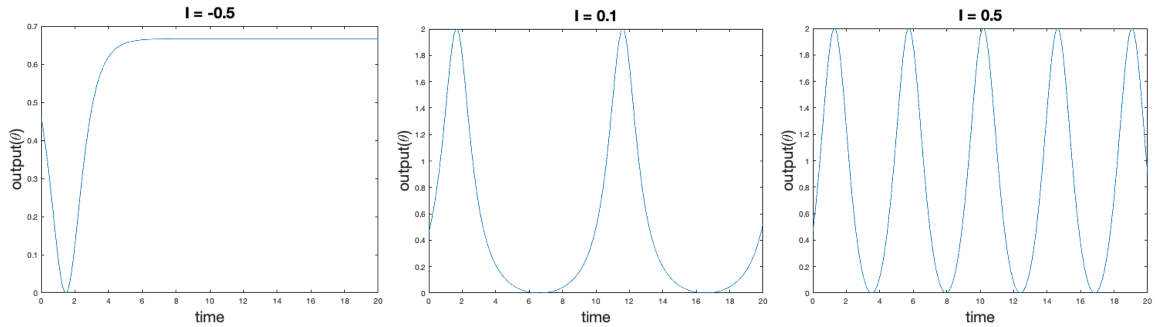
Solutions of Practical session 2: Modelling the emergence of seizures in networks

Part 1: Simulate a deterministic phase oscillator

1. The solution corresponds to the script `ThetaModel.m` (see the directory 'intro_to_modelling/practical2/solutions/' in github).

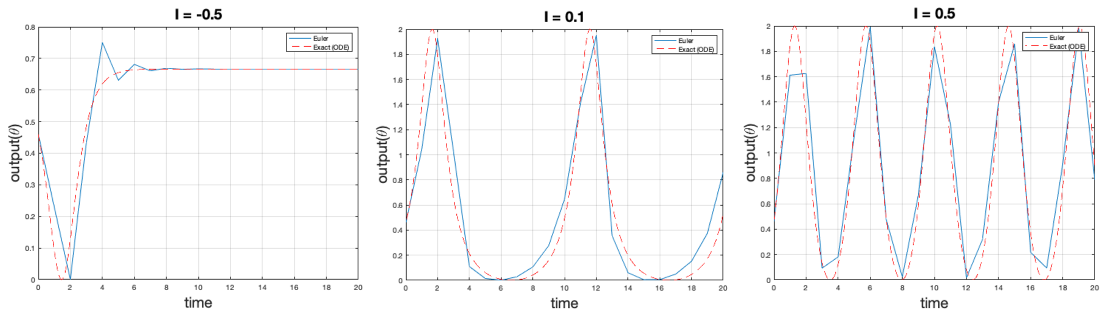
2. By running the script `practical2_part1.m` for $I = -0.5$, $I = 0.1$, and $I = 0.5$, we obtain:



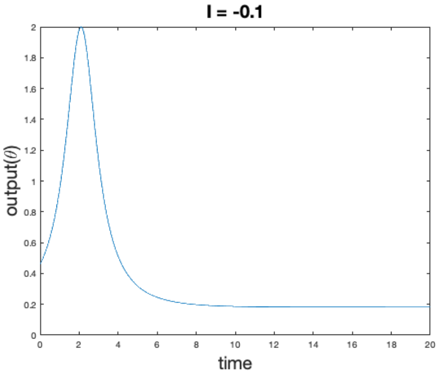
As referred in the presentation, $I = 0$ defines the bifurcation, i.e. the transition from resting at $I < 0$ to oscillations $I > 0$. Furthermore, as the plots above suggest, higher positive values of I imply oscillations with higher frequencies.

