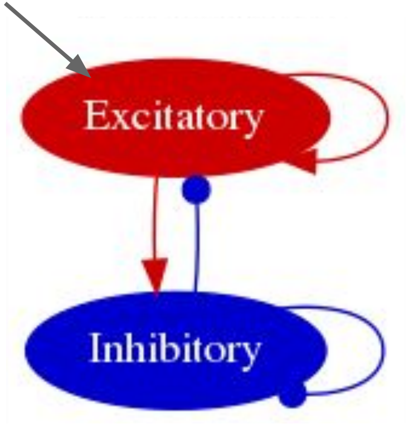
Andronov-Hopf bifurcation

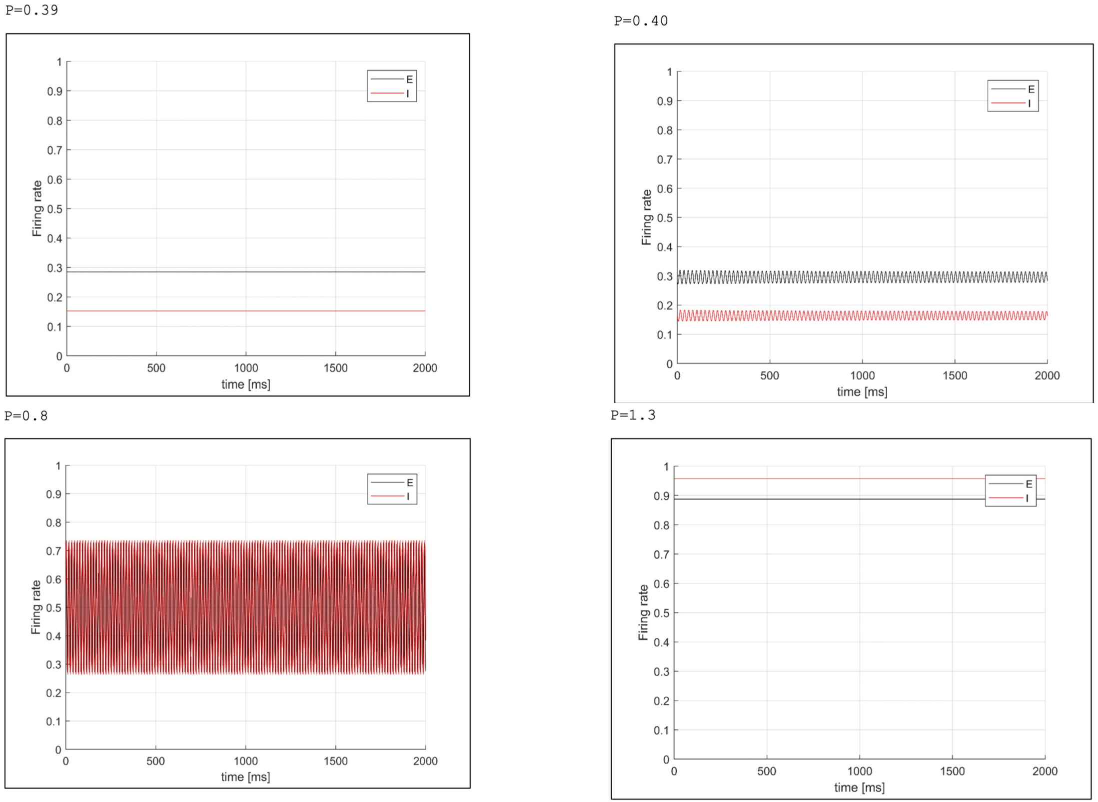


scholarpedia.org

Wilson-Cowan model

Parameter P

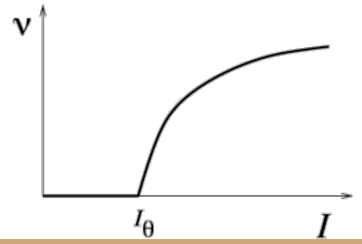


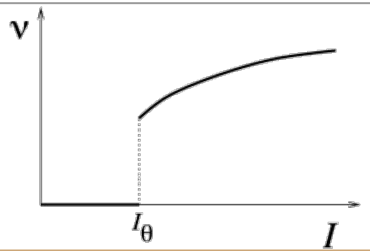


$$\tau_E \dot{E} = -E + S_E(P + c_{EE}E - c_{IE}I)$$
$$\tau_I \dot{I} = -I + S_I(c_{EI}E - c_{II}I)$$

