Divisibility tests for 2,3,4,5,6,7,8,9,10 and 11

Test for 2
A number is divisible by 2 if the digit in ones place is even (0, 2, 4, 6 or 8) e.g. 3978, 9750, 4326, 5268, 21474390, 4257692, 10092574 etc.

Test for 3
A number is divisible by 3 if the sum of its digits is a multiple of 3.
Multiples of 3 include 3, 6, 9,12,15,18, 21, 24, 27 …

Example 1
6435
$$6435 = 6 + 4 + 3 + 5$$
$$= 18$$
$$= 1 + 8$$
$$= 9$$
9 is divisible by 3 so, 6435 is divisible by 3

Example 2
932
$$932 = 9 + 3 + 2$$
$$= 14$$
$$= 1+4$$
$$= 5$$
5 is not divisible by 3 so, 932 is not divisible by 3

Test for 4
A number is divisible by 4 if the last two digits are zero(00) or form a number which is a multiple of 4

All numbers ending with 00, 04, 08, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 72, 76, 80, 84, 88, 92 and 96 are divisible by 4
e.g. 13592

In the number 13592, the number formed by the last two digits (92) is a multiple of 4, so, 13592 is divisible by 4

Test for 5
A number is divisible by 5 if the digit in the ones place is either 0 or 5
e.g. 3495

The digit in ones place is 5, so 3495 is divisible by 5

Test for 6
A number is divisible by 6 if the digit in ones place is even and the sum of its digit is a multiple of 3 e.g. 498

498 has an even digit in ones place and the sum of the digits 4+9+8 = 21 is divisible by 3
So, 498 is divisible by 6.

Test for 7

Double the last digit. Take away the result from the number formed by the remaining digits. If the outcome is 0 or a multiple of 7, then that number is divisible by 7
e.g. 308

4 is the last digit, let us double it (2x8) or 8+8 = 16
The number formed by the remaining digits is 31. Let us take away 16 from 30.
30 – 16 = 14
Since 14 is a multiple of 7 so, 308 is divisible by 7.

Test for 8

A number is divisible by 8 if the number formed by the last three digits is zero(000) or a multiple of 8
e.g.135416

416 is the number formed by the last three digits. 416 is divisible by 8, so 135416 is divisible by 8

Test for 9

A number is divisible by 9 if the sum of its digits is a multiple of 9
e.g. 72135

7+2+1+3+5 = 18
18 is a multiple of 9, so, 72135 is divisible by 9

Test for 10

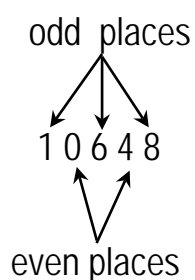A number is divisible by 10 if the digit in ones place is zero(0)
e.g. 34890

34890 has 0 in ones place, so, 34890 is divisible by 10

Test for 11

A number is divisible by 11 if the difference between the sum of the digits in the even places and the sum of the digits in the odd places is zero(0) or a multiple of 11
e.g. 10648

odd places

10 6 4 8

even places

Sum of the digits in even places 0+4 = 4
Sum of the digits in odd places 1+6+8 = 15
Difference 15 – 4 = 11

Since the difference between the sum of the digits in the even places and the sum of the digits in odd places is multiple of 11, so, 10648 is divisible by 11

By Sseggayi Benjamin Kasuka

Exercise 4:1

1. Without division, which of the following numbers is divisible by;

   a) 2?
   36, 15, 3015, 2020, 1817, 90010

   b) 3?
   18, 361, 144, 12032, 5034, 3332

   c) 4?
   2020, 148, 953, 380, 352, 970

   d) 5?
   558, 3382, 380, 542, 670, 485

   e) 6?
   382, 935, 266, 1470, 2376, 361

   f) 7?
   568, 382, 1392, 427, 167, 364

   g) 8?
   9800, 1358, 900000, 76344

   h) 9?
   893, 456, 935, 1890, 1623, 813

   i) 10?
   893, 456, 935, 1890, 163, 2020

   j) 11?
   825, 234, 8624, 1617, 4712

2. List the first 5 multiples of
   a) 2   b) 3   c) 4   d) 5   e) 7

TYPES OF NUMBERS
Identifying whole numbers and counting numbers

A set of numbers including zero is a set of whole numbers
Whole numbers = {0, 1, 2, 3, 4, 5, 6, 7, 8 …}

Note: - From whole numbers we can obtain counting/natural numbers
- Natural numbers and counting numbers are the same.
- Zero is not a counting number
Natural numbers = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10 …}

Exercise 4:2

1. Write a set of the first six whole numbers
2. List a set of counting numbers from 11 to 19
3. Given that P = {natural numbers between 5 and 13} . Find n(P)
4. If K = {counting numbers less than 15 but divisible by 3}. List all members of K
5. Work out the sum of the 6th and the 13th natural numbers.
6. Find the difference between the 8th and the 23rd whole number.
7. Form the largest number using the following digits: 7, 3, 6, 8, 5 and 2
8. Write the smallest 5-digit numeral that can be formed using the digits: 5, 9, 6, 0 and 3
9. Given the number: 513678
   a) Without division, show that the number given above is divisible by 3.
   b) Round off the given number to the nearest thousands
   c) Express the given number in standard form
   d) Work out the difference between the value of 3 and the value of 7 in the given number.

By Sseggayi Benjamin Kasuka

Finding consecutive counting numbers

Example

The sum of three consecutive counting numbers is 42. What are these numbers?

