InvarGenT – Manual

by Łukasz Stafiniak

Institute of Computer Science
University of Wrocław

Abstract

InvarGenT is a proof-of-concept system for invariant generation by full type inference with Guarded Algebraic Data Types and existential types encoded as automatically generated GADTs. This user manual discusses motivating examples, briefly presents the syntax of the InvarGenT language, and describes the parameters of the inference process that can be passed to the InvarGenT executable.

1 Introduction

Type systems are an established natural deduction-style means to reason about programs. Dependent types can represent arbitrarily complex properties as they use the same language for both types and programs, the type of value returned by a function can itself be a function of the argument. Generalized Algebraic Data Types bring some of that expressivity to type systems that deal with data-types. Type systems with GADTs introduce the ability to reason about return type by case analysis of the input value, while keeping the benefits of a simple semantics of types, for example deciding equality can be very simple. Existential types hide some information conveyed in a type, usually when that information cannot be reconstructed in the type system. A part of the type will often fail to be expressible in the simple language of types, when the dependence on the input to the program is complex. GADTs express existential types by using local type variables for the hidden parts of the type encapsulated in a GADT.

The InvarGenT type system for GADTs differs from more pragmatic approaches in mainstream functional languages in that we do not require any type annotations on expressions, even on recursive functions. The implementation also includes linear equations and inequalities over rational numbers in the language of types, with the possibility to introduce more domains in the future.

2 Tutorial

The concrete syntax of InvarGenT is similar to that of OCaml. However, it does not currently cover records, the module system, objects, and polymorphic variant types. It supports higher-order functions, algebraic data-types including built-in tuple types, and linear pattern matching. It supports conjunctive patterns using the `as` keyword, but it does not support disjunctive patterns. It does not currently support guarded patterns, i.e. no support for the `when` keyword of OCaml.

The sort of a type variable is identified by the first letter of the variable. `a`,`b`,`c`,`r`,`s`,`t`,`a1`,... are in the sort of terms called `type`, i.e. "types proper". `i`,`j`,`k`,`l`,`m`,`n`,`i1`,... are in the sort of linear arithmetics over rational numbers called `num`. Remaining letters are reserved for sorts that may be added in the future. Value constructors (like in OCaml) and type constructors (unlike in OCaml) have the same syntax: capitalized name followed by a tuple of arguments. They are introduced by `newtype` and `newcons` respectively. The `newtype` declaration might be misleading in that it only lists the sorts of the arguments of the type, the resulting sort is always `type`. Values assumed into the environment are introduced by `external`. There is a built-in type corresponding to declaration `newtype Num : num` and definitions of numeric constants `newcons 0 : Num 0 newcons 1 : Num 1`... The programmer can use `external` declarations to give the semantics of choice to the `Num` data-type.

In examples here we use Unicode characters. For ASCII equivalents, take a quick look at the tables in the following section.

We start simple, with a function that can compute a value from a representation of an expression – a ready to use value whether it be `Int` or `Bool`. Prior to the introduction of GADT types, we could only implement a function `eval : ∀a. Term a → Value` where, using OCaml syntax, `type value = Int of int | Bool of bool`.

```
newtype Term : type
external plus : Int → Int → Int
external is_zero : Int → Bool
external if_then : ∀a. Bool → a → a → a
newcons Lit : Int ⟶ Term Int
newcons Plus : Term Int * Term Int ⟶ Term Int
newcons IsZero : Term Int ⟶ Term Bool
newcons If : ∀a. Term Bool * Term a * Term a ⟶ Term a

let rec eval = function
  | Lit i -> i
  | IsZero x -> is_zero (eval x)
  | Plus (x, y) -> plus (eval x) (eval y)
  | If (b, t, e) -> if_then (eval b) (eval t) (eval e)
```

