practice_housingprice_belief

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Dataset

In this case study, I'm going to fit a Beyesian approach based linear regression model. First I'll explain the dataset. The project is part of my PhD study, and I collected data from mathematics teachers in China's secondary school via questionniare. There is 155 respondants in total. In MLM_data, it contains common demographic information.It also contains 8 set of plausible values of theta_P (i.e., teachers' connectionist tendency- how often they conduct conduct connectionist pedagogies) and thetaB_G (i.e., teachers' belief about equity, how inclusive they are in terms of teaching and learning in mathematics education.) Recall that we've also run Bayesian Item Response Model elsewhere, and here the 8 sets of plausible values is extrated randomly from the posterior of the two IRTs. Specically, it is the posterior of no. 1100/1500 iterations in chain 1; no. 1200/1600 iterations in chain 2; no, 1300/1700 iterations in chain 3; and 1400/1800 iterations in chain 4. picking them randomly is for caputring the uncertainties in previous IRT models. The school_data contains housing price and school resouce data. Not all these demographic info will be used here, but it is not because I didn't try - I tried so many different combinations, and the case study here after is the relatively best choice I can think of. Here is the brief variable description we are using later:

Variable	**Level**	**Description**
Dependent Variable (DV)		
_P (theta_P)	Teacher	Teacher's connectionist tendency. Higher scores indicate more frequent use of connectionist pedagogy.
Independent Variables (IVs)		
Teacher-Level Variables		
B_G (thetaB_G)	Teacher	Teacher's belief about equity. Higher scores indicate a more inclusive belief in mathematics education.
Sex	Teacher	Gender: 1 for male, 2 for female.
Attain_Lvl	Teacher	Self-reported average attainment level of students in their class (1-5 scale). Higher values indicate stronger perceived attainment.
YearG	Teacher	The year group that the teacher teaches.
School-Level Variables		
Sch_id	School	School ID.
zlog_hp	School	Standardized log average housing price around the school.
z_resource	School	Availability of educational resources on a standardised scale.
District	School	School district classification (1/2/3) based on SES ranking, where 1 is the highest.

(It should be noted that here I adopt a two-step Bayesian approach, in which I didn't 'take away' whole information from previous IRT models. The ideal Bayesian tradition is to jointly fit all the models, say including IRT fit and LM fit in a same Bayesian model. I gave up this full bayesian appraoch for serveral reasons: i. The package I use (brms) does not support a explicit joint model that tranferring latent variables between each other; ii. Additionally, the package that supports full Bayesian modeling in this case (e.g., rstan) requires extremely high computational power and highly optimized code logic. I attempted multiple times to construct a fully joint model, but I consistently encountered convergence issues. One major reason is the significant difference in data structures between the two modeling layers: in the IRT model, each respondent answered more than 50 items, resulting in over 7,500 response observations, meaning the data is relatively rich; however, in the LM model, only 155 teachers provided 155 observations. This imbalance in data structure makes it extremely difficult to achieve simultaneous convergence of both layers within a single MCMC simulation.)

Data Cleaning

Let's do some data cleaning. I was already aware that my data structure contained some missing values, so I standardized their representation by displaying them uniformly as NA. Then, I matched my eight sets of plausible teacher theta values with the cleaned dataset, resulting in eight corresponding plausible datasets.

```
## Subset data for plausible value set 1 created with 155 rows and 7 columns
## Subset data for plausible value set 2 created with 155 rows and 7 columns
## Subset data for plausible value set 3 created with 155 rows and 7 columns
## Subset data for plausible value set 4 created with 155 rows and 7 columns
## Subset data for plausible value set 5 created with 155 rows and 7 columns
## Subset data for plausible value set 6 created with 155 rows and 7 columns
## Subset data for plausible value set 7 created with 155 rows and 7 columns
## Subset data for plausible value set 8 created with 155 rows and 7 columns
```

```
## tibble [155 x 7] (S3: tbl_df/tbl/data.frame)
##  $ District  : Factor w/ 3 levels "1","2","3": 2 2 2 2 2 2 2 3 3 3 ...
##  $ Sch_ID    : Factor w/ 36 levels "0","101","102",..: 25 25 23 24 18 26 23 31 31 35 ...
##  $ Sex       : Factor w/ 2 levels "1","2": 1 2 1 1 2 2 1 1 1 1 ...
##  $ YearG     : Factor w/ 3 levels "1","2","3": 1 3 3 2 2 1 1 1 1 2 ...
##  $ Attain_Lvl: Factor w/ 5 levels "1","2","3","4",..: 4 2 3 3 4 2 4 2 2 3 ...
##  $ theta_P_1 : num [1:155] 1.081 0.146 -1.197 -1.899 0.873 ...
##  $ thetaB_G_1: num [1:155] 0.379 0.328 1.558 1.565 -0.172 ...
```

Multiple imputation for missing data

Ideally in Bayesian tradition, the missing data is best estimated jointly with the main model. However my missing data are mainly discrete data (e.g., gender, district, attainlvl etc.), the flagship algorithm HMC, which is built into brms, faces significant challenges in estimating discrete data as missing values simultaneously. Therefore, I chose to handle these missing data externally using multiple imputation before fitting the model. Here, I perform multiple imputations separately for each set of plausible data. Within each dataset, I establish a global prediction strategy, meaning that all available observed data are used to estimate the missing values. It is important to note that _P (theta_P) is the dependent variable (DV) and has no missing data, so it is excluded from the imputation process. Additionally, Sch_ID is also not imputed. A small number of teachers did not report their school ID, and rather than predicting their likely school assignment, I chose to place them into a placeholder school category labeled as "0" instead of imputing a potentially incorrect school group. For each set of plausible data, the imputation process undergoes 20 iterations using the built-in algorithm in the mice package to ensure stability and convergence. After these iterations, five different imputed versions are generated for each plausible dataset, incorporating natural variability in missing data estimation. Given that there are eight plausible datasets in total, this process results in the creation of 40 fully imputed datasets for further analysis.