Let the first number be n

1st No.	2nd No.	3rd No.
n	n+1	n+2

Value of n

$n + (n+1) + (n+2) = 42$

$n + n + 1 + n + 2 = 42$

$n + n + n + 1 + 2 = 42$

$3n + 3 = 42$

$3n + 3 - 3 = 42 - 3$

$3n = 39$

$\dfrac{3n}{3} = \dfrac{39}{3}$

$n = 13$

So,

The first number (n)   = 13

The second number (n+1) = 13+1 = 14

The third number (n+2) = 13+2   = 15

The numbers are 13, 14 and 15

Exercise 4:3

1. Find the three consecutive counting numbers whose sum is

   a) 15   d) 18   g) 36   j) 96
   b) 9   e) 21   h) 54   k) 39
   c) 24   f) 30   i) 84   l) 270

2. The sum of three consecutive counting numbers is 51. Find the numbers.

3. Find the three consecutive natural numbers whose total is 63.

4. The sum of 4 consecutive natural numbers is 98. Find these numbers.

5. Mukulima planted trees in three consecutive days. A day, he planted one tree more than the number of trees planted the previous day. If he planted 72 trees altogether, how many trees did he plant on the second day?

6. A poultry farmer collected 138 eggs in 3 consecutive days. A day, she collected 1 egg less than the previous day. How many eggs were collected on the third day?

7. The sum of three consecutive natural numbers is 45. The first number is d
   a) Find the value of d
   b) Find the actual numbers.
   c) Work out the range of the numbers.

8. The total of three consecutive counting numbers is 57. If the second number is t,
   a) Find the value of t
   b) What are the numbers?

9. The sum of 4 consecutive counting numbers is 38. If the largest number is h
   a) Find the value of h
   b) Work out the product of the 3rd and the 4th number.

By Sseggayi Benjamin Kasuka

Identifying even and odd numbers

When a whole number is doubled (multiplied by 2), the result is an even number

Whole numbers = {0,   1,   2,   3,   4,   5,   6,   7,   8,   9 …}

0x2 1x2 2x2 3x2 4x2 5x2 6x2 7x2 8x2 9x2

Even numbers = {0,   2,   4,   6,   8,   10, 12 , 14, 16, 18 …}

When an even number is increased by 1, the result is an odd number

Even numbers = {0,   2,   4,   6,   8,   10, 12 , 14,   16, 18 …}

0+1 2+1 4+1 6+1 8+1 10+1 12+1 14+1 16+1 18+1

Odd numbers = {1,   3,   5,   7,   9,   11, 13, 15,   17, 19 …}

Exercise 4:4

1. Complete:
    a) 1, 3, 5, 7, _____, _____
    b) 0, 2, 4, 6, 8, ____, _____
    c) 13, 15, 17, 19, _____, _____
    d) 24, ____, 28, _____, _____, _____, 36
2. Work out the product of fifth and the eleventh odd number.
3. Given that P = {Odd numbers less than 11}. Find n(P).
4. Work out the sum of the 2nd and 7th even number.
5. Work out the sum of the first 5 even numbers.

Finding consecutive even numbers and consecutive odd numbers.

Example 1

The sum of three consecutive even numbers is 72. Find the numbers

Let the first number be y

1st no.	2nd no.	3rd no.
y	(y + 2)	(y + 4)

Value of y

$y + (y + 2) + (y + 4) = 72$

$y + y + 2 + y + 4 = 72$

$y + y + y + 2 + 4 = 72$

$3y + 6 = 72$

$3y + 6 - 6 = 72 - 6$

$3y = 66$

$\dfrac{3y}{3} = \dfrac{66}{3}$

$y = 22$

1st no.	2nd no.	3rd no.
y	y + 2	y + 4
22	22 + 2	22 + 4
	24	26

The numbers are 22, 24 and 26

Example 2

The sum of three consecutive odd numbers is 57. Find the numbers

Let the first number be y

1st no.	2nd no.	3rd no.
y	(y + 2)	(y + 4)

<u>Value of y</u>

$y + (y + 2) + (y + 4) = 57$

$y + y + 2 + y + 4 = 57$

$y + y + y + 2 + 4 = 57$

$3y + 6 = 57$

$3y + 6 - 6 = 57 - 6$

$3y = 51$

$\dfrac{3y}{3} = \dfrac{51}{3}$

$y = 17$

1st no.	2nd no.	3rd no.
y	y + 2	y + 4
17	17 + 2	17 + 4
	19	21

The numbers are 17, 19 and 21

Example 3

The sum of four consecutive odd numbers is 136. The largest number is h.

a) Find the value of h

1st no.	2nd no.	3rd no.	4th no.
(h – 6)	(h – 4)	(h – 2)	h

Value of h

$(h - 6)+(h - 4)+(h - 2)+h = 136$

$h - 6 + h - 4 + h - 2 + h = 136$

$h + h + h + h - 6 - 4 - 2 = 136$

$4h - 12 = 136$

$4h - 12 + 12 = 136 + 12$

$\dfrac{4h}{4} = \dfrac{148}{4}$

$h = 37$

b) Find the numbers

1st no.	2nd no.	3rd no.	4th no.
h – 6	h – 4	h – 2	h
37 – 6	37 – 4	37 – 2	37
31	33	35	

The numbers are 31, 33, 35 and 37

Exercise 4:5

1. Find the consecutive three odd numbers whose sum is;

   a) 39   d) 33   g) 63   j) 51

   b) 27   e) 45   h) 81   k) 93

   c) 15   f) 69   i) 75   l) 117

2. The sum of three consecutive odd numbers is 87. Find the numbers.

3. The sum of three consecutive odd numbers is 57.

   a) Find the numbers

   b) Find the range of numbers

4. Find the consecutive three even numbers whose sum is;

   a) 24   d) 30   g) 36   j) 84

   b) 18   e) 54   h) 66   k) 96

   c) 12   f) 48   i) 60   l) 72

5. The sum of three consecutive even numbers is 56.

   a) Find the numbers

   b) Find the range of the numbers

7. The sum of three consecutive odd numbers is 99.
   a) Find the actual numbers.
   b) Find the range of the numbers.
   c) Work out the product of the second and the third number.

8. Find the four consecutive odd numbers whose sum is 56.

9. The sum of three consecutive even numbers is 36. The first number is m
   a) Find the value of m
   b) Find the numbers
   c) Find the range of the numbers.

10. The sum of 3 consecutive odd numbers is 93. Find the other number if two of them are 29 and 31

11. The sum of three consecutive odd numbers is 69. The second number is b.
    a) Find the value of b
    b) Find the numbers.