Let us look at the corresponding generated, also called *exported*, OCaml source code:

```
type _ term =
  | Lit : int -> int term
  | Plus : int term * int term -> int term
  | IsZero : int term -> bool term
  | If : (*∀'a.*)bool term * 'a term * 'a term -> 'a term

external plus : (int -> int -> int) = "plus"
external is_zero : (int -> bool) = "is_zero"
external if_then : (bool -> 'a -> 'a -> 'a) = "if_then"
let rec eval : type a . (a term -> a) =
  (function Lit i -> i | IsZero x -> is_zero (eval x)
    | Plus (x, y) -> plus (eval x) (eval y)
    | If (b, t, e) -> if_then (eval b) (eval t) (eval e))
```

The `Int`, `Num` and `Bool` types are built-in. `Int` and `Bool` follow the general scheme of exporting a datatype constructor with the same name, only lower-case. However, numerals `0`, `1`, ... are always type-checked as `Num 0`, `Num 1`... `Num` can also be exported as a type other than `int`, and then numerals are exported via an injection function (ending with) `of_int`.

The type inferred is `eval : ∀a. Term a→a`. GADTs make it possible to reveal that `IsZero x` is a `Term Bool` and therefore the result of `eval` should in its case be `Bool`, `Plus (x, y)` is a `Term Num` and the result of `eval` should in its case be `Num`, etc. InvarGenT does not provide an `if`...`then`...`else`... syntax to stress that the branching is relevant to generating postconditions, but it does export `match`/`ematch` ... `with True ->` ... `| False ->` ... using `if` expressions.

`equal` is a function comparing values provided representation of their types:

```
newtype Ty : type
newtype Int
newtype List : type
newcons Zero : Int
newcons Nil : ∀a. List a
newcons TInt : Ty Int
newcons TPair : ∀a, b. Ty a * Ty b ⟶ Ty (a, b)
newcons TList : ∀a. Ty a ⟶ Ty (List a)
newtype Boolean
```

```
newcons True : Boolean
newcons False : Boolean
external eq_int : Int → Int → Bool
external b_and : Bool → Bool → Bool
external b_not : Bool → Bool
external forall2 : ∀a, b. (a → b → Bool) → List a → List b → Bool

let rec equal = function
  | TInt, TInt -> fun x y -> eq_int x y
  | TPair (t1, t2), TPair (u1, u2) ->
    (fun (x1, x2) (y1, y2) ->
        b_and (equal (t1, u1) x1 y1)
              (equal (t2, u2) x2 y2))
  | TList t, TList u -> forall2 (equal (t, u))
  | _ -> fun _ _ -> False
```

InvarGenT returns an unexpected type: `equal`:∀`a,b.(Ty a, Ty b)`→`a`→`a`→`Bool`, one of four maximally general types of `equal1`. The other maximally general "wrong" types are ∀`a,b.(Ty a, Ty b)`→`b`→`b`→`Bool` and ∀`a,b.(Ty a, Ty b)`→`b`→`a`→`Bool`. This illustrates that unrestricted type systems with GADTs lack principal typing property.

InvarGenT commits to a type of a toplevel definition before proceeding to the next one, so sometimes we need to provide more information in the program. Besides type annotations, there are two means to enrich the generated constraints: `assert false` syntax for providing negative constraints, and `test` syntax for including constraints of use cases with constraint of a toplevel definition. To ensure only one maximally general type for `equal`, we use both. We add the lines:

```
  | TInt, TList l -> (function Nil -> assert false)
  | TList l, TInt -> (fun _ -> function Nil -> assert false)
test b_not (equal (TInt, TList TInt) Zero Nil)
```

Actually, InvarGenT returns the expected type `equal`:∀`a,b.(Ty a, Ty b)`→`a`→`b`→`Bool` when either the two `assert false` clauses or the `test` clause is added. When using `test`, the program should declare the type `Boolean` and constant `True`. In a future version, this will be replaced by a built-in type `Bool` with constants `True` and `False`, exported into OCaml as type `bool` with constants `true` and `false`.