School data processing

For the school level data, some pre-processing is needed here. According to the literature, I aggregate educational resource variable from number of students/number of teachers/building area/financial expenditure. Specifically, I first calculate the area/teacher/financial expenditure per student, and then normalised each variable to unify the scale, and use the weighted adding strategy (used in literature: n_resource = 0.2 * norm_area + 0.5 * norm_teacher + 0.3 * norm_finance) to get the variable n_resource. For housing price, logarithmic transformation before standardisation is also standard practice in the literature (to handle skewed distribution nature of hp).

```
##   District Sch_ID Sex YearG Attain_Lvl theta_P thetaB_G z_resource    zlog_hp
## 1        2    209   1     1          4   1.081    0.379  0.3171210  1.1067795
## 2        2    209   2     3          2   0.146    0.328  0.3171210  1.1067795
## 3        2    207   1     3          3  -1.197    1.558  2.0714826  1.0310756
## 4        2    208   1     2          3  -1.899    1.565 -0.5230419  0.3519804
## 5        2    201   2     2          4   0.873   -0.172 -0.2380654  0.2173909
## 6        2    213   2     1          2   1.289    3.319 -0.4872584 -0.8332454

##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## -1.021263 -0.529695 -0.239422 -0.005045  0.348904  2.071483

##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## -1.940134 -0.542190  0.184960 -0.007747  0.621380  1.370986
```

Model test

To recall - now I have 40 plausible datasets with no missing data, and I would like to know the mechanism for teacher conducting connectionist practice.

Model iteration

I have done way more work than it presented here, but a logical testing/thinking procedure can be this: 1. (test-simple): P ~ B - we want a simple model including the main variable only.

2. (test_basic_lm): P ~ B + District + Sex + mo(YearG) + mo(Attain_Lvl) + zlog_hp + z_resource

- We want to add in all variables that we are interested in.

3. (test_mlm): P ~ B + District + Sex + mo(YearG) + mo(Attain_Lvl) + zlog_hp + z_resource + (1|school)

- We want to introduce the multilevel nature of the data in. Note that even if statistically the multilevel choice works slightly worse than single level choice, I'd still prefer mlm (unless it's 'incredibly' bad) - because it provides irreplaceable explanatory power, as in reality teachers are nested in schools, and we'd like to know if, and to what extent, the intercept is different from school to school (i.e., even if there is no pattern found, it still provides important information).

4. (test_BgAttainonly_mlm): P ~ B * mo(Attain_Lvl) + District + Sex + mo(YearG) + zlog_hp + z_resource + (1|school)
5. (test_hpYearG_mlm): P ~ B + mo(Attain_Lvl) + District + Sex + mo(YearG) * zlog_hp + z_resource + (1|school)

- If we are lucky that our dataset/model is good enough, and our assumption is simple enough, we can stop at model3. Simple models always have clearer presentation and better explanation power. But there is no harm to try some other settings, like interactions, here. However, mathematically we can try so many combinations here, we need to choose the ones guided by theory/assumption. To be very honest, I almost tried everything - It seems that my various combinations of variables have pretty good explanatory power in the sense of folk psychology, This can be good or/and bad. Here, I mainly look into:

i. the interaction between B and mo(Attain_Lvl;
ii. the interaction between zlop_hp and mo(YearG).

6. (test_final2_mlm): P ~ B * mo(Attain_Lvl) + District + Sex + mo(YearG) * zlog_hp + n_resource + (1|school)

For small scale dataset, we should always consider whether our model can include the complexity we want. I then tried three combinations to include information from model 4/5.

(Note that it is also common, and yet powerful(!), to include random slope in higher level, e.g., (1+B|school). I've tried different combinations, but as my dataset scale is too small, that some level/schools contains less than 5 teachers, it makes the random slope unstable. That's why I gave up, but that doesn't mean there's no need to test this step.)

Model Comparison

Let's first be clear about the purpose of model comparison here - we are not aim to analyse each model's output (we'll do this later), but instead, we want to know whether the models are stable enough, and which provides more information, and is better. Three model evaluation metrics are used here: Bayes R^2 (similar to R^2 in frequentism); LOO-CV (Leave-One-Out Cross-Validation, a model comparison criterion used to estimate out-of-sample predictive accuracy); Model weights. As we can see in the output, final1_mlm performs relatively the best in all metrics.

	Model	ELPD.diff..95..CI.	Weight	Bayesian.R...95..CI.
final1_mlm.final1_mlm	Final (both interactions)	0.00 [0.00, 0.00]	59.601%	0.207 [0.061, 0.357]
hpYearG_mlm.hpYearG_mlm	YearG×hp MLM	-2.54 [-10.01, 4.92]	7.014%	0.170 [0.056, 0.302]
simple.simple	Simple	-3.91 [-13.91, 6.09]	33.385%	0.051 [0.006, 0.115]
BgAttainonly_mlm.BgAttainonly_mlm	B×Attain MLM	-5.04 [-11.61, 1.53]	0.000%	0.158 [0.053, 0.282]
basic_lm.basic_lm	Basic LM	-8.37 [-17.86, 1.12]	0.000%	0.099 [0.040, 0.192]
mlm.mlm	MLM	-9.22 [-18.63, 0.19]	0.000%	0.112 [0.043, 0.212]

Test different likelihood family

Though we already staandardised all variables, considering the sample size is relatively (very) small, I test different likelihood family to see if choices other than normal (i.e., student_t and skew_normal) can have better performance. The result shows there is almost no difference between the three settings. Then we can comfortably continue analysing with default normal likelihood.

```
##                         elpd_diff se_diff
## test_final1_skewnormal   0.0       0.0
## test_final1_mlm         -0.8       1.4
## test_final1_student     -1.5       1.3
```

Final model presentation

According to the test model comparison result, I choose the first three models (final MLM, MLM with HP*YearG, simple P~B). I also add the base MLM model for reference.

