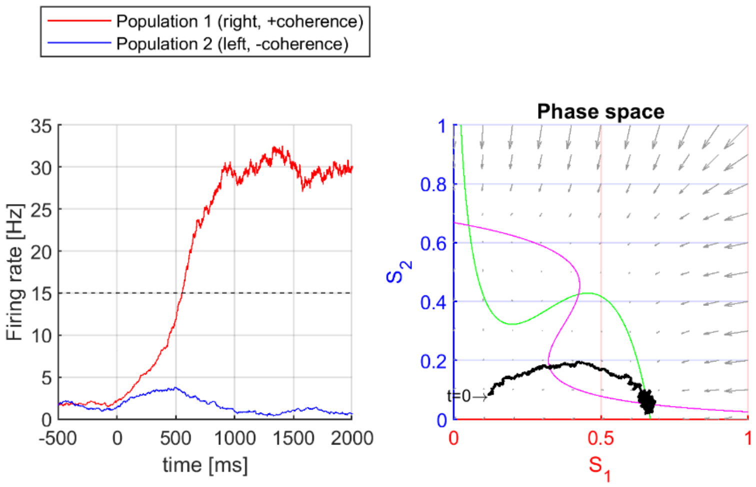
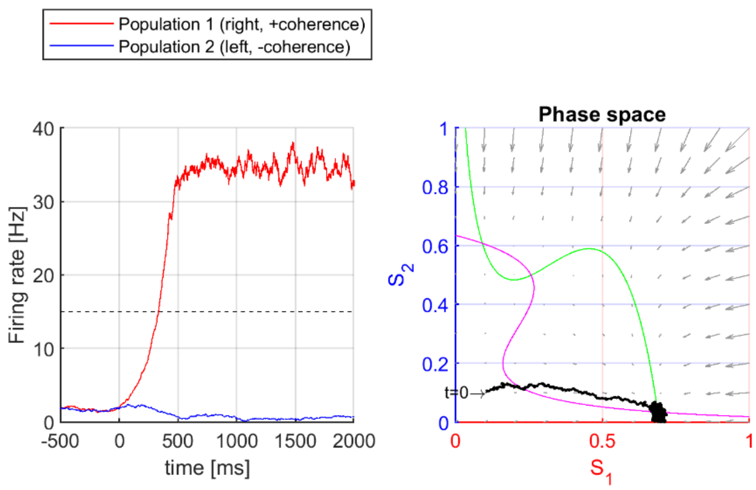
# Solutions to Practical Session 3

## Part 1: Simulating decision-making in the brain

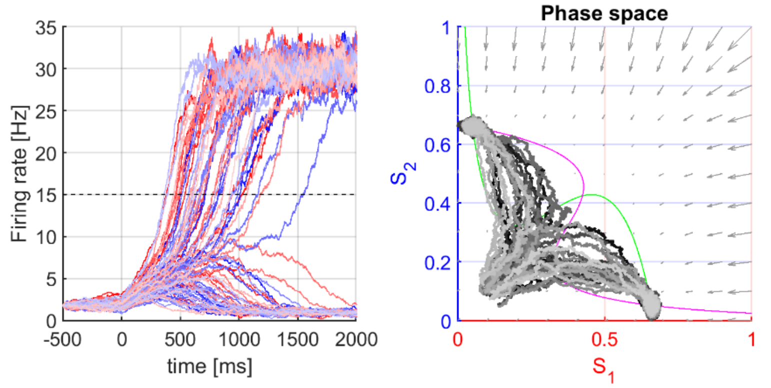
1. Since there is no coherence, it is random guesswork with two choices. Hence, we expect the unbiased participant to guess right 50% of the time (and left the other 50% of the time). When running the simulations, approximately 50% of the time it should look something like the figure below. In the left plot, the red (right selective) population tends to a firing rate of ~30 Hz, while the blue (left selective) population tends to a firing rate of ~1 Hz. On the phase plane, the dynamics move to the steady state at the bottom right, which is high on the red x-axis and low on the blue y-axis. This means the red (right selective) population was selected, so the decision is that the dots are moving right. The other half of the time the blue population should go to 30 Hz and the red to 1 Hz, which is the top left steady state in the phase space. This would be a decision for left.



