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Assignment 9

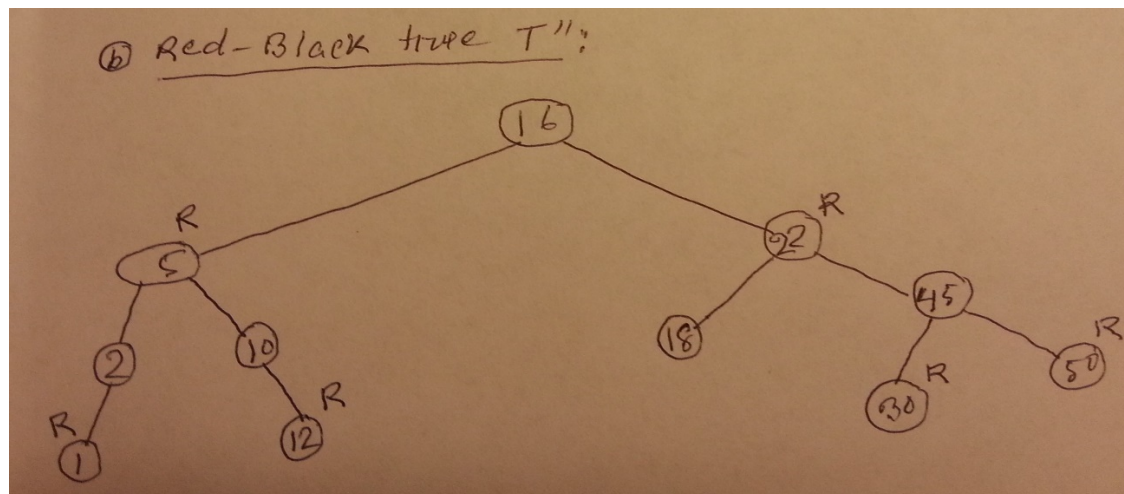
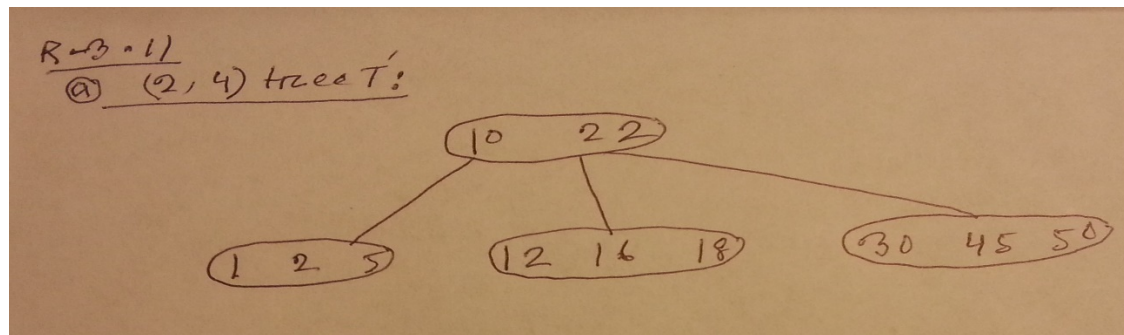
R-3.11 Consider the following sequence of keys:
(5, 16, 22, 45, 2, 10, 18, 30, 50, 12, 1)

Consider the insertion of items with this set of keys, in the order given, into: a. an initially empty (2,4) tree T' .

b. an initially empty red-black tree T'' .

Draw T' and T'' after each insertion.

Answer:



R-3.14 For each of the following statements about red-black trees, determine whether it is true or false. If you think it is true, provide a justification. If you think it is false, give a counterexample.

| | |
|---|---|
| a | False. Reason: To be a red-black root has to be black but there is no guarantee root of subtree will be black. It might be red or black. |
| b | True. Reason: if by another external node it is a black external nodes, because there's a rule that all external nodes are black. And also it can be red, because when we insert, or doing recolor, or restructure, the node will still be red. |
| c | False. Reason: Every red-black tree can become (2,4) tree, and the other way around. But we can make the tree with the same red black tree and produce a different (2,4) tree, even though it's result will be the same. So basically, it's not unique. |
| d | False. Reason: Every (2,4) tree can become red-black tree, and the other way around. But we can make the tree with the same (2,4) tree and produce a different red black tree, even though it's result will be the same. So basically, it's not unique. |

- a. a subtree of a red-black tree is itself a red-black tree.
b. the sibling of an external node is either external or it is red.
c. given a red-black tree T, there is an unique (2,4) tree T' associated with T. d. given a (2,4) tree T, there is an unique red-black tree T' associated with T.

Design a pseudo code algorithm **isValidAVL(T)** that decides whether or not a binary tree is a valid AVL tree. For this problem, we define valid to mean that the height of the left and right sub-trees of every node do not differ by more than one.

What is the time complexity of your algorithm?

Design an algorithm, **isPermutation(A,B)** that takes two sequences A and B and determines whether or not they are permutations of each other, i.e., they contain same elements but possibly occurring in a different order. Assume the elements in A and B cannot be sorted. **Hint:** A and B

may contain duplicates. Same problem as in previous homework, but this time use a dictionary to solve the problem.

What is the worst case time complexity of your algorithm? Justify your answer.

C-3.10 Let D be an ordered dictionary with n items implemented by means of an AVL tree (or a Red-Black tree). Show how to implement the following operation on D in time $O(\log n + s)$, where s is the size of the iterator returned:

FindAllInRange(k_1, k_2):

Return an iterator of all the elements in D with key k such that $k_1 < k < k_2$.

Answer:

```
Algorithm findAllInRange( $k_1, k_2$ )
  Input: key  $k_1, k_2$ 
  Output: return iterator for all
the elements in  $D$  within the range of
 $k_1$  and  $k_2$ 
   $T \leftarrow$  tree of  $D$ 
   $S \leftarrow$  findElements( $T, T.root(), k_1, k_2$ )
  return  $S.iterator()$ 
```

```
Algorithm findElements( $T, p, k_1, k_2$ )
  Input: Tree  $T$ , position of a node
 $p$ , key  $k_1, k_2$ 
  Output: Sequence  $S$  with all the
elements between the range of  $k_1$  and
 $k_2$  inclusive.
   $S \leftarrow$  new Sequence
   $k \leftarrow T.key(p)$ 
  if  $k_1 \leq k \wedge k \leq k_2$  then
     $S.insertLast(D.findElement(k))$ 

  findElements( $T, T.leftChild(p), k_1, k_2$ )

  findElements( $T, T.rightChild(p), k_1, k_2$ )
    else if  $k < k_1$  then
      return
  findElements( $T, T.leftChild(p), k_1, k_2$ )
  return  $S$ 
```