12. The total of three consecutive even numbers is 54. The largest number is m.
    a) Find the value of m
    b) Find the range of the numbers.

13. The sum of 4 consecutive even numbers is 92. If the largest number is k. Find the value of k then find the numbers.

14. The total age of three children is 45 years. The children who were produced at intervals 2 years. How old is each child?

15. Kiviiri picked 60 mangoes in 4 consecutive days. In a day, he picked 2 mangoes more than the mangoes picked the previous day.
    a) How many mangoes did he pick on the first day?
    b) Find the total number of mangoes he picked on the third and the fourth day.

16. In a certain school, 54 kg of rice were consumed by pupils in 3 consecutive days. In a day, they consumed 2kg more than what was consumed the previous day. Find in kilograms, the mass of rice consumed by pupils on the third day.

17. Makumbi harvested 105 sacks of maize in three consecutive seasons. In a season, he harvested 2 sacks more than the previous season.
    a) How many sacks of maize were harvested in the first season?
    b) Given that a sack of maize cost sh. 45,000. How much money did Makumbi get in the third season?

18. A factory produced 480 crates of soda in 4 consecutive weeks. The production decreased by 2 crates for every week that past.
    a) How many crates of soda were produced in the third week?
    b) If a crate of soda was sold at sh. 20,000, how much money was collected in the first week?

19. In a market, the cost of a mango is sh. 500. A trader sold some mangoes at sh. 33000 in 3 consecutive hours. In an hour, she sold 2 mangoes more than the previous hour.
    a) How many mangoes did she sell in the first hour?
    b) Find the amount of money she had by the end of the first hour.
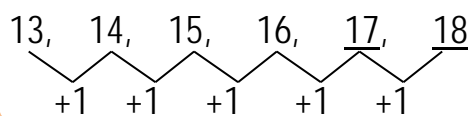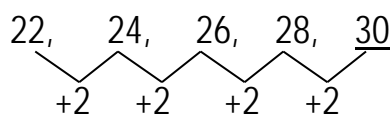
Sequences of counting, odd and even numbers

Example 1
Find the next two numbers in the
sequence: 13, 14, 15, 16, ___, ___

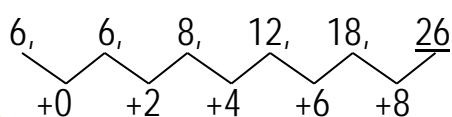13,   14,   15,   16,   <u>17</u>,   <u>18</u>
  +1   +1   +1   +1   +1

Example 2
Find the next number in the sequence
22, 24, 26, 28, ___

22,   24,   26,   28,   <u>30</u>
    +2   +2   +2   +2

Example 3
Find the next number in the sequence:
6, 6, 8, 12, 18, _____

6,   6,   8,   12,   18,   <u>26</u>
  +0   +2   +4   +6   +8

Example 4
Find the next number in the sequence:
1, 4, 9, 16, 25, ___

1,   4,   9,   16,   25   <u>36</u>
  +3   +5   +7   +9   +11
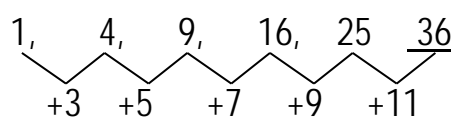
Exercise 4:6
Find the missing numbers in the sequences below.

a) 1, 2, 3, 4, 5, 6, ____, ____
b) 0, 1, 2, 3, 4, _____, ____
c) 0, 2, 4, 6, 8, ____
d) 1, 3, 5, 7, 9, ____, ____
e) 19, 21, 23, 25, 27, ____, ____
f) 23, 24, 25, 26, ____, ____
g) 13, 14, 16, 19, 23, ____, ___

h) 13, 13, 14, 16, 19, ____, ____
i) 17, 18, 20, 23, 27, ____
j) 34, 35, 38, 43, ____
k) 9, 9, 10, 12, 15, ____
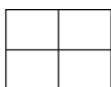l) 5, 16, 25, 32, 37, ____, ____
m) 49, 51, 55, 61, 69, ____
n) 23, 35, 45, 53, 59, ____

Identifying square numbers
Study the squares below

1 x 1
1

2 x 2
4

3 x 3
9

When a number is multiplied by itself, the product is the square of that number.
The first 20 square numbers

1 x 1 = 1	2 x 2 = 4	3 x 3 = 9	4 x 4 = 16	5 x 5 = 25	6 x 6 = 36	7 x 7 = 49	8 x 8 = 64	9 x 9 = 81	10 x 10 = 100

11 x 11 = 121	12 x 12 = 144	13 x 13 = 169	14 x 14 = 196	15 x 15 = 225	16 x 16 = 256	17 x 17 = 289	18 x 18 = 324	19 x 19 = 361	20 x 20 = 400

A set of square numbers = {1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196 …}

By Sseggayi Benjamin Kasuka

Square numbers can also be obtained by adding consecutive odd numbers

$1 = 1$
$1+3 = 4$
$1+3+5 = 9$
$1+3+5+7 = 16$
$1+3+5+7+9 = 25$
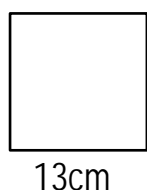$1+3+5+7+9+11 = 36$
$1+3+5+7+9+11+13 = 49$ …

A set of square numbers less than 50 = {1, 4, 9, 16, 25, 36, 49}

Example 1
Work out the square of 9
$9^2 = 9 \times 9$
  $= 81$

Example 2
Calculate the area of the square below

13cm

13cm

$A = s \times s$
$A = 13cm \times 13cm$
$A = 169cm^2$

Example 3
Work out the difference between the square of 8 and the square of 4

Approach 1

Square of 8
$8 \times 8 = 64$

Square of 4
$4 \times 4 = 16$

Difference
$64 – 16 = 48$

Approach 2

$8^2 – 4^2$
$(8 \times 8) – (4 \times 4)$
$64 – 16$
48

Exercise 4:7

1. Find the square of the following:
   a) 6   e) 14   i) 12   m) 23   q) 78
   b) 9   f) 10   j) 5   n) 36   r) 123
   c) 7   g) 4   k) 19   o) 21   s) 56
   d) 8   h) 16   l) 15   p) 49   t) 459

