EI-10

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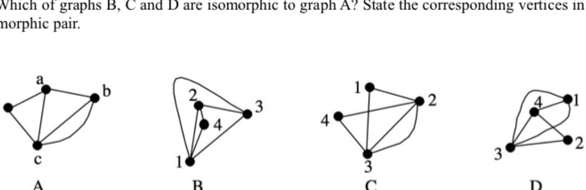
Exercises

Graph Theory, Basics:

1) Draw the graphs whose vertices and edges are as follows. In each case say if the graph is a simple graph.

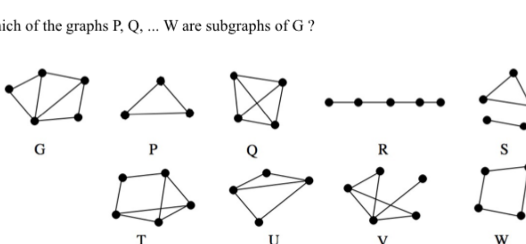
a) V = {u, v, w, x}, E = {uv, vw, wx, vx}
b) V = {1, 2, 3, 4, 5, 6, 7, 8}, E = {12, 22, 23, 34, 35, 67, 68, 78}
c) V = {n, p, q, r, s, t}, E = {np, nq, nt, rs, rt, st, pq}

2) Which of graphs B, C and D are isomorphic to graph A? State the corresponding vertices in each isomorphic pair.



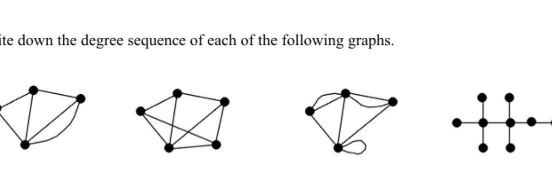
A B C D

3) Which of the graphs P, Q, ... W are subgraphs of G ?



G P Q R S T U V W

4) Write down the degree sequence of each of the following graphs.



A B C D

5) Graphs G_1 and G_2 have the same degree sequence - are they necessarily isomorphic? If your answer is no, give a counter example.

6) Graphs G_1 and G_2 are isomorphic. Do they necessarily have the same degree sequence? If your answer is no, give a counter example.

7) Draw simple connected graphs with the degree sequences:

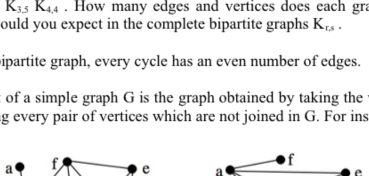
a) (1, 1, 2, 3, 3, 4, 4, 6)
b) (3, 3, 3, 3, 3, 5, 5, 5)
c) (1, 2, 3, 3, 3, 4, 4)

8) Draw the graphs K_5, N_5 and C_5 (K_i is the fully connected graph with i nodes, N_i is the graph with i nodes and no edges, and C_i is the cycle with i nodes).

9) A graph is complete bipartite, if bipartite and every node has full adjacency. Draw the complete bipartite graphs $K_{2,3}$ $K_{3,5}$ $K_{4,4}$. How many edges and vertices does each graph have? How many edges and vertices would you expect in the complete bipartite graphs $K_{r,s}$.

10) Show that, in a bipartite graph, every cycle has an even number of edges.

11) The complement of a simple graph G is the graph obtained by taking the vertices of G (without the edges) and joining every pair of vertices which are not joined in G. For instance:



G complement of G

a) Verify that the complement of the path graph P_4 is P_4.
b) What are the complements of K_4 , $K_{3,3}$, C_5 ?
c) What is the relationship between the degree sequence of a graph and that of its complement?
d) Show that if a simple graph G is isomorphic to its complement then the number of vertices of G has the form 4k or 4k + 1 for some integer k.
e) Find all the simple graphs with 4 or 5 vertices which are isomorphic to their complements.
f) Construct a graph with eight vertices which is isomorphic to its complement.

Dynamic Programming:

1) Knapsack problem: We have n items we want to put in our knapsack. Each item i has a size s_i and a value v_i . The space in the knapsack is limited, so the sum of the sizes of the items in the knapsack can not exceed S. Our objective is to put those items in the knapsack which have the maximal combined value.

a)

value	10	40	30	50
size	5	4	6	3
limit	10			

b)

value	12	4	3	6	1	3
size	7	5	2	2	1	3
limit	12					

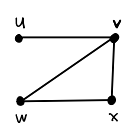
Hint: create a 2D table T, where T(i,j) stores the maximal value for the size limit j, when we only consider the first i items. T can be computed recursively: T(i,j) = max{ T(i-1,j), v_i + T(i-1,j-s_i)}

2) Longest Common Subsequence (LCS): A subsequence is a sequence that can be derived from another sequence by deleting some elements without changing the order of the remaining elements. For example "ABH" is a subsequence of "ANRBHC". Find the longest common subsequence LCS(x,y) in the following two (character) sequences x and y:

"ABCBDAB" and "BDCABA"

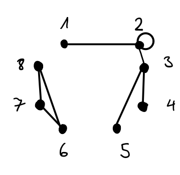
Hint: Create table c(i,j) which contains the length of the LCS of the prefixes of the input sequences.
c(i,j) = | LCS(x[1..i], y[1..j]) |
c(m,n) = | LCS(x,y) |, where |x| = m and |y| = n
We can compute c(i,j) recursively:
c(i,j) = c(i-1,j-1) + 1 if x[i] = y[j],
max{c(i-1,j), c(i,j-1)} otherwise

