SENIOR RESEARCH

# Topic: Regime-Switching Optimization in The Stock Exchange of Thailand

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## Abstract

The purpose of this paper is to check robustness of Regime-Switching Model by Ang and Bekaert, 2003. Stock market conditions can be divided into at least 2 regimes, a bull market regime and a bear market regime. Equity's return normally has more volatility and highly correlation in a bear market. The Regime-Switching Model with 2 regimes was developed to match the patterns. However, I believe that 3 regimes condition, including bull, normal and bear markets, is possible. Thus, I extend a Regime-Switching model with 3 regimes. Then, I test the models in the case of The Stock Exchange of Thailand. There are 3 key parameters for checking robustness of the model namely, number of regimes, frequency of portfolio rebalancing and degree of risk aversion. I compare the best regime-switching strategy to a standard mean-variance strategy and an equal weight strategy. I found that a 2-regime model with 3-month rebalancing period is the most appropriate model among regime-switching models for The Stock Exchange of Thailand. Although the 2-regime model has lower return than a 3-regime model, the 2-regime model has better AIC, Schwartz criterion and practical level (stable weight of each asset) which gives lower transaction cost. Among 2-regime models, a model which rebalances every 3 months leads to the highest wealth. The 2-regime model most benefit to normal risk aversion investors. Finally, the result shows that the 2-regime model loses to the standard mean-variance but wins the equal weight strategy in this case. Result in this paper contrasts with that of Ang and Bekaert,2003. However, we know that asset allocation is still necessary since both a regime-switch model and a standard mean-variance strategy beat an equal weight strategy.

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# 1. Introduction

In 1952, Harry Markowitz proposed a mean-variance optimization in his paper, "portfolio selection" published by the Journal of Finance. It is the beginning of a modern portfolio theory. Portfolio managements have mainly based on asset allocations since that time. Overtime, the modern portfolio theory has been developed. Many strategies were proposed such as a risk parity model and a regime-switching model. Although technology has been highly developed, portfolio managements, such as Robo advisor, still use the mean-variance optimization as an optimization background (Lam, 2016).

It has been known that equities' returns are more correlated with each other in a bear regime than in a bull regime (Rachel Campbell, 2002). Because of asymmetric correlations, covariance matrices that is an input for portfolio optimization processes should be different among regimes. This result supports that a regime-switching strategy is important.

The Regime-Switching model by Ang and Bekaert in 2003 adapt the Markov chain process to assign a regime to each period by computing regime and transition probabilities for each period. Market's returns data are used to define market regimes. It captures differences between 2 market regimes very well. A bull regime has more average return and less volatility than a bear market. Moreover, its performance is better than that of the static mean-variance strategy in the universe of developed equity market around the world. There is another evidence (Kun Yu, 2017) shows that the Regime-Switching model beats the Non-Regime strategy in the world market in period of 1994-2017.

In addition to the Regime-Switching model by Ang and Bekaert which has 2 regimes, a 6-regimes with 4-lags model is the best one to describe stock market returns (Chia-Shang James Chu, 1996). The result shows that 6 regimes have significantly different returns and volatilities. However, the result is consistent with the Regime-Switching Model, the higher return in a regime, the lower volatility in that regime.

In Ang and Bekaert's work, they rebalance the portfolio every month because the period is consistent with their data frequency. They didn't mention how long the best rebalancing period is. This is one of the gap that will be filled by this research

Moreover, the risk aversion index affects returns and risks but has no effect on the Sharpe ratio. It means that the risk aversion index doesn't affect the risky portfolio (all-equity portfolio). However, the risk aversion index should affect the portfolio's performance if risk-free assets were added to the portfolio.

In this paper, I revisit the Regime-Switching Model by Ang and Bekaert in 2003 in the case of The Stock Exchange of Thailand and check robustness of the model. Key parameters of robustness are number of regimes, frequency of portfolio rebalancing and degree of risk aversion. To compare performance, I compare the regime-switching model to the standard mean-variance strategy and the equal weight strategy with the same rebalancing period and degree of risk aversion.

I found that a 2-regime model is the most suitable model in this case. Although a 3-regime model leads to higher wealth, it has worse statistic and practical level (weight of each index) which leads to higher transaction cost. Within the 2-regime model, a 3-month rebalancing period has the highest return. With short constraint, the 2-regime model can't beat standard mean-variance and equal weight strategies but it has the highest Sharpe ratio. The 2-regime model with 3-month rebalancing period most benefits to investors with normal risk aversion. Higher risk aversion from low to normal level leads to higher returns per a unit of risk while doesn't dilute benefit of the Regime-Switching model.

The rest of this paper is organized as follows. I explain data with sample statistic in section 2 and methodology in section 3. Main empirical results are shown in section 4. I provide further discussions in section 5 and conclude in section 6.

## 2. Data

I use the SET index to be the indicator for switching the regime. In addition, I select 5 sectors that are the biggest sectors of The Stock Exchange of Thailand including energy, banking, property, communication and commerce namely SETENERG, SETBANK, SETPROP, SETCOMMU, SETCOM. I collect the data from Bloomberg terminal. I use the data from 1995 to the end of 2017. Data starting from 1995 is because 1995 is the year that data of all 5 indexes are available. A risk-free rate used in this work is short term Thai government bond rate provided by Bank of Thailand (Bank of Thailand Website) and IMF (IMF international financial statistics book). The collected data are monthly. An initial window for optimization process is 10 years following Ang and Bekaert, 2003. Then, I roll the data with a fixed initial approach. Thus, an investing period is January 2005 to December 2017.

Table 1 shows that there are 2 sectors, energy and commerce, that are overperform the market, SET index. This could play an important role in an asset allocation process. It also shows that every sector is highly correlate with the SET index so the index is appropriate to be a regime defining index in the Markov chain process. In addition, the banking and property sectors have higher correlation than others.

Table 1: characteristic of sample (annualized)

| | SET | SETENERG | SETBANK | SETPROP | SETCOMMU | SETCOM |
|---|---|---|---|---|---|---|
| mean return | 5.76 | 11.95 | 5.18 | 5.49 | 7.02 | 13.28 |
| Stdev | 28.23 | 31.01 | 39.28 | 44.01 | 39.08 | 25.16 |
| beta | 1 | 0.7768 | 1.2995 | 1.3001 | 1.1681 | 0.6137 |
| Stdev | | 0.06 | 0.0481 | 0.0142 | 0.076 | 0.0223 |
| correlation | SET | SETENERG | SETBANK | SETPROP | SETCOMMU | SETCOM |
| SET | 1 | | | | | |
| SETENERG | 0.764269 | 1 | | | | |
| SETBANK | 0.898929 | 0.577 | 1 | | | |
| SETPROP | 0.828938 | 0.476429 | 0.80299 | 1 | | |
| SETCOMMU | 0.790627 | 0.589451 | 0.647972 | 0.568953 | 1 | |
| SETCOM | 0.719421 | 0.560931 | 0.604352 | 0.593263 | 0.591687 | 1 |

Mean returns are shown in percentage

## 3. Methodology

### 3.1 2-Regime-Switching Beta model

I follow the Regime-Switching Beta model of Ang and Bekaert,2003. At first, market excess returns are inputted into the Markov chain process to get transition probability matrices and probabilities of each regime for a period. Notation of transition probabilities are

$$P = p(s_t = 1 | s_{t-1} = 1)$$

$$Q = p(s_t = 2|s_{t-1} = 2)$$

While $s_t$ denotes the regime of the period. For example, $P$ is a transition probability from state 1 in period t-1 to state 1 in period t. Thus, a transition probability from state 1 in period t-1 to state 2 in period t is $1 - P$.

If the recent period's regime is state 1, the expected market excess return is

$$e_1^w = P\mu^w(s_{t+1} = 1) + (1 - P)\mu^w(s_{t+1} = 2)$$

If the recent period's regime is state 2, the expected market excess return is

$$e_2^w = (1 - Q)\mu^w(s_{t+1} = 1) + Q\mu^w(s_{t+1} = 2)$$

After this, the model uses the CAPM to compute a beta matrix that contain betas for each security. The betas explain co-moving between each security and the market. Let

$$\beta = \begin{pmatrix} \beta_1 \\ . \\ . \\ . \\ \beta_n \end{pmatrix}$$

In the case of n securities considered for a portfolio. Hence, the expected return vector for each regime is

$$e_i = (1 - \beta)\mu^z + \beta e_i^w, \qquad i = 1,2$$

Before computing conditional covariance matrix, I must introduce matrix $V$. $V$ is a matrix of zeros with $(\sigma^j)^2$, variance of each security, along the diagonal. There will be two possible variance matrices for unexpected return next period

$$\Omega_i = (\beta\beta')(\sigma^w(s_{t+1} = i))^2 + V, \qquad i = 1,2$$

So conditional variance matrices are computed by combining the possible variance matrices and the transition probabilities. The conditional variance matrices are

$$\Sigma_1 = P\Omega_1 + (1 - P)\Omega_2 + P(1 - P)(e_1 - e_2)(e_1 - e_2)'$$

$$\Sigma_2 = (1 - Q)\Omega_1 + Q\Omega_2 + Q(1 - Q)(e_1 - e_2)(e_1 - e_2)'$$

The standard optimal mean-variance portfolio is

$$w_i = \frac{1}{\gamma}\Sigma_i^{-1}e_i$$

Where, $\gamma$ is an investor's risk aversion. To compute securities' weights, there are 2 conditional covariance matrices. Use of conditional covariance matrix depends on the probability for each state. If the probability for state 1 is more than 0.5, the first conditional covariance matrix will be used. The condition is also applied for state 2.

