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B3 - Time Series Analysis - Fall 2017
Home Work Assignment 1

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3 Theoretical part

The first task is to derive some statistical properties of a set of models, the models are

1. MA(1)

$$Y_t = e_t - \theta e_{t-1}. \tag{1}$$

2. MA(2)

$$Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}. \tag{2}$$

3. AR(1)

$$Y_t = \phi Y_{t-1} + e_t \tag{3}$$

4. AR(2)

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t \tag{4}$$

5. ARMA(1,1)

$$Y_t = \phi Y_{t-1} + e_t - \theta e_{t-1} \tag{5}$$

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Note that, for ease of notation and with no loss of generality, we do not have any constants in these models. For all the processes above, we have

$$e_t \sim NID\left(0,1\right). \tag{6}$$

For each of the models, derive and report the following theoretical properties:

1. The Mean function $\mu_t = E\left(Y_t\right)$
2. The Variance function $\gamma_0 = V\left(Y_t\right)$
3. The First autocovariance $\gamma_1 = Cov\left(Y_t, Y_{t-1}\right)$
4. The Second autocovariance $\gamma_2 = Cov\left(Y_t, Y_{t-2}\right)$
5. The First Autocorrelation $\rho_1 = \frac{\gamma_1}{\gamma_0}$
6. The Second Autocorrelation $\rho_2 = \frac{\gamma_2}{\gamma_0}$
7. A general expression for the the Autocorrelation function as a function of the parameters of the process ρ_k, $k \geq 1$

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Thus, you need to, sort of, check all the entries in the table

	Statistical property						
Model	μ_t	γ_0	γ_1	γ_2	ρ_1	ρ_2	ρ_k
MA(1)							
MA(2)							
AR(1)							
AR(2)							
ARMA(1,1)							

(Note already: All the *derivations* **must be in an appendix.**)

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1. MA_Simulate_Plot.R
2. AR_Simulate_Plot.R
3. ARMA_Simulate_Plot.R

4.1 How to simulate a realization

To simulate a realization of a process, run the corresponding program. For example, for the MA(1) model.

1. MA_Simulate_Plot.R
2. Make sure the *parameters* are set to the correct value(s).
3. Set the random number generator seed to the birthdate of one of the students in the group e.g. set.seed(960231).
4. Simulate $n = 5\ 000$ observations (make sure NumObsSim is set to the correct value)
5. Plot the first, say 500 observations (set numObsToPlot=500)
6. See below on how to report these two graphs.

4.2 Parameter values in the simulations

Below are the parameter values or combinations of parameter values you should use when simulating the realizations of the different stochastic processes. These parameter values you need to change in the program code for each set of parameters.

For the MA(1) process, simulate data using the following parameter values:

		θ			
-1.0	-0.50	0.0	0.50	1.0	2

. (7)

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For the MA(2) process, simulate data using the following parameter combinations:

		$\theta_1 =$		
		-0.4	0.0	0.4
θ_2	0.0			
	0.7			
	1.0			

.

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of course, this means that in each cell, we have the combination (θ_1, θ_2) so that

		$\theta_1 =$		
		-0.4	0.0	0.4
θ_2	0.0	$\theta_1 = -0.4, \theta_2 = 0.0$	$\theta_1 = 0.0, \theta_2 = 0.0$	etc
	0.7	etc		
	1.0			

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