Final model summary

Simple model P ~ B

```
##
##
## SIMPLE MODEL COMPARISON

## ======================================

## TEST MODEL (single dataset)

## --------------------------------------

##  Family: gaussian
##   Links: mu = identity; sigma = identity
## Formula: ztheta_P ~ zthetaB_G
##    Data: final_imputed_datasets[[1]] (Number of observations: 155)
##   Draws: 4 chains, each with iter = 1000; warmup = 500; thin = 1;
##          total post-warmup draws = 2000
##
## Regression Coefficients:
##           Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## Intercept    -0.00      0.08    -0.14     0.15 1.00     1496     1326
## zthetaB_G     0.21      0.08     0.06     0.37 1.00     1795     1128
##
## Further Distributional Parameters:
##       Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma     0.98      0.06     0.88     1.10 1.00     1431     1164
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

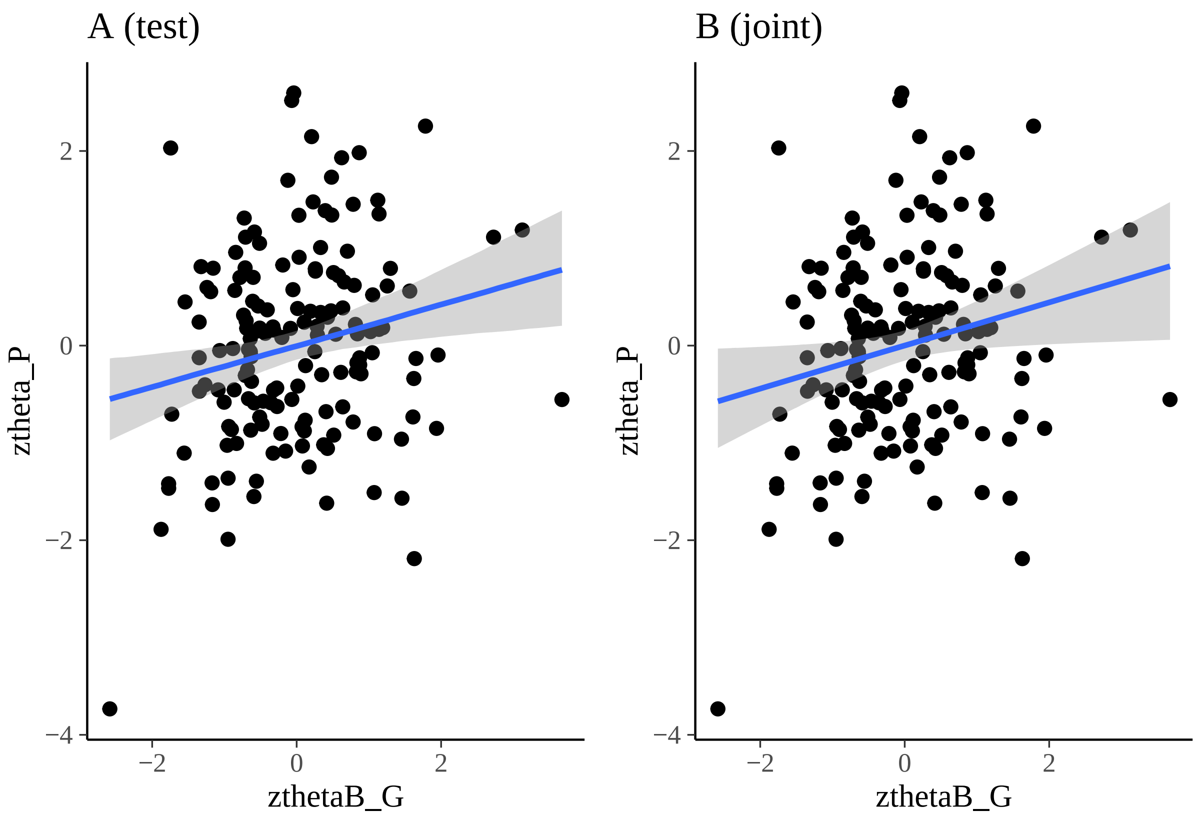
```
##
## MULTIPLE MODEL (40 imputed datasets)

## --------------------------------------

##  Family: gaussian
##   Links: mu = identity; sigma = identity
## Formula: ztheta_P ~ zthetaB_G
##    Data: final_imputed_datasets (Number of observations: 155)
##   Draws: 160 chains, each with iter = 1000; warmup = 500; thin = 1;
##          total post-warmup draws = 80000
##
## Regression Coefficients:
##           Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## Intercept     0.00      0.08    -0.15     0.16 1.00    59758    50231
## zthetaB_G     0.22      0.10     0.02     0.40 1.20      537     1013
##
## Further Distributional Parameters:
##       Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma     0.98      0.06     0.88     1.11 1.02     4647    45550
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

The simple model indicates there is a small to medium positive association between B and P, with a posterior mean of = 0.21 [0.06, 0.37] in test model, and = 0.22 [0.02, 0.40] in the joint model. The entire credible interval is positive, providing strong evidence for a meaningful positive relationship. We can also visualise this as below:

P ~ B presentation But as we can see, it is off huge uncertainty. This is also why Bayes R^2 of simple model is extremely low, indicating though it fits well, it can only explain little of the model variance.