Now we demonstrate numerical invariants:

```
newtype Binary : num
newtype Carry : num
newcons Zero : Binary 0
newcons PZero : ∀n[0≤n]. Binary(n) ⟶ Binary(n+n)
newcons POne : ∀n[0≤n]. Binary(n) ⟶ Binary(n+n+1)
newcons CZero : Carry 0
newcons COne : Carry 1

let rec plus =
  function CZero ->
    (function Zero -> (fun b -> b)
      | PZero a1 as a ->
        (function Zero -> a
          | PZero b1 -> PZero (plus CZero a1 b1)
          | POne b1 -> POne (plus CZero a1 b1))
      | POne a1 as a ->
        (function Zero -> a
          | PZero b1 -> POne (plus CZero a1 b1)
```

```
          | POne b1 -> PZero (plus COne a1 b1)))
    | COne ->
    (function Zero ->
        (function Zero -> POne(Zero)
          | PZero b1 -> POne b1
          | POne b1 -> PZero (plus COne Zero b1))
      | PZero a1 as a ->
        (function Zero -> POne a1
          | PZero b1 -> POne (plus CZero a1 b1)
          | POne b1 -> PZero (plus COne a1 b1))
      | POne a1 as a ->
        (function Zero -> PZero (plus COne a1 Zero)
          | PZero b1 -> PZero (plus COne a1 b1)
          | POne b1 -> POne (plus COne a1 b1)))
```

We get `plus`: ∀`i,j,k.Carry i`→`Binary j`→`Binary k`→`Binary (i+j+k)`.

We can introduce existential types directly in type declarations. To have an existential type inferred, we have to use `efunction` or `ematch` expressions, which differ from `function` and `match` only in that the (return) type is an existential type. To use a value of an existential type, we have to bind it with a `let..in` expression. Otherwise, the existential type will not be unpacked. An existential type will be automatically unpacked before being "repackaged" as another existential type.

```
newtype Room
newtype Yard
newtype Village
newtype Castle : type
newtype Place : type
newcons Room : Room ⟶ Castle Room
newcons Yard : Yard ⟶ Castle Yard
newcons CastleRoom : Room ⟶ Place Room
newcons CastleYard : Yard ⟶ Place Yard
newcons Village : Village ⟶ Place Village

external wander : ∀a. Place a → ∃b. Place b

let rec find_castle = efunction
  | CastleRoom x -> Room x
  | CastleYard x -> Yard x
  | Village _ as x ->
    let y = wander x in
    find_castle y
```

We get `find_castle`: ∀`a. Place a`→ ∃`b. Castle b`.

A more practical existential type example:

```
newtype Bool
newcons True : Bool
newcons False : Bool
newtype List : type * num
newcons LNil : ∀a. List(a, 0)
newcons LCons : ∀n,a[0≤n]. a * List(a, n) ⟶ List(a, n+1)

let rec filter = fun f ->
  efunction LNil -> LNil
    | LCons (x, xs) ->
```

```
      ematch f x with
        | True ->
          let ys = filter f xs in
          LCons (x, ys)
        | False ->
          filter f xs
```

We get `filter`:∀a,i.(a→Bool)→List (a, i)→ ∃j[j≤i].List (a, j). Note that we need to use both `efunction` and `ematch` above, since every use of `function` or `match` will force the types of its branches to be equal. In particular, for lists with length the resulting length would have to be the same in each branch. If the constraint cannot be met, as for `filter` with either `function` or `match`, the code will not type-check.