Andronov-Hopf vs Saddle-node bifurcation

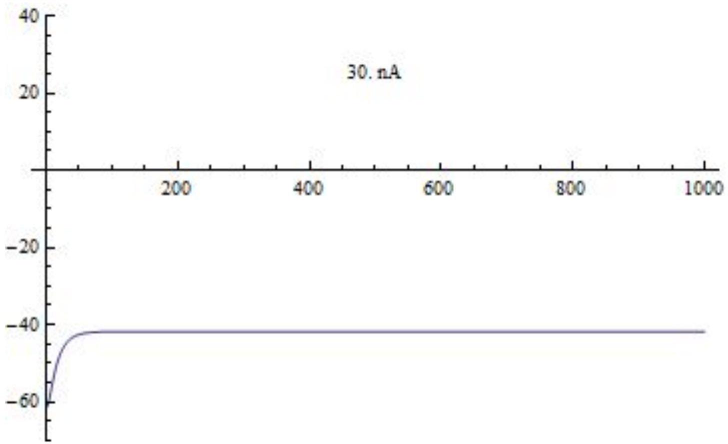
Saddle-node	**Andronov-Hopf**
Can generate infinitely slow oscillations	Generates oscillations at fixed frequency
Can be used for modelling epilepsy	Can be used for gamma oscillations
Emerge with fixed amplitude	Can emerge with low amplitude
Theta model	Wilson-Cowan model
Type I neurons	**Type II neurons**

