TOPIC 2: WHOLE NUMBERS

Introduction

A **place value** is a position name of a digit in a numeral.

A digit

A symbol used in writing numbers in their decimal representation. In decimal notation (base ten), the digits used are; 0,1,2,3,4,5,6,7,8 and 9. In binary system (base two), there are just two digits used; 0 and 1.

Numeral

A symbol used to donate a number. The Roman numerals I,V,X,L,C,D and M represent 1,5,10,50,100,500 and 1000 in the Hindu Arabic number system. The Arabic numerals 0,1,2,3,4,5,6,7,8 and 9 are used in the Hindu Arabic number system to give numbers in the form generally familiar today.

Formation of numerals

Examples

1.Using digits 4, 7 and 2, form the

i) the least 3-digit numeral

247

ii) largest 3-digit numeral

742

2. Write down all the possible 3-digit numerals that can be formed from the following; 3,6,1

361, 316, 631, 613, 136, 163

3. Nsontwa wrote digits on the cards as shown below.

6	0	5	3

a) Form the least four digit numeral

3056

b) Form the greatest four digit numeral.

6530

c) Work out the difference between the greatest and the least 3-digit numeral that can be formed using the above digits.

$$\begin{array}{r} 653 \\ -\ 305 \\ \hline 348 \end{array}$$

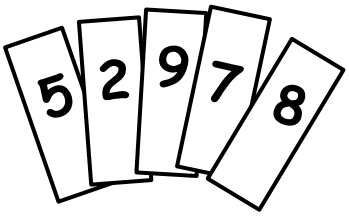
Exercise 2:1

1. Given the digits; 5,1 and 9.
   a) Form the greatest and the least 3-digit numeral using the above digits
   b) Work out the sum of the largest and the least 3-digit numerals formed above

2. Given the digits 7, 5, 9, 1 and 3, form the smallest and the largest number using the above digits.

3. What is the smallest 5-digit numeral that can be formed from the digits; 6, 8, 0, 4 and 2

4. Given the digits on cards:



(a) Form the smallest and the largest numbers.
(b) Find the sum of the numbers you have formed above.
(c) Find the difference of the smallest and the largest numbers.

5. A P.7 pupil forms three digit numerals using the digits; 3, 9 and 2. What is the probability that the number to be formed is even?

6. Work out the difference between the largest and the smallest 3-digit numeral that can be formed using 0, 9 and 7

Place values of digits in whole numbers up to 99,999,999

A place value is the position name of a digit in the given numeral.
To find place values of digits, draw a table of place values and place in the digits starting from the right.

Example

Given the number 96,871,043, write the place value of each digit.

MILLIONS			THOUSANDS			UNITS		
H	T	O	H	T	O	H	T	O
	9	6	8	7	1	0	4	3

The place value of 9 is ten millions.
The place value of 6 is millions.
The place value of 8 is hundred thousands.
The place value of 7 is ten thousands.
The place value of 1 is thousands.
The place value of 0 is hundreds.
The place value of 4 is tens.
The place value of 3 is ones.

Exercise 2:2

1. Give the place value of each digit in the numerals.

i) 248
ii) 3,926
iii) 16
iv) 84,629
v) 4,896,217
vi) 76,429,830
vii) 542,869
viii) 2,517,960
ix) 6,538
x) 217,043
xi) 3,087,921
xii) 90,841,236

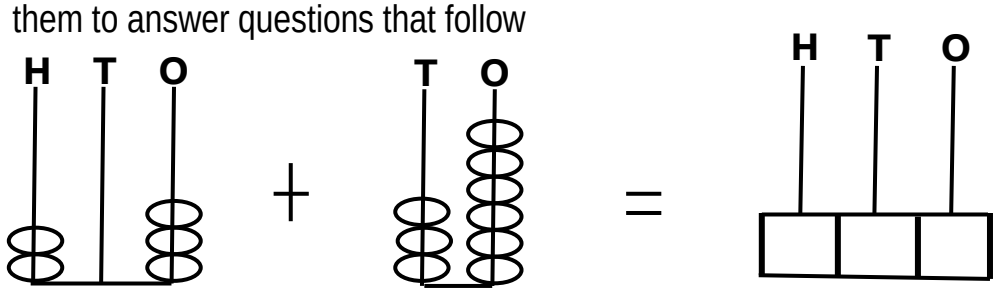
2. What is the place value of the underlined digit?
   i) 6<u>8</u>2 ii) <u>4</u>684 iii) 68<u>9</u> iv) 4<u>3</u>24692

3. Underline the digit in the hundreds place in each of the given numerals.
   i) 132 iii) 486 vi) 6,426
   ii) 3,048 iv) 3,648,932 v) 42,689

4. Work out the difference between the place value of 8 in 68 and the place value of 3 in 23842.

5. Represent 3048 on the abacus

6. The diagram below represents two numbers on the abacus. Study the diagrams and use them to answer questions that follow



a) Write the numbers represented in the addition.
b) Work out the addition and represent your answer on the third abacus.

Values of digits in whole numbers up to 99,999,999

The value of a digit is the product of a digit and its place value in a numeral.

