## InvarGenT: Manual

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#### Abstract

InvarGenT is a proof-of-concept system for invariant generation by full type inference with Guarded Algebraic Data Types and existential types encoded as automatically generated GADTs. This user manual discusses motivating examples, briefly presents the syntax of the InvarGenT language, and describes the parameters of the inference process that can be passed to the InvarGenT executable.

### 1 Introduction

Type systems are an established natural deduction-style means to reason about programs. Dependent types can represent arbitrarily complex properties as they use the same language for both types and programs, the type of value returned by a function can itself be a function of the argument. Generalized Algebraic Data Types bring some of that expressivity to type systems that deal with data-types. Type systems with GADTs introduce the ability to reason about return type by case analysis of the input value, while keeping the benefits of a simple semantics of types, for example deciding equality can be very simple. Existential types hide some information conveyed in a type, usually when that information cannot be reconstructed in the type system. A part of the type will often fail to be expressible in the simple language of types, when the dependence on the input to the program is complex. GADTs express existential types by using local type variables for the hidden parts of the type encapsulated in a GADT.

The InvarGenT type system for GADTs differs from more pragmatic approaches in mainstream functional languages in that we do not require any type annotations on expressions, even on recursive functions. The implementation also includes linear equations and inequalities over rational numbers in the language of types, with the possibility to introduce more domains in the future.

### 2 Tutorial

The concrete syntax of InvarGenT is similar to that of OCaml. However, it does not currently cover records, the module system, objects, and polymorphic variant types. It supports higher-order functions, algebraic data-types including built-in tuple types, and linear pattern matching. It supports conjunctive patterns using the as keyword, but it does not support disjunctive patterns. It does not currently support guarded patterns, i.e. no support for the when keyword of OCaml.

The sort of a type variable is identified by the first letter of the variable. a,b,c,r,s,t,a1,... are in the sort of terms called type, i.e. "types proper". i,j,k,l,m,n,i1,... are in the sort of linear arithmetics over rational numbers called num. Remaining letters are reserved for sorts that may be added in the future. Value constructors (like in OCaml) and type constructors (unlike in OCaml) have the same syntax: capitalized name followed by a tuple of arguments. They are introduced by newtype and newcons respectively. The newtype declaration might be misleading in that it only lists the sorts of the arguments of the type, the resulting sort is always type. Values assumed into the environment are introduced by external. There is a built-in type corresponding to declaration newtype Num: num and definitions of numeric constants newcons 0: Num 0 newcons 1: Num 1... The programmer can use external declarations to give the semantics of choice to the Num data-type.

In examples here we use Unicode characters. For ASCII equivalents, take a quick look at the tables in the following section.

We start simple, with a function that can compute a value from a representation of an expression – a ready to use value whether it be Int or Bool. Prior to the introduction of GADT types, we could only implement a function eval :  $\forall a$ . Term  $a \rightarrow Value$  where, using OCaml syntax, type value = Int of int | Bool of bool.

Let us look at the corresponding generated, also called *exported*, OCaml source code:

The Int, Num and Bool types are built-in. Int and Bool follow the general scheme of exporting a datatype constructor with the same name, only lower-case. However, numerals 0, 1, ... are always type-checked as Num 0, Num 1... Num can also be exported as a type other than int, and then numerals are exported via an injection function (ending with) of\_int.

The syntax external let allows us to name an OCaml library function or give an OCaml definition which we opt-out from translating to InvarGenT. Such a definition will be verified against the rest of the program when InvarGenT calls ocamlc -c (or Haskell in the future) to verify the exported code. Another variant of external (omitting the let keyword) exports a value using external in OCaml code, which is OCaml source declaration of the foreign function interface of OCaml. When we are not interested in linking and running the exported code, we can follow the convention of reusing the name in the FFI definition: external f: ... = "f".