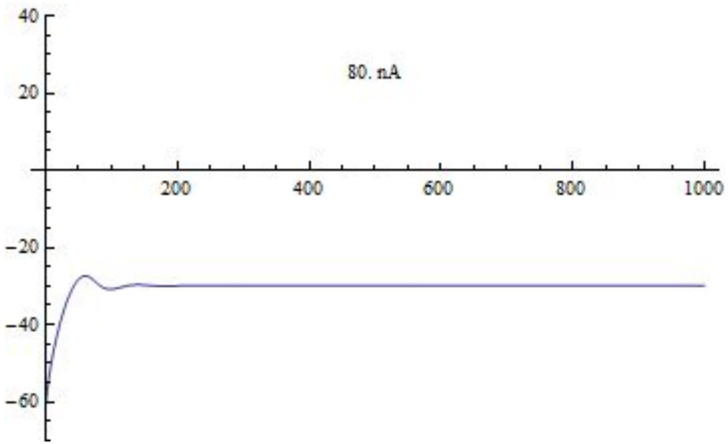




Andronov-Hopf vs Saddle-node bifurcation

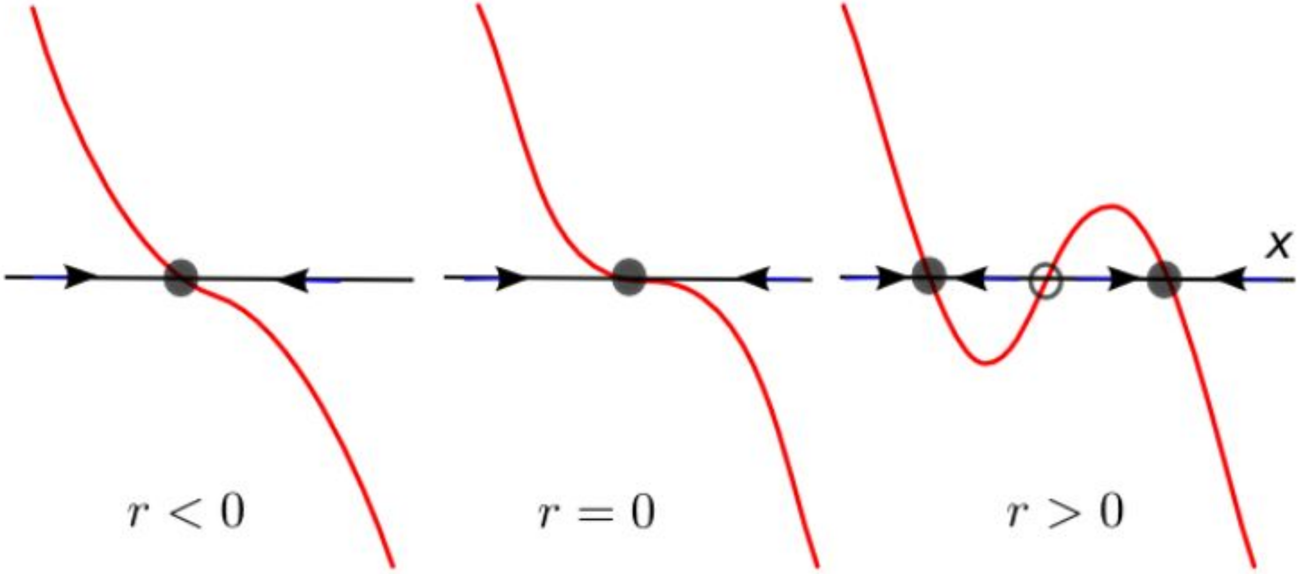
Saddle-node	Andronov-Hopf





Pitch-fork bifurcation

$$\dot{x} = rx - x^3$$



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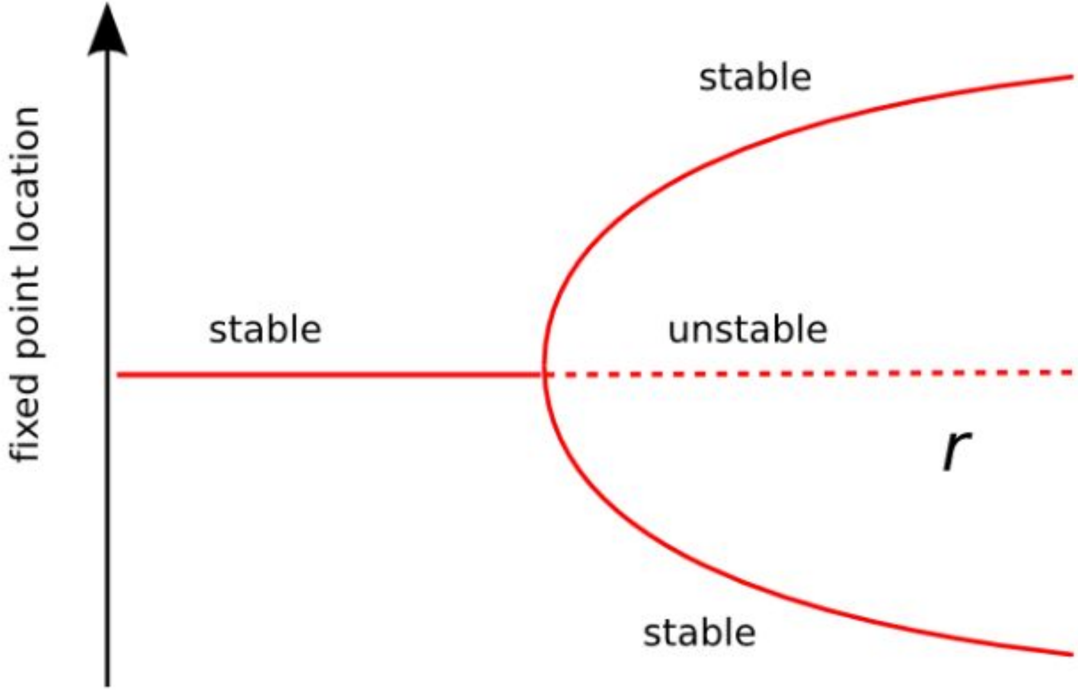