2. Fill in the missing numbers.
   a) 1, 4, 9, 16, 25, ____, ___   d) 5, 6, 10, 19, 44, ____, ____
   b) 49, 36, 25, 16, ____, ___   e) 2, 51, 87, 112, 128, ____, ___
   c) 9, 16, 25, 36, 49, ____, ___   f) 80, 79, 75, 66, 50, ____, ___

3. Write down the square numbers between 10 and 70

4. Work out the sum of the square of 4 and the square of 6.

5. In the list below, circle all square numbers
   44,   25,   16,   78,   90,   166,   144,   1,   6

6. Calculate the area of a square whose side is:
   a) 14m   c) 13cm   e) 24cm   g) 9mm   i) 50m
   b) 25cm   d) 20cm   f) 17dm   h) 123m   j) 654cm

Finding square roots of numbers

Any number multiplied by itself to get a square, the number becomes the square root of that square

e.g.

Square root ◆——— 7 x 7 = 49 ◆———◆ Square

Think in pairs
What number can be multiplied by itself to get?

9, 36, 64, 81, 100, 25, 16, 144, 49, 169, 196, 225, 400, 289

Example 1	Example 2

Example 1

Work out the square root of 144

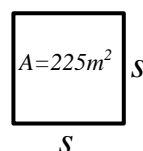| 2	144
2	72
2	36
2	18
3	9
3	3
	1

$\sqrt{144} = \sqrt{(2\times2)\times(2\times2)\times(3\times3)}$
$\sqrt{144} = 2\times2\times3$
$\sqrt{144} = 12$

Example 2

The area of a square in 225m²
Find the length of each side of the square

$A=225m^2$ s

s

$s\times s = A$
$s^2 = 225m^2$
$\sqrt{s^2} = \sqrt{225m^2}$
$\sqrt{(s\times s)} = \sqrt{(3\times3)\times(5\times5)\times(m\times m)}$
$s = 3\times5\times m$
$s = 15m$

3	225
3	75
5	25
5	5
	1

Exercise 4:8

1. Work out the square root of;
   - a) 9
   - b) 36
   - c) 64
   - d) 81
   - e) 196
   - f) 49
   - g) 144
   - h) 16
   - i) 169
   - j) 100
   - k) 400
   - l) 289

2. Find the length of each side of a square whose area is
   - a) 25dm²
   - b) 256cm²
   - c) 4m²
   - d) 100dm²
   - e) 81cm²
   - f) 576m²
   - i) 361cm²
   - j) 121cm²
   - k) 625m²

3. A road was constructed along one side of a square garden. The area of the square garden is 36km². Calculate the length of the road.

4. Pupils in P.7 class shared 225 books. Each pupil got the number of books equal to the number of pupils in the class. Find the number of pupils in P.7 class.

5. Muwanika had oranges in boxes. The number of oranges in each box was the same as the number of boxes he had. When he sold 99 oranges, he remained with 97 oranges. How many oranges were in each box?

By Sseggayi Benjamin Kasuka

Identifying cube numbers

When a number is multiplied by its itself thrice, a cube number is formed

1x1x1	2x2x2	3x3x3	4x4x4	5x5x5	6x6x6	7x7x7	8x8x8	9x9x9	10x10x10
= 1	= 8	= 27	= 64	= 125	= 216	= 343	= 512	= 729	= 1000

A set of cube numbers = {1, 8, 27, 125, 216, 343, 512, 729, 1000 …}

Example 1

Find the next two numbers in the sequence: 1, 8, 27, 64, __, __

1, 8, 27, 64, 125, 216

1^3 2^3 3^3 4^3 5^3 6^3

5^3 = 5 x 5 x 5 = 125
6^3 = 6 x 6 x 6 = 216

Example 2

What is the sum of the first 3 cube numbers?

1 x 1 x 1 = 1
2 x 2 x 2 = 8
3 x 3 x 3 = 27

Sum
1+8+27
9+27
36

Example 3

Work out the cube of 9

9^3
9 x 9 x 9
81 x 9
729

Exercise 4:9

1. Find the cube of;
   a) 2   d) 4   g) 5
   b) 7   e) 9   h) 12
   c) 10   f) 8   i) 15

2. Fill in the missing numbers
   i) 1, 8, 27, 64, ___, ___   iv) 3, 4, 12, 39, ____, ____
   ii) 216, 125, 64, 27, ___, ___   iv) 2, 66, 93, 101, ____
   iii) 27, 64, 125, 216, 343, ___, ____   v) 5, 6, 14, 41, ___, ___

3. List the first 4 cube numbers

4. Find the difference between the 8th and 12th cube numbers.

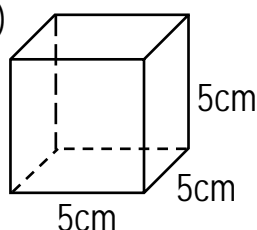5. Work out the sum of the first five cube numbers.

6. Given that A = {cube numbers less than 100}. Find n(A)

7. If D = {cube numbers between 1 and 100}. How many proper subsets are in D

8. Given that H = {1,8, 27, 64}. Name set H
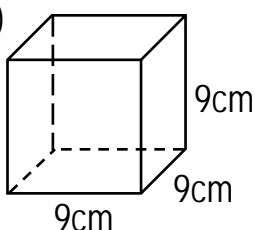
9. Calculate the volume of each of the cubes below.

a)

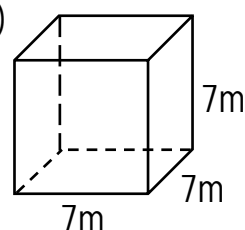5cm
5cm
5cm

b)

9cm
9cm
9cm

c)

7m
7m
7m

By Sseggayi Benjamin Kasuka

Identifying triangular numbers
They are obtained by adding consecutive counting numbers

1	=	1
1+2	=	3
1+2+3	=	6
1+2+3+4	=	10
1+2+3+4+5	=	15
1+2+3+4+5+6	=	21
1+2+3+4+5+6+7	=	28
1+2+3+4+5+6+7+8	=	36
1+2+3+4+5+6+7+8+9	=	45
1+2+3+4+5+6+7+8+9+10	=	55

A set of triangular numbers ={1, 3, 6, 10, 15, 21, 28, 36, 45, 55 …}

Example 1

Find the 8th triangular number.