The type inferred is eval:  $\forall a$ . Term  $a \rightarrow a$ . GADTs make it possible to reveal that IsZero x is a Term Bool and therefore the result of eval should in its case be Bool, Plus (x, y) is a Term Num and the result of eval should in its case be Num, etc. InvarGenT does not provide an if...then...else... syntax to stress that the branching is relevant to generating postconditions, but it does export match/ematch ... with True -> ... | False -> ... using if expressions.

equal is a function comparing values provided representation of their types:

```
newtype Ty : type
newtype Int
newtype List : type
newcons Zero : Int
```

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```
newcons Nil : \forall a. List a
newcons TInt : Ty Int
newcons TPair : \forall a, b. Ty a * Ty b \longrightarrow Ty (a, b)
newcons TList : \forall a. \ Ty \ a \longrightarrow Ty \ (List \ a)
newtype Boolean
newcons True : Boolean
newcons False : Boolean
external eq_int : Int 
ightarrow Int 
ightarrow Bool
external b_and : Bool 
ightarrow Bool 
ightarrow Bool
external b_not : Bool \rightarrow Bool
external forall2 : \forall a, b. (a \rightarrow b \rightarrow Bool) \rightarrow List a \rightarrow List b \rightarrow Bool
let rec equal = function
  | TInt, TInt -> fun x y -> eq_int x y
  | TPair (t1, t2), TPair (u1, u2) ->
     (fun (x1, x2) (y1, y2) \rightarrow
          b_and (equal (t1, u1) x1 y1)
                  (equal (t2, u2) x2 y2))
   | TList t, TList u -> forall2 (equal (t, u))
  | _ -> fun _ _ -> False
```

InvarGenT returns an unexpected type: equal:  $\forall a,b.$  (Ty a, Ty b)  $\rightarrow a \rightarrow a \rightarrow Bool$ , one of four maximally general types of equal as defined above. The other maximally general "wrong" types are  $\forall a,b.$  (Ty a, Ty b)  $\rightarrow b \rightarrow b \rightarrow b \rightarrow Bool$  and  $\forall a,b.$  (Ty a, Ty b)  $\rightarrow b \rightarrow a \rightarrow Bool$ . This illustrates that unrestricted type systems with GADTs lack principal typing property.

InvarGenT commits to a type of a toplevel definition before proceeding to the next one, so sometimes we need to provide more information in the program. Besides type annotations, there are two means to enrich the generated constraints: assert false syntax for providing negative constraints, and test syntax for including constraints of use cases with constraint of a toplevel definition. To ensure only one maximally general type for equal, we use both. We add the lines:

```
| TInt, TList l -> (function Nil -> assert false)
| TList l, TInt -> (fun _ -> function Nil -> assert false)
test b_not (equal (TInt, TList TInt) Zero Nil)
```

Actually, InvarGenT returns the expected type equal:  $\forall a,b. (Ty a, Ty b) \rightarrow a \rightarrow b \rightarrow Bool$  when either the two assert false clauses or the test clause is added.

Now we demonstrate numerical invariants:

```
newtype Binary : num
newtype Carry : num
newcons Zero : Binary 0
newcons PZero : \forall n [0 \le n]. Binary n \longrightarrow Binary(2 n)
newcons POne : \forall n[0 \le n]. Binary n \longrightarrow Binary(2 n + 1)
newcons CZero : Carry 0
newcons COne : Carry 1
let rec plus =
  function CZero ->
     (function Zero -> (fun b -> b)
       | PZero a1 as a ->
         (function Zero -> a
           | PZero b1 -> PZero (plus CZero a1 b1)
           | POne b1 -> POne (plus CZero a1 b1))
       | POne a1 as a ->
         (function Zero -> a
```

```
| PZero b1 -> POne (plus CZero a1 b1)
| POne b1 -> PZero (plus COne a1 b1)))
| COne ->
(function Zero -> POne(Zero)
| PZero b1 -> POne b1
| POne b1 -> PZero (plus COne Zero b1))
| PZero a1 as a ->
(function Zero -> POne (a1)
| PZero b1 -> POne (plus CZero a1 b1)
| PZero b1 -> PZero (plus COne a1 b1))
| POne a1 as a ->
(function Zero -> PZero (plus COne a1 b1))
| POne a1 as a ->
(function Zero -> PZero (plus COne a1 Zero)
| PZero b1 -> PZero (plus COne a1 b1))
| POne b1 -> PZero (plus COne a1 b1))
```

We get plus:  $\forall i,k,n.Carry i \rightarrow Binary k \rightarrow Binary n \rightarrow Binary (n + k + i).$ 

We can introduce existential types directly in type declarations. To have an existential type inferred, we have to use efunction or ematch expressions, which differ from function and match only in that the (return) type is an existential type. To use a value of an existential type, we have to bind it with a let..in expression. Otherwise, the existential type will not be unpacked. An existential type will be automatically unpacked before being "repackaged" as another existential type.

