# Microscopic Foundations of the Curvature Feedback Model

## From Quantum Geometry to Macroscopic Saturation

### The Lagrangian Derivation and Quantum Gravity Connection

Lukas Geiger*[1]

[1]Independent Researcher, Bernau im Schwarzwald

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*Working Paper – Paper III in the CFM series [1, 2]*

**Zusammenfassung**

Papers I and II of this series established the Curvature Feedback Model (CFM) as a phenomenologically successful alternative to ΛCDM, eliminating the entire dark sector through a geometric curvature return mechanism. The present paper addresses the outstanding theoretical challenge: *What is the microscopic origin of the saturation ODE?* We seek the quantum system whose macroscopic (thermodynamic) limit yields the curvature return equation $d\Omega_\Phi/da = k\,[1-(\Omega_\Phi/\Phi_0)^2]$. We explore four candidate frameworks: (1) a scalar field with a double-well potential yielding tanh-type saturation via spontaneous symmetry breaking; (2) Loop Quantum Gravity, where holonomy corrections produce bounded curvature invariants; (3) Finsler geometry, where direction-dependent metrics naturally generate scale-dependent gravitational effects; and (4) information-theoretic spacetime, where the saturation ODE emerges from a maximum-entropy principle on causal sets. We derive the effective Lagrangian $\mathscr{L}_{\rm CFM}$ that reproduces the extended Friedmann equation and discuss the implications for quantum gravity.

**Keywords:** Curvature Feedback Model, quantum gravity, Lagrangian formulation, Loop Quantum Gravity, Finsler geometry, saturation mechanism, modified gravity

**Subject areas:** Theoretical Physics, Quantum Gravity, Mathematical Physics

*Correspondence: Lukas Geiger, Geißbühlweg 1, 79872 Bernau, Germany.

## Inhaltsverzeichnis

## AI Disclosure

This paper was developed with intensive use of AI systems. Their contributions are disclosed in detail:

**Claude Opus 4.6 (Anthropic)**
Co-writer: Text generation, mathematical derivations, code development.

**Gemini (Google DeepMind)**
Reviewer: Critical feedback, quantum gravity connections, strategic recommendations.

*Note:* Despite the substantial machine contribution, final responsibility for the scientific content and interpretation rests with the human author.

# 1 Introduction: The Central Question

The Curvature Feedback Model (CFM) [1] and its MOND-compatible extension [2] have demonstrated remarkable phenomenological success:

- **Paper I:** The standard CFM replaces dark energy with a curvature return potential, achieving $\Delta\chi^2 = -12.2$ vs. $\Lambda$CDM on Pantheon+ data.
- **Paper II:** The extended CFM eliminates the entire dark sector (both dark energy and dark matter) in a baryon-only universe, achieving $\Delta\chi^2 = -26.3$ with a geometric "dark matter" term that scales as spatial curvature ($\beta = 2.02 \pm 0.20$).

Both results derive from a single dynamical equation – the *saturation ODE*:

$$\frac{d\Omega_\Phi}{da} = k\left[1 - \left(\frac{\Omega_\Phi}{\Phi_0}\right)^2\right] \tag{1}$$

whose solution is the tanh function that provides the late-time acceleration. The extended model adds a power-law term $\alpha \cdot a^{-\beta}$ representing the unsaturated (early-time) phase of the same geometric process.

The central question of this paper is:

> *Which microscopic (quantum) system has the property that its macroscopic (thermodynamic) limit yields the saturation ODE* (1)*? And can the full extended Friedmann equation be derived from a Lagrangian?*

This question is not merely academic. Without a Lagrangian formulation, the CFM cannot:

1. Be consistently coupled to matter fields
2. Generate perturbation equations for $C_\ell$ and $P(k)$ predictions
3. Be connected to known quantum gravity frameworks
4. Be considered a complete physical theory

# 2 The Effective Lagrangian

## 2.1 Requirements

The effective Lagrangian $\mathscr{L}_{\text{CFM}}$ must satisfy:

1. **Background:** The Euler-Lagrange equations, evaluated on the FLRW metric, must yield the extended Friedmann equation:

$$H^2(a) = H_0^2\left[\Omega_b a^{-3} + \Phi_0 \cdot f_{\text{sat}}(a) + \alpha \cdot a^{-\beta}\right] \tag{2}$$

2. **Saturation dynamics:** The scalar field equation of motion must reduce to $d\Omega_\Phi/da = k[1-(\Omega_\Phi/\Phi_0)^2]$ on the FLRW background.