## 3.2 3-Regime-Switching Beta model

I extend the formula of Ang and Bekaert, 2003 because they assume that the market can only have 2 regimes. But a 3-regime model including a bear regime, a normal regime and a bull regime, also sounds make sense. The calculation is a bit complex than previous model because a transition probability matrix's dimension is 3x3 instead of 2x2. The model is as follow.

Transition probability matrix is

$$\begin{vmatrix} P1 = p(s_t = 1|s_{t-1} = 1) & P2 = p(s_t = 2|s_{t-1} = 1) & P3 = p(s_t = 3|s_{t-1} = 1) \\ Q1 = p(s_t = 1|s_{t-1} = 2) & Q2 = p(s_t = 2|s_{t-1} = 2) & Q3 = p(s_t = 3|s_{t-1} = 2) \\ R1 = p(s_t = 1|s_{t-1} = 3) & R2 = p(s_t = 2|s_{t-1} = 3) & R3 = p(s_t = 3|s_{t-1} = 3) \end{vmatrix}$$

If the recent period's regime is state 1, the expected market excess return is

$$e_1^w = P1\mu^w(s_{t+1} = 1) + P2\mu^w(s_{t+1} = 2) + P3\mu^w(s_{t+1} = 3)$$

If the recent period's regime is state 2, the expected market excess return is

$$e_2^w = Q1\mu^w(s_{t+1} = 1) + Q2\mu^w(s_{t+1} = 2) + Q3\mu^w(s_{t+1} = 3)$$

If the recent period's regime is state 3, the expected market excess return is

$$e_3^w = R1\mu^w(s_{t+1} = 1) + R2\mu^w(s_{t+1} = 2) + R3\mu^w(s_{t+1} = 3)$$

Next, I compute $\beta$ using the CAPM and the expected return vector for each regime. It is similar to that of the 2-Regime-Switching Beta model but different in number of state.

$$e_i = (1-\beta)\mu^z + \beta e_i^w, \qquad i = 1,2,3$$

In this step we also have 3 variance matrices as follow

$$\Omega_i = (\beta\beta')(\sigma^w(s_{t+1} = i))^2 + V, \qquad i = 1,2,3$$

Also, there will be 3 conditional covariance matrices

$$\Sigma_1 = P1\Omega_1 + P2\Omega_2 + P3\Omega_3 + P1(P2)(e_1 - e_2)(e_1 - e_2)' + P1(P3)(e_1 - e_3)(e_1 - e_3)' \\ + P2(P3)(e_2 - e_3)(e_2 - e_3)'$$

$$\Sigma_2 = R1\Omega_1 + R2\Omega_2 + R3\Omega_3 + R1(R2)(e_1 - e_2)(e_1 - e_2)' + R1(R3)(e_1 - e_3)(e_1 - e_3)' \\ + R2(R3)(e_2 - e_3)(e_2 - e_3)'$$

$$\Sigma_3 = Q1\Omega_1 + Q2\Omega_2 + Q3\Omega_3 + Q1(Q2)(e_1 - e_2)(e_1 - e_2)' + Q1(Q3)(e_1 - e_3)(e_1 - e_3)' \\ + Q2(Q3)(e_2 - e_3)(e_2 - e_3)'$$

Same as the 2-Regime-Switching Beta model, the standard optimal mean-variance portfolio is

$$w_i = \frac{1}{\gamma}\Sigma_i^{-1}e_i$$

Where $\gamma$ is investor's risk aversion. The difference is 0.5 isn't the condition for determining the state. In this model, which regime has the highest probability for each period, that regime will be assigned to the period.

## 3.3 Determining the number of regimes in Regime-Switching Beta model

I compare precision of the Markov chain process in the model that contain 2-7 regime following (Chia-Shang James Chu, 1996). Both Akaike's information criterion (AIC) and Schwarz's criterion can be used to consider for better model (Sclove, 1983). The best number of regimes is the model with the lowest AIC and Schwarz's criterion. Normally, AIC and Schwarz's criterion should give the same way of suggestion because of similar criterion's formulas.

I discuss how probability is consistent with the market returns by graphing both probabilities and cumulative market returns in the same panel. Then, I compare returns, standard deviations and correlations of each regime whether it consistent with the result from graph and the result of other literature.

In addition, I compare the performances of each strategy by considering cumulative returns and ratios such as Sharpe ratio and Sortino ratio. Moreover, I consider whether the strategy is practical or not by stability of the proportion to each asset. If the weight is stable, the model is likely to be practical.

## 3.4 How risk aversion $A$ affect capital allocation between risky asset and risk-free asset

As mentioned previously, the risk aversion $\gamma$ doesn't affect Sharpe ratio of portfolios (Andrew Ang, 2003). I will adapt a risk aversion $A$ in the step of capital allocation to a complete portfolio containing both risky assets, which I get from Regime-Switching Beta model, and risk-free assets.

According to (Zvi Bodie, 2014), The step begins with utility score function:

$$U = E(r) - \frac{1}{2}A\sigma^2$$

Where, $U$ is the utility value and $A$ is an investor's risk aversion index. $E(r)$ and $\sigma^2$ are a portfolio return and a variance of return respectively. If $A$ is a positive number, it means investors are risk averse. Thus, their utility is enhanced by expected return but risk (variance of return) decrease their utility. The higher risk aversion index, the more risk averse and the more penalty of risk.

In contrast, if $A$ is negative, it means that investors are risk lover. Their utility is driven by risk. Finally, investors are risk neutral if $A$ is equal to 0. They feel indifferent with varying risk so their utility depends only on expected return.

In this step, investors already decide the composition of the risky portfolio $P$ by the Regime-Switching Beta model. Now, the concern is the proportion in complete portfolio. Investors must decide a proportion investment budget $y$, allocated to the risky portfolio $P$. The rest, $1 - y$, goes to risk-free assets. Thus, the expected return of complete portfolio is

$$E(r_c) = yE(r_p) + (1 - y)r_f = r_f + y[E(r_p) - r_f]$$

Where $r_p$ is return of the risky portfolio and $r_f$ is a risk-free rate.

Then, substitute $E(r_c)$ into $E(r)$ in the utility score function and maximize the utility score function as follow

$$\max_y U = E(r_c) - \frac{1}{2}A\sigma_c^2 = r_f + y[E(r_p) - r_f] - \frac{1}{2}Ay^2\sigma_p^2$$

After solving the problem, the optimal proportion in risky portfolio $y^*$ for risk averse investors is

$$y^* = \frac{E(r_p) - r_f}{A\sigma_p^2}$$

Normally, investor's risk aversion index is in the range of 2-4. Thus, I divide investors into 3 groups to compare the effect of the risk aversion index across types of investors. The first group is low risk aversion group with $A = 2$. The second group is normal risk aversion group with $A = 3$. The last group is high risk aversion group, $A = 4$.

### 3.5 Implementation

I adapt a MATLAB code provided by Kun Yu and Luoyi Zou for their work in 2017. The MATLAB code can be downloaded via this link: https://github.com/DuPupu/Regime-Switching , access in February 2018. In order to estimate transition probabilities and probabilities, a MATLAB package for Markov regime switching models provided by Marcelo Perlin is used in the script. The script follows the equation in work by Ang and Bekaert, 2003. The code is provided for only 2-regime model so I added 3-regime model's equation mentioned before into the original code. Then, I do back testing to compare models' performances.

In addition, I put a short constraint to the MATLAB code because most of retail investors have it. This is a big difference between my work and work of Ang and Bekaert,2003 and Kun Yu and Luoyi Zou, 2017. Another difference is that I vary rebalance period. Both works I referred rebalance the portfolio every month. In my work, I do 4 rebalancing periods including 1 month, 3 months (a quarter), 6 months (half year), and 12 months (a year).

To compute AIC and Schwarz criterion, I load the same data set as in MATLAB into Eviews programs and run the Markov switching regression in that. The program gives AIC and Schwarz criterion automatically. I run the Markov switching regression range 2-7 regimes following Chia-Shang James Chu,Gary J. Santoni and Tung Liu, 1996

After all, I choose the best risky portfolio and plug in expected return and variance of return to calculate the proportion of porfolio investing in the risky portfolio $y^*$ via MATLAB. The rest goes to risk-free assets. Then, I do backtesting again to compare across different risk aversion investors.

## 4. Result

### 4.1 2-regime model vs. 3-regime model

Table 2: AIC and Schwartz criterion of 2-7 regime Markov switching models

| number of regime in Markov switching | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| AIC | -2.6322* | -2.6272 | -2.6098 | -2.6041 | -2.3140 | -2.3367 |
| Schwartz | -2.5562* | -2.4753 | -2.3517 | -2.2093 | -1.7523 | -1.5776 |

*indicates the lowest value

As I mentioned, the lower AIC and Schwartz criterion, the better Markov switching model. Thus, the 2-regime Markov switching model seem to be the best fitted model to market regime since it has the lowest both AIC and Schwartz criterion among 2-7 number of regimes. More-than-7-regime model is also worse than the 2-regime Markov switching model because the more number of regime, the higher AIC and Schwartz criterion. Although the 3-regime model has higher AIC and Schwartz criterion, it is make sense that market contains 3 regimes including a bear regime, a normal regime and a bull regime. Thus, I will compare the 2-regime model and the 3-regime model more in detail.

