CS 170 NOTES

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NOTES FROM A COURSE BY AARON COTE

Abstract. These notes were taken during CS 171 (Discrete Methods in Computer Science) taught by Aaron Cote in Spring 2014 at University of Southern California. They were live-LaTeXed during lectures in `TeXShop` and compiled using `xelatex`. Each lecture gets its own section. The notes are not edited afterward, so there may be typos; please email corrections to `yifeiyan@usc.edu`.

1. Lecture 1, Monday, January 13

Course goals: Discrete math and problem solving skills with a wide range of topics.
Review: Sets, functions, sequences

Definition 1.1. A set is an unordered collection of objects.

Definition 1.2. Two sets are equal if and only if they have the same elements (aka objects, or members).

Venn graph
Universal set U contains all objects under consideration.

Definition 1.3. The intersection of sets S_1 and S_2, denoted by $S_1 \cap S_2$, is the set that contains those elements in both S_1 and S_2. $S_1 \cap S_2 = \{x | x \in S_1 \wedge x \in S_2\}$

Definition 1.4. The union of sets S_1 and S_2, denoted by $S_1 \cup S_2$, is the set that contains those elements that are either in S_1 or S_2, or both. $S_1 \cup S_2 = \{x | x \in S_1 \vee x \in S_2\}$

Definition 1.5. The complement of set S, denoted by $\overline{S}$, is the set that contains those elements that are in the universal set U but not in S. $\overline{S} = \{x | x \notin S\}$

Empty set $\emptyset = \overline{U}$

Definition 1.6. Two sets S_1 and S_2 are disjoint if $S_1 \cap S_2 = \emptyset$.

Generalized intersection

$$\bigcap_{i=1}^{n} S_i = S_1 + S_2 + \cdots + S_n$$

Generalized union

$$\bigcup_{i=1}^{n} S_i$$

Definition 1.7. Set A is a subset of set B, denoted by $A \subseteq B$, if and only if everything in A is in B.

Theorem 1.8. *Any set is a subset of itself. $\emptyset$ is a subset of any set.*

If two sets are subsets of each other, two sets are equal.

Definition 1.9. Set A is a strict subset of set B, denoted by $A \subset B$, if $A \subseteq B$ and $A \neq B$.

$A \subset B \Rightarrow A \subseteq B$, and its opposite is not true.

Example 1.10. 7 stamps: 2 red, 2 yellow, 3 green. A, B, C, are three perfect logicians. A removes blindfold and can't tell any conclusions about the colors about who's wearing what. B can't either.

From what A said, B and C can't wear red or yellow together. C wears green.

Example 1.11. 3 Truth machines are in stock. A machine corresponds true/false to red/green, but patterns for different machines can be different. 1 machine is broken (it answers arbitrarily) and 2 are working. Ask one single question (with a single yes/no answer) to one machine and determine which two are working.

2. Lecture 2, Thursday, January 15

Definition 2.1. The cardinality of a set A, denoted by $|A|$, is the number of distinct elements in A.

Example 2.2. $|\{\text{cake, pie, cake}\}| = 2$

Example 2.3. $|\emptyset| = |\{\}| = 0$

Example 2.4. $|\{\emptyset\}| = 1$

Example 2.5. $|\{\mathbf{Z}, \mathbf{N}\}| = 2$

Example 2.6. $|\mathbf{Z}| = \infty$

Definition 2.7. The power set of S, denoted by $P(S)$, is the set of all subsets of S.

Example 2.8. $S = \{\text{cake, pie}\}$, $P(S) = \{\emptyset, \{\text{cake}\}, \{\text{pie}\}, \{\text{cake, pie}\}\}$

Example 2.9. $P(\emptyset) = \{\emptyset\}$

Example 2.10. $P(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$

Example 2.11. $P(P(\{a\})) = P(\{\emptyset, \{a\}\}) = \{\emptyset, \{\emptyset\}, \{\{a\}\}, \{\emptyset, \{a\}\}\}$

$P(A) = P(B) \rightarrow A = B$
$|A| = n \rightarrow |P(A)| = 2^n$

Definition 2.12. An ordered n-tuple is a collection of elements where order matters, i.e., an ordered set.

Example 2.13. $(a, b) \neq (b, a)$

Example 2.14. $(a, a, b) \neq (a, b)$

Definition 2.15. The Cartesian product of two sets A and B, denoted by $A \times B$, is an unordered set that consists all ordered pairs of (a, b) such that a is in A and b is in B.

$$A \times B = \{(a, b) | a \in A \wedge b \in B\}$$

Example 2.16. $\{1, 2\} \times \{1, 3, 4\} = \{(1, 1), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4)\}$

Example 2.17. $\{(1,1),(1,3),(1,4),(2,1),(2,3),(2,4)\} \times \{1,2\} = \{((1,1),1),((1,3),1),\cdots\}$

Definition 2.18. A function for A to B, denoted by $f : A \to B$, takes as input an element from set A and outputs an element from set B.

Definition 2.19. A function $f : A \to B$ is injective or one-to-one if every input maps to a distinct output, i.e., for an injective function f, $f(a) = f(b) \to a = b$

Example 2.20. $f(x) : \mathbf{R} \to \mathbf{Z}, f(x) = \lfloor x \rfloor$ is not injective.

Remark. Floor function $\lfloor x \rfloor$, ceiling function $\lceil x \rceil$.

Definition 2.21. A function $f : A \to B$ is surjective or onto if every element in B can be produced.

Example 2.22. $f(x) : \mathbf{R} \to \mathbf{Z}, f(x) = \lfloor x \rfloor$ is surjective.

Definition 2.23. A function f is bijective or one-to-one correspondence if it's both injective and surjective.

Example 2.24. $f(x) : \mathbf{Z} \to$ even integers, $f(x) = 2x$ is both injective and surjective, thus is bijective.

Definition 2.25. A sequence is a function from $\mathbf{N}$ to some set S.

Example 2.26. $f_0 = 0, f_1 = 1, f_2 = 2, f_3 = 3, f_4 = 5, f_5 = 8, \cdots$

Example 2.27. $1, 2, 3, 4, \cdots$

Example 2.28. $1, 4, 9, 16, \cdots$

Example 2.29. $f_n = f_{n-1} + f_{n-2},\ f_0 = 0,\ f_1 = 1$

Remark. Recurrence relations: recursive definitions of sequences.

Example 2.30. 3, 3, 3, 3, $\cdots$: $f_0 = 3,\ f_n = f_{n-1}$

Example 2.31. $f_n = 2n$: $f_n = 2 + f_{n-1}$

Example 2.32. $f_n = n^2$: $f_n = f_{n-1} + 2n - 1$

Example 2.33. $f_n = n + (-1)^n$: $f_n = f_{n-2} + 2$

Example 2.34. The polulation of world in 2010 is 6.9 billion, assume it grows at an annual rate of 11%. $f_n = 1.011 f_{n-1},\ f_0 = 6.9\text{billion}$

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