3. **General covariance:** The action must be diffeomorphism-invariant.

4. **Correct limits:** In the limit $k \to 0$, $\alpha \to 0$, the theory must reduce to GR with cosmological constant.

## 2.2 Scalar Field Approach

The most natural Lagrangian formulation introduces a scalar field $\phi$ with a potential $V(\phi)$:

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi) + \mathscr{L}_m\right] \tag{3}$$

For the saturation ODE to emerge, we require $V(\phi)$ such that the homogeneous field equation on FLRW yields tanh-type solutions.

**Proposition 1** (Double-Well Saturation Potential)**.** *The potential*

$$V(\phi) = V_0\left[1 - \tanh^2\left(\frac{\phi}{\phi_0}\right)\right] = \frac{V_0}{\cosh^2(\phi/\phi_0)} \tag{4}$$

*produces a scalar field equation whose late-time solution on the FLRW background is* $\phi(a) \propto \tanh(k(a - a_{\mathrm{trans}}))$*, reproducing the saturation term of the CFM.*

*Sketch of proof:* The Klein-Gordon equation on FLRW,

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \tag{5}$$

with $V'(\phi) = -2V_0\tanh(\phi/\phi_0)/(\phi_0\cosh^2(\phi/\phi_0))$, admits the solution $\phi = \phi_0\tanh(\lambda t)$ in the slow-roll regime where $\ddot{\phi} \ll 3H\dot{\phi}$, with $\lambda$ related to $k$ and $H_0$. The energy density $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ then maps to $\Omega_\Phi(a) = \Phi_0 \cdot f_{\mathrm{sat}}(a)$. □

*Note:* The $\cosh^{-2}$ potential is well known in quantum mechanics as the Pöschl-Teller potential. Its appearance here suggests a deep connection between quantum bound states and cosmological saturation.

## 2.3 The Power-Law Term: Geometric Origin

The geometric "dark matter" term $\alpha \cdot a^{-\beta}$ with $\beta \approx 2$ requires a separate origin. Two approaches are possible:

**Approach 1: Curvature-squared terms.** Adding a Gauss-Bonnet or $R^2$ term to the action:

$$S_{\mathrm{geom}} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + \gamma R^2 + \delta\, R_{\mu\nu}R^{\mu\nu}\right] \tag{6}$$

produces corrections to the Friedmann equation that scale as $a^{-2}$ in the radiation-to-matter transition era. The coefficient $\gamma$ can be related to $\alpha$.

**Approach 2: Vector field (AeST-inspired).** Following Skordis & Złośnik [5], a timelike vector field $A_\mu$ constrained by $g^{\mu\nu}A_\mu A_\nu = -1$ contributes an effective energy density that scales non-standardly with $a$. The CFM power-law term may emerge as the cosmological background of such a vector field.

### 2.4 The Combined Action

Combining both contributions, the full CFM action reads:

$$\boxed{S_{\mathrm{CFM}} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + \gamma R^2 - \frac{1}{2}(\partial\phi)^2 - \frac{V_0}{\cosh^2(\phi/\phi_0)} + \mathscr{L}_m\right]} \tag{7}$$

where the $R^2$ term generates the power-law ("dark matter") contribution and the scalar field generates the saturation ("dark energy") contribution. The game-theoretic equilibrium between null space and spacetime bubble is encoded in the balance between $\gamma$ and $V_0$.

*Status:* This is a candidate action. Its consistency (ghost freedom, stability, correct Newtonian limit) must be verified. The full perturbation equations derived from (7) will determine whether the model can reproduce CMB and LSS observations.

## 3 Quantum Gravity Connections

### 3.1 Why the Saturation ODE?

The central puzzle is the specific form of the saturation ODE (1): $dX/da = k(1-X^2)$. This equation has two fixed points ($X = \pm 1$), of which $X = +1$ is stable. The tanh solution is the unique trajectory connecting $X = 0$ (zero curvature return) to $X = 1$ (full saturation). We survey four frameworks that naturally produce such dynamics.

### 3.2 Approach 1: Loop Quantum Gravity

In Loop Quantum Gravity (LQG) [6, 7], spacetime is quantized into discrete spin network states. The key feature for our purposes is the *bounded curvature* property: holonomy corrections replace curvature invariants $R$ with bounded functions $\sin(\mu R)/\mu$ (where $\mu$ is related to the Planck area).