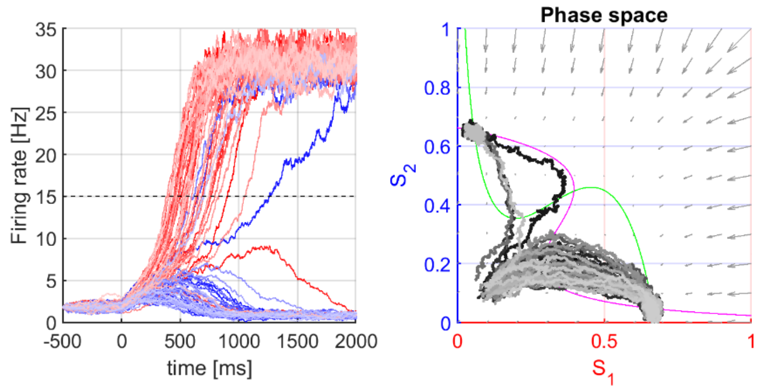

2. The previous question discussed the case when coherence is 0%. The figure below shows an example simulation for coherence=51.2%. You should notice that increasing coherence will increase the fraction of correct decisions and reduce the reaction time. This is in line with data. The results for 50 simulations at 0%, 10% and 51.2% coherence are shown on the next page. These results were plotted using the script solutions/loop_partical3_part1.m in the solutions folder. The simulated decisions and reaction times are also saved in this folder.