Pitch-fork bifurcation

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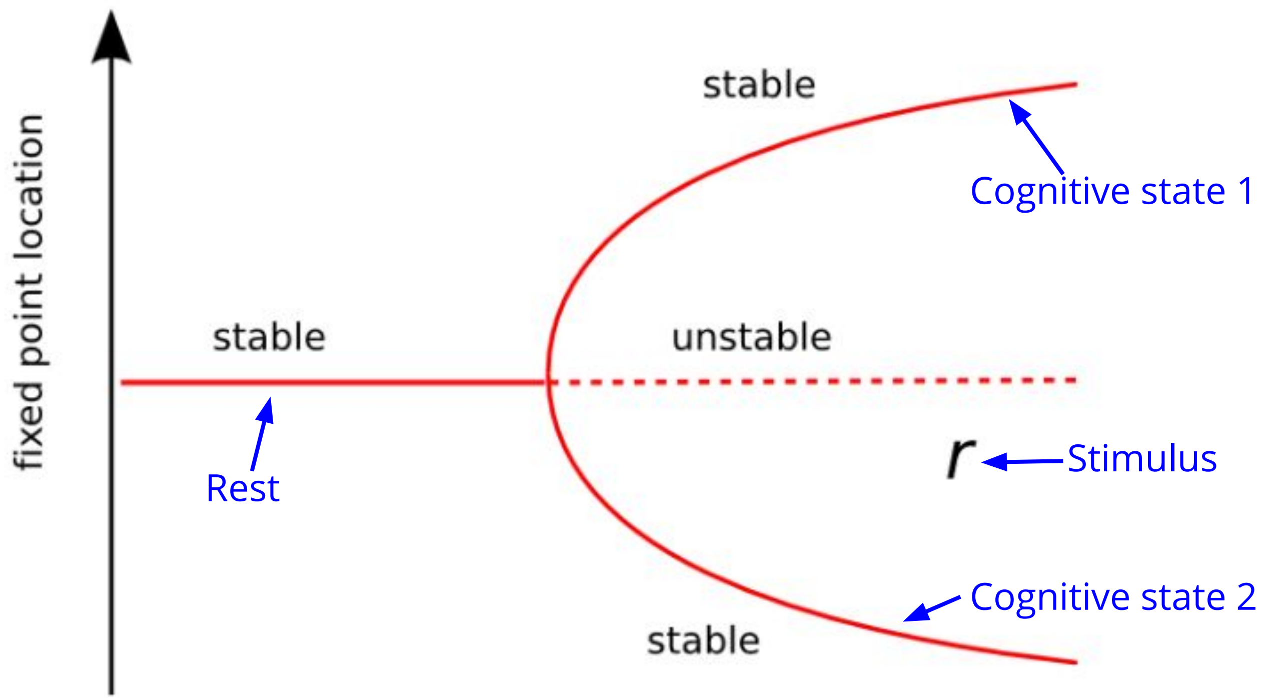
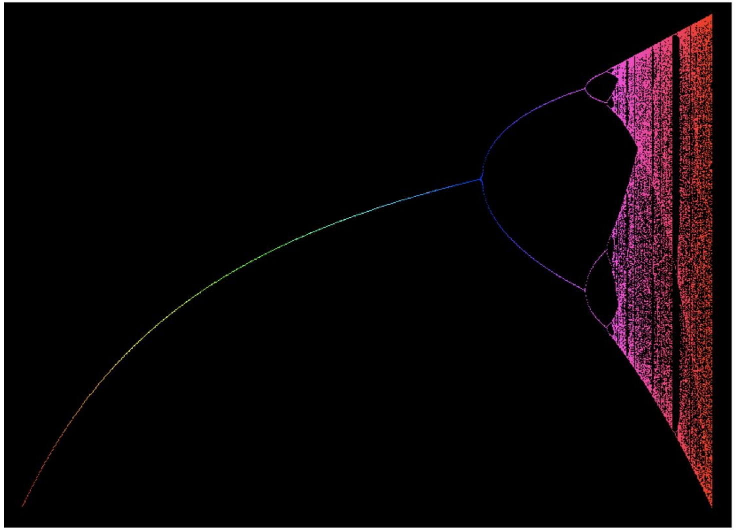


Types of bifurcations

- Saddle-node
- Pitch-fork
- Andronov-Hopf

And many others!

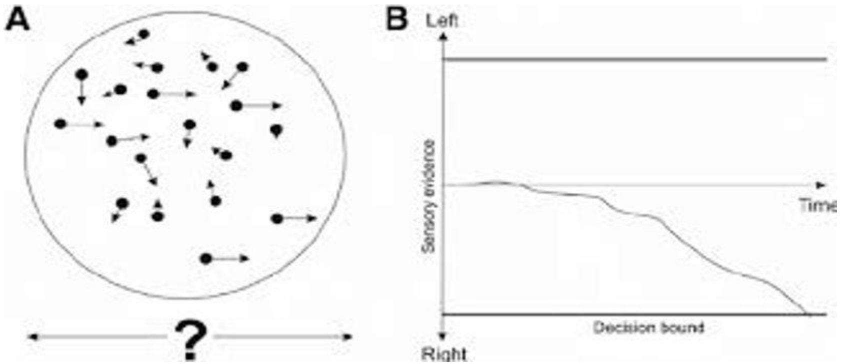


http://virtualmathmuseum.org

More: http://scholarpedia.org/article/Bifurcation

Wong & Wang model

Perceptual Decision making Task



Shadlen and Newsome, 1996

2D Wong and wang model

$$\boxed{\frac{dS_i}{dt} = -\frac{S_i}{\tau_{NMDA}} + (1 - S_i)\gamma H_i}$$

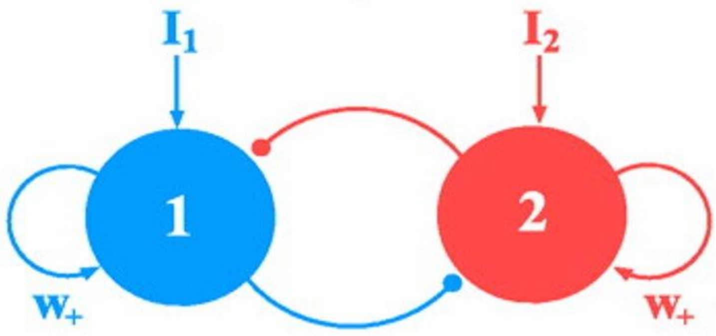
$$H_i = \frac{ax_i - b}{1 - exp(-d(ax_i - b))}$$