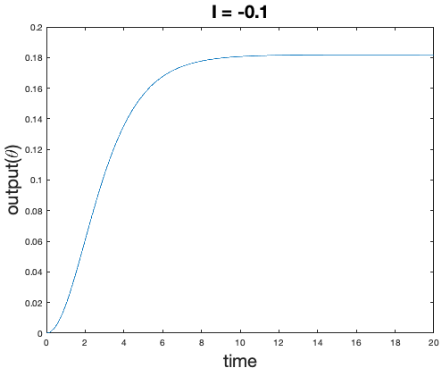
Note that the sudden emergence of oscillations as a parameter reaches a critical value corresponds to a bifurcation. It is the saddle-node on an invariant circle (SNIC) bifurcation mentioned in the presentation. We have seen another bifurcation in practical session 1, task 6 (Hopf bifurcation). In the case of the SNIC bifurcation, oscillations emerge with large amplitude and very low frequency (zero at the bifurcation point), whereas in the (supercritical) Hopf bifurcation oscillations emerge with low amplitude (zero at the bifurcation point) and high frequency (i.e. non-zero).

3. As in the first practical session (part 1, task 3), the smaller the time step, the more time-consuming the computation will be. On the other hand, if the time step is too large, then the numerical simulation won't be accurate. For example, if instead of using $dt = 0.01$ as above, we use $dt = 1$ we get



4. Depending on the initial condition we may or may not see a spike:



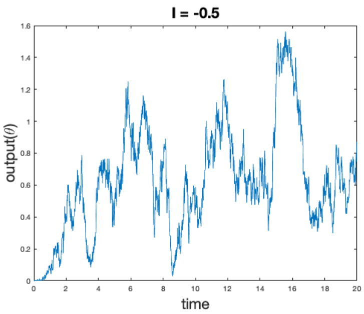


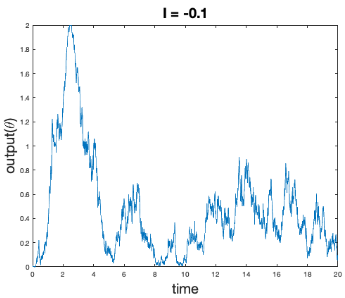
On the left, the initial condition is $\theta(t = 0) = 1$, whereas on the right it is $\theta(t = 0) = 0$. Seeing a spike depends on whether the initial condition starts “above” the unstable point. If it does, then the unstable point repels the oscillator and makes it describe an oscillation before it reaches the stable point.
The long-term behaviour is the same, i.e. the oscillator tends to the same stable point (i.e. same phase).

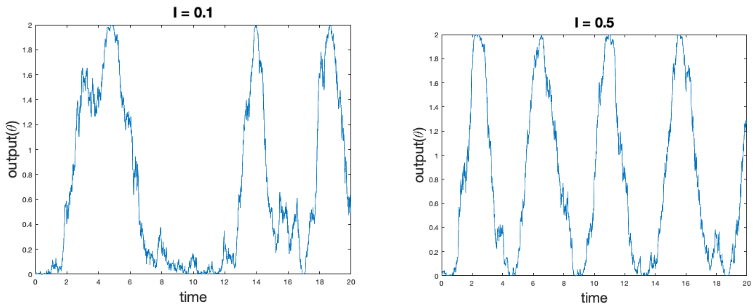
Part 2: Simulate a stochastic phase oscillator

5. The solution corresponds to the script `practical2_part2.m` (see the directory ‘intro_to_modelling/practical2/solutions/’ in github).

6. A fixed σ can produce different kinds of perturbations on the dynamics depending on I. The closer I is to zero from negative values, the easier is for the noise to perturb the dynamics over the unstable point and produce a large amplitude oscillation (spike). On the other hand, in the regime with oscillations, we observe that oscillations with higher frequency (higher I) are less affected by the noise (in their frequency and shape). See some examples below. Note that in order to obtain this intuition you may have to run the script several times for the same parameter values. Increasing the total time (i.e. number of time steps) can also be helpful to see longer time series, where these effects become clearer.



