Extending the Iris Proof Mode with Inductive Predicates using Elpi

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Program verification

- Verify programs by specifying pre and post conditions
- Specification happens in separation logic
- We make use of embeddings of separation logic in a proof assistant
- Iris (Jung et al. 2018) & Coq (Huet, Kahn, and Paulin-Mohring 2002)

Separation logic with Hoare triples

$$[\mathrm{isD}\ d\ y]\ \mathrm{op}\ d\ x\ [\mathrm{isD}\ d\ (\mathrm{f}\ x\ y)]$$

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Imperative
program

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Imperative program

Functional program

Separation logic with Hoare triples

Representation predicate

$[\mathrm{isD}\ d\ y]\ \mathrm{op}\ d\ x\ [\mathrm{isD}\ d\ (\mathrm{f}\ x\ y)]$

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$$[\mathrm{isList}\ hd\ \vec{v}]\ \mathrm{delete}\ hd\ i\ [\mathrm{isList}\ hd\ (\mathrm{remove}\ i\ \vec{v})]$$

Representation predicates

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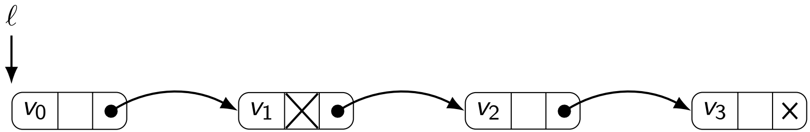
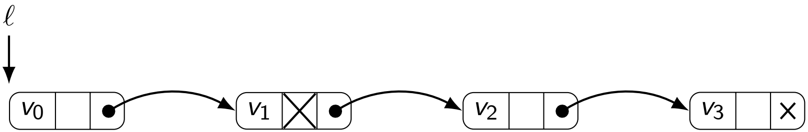


Harris (2001)

Representation predicates



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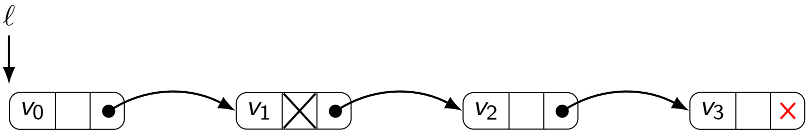

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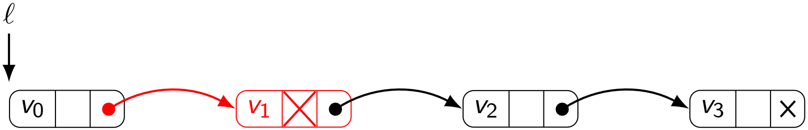

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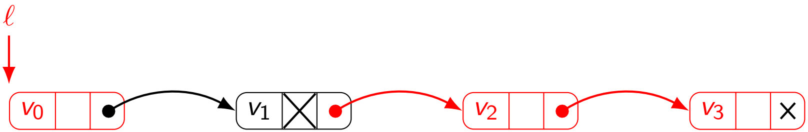

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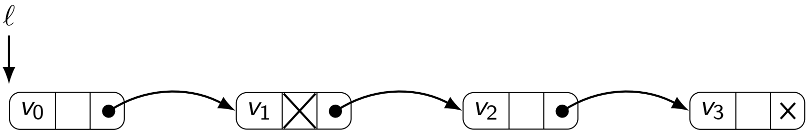

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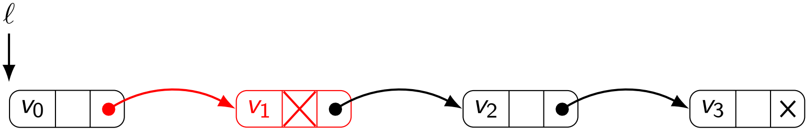

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Outline of our solution

Coq

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eiInd
Inductive is_MLL : val → list val → iProp :=
    | empty_is_MLL : is_MLL NONEV []
    | mark_is_MLL v vs l tl :
      l ↦ (v, #true, tl) -∗ is_MLL tl vs -∗
      is_MLL (SOMEV #l) vs
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- Proof of constructors, `empty_is_MLL`, `mark_is_MLL`, `cons_is_MLL`

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- Definition of `is_MLL`
- Proof of constructors, `empty_is_MLL`, `mark_is_MLL`, `cons_is_MLL`
- Proof of induction principle
- Integration with IPM tactics

Approach

Theory

Challenges in practice

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- Define the pre fixpoint function

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Challenges in practice

- Deal with n-ary predicates
- Proof search for monotonicity
- Integrating resulting definitions and lemmas into the Iris tactics language

Contributions

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- Posed a strategy for defining modular tactics in Elpi
- Posed a syntactic proof search algorithm for finding a monotonicity proof of a pre fixpoint function
- Evaluated Elpi as a meta-programming language for the IPM

Monotone pre fixpoint function

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\begin{aligned}
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Definition (Monotone predicate)

Function $\mathsf{F} : (A \to iProp) \to A \to iProp$ is *monotone* when, for any $\Phi, \Psi : A \to iProp$, it holds that

$$\Box(\forall y.\, \Phi\, y \twoheadrightarrow \Psi\, y) \vdash \forall x.\, \mathsf{F}\,\Phi\, x \twoheadrightarrow \mathsf{F}\,\Psi\, x$$

Monotone signatures

Connective	Type	Signature
$*$	$iProp \to iProp \to iProp$	$(\twoheadrightarrow) \Longrightarrow (\twoheadrightarrow) \Longrightarrow (\twoheadrightarrow)$
$\vee$	$iProp \to iProp \to iProp$	$(\twoheadrightarrow) \Longrightarrow (\twoheadrightarrow) \Longrightarrow (\twoheadrightarrow)$
$\twoheadrightarrow$	$iProp \to iProp \to iProp$	$\text{flip}(\twoheadrightarrow) \Longrightarrow (\twoheadrightarrow) \Longrightarrow (\twoheadrightarrow)$
$\exists$	$(A \to iProp) \to iProp$	$\gtrdot(\twoheadrightarrow) \Longrightarrow (\twoheadrightarrow)$

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$\mathrel{-\!\!*}$	$iProp \to iProp \to iProp$	$\mathrm{flip}(\mathrel{-\!\!*}) \Longrightarrow (\mathrel{-\!\!*}) \Longrightarrow (\mathrel{-\!\!*})$
$\exists$	$(A \to iProp) \to iProp$	$\gtrdot(\mathrel{-\!\!*}) \Longrightarrow (\mathrel{-\!\!*})$

Definition (Respectful relation)

The *respectful relation* $R \Longrightarrow R' : iRel\ (A \to B)$ of two relations $R : iRel\ A$, $R' : iRel\ B$ is defined as

$$R \Longrightarrow R' \triangleq \lambda f, g.\ \forall x, y.\ R\,x\,y \mathrel{-\!\!*} R'\,(f\,x)\,(g\,y)$$

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Definition (Proper element of a relation)

Given a relation R: *iRel* A and an element $x \in A$, x is a proper element of R if $R\,x\,x$.

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$(\gtrdot (-\!\!*) \Longrightarrow (-\!\!*))$	$\forall \Phi, \Psi.\, (\forall x.\, \Phi\, x -\!\!* \Psi\, x) -\!\!* (\exists x.\, \Phi\, x) -\!\!* (\exists x.\, \Psi\, x)$

Monotonicity as a signature

Proof search

Elpi

Demo

Conclusion

Future work