$$x_1 = J_{11}S_1 - J_{12}S_2 + I_0 + I_1 + I_{noise,1}$$

$$x_2 = J_{22}S_2 - J_{21}S_1 + I_0 + I_2 + I_{noise,2}$$

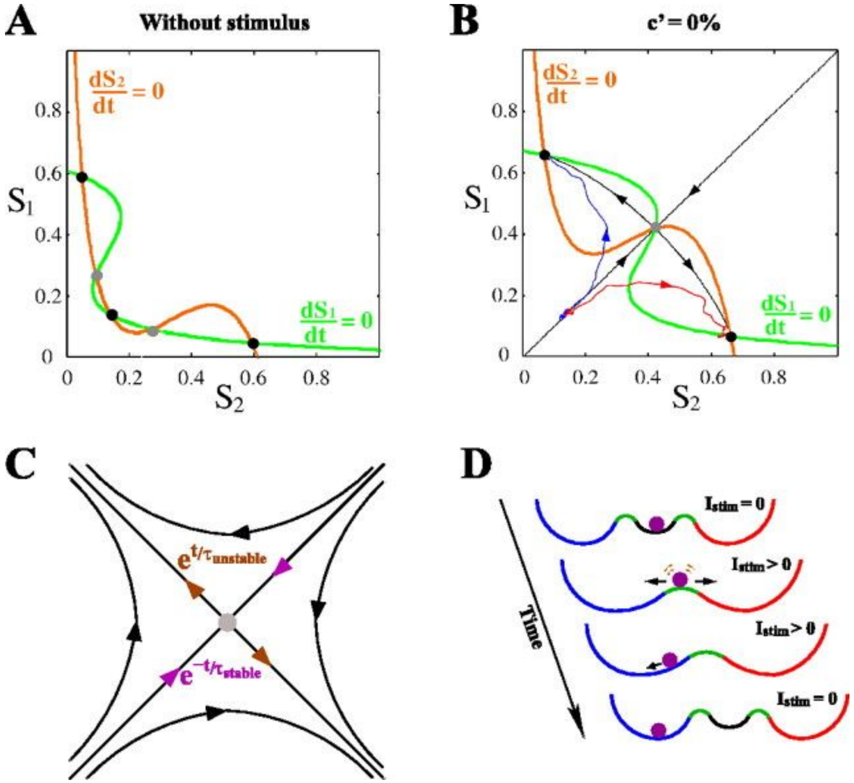
$$I_i = J_{A,ext}\mu_0(1 \pm \frac{c}{100\%})$$

$$\tau_{AMPA}\frac{dI_{noise,i}(t)}{dt} = -I_{noise,i}(t) + \eta_i(t)\sqrt{\tau_{AMPA}\sigma^2_{noise}}.$$



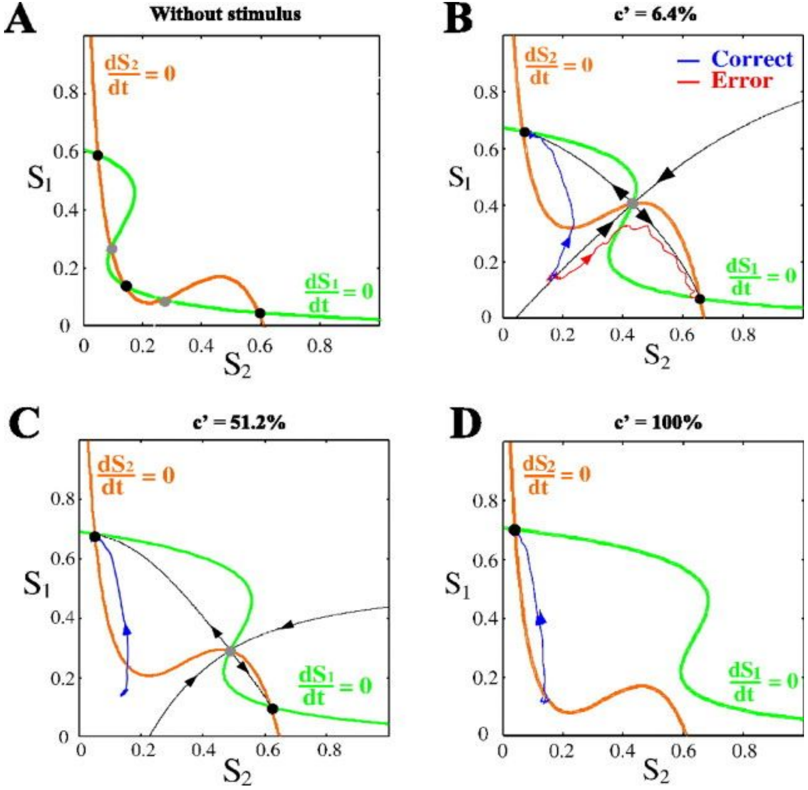
Wong & Wang, A Recurrent Network Mechanism of Time Integration in Perceptual Decisions, 2006