```
newtype Yard
newtype Village
newtype Castle : type
newtype Place : type
\mathtt{newcons}\ \mathtt{Room}\ :\ \mathtt{Room}\ \longrightarrow\ \mathtt{Castle}\ \mathtt{Room}
{\tt newcons} \ {\tt Yard} \ : \ {\tt Yard} \ \longrightarrow \ {\tt Castle} \ {\tt Yard}
{\tt newcons} \ {\tt CastleRoom} \ : \ {\tt Room} \ \longrightarrow \ {\tt Place} \ {\tt Room}
newcons CastleYard : Yard \longrightarrow Place Yard
{\tt newcons} \ {\tt Village} \ : \ {\tt Village} \ \longrightarrow \ {\tt Place} \ {\tt Village}
external wander : \forall a. Place a \rightarrow \exists b. Place b
let rec find_castle = efunction
   | CastleRoom x -> Room x
   | CastleYard x -> Yard x
   | Village _ as x ->
      let y = wander x in
      find_castle y
    We get find_castle: \forall a. Place a \rightarrow \exists b. Castle b.
    A more practical existential type example:
newtype Bool
newcons True : Bool
newcons False : Bool
newtype List : type * num
newcons LNil : \foralla. List(a, 0)
newcons LCons : \forall n,a[0 \le n]. a * List(a, n) \longrightarrow List(a, n+1)
let rec filter = fun f ->
   efunction LNil -> LNil
```

newtype Room

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We get filter:  $\forall n$ , a.(a $\rightarrow$ Bool) $\rightarrow$ List (a, n) $\rightarrow \exists k [0 \le n \land 0 \le k \land k \le n]$ .List (a, k). Note that we need to use both efunction and ematch above, since every use of function or match will force the types of its branches to be equal. In particular, for lists with length the resulting length would have to be the same in each branch. If the constraint cannot be met, as for filter with either function or match, the code will not type-check.

A more complex example that computes bitwise or – ub stands for "upper bound":