Figure 1: probabilities of the 2-regime model

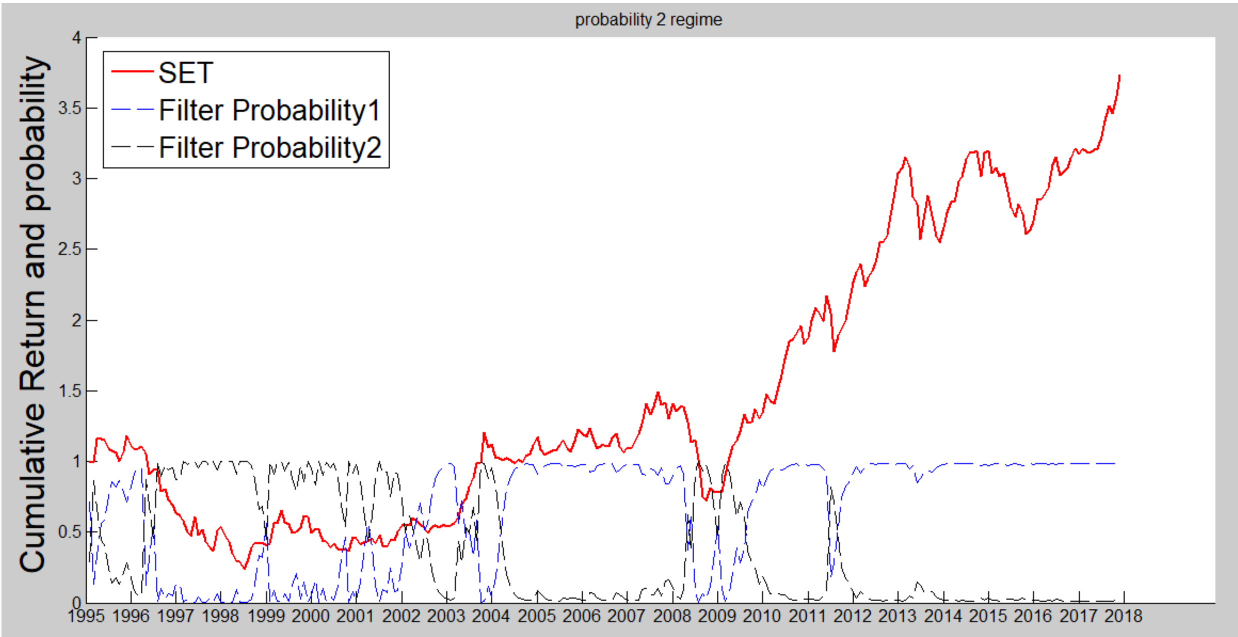



Figure 2: probabilities of the 3-regime model

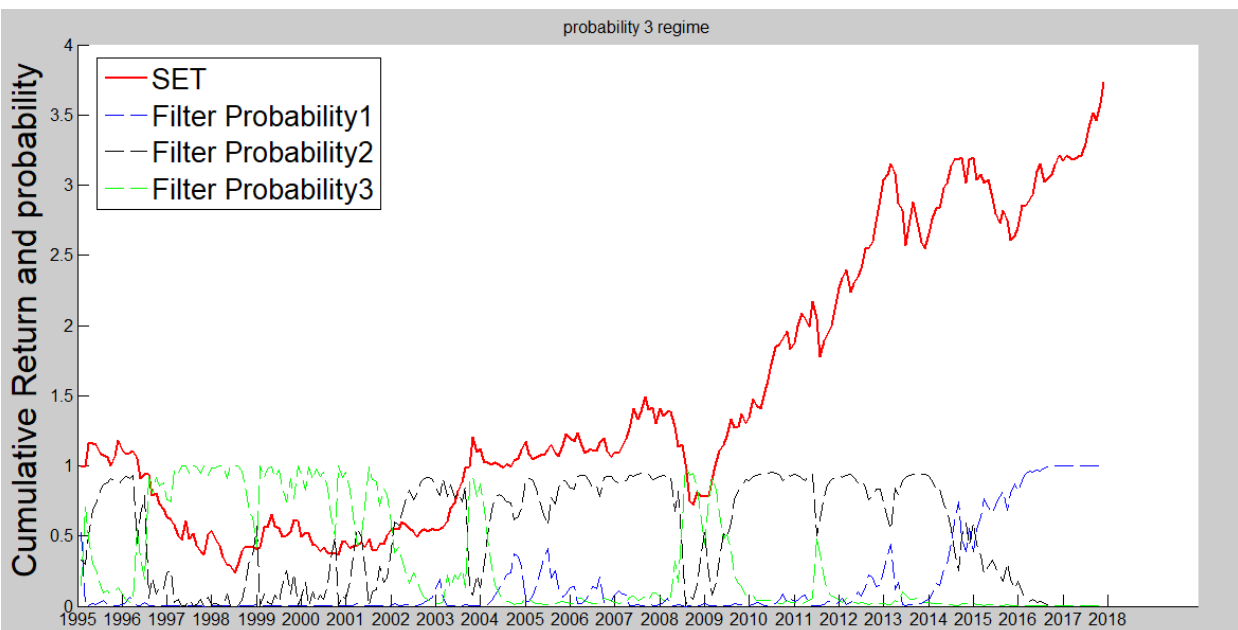


Figure 1 and figure 2 show the probabilities of the 2-regime model and the 3-regime model. Filter probabilities tell that the next month period should be in which regime using current information. The market will be in the regime that has the highest probability. Thus, the market will be in the period that has probability more than 0.5 in the case of 2-regime model.

In 2-regime model, the filter probability 1 is the probability for a bull market and the filter probability 2 is for a bear market. The probabilities from the 2-regime model is well-captured the market regime as you see in the figure 1. The filter probability 1 is high when the SET index has an upward trend. Also, the filter probability 2 is high when the SET index has a downward trend.

On the other hand, the probability of the 3-regime model doesn't capture the market regime well. Although the filter probability 3 is well fit to a bear market, the filter probability 1 and the filter probability 2 don't well fit to a bull market and a normal market. The filter probability 1 seem to be representative of a bull market but it fit to a bull market only in the year 2015-2017. In addition, the filter probability 2 is the highest probability both in a normal market (2004-2008) and a bull market (2010-2014) while the slope of the SET index in both periods are clearly different.

Table 3 shows that 2 regimes are different. Regime 1 is a bull market. It has positive excess return with low volatility. In contrast, regime 2 has negative excess return but higher volatility. Moreover, there is higher correlation in regime 2 than in regime 1. It is consistent with the work by Campbell, 2002.

Table 3: the 2-regime model's mean excess return and correlation (annualized)

| | | | | | | |
|---|---|---|---|---|---|---|
| Excess return | | | | | | |
| | | SETENERG | SETBANK | SETPROP | SETCOMMU | SETCOM |
| regime1 | Mean | 13.51 | 3.36 | 11.17 | 4.04 | 16.94 |
| | Stdev | 25.49 | 23.78 | 27.37 | 25.72 | 18.73 |
| regime2 | Mean | -13.47 | -10.69 | -29.35 | -5.53 | -16.29 |
| | Stdev | 41.93 | 64.41 | 70.97 | 62.04 | 36.75 |
| Regime 1 correlation matrix | | | | | | |
| | | SETENERG | SETBANK | SETPROP | SETCOMMU | SETCOM |
| | SETENERG | 1 | | | | |
| | SETBANK | 0.565589 | 1 | | | |
| | SETPROP | 0.430464 | 0.706462 | 1 | | |
| | SETCOMMU | 0.492057 | 0.610417 | 0.549011 | 1 | |
| | SETCOM | 0.404688 | 0.561504 | 0.642146 | 0.464984 | 1 |
| Regime 2 correlation matrix | | | | | | |
| | | SETENERG | SETBANK | SETPROP | SETCOMMU | SETCOM |
| | SETENERG | 1 | | | | |
| | SETBANK | 0.60863 | 1 | | | |
| | SETPROP | 0.512746 | 0.840714 | 1 | | |
| | SETCOMMU | 0.672463 | 0.663561 | 0.5785 | 1 | |
| | SETCOM | 0.678787 | 0.63823 | 0.564397 | 0.671664 | 1 |

Excess returns are shown in percentages

Table 4: the 3-regime model's mean excess return and correlation (annualized)

| | | | | | | |
|---|---|---|---|---|---|---|
| Excess return | | | | | | |
| | | SETENERG | SETBANK | SETPROP | SETCOMMU | SETCOM |
| regime1 | Mean | 10.82 | -0.57 | 2.05 | -8.95 | 11.29 |
| | Stdev | 16.09 | 14.10 | 11.41 | 20.78 | 11.78 |
| regime2 | Mean | 12.32 | 6.93 | 9.80 | 4.83 | 16.05 |
| | Stdev | 22.85 | 20.84 | 28.58 | 23.22 | 19.52 |
| regime3 | Mean | -6.99 | -14.25 | -19.93 | -0.09 | -9.87 |
| | Stdev | 46.34 | 65.85 | 70.50 | 63.17 | 36.78 |
| Regime 1 correlation matrix | | | | | | |
| | | SETENERG | SETBANK | SETPROP | SETCOMMU | SETCOM |
| | SETENERG | 1 | | | | |
| | SETBANK | 0.358264 | 1 | | | |
| | SETPROP | 0.505575 | 0.277562 | 1 | | |
| | SETCOMMU | 0.471571 | 0.615663 | 0.404195 | 1 | |
| | SETCOM | 0.425394 | 0.385566 | 0.475247 | 0.399551 | 1 |
| Regime 2 correlation matrix | | | | | | |
| | | SETENERG | SETBANK | SETPROP | SETCOMMU | SETCOM |
| | SETENERG | 1 | | | | |
| | SETBANK | 0.546277 | 1 | | | |
| | SETPROP | 0.445047 | 0.725757 | 1 | | |
| | SETCOMMU | 0.376881 | 0.460934 | 0.514816 | 1 | |
| | SETCOM | 0.431096 | 0.562175 | 0.63257 | 0.417429 | 1 |
| Regime 3 correlation matrix | | | | | | |
| | | SETENERG | SETBANK | SETPROP | SETCOMMU | SETCOM |
| | SETENERG | 1 | | | | |
| | SETBANK | 0.605407 | 1 | | | |
| | SETPROP | 0.488366 | 0.834755 | 1 | | |
| | SETCOMMU | 0.678639 | 0.694917 | 0.594629 | 1 | |
| | SETCOM | 0.62782 | 0.642359 | 0.582932 | 0.683341 | 1 |

Excess returns are shown in percentages

Table 4 shows that regime 3 is a bear market. It has negative excess returns and the highest volatilities. It also has the highest overall correlation among indexes. This result is consistent with the result from figure 2. On the other hand, figure 2 suggest that regime 1 should be a bull market but it shows ambiguous excess returns though there are the lowest volatility and correlation. Although regime 2 seems to be a normal regime from figure 2, it has the highest mean excess return which should be a property of a bull market according to table 4. Thus, the 3-regime model has conflict with itself.

