Practical Session 2: Bifurcations in a Model of Decision Making

This practical session aims to introduce you to stability analysis and bifurcations in a model of decision-making in the brain. The practical is split into three parts, which take you through the steps to simulate and analyse the Wong-Wang model of decision making. The parts are outlined below:

1. Simulating decision-making in the brain
2. Stability analysis of the undecided steady state; how are decisions forced?
3. Bifurcation analysis of decision making and working memory in neurodegeneration

In this session, most of the coding has been done for you. However, for background on how we solve dynamical systems on a computer, it is strongly advised you complete the previous practical sessions, which can be found online at https://github.com/lukewtait/intro_to_modelling. In particular, this session will use the function EulerSDE.m, which you wrote in practical session 1, to simulate the model.

For a guide on Matlab syntax, see:
https://github.com/lukewtait/intro_to_modelling/blob/master/practical1/SyntaxGuide.pdf

If you have any questions after the practical is finished, you can ask at:
https://brainmodelworkshop.freeforums.net (no registration required)

To learn more about the Wong-Wang model, see the original paper:
Wong and Wang (2006), J Neurosci 26(4):1314-1328 https://doi.org/10.1523/JNEUROSCI.3733-05.2006

To learn more about modelling decision making in CUBRIC, visit the webpage for the Cognition and Computation Brain Lab: https://ccbrain.org

Part 1: Simulating decision-making in the brain

In this section you will simulate the Wong-Wang model of decision-making in the brain. Specifically, you will simulate the response of two selective populations of neurons to a random dot motion stimulus; Population 1 is selective to dots moving left, and Population 2 is selective to dots moving right. You will study how altering the amount of coherence in the direction of movement of the dots alters the response of the network.

This part will use the script `practical3_part1.m`. Line 4 reads `coherence=0`. You can edit this line to edit the strength of the motion coherence; `coherence` can range from `-100` (all dots moving left) to `100` (all dots moving right). `coherence=0` is random motion. Each time the script is run, a figure should be generated, showing the firing rates of red (right selective) and blue (left selective) populations of neurons. A decision is made when the firing rate of a population exceeds 15 Hz.

1) A participant is required to decide whether the dots are moving left or right. Given a motion coherence of 0% (the dots are moving randomly), how often do you expect the participant to decide the dots are moving right (assuming no bias)? Hint: this is essentially random guesswork, with two choices. Test your prediction using `practical3_part1.m` by setting motion coherence to 0%, and running the script several times, estimating how often the red (right selective) population is selected.
2) At the end of this worksheet are several figures. Figure 1 shows the difference between simulations at 0% and 51.2% coherence. Check that you get similar results when you vary the motion coherence from 0-100%. What effect does increasing coherence have on the

percentage of times the correct (in this case the red right selective population) is chosen? What about the time taken for a response? Does this align with experimental data shown in Figure 2?

3) Set the motion coherence to 0, and simulate. Consider the plot of the phase portrait on the right of the generated figure. How many steady states exist? How many steady states exist when coherence is 100%? Are we ever *guaranteed* a correct response, and if so how large does the coherence have to be?

Part 2: Stability analysis of the undecided state; how are decisions forced?

In part 1, the simulated participant always made a decision. In this section, we will consider the conditions required to force a decision by performing a stability analysis on the symmetric steady state; i.e. the steady state corresponding to both populations having approximately equal firing rates, corresponding to no decision having been made. For this section, motion coherence will always be set to zero. This part will use the script `practical3_part2.m`.

The script `practical3_part2.m` is similar to `practical3_part1.m`, except line 4 allows you to vary stimulus strength instead of coherence (coherence is automatically fixed at 0%). Additionally, it will output the Jacobian eigenvalues of the symmetric steady state into the Matlab console, allowing you to assess stability.

1) Run the script `practical3_part2.m`, ensuring stimulus=30 (default value) on line 4. The results should be the same as simulations in task 1 when coherence was 0%. Even though the simulation always starts at the symmetric/undecided steady state (line 20), a decision is always made. Does this suggest the symmetric state is stable or unstable? Validate this using the Jacobian eigenvalues (printed in the console). If the symmetric state is unstable, are both eigenvalues positive, or just one? What is it called when some eigenvalues are positive and some are negative?
2) Set stimulus=0 on line 4. This means that the participant receives no stimulus to initiate decision-making. Run the script again. Does the participant make a decision, or stay at the symmetric steady state? Is the symmetric steady state stable or unstable? Validate this using the eigenvalues. To help, you may also wish to look at the phase portraits for stimulus=30 and stimulus=0 in Figure 3, which is taken from the original Wong-Wang paper.
3) Uncomment lines 58-79 (you can highlight these lines and click Ctrl+T in Windows). Now run practical3_part2.m again. A new plot should appear, showing the value of the two eigenvalues as the strength of the stimulus is varied from 0-30 Hz. At what stimulus strength does the symmetric steady state become unstable? Do you think a decision will be made for stimuli of 5, 15, and 25 Hz? Check your predictions by simulating (you can comment out lines 58-79 by highlighting them and typing Ctrl+R).
4) If a steady state changes from stable to unstable as a parameter is varied, a ____________ has occurred. (Hint: it's in the title of the workshop!)