Modifying parameter *c*



Wong & Wang (2006) https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6674568/

Modifying parameter *c*



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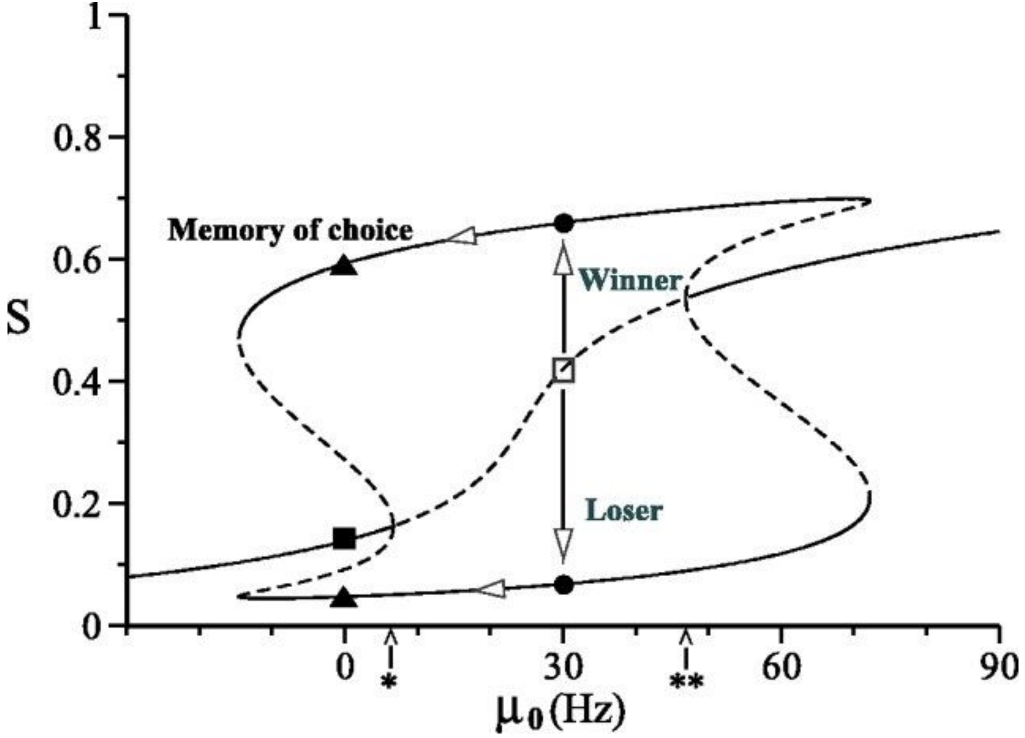
Working memory



Wong & Wang (2006) https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6674568/

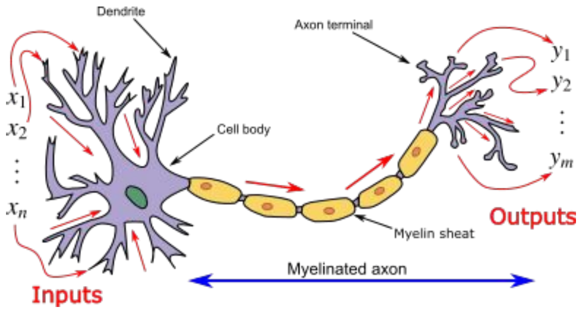
Biological realism

$$I = C_m \frac{\mathrm{d}V_m}{\mathrm{d}t} + \bar{g}_\mathrm{K} n^4 (V_m - V_K) + \bar{g}_\mathrm{Na} m^3 h (V_m - V_{Na}) + \bar{g}_l (V_m - V_l),$$