Figure 3 to figure 6 show performance of the 2-regime model compared to the 3-regime model. In the case of 1-month and 12-month rebalancing period, the 3-regime model is better than the 2-regime model, clearly. While in the case of 6-month rebalancing, the 3-regime model is only a bit better than the

2-regime model. In the other hand, the 2-regime model is better than the 3-regime model in the case of 3-month rebalancing period. Thus, it's not clear to say that which model is the better one. Then, I will compare practical levels between the 2-regime and the 3-regime model using the weight given to each index.

Figure 3: cumulative return of the regime-switching model with 1-month rebalancing period

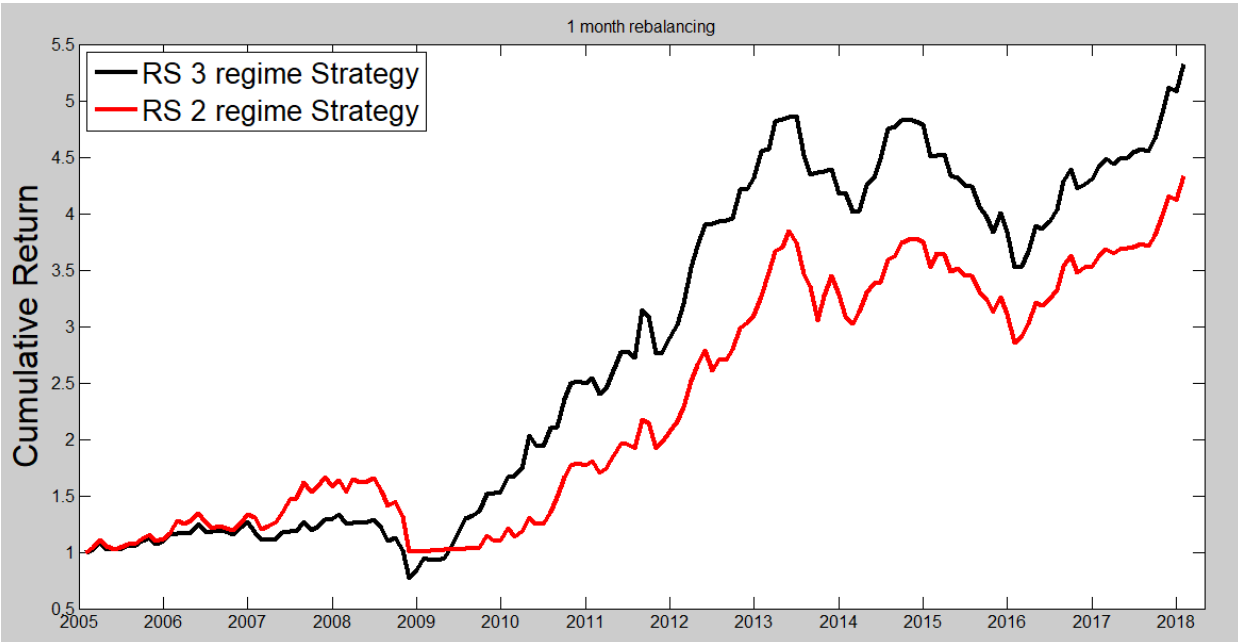



Figure 4: cumulative return of the regime-switching model with 3-month rebalancing period

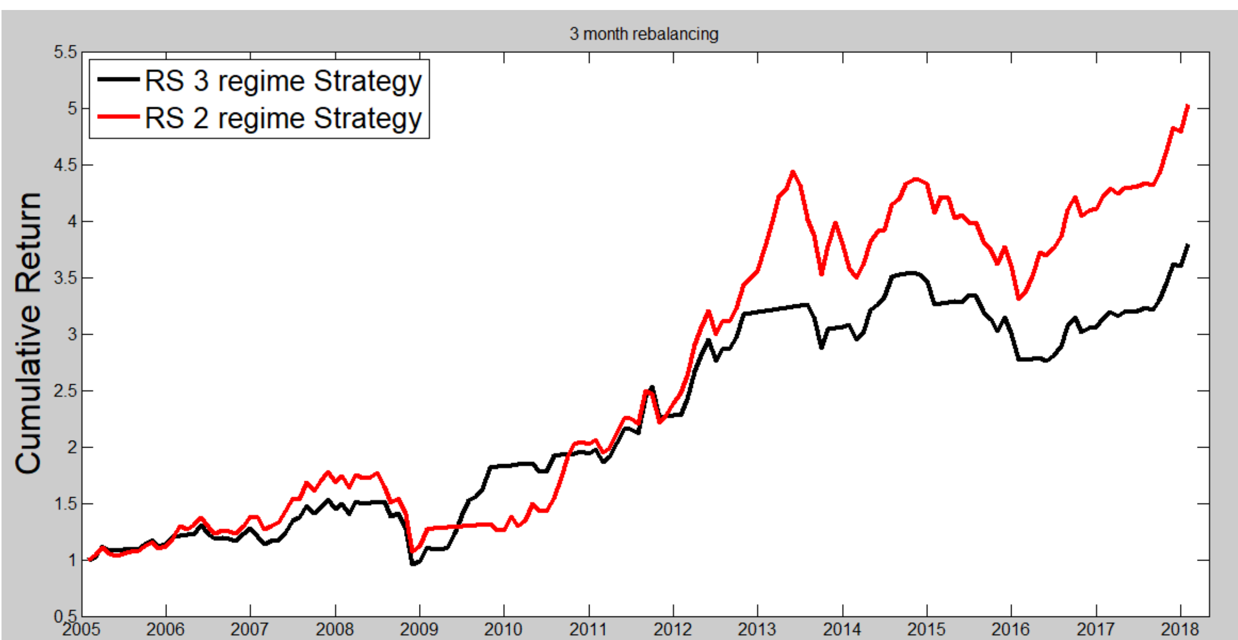


Figure 5: cumulative return of the regime-switching model with 6-month rebalancing period

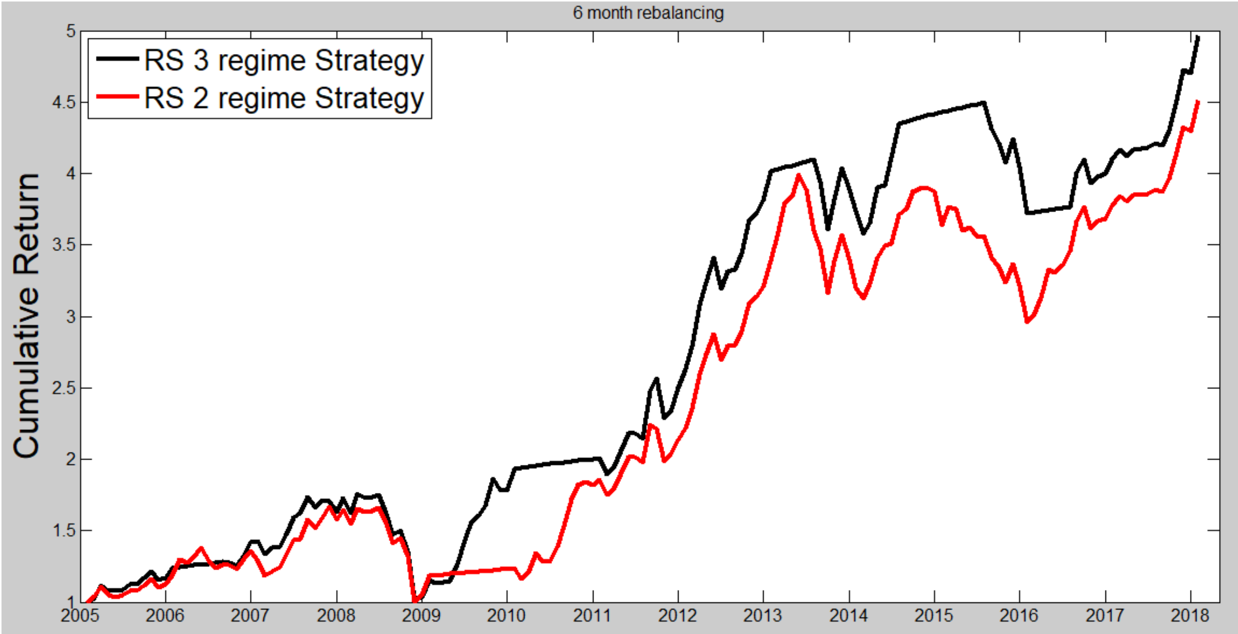



Figure 6: cumulative return of the regime-switching model with 12-month rebalancing period