Value = Digit x Place value

Example 1

1. Work out the value of 7 in 82,762

TTh	Th	H	T	O
8	2	<u>7</u>	6	2

7 x 100 = 700

Example 2

2. Work out the sum of the value of 8 and the value of 5 in 7,028,453

M	HTh	TTh	Th	H	T	O
7	0	2	<u>8</u>	4	<u>5</u>	3

5x10 = 50

8x1,000 = 8,000

<u>Sum</u>

8,000 + 50 = 8,050

Exercise 2:3

1. Work out the value of the underlined digits.

i) 6<u>3</u>	iv) 643,6<u>8</u>9	vii) 6,2<u>8</u>9,021
ii) 32<u>4</u>	v) 436,892	viii) <u>2</u>7,896,540
iii) <u>8</u>,624	vi) <u>3</u>,168	ix) 5<u>0</u>,971,234

2. Find the value of 9 in the number
   i) 98
   ii) 8963
3. Find the value of the digit in the thousands place in the number; 850,634
4. Work out the sum of the value of 9 and the value of 7 in 87,692
5. Find the product of the value of 6 and the place value of 9 in 193,462.
6. Work out the sum of the value of 8 in 6,804 and the value of 6 in 7,692
7. The value of 6 in a number is 60,000. What is the place value of 6 in the same number?
8. The difference between the value of 6 and the value of 4 in a numeral is 56000. The place value of 6 is ten thousands. Find the place value of 4 in the numeral.
9. The quotient of the value of 4 and the value of 5 in a numeral is 80. The place value of 5 in the same numeral is hundreds. Find the place value of 4.
10. The difference between the place value of 5 and the value of 7 in a certain number is 9930. If the place value of 7 is tens, work out the place value of 5.
11. The product of the value of 7 and the digit in thousands place of the same number is 630,000. The place value of 7 is tens. Find the unknown digit.
12. The difference between the value of the digit in the ten thousands place and the value of 6 is 89400. The place value of 6 is hundreds. Find the unknown digit.

Writing whole numbers up to 99,999,999 in words

Example 1

1. Write 690,014 in words

Thousands	Units
6 9 0	0 14

Six hundred ninety thousand, fourteen.

Example 2

2. Write 93,602,919 in words

Millions	Thousands	Units
9 3	6 0 2	9 19

Ninety three million, six hundred two thousand, nine hundred nineteen.

Exercise 2:4

1. Write the following numbers in words,
   i) 842 ii) 693 iii) 9048 iv) 320813 v) 86432 vi) 98014

2. Express in words

i) 83,307	iii) 6,404,891	v) 99,999,999	vii) 42,000,019
ii)170,013	iv) 643,279	vi) 4,098,045	viii) 2,049,093

Writing whole numbers up to 99,999,999 in figures

Examples

1. Write 'Ninety thousand, thirty six' in figures.

Ninety thousand - 90,000
thirty six - + 36
90,036

2. Write in figures: Three hundred two thousand, eight hundred ninety five.

Three hundred two thousand - 302,000
eight hundred - 800
ninety five - + 95
302,895

Exercise 2:5

1. Write the following in figures;
   a) Ninety five.
   b) Three hundred forty two.
   c) Fourteen thousand, nine.
   d) One thousand, thirteen.
   e) Forty two thousand, eight.
   f) Eighty thousand, ten
   g) Eight hundred six thousand, four hundred eight.
   h) Two hundred twenty thousand, twenty four.
   i) Five hundred one thousand, twelve
   j) Eight hundred six million, four hundred two thousand, seventeen.

2. Write in figures.
   i) Half a million.
   ii) Three quarters a million.
   iii) Seventy five million, two hundred eight thousand, twelve
   iv) A quarter a million.
   v) Seven million, two thousand, twenty three.

Expanding whole numbers using place values, values and indices

Examples

Expand 20198 using

i) place values

TTh	Th	H	T	O
2	0	1	9	8

(2x10,000) + (1x100) + (9x10) +(8x1)

ii) values

TTh	Th	H	T	O
2	0	1	9	8

(2x10,000) + (1x100) + (9x10) +(8x1)

20000 + 100 + 90 + 8

iii) indices / powers of ten / exponents / standard decimal notation

TTh	Th	H	T	O
2	0	1	9	8

(2x10,000) + (1x100) + (9x10) +(6x1)

(2x10x10x10x10) + (1x10x10) + (9 x 10) + (6x1)

$(2x10^4) + (1x10^2) + (9x10^1) + (6x10^0)$

Exercise 2:6

1. Expand the following using values

i) 29
ii) 3,002
iii) 2,023
iv) 645,607
v) 9,080,523
vi) 32,579
vii) 3,093,418
viii) 8,176,894
ix) 65,809,231

2. Expand the following using place values.

i) 827
ii) 8,625
iii)30,215
iv) 3,248,207
v) 408,978
vi) 72,307,973
vii) 709,812
viii) 1,246,379
ix) 56,000,073

3. Expand the following using powers of ten.

i) 63,023
ii) 405
iii) 86,793
iv) 340,208
v) 318,965
vi) 95,021,346

Finding the expanded number

Example 1

What number has been
expanded to give 20000+500+9?