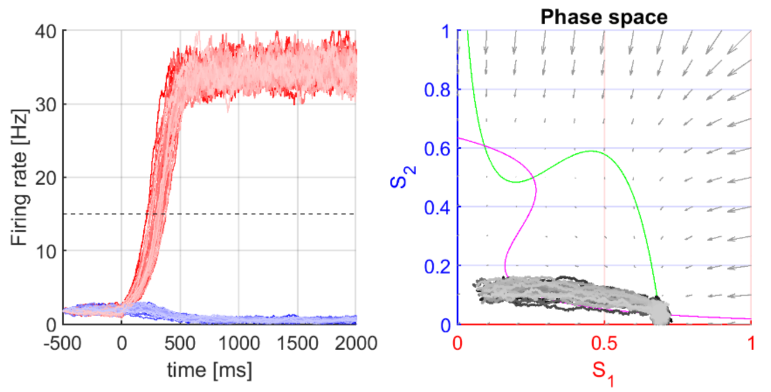
0% Coherence

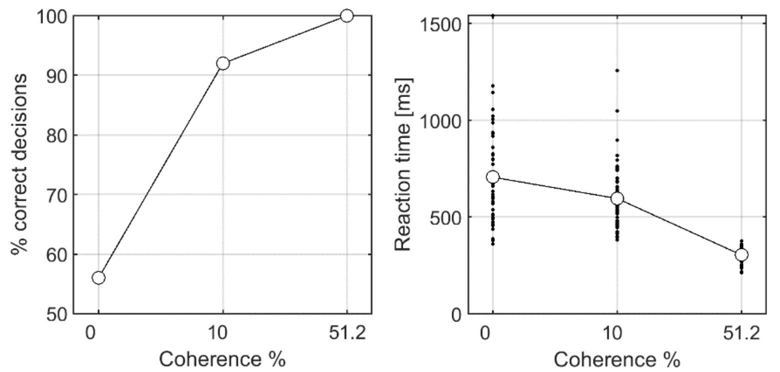



10% Coherence

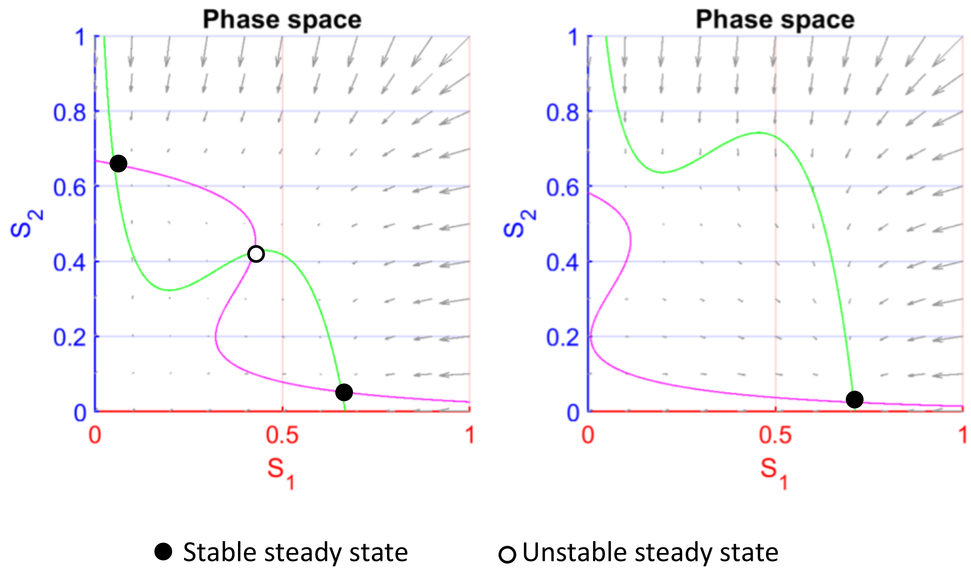



51.2% Coherence



3. Steady states are the points where the nullclines meet. Pictures of the phase space for 0% and 100% coherence are shown below, with steady states marked. For 0% coherence there are 3 steady states. For 100% coherence there is one steady state. You are guaranteed a correct response only if there is one steady state, so this is true for above approximately 70% coherence. Note: It becomes *very* unlikely to get a wrong decision at much lower values (in the previous solution we saw that at 51.2% coherence 50 out of 50 decisions were correct), however there is still a very small chance of an incorrect decision as there exists a stable steady corresponding to the wrong decision.



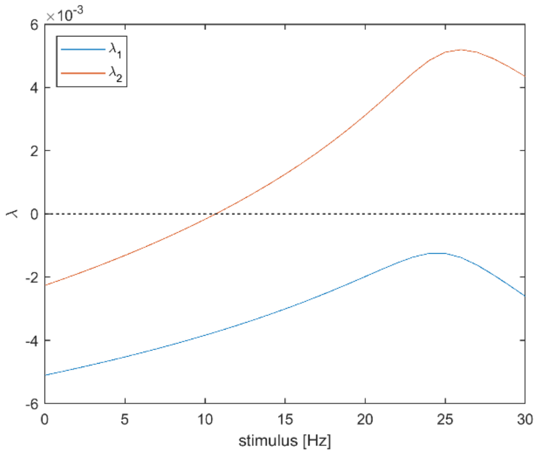

## Part 2: Stability analysis of the undecided state; how are decisions forced?

1. The symmetric steady state is unstable, since the firing rates are repelled away from the symmetric steady state value (shown by a black line on the plot). The eigenvalues should be -0.0026 and 0.0043. Since all eigenvalues have to be negative for the steady state to be stable, and 0.0043 is positive, this validates that the steady state is unstable. A steady state with some negative and some positive eigenvalues is called a 'saddle', and is attractive in directions corresponding to negative eigenvalues and repulsive in directions corresponding to positive eigenvalues. This can be seen in Figure 3B of the worksheet, where arrows point along the stable and unstable directions. This explains why, in the phase plane plots on the previous page, the dynamics start off moving toward the steady state before getting repelled away; they begin along the stable direction.

2. With no stimulus, the participant does not make a decision. Instead, the firing rates remain at the symmetric steady state. This suggests the symmetric steady state is stable. The eigenvalues are -0.0051 and -0.0023, which validates it is stable (both are negative). The phase plane plots in Figure 3 of the worksheet show that the symmetric steady state is stable for stimulus=0, and is surrounded by two unstable steady states (five in total). Somewhere between stimulus=0 and stimulus=30, the two unstable steady states collide with the stable symmetric steady state, which becomes unstable.