In Loop Quantum Cosmology (LQC) [8], the Friedmann equation becomes:

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right) \tag{8}$$

where $\rho_c \sim \rho_{\mathrm{Pl}}$ is a critical density. This has the structure of a saturation equation: the expansion rate is bounded as $\rho \to \rho_c$.

**Conjecture 1** (LQG–CFM Connection)**.** *The saturation ODE* (1) *is the late-time, low-energy residual of the LQC curvature bound. In the early universe, the bound prevents singularities; in the late universe, the same mechanism produces the curvature return saturation. The parameters $k$ and $\Phi_0$ are related to the LQG area gap $\Delta$ and the Barbero-Immirzi parameter $\gamma_{\mathrm{BI}}$.*

*Evidence:* Both equations share the structure $dX/dt \propto (1-X^2)$. In LQC, $X$ is the curvature; in CFM, $X$ is the curvature return potential. The mapping requires identifying $\Omega_\Phi/\Phi_0$ with a normalized curvature invariant.

### 3.3 Approach 2: Finsler Geometry

Finsler geometry generalizes Riemannian geometry by allowing the metric to depend on both position and direction: $F(x,\dot{x})$ instead of $g_{\mu\nu}(x)\,dx^\mu\,dx^\nu$ [9]. This direction dependence can produce:

- Scale-dependent gravitational effects (mimicking MOND at galactic scales)
- Non-standard cosmological scaling (the $a^{-\beta}$ term)
- A natural saturation mechanism when the directional dependence reaches a geometric bound

**Conjecture 2** (Finsler–CFM Connection)**.** *The extended CFM Friedmann equation corresponds to a Finsler spacetime with a specific choice of Finsler function $F$. The "dark matter" term $\alpha \cdot a^{-2}$ arises from the osculating Riemannian curvature of the Finsler metric, and the saturation term arises from the Finsler analog of the Ricci scalar reaching a geometric bound.*

*Note:* Finsler geometry has been applied to MOND [10] and to modified dispersion relations in quantum gravity [11]. The CFM may provide the cosmological realization of a Finsler spacetime.

### 3.4 Approach 3: Information-Theoretic Spacetime

If spacetime is fundamentally information-theoretic (as suggested by the holographic principle [12] and the ER=EPR conjecture [13]), then the saturation ODE can be reinterpreted as a *maximum entropy principle*:

- The curvature return potential $\Omega_\Phi$ represents the "processed information" of the spacetime system.
- The saturation limit $\Phi_0$ represents the maximum information capacity (holographic bound).
- The ODE $dX/da = k(1-X^2)$ is the logistic-type growth equation for information processing, where the rate of information gain decreases as the system approaches its capacity.

In this picture, the game-theoretic interpretation of Paper I [1] becomes literal: the null space and spacetime bubble are two subsystems of a quantum information network, and their Nash equilibrium is determined by the information-theoretic constraints of the holographic bound.

### 3.5 Approach 4: Causal Set Theory

Causal set theory [14, 15] models spacetime as a discrete partial order of events. The key result for our purposes is the *Sorkin cosmological constant* [16]: in a causal set universe with $N$ elements, the cosmological constant has fluctuations of order $\Lambda \sim 1/\sqrt{N}$, providing a natural explanation for the observed smallness of $\Lambda$.

**Conjecture 3** (Causal Set–CFM Connection)**.** *In a dynamically evolving causal set, the curvature return potential $\Omega_\Phi$ corresponds to the "effective cosmological constant" that changes as new elements are added to the set. The saturation at $\Phi_0$ corresponds to the causal set reaching its equilibrium density. The power-law term $\alpha \cdot a^{-2}$ reflects the initial transient before the set reaches equilibrium.*

# 4 The Geometric Phase Transition

## 4.1 From Dark Matter Phase to Dark Energy Phase

Paper II [2] introduced the concept of a geometric phase transition: at early times, the curvature return potential behaves like dark matter ($\alpha \cdot a^{-2}$), and at late times, it saturates into dark energy ($\Phi_0 \cdot f_{\text{sat}}$). This section provides the theoretical underpinning.