20000
500
+ 9
20509

Example 2

Write in short; (7x10,000)+(4x1000)+(9x10)

(7x10,000)+(4x1,000)+(9x10)
70,000 + 4,000 + 90
70,000
4,000
+ 90
74,090

Example 3

Find the number whose expanded form is $(2x10^4)+(8x10^3)+(3x10^1)+(9x10^0)$

$(2x10^4)+(8x10^3)+(3x10^1)+(9x10^0)$
(2x10x10x10x10) + (8x10x10x10) + (3x10) + (9x1)
(2x10000)+(8x1000)+(3x10)+(9x1)
20000 + 8000 + 30 + 9

20000
8000
30
+ 9
28039

Exercise 2:7

1. What number has been expanded below
   i) (2x100)+(8x10)+(6x1)
   ii) (3x1,000)+(2x10)+(3x1)
   iii) (8x100,000)+(5x100)+(2x10)
   iv) (6x100)+(4x1,000)+(7x1)
   v) (9x10,000,000) + (5x10,000) + (6x1)
   vi) (4x100,000) + (8x10,000) + (3x100)

2. Write in short;
   i) 800 + 1
   ii) 600 + 60 + 6
   iii) 80000 + 90 + 5
   iv) 2000 + 20 + 3
   v) 300000 + 70000 + 900 + 10+8
   vi) 500,000 + 90
   vii) 600,000 + 30,000 + 200 + 5,000 + 9
   viii) 30,000 + 4,000 + 800 + 6 + 80
   ix) 70,000,000 + 70,000 + 700 + 7
   xi) 9,000,000 + 80,000 + 4000 + 5

3. What number has been expanded below?

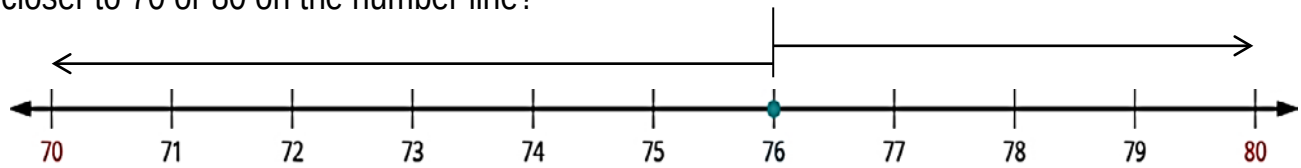
i) $(5x10^2)+(3x10^1)+(4x10^0)$
ii) $(2x10^3)+(2x10^1)+(2x10^0)$
iii) $(3x10^4)+(3x10^3)+(2x10^2)$
iv) $(6x10^7)+(8x10^4)+(4x10^3)+(9x10^0)$
v) $(4x10^5) + (5x10^3) + (7x10^2)$
vi) $(6x10^6) + (2x10^4) + (9x10^2) + (5x10^0)$
vii) $(2x10^7) + (9x10^6) + (3x10^5) + (9x10^1)$
ix) $(8x10^6) + (7x10^3) + (4x10^1) + (7x10^0)$

Rounding off whole numbers

In 2014, the Uganda Census Bureau reported the population of Bukomansimbi district as 191,127people. It might be enough to say that the population is approximately 200,000. The word approximately means that 200,000 is not the exact population, but is close to the exact value.

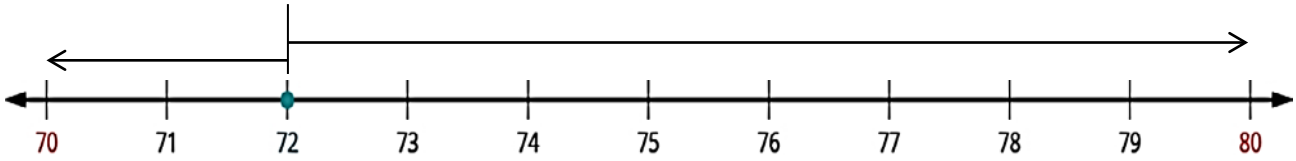
The process of approximating a number is called rounding. Numbers are rounded to a specific place value depending on how much accuracy is needed. Saying that the population of Bukomansimbi district is approximately 200,000 means we rounded to the hundred thousands place. The place value to which we round to depends on how we need to use the number.

Using the number line can help you visualize and understand the rounding process. Look at the number line below. Suppose we want to round the number 76 to the nearest ten. Is 76 closer to 70 or 80 on the number line?



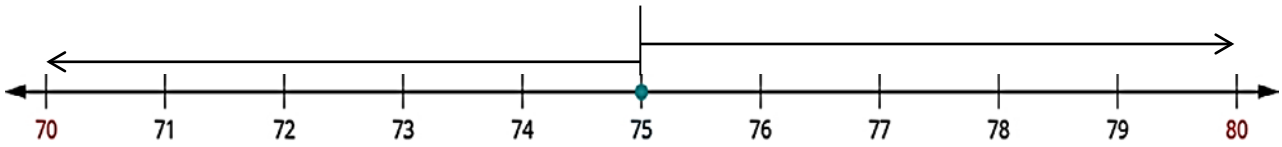
We can see that 76 is closer to 80 than to 70. So 76 rounded to the nearest ten is 80.

Now consider the number 72.
Find 72 on the number line.



We can see that 72 is closer to 70, so 72 rounded to the nearest ten is 70.

How do we round 75 to the nearest ten?
Find 75 on the number line.



The number 75 is exactly midway between 70 and 80.

So that everyone rounds the same way in cases like this, mathematicians have agreed to round to the higher number, 80. So, 75 rounded to the nearest ten is 80.

Now that we have looked at this process on the number line, we can introduce a general procedure. To round a number to a specific place, look at the number to the right of that place. If the number is less than 5, round down. If it is greater than or equal to 5, round up.

So, for example, to round 76 to the nearest ten, we look at the digit in the ones place. The digit in the ones place is a 6. Because 6 is greater than or equal to 5, we increase the digit in the tens place by one. So the 7 in the tens place becomes an 8. Now, replace any digits to the right of the 8 with zeros. So, 76 rounds to 80.

Let's look again at rounding 72 to the nearest ten. Again, we look to the ones place. The digit in the ones place is 2. Because 2 is less than 5, we keep the digit in the tens place the same and replace the digits to the right of it with zero. So, 72 rounded to the nearest ten is 70.