Basic MLM summary

```
##
##
## BASIC MULTILEVEL MODEL COMPARISON

## ======================================

## TEST MODEL (single dataset)

## --------------------------------------

##  Family: gaussian
##   Links: mu = identity; sigma = identity
## Formula: ztheta_P ~ zthetaB_G + District + Sex + mo(YearG) + zlog_hp + z_resource + mo(Attain_Lvl) +
##    Data: final_imputed_datasets[[1]] (Number of observations: 155)
##   Draws: 4 chains, each with iter = 1000; warmup = 500; thin = 1;
##          total post-warmup draws = 2000
##
## Multilevel Hyperparameters:
## ~Sch_ID (Number of levels: 36)
##               Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)     0.13      0.10     0.01     0.35 1.00      856      821
##
```

```
## Regression Coefficients:
##              Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## Intercept       -0.02      0.24    -0.49     0.48 1.00     1635     1441
## zthetaB_G        0.20      0.08     0.03     0.36 1.00     2950     1531
## District2       -0.31      0.21    -0.73     0.10 1.00     2359     1492
## District3       -0.14      0.22    -0.57     0.30 1.00     1534     1294
## Sex2             0.05      0.16    -0.26     0.38 1.01     2857     1526
## zlog_hp          0.09      0.10    -0.11     0.29 1.00     2139     1634
## z_resource      -0.06      0.13    -0.31     0.20 1.00     2043     1651
## moYearG         -0.03      0.10    -0.23     0.16 1.00     1902     1556
## moAttain_Lvl     0.08      0.09    -0.12     0.26 1.00     1483     1163
##
## Monotonic Simplex Parameters:
##                  Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## moYearG1[1]          0.52      0.29     0.03     0.98 1.00     1952     1180
## moYearG1[2]          0.48      0.29     0.02     0.97 1.00     1952     1180
## moAttain_Lvl1[1]     0.25      0.19     0.01     0.69 1.00     2138     1101
## moAttain_Lvl1[2]     0.21      0.17     0.01     0.64 1.00     2106      900
## moAttain_Lvl1[3]     0.26      0.19     0.01     0.69 1.00     1946     1387
## moAttain_Lvl1[4]     0.27      0.20     0.01     0.72 1.00     2575     1375
##
## Further Distributional Parameters:
##       Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma     0.99      0.06     0.88     1.10 1.00     2356     1549
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

```
##
## MULTIPLE MODEL (40 imputed datasets)
```

```
## --------------------------------------
```

```
##  Family: gaussian
##   Links: mu = identity; sigma = identity
## Formula: ztheta_P ~ zthetaB_G + District + Sex + mo(YearG) + zlog_hp + z_resource + mo(Attain_Lvl) +
##    Data: final_imputed_datasets (Number of observations: 155)
##   Draws: 160 chains, each with iter = 1000; warmup = 500; thin = 1;
##          total post-warmup draws = 80000
##
## Multilevel Hyperparameters:
## ~Sch_ID (Number of levels: 36)
##               Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)     0.14      0.10     0.00     0.38 1.02     5894    21435
##
## Regression Coefficients:
##              Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## Intercept        0.05      0.27    -0.50     0.59 1.03     2711    11837
## zthetaB_G        0.20      0.10     0.01     0.39 1.16      635     1392
## District2       -0.21      0.22    -0.64     0.23 1.06     1643     6257
## District3       -0.18      0.24    -0.64     0.29 1.08     1141     5615
## Sex2            -0.02      0.17    -0.35     0.32 1.04     2139    16610
```

```
## zlog_hp          0.09      0.11    -0.12     0.30 1.03     3220    16535
## z_resource      -0.04      0.13    -0.30     0.23 1.06     1561     6690
## moYearG         -0.04      0.10    -0.24     0.16 1.02     5192    58360
## moAttain_Lvl     0.05      0.10    -0.16     0.25 1.04     2553    18745
##
## Monotonic Simplex Parameters:
##                  Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## moYearG1[1]          0.51      0.28     0.03     0.97 1.00   106537    51163
## moYearG1[2]          0.49      0.28     0.03     0.97 1.00   106537    51163
## moAttain_Lvl1[1]     0.26      0.19     0.01     0.71 1.00    82184    44457
## moAttain_Lvl1[2]     0.22      0.18     0.01     0.66 1.00    92649    47554
## moAttain_Lvl1[3]     0.26      0.19     0.01     0.71 1.00    33392    56413
## moAttain_Lvl1[4]     0.26      0.19     0.01     0.71 1.00    92214    56896
##
## Further Distributional Parameters:
##       Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma     0.99      0.06     0.88     1.11 1.01     7825    57200
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

The MLM did not perform well in the previous test model comparison, but it is included here as a baseline multilevel model for reference. After adding the (1 | school) random effect, the coefficient for B remained largely unchanged. Furthermore, most of the variables we are interested in have minimal effects on P, with posterior mean coefficients all below 0.1. While District shows a small negative effect on P on average compared to the baseline level, its posterior credible interval crosses 0, indicating a high degree of uncertainty.

