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CSCI 170 MW afternoon lecture
PROFESSOR Aaron COTE
Due Thursday, February 6, 2014
Homework #2

2.
(a)
In the best situation under the worst case, each question eliminates one half of all number under current consideration. This strategy takes $\lceil \log_2 1000 \rceil = 10$ steps, which is the least number of steps in the worst case.
(b)
As long as the initial pool of candidate integers is continuous, it does not matter what the boundary numbers exactly are, since one can always apply binary search to get best result in the worst case, which would take $\lceil \log_2(4642 - 585) \rceil = 12$ questions.
3.

$$
\begin{aligned}
f_a(n) &= \frac{n^3}{\log n} = o(n^3) = n^{\frac{8}{3}} \frac{n^{\frac{1}{3}}}{\log n} = \omega(n^{\frac{8}{3}}) \\
f_b(n) &= n^3 = \Theta(n^3) \\
f_c(n) &= \log^2 n = \Theta(\log^2 n) \\
f_d(n) &= \sum_{i=1}^{n} \sum_{j=1}^{i} j = \sum_{i=1}^{n} \frac{i(i+1)}{2} = \frac{1}{2} \sum_{i=1}^{n} \left(i^2 + i\right) = \frac{1}{2} \left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right] \\
&= \Theta(n^3) \\
f_e(n) &= \log_{1.5} n^2 = 2 \log_{1.5} n = \Theta(\log n) \\
f_f(n) &= 2^{\log n} = n = \Theta(n) \\
f_g(n) &= n^{\frac{8}{3}} = \Theta\left(\frac{n^3}{\log n}\right) \\
f_h(n) &= 1.001^n \text{ is exponential.}
\end{aligned}
$$

According to this simplification, it is clear that, $f_h(n) > f_b(n) = f_d(n) > f_a(n) > f_g(n) > f_f(n) > f_c(n) > f_e(n)$, in terms of rate of growth.
$f_d(n)$ and $f_b(n)$ satisfy $f_d(n) = \Theta(f_b(n))$.
4.
(a)
True. $f(n) \le cg(n)$ for some constant c, then $\log f(n) \le \log cg(n) = \log c + \log g(n) \le c' \log g(n)$.
(b)
False. Set $f(n) = n^3$, $g(n) = n$, $s(n) = n^3$, $r(n) = n^2$. Then $n^3 = O(n^3)$ and $n = O(n^2)$, but apparently $\frac{f(n)}{g(n)} = n^2 \neq O\left(\frac{s(n)}{r(n)}\right) = O(n)$.
(c)

False. Set $f(n) = n^3 - n$ and $g(n) = n^3$, then $n^3 - n = \Theta(n^3)$ and $g(n) = n^3$ is a bijective function on $\mathbf{R} \to \mathbf{R}$, but apparently $f(n) = n^3 - n$ on $\mathbf{R} \to \mathbf{R}$ is not injective, thus isn't bijective either.

(d)

True. $f(n) \leq c_1 g(n) \leq c_1 g(n)^2 \Rightarrow \sqrt{f(n)} \leq c_2 g(n)$

$h(n) \leq c_3 f(n) \Rightarrow \sqrt{h(n)} \leq c_4 \sqrt{f(n)} \leq c_5 g(n) \leq c_6 g(n)^2 \Rightarrow g(n)^2 = \Omega\left(\sqrt{h(n)}\right)$

5.

(a)

$f(n) = \omega(g(n))$ means $g(n)$ is a lower bound of $f(n)$ but is not asymptotically tight.

(b)

$g(n) = \log n$ and $g(n) = n^{1.5}$

(c)

Yes, because when $\lim_{n\to\infty} \dfrac{h(n)}{f(n)} = 0$, $\lim_{n\to\infty} \dfrac{2h(n)}{f(n)} = 0$ as well.

(d)

No. For example, when $f(n) = n^2$ and $g(n) = n$, $n^2 = \omega(n)$; $\log f(n) = \log n^2 = 2 \log n$ and $\log g(n) = \log n$ but $2 \log n \neq \omega(\log n)$.