3. After uncommenting the code you should see a plot like the one below. This plot shows the two eigenvalues of the model (y-axis; one in blue, one in orange) as the stimulus strength is varied (x-axis).

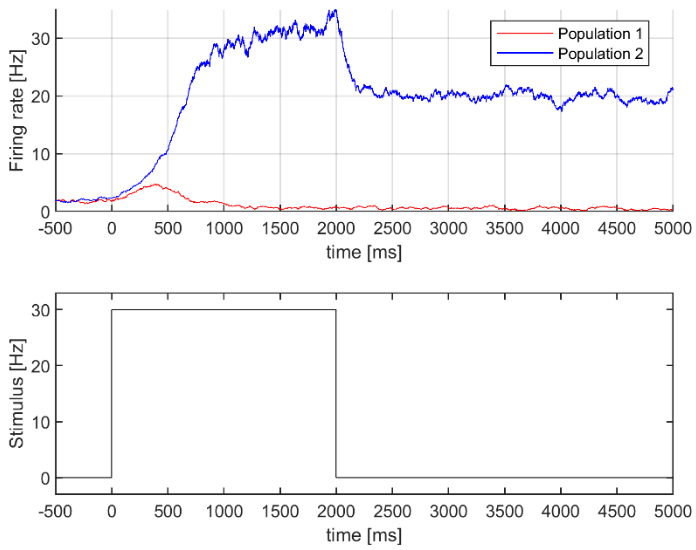
For all stimulus strengths, one eigenvalue ($\lambda_1$) is negative, so there is always one 'stable' direction. Below approximately 10 Hz the second eigenvalue ($\lambda_2$) is negative, so for these stimulus strengths the symmetric steady state is stable (meaning no decision will be made). However, above 10 Hz $\lambda_2$ is positive, so the symmetric steady state is unstable (it is a saddle) and a decision is made. Therefore no decision is made for 5 Hz, but a decision will be made for 15 and 25 Hz.