```
newtype Binary : num
newcons Zero : Binary 0
newcons PZero : \forall n \ [0 \le n]. Binary n \longrightarrow Binary(2 \ n)
newcons POne : \forall n \ [0 \le n]. Binary n \longrightarrow Binary(2 n + 1)
let rec ub = efunction
  | Zero ->
       (efunction Zero -> Zero
         | PZero b1 as b -> b
         | POne b1 as b -> b)
  | PZero a1 as a ->
       (efunction Zero -> a
         | PZero b1 ->
           let r = ub a1 b1 in
           PZero r
         | POne b1 ->
           let r = ub a1 b1 in
           POne r)
  | POne a1 as a ->
       (efunction Zero -> a
         | PZero b1 ->
           let r = ub a1 b1 in
           POne r
         | POne b1 ->
           let r = ub a1 b1 in
           POne r)
```

 $ub: \forall k, n. Binary k \rightarrow Binary n \rightarrow \exists: i[0 \le n \land 0 \le k \land n \le i \land k \le i \land i \le n+k]. Binary i.$ 

Why cannot we shorten the above code by converting the initial cases to Zero -> (efunction b -> b)? Without pattern matching, we do not make the contribution of Binary n available. Knowing n=i and not knowing  $0 \le n$ , for the case k=0, we get:  $ub: \forall k, n.Binary k \to Binary n \to \exists:i[0 \le k \land n \le i \land i \le n+k]$ . Binary i.  $n \le i$  follows from n=i,  $i \le n+k$  follows from n=i and  $0 \le k$ , but  $k \le i$  cannot be inferred from k=0 and n=i without knowing that  $0 \le n$ .

Besides displaying types of toplevel definitions, InvarGenT can also export an OCaml source file with all the required GADT definitions and type annotations.

# 3 Syntax

Below we present, using examples, the syntax of InvarGenT: the mathematical notation, the concrete syntax in ASCII and the concrete syntax using Unicode.

type variable: types	$\alpha, \beta, \gamma, \tau$	a,b,c,r,s,t,a1,	
type variable: nums	k, m, n	i,j,k,l,m,n,i1,	
type var. with coef.	$\frac{1}{3}n$	1/3 n	
type constructor	List	List	
number (type)	7	7	
numerical sum (type)	m+n	m+n	
existential type	$\exists k, n[k \leqslant n].\tau$	$ex k, n [k \le n].t$	$\exists k, n[k \le n].t$
type sort	$s_{ m ty}$	type	
number sort	$s_R$	num	
function type	$\tau_1 \rightarrow \tau_2$	t1 -> t2	$t1 \rightarrow t2$
equation	a = b	a = b	
inequation	$k \leqslant n$	k <= n	$k \leq n$
conjunction	$\varphi_1 \wedge \varphi_2$	a=b && b=a	a=b ∧ b=a

For the syntax of expressions, we discourage non-ASCII symbols. Below e,  $e_i$  stand for any expression, p,  $p_i$  stand for any pattern, x stands for any lower-case identifier and K for an upper-case identifier.

COOR TOTAL TOTAL			
named value	x	x –lower-case identifier	
numeral (expr.)	7 7		
constructor	K	K –upper-case identifier	
application	$e_1 e_2$	e1 e2	
non-br. function	$\lambda(p_1.\lambda(p_2.e))$	fun (p1,p2) p3 -> e	
branching function	$\lambda(p_1.e_1p_n.e_n)$	function p1->e1     pn->en	
pattern match	$\lambda(p_1.e_1p_n.e_n)e$	match e with p1->e1     pn->en	
postcond. function	$\lambda[K](p_1.e_1p_n.e_n)$	efunction p1->e1	
postcond. match	$\lambda[K](p_1.e_1p_n.e_n)e$	ematch e with p1->e1	
rec. definition	$\mathbf{letrec}x = e_1\mathbf{in}e_2$	let rec x = e1 in e2	
definition	$\mathbf{let}\ p = e_1  \mathbf{in}  e_2$	let p1,p2 = e1 in e2	
asserting dead br.	$\boldsymbol{F}$	assert false	
assert equal types	assert $\tau_{e_1} = \tau_{e_2}$ ; $e_3$	assert = type e1 e2; e3	
assert inequality	$\mathbf{assert}\ e_1 \leqslant e_2; e_3$	assert e1 <= e2; e3	

Toplevel expressions (corresponding to structure items in OCaml) introduce types, type and value constructors, global variables with given type (external names) or inferred type (definitions).

type constructor	newtype List : type * num
value constructor	newcons Cons : all n a. a * List(a,n)> List(a,n+1)
	$\texttt{newcons Cons} \; : \; \forall \texttt{n,a. a * List(a,n)} \; \longrightarrow \; \texttt{List(a,n+1)}$
declaration	$external \ filter : \ \forall n,a. \ List(a,n) \rightarrow \exists k [k \le n] \ . List(a,k) = "filter"$
let-declaration	external let mult : $\forall$ n,m. Num n $\rightarrow$ Num m $\rightarrow \exists$ k.Num k = "( * )"
rec. definition	let rec f =
non-rec. definition	let a, b =
definition with test	let rec f = test e1;; en
	let p1,p2 = test e1;; en

Tests list expressions of type Boolean that at runtime have to evaluate to True. Type inference is affected by the constraints generated to typecheck the expressions.

Like in OCaml, types of arguments in declarations of constructors are separated by asterisks. However, the type constructor for tuples is represented by commas, like in Haskell but unlike in OCaml.

For simplicity of theory and implementation, mutual non-nested recursion and or-patterns are not provided. For mutual recursion, nest one recursive definition inside another.

At any place between lexemes, regular comments encapsulated in (\*...\*) can occur. They are ignored during lexing. In front of all toplevel definitions and declarations, e.g. before a newtype, newcons, external, let rec or let, and in front of let rec .. in and let .. in nodes in expressions, documentation comments (\*\*...\*) can be put. Documentation comments at other places are syntax errors. Documentation comments are preserved both in generated interface files and in exported source code files.

## 4 Solver Parameters and CLI

The default settings of InvarGenT parameters should be sufficient for most cases. For example, after downloading InvarGenT source code and changing current directory to invargent, we can enter, assuming a Unix-like shell:

- \$ make main
- \$ ./invargent examples/binary\_upper\_bound.gadt

To get the inferred types printed on standard output, use the -inform option:

\$ ./invargent -inform examples/binomial\_heap\_nonrec.gadt

In some situations, hopefully unlikely for simple programs, the default parameters of the solver algorithms do not suffice. Consider this example, where we use -full\_annot to generate type annotations on function and let..in nodes in the .ml file, in addition to annotations on let rec nodes:

```
$ ./invargent -inform -full_annot examples/equal_assert.gadt
File "examples/equal_assert.gadt", line 20, characters 5-103:
No answer in type: term abduction failed
Perhaps increase the -term_abduction_timeout parameter.
Perhaps increase the -term_abduction_fail parameter.
```

The Perhaps increase suggestions are generated only when the corresponding limit has actually been exceeded. Remember however that the limits will often be exceeded for erroneus programs which should not type-check. Here the default number of steps till term abduction timeout, which is just 700 to speed up failing for actually erroneous programs, is too low. The complete output with timeout increased:

```
\ ./invargent -inform -full_annot -term_abduction_timeout 4000 \ examples/equal_assert.gadt val equal : \forall a, b. (Ty a, Ty b) \rightarrow a \rightarrow b \rightarrow Bool InvarGenT: Generated file examples/equal_assert.gadti InvarGenT: Generated file examples/equal_assert.ml InvarGenT: Command "ocamlc -c examples/equal_assert.ml" exited with code 0
```

To understand the intent of the solver parameters, we need a rough "birds-eye view" understanding of how InvarGenT works. The invariants and postconditions that we solve for are logical formulas and can be ordered by strength. Least Upper Bounds (LUBs) and Greatest Lower Bounds (GLBs) computations are traditional tools used for solving recursive equations over an ordered structure. In case of implicational constraints that are generated for type inference with GADTs, constraint abduction is a form of LUB computation. *Disjunction elimination* is our term for computing the GLB wrt. strength for formulas that are conjunctions of atoms. We want the invariants of recursive definitions – i.e. the types of recursive functions and formulas constraining their type variables – to be as weak as possible, to make the use of the corresponding definitions as easy as possible. The weaker the invariant, the more general the type of definition. Therefore the use of LUB, constraint abduction. For postconditions – i.e. the existential types of results computed by efunction expressions and formulas constraining their type variables – we want the strongest possible solutions, because stronger postcondition provides more information at use sites of a definition. Therefore we use GLB, disjunction elimination, but only if existential types have been introduced by efunction or ematch.

Below we discuss all of the InvarGenT options.

- -inform. Print type schemes of toplevel definitions as they are inferred.
- -no\_sig. Do not generate the .gadti file.
- -no\_ml. Do not generate the .ml file.

- -no\_verif. Do not call ocamlc -c on the generated .ml file.
- -num\_is. The exported type for which Num is an alias (default int). If -num\_is bar for bar different than int, numerals are exported as integers passed to a bar\_of\_int function. The variant -num\_is\_mod exports numerals by passing to a Bar.of\_int function.
- -full\_annot. Annotate the function and let..in nodes in generated OCaml code. This increases the burden on inference a bit because the variables associated with the nodes cannot be eliminated from the constraint during initial simplification.
- -keep\_assert\_false. Keep assert false clauses in exported code. When faced with multiple maximally general types of a function, we sometimes want to prevent some interpretations by asserting that a combination of arguments is not possible. These arguments will not be compatible with the type inferred, causing exported code to fail to typecheck. Sometimes we indicate unreachable cases just for documentation. If the type is tight this will cause exported code to fail to typecheck too. This option keeps pattern matching branches with assert false in their bodies in exported code nevertheless.
- -term\_abduction\_timeout. Limit on term simple abduction steps (default 700). Simple abduction works with a single implication branch, which roughly corresponds to a single branch an execution path of the program.
- -term\_abduction\_fail. Limit on backtracking steps in term joint abduction (default 4). Joint abduction combines results for all branches of the constraints.
- -no\_alien\_prem. Do not include alien (e.g. numerical) premise information in term abduction.