1+2+3+4+5+6+7+8
6+9+13+8
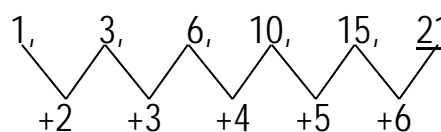15+21
36

Example 2

Write the next number in the sequence:
1, 3, 6, 10, 15, ___

1,   3,   6,   10,   15,   21
  +2   +3   +4   +5   +6

→ We can also use the formula $\frac{n}{2}(n+1)$ to find triangular numbers.

Example 3

Use $\frac{n}{2}(n+1)$ to find the 29th triangular number.

$$\frac{n}{2}(n+1) = \frac{29}{2}(29+1)$$
$$= \frac{29}{2} \times 30$$
$$= 29 \times 15$$
$$= 435$$

Exercise 4:10
1. Fill in the missing triangular numbers
   1, 3, 6, 10, 15, ____, ____
2. Write down all triangular numbers between 10 and 40.
3. What is the 15th triangular number?
4. Work out the difference between the 8th and the 5th triangular numbers.
5. In the list below, circle all triangular numbers;
   2, 4, 7, 1, 15, 20, 10
6. Find the 21st triangular number.

Factors and common factors

A number that can divide another number the exact number of times is a factor.
A common factor is a number that can divide two or more numbers leaving no remainder.

Example 1

List all factors of 12

12 ÷ 1 = 12
12 ÷ 2 = 6
12 ÷ 3 = 4
12 ÷ 4 = 3
12 ÷ 6 = 2
12 ÷ 12 = 1
Factors of 12 = {1,2,3,4,6,12}

Example 2

Find the common factors of 18 and 24

Factors of 18
18 ÷ 1 = 18
18 ÷ 2 = 9
18 ÷ 3 = 6
18 ÷ 6 = 3
18 ÷ 9 = 2
18 ÷ 18 = 1
{1,2,3,6,9,18}

Factors of 24
24 ÷ 1 = 24
24 ÷ 2 = 12
24 ÷ 3 = 8
24 ÷ 4 = 6
24 ÷ 6 = 4
24 ÷ 8 = 3
24 ÷ 12 = 2
24 ÷ 24 = 1
{1,2,3,4,6,8,12,24}

Common factors of 18 and 24 are {1,2,3,6}

→ 1 is a factor of any counting number

Exercise 4:11

1. List all factors of;

a) 6	c) 14	e) 27	g) 42	i) 54	k) 20	m) 40
b) 15	d) 20	f) 30	h) 90	j) 48	l) 84	n) 64

2. Find the common factors of the following:

a) 12 and 8	e) 12 and 15	i) 20, 30 and 40
b) 16 and 20	f) 18 and 30	j) 25, 30 and 35
c) 45 and 35	g) 84 and 90	k) 12, 18 and 30
d) 60 and 24	h) 24, 54 and 90	l) 15, 27 and 36

Finding the highest common factor of numbers

The highest common factor (HCF) of two or more numbers is the greatest whole number from the common factors

Example 1

Find the highest common factor of 30 and 36

Factors of 30 = {1,2,3,5,6,10,15,30}

Factors of 36 = {1,2,3,4,6,9,12,18,36}

Common factors = {1,2,3,6}

The highest common factor is 6

Example 2

What is the highest common divisor of 27 and 18

Factors of 27 = {1,3,9,27}

Factors of 18 = {1,2,3,6,9,18}

Common factors = {1,3,9}

The highest common divisor is 9

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Exercise 4:12

1. Find the highest common factor (HCF) of:

a) 12 and 8	e) 12 and 15	i) 20, 30 and 40
b) 16 and 20	f) 18 and 30	j) 25, 30 and 35
c) 45 and 35	g) 84 and 90	k) 12, 18 and 30
d) 60 and 24	h) 24, 54 and 90	l) 15, 27 and 36

2. Work out the highest common divisor of 36 and 48

The lowest common factor (LCF) of two or more numbers is always 1

Identifying prime numbers
→ Prime numbers have only two factors

The table below shows numbers and their factors.

Counting number	Factors	No. of factors	Type of number
1	{1}	1	not prime number
2	{1,2}	2	prime number
3	{1,3}	2	prime number
4	{1,2,4}	3	not prime number
5	{1,5}	2	prime number
6	{1,2,3,6}	4	not prime number
7	{1,7}	2	prime number
8	{1,2,4,8}	4	not prime number
9	{1,3,9}	3	not prime number
10	{1,2,5,10}	4	not prime number
11	{1,11}	2	prime number
12	{1,2,3,4,6,12}	6	not prime number

From the table above, numbers with only two factors are {2,3,5,7,11}. They are called prime numbers

Example 1

What is the sum of prime numbers between 70 and 80?

Here, we first list all numbers between 70 and 80 then cross out all multiples of 2, 3, 5 and 7. All the remaining numbers are prime numbers.