## 4.2 Order Parameter and Symmetry Breaking

The saturation variable $X = \Omega_\Phi/\Phi_0 \in [0,1]$ can be interpreted as an *order parameter*:

- $X = 0$: Disordered phase (no curvature return, geometric "DM" dominates)
- $X = 1$: Ordered phase (full saturation, geometric "DE" dominates)
- The transition at $a_{\text{trans}}$: The crossover between phases

The saturation ODE $dX/da = k(1 - X^2)$ has the form of a Ginzburg-Landau equation for a second-order phase transition with a double-well free energy $F(X) = -k(X - X^3/3)$. The "temperature" parameter is the scale factor $a$, and the transition occurs as $a$ increases past $a_{\text{trans}}$.

## 4.3 Analogy to Spontaneous Magnetization

The mathematical structure is identical to the mean-field theory of ferromagnetism:

| **Ferromagnetism** | **CFM Cosmology** | **Variable** |
|---|---|---|
| Magnetization $M$ | Curvature return $\Omega_\Phi$ | Order parameter |
| Temperature $T$ | Scale factor $a$ | Control parameter |
| Curie point $T_c$ | Transition $a_{\text{trans}}$ | Critical point |
| Spin interaction $J$ | Curvature coupling $k$ | Interaction strength |
| Saturation $M_s$ | Saturation $\Phi_0$ | Maximum value |
| $\tanh(J/k_BT)$ | $\tanh(k(a - a_{\text{trans}}))$ | Solution |

This analogy suggests that the curvature return is driven by *cooperative phenomena*: individual spacetime degrees of freedom (area quanta in LQG, causal set elements, etc.) align collectively, producing a macroscopic saturation effect. The game-theoretic "equilibrium" of Paper I is the cosmological analog of thermal equilibrium in statistical mechanics.

## 4.4 Critical Exponents and Universality

If the analogy to phase transitions is more than formal, the CFM should exhibit *universality*: the saturation exponent and the transition shape should be robust against microscopic details. This would explain why the phenomenological tanh function fits the data well – it is the universal scaling function for a mean-field phase transition, regardless of the microscopic mechanism.

**Conjecture 4** (Universality of the Saturation Mechanism)**.** *The* tanh *form of the curvature return potential is a universal consequence of any microscopic theory with:*

*1. A bounded curvature return (saturation limit $\Phi_0$)*

*2. A cooperative interaction between spacetime degrees of freedom (coupling $k$)*

*3. A single relevant direction (the scale factor $a$)*

*The specific microscopic mechanism (LQG, Finsler, causal sets) affects only the values of $k$ and $\Phi_0$, not the functional form.*

# 5 Testable Predictions from the Lagrangian

The effective action (7) generates specific predictions beyond the background expansion history:

## 5.1 Perturbation Equations

Linearizing the action around the FLRW background yields coupled equations for:

- The metric perturbations $\Phi_N$ (Newtonian potential) and $\Psi$ (curvature perturbation)
- The scalar field perturbation $\delta\phi$
- The matter perturbations $\delta_m$ and $v_m$

The $R^2$ term produces an *anisotropic stress* ($\Phi_N \neq \Psi$), which is a testable prediction distinguishing the CFM from $\Lambda$CDM and from simple quintessence models.

## 5.2 Gravitational Slip Parameter

The ratio $\eta = \Phi_N/\Psi$ is predicted to deviate from unity:

$$\eta(a,k) = 1 + \delta\eta(a,k) \tag{9}$$

where $\delta\eta$ depends on the $R^2$ coupling $\gamma$ and is scale-dependent. This can be tested by comparing weak lensing (sensitive to $\Phi_N + \Psi$) with galaxy clustering (sensitive to $\Psi$ alone).

## 5.3 Scalar Field Oscillations

The Pöschl-Teller potential (4) supports a discrete spectrum of bound states. In the cosmological context, these correspond to oscillatory corrections to the expansion rate at late times:

$$H^2(a) = H^2_{\text{smooth}}(a)\left[1 + \varepsilon \cdot e^{-\Gamma a}\cos(\omega a + \delta)\right] \tag{10}$$

with amplitude $\varepsilon \ll 1$. These oscillations, if detectable in high-precision BAO or SN data, would provide direct evidence for the quantum nature of the saturation mechanism.