Part 3: Bifurcation analysis of decision-making and working memory in neurodegeneration

In this section, we will study how working memory of a decision arises based on a bifurcation diagram. Then, we will reduce the strength of NMDA synapses and see how this changes the bifurcation diagram and alters decision-making and working memory. This section will rely heavily on the bifurcation diagrams in Figure 4, and will use the script `practical3_part3.m`. As in the previous section, for all simulations the motion coherence is 0%.

In `practical3_part3.m`, you can control the strength of NMDA synapses using the parameter `nmda_ratio` on line 4. All NMDA synapses are multiplied by `nmda_ratio`, so `nmda_ratio=1` is the default 'healthy' parameter used so far, and `nmda_ratio<1` reduces the strength of NMDA (e.g. synaptic degeneration or pharmacological blockade). Line 5 allows us to control the strength of the stimulus, as in the previous section. The stimulus is on for $0<t<2000$ ms, for all other times stimulus strength is zero.

1) Consider the bifurcation diagram in Fig 4A. We have seen in the previous two sections that for stimulus=30 Hz and coherence=0%, the participant always makes a decision. Can you explain this from the bifurcation diagram? Based on the bifurcation diagram, do you think the participant will remember their decision (working memory) if the stimulus is subsequently removed? To test your prediction, run `practical3_part3.m` with `stimulus=30` and `nmda_ratio=1`. Hint: The participant remembers their decision if they remain at the decision steady state instead of returning to the symmetric steady state.

2) The bifurcation diagram for `nmda_ratio=0.78` is shown in Fig 4B. From this bifurcation diagram, decide whether each of the following statements is true or false when `nmda_ratio=0.78`.

> A) The participant will make a decision with a stimulus of 30 Hz.
> B) The participant will make a decision with a stimulus of 90 Hz.
> C) If a participant makes a decision, they will have working memory of the decision when the stimulus is removed.

Test your predictions using the script.

3) Repeat question 2 for `nmda_ratio=0.72`, using the bifurcation diagram in Fig 4C.

4) Consider a hypothetical disease in which NMDA synapses are progressively impaired. According to the Wong-Wang model, what is the predicted order of symptoms in the disease progression?

Figures

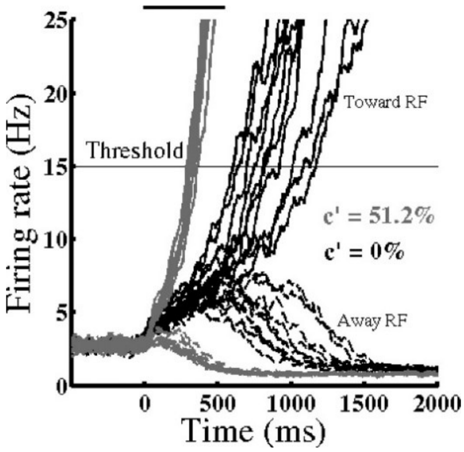


Figure 1: Fig 2 of Wong & Wang (2006) J Neurosci 26(4):1314-1328. Time course with two different motion strengths. Motion coherence of 0% (black traces) and 51.2% (light gray traces) each with 10 sample trials. Firing rates that ramp upward (bold traces) are for saccades made toward the RF of the neuron, whereas downward (dashed traces) are for saccades away from RF. Ramping is steeper for higher coherence level. The prescribed threshold is fixed at 15 Hz. Once the firing rate crosses the threshold, a decision is made, and the decision time is the time it takes from stimulus onset (0 ms) until the threshold is crossed. The reaction time is defined as the decision time plus a nondecision latency of 100 ms. The bold horizontal line at the top of the figure denotes the duration, at zero coherence, where the firing rates toward and away from RF are indistinguishable.

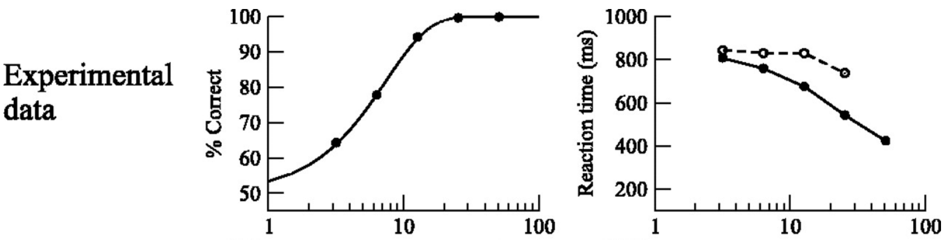


Figure 2: Modified from Fig 3 of Wong & Wang (2006) J Neurosci 26(4):1314-1328. Performance and reaction time in the experiment of Roitman and Shadlen (2002). First column, Psychometric data from experiment (data are fit with a Weibull function). Second column, Reaction time from experiment. Open circles joined by dashed lines, Mean reaction of error trials; filled circles, correct trials. Experimental data are adapted from Mazurek et al. (2003).