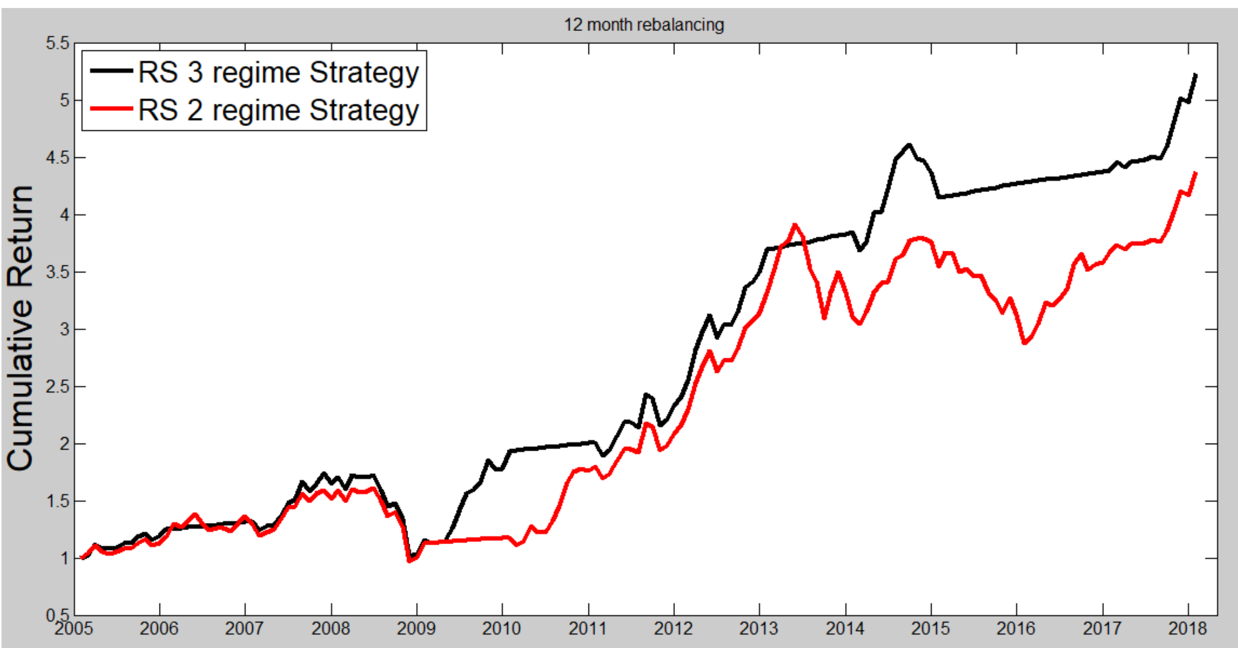


Figure 7: weight given to indexes of the 2-regime-switching model with 1, 3, 6, 12-month rebalancing period

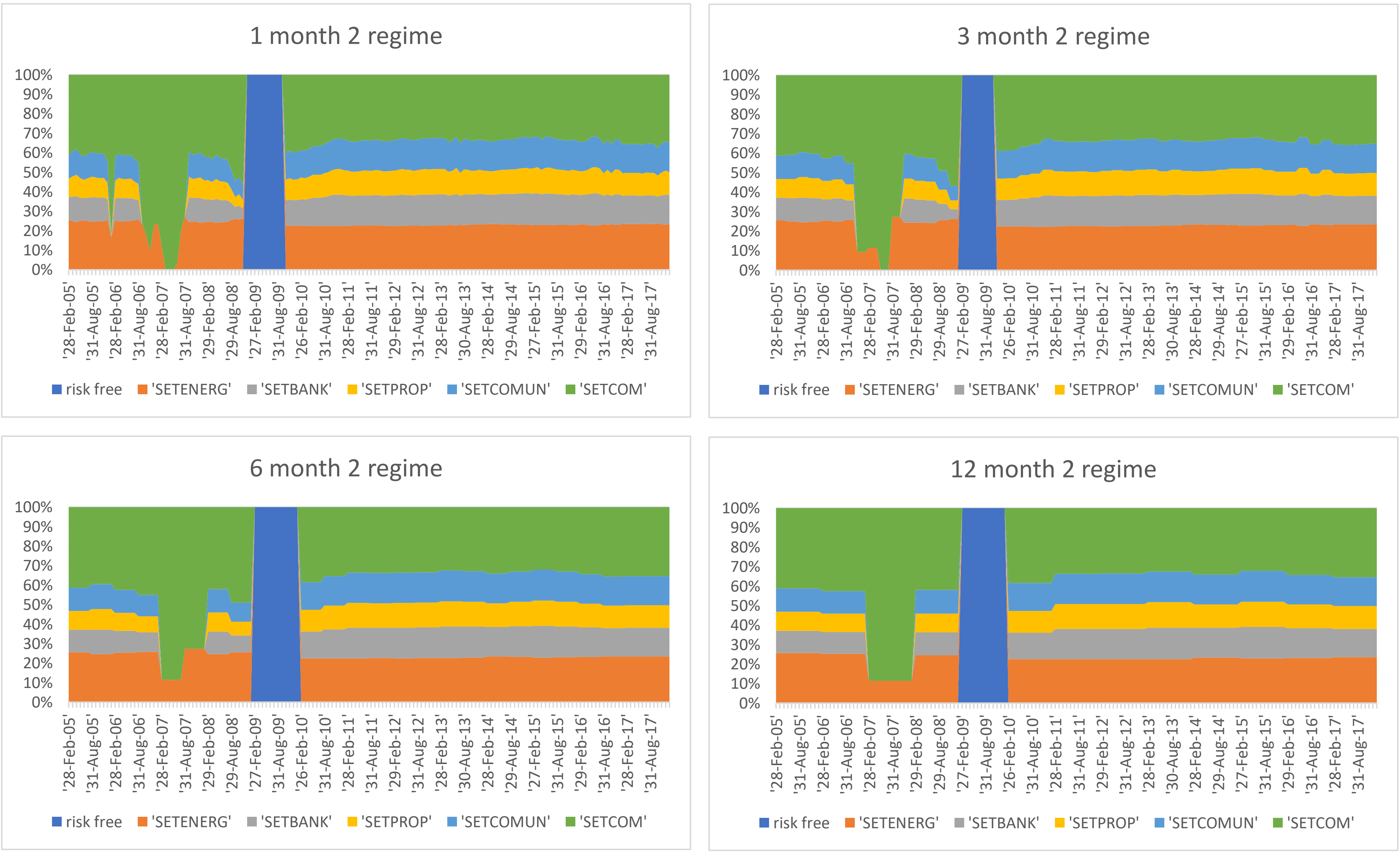
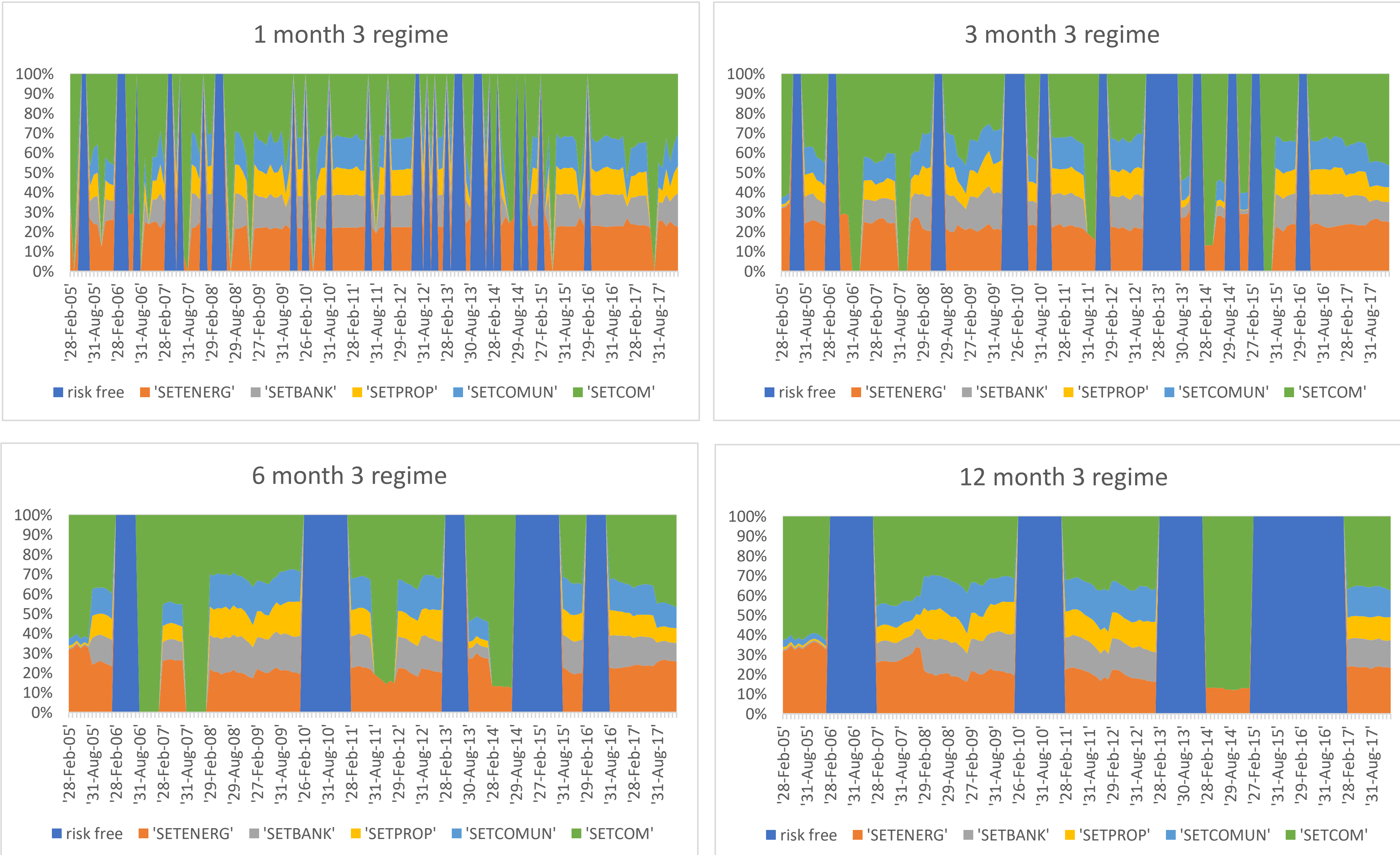


Figure 8: weight given to indexes of the 3-regime-switching model with 1, 3, 6, 12-month rebalancing period



According to figure 7 and figure 8, the 2-regime-switcing is more practical than the 3-regime model since the weight given to each index is less volatile. It is hard to adjust the proportion of the portfolio heavily all the time. Especially for large portfolios, it is impossible to sell other indexes and hold only one index frequently. In addition, higher volatility of weight causes higher transaction costs since investors must trade assets more frequently. Consequently, it leads to lower wealth if transaction cost is taken into the account.