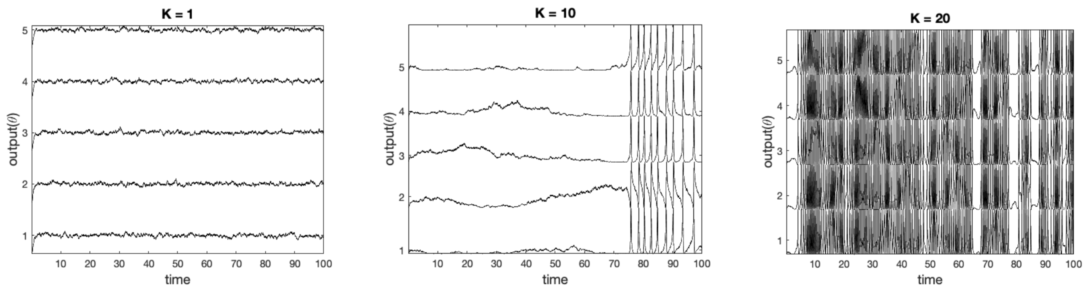




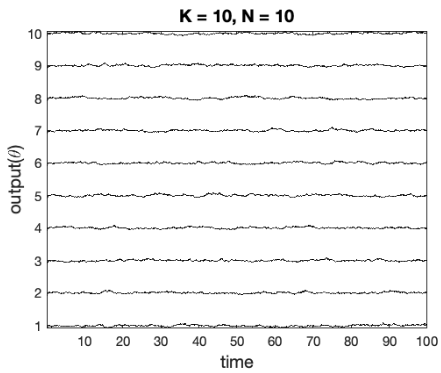
Part 3: Simulate a network of interacting phase oscillators

7. The solution corresponds to the script `ThetaModel_N.m` (see the directory ‘intro_to_modelling/practical2/solutions/’ in github).

8. The larger the K, the higher is the propensity of the network to generate oscillatory activity (which we interpret as seizure activity). Stronger connections make oscillators more likely to excite others to the oscillatory regime. (See the script `practical2_part3.m` in the solutions.)

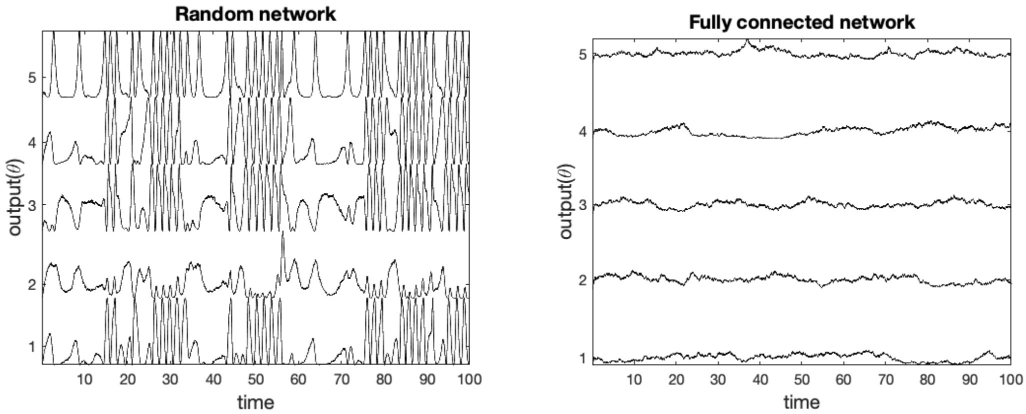


9. The bigger the network, the less likely it is to generate oscillations (all else being equal):



This is essentially due to the ‘normalisation’ of the interaction term by N. Higher N makes the contribution of each oscillator less likely to induce a transition to oscillations in other nodes.

10. We obtain the following results for the two networks (see the script in the solutions):



Note that different random network realisations will give different results. Nevertheless, in general we observe oscillations in the random network and no oscillations in the fully connected network. Given that we relate oscillations to seizure activity, the random network would correspond to the individual with epilepsy and the fully connected network to the healthy individual. Note also that the comparison between two networks is only meaningful if all parameters are the same in the two simulations.