A more complex example that computes bitwise *or* – `ub` stands for "upper bound":

```
newtype Binary : num
newcons Zero : Binary 0
newcons PZero : ∀n [0≤n]. Binary(n) ⟶ Binary(n+n)
newcons POne : ∀n [0≤n]. Binary(n) ⟶ Binary(n+n+1)

let rec ub = efunction
  | Zero ->
      (efunction Zero -> Zero
        | PZero b1 as b -> b
        | POne b1 as b -> b)
  | PZero a1 as a ->
      (efunction Zero -> a
        | PZero b1 ->
          let r = ub a1 b1 in
          PZero r
        | POne b1 ->
          let r = ub a1 b1 in
          POne r)
  | POne a1 as a ->
      (efunction Zero -> a
        | PZero b1 ->
          let r = ub a1 b1 in
          POne r
        | POne b1 ->
          let r = ub a1 b1 in
          POne r)
```

Type: `ub`:∀k,n.Binary k→Binary n→∃:i[n≤i∧k≤i∧i≤k+n∧0≤n∧0≤k].Binary i.

Why cannot we shorten the above code by converting the initial cases to `Zero -> (efunction b -> b)`? Without pattern matching, we do not make the contribution of `Binary n` available. Knowing n=i and not knowing 0≤n, for the case k=0, we get: `ub`:∀k,n.Binary k→Binary n→ ∃:i[n≤i∧i≤k+n∧0≤k].Binary i. n≤i follows from n=i, i≤k+n follows from n=i and 0≤k, but k≤i cannot be inferred from k=0 and n=i without knowing that 0≤n.

Besides displaying types of toplevel definitions, InvarGenT can also export an OCaml source file with all the required GADT definitions and type annotations.

3 Syntax

Below we present, using examples, the syntax of InvarGenT: the mathematical notation, the concrete syntax in ASCII and the concrete syntax using Unicode.

type variable: types	$\alpha, \beta, \gamma, \tau$	`a,b,c,r,s,t,a1,...`	
type variable: nums	k, m, n	`i,j,k,l,m,n,i1,...`	
type constructor	List	`List`	
number (type)	7	`7`	
numerical sum (type)	$m+n$	`m+n`	
existential type	$\exists k, n[k \leqslant n].\tau$	`ex k n [k<=n].t`	`∃k,n[k≤n].t`
type sort	s_{ty}	`type`	
number sort	s_R	`num`	
function type	$\tau_1 \to \tau_2$	`t1 -> t2`	`t1 → t2`
equation	$a \doteq b$	`a = b`	
inequation	$k \leqslant n$	`k <= n`	`k ≤ n`
conjunction	$\varphi_1 \wedge \varphi_2$	`a=b && b=a`	`a=b ∧ b=a`

For the syntax of expressions, we discourage non-ASCII symbols. Below e, e_i stand for any expression, p, p_i stand for any pattern, x stands for any lower-case identifier and K for an upper-case identifier.

named value	x	`x` –lower-case identifier
numeral (expr.)	7	`7`
constructor	K	`K` –upper-case identifier
application	$e_1\, e_2$	`e1 e2`
non-br. function	$\lambda(p_1.\lambda(p_2.e))$	`fun (p1,p2) p3 -> e`
branching function	$\lambda(p_1.e_1 ... p_n.e_n)$	`function p1->e1 | ... | pn->en`
pattern match	$\lambda(p_1.e_1 ... p_n.e_n)\, e$	`match e with p1->e1 | ... | pn->en`
postcond. function	$\lambda[K](p_1.e_1 ... p_n.e_n)$	`efunction p1->e1 | ...`
postcond. match	$\lambda[K](p_1.e_1 ... p_n.e_n)\, e$	`ematch e with p1->e1 | ...`
rec. definition	$\mathbf{letrec}\, x = e_1\, \mathbf{in}\, e_2$	`let rec x = e1 in e2`
definition	$\mathbf{let}\, p = e_1\, \mathbf{in}\, e_2$	`let p1,p2 = e1 in e2`
asserting dead br.	$\boldsymbol{F}$	`assert false`
assert equal types	$\mathbf{assert}\, \tau_{e_1} \doteq \tau_{e_2}; e_3$	`assert = type e1 e2; e3`
assert inequality	$\mathbf{assert}\, e_1 \leqslant e_2; e_3$	`assert e1 <= e2; e3`