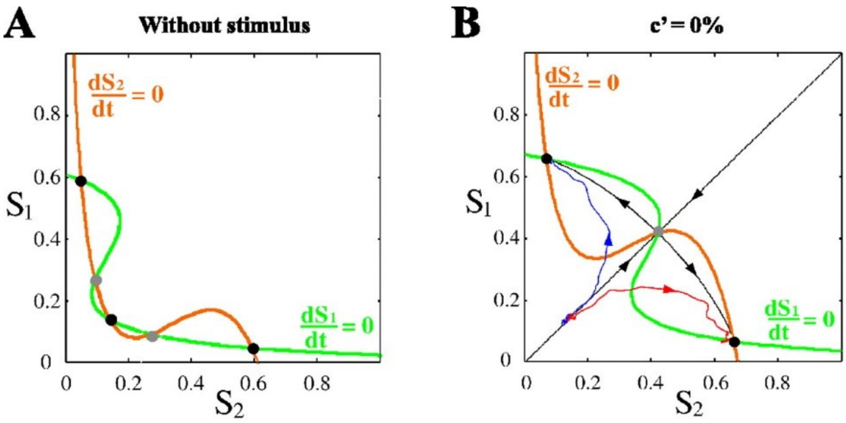
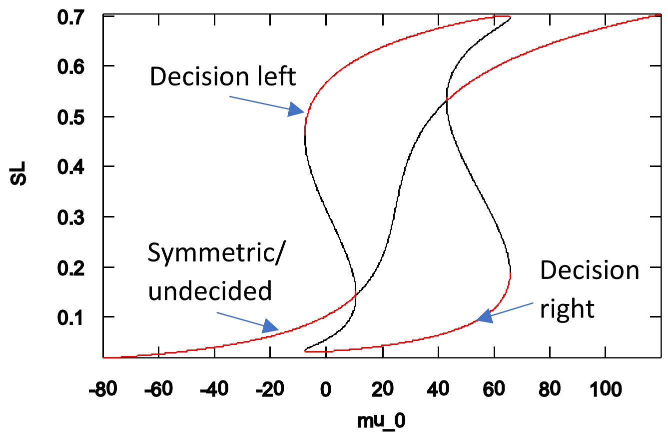
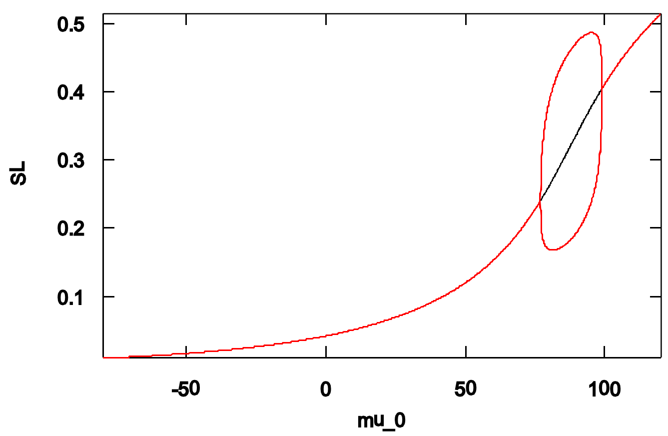


Figure 3: Modified from Fig 4 of Wong & Wang (2006) J Neurosci 26(4):1314-1328. Random choice with stimulus at zero coherence. A, Phase-plane without stimulus. Black circles, Stable steady states; gray circles, saddle-type unstable steady states. The green and orange lines are the nullclines for the synaptic dynamical variables S_1 *and* S_2*. A threshold at 15 Hz would correspond to* S *= 0.49 in phase space.* ***B****, With an unbiased stimulus of 30 Hz, the two unstable steady states together with the low stable steady state disappear, and a new symmetric unstable steady state is formed. Superimposed are two typical single-trial trajectories (blue and red lines) of the state of the system from simulations.*

A



B



C

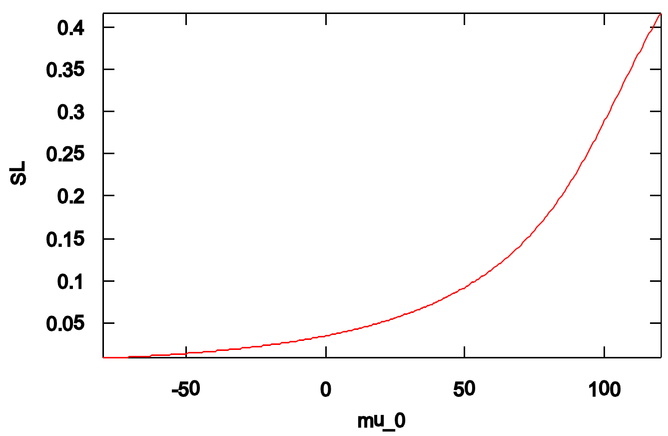


Figure 4: Bifurcation diagrams as stimulus strength (mu_0) is changed. Y-axis shows steady state values of the left population (SL, also written S_1 in the slides). Red lines are stable steady states, black lines are unstable. (A) NMDA-ratio = 100%. (B) NMDA-ratio = 78%. (C) NMDA-ratio = 72%.