MLM with housing_price * YearGroup interaction

```
##
##
## hpYearG INTERACTION MODEL COMPARISON

## ======================================

## TEST MODEL (single dataset)

## --------------------------------------

##  Family: gaussian
##   Links: mu = identity; sigma = identity
## Formula: ztheta_P ~ zthetaB_G + District + Sex + mo(YearG) * zlog_hp + z_resource + mo(Attain_Lvl) +
##    Data: final_imputed_datasets[[1]] (Number of observations: 155)
##   Draws: 4 chains, each with iter = 1000; warmup = 500; thin = 1;
##          total post-warmup draws = 2000
##
## Multilevel Hyperparameters:
## ~Sch_ID (Number of levels: 36)
##               Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)     0.16      0.11     0.01     0.42 1.01      586      713
##
```

```
## Regression Coefficients:
##                 Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## Intercept           0.04      0.23    -0.41     0.51 1.00     1315     1099
## zthetaB_G           0.19      0.07     0.04     0.34 1.00     1769     1623
## District2          -0.36      0.20    -0.76     0.05 1.00     1663     1568
## District3          -0.20      0.21    -0.62     0.22 1.00     1496     1424
## Sex2                0.03      0.16    -0.28     0.34 1.00     2684     1634
## zlog_hp             0.48      0.15     0.19     0.78 1.00     1060     1396
## z_resource         -0.05      0.13    -0.32     0.19 1.00     1680     1264
## moYearG            -0.02      0.10    -0.22     0.18 1.00     1469     1537
## moAttain_Lvl        0.07      0.09    -0.11     0.25 1.00     1171     1105
## moYearG:zlog_hp    -0.35      0.10    -0.54    -0.16 1.00     1138     1645
##
## Monotonic Simplex Parameters:
##                     Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## moYearG1[1]             0.51      0.29     0.02     0.98 1.00     1536     1190
## moYearG1[2]             0.49      0.29     0.02     0.98 1.00     1536     1190
## moAttain_Lvl1[1]        0.24      0.19     0.01     0.68 1.00     2177     1081
## moAttain_Lvl1[2]        0.20      0.17     0.01     0.61 1.01     1692      754
## moAttain_Lvl1[3]        0.28      0.20     0.01     0.73 1.00     1753     1274
## moAttain_Lvl1[4]        0.28      0.20     0.01     0.72 1.00     2506     1328
## moYearG:zlog_hp1[1]     0.87      0.11     0.58     1.00 1.00     2614     1193
## moYearG:zlog_hp1[2]     0.13      0.11     0.00     0.42 1.00     2614     1193
##
## Further Distributional Parameters:
##       Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma     0.94      0.06     0.83     1.06 1.00     1874     1507
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

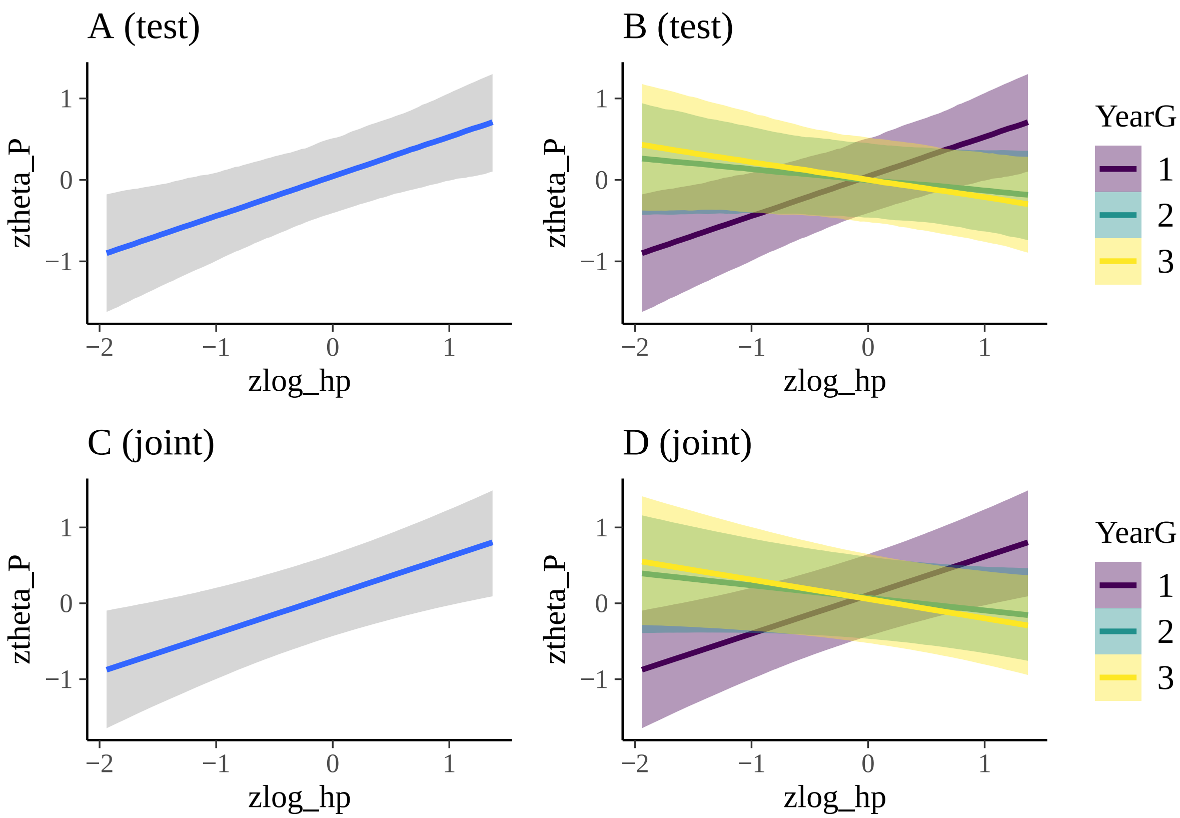
```
##
## MULTIPLE MODEL (40 imputed datasets)
```

```
## --------------------------------------
```

```
##  Family: gaussian
##   Links: mu = identity; sigma = identity
## Formula: ztheta_P ~ zthetaB_G + District + Sex + mo(YearG) * zlog_hp + z_resource + mo(Attain_Lvl) +
##    Data: final_imputed_datasets (Number of observations: 155)
##   Draws: 160 chains, each with iter = 1000; warmup = 500; thin = 1;
##          total post-warmup draws = 80000
##
## Multilevel Hyperparameters:
## ~Sch_ID (Number of levels: 36)
##               Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)     0.16      0.12     0.01     0.43 1.03     3500    10466
##
## Regression Coefficients:
##                 Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## Intercept           0.11      0.27    -0.43     0.65 1.05     2039     6878
## zthetaB_G           0.21      0.10     0.02     0.39 1.20      541     1502
```