Example 1

Round off 5436 to the nearest hundreds.

Required place value

Th	H	T	O
5	4	3	6

5 4 0 0
+ 0 0 0
5 4 0 0

5436 ≈ 5400

Example 2

Round off 796 to the nearest tens.

Required place value

H	T	O
7	9	6

7 9 0
+ 1 0
8 0 0

790 ≈ 800

Exercise 2:8

1. Round off to the nearest tens.
   a) 82 c) 5476 e) 27496
   b) 524 d) 19530 f) 83785

2. Round off the following to the nearest hundreds.
   a) 3,824 c) 425,607 e) 42,575
   b) 435 d) 36,749 f) 136,743

3. Round off the following to the nearest ten thousands.
   i) 346,580 iii) 326,854 v) 842,890
   ii) 9,836,234 iv) 7,475,854 vi) 6,316,987

4. Round off 3865 to the nearest thousands.
5. Round off 29995 to the nearest tens.

6. Round off to the nearest hundred thousands.

a) 32,546,742 c) 54,783,965 e) 6,279,432
b) 27,610,650 d) 8,268,501 f) 999,999

7. Round off to the nearest millions.

a) 4,238,906 c) 1,363,796 e) 99,999,999
b) 26,985,290 d) 2,790,654 f) 37,243,276

NUMBER SYSTEMS

The commonly used number systems in Uganda are;

- Hindu Arabic numerals
- Roman numerals

Study the tables below.

Table A

Hindu Arabic numerals	Roman numerals
1	I
5	V
10	X
50	L
100	C
500	D
1000	M

Table B

1 = I
2 = II
3 = III
4 = IV
5 = V
6 = VI
7 = VII
8 = VIII
9 = IX

Table C

10 = X
20 = XX
30 = XXX
40 = XL
50 = L
60 = LX
70 = LXX
80 = LXXX
90 = XC

Table D

100 = C
200 = CC
300 = CCC
400 = CD
500 = D
600 = DC
700 = DCC
800 = DCCC
900 = CM

Table E

1000 = M
2000 = MM
3000 = MMM

More Roman numerals

4000	5000	6000	7000	8000	9000	10000	1000000
$\overline{IV}$	$\overline{V}$	$\overline{VI}$	$\overline{VII}$	$\overline{VIII}$	$\overline{IX}$	$\overline{X}$	$\overline{M}$

Expressing Hindu Arabic numerals as Roman numerals

Example 1

Express 39 in Roman numerals

39 = 30 + 9
↓ ↓
XXX IX

39 = XXXIX

Example 2

Write 789 in Roman numerals

789 = 700 + 80 + 9
↓ ↓ ↓
DCC LXXX IX

789 = DCCLXXXIX

Example 3

Basudde died in 1997. Write the year he died in Roman numerals.

1997 = 1000 + 900 + 90 + 7
↓ ↓ ↓ ↓
M CM XC VII

1997 = MCMXCVII

Example 4

I was born in 1982. Write this year in Roman numerals.

1982 = 1000 + 900 + 80 + 2
↓ ↓ ↓ ↓
M CM LXXX II

1982 = MCMLXXXII

Exercise 2:9

1. Express the following in Roman numerals.

a) 15	e) 284	i) 342	m) 259	q) 738	u) 2345
b) 24	f) 649	j) 983	n) 1897	r) 123	v) 3696
c) 46	g) 2023	k) 720	o) 1453	s) 548	w) 1999
d) 97	h) 1459	l) 1962	p) 689	t) 901	x) 1427

2. Sseggayi is 28 years old. Express his age in Roman numerals.
3. Matsiko harvested 984 tomatoes. Write in Roman numerals, the number of tomatoes Matsiko harvested.
4. L'okori died in 2022. Express the year he died in Roman numerals.
5. The total mass of pupils in a class is 2984kg. Write this mass in Roman numerals.
6. Abduna saves 3050 shillings every day. Express his daily saving in Roman numerals.
7. A factory produces 2792 sweets every day. Express its daily production in Roman numerals.
8. A farmer collected 798 eggs. Express in Roman numerals, the number of eggs the farmer collected.
9. Kisuule was born in 2001. How old is he in Roman numerals?
10. Rehema had 52 goats, 8goats died. Express the remaining number of goats in Roman numerals.

Expressing Roman numerals to Hindu Arabic numerals

Example 1

Express XCVI in Hindu Arabic numerals.

XCVI = XC VI
↓ ↓
90 + 6

XCVI = 96

Example 2

Write the Hindu Arabic numeral for MMCDLXXII

MMCDLXXII = MM CD LXX II
↓ ↓ ↓ ↓
2000 + 400 +70 + 2

MMCDLXXII = 2472

Exercise 2:10

1. Write the following in Hindu Arabic numerals.

a) XVI	e) LI	i) XLII	m) XCIX
b) XIV	f) XLIII	j) XLIX	n) CIV
c) XIX	g) XXXVIII	k) LXVII	o) CXXIX
d) XXVI	h) LXXXIV	l) LXXXIV	p) CCCIX

2. Express the following in Hindu Arabic numerals.

a) CDIV	e) CDXCVII	i) MIX	m) MMCCXLI
b) DCCXI	f) DCCCIX	j) MCII	n) MMMDCCCXII
c) CMXVI	g) DCXXXIX	k) MCCCVI	o) MCMLXII
d) CDXLIII	h) CMXLIV	l) MCDXCII	p) CDLXXIV