### 5.4 Modified Gravitational Waves

The $R^2$ term modifies the gravitational wave propagation equation:

$$\ddot{h}_{ij} + (3H + \Gamma_{\rm GW})\dot{h}_{ij} + \left(\frac{k^2}{a^2} + m_{\rm GW}^2\right) h_{ij} = 0 \tag{11}$$

where $\Gamma_{\rm GW}$ and $m_{\rm GW}^2$ are corrections from the curvature-squared term. This predicts:

- A frequency-dependent gravitational wave speed ($c_{\rm GW} \neq c$ at high frequencies)
- A massive graviton mode with $m_{\rm GW} \propto \sqrt{\gamma}$

The LIGO/Virgo/KAGRA constraint $|c_{\rm GW}/c - 1| < 10^{-15}$ [19] places an upper bound on $\gamma$.

## 6 Connection to Known Frameworks

### 6.1 Relation to $f(R)$ Gravity

The action (7) with the $R^2$ term is a special case of $f(R) = R + \gamma R^2$ gravity (Starobinsky model) [17]. The CFM adds the scalar field with the Pöschl-Teller potential, breaking the degeneracy between $f(R)$ models.

### 6.2 Relation to AeST

The relativistic MOND theory AeST [5] contains a scalar field $\phi$ and a constrained vector field $A_\mu$. The CFM scalar field may be identified with (or related to) the AeST scalar field, while the $R^2$ term may encode the cosmological effect of the AeST vector field. A precise mapping between the two theories is a key objective.

### 6.3 Relation to Emergent Gravity

Verlinde's emergent gravity proposal [18] derives MOND-like effects from the entanglement entropy of de Sitter space. The CFM's game-theoretic framework shares the core idea that gravity (and its "dark" extensions) are emergent phenomena, not fundamental forces. The saturation mechanism may be the cosmological realization of Verlinde's entropy-area relation.

## 7 Discussion and Outlook

### 7.1 Summary of the Three-Paper Program

The CFM program now spans three papers:

1. **Paper I** [1]: Game-theoretic foundation, standard CFM, dark energy replacement. Validated against Pantheon+.
2. **Paper II** [2]: MOND unification, extended CFM, baryon-only universe, Decaying Dark Geometry hypothesis. Validated against Pantheon+.

3. **Paper III** (this work): Lagrangian formulation, quantum gravity connections, phase transition interpretation, testable predictions.

Together, these papers propose a *complete cosmological framework* in which:

- The dark sector is eliminated (Paper II)
- The expansion history is explained by geometric curvature return (Papers I, II)
- The microscopic origin is a tanh-type phase transition of spacetime geometry (Paper III)
- The Lagrangian is $R + \gamma R^2$ plus a Pöschl-Teller scalar field (Paper III)

## 7.2 What Remains

Despite the theoretical progress, critical steps remain:

1. **CMB power spectrum:** Computing $C_\ell$ from the perturbation equations of the full action (7). This is the single most important test.
2. **Ghost analysis:** Verifying that the action (7) is free of ghost instabilities (negative kinetic energy modes). The $R^2$ term introduces the scalaron, which must be checked for stability.
3. **Solar system tests:** The $R^2$ modification produces a Yukawa correction to Newton's law. The coupling $\gamma$ must be small enough to satisfy solar system constraints.
4. **Numerical verification:** Solving the full perturbation equations numerically (using a modified CLASS/CAMB code) to predict $C_\ell$, $P(k)$, and $f\sigma_8$.
5. **Quantum gravity:** Deriving $k$, $\Phi_0$, $\alpha$, and $\beta$ from one of the microscopic frameworks (LQG, Finsler, causal sets) – or from a new framework suggested by the tanh structure.

## 7.3 The Vision: Cosmology as Phase Transition

If the program succeeds, the history of the universe becomes a *geometric phase transition*:

1. **Big Bang:** Emergence of the spacetime bubble from the null space (game-theoretic nucleation).
2. **Early universe:** Unsaturated curvature return dominates – geometry behaves like "dark matter" ($a^{-2}$), providing gravitational scaffolding for structure formation.
3. **Transition:** The curvature return approaches saturation ($a \approx a_{\text{trans}}$) – the geometric phase transition from DM-like to DE-like behavior.
4. **Late universe:** Saturated curvature return dominates – geometry behaves like "dark energy" (accelerated expansion).
5. **Far future:** Full saturation $\Omega_\Phi \to \Phi_0$ – the Nash equilibrium is reached, the null space gradient is neutralized, and expansion approaches de Sitter.

The entire history of cosmic acceleration and structure formation is then described by a single equation – the saturation ODE – whose form is universal (a consequence of mean-field phase transition theory) and whose parameters are determined by quantum gravity.

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