{71, ~~72~~, 73, ~~74~~, ~~75~~, ~~76~~, ~~77~~, ~~78~~, 79}

Prime numbers between 70 and 80 are {71, 73, 79}
Sum = 71+73+79
Sum = 223

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Example 2

Find the product of the next two numbers in the sequence: 2,3,5,7, __, ___

2,3,5,7, <u>11</u>, <u>13</u> (Prime numbers)

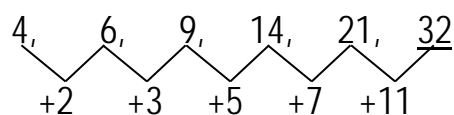*Product*
11 x 13 = 143

Example 3

Find the next number in the sequence:
4, 6, 9, 14, 21, ___

4, 6, 9, 14, 21, <u>32</u>
  +2   +3   +5   +7   +11

Exercise 4:13

1. List the first seven prime numbers
2. List all prime numbers between 60 and 80
3. Work out the sum of the 4th and the 6th prime number
4. Write the next four prime numbers; 2, 3, 5, 7, ___, ___, ___, ____
5. List all prime numbers less than 20
6. Write all prime numbers between 30 and 45
7. Work out the sum of the prime numbers between 80 and 90.
9. Find the next numbers in the sequences below:

a) 2, 3, 5, 7, ___, ___

b) 13, 17, 19, 23, ___, ___

c) 23, 19, 17, 13, ___, ___

d) 43, 47, 53, 59, ___, ___

e) 6, 8, 11, 16, 23, __, __

f) 23, 25, 28, 33, 40, __, __

g) 7, 12, 19, 30, 43, __, __

h) 2, 21, 38, 51, 62, __, __

10. In the table below, shade all prime numbers.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	29	40
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

Identifying composite numbers

→ Composite numbers have more than 2 factors. The first composite number is 4
A set of composite numbers is {4, 6, 8, 9, 10, 12, 14, 15, 16, …}

Note: 1 is neither a prime number nor a composite number because it has only one factor.

Exercise 4:14

1. List the first 6 composite numbers.
2. Work out the sum of the first 4 composite numbers.
3. Work out the square of the 4th composite number.
4. Write the next composite numbers.

4, 6, 8, 9, __, __

5. Find the next number in the sequences:

a) 6, 10, 16, 24, 33, ___

b) 12, 16, 22, 30, 39, ___

c) 7, 16, 26, 38, 52, ___

d) 3, 23, 41, 57, 72, ___

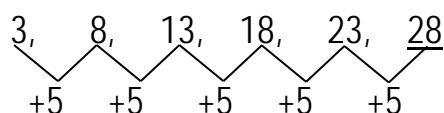*By Sseggayi Benjamin Kasuka*

Forming patterns using increasing order (progression)

We add or multiply where the sequence is increasing

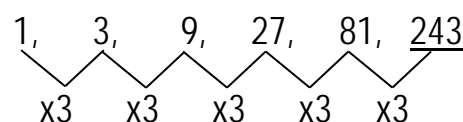**Example 1**

Find the next two numbers in the sequence:
a) 3, 8, 13, 18, ___, ___

3,   8,   13,   18,   23,   28

+5   +5   +5   +5   +5

Example 2

Write the next number in the sequence:
1, 3, 9, 27, 81, ___

1,   3,   9,   27,   81,   243

x3   x3   x3   x3   x3

Exercise 4:15

1. Write the next number in each of the sequences below:

a) 0, 2, 4, 8, 10, __
b) 2, 5, 7, 10, 12, __
c) 0, 3, 6, 12, __
d) 3, 5, 8, 13, 20, __
e) 7, 12, 17, 22, __
f) 3, 6, 12, 24, 48, __
g) 5, 6, 7, 11, 15, 16, 17, __
h) 8, 12, 16, 20, __
i) 4, 5, 8, 14, 24, ___
j) 7, 9, 10, 13, 15, 19, __
k) 1, 2, 10, 37, __

l) 11, 17, 23, 29, __
m) 10, 21, 32, 43, __
n) 33, 40, 47, 54, __
o) 46, 51, 56, 61, __
p) 81, 91, 101, 111, __
q) 118, 138, 158, 178, __
r) 2, 2, 3, 5, 8, __
s) 1, 4, 9, 16, 25, __
t) 1, 3, 6, 10, 15, __
u) 2, 9, 16, 23, __
v) 10, 17, 24, 31, ___

w) 2, 3, 6, 12, 22, __
x) 3, 4, 8, 17, __
y) 5, 6, 9, 15, 25, __
z) 3, 4, 12, 39, __
aa) 1, 5, 11, 19, 28, __
ab) 6, 8, 11, 16, 27, __
ac) 5, 6, 9, 14, 21, __
ad) 13,17,23,31,40, __
ae) 6, 8, 11, 16, 23, __
af) 8, 9, 13, 22, 38, __
ag) 10, 11, 19, 46, __

2. Find the next number in sequence:

a) 2, 2, 4, 12, 48, __
b) 1, 3, 9, 27, __
c) 1, 5, 20, 60, __

d) 10, 20, 40, 80, __
e) 1, 4, 16, 64, __
f) 3, 6, 12, 24, __
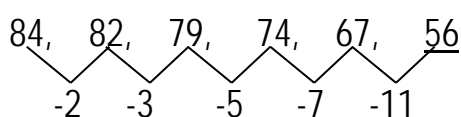
g) 1, 1, 2, 6, 24, __
h) 5, 10, 20, 40, __
i) 2, 8, 34, 128, __

Forming patterns using increasing order (progression)

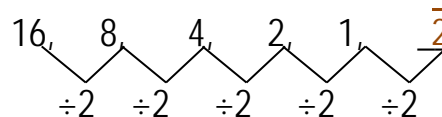*We subtract or divide where the sequence is decreasing*