3. Mawanda sat for PLE in MMXXII. Express this year in Hindu Arabic numerals.
4. Tumuhimbise is XIV years old. Express his age in Hindu Arabic numerals.
5. Abaho a farmer in Sironko collected CDLVI eggs. Express in Hindu Arabic numerals, the number of eggs he collected.
6. Akiror has XLIX books in her bag. Write this in Hindu Arabic numerals.
7. A farmer sold CMLXXXIX litres of milk. How many litres of milk were sold in Hindu Arabic numerals?
8. I joined St Sebastian Bethlehem SS in MMIX. Write this year in Roman numerals.
9. Kalaveri's father deid in MMIV. Which year was it in Hindu Arabic numerals?
10. Ainembabazi has CLXI animals on her farm. Express the number of animals on her farm in Hindu Arabic numerals.
11. What Hindu Arabic numeral must be subtracted from XCIV to become XLIII.
12. Expand the Hindu Arabic numeral for MCMLXXXVI using powers of ten.

Number bases

Study the table below.

Base	Name	Digits used	Example	Read as
Base two	Binary	0,1	101_{two}	One zero one base two
Base three	Ternary	0,1,2	21_{three}	Two one base three
Base four	Quaternary	0,1,2,3	103_{four}	One zero three base four
Base five	Quinary	0,1,2,3,4	312_{five}	Three one two base five
Base six	Senary	0,1,2,3,4,5	25_{six}	Two five base six
Base seven	Septenary	0,1,2,3,4,5,6	140_{seven}	One four zero base seven
Base eight	Octal	0,1,2,3,4,5,6,7	73_{eight}	Seven three base eight
Base nine	Nonary	0,1,2,3,4,5,6,7,8	120_{nine}	One two zero base nine
Base ten	Decimal/Denary	0,1,2,3,4,5,6,7,8,9	231_{ten}	Two hundred thirty one

Identifying place values in number bases

Examples

Study the table below.

Base	Number	Place value	Read as
Base two	1011_{two}	1011_{two} → ones → twos → two twos → two two twos	One zero one one base two
Base three	102_{three}	102_{three} → ones → threes → three threes	One zero two base three
Base four	213_{four}	213_{four} → ones → fours → four fours	Two one three base four
Base five	2314_{five}	2314_{five} → ones → fives → five fives → five five fives	Two three one four base five

Exercise 2:11

1. Write the place value of each digit in each of the following numbers.
   a) 11_{two} d) 11011_{two} g) 1001_{two}
   b) 101_{two} e) 1111_{two} h) 110110_{two}
   c) 110_{two} f) 10011_{two} i) 1110_{two}

2. Underline the digit in the two twos place in the following:
   a) 100_{two} b) 1011_{two} c) 11001_{two}

3. Write the following in words.
   a) 110_{two} c) 10011_{two} e) 1110_{two}
   b) 100_{two} d) 1011_{two} f) 10111_{two}

4. Write the place value of the underlined digit.

a) $1\underline{1}01_{two}$ d) $1\underline{2}01_{three}$ g) $\underline{1}302\underline{}_{seven}$
b) $201\underline{}_{four}$ e) $1\underline{1}1_{two}$ h) $1\underline{8}3_{nine}$
c) $3\underline{1}50_{six}$ f) $\underline{1}24_{five}$ i) $1\underline{2}03_{eight}$

5. Write the following in words.

a) 110_{two} d) 121_{three} g) 52_{seven}
b) 203_{four} e) 1101_{two} h) 820_{nine}
c) 140_{six} f) 143_{five} i) 1102_{eight}

Expressing base ten to non-decimal bases

Example 1

Change 19_{ten} to binary.

Base	No.	
2	19	Rem
2	9	1
2	4	1
2	2	0
2	1	0
	0	1

$19_{ten} = 10011_{two}$

Example 2

Express 42_{ten} into base five.

Base	No.	
5	42	Rem
5	8	2
5	1	3
	0	1

$42_{ten} = 132_{five}$

Example 3

Change 28_{ten} into ternary.

Base	No.	
3	28	Rem
3	9	1
3	3	0
3	1	0
	0	1

$28_{ten} = 1001_{three}$

Example 4

Change 21_{ten} to base two.

Base	No.	
2	21	Rem
2	10	1
2	5	0
2	2	1
2	1	0
	0	1

$21_{ten} = 10101_{two}$

Exercise 2:12

1. Change the following to base two.

a) 9_{ten} c) 8_{ten} e) 27_{ten} g) 15_{ten}
b) 11_{ten} d) 39_{ten} f) 13_{ten} h) 73_{ten}

2. Change the following into base five.

a) 19_{ten} c) 28_{ten} e) 37_{ten} g) 91

b) 31_{ten} d) 49_{ten} f) 69_{ten} h) 37

3. Convert as instructed:

a) 13 to base six

b) 65_{ten} to base seven

c) 45_{ten} to base four

d) 23 to base three

e) 87_{ten} to base nine

f) 51_{ten} to base three

Expanding numbers in non-decimal systems

Example 1

Expand 1011_{two} using powers of 2

2^3	2^2	2^1	2^0
1	0	1	1

$(1x2^3) + (0x2^2) + (1x2^1) + (1x2^0)$

Example 2

Expand 203_{five}

5^2	5^1	5^0
2	0	3

$(2x5^2) + (0x5^1) + (3x5^0)$

Exercise 2:13

Expand the following:

a) 11_{two} d) 1011_{two} g) 123_{four} j) 124_{six}

b) 110_{two} e) 10111_{two} h) 210_{three} k) 1202_{three}

c) 1111two f) 102_{three} i) 124_{five} l) 210_{seven}

Changing non-decimal base numbers to base ten (decimal base system)

Example 1

Change 11010_{two} to denary base.