- -early\_num\_abduction. Include recursive branches in numerical abduction from the start. By default, in the second iteration of solving constraints, which is the first iteration that numerical abduction is performed, we only pass non-recursive branches to numerical abduction. This makes it faster but less likely to find the correct solution.
- -early\_postcond\_abd. Include postconditions from recursive calls in abduction from the start. We do not derive requirements put on postconditions by recursive calls on first iteration. The requirements may turn smaller after some derived invariants are included in the premises. This option turns off the special treatment of postconditions on first iteration.
- -num\_abduction\_rotations. Numerical abduction: coefficients from  $\pm 1/N$  to  $\pm N$  (default 3). Numerical abduction answers are built, roughly speaking, by adding premise equations of a branch with conclusion of a branch to get an equation or inequality that does not conflict with other branches, but is equivalent to the conclusion equation/inequality. This parameter decides what range of coefficients is tried. If the highest coefficient in correct answer is greater, abduction might fail.
- -num\_prune\_at. Keep less than N elements in abduction sums (default 6). By elements here we mean distinct variables lack of constant multipliers in concrete syntax of types is just a syntactic shortcoming.
- -num\_abduction\_timeout. Limit on numerical simple abduction steps (default 1000).
- -num\_abduction\_fail. Limit on backtracking steps in numerical joint abduction (default 10).
- -disjelim\_rotations. Disjunction elimination: check coefficients from 1/N (default 3). Numerical disjunction elimination is performed by approximately finding the convex hull of the polytopes corresponding to disjuncts. A step in an exact algorithm involves rotating a side along a ridge an intersection with another side until the side touches yet another side. We approximate by trying out a couple of rotations: convex combinations of the inequalities defining the sides. This parameter decides how many rotations to try.
- -iterations\_timeout. Limit on main algorithm iterations (default 6). Answers found in an iteration of the main algorithm are propagated to use sites in the next iteration. However, for about four initial iterations, each iteration turns on additional processing which makes better sense with the results from the previous iteration propagated. At least three iterations will always be performed.

- -richer\_answers. Keep some equations in term abduction answers even if redundant. Try keeping an initial guess out of a list of candidate equations before trying to drop the equation from consideration. We use fully maximal abduction for single branches, which cannot find answers not implied by premise and conclusion of a branch. But we seed it with partial answer to branches considered so far. Sometimes an atom is required to solve another branch although it is redundant in given branch. -richer\_answers does not increase computational cost but sometimes leads to answers that are not most general. This can always be fixed by adding a test clause to the definition which uses a type conflicting with the too specific type.
- -passing\_ineq\_trs. Include inequalities in conclusion when solving numerical abduction. This setting leads to more inequalities being tried for addition in numeric abduction answer.
- -not\_annotating\_fun. Do not keep information for annotating function nodes. This may allow eliminating more variables during initial constraint simplification.
- -annotating\_letin. Keep information for annotating let..in nodes. Will be set automatically anyway when -full\_annot is passed.
- -let\_in\_fallback. Annotate let..in nodes in fallback mode of .ml generation. When verifying the resulting .ml file fails, a retry is made with function nodes annotated. This option additionally annotates let..in nodes with types in the regenerated .ml file.

Let us see another example where parameters allowing the solver do more work are needed:

 $\$  ./invargent -inform -num\_abduction\_rotations 4 -num\_abduction\_timeout 2000 \ examples/flatten\_quadrs.gadt val flatten\_quadrs :  $\forall n$ , a. List ((a, a, a, a), n)  $\rightarrow$  List (a, n + n + n + n) InvarGenT: Generated file examples/flatten\_quadrs.gadti InvarGenT: Generated file examples/flatten\_quadrs.ml InvarGenT: Command "ocamlc -c examples/flatten\_quadrs.ml" exited with code 0

Based on user feedback, we will likely increase the default values of parameters in a future version.