Example 1

Find the square of the next number in the sequence: 84, 82, 79, 74, 67, __

84,   82,   79,   74,   67,   56

-2   -3   -5   -7   -11

Square = 56^2
= 56 x 56
= 3136

Example 2

Find the next two numbers in the sequence: 16, 8, 4, 2, 1, __

16,   8,   4,   2,   1, $\frac{1}{2}$

÷2   ÷2   ÷2   ÷2   ÷2

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Exercise 4:16

1. Find the next number in the sequence:

a) 40, 35, 30, 25, __   g) 20, 18, 15, 11, __   m) 59,58,54,45,29, __

b) 29, 25, 21, 17, __   h) 35, 33, 31, 29, __   n) 72,71,68,58,43, __

c) 20, 17, 14, 11, __   i) 17, 15, 13, 11, 9, __   o) 101,100,92,65, __

d) 48, 41, 34, 27, __   j) 90, 80, 70, 60, 50, __   p) 50,46,40,32,23 __

e) 82, 72, 62, 52, __   k) 73, 69, 63, 55, 46, __   q) 45,43,40,35,28, __

f) 108, 97, 86, 75, __   l) 45, 43, 40, 35, 28, __   r) 40,38,35,33,30, __

2. Write the missing numbers in the sequence:

a) 1024, 256, 64, __, __   e) 256, 128, __, 32, __, 8, __, 2, __

b) 729, 243, 81, __, 9, __, 1   f) 3125, 625, 125, __, __, __

c) 320, 160, __, 40, __, __, 10, __   g) 3125, 625, 125, __, __, __

d) 896, 448, 224, __, __, 28, 14, __   h) 81, 27, 9, 3, __, __, __

Try these also

1. Fill in the missing numbers in the sequence:

a) $\dfrac{1}{13}, \dfrac{3}{13}, \dfrac{5}{13}, \dfrac{7}{13}$ —, —   e) $\dfrac{1}{4}, \dfrac{1}{9}, \dfrac{1}{16}, \dfrac{1}{25}$ —, —   i) 4.6, 4.3, 4, 3.7, __, __

b) $\dfrac{1}{2}, \dfrac{1}{3}, \dfrac{1}{5}, \dfrac{1}{7}$ —, —   f) $\dfrac{16}{19}, \dfrac{13}{19}, \dfrac{10}{19}, \dfrac{7}{19}$ —, —   j) (2,3,4), (3,4,5), (4,5,6), __, __

c) $\dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}, \dfrac{1}{16}$ —, —   g) $\dfrac{1}{32}, \dfrac{1}{16}, \dfrac{1}{8}, \dfrac{1}{4}$ —, —   k) (1,4,5), (2,5,6), 3,6,7), (4,7,8), __

d) $\dfrac{1}{3}, \dfrac{1}{6}, \dfrac{1}{9}, \dfrac{1}{18}$ —, —   h) 1, 1, 2, 3, 5, 8, __   l) 9, 6, 7, 1, 9, 0, 27, __, __

2. Find the sum of the first 4 rectangular numbers.

3. Find the next number in the sequence:

a) 1, 3, 9, 21, 41, ___

b) 2, 6, 12, 20, 30, ___

Prime factorization

→A number can be written as a product of its prime factors, in subscript form/set notation or in form of powers

→Prime factors are always written in ascending order.

Example 1	Example 2
Prime factorise 72	Find the missing numbers.

Example 1

Prime factorise 72

2	72
2	36
2	18
3	9
3	3
	1

72 as a product of its prime factors

72 = 2x2x2x3x3

In power form
72 = $2^3 \times 3^2$

In set notation / subscript form
F_{72} = {$2_1, 2_2, 2_3, 3_1, 3_2$}

Example 2

Find the missing numbers.

90 ÷ 2 = 45
or
3 x 15 = 45

15 ÷ 5 = 3

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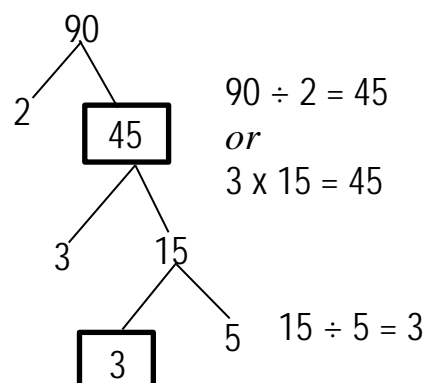
Example 3

Prime factorize 18 and 24 and represent the prime factors on the Venn diagram.

2	18
3	9
3	3
	1

2	24
2	12
2	6
3	3
	1
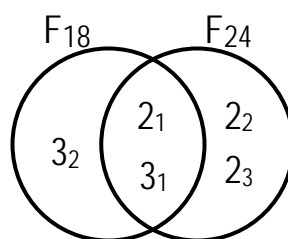
$F_{18} = \{2_1, 3_1, 3_2\}$ $F_{24} = \{2_1, 2_2, 2_3, 3_1\}$

b) Use the above Venn diagram to find;

i) the GCF of 18 and 24
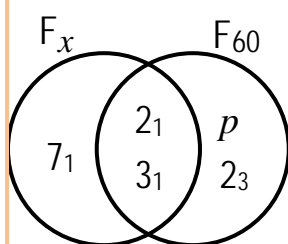
$GCF = 2 \times 3$
$GCF = 6$

ii) the LCM of 18 and 24

$LCM = 3 \times 3 \times 2 \times 2 \times 2$
$LCM = 9 \times 8$
$LCM = 72$

Example 4

Study the Venn diagram below and use it to answer questions that follow.

a) Find the value of p

$p \times 2 \times 3 \times 2 = 60$
$12p = 60$
$\dfrac{12p}{12} = \dfrac{60}{12}$
$p = 5$

b) Find the value of x

$x = 7 \times 2 \times 3$
$x = 42$

c) Find the GCF of x and 30

GCF = 2x3
GCF = 6

d) Work out the LCM of x and 30

LCM = 7x2x3x5x2
LCM = 420

Example 5

The prime factors of 24 and 90 are given below.