Toplevel expressions (corresponding to structure items in OCaml) introduce types, type and value constructors, global variables with given type (external names) or inferred type (definitions).

type constructor	`newtype List : type * num`
value constructor	`newcons Cons : all n a. a * List(a,n) --> List(a,n+1)`
	`newcons Cons : ∀n,a. a * List(a,n) ⟶ List(a,n+1)`
declaration	`external filter : ∀n,a. List(a,n)→∃k[k<=n].List(a,k)`
rec. definition	`let rec f =...`
non-rec. definition	`let a, b =...`
definition with test	`let rec f =... test e1; ...; en`
	`let p1,p2 =... test e1; ...; en`

Tests list expressions of type `Boolean` that at runtime have to evaluate to `True`. Type inference is affected by the constraints generated to typecheck the expressions.

Like in OCaml, types of arguments in declarations of constructors are separated by asterisks. However, the type constructor for tuples is represented by commas, like in Haskell but unlike in OCaml.

For simplicity of theory and implementation, mutual non-nested recursion and or-patterns are not provided. For mutual recursion, nest one recursive definition inside another.

4 Solver Parameters and CLI

The default settings of InvarGenT parameters should be sufficient for most cases. For example, after downloading InvarGenT source code and changing current directory to `invargent`, we can enter, assuming a Unix-like shell:

```
$ make main
$ ./invargent examples/binary_upper_bound.gadt
```

To get the inferred types printed on standard output, use the `-inform` option:

```
$ ./invargent -inform examples/binomial_heap_nonrec.gadt
```

In some situations, hopefully unlikely for simple programs, the default parameters of the solver algorithms do not suffice. Consider this example, where we use `-full_annot` to generate type annotations on `function` and `let..in` nodes in the `.ml` file, in addition to annotations on `let rec` nodes:

```
$ ./invargent -inform -full_annot examples/equal_assert.gadt
File "examples/equal_assert.gadt", line 20, characters 5-103:
No answer in type: term abduction failed

Perhaps increase the -term_abduction_timeout parameter.
Perhaps increase the -term_abduction_fail parameter.
```

The `Perhaps increase` suggestions are generated only when the corresponding limit has actually been exceeded. Remember however that the limits will often be exceeded for erroneus programs which should not type-check. Here the default number of steps till term abduction timeout, which is just `700` to speed up failing for actually erroneous programs, is too low. The complete output with timeout increased:

```
$ ./invargent -inform -full_annot -term_abduction_timeout 4000 \
examples/equal_assert.gadt
val equal : ∀a, b. (Ty a, Ty b) → a → b → Boolean
InvarGenT: Generated file examples/equal_assert.gadti
InvarGenT: Generated file examples/equal_assert.ml
InvarGenT: Command "ocamlc -c examples/equal_assert.ml" exited with code 0
```

To understand the intent of the solver parameters, we need a rough "birds-eye view" understanding of how InvarGenT works. The invariants and postconditions that we solve for are logical formulas and can be ordered by strength. Least Upper Bounds (LUBs) and Greatest Lower Bounds (GLBs) computations are traditional tools used for solving recursive equations over an ordered structure. In case of implicational constraints that are generated for type inference with GADTs, constraint abduction is a form of LUB computation. *Disjunction elimination* is our term for computing the GLB wrt. strength for formulas that are conjunctions of atoms. We want the invariants of recursive definitions – i.e. the types of recursive functions and formulas constraining their type variables – to be as weak as possible, to make the use of the corresponding definitions as easy as possible. The weaker the invariant, the more general the type of definition. Therefore the use of LUB, constraint abduction. For postconditions – i.e. the existential types of results computed by `efunction` expressions and formulas constraining their type variables – we want the strongest possible solutions, because stronger postcondition provides more information at use sites of a definition. Therefore we use LUB, disjunction elimination, but only if existential types have been introduced by `efunction` or `ematch`.

Below we discuss all of the InvarGenT options.

