CSCI 170 Homework #1

Due Date: Tuesday, January 28th, 2:30pm

Submit in class or the dropbox (Box 11, first floor of SAL, opposite the Men's bathrooms).

1. Write your name, student ID Number, which lecture you attend (MW morning, MW afternoon, or TTh afternoon), and which discussion section you attend (Mon 4pm, Mon 6pm, Tues, or Wed). Multi-page submissions must be stapled.

2. DotA 2, League of Legends, and StarCraft 2 are three popular video games right now. Let the set D contain the people who play DotA 2, let the set L contain the people who play League of Legends, and let the set S contain the people who play StarCraft 2.

   (a) Consider the set of people who play DotA 2 but not StarCraft 2. Express this set in terms of D, L, and S using set notation. Express this set again using a Venn Diagram.

   (b) Consider the set of people whom either (1) play DotA 2 but not StarCraft 2, or (2) play StarCraft 2 but not League of Legends. Express this set in terms of D, L, and S using set notation. Express this set again using a Venn Diagram.

   (c) In part b), it does not matter if the **or** is an inclusive 'or' or an exclusive 'or'. Explain why.

   (d) Explain why $|\overline{L} \cap \overline{S}| = |U| - |L| - |S| + |L \cap S|$.

3. Consider a sequence indicating the cost to produce a bunch of satellites. Let C_n denote the cost to produce the nth satellite. Assumedly each successive unit will cost less as we improve efficiency, which will be represented by the value of B. The cost to produce the first n satellites is then given by the following summation:

$$\sum_{i=1}^{n} C_i = C_1 n^B, \text{ where } B = 1 - \log_2(\frac{1}{0.8})$$

   (a) How much does it cost to produce 8 satellites, assuming the first satellite costs 10 million dollars?

   (b) Give a recurrence relation indicating the cost of the nth unit C_n in terms of n, B, and C_i for $i \leq n-1$.

   (c) Give a closed formula indicating the cost of the nth unit C_n in terms of n, B, and C_1.

   (d) What is the difference in cost between the 8th satellite and the first satellite?

4. For each of the following functions, $\mathbf{O}$ denotes the set of odd integers, and $\mathbf{E}$ denotes the set of even integers.
   Indicate whether each of these functions are injective (one-to-one), surjective (onto), neither, or both (bijective). Explain your reasoning.

   (a) $f : \mathbf{O} \to \mathbf{E}$, where $f(x) = 4x$

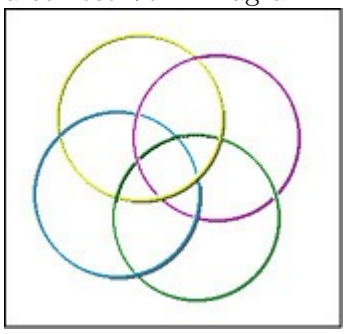
   (b) $f : \mathbf{Z} \to \mathbf{Z}$, where $f(x) = \lfloor \frac{x}{2} \rfloor$

   (c) $f : \mathbf{Z} \to \mathbf{Z}$, where $f(x) = x + 2$ if $x \in \mathbf{E}$, and $f(x) = x + 4$ if $x \in \mathbf{O}$

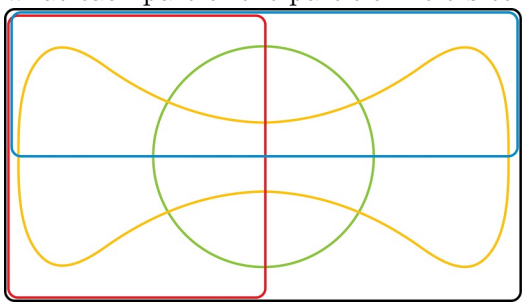
   (d) $f : \mathbf{Z} \to \mathbf{N}$, where $f(x) = 1 + |x|$ (1 plus the absolute value of x)

5. We will talk about 4-set Venn Diagrams in this problem.

   (a) Consider the following attempt at a 4-set Venn Diagram. Clearly explain why it is not a correct Venn Diagram.



   (b) Here is another attempt which works. Let set A be the red rectangle on the left. Let set B be the blue rectangle on the top. Let set C be the green circle. Let set D by the yellow hourglass. Explain why this **is** a correct 4-set Venn Diagram by clearly labeling what each part of the partition refers to.



   (c) Using the correct 4-set Venn Diagram from part b), redraw the diagram and indicate each of the following sets (by shading them).

      i. $(B \cap C) - D$

      ii. $[A - (A \cap B)] \cup (B \cap C)$

   (d) Is it true that $B \subseteq (B \cap A) \cup (B \cap C) \cup (B \cap D)$? Explain your reasoning.

If you need some extra practice, try the following problems. Do not submit them, as they will not be graded. All practice problems and solutions are in the Rosen textbook.

Chapter 2.1: exercises 5, 11, 19, 23
Chapter 2.2: exercises 3, 27, 51
Chapter 2.3: exercises 5, 23, 57, 69
Chapter 2.4: exercises 5, 17, 19, 29, 35