I could say that the 2-regime model is more suitable to the Stock Exchange of Thailand than 3-regime model. Although, the 3-regime perform better than the 2-regime model in most of rebalancing period, the 2-regime weight is more practical. Moreover, AIC and Schwartz criterion suggest that the 2-regime model is the best fit model. Its probability and excess return are consistent while there is a conflict of result of the 3-regime model.

Figure 9: the 2-regime model efficient frontier

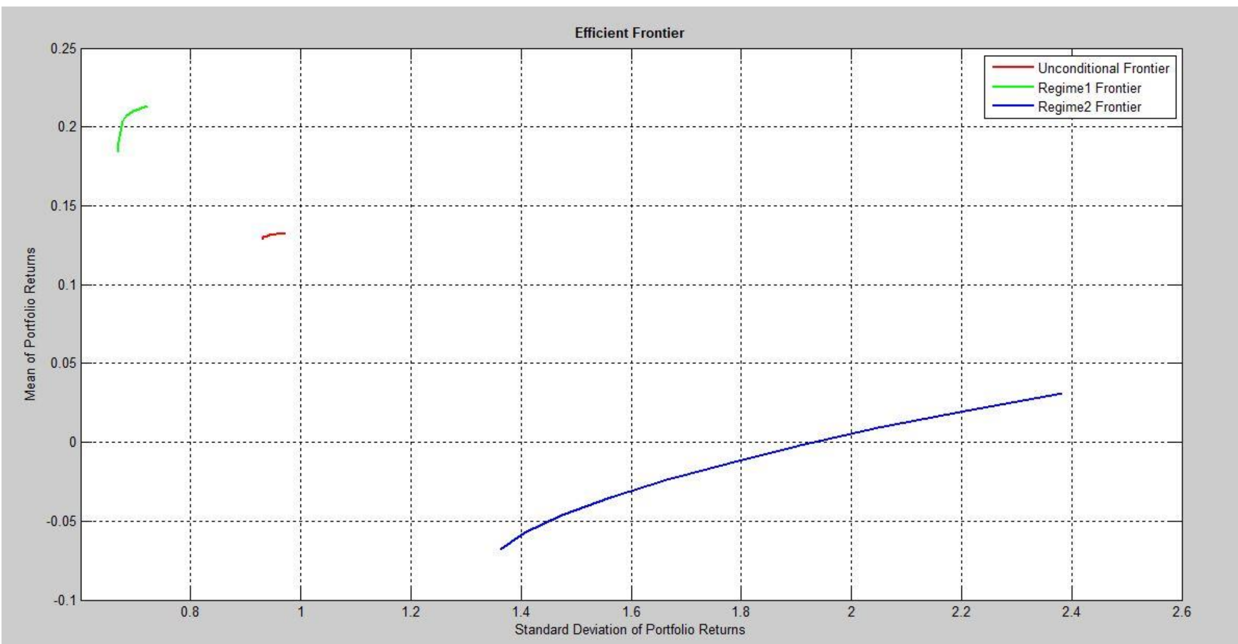



Figure 9 shows that the regime-switching model is important to be added to the standard mean-variance optimization. The 2-regime model can separate market regimes clearly. Regime 1 frontier is a bull market efficient frontier which has high returns and low volatilities. It means that in the bull market, trade-off between returns and risks is better than that of a bear market so an optimization should be different between regimes. If you use the unconditional efficient frontier meaning ignoring the regime-switching model, you can’t take this advantage.

### 4.2 Assess rebalancing period of 2-regime model

In previous section, I discussed that the 2-regime model is better than the 3-regime model. In this section, I discuss about suitable rebalancing period for the regime-switching model. By comparing cumulative Return in figure 10, the 3-month rebalancing period is the most suitable rebalancing period since it makes the highest wealth to investors. Thus, model I used after this is the 2-regime model with 3-month rebalancing period.

Figure 10: the 2-regime model performance varying rebalancing period

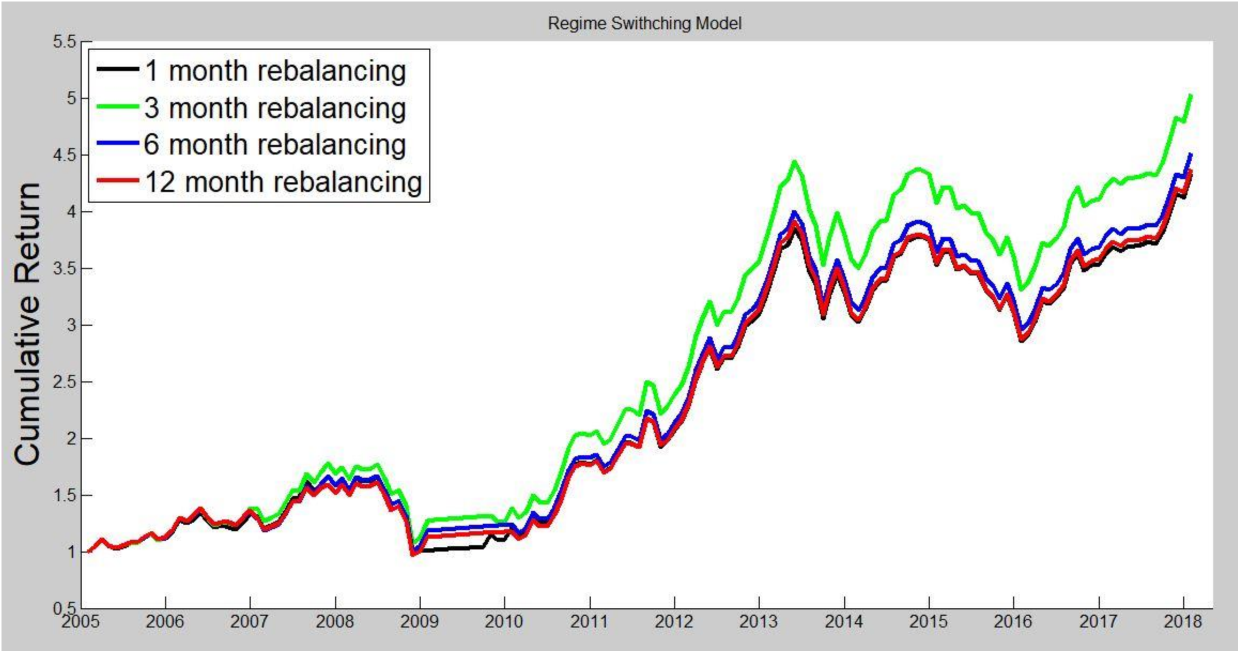



Figure 11: the 2-regime model with 3-month rebalancing compared to other strategies

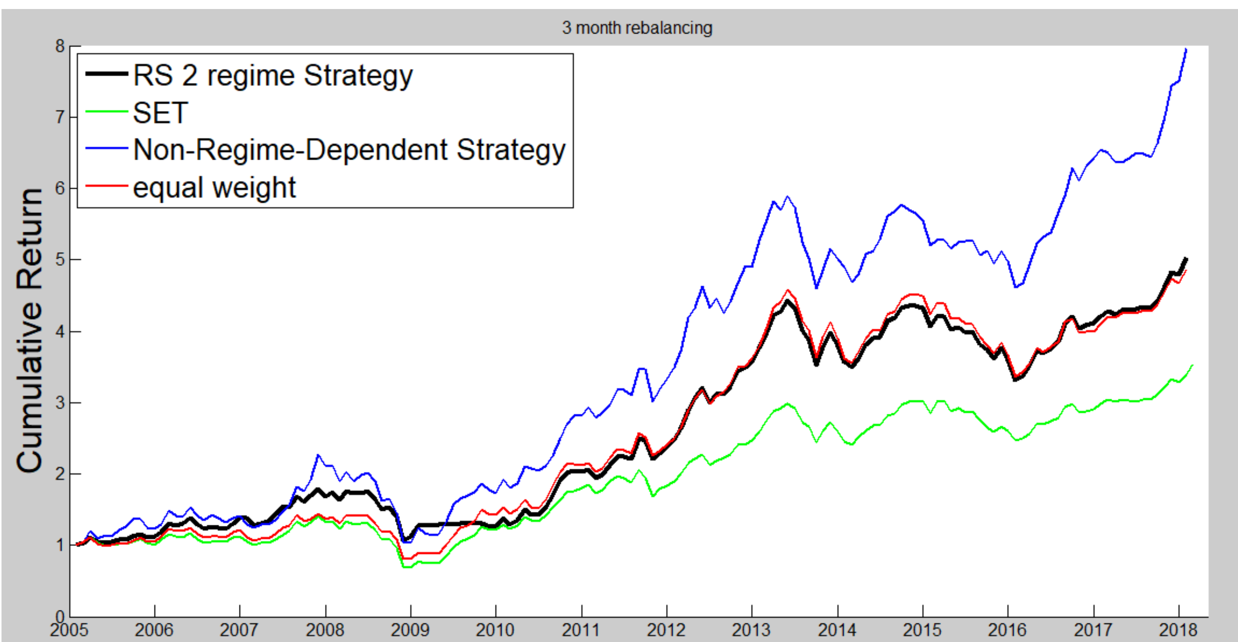


Table 5: all equity portfolio performance (annualized)

| | SET index | Regime-switching | Non-Regime-switching | equal weight |
|---|---|---|---|---|
| mean return | 9.65 | 12.43 | 15.97 | 12.18 |
| stdev | 19.23 | 17.97 | 22.98 | 19.42 |
| Sharpe ratio | 0.4897 | 0.6791 | 0.5739 | 0.484 |
| maximum drawdown | 51.62 | 40.13 | 54.18 | 4.38 |
| downside deviation | 16.33 | 14.89 | 17.64 | 15.3 |
| Sortino ratio | 0.5907 | 0.8347 | 0.9051 | 0.7961 |

returns, standard deviation, maximum drawdown and downside deviation are shown in percentages