```
## District2          -0.26      0.22    -0.69     0.17 1.06     1462     4505
## District3          -0.23      0.23    -0.69     0.24 1.10      961     4269
## Sex2               -0.04      0.16    -0.36     0.28 1.05     1811    10733
## zlog_hp             0.51      0.16     0.20     0.81 1.05     1784     8734
## z_resource         -0.04      0.13    -0.30     0.22 1.07     1414     7627
## moYearG            -0.02      0.10    -0.21     0.17 1.02     4993    39848
## moAttain_Lvl        0.04      0.10    -0.17     0.23 1.05     1997    13494
## moYearG:zlog_hp    -0.38      0.10    -0.58    -0.18 1.07     1366     5183
##
## Monotonic Simplex Parameters:
##                    Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## moYearG1[1]            0.50      0.29     0.03     0.97 1.00   104513    51460
## moYearG1[2]            0.50      0.29     0.03     0.97 1.00   104513    51460
## moAttain_Lvl1[1]       0.26      0.19     0.01     0.71 1.00    81877    45027
## moAttain_Lvl1[2]       0.22      0.18     0.01     0.66 1.00    98984    48370
## moAttain_Lvl1[3]       0.26      0.20     0.01     0.72 1.01    12320    55664
## moAttain_Lvl1[4]       0.26      0.20     0.01     0.71 1.00    90087    56457
## moYearG:zlog_hp1[1]    0.88      0.11     0.60     1.00 1.01     9515    23878
## moYearG:zlog_hp1[2]    0.12      0.11     0.00     0.40 1.01     9515    23878
##
## Further Distributional Parameters:
##       Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma     0.93      0.06     0.82     1.06 1.05     1728     7735
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

However, after adding housing price * YearGroup term into the MLM model, some interesting patterns occur. The housing price variable (zlog_hp) already shows a marginally large effect on P at the baseline level.At the same time, this effect is further moderated by YearG, as indicated by the negative interaction term (moYearG:zlog_hp = -0.38, 95% CI: -0.58, -0.18). The results suggest that the positive effect of housing price on P decreases as YearG increases and may even reverse. In other words, while higher housing prices are generally associated with a greater connectionist teaching tendency in lower year groups, this effect diminishes and could become negative for teachers working with older students. This pattern well-occured in both test model and joint model, as visualised below: ### Presentation of zlog_hp * YearG



Final MLM with two interactions

```
##
##
## FINAL MODEL COMPARISON

## ======================================

## TEST MODEL (single dataset)

## --------------------------------------

##  Family: gaussian
##   Links: mu = identity; sigma = identity
## Formula: ztheta_P ~ zthetaB_G * mo(Attain_Lvl) + District + Sex + mo(YearG) * zlog_hp + z_resource +
##    Data: final_imputed_datasets[[1]] (Number of observations: 155)
##   Draws: 4 chains, each with iter = 1000; warmup = 500; thin = 1;
##          total post-warmup draws = 2000
##
## Multilevel Hyperparameters:
## ~Sch_ID (Number of levels: 36)
##               Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)     0.18      0.12     0.01     0.46 1.00      412      463
##
```

```
## Regression Coefficients:
##                        Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS
## Intercept                 -0.02      0.24    -0.50     0.39 1.01     1672
## zthetaB_G                 -0.19      0.23    -0.78     0.16 1.00     1053
## District2                 -0.32      0.20    -0.71     0.07 1.00     1791
## District3                 -0.25      0.21    -0.66     0.18 1.00     1654
## Sex2                       0.05      0.15    -0.24     0.34 1.00     2602
## zlog_hp                    0.44      0.15     0.16     0.72 1.00     1317
## z_resource                -0.05      0.13    -0.31     0.20 1.00     1753
## moAttain_Lvl               0.10      0.09    -0.06     0.27 1.01     1332
## moYearG                   -0.01      0.10    -0.20     0.18 1.00     2121
## moAttain_Lvl:zthetaB_G     0.27      0.11     0.07     0.50 1.00      853
## moYearG:zlog_hp           -0.32      0.09    -0.49    -0.15 1.00     1726
##                        Tail_ESS
## Intercept                  1326
## zthetaB_G                   864
## District2                  1376
## District3                  1245
## Sex2                       1630
## zlog_hp                    1416
## z_resource                 1359
## moAttain_Lvl               1394
## moYearG                    1437
## moAttain_Lvl:zthetaB_G     1164
## moYearG:zlog_hp            1532
##
## Monotonic Simplex Parameters:
##                            Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS
## moAttain_Lvl1[1]               0.23      0.18     0.01     0.66 1.00     1763
## moAttain_Lvl1[2]               0.20      0.17     0.01     0.61 1.00     2714
## moAttain_Lvl1[3]               0.29      0.20     0.01     0.73 1.00     1371
## moAttain_Lvl1[4]               0.28      0.20     0.02     0.71 1.00     1465
## moYearG1[1]                    0.50      0.29     0.02     0.97 1.00     1998
## moYearG1[2]                    0.50      0.29     0.03     0.98 1.00     1998
## moAttain_Lvl:zthetaB_G1[1]     0.20      0.15     0.01     0.54 1.00     1533
## moAttain_Lvl:zthetaB_G1[2]     0.07      0.08     0.00     0.27 1.00     1717
## moAttain_Lvl:zthetaB_G1[3]     0.28      0.17     0.03     0.68 1.00     1319
## moAttain_Lvl:zthetaB_G1[4]     0.45      0.20     0.05     0.80 1.00     1456
## moYearG:zlog_hp1[1]            0.86      0.13     0.54     1.00 1.00     1845
## moYearG:zlog_hp1[2]            0.14      0.13     0.00     0.46 1.00     1845
##                            Tail_ESS
## moAttain_Lvl1[1]                975
## moAttain_Lvl1[2]               1250
## moAttain_Lvl1[3]                995
## moAttain_Lvl1[4]               1448
## moYearG1[1]                    1211
## moYearG1[2]                    1211
## moAttain_Lvl:zthetaB_G1[1]     1091
## moAttain_Lvl:zthetaB_G1[2]     1162
## moAttain_Lvl:zthetaB_G1[3]     1119
## moAttain_Lvl:zthetaB_G1[4]      850
## moYearG:zlog_hp1[1]             826
## moYearG:zlog_hp1[2]             826
##
```

```
## Further Distributional Parameters:
##       Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma     0.91      0.06     0.80     1.02 1.00     2267     1426
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