1.) a) V = {u,v,w,x}, E = {uv, vw, wx, vx}



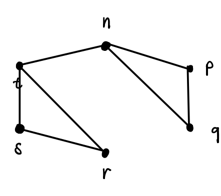
simple graph

b) V = {1,2,3,4,5,6,7,8}, E = {12, 22, 23, 34, 35, 67, 68, 78}



Pseudograph

c) V = {n,p,q,r,s,t}, E = {np, nq, nt, rs, rt, st, pq}



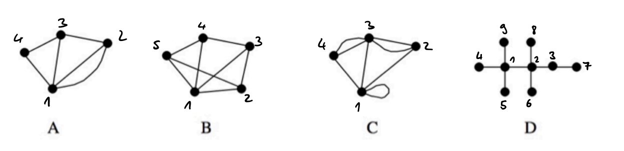
simple graph

2.) E_A = { ab, ac, ad, bc, bc, cd }
E_B = { 12, 13, 13, 14, 23, 24 }
E_C = { 12, 13, 23, 23, 24, 34 }
E_D = { 13, 13, 14, 23, 24, 34 }

D is Isomophic to A

3.) SUBGRAPHS OF G ARE: R, S, T, U

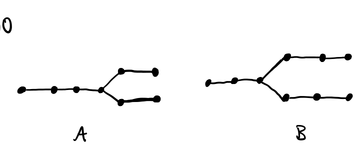
4.) 4) Write down the degree sequence of each of the following graphs.



A B C D

A = (4,3,3,2) B = (4,3,3,3,3) C = (5,5,3,3) D = (4,4,2,1,1,1,1,1,1)

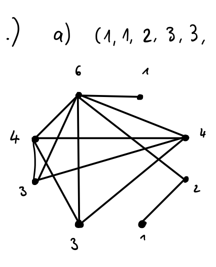
5.) NO



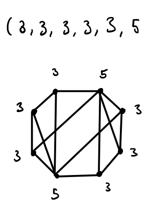
A B

6.) Yes

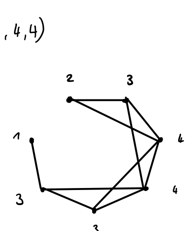
7.) a) (1, 1, 2, 3, 3, 4, 4, 6)



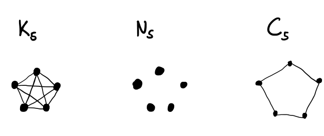
b) (3, 3, 3, 3, 3, 5, 5, 5)



c) (1, 2, 3, 3, 3, 4, 4)

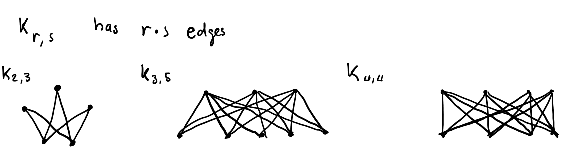


8.) K_5 N_5 C_5



9.) $K_{r,s}$ has r·s edges

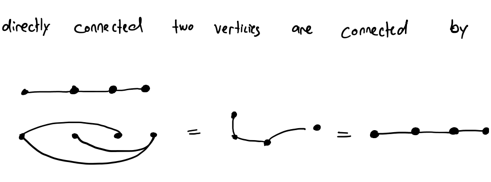
$K_{2,3}$ $K_{3,5}$ $K_{4,4}$



10.) Every not directly connected two vertices are connected by two edges.

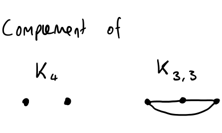
11.) a) P_4 =

component

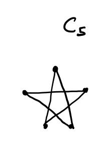


b.) Complement of

K_4 $K_{3,3}$



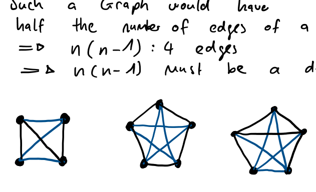
C_5



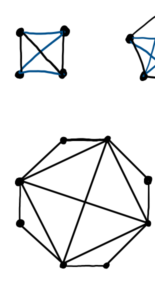
c) 3,3,3,3 → 0,0,0,0
3,3,3 → 2,2,2
2,2,2,2,2 → 2,2,2,2,2
$\Rightarrow ((i-1)-n_n, \ldots, (i-1)-n_1)$
i = number of nodes
n_n = n-th element

d) Such a Graph would have half the number of edges of a compete Graph
$\Rightarrow$ $n(n-1):4$ edges
$\Rightarrow$ $n(n-1)$ must be a devidant of 4 □

e)



f.)



Dynamic Programming:

1.) a)

	0	1	2	3	4
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	50
4	0	0	40	40	50
5	0	10	40	40	50
6	0	10	40	40	50
7	0	10	40	40	90
8	0	10	40	40	90
9	0	10	50	50	90
10	0	10	50	70	90

take items 4 & 2

b)

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	0	0	1	1
2	0	0	0	3	6	6	3
3	0	0	0	3	6	7	7
4	0	0	0	3	9	9	9
5	0	0	0	3	9	10	10
6	0	0	4	4	9	10	10
7	0	12	12	12	12	12	12
8	0	12	12	12	12	13	13
9	0	12	12	15	18	19	18
10	0	12	12	15	18	19	19
11	0	12	12	15	21	21	21
12	0	12	16	16	21	32	32

take items 1, 2 & 4

2.) LCS (x,y) of ABCBDAB and BDCABA

	A	B	C	B	D	A	B
B	0	1	1	1	1	1	1
D	0	1	1	1	2	2	2
C	0	1	2	2	2	2	2
A	1	1	2	2	2	3	3
B	1	2	2	3	3	3	4
A	1	2	2	3	3	4	4

LCS: ABCBDAB & BDCABA
⇓
BCAB & BCDA