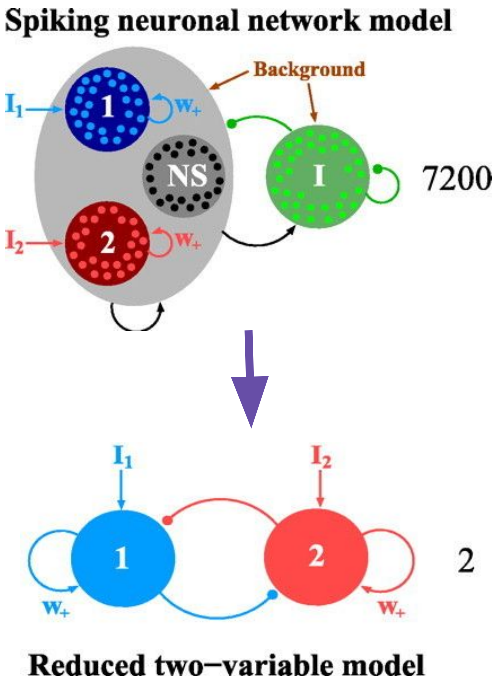
$$\frac{dn}{dt} = \alpha_n(V_m)(1-n) - \beta_n(V_m)n$$

$$\frac{dm}{dt} = \alpha_m(V_m)(1-m) - \beta_m(V_m)m$$

$$\frac{dh}{dt} = \alpha_h(V_m)(1-h) - \beta_h(V_m)h$$

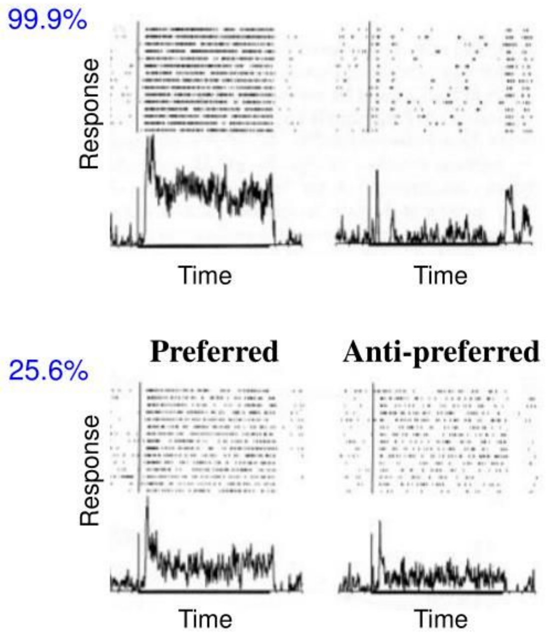


LIF neurons

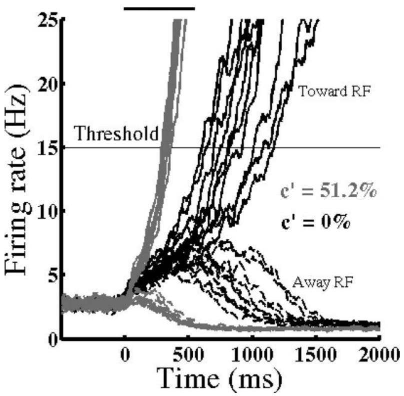


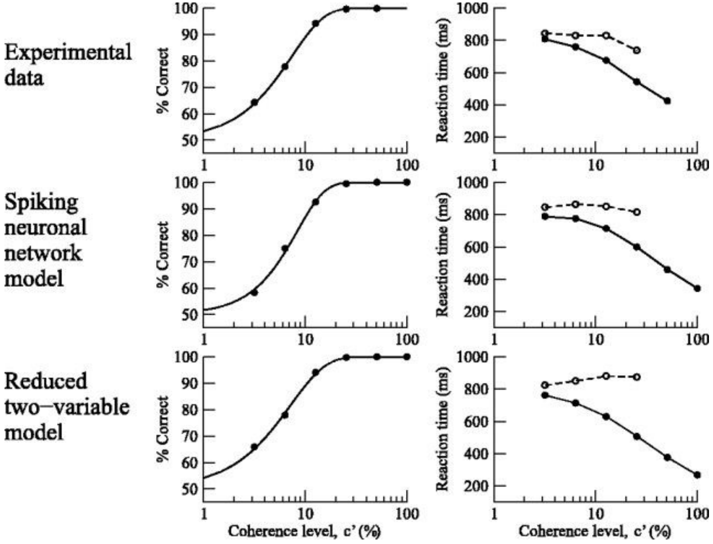
Wong & Wang (2006) https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6674568/

Biological realism



(Britten, Shadlen, Newsome, & Movshon, 1993)