```
##
## MULTIPLE MODEL (40 imputed datasets)
```

```
## --------------------------------------
```

```
##  Family: gaussian
##   Links: mu = identity; sigma = identity
## Formula: ztheta_P ~ zthetaB_G * mo(Attain_Lvl) + District + Sex + mo(YearG) * zlog_hp + z_resource +
##    Data: final_imputed_datasets (Number of observations: 155)
##   Draws: 160 chains, each with iter = 1000; warmup = 500; thin = 1;
##          total post-warmup draws = 80000
##
## Multilevel Hyperparameters:
## ~Sch_ID (Number of levels: 36)
##               Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)     0.17      0.12     0.01     0.44 1.03     3338    12411
##
## Regression Coefficients:
##                       Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS
## Intercept                 0.06      0.27    -0.49     0.59 1.04     2329
## zthetaB_G                 0.03      0.28    -0.59     0.56 1.21      515
## District2                -0.25      0.22    -0.68     0.19 1.06     1449
## District3                -0.23      0.24    -0.69     0.24 1.10      961
## Sex2                     -0.03      0.17    -0.35     0.29 1.06     1470
## zlog_hp                   0.50      0.16     0.19     0.81 1.06     1557
## z_resource               -0.04      0.13    -0.30     0.22 1.06     1520
## moAttain_Lvl              0.06      0.10    -0.15     0.25 1.06     1542
## moYearG                  -0.02      0.10    -0.21     0.17 1.02     5301
## moAttain_Lvl:zthetaB_G    0.12      0.15    -0.19     0.44 1.36      358
## moYearG:zlog_hp          -0.37      0.10    -0.57    -0.17 1.07     1372
##                       Tail_ESS
## Intercept                10163
## zthetaB_G                 1021
## District2                 5002
## District3                 4611
## Sex2                      7568
## zlog_hp                   7464
## z_resource                7389
## moAttain_Lvl              7409
## moYearG                  29232
## moAttain_Lvl:zthetaB_G     553
## moYearG:zlog_hp           6891
##
## Monotonic Simplex Parameters:
##                           Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS
```

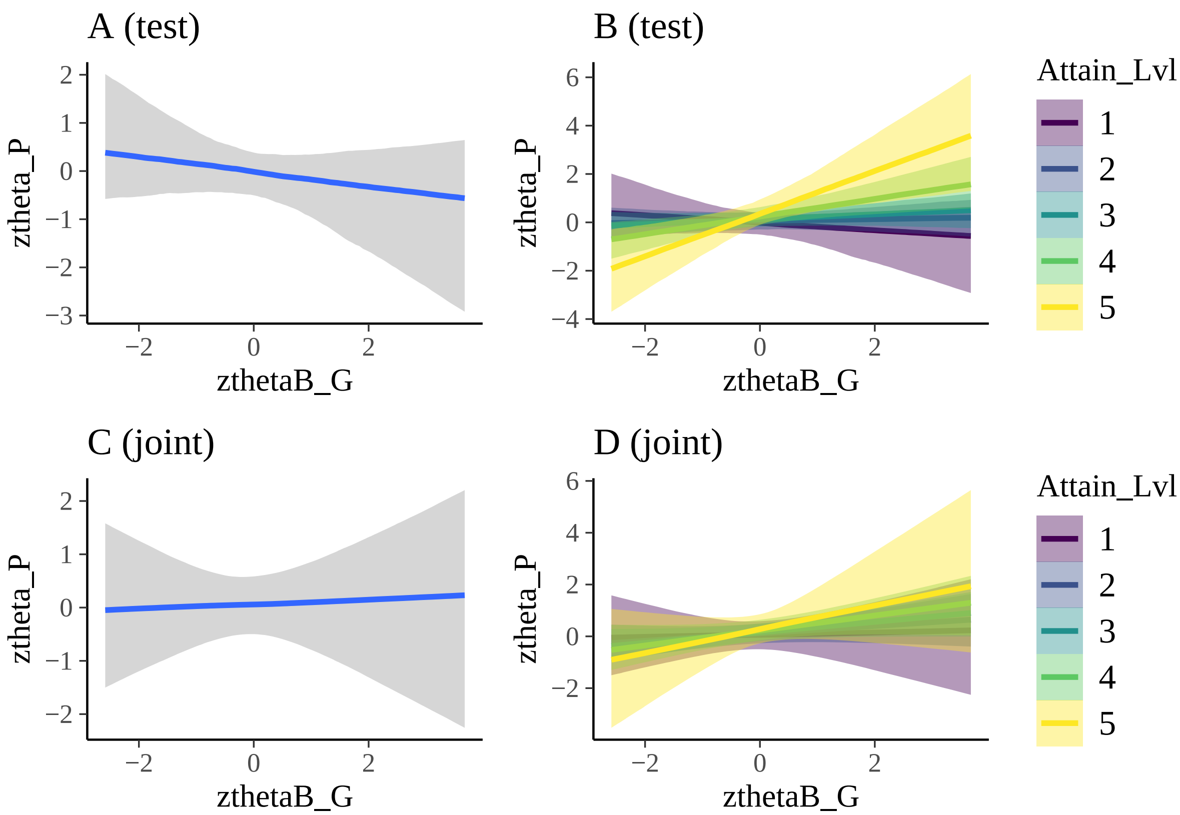
```
## moAttain_Lvl1[1]                0.26      0.19     0.01     0.71 1.00    81426
## moAttain_Lvl1[2]                0.21      0.17     0.01     0.64 1.00    96902
## moAttain_Lvl1[3]                0.27      0.20     0.01     0.71 1.01    10550
## moAttain_Lvl1[4]                0.26      0.20     0.01     0.72 1.00    85955
## moYearG1[1]                     0.50      0.28     0.03     0.97 1.00   116585
## moYearG1[2]                     0.50      0.28     0.03     0.97 1.00   116585
## moAttain_Lvl:zthetaB_G1[1]      0.26      0.19     0.01     0.70 1.02     4076
## moAttain_Lvl:zthetaB_G1[2]      0.19      0.17     0.01     0.62 1.05     1957
## moAttain_Lvl:zthetaB_G1[3]      0.20      0.17     0.01     0.63 1.04     2418
## moAttain_Lvl:zthetaB_G1[4]      0.35      0.23     0.02     0.81 1.06     1608
## moYearG:zlog_hp1[1]             0.87      0.11     0.58     1.00 1.01     9602
## moYearG:zlog_hp1[2]             0.13      0.11     0.00     0.42 1.01     9602
##                             Tail_ESS
## moAttain_Lvl1[1]               45460
## moAttain_Lvl1[2]               49531
## moAttain_Lvl1[3]               55442
## moAttain_Lvl1[4]               54262
## moYearG1[1]                    54630
## moYearG1[2]                    54630
## moAttain_Lvl:zthetaB_G1[1]     17811
## moAttain_Lvl:zthetaB_G1[2]     21913
## moAttain_Lvl:zthetaB_G1[3]     11236
## moAttain_Lvl:zthetaB_G1[4]      3377
## moYearG:zlog_hp1[1]            28331
## moYearG:zlog_hp1[2]            28331
##
## Further Distributional Parameters:
##       Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sigma     0.92      0.06     0.81     1.05 1.06     1627     6739
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