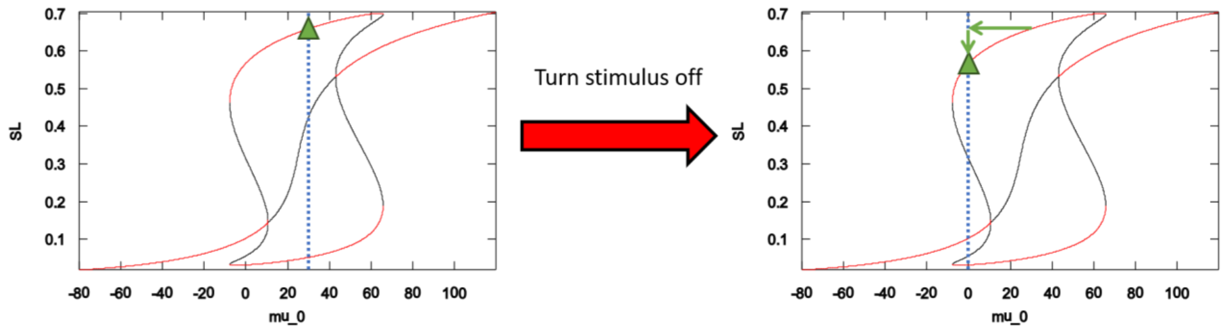



4. Bifurcation.

## Part 3: Bifurcation analysis of decision-making and working memory in neurodegeneration

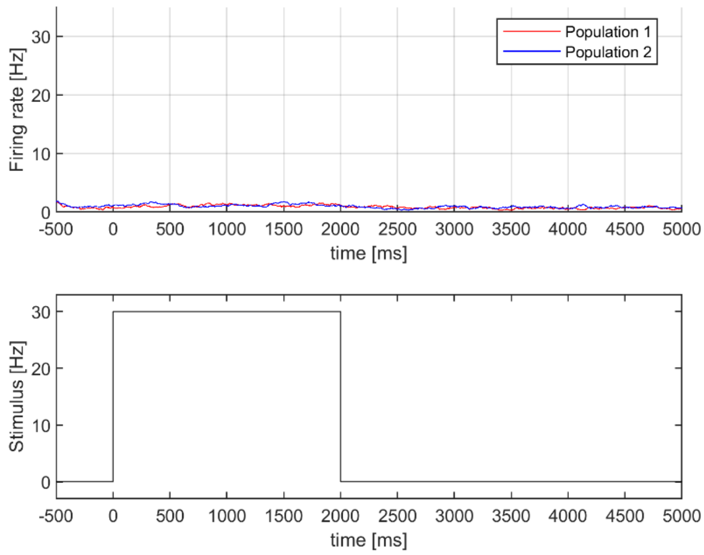
1. In the bifurcation diagram, for stimulus=30 Hz there are three steady states. The middle (symmetric) state is unstable, while the two unsymmetric states are stable. The system is therefore repelled from the symmetric steady state to one of the decision states. From the bifurcation diagram, we can see that there is working memory (explained on the next page). Working memory is shown in the simulation below, since the winner (blue) remains at a much higher firing rate than the loser (red) even after the stimulus is removed at t=2000 ms.



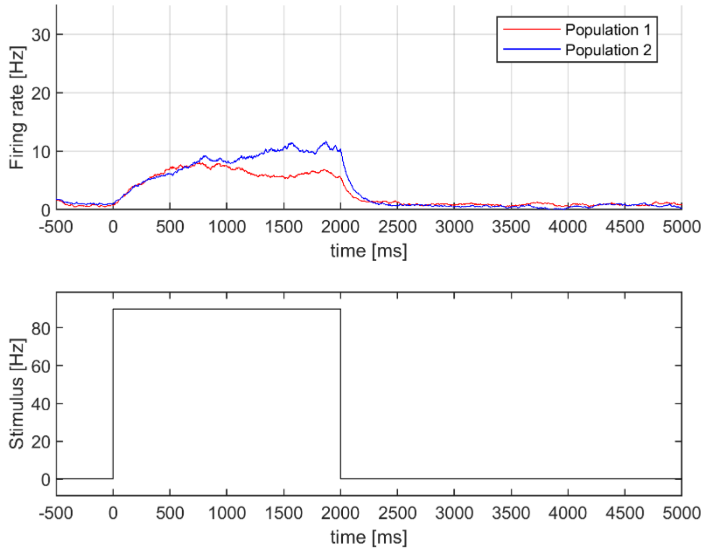



Working memory is explained by this figure. The dashed blue line shows the value of the stimulus, and the green triangle shows the value of the left population's NMDA synapses, SL. On the left, we start at the point where a decision has been made, and stimulus=30. In this case, the decision was 'left', since SL is high. If we were at the lower branch, the decision would be 'right', but the origin of working memory is the same. The stimulus is turned off instantaneously, but so the system jumps from stimulus=30 to stimulus=0 while maintaining the value of SL (horizontal green arrow in the right figure). The system then moves to the nearest stable steady state (vertical green arrow), which is still the 'left' branch. Hence, the system remains on this branch even after the stimulus is removed.

2. A: false. At stimulus=30, the only stable steady state is the symmetric branch, so no decision is made. B: true. At stimulus=90, the symmetric state is unstable and there are two stable decision states. C: false. At stimulus=0, the only stable steady state is the symmetric one, so if the system starts at a decision branch (e.g. stimulus=90) and the stimulus is removed (stimulus=0), the system will tend to the stable symmetric steady state. Below is a simulation for 30 Hz, with no decision making.




On the next page is a simulation for 90 Hz. A decision is made (blue population has higher firing rate than red), but the winner is less clear-cut than for NMDA ratio=1 as the decision steady states are closer together. However, once the stimulus is removed the system returns to the symmetric steady state.




3. A+B: false. There is only ever a symmetric branch, so no decision making can be made. C: N/A as no decision is made.

4. As the disease progresses, decision making and working memory will both be impaired. At early stages, working memory will be impaired first, while decisions will still be made but will require a stronger stimulus to initiate decision making. At later stages, all decision making will be lost.