2^4	2^3	2^2	2^1	2^0
1	1	0	1	0

$(1x2^4)+(1x2^3)+(0x2^2)+(1x2^1)+(0x2^0)$

(1x2x2x2x2) +(1x2x2x2)+(0x2x2)+(1x2)+(0x1)

16 + 8 + 0+ 2 + 0

26_{ten}

Example 2

Change 342_{five} to decimal base

5^2	5^1	5^0
3	4	2

$(3x5^2)+(4x5^1)+(2x5^0)$

(3x5x5)+(4x5)+(2x1)

75 + 20 + 2

97_{ten}

Exercise 2:14

1. Change the following to base ten.

a) 101_{two} d) 10001_{two} g) 11010_{two}

b) 1101_{two} e) 1011_{two} h) 111_{two}

c) 1100_{three} f) 1111_{two} i) 11100_{two}

2. Change the following to base ten.

a) 23_{five} d) 401_{five} g) 342_{seven}
b) 102_{four} e) 123_{six} h) 45_{nine}
c) 211_{three} f) 112_{four} i) 203_{eight}

Finding the number expanded in non decimal bases

Example

What number has been expanded to give $(1x2^3)+(1x2^2)+(1x2^0)$?

$(1x2^3)+(1x2^2)+(1x2^0)$
$(1x2x2x2)+(1x2x2)+(1x1)$
$8 + 4 + 1$
13_{ten}

Base	No.	
2	13	Rem
2	6	1
2	3	0
2	1	1
	0	1

1101_{two}

Exercise 1:15

1. Find the number which has been expanded.

i) $(1x2^4)+(1x2^2)+(1x2^1)+(1x2^0)$
ii) $(1x2^2)+(0x2^1)+(1x2^0)$
iii) $(1x2^4)+(1x2^2)+(1x2^0)$
iv) $(1x2^3)+(1x2^2)+(1x2^1)+(1x2^0)$
v) $(1x2^3)+(1x2^2)+(1x2^0)$
vi) $(1x2^3)+(1x2^1)+(1x2^0)$
vii) $(1x2^3)+(1x2^2)+(1x2^1)$
viii) $(1x2^5)+(1x2^2)+(1x2^0)$
ix) $(1x2^3)+(1x2^0)$
xi) $(3x5^3) + (4x5^2) + (1x5^0)$
xi) $(1x5^2) + (3x5^1) + (2x5^0)$
xii) $(1x3^2) + (1x3^1)$
xiii) $(1x4^3) + (2x4^2) + (1x4^1) + (2x4^0)$
xiv) $(2x7^2) + (1x7^1) + (1x7^0)$
xv) $(1x3^3) + (2x3^1) + (2x3^0)$
xvi) $(1x9^2) + (6x9^0)$
xvii) $(2x6^2) + (0x6^1) + (3x6^0)$
xviii) $(3x4^2) + (2x4^1) + (1x4^0)$

2. Write in short: $(3x5^2) + (4x5^1) + (2x5^0)$
3. Express $(1x8^2) + (4x8^0)$ as a single number.
4. What number has been expanded to give; $(2x5x5x5) + (4x5x5) + (1x5) + (3x1)$?

Changing non-decimal base numbers to other non-decimal bases

Revision tips

→ First change any number in non-decimal base into base ten.

→ Then change the number in base ten to the required base.

Example 1

Change 10011_{two} to base three

10011_{two} to base ten

2^4	2^3	2^2	2^1	2^0
1	0	0	1	1

$(1x2^4)+(0x2^3)+(0x2^2)+(1x2^1)+(1x2^0)$

$(1x2x2x2x2)+(0x2x2x2)+(0x2x2)+(1x2)+(1x1)$

$16 + 0 + 0 + 2 + 1$

19_{ten}

19_{ten} to base three

Base	No.	
3	19	Rem
3	6	1
3	2	0
	0	2

$10011_{two} = 201_{three}$

Example 2

Change 203_{four} to base six

203_{four} to base ten

4^2	4^1	4^0
2	0	3

$(2x4^2) + (0x4^1) + (3x4^0)$

$(2x4x4) + (0x4) + (3x1)$

$32 + 0 + 3$

35_{ten}

35_{ten} to base six

Base	No.	
6	35	Rem
6	5	5
	0	5

$203_{four} = 55_{six}$

Exercise 2:16

1. Change the following into base two.
   a) 23_{four}
   b) 120_{five}
   c) 14_{six}
   d) 26_{seven}
   e) 102_{three}
   f) 38_{nine}
   g) 211_{four}
   h) 103_{five}
   i) 112_{three}

\2. Convert 23_{four} into base five.

3. Change 11011_{two} to base three.

4. Find the base five number equivalent to $302_{four.}$

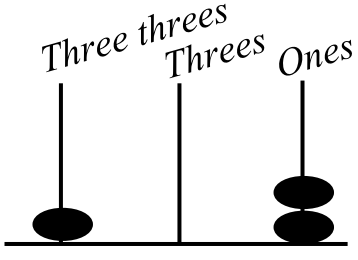
5. Change 14_{five} to base four.

6. Convert 32_{four} into binary.

7. Convert 10111_{two} to base four.

8. Change 231_{four} to base six.

9. Change to base two, the number shown on the abacus below.



Finding the unknown base

Revision tips

→ First change any number in non-decimal base into base ten.

→ Then solve the resulting equation.