According to figure 11, although the Regime-Switching model can beat SET index, it can't beat the non-regime-switching model (the standard mean-variance optimization). It also can't beat the equal weight strategy clearly. Table 5 shows that the 2-regime model is worse than other strategies in term of mean return but it has the lowest standard deviation and the highest Sharpe ratio. In addition, it has the lowest maximum drawdown and downside risk but its Sortino ratio is between the non-regime-switching strategy and the equal weight strategy. Thus, it doesn't clear whether the regime-switching model is useful. Next section, I am going to adjust risk aversion parameter to find that who has the most effect of the regime-switching model

## 4.3 Adjusting risk aversion $A$

Table 6: weight to risky and risk-free asset

| Risk aversion | 2-regime-switching | | Non-regime-switching | | Equal weight | |
|---|---|---|---|---|---|---|
| | risky | Risk-free | risky | Risk-free | risky | Risk-free |
| 2 | 106.50 | 0 | 98.64 | 1.36 | 87.88 | 12.12 |
| 3 | 71.00 | 29.00 | 65.76 | 34.24 | 58.58 | 41.42 |
| 4 | 53.25 | 46.75 | 49.32 | 50.68 | 43.94 | 56.06 |

Weights are shown in percentages

From table 6, the higher risk aversion, the lower weight given to risky asset or the higher weight given to risk-free asset. Comparing weight in the same level of risk aversion, the 2-regime strategy has the highest weight given to risky asset, followed by the non-regime strategy and equal weight strategy. It is because the 2-regime switching has the highest expected return per a unit of risk (Sharpe ratio). The 2-regime model with risk aversion of 2 suggests that investors should invest 106% of their wealth in the risky portfolio. It means that investors must borrow money equal to 6% of their wealth to invest in the risky portfolio. To be consistent with retail investors, I put a borrowing constraint to investors so they invest only 100% of their wealth in the risky portfolio and don't invest in a risk-free asset.

Figure 12: cumulative return of the 2-regime model with risk aversion index = 2

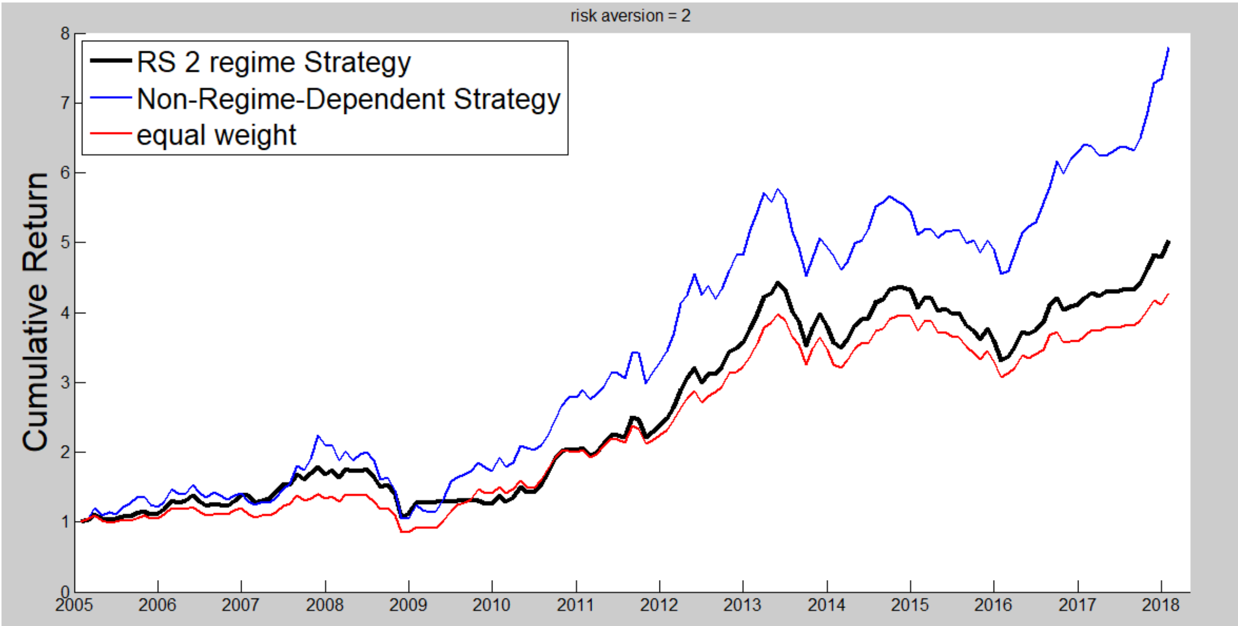



Figure 13: cumulative return of the 2-regime model with risk aversion index = 3

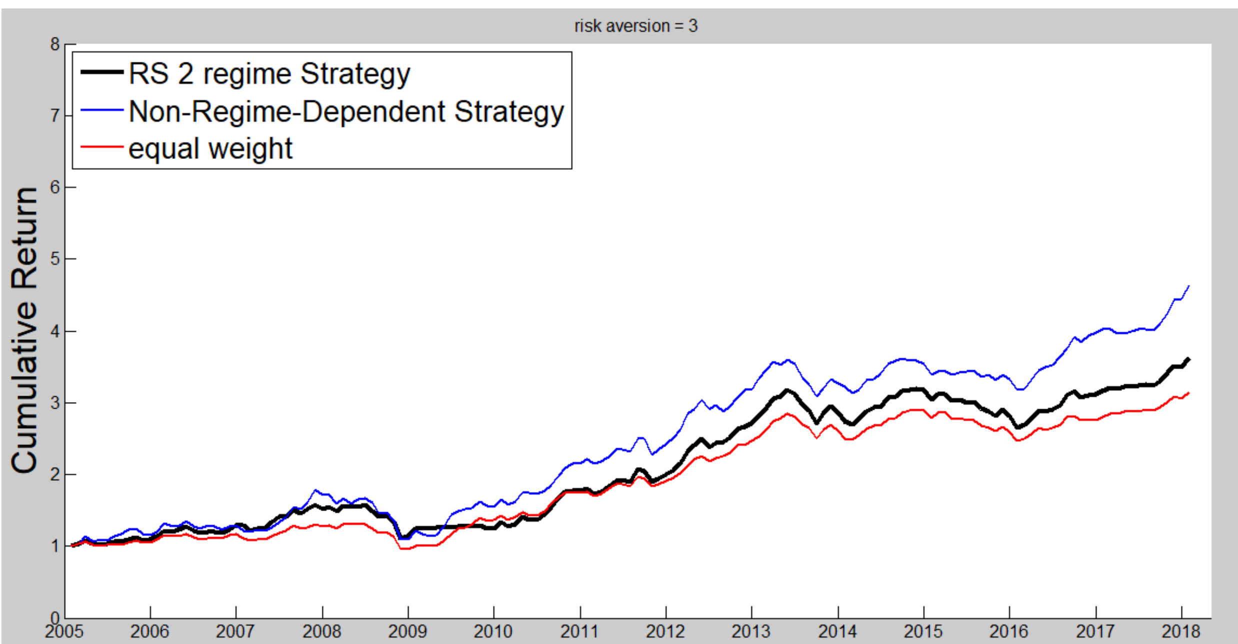
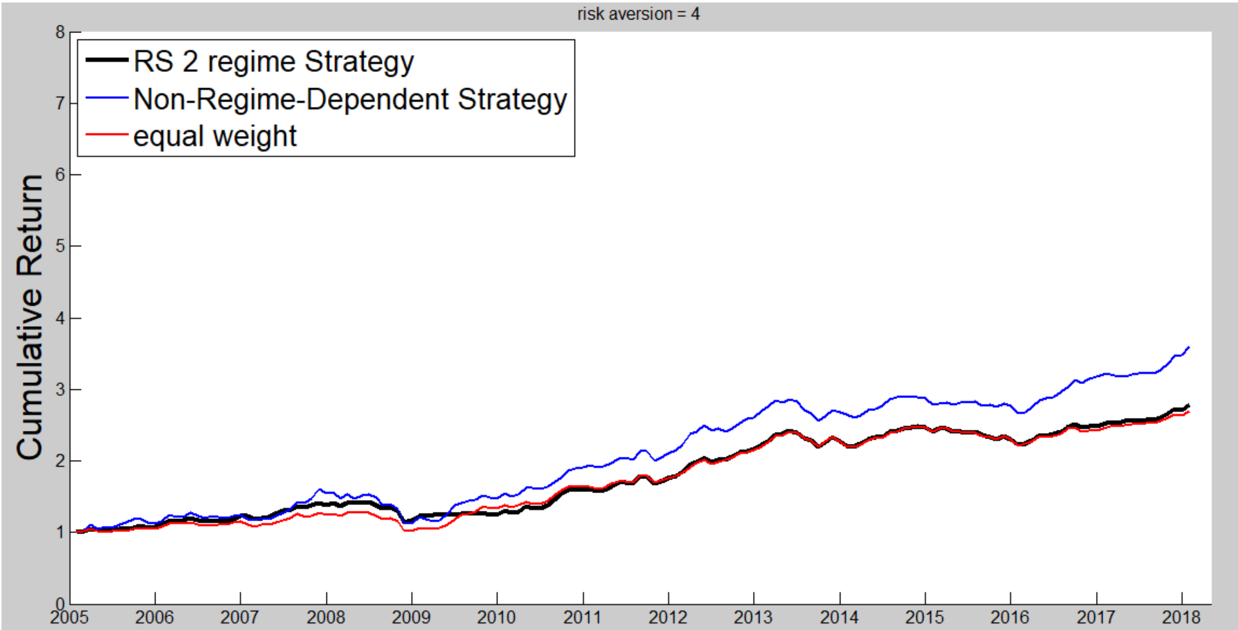


Figure 14: cumulative return of the 2-regime model with risk aversion index = 4




In the case of low risk aversion (risk aversion = 2), the non-regime strategy gives investors the highest final wealth, followed by the 2-regime strategy and the equal weight strategy. Then the risk aversion increased to 3, the performance order remains the same but a gap between the non-regime strategy and the 2-regime strategy drops. Also, a gap between the 2-regime strategy and the equal weight strategy decreases. Finally, in the case of high risk aversion (risk aversion = 4), a gap between the 2-regime strategy and the non-regime strategy drops a bit while performance of the 2-regime strategy and the equal weight strategy are rarely different. Thus, there is no benefit of the regime-switching model in the case of high risk aversion.