`-inform`. Print type schemes of toplevel definitions as they are inferred.

`-no_sig`. Do not generate the `.gadti` file.

`-no_ml`. Do not generate the `.ml` file.

`-no_verif`. Do not call `ocamlc -c` on the generated `.ml` file.

`-num_is`. The exported type for which `Num` is an alias (default `int`). If `-num_is bar` for `bar` different than `int`, numerals are exported as integers passed to a `bar_of_int` function. The variant `-num_is_mod` exports numerals by passing to a `Bar.of_int` function.

- `-full_annot.` Annotate the `function` and `let..in` nodes in generated OCaml code. This increases the burden on inference a bit because the variables associated with the nodes cannot be eliminated from the constraint during initial simplification.
- `-term_abduction_timeout.` Limit on term simple abduction steps (default 700). Simple abduction works with a single implication branch, which roughly corresponds to a single branch – an execution path – of the program.
- `-term_abduction_fail.` Limit on backtracking steps in term joint abduction (default 4). Joint abduction combines results for all branches of the constraints.
- `-no_alien_prem.` Do not include alien (e.g. numerical) premise information in term abduction.
- `-early_num_abduction.` Include recursive branches in numerical abduction from the start. By default, in the second iteration of solving constraints, which is the first iteration that numerical abduction is performed, we only pass non-recursive branches to numerical abduction. This makes it faster but less likely to find the correct solution.
- `-num_abduction_rotations.` Numerical abduction: coefficients from $\pm 1/N$ to $\pm N$ (default 3). Numerical abduction answers are built, roughly speaking, by adding premise equations of a branch with conclusion of a branch to get an equation or inequality that does not conflict with other branches, but is equivalent to the conclusion equation/inequality. This parameter decides what range of coefficients is tried. If the highest coefficient in correct answer is greater, abduction might fail.
- `-num_prune_at.` Keep less than N elements in abduction sums (default 6). By elements here we mean distinct variables – lack of constant multipliers in concrete syntax of types is just a syntactic shortcoming.
- `-num_abduction_timeout.` Limit on numerical simple abduction steps (default 1000).
- `-num_abduction_fail.` Limit on backtracking steps in numerical joint abduction (default 10).
- `-disjelim_rotations.` Disjunction elimination: check coefficients from $1/N$ (default 3). Numerical disjunction elimination is performed by approximately finding the convex hull of the polytopes corresponding to disjuncts. A step in an exact algorithm involves rotating a side along a ridge – an intersection with another side – until the side touches yet another side. We approximate by trying out a couple of rotations: convex combinations of the inequalities defining the sides. This parameter decides how many rotations to try.
- `-passing_ineq_trs.` Include inequalities in conclusion when solving numerical abduction. This setting leads to more inequalities being tried for addition in numeric abduction answer.
- `-not_annotating_fun.` Do not keep information for annotating `function` nodes. This may allow eliminating more variables during initial constraint simplification.
- `-annotating_letin.` Keep information for annotating `let..in` nodes. Will be set automatically anyway when `-full_annot` is passed.
- `-let_in_fallback.` Annotate `let..in` nodes in fallback mode of `.ml` generation. When verifying the resulting `.ml` file fails, a retry is made with `function` nodes annotated. This option additionally annotates `let..in` nodes with types in the regenerated `.ml` file.

Let us see another example where parameters allowing the solver do more work are needed:

```
$ ./invargent -inform -num_abduction_rotations 4 -num_abduction_timeout 2000 \
examples/flatten_quadrs.gadt
val flatten_quadrs :
  ∀n, a. List ((a, a, a, a), n) → List (a, n + n + n + n)
InvarGenT: Generated file examples/flatten_quadrs.gadti
InvarGenT: Generated file examples/flatten_quadrs.ml
InvarGenT: Command "ocamlc -c examples/flatten_quadrs.ml" exited with code 0
```

Based on user feedback, we will likely increase the default values of parameters in a future version.