Example 1

Given that $24_m = 14_{ten}$. Find the value of m

$$24_m = 14_{ten}$$
$$(2xm^1) + (4xm^0) = 14$$
$$(2xm) + (4x1) = 14$$
$$2m + 4 = 14$$
$$2m + 4 - 4 = 14 - 4$$
$$2m = 10$$
$$\frac{2m}{2} = \frac{10}{2}$$
$$m = 5$$

Example 2

Given that $34_n = 112_{four}$. Find the base represented by n

$$34_n = 112_{four}$$
$$(3xn^1)+(4xn^0) = (1x4^2)+(1x4^1)+(2x4^0)$$
$$(3xn)+(4x1) = (1x4x4)+(1x4)+(2x1)$$
$$3n + 4 = 16 + 4 + 2$$
$$3n + 4 = 22$$
$$3n + 4 - 4 = 22 - 4$$
$$\frac{3n}{3} = \frac{18}{3}$$
$$n = 6$$

n is base six

Exercise 2:17

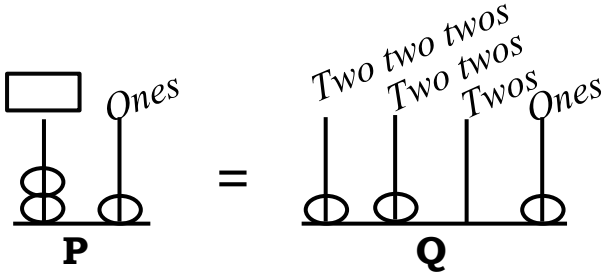
1. Find the value of the unknown base.

a) $11_y = 3_{ten}$
b) $32_r = 17_{ten}$
c) $43_h = 23_{ten}$
d) $12_f = 101_{two}$
e) $24_k = 32_{four}$
f) $23_y = 31_{four}$
g) $34_d = 31_{eight}$
h) $31six = 34_b$
i) $23_{six} = 21_f$
j) $43_n = 123_{four}$
k) $33_m = 102_{four}$
l) $25_w = 122_{three}$
m) $32_g = 104_{five}$
n) $32_p = 113_{four}$
o) $112_{three} = 32_r$
p) $211_{three} = 34_y$
q) $25_p = 201_{three}$
r) $34_p = 112_{four}$
s) $22_a = 110_{four}$
t) $23_q = 1101_{two}$
u) $23_t = 10011_{two}$
v) $51_n = 1011_{three}$
w) $32_x = 110_{four}$
x) $10011_{three} = 85_p$

2. Find the value of the unknown base.

a) $101_p = 12_{three}$
b) $102_h = 21_{five}$
c) $201_n = 53_{six}$
d) $32_{nine} = 104_b$
e) $302_t = 122_{six}$
f) $204_g = 1030_{four}$

3. Study the abaci; **P** and **Q** below and use them to answer questions that follow.



a) Write the number shown on abacus **Q**.
b) Find the missing place value on abacus **P**.

4. Solve:

a) $y^2 = 10000_{two}$
b) $b^2 = 21_{four}$
c) $2p^2 = 101_{seven}$
d) $3g^2 = 143_{five}$
e) $y^2 - 12_{three} = 21_{five}$
f) $2h^2 + 111_{two} = 121_{four}$

Addition of numbers in non decimal bases

Example 1

Add: $101_{two} + 11_{two}$

101_{two} 1+1 = 2
$+11_{two}$ 2÷2 = 1rem0
1000_{two}

Example 2

Work out: $1101_{two} + 101_{two}$

1101_{two} 1+1 = 2
$+101_{two}$ 2÷2 = 1rem0
10010_{two}

Example 3

Add: $123_{five}+43_{five}$

123_{five} 6÷5=1rem1
$+43_{five}$ 7÷5=1rem2
221_{five}

Exercise 2:18

1. Work out the following:
   a) $11_{two} + 1_{two}$
   b) $100_{two} + 11_{two}$
   c) $101_{two} + 1_{two}$
   d) $101_{two} + 11_{two}$
   e) $110_{two} + 11_{two}$
   f) $101_{two} + 111_{two}$
   g) $1011_{two} + 101_{two}$
   h) $11011_{two} + 111_{two}$
   i) $1011_{two} + 1001_{two}$

2. Add:
   a) 111_{two} $+11_{two}$
   b) 1011_{two} $+ 1_{two}$
   c) 101_{two} $+101_{two}$
   d) 11100_{two} $+ 100_{two}$
   e) 11011_{two} $+ 101_{two}$
   f) 10110_{two} $+ 10_{two}$
   g) 11111_{two} $+ 100_{two}$
   h) 101_{two} $+11111_{two}$

3. Work out:
   a) $231_{five} + 23_{five}$
   b) $42_{six} + 33_{six}$
   c) $213_{four} + 21_{four}$
   d) $231_{four} + 33_{four}$
   e) $123_{five} + 324_{five}$
   f) $410_{seven} + 66_{seven}$
   g) $403_{six} + 15_{six}$
   h) $222_{three} + 22_{three}$
   i) $34_{five} + 4_{five}$
   j) $55_{six} + 45_{six}$
   k) $125_{seven} + 603_{seven}$
   l) $102_{three} +211_{three}$

4. Alex has 1101_{two} books, Kisuule has 11_{two} books. How many books do they have altogether?
5. Akiki is 111_{three} years now. How old will Akiki be 12_{three} years from now?