The final MLM model includes two interaction terms: Attain_Lvl × thetaB_G and zlog_hp × YearG. For zlog_hp × YearG, the same pattern is observed in both the test model (which uses a single plausible dataset) and the joint model (which aggregates results across 40 plausible datasets). At the baseline level, housing price (zlog_hp) has a marginally large positive effect on P, but this effect is negatively moderated by YearG.

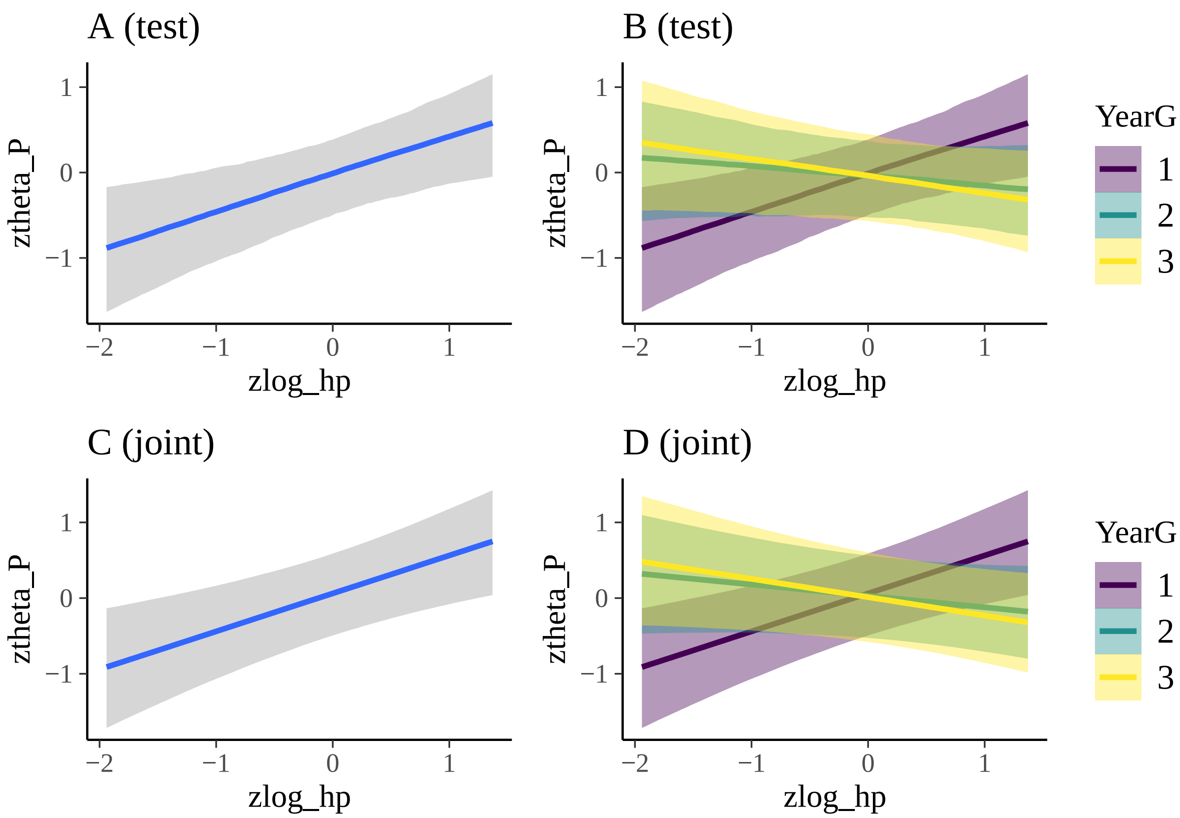
For thetaB_G × Attain_Lvl, the results show notable differences between the test and joint models. In the joint model, the originally small fixed effect of thetaB_G disappears at the baseline level, showing no meaningful effect. In contrast, in the test model, this effect shifts toward an uncertain negative direction, as indicated by a credible interval that crosses zero. Importantly, in the test model, Attain_Lvl moderates the effect of thetaB_G on P, meaning that at the same level of teacher belief (thetaB_G), teachers who perceive their students as having higher average attainment (Attain_Lvl) tend to use connectionist pedagogy more frequently. However, this interaction effect is weakened in the joint model, with the coefficient for moAttain_Lvl:zthetaB_G (0.12, Est. Error = 0.15, 95% CI: -0.19 to 0.44) becoming statistically uncertain, as its credible interval includes zero. This suggests that when considering the full distribution of plausible datasets, the moderating effect of Attain_Lvl becomes less pronounced, possibly due to increased variability in the estimated relationships.

The random intercepts across schools show small variation on average (.17[.01, .44]), indicating that while teaching practices do differ between schools, these differences are not substantial.

final MLM thetaB presentation



final MLM hp*yearG presentation



final school random effect presentation