Table 17: performance by risk aversion index (annualized)

| Risk aversion | 2 | | | 3 | | | 4 | | |
|---|---|---|---|---|---|---|---|---|---|
| strategy | Regime-switch | Non-regime | Equal weight | Regime-switch | Non-regime | Equal weight | Regime-switch | Non-regime | Equal weight |
| mean return | 12.43 | 15.80 | 11.18 | 9.91 | 11.80 | 8.80 | 7.87 | 9.85 | 7.62 |
| stdev | 17.97 | 22.66 | 17.07 | 12.73 | 15.09 | 11.39 | 8.38 | 11.32 | 8.55 |
| Sharpe ratio | 0.6791 | 0.5745 | 0.4923 | 0.7603 | 0.5982 | 0.5284 | 0.9125 | 0.6245 | 0.5657 |
| maximum drawdown | 40.13 | 53.65 | 39.31 | 29.53 | 38.89 | 27.04 | 19.94 | 29.89 | 20.51 |
| downside deviation | 14.89 | 17.40 | 13.44 | 10.56 | 11.61 | 8.96 | 6.92 | 8.70 | 6.70 |
| Sortino ratio | 0.8347 | 0.9079 | 0.8320 | 0.9387 | 1.0168 | 0.9819 | 1.1379 | 1.1313 | 1.1378 |

returns, standard deviation, maximum drawdown and downside deviation are shown in percentages

Table 17 gives performance statistics of each investing strategy varying risk aversion indexes. Although, a higher risk aversion leads to lower mean returns, it also leads to lower risks. In addition, a higher risk aversion leads to higher returns per a unit of risk, see Sharpe ratio and Sortino ratio. However, too high-risk aversion cause indifference between the 2-regime strategy and the equal weight strategy. On the other hand, the 2-regime strategy highly underperform the non-regime strategy in the case of low risk aversion. Thus, 2-regime model mostly benefits to investors with normal risk aversion.

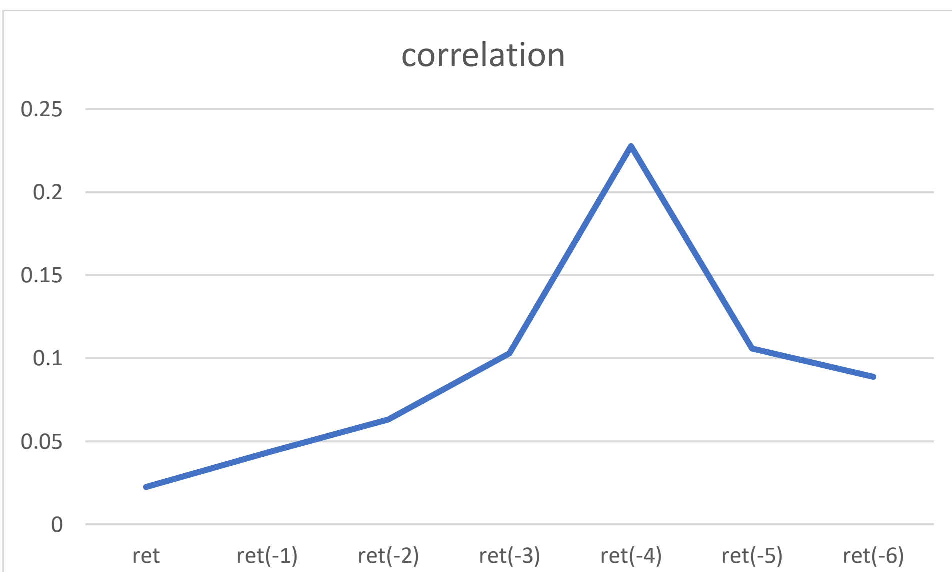
## 5. Discussion

The result of this paper clearly differs from (Andrew Ang, 2003) and (Kun Yu, 2017) which show that the Regime-Switching model beats the non-regime model. There are 2 reasons that these 2 research papers might not hold. Firstly, instead of comparing to dynamic mean-variance strategy, they compare the Regime-Switching model with 1-month rebalancing to a static mean-variance strategy. Thus, Regime-Switching may be better because of worse performance of a static strategy than a dynamic strategy.

Secondly, they allow short order while I don't. Short constraint is more consistent with retail investors' situation. Thus, investors can't make profits when returns are negative. That is why the Regime-Switching model in this paper is perform worse than that of 2 referred research papers.

Finally, I found that there are lags between probability and market return (SET index). According to figure 15, correlation between probability and 4-month lagged market return is the highest. It shows that Regime probability is slower than the real regime. It may be because the Markov chain process in the Regime-Switching Beta model doesn't well perform enough to predict regime for the next period. Thus, Markov model should be improved.

Figure 15: correlation between probability and lagged market return



## 6. Conclusion

The 2-regime model is more suitable to The Stock Exchange of Thailand than the 3-regime model. Although the 3-regime perform better in most rebalancing, few criteria indicates that the third regime is unnecessary. First, the 3-regime model has higher AIC and Schwartz criterion. In addition, the probabilities don't represent regimes clearly. Probability for regime 2 is the highest both in normal and bull markets. Regime 1 which be seen for representing a bull market shows ambiguous returns of each index. It has both positive and negative returns in the regime. Finally, weights given to each index are more volatile so it is less practical than the 2-regime model. Moreover, higher weight volatility will cause lower wealth if transaction cost is considered.

The 2-regime model divides market regimes very well. The first regime stands for a bull market. It has positive returns and lower volatilities. Moreover, it has less correlations among each index. On the other hand, the second regime has negative returns, higher volatilities and higher correlations among each index. Thus, the efficient frontiers of each regime can be drawn separately. Optimization should be different between regimes.

Compared all equity portfolio, the 3-month rebalancing period is the most effective since it leads to the highest wealth. However, it loses to the standard mean-variance strategy with the same rebalancing period. It is indifferent from the equal weight strategy. By the way, it gives investors the highest Sharpe ratio and it Sortino ratio is between other 2 strategies.

After adjusting the risk aversion, the normal risk aversion investors (risk aversion = 3) most benefit from the 2-regime model. Although they get lower returns than low risk aversion investors, they can raise Sharpe ratio and Sortino ratio. However, too high-risk aversion investors lost benefit of the 2-regime model because it makes the 2-regime strategy and the equal weight strategy indifferent.

Although the 2-regime model which is the best model among regime-switching models underperforms the standard mean-variance strategy, it is better than the equal weight strategy. It shows that optimization process is still necessary since both the regime-switching model, which uses the mean-variance optimization, and the standard mean-variance strategy can beat the equal weight strategy.

There are 2 reasons make this paper's result differ from that of others which show that the regime-switching can beat the standard mean-variance strategy. Firstly, I compare the regime-switching model with a dynamic mean-variance strategy instead of a static one. Secondly, I put a short constraint to the model.

Finally, I found that the probabilities predicted by the model is 4-month lagged after the market return (SET index). According to figure 15, correlation between probabilities and 4-month lagged market returns is the highest.

There are several extensions to improve the framework. The First extension is to test a short constraint with other markets. It is to robust the result that the regime-switching model doesn't work with short constraint compared to a standard mean-variance strategy.

The second extension is to improve a Markov chain process in the regime-switching model. It may be more precisely predict regimes. It is interesting to add lagged return to Markov chain process since a 6-regime and 4-lag model is the most fit to American market, (Chia-Shang James Chu, 1996).

# References

Andrew Ang, G. B., 2003. How Do Regimes Affect Asset Allocation?. *NBER working paper series.*

Chia-Shang James Chu, G. J. S. T. L., 1996. Stock Market Volatility and Regime Shifts in Returns. *Information Sciences,* pp. 170-190.

Kun Yu, L. Z., 2017. *Regime Switching,* Zurich: university of Zurich.

Lam, J. W., 2016. *Robo-Advisors: A Portfolio Management Perspective,* s.l.: Department of Economics, Yale College.

Perlin, M., 2015. *The MATLAB Package for Markov Regime Swithcing Models.* s.l.:s.n.

Rachel Campbell, K. K. P. K., 2002. Increased Correlation in Bear Markets. *Financial Analysts Journal,* pp. 87-94.

Sclove, S. L., 1983. Time-Series Segmentation: A Model and a Method. *Infomation Science,* Volume 29, pp. 7-25.

Zvi Bodie, A. K. A. J. M., 2014. *Investments.* 10 ed. New York: McGraw-Hill Education.