Subtraction of numbers in non decimal bases

Example 1

Subtract: $11101_{two} – 1100_{two}$

11101_{two}
$- 1100_{two}$
10001_{two}

Example 2

Work out: $1100_{two} – 101_{two}$

1100_{two}
$- 101_{two}$
111_{two}

Example 3

Work out: $320_{five}-23_{five}$

320_{five}
$- 23_{five}$
242_{five}

Exercise 2:19

1. Work out the following:

a) $11_{two} - 1_{two}$
b) $100_{two} - 11_{two}$
c) $101_{two} - 11_{two}$
d) $100_{two} - 11_{two}$
e) $110_{two} - 11_{two}$
f) $1110_{two} - 101_{two}$
g) $1011_{two} - 101_{two}$
h) $11011_{two} - 111_{two}$
i) $11010_{two} - 1001_{two}$

2. Subtract:

a) $111_{two} - 11_{two}$
b) $1011_{two} - 110_{two}$
c) $101_{two} - 11_{two}$
d) $11100_{two} - 101_{two}$
e) $11011_{two} - 101_{two}$
f) $10100_{two} - 10_{two}$
g) $11111_{two} - 100_{two}$
h) $100000_{two} - 1111_{two}$

3. Work out:

a) $231_{five} - 23_{five}$
b) $52_{six} - 33_{six}$
c) $213_{four} - 21_{four}$
d) $231_{four} - 33_{four}$
e) $423_{five} - 324_{five}$
f) $410_{seven} - 66_{seven}$
g) $403_{six} - 15_{six}$
h) $212_{three} - 21_{three}$
i) $33_{five} - 4_{five}$
j) $54_{six} - 45_{six}$
k) $225_{seven} - 53_{seven}$
l) $1102_{three} - 211_{three}$

2. Take away 12_{five} from 311_{five}
3. Tr. Odomelo had 432_{five} sweets. He gave 41_{five} sweets to John and 42_{five} to Alex. How many sweets did Tr. Odomelo remain with?
4. Elungat is 123_{four} years old now. How many years ago was Elungat 33_{four} years old?
5. Find the next number in the sequence;
   11_{two}, 1000_{two}, 1101_{two}, 10010_{two}, _____
6. Solve: $d + 11_{two} = 1101_{two}$

Multiplication of numbers in non decimal bases

Example 1

Work out: $1101_{two} \times 11_{two}$

$$\begin{array}{r} 1101_{two} \\ \times\ 11_{two} \\ \hline 1101_{two} \\ +11010_{two} \\ \hline 100111_{two} \end{array}$$

Example 2

Multiply: $42_{five} \times 12_{five}$

$$\begin{array}{r} 42_{five} \\ \times 12_{five} \\ \hline 134_{five} \\ +420_{five} \\ \hline 1104_{five} \end{array}$$

Exercise 2:20

1. Work out:

a) $110_{two} \times 11_{two}$
b) $111_{two} \times 11_{two}$
c) $1101_{two} \times 11_{two}$
d) $111_{two} \times 10_{two}$
e) $101_{two} \times 10_{two}$
f) $101_{two} \times 11_{two}$
g) $11_{two} \times 11_{two}$
h) $10_{two} \times 10_{two}$
i) $1010_{two} \times 11_{two}$
j) $1011_{two} \times 11_{two}$
k) $1010_{two} \times 101_{two}$
l) $1001_{two} \times 100_{two}$

2. Multiply:

a) $11_{two} \times 11_{two}$
b) $11011_{two} \times 11_{two}$
c) $11010_{two} \times 111_{two}$

3. Multiply:

a) $120_{three} \times 11_{three}$
b) $14_{five} \times 2_{five}$
c) $431_{five} \times 21_{five}$
d) $402_{five} \times 33_{five}$
e) $24_{six} \times 5_{six}$
f) $112_{three} \times 20_{three}$
g) $34_{five} \times 13_{five}$
h) $231_{four} \times 22_{four}$
i) $123_{five} \times 32_{five}$
j) $45_{six} \times 2_{six}$
k) $234_{seven} \times 31_{seven}$
l) $1023_{eight} \times 23_{eight}$

4. Find the product of;

a) 231_{six} and 32_{six}
b) 124_{five} and 12_{five}
c) 88_{nine} and 67_{nine}

"Think as a mathematician"

1. Expand 30482 using standard decimal notation.
2. Round off 39964 to the nearest hundred.
3. Express the sum of 827 and 438 in Roman numerals.
4. What must be taken away from CDXCVI to become CDXLIV?
5. Change 3432_{five} into denary base.
6. Change 2244_{five} to ternary system.
7. Find the value of the unknown base.
   i) $46_m = 1021_{three}$
   ii) $607_k = 12013_{four}$
   iii) $3002_g = 1234_{five}$
8. Work out: $1011_{two} - 111_{two} + 101_{two}$
9. Find the original number:
   $(3x7^4) + (6x7^2) + (4x7^1) + (5x7^0)$
10. Work out: $134_{six} \times 23_{five}$ answer in base three
11. Fill in the missing number

$$\begin{array}{r} \square\,3\,4\,2_{five} \\ +\ 1\,\square\,4_{five} \\ \hline 3\,0\,3\,\square_{five} \end{array}$$

12. Work out: $10101_{two} \div 111_{two}$
13. Simplify: $\dfrac{110_{two} \times 11_{two}}{1001_{two}}$
14. Divide 21_{seven} by 12_{three} and leave your answer in binary system.
15. Given that $3\square2_{five} = 117_{nine}$. Find the missing digit in the box.
16. Given that 23abcd is a 6-digit numeral where letters a,b,c and d stand for the unkown digits. Find b if the difference between the value of 3 and the value of b in the numeral is 2900.
17. Find the unknown digit: $2k_{five} = 15_{six}$