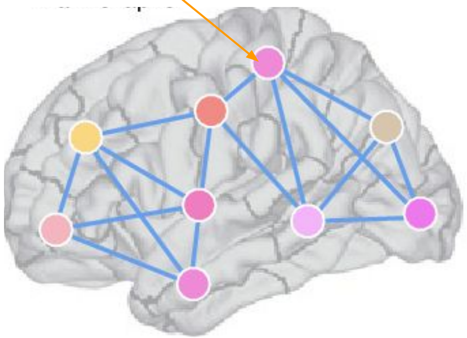


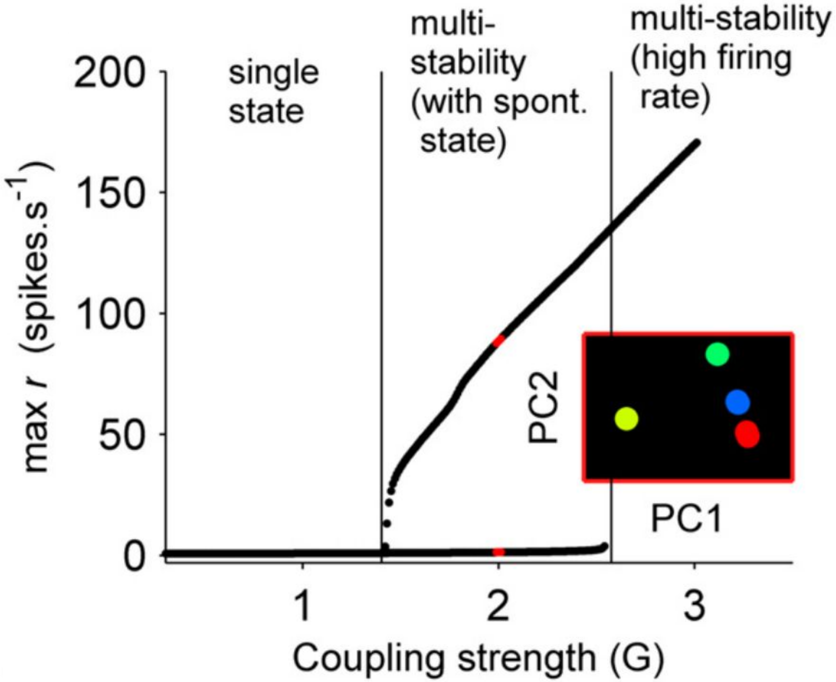


Wong & Wang (2006) https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6674568/

Deco's model

$$\frac{dS_i(t)}{dt} = -\frac{S_i}{\tau_S} + (1 - S_i)\gamma H(x_i) + \sigma v_i(t)$$



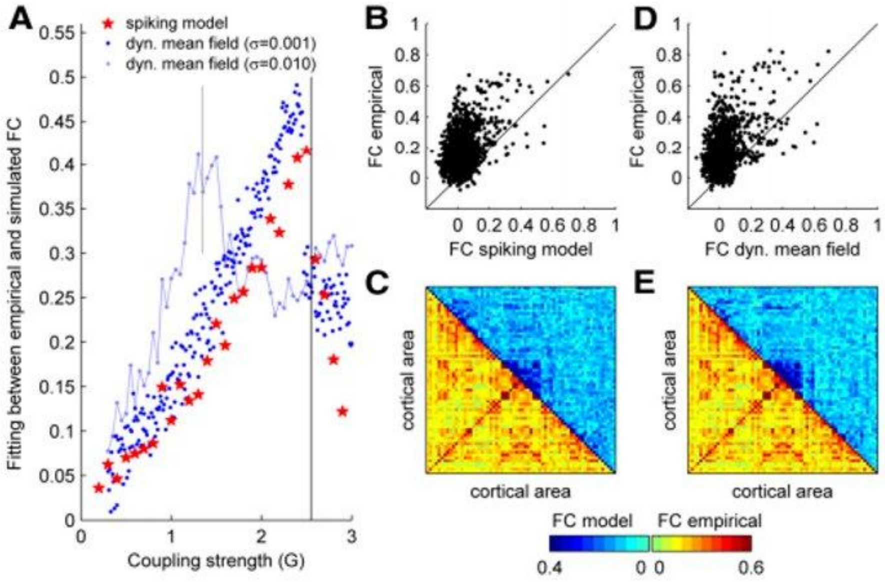


Deco et al. *Resting-State Functional Connectivity Emerges from Structurally and Dynamically Shaped Slow Linear Fluctuations*. 2013.

Deco’s model

$$\frac{dS_i(t)}{dt} = -\frac{S_i}{\tau_S} + (1 - S_i^{(0)})\gamma H(x_i^{(0)})$$

The optimal operating point for explaining the emergence of RSN is near a critical point.



Deco et al. *Resting-State Functional Connectivity Emerges from Structurally and Dynamically Shaped Slow Linear Fluctuations*. 2013.

Thank you!

Questions?

Let's move on to the practical session: github.com/lukewtait/intro_to_modelling

Clone the repo or pull the changes if you have it.


$\frac{dV}{dt} = f(V)$
Practical session