$24 = 2^3 \times 3$
$90 = 2 \times 3^2 \times 5$

Use the above prime factors to find the lowest common multiple (LCM) of 24 and 90

$24 = 2^3 \times 3$
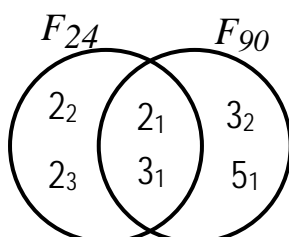$24 = 2 \times 2 \times 2 \times 3$
$F_{24} = \{2_1, 2_2, 2_3, 3_1\}$

$90 = 2 \times 3^2 \times 5$
$90 = 2 \times 3 \times 3 \times 5$
$F_{90} = \{2_1, 3_1, 3_2, 5_1\}$

$LCM = 2 \times 2 \times 2 \times 3 \times 3 \times 5$
$LCM = 360$

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Example 6

Find the number that has been prime factorized to give: $\{2_1, 2_2, 3_1, 5_1, 5_2\}$

$\{2_1, 2_2, 3_1, 5_1, 5_2\}$

2x2x3x5x5

300

The number is 300

Exercise 4:17

1. Express each of the following numbers as a product of its prime factors.

a) 36   e) 72   i) 30

b) 42   f) 50   j) 108
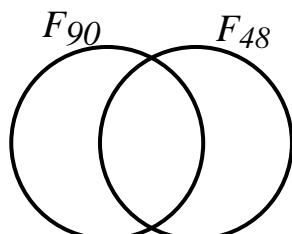
c) 144   g) 45   k) 90

d) 180   h) 120   l) 48

2. Prime factorise 54 and leave your answer in power form.

3 a) Prime factorise 90 and 48 and represent the prime factors in the Venn diagram below.
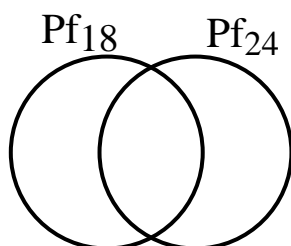
F_{90} F_{48}

b) Use the above Venn diagram to find the
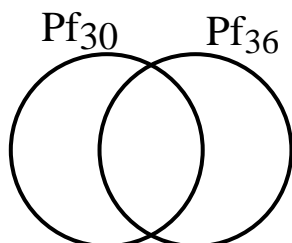   i) LCM of 90 and 48
     ii) GCF of 90 and 48

4 a) Prime factorise 18 and 24 and represent the prime factors on the Venn diagram below.

Pf_{18} Pf_{24}

b) Find the GCF of 18 and 24

c) Calculate the LCM of 18 and 24

5 a) Prime factorise 30 and 36 and represent the prime factors on the Venn diagram below.
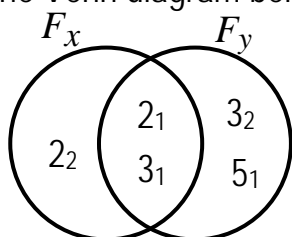
Pf_{30} Pf_{36}

b) Find the Greatest Common Factor (GCF) of 30 and 36.

c) Calculate the Lowest Common Multiple (LCM) of 30 and 36.

6. Given that Fp = $\{2_1, 2_2, 2_3, 5_1\}$. Find the value of p.

7. The Venn diagram below shows the prime factors of x and y

F_x F_y

2_1 3_2
2_2 3_1 5_1

a) Find the value of y
b) Find the value of x
c) Find the GCF of x and y
d) Find the LCM of y and x

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8. The prime factors of 48 and 60 are given below.

$48 = 2^4 \times 3$

$60 = 2^2 \times 3 \times 5$

Use the above prime factors to find,

i) the lowest common multiple (LCM) of 48 and 60

ii) the highest common factor of 48 and 60

9. Given that $F_{18} = \{2_1, 3_1, 3_2\}$ and $F_m = \{2_1, 2_2, 2_3, 3_1\}$. Use the given information to find;

i) the value of m

ii) the GCF of 18 and m

iii) the LCM of 18 and m

10. The prime factors of 36 and 150 are given below.

$36 = 2^2 \times 3^2$

$150 = 2 \times 3 \times 5^2$

Use the given prime factors above to find

a) the Lowest Common Multiple (LCM) of 36 and 150

b) the GCF of 36 and 150.

11. Prime factorise 90 and give your answer using subscript form.

12. The prime factors of x and 36 are $\{2_1, 2_2, 3_1, 5_1\}$ and $\{2_1, 2_2, 3_1, y\}$ respectively.
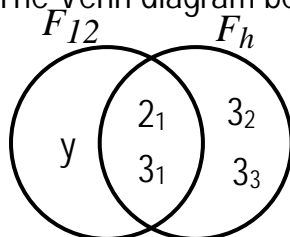
a) Find the value of;

i) x

ii) y

b) Find the GCF of x and 36

c) Work out the LCM of x and 36

13. The Venn diagram below shows the prime factors of 12 and h
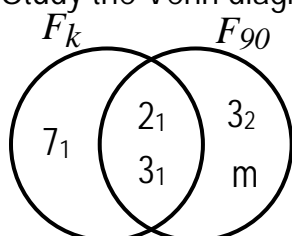
F_{12} F_h

a) Find the value of y

b) Find the value of h

c) Find the GCF of 12 and h

d) Find the LCM of 12 and h

14. Study the Venn diagram below and use it to answer the questions that follow.
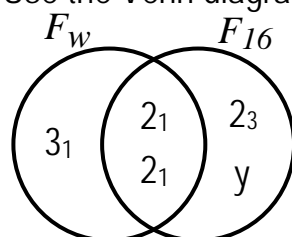
F_k F_{90}

a) Find the value of k

b) Find the value of m

c) Find a single number for $F_k \cup F_{90}$

d) Work out the GCF of k and 90

15. Use the Venn diagram below to answer questions that follow.

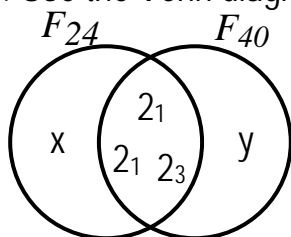F_w F_{16}

a) Find the value of w

b) Find the value of y

c) Find the GCF of w and 16

d) Work out the LCM of w and 16

16. Use the Venn diagram below to answer questions that follow.

F_{24} F_{40}

x, 2_1, 2_1 2_3, y

a) Find the value of x
b) Find the value of y
c) Find the GCF of 24 and 40
d) Work out the LCM of 24 and 40

17. Study the Venn diagram below and use it to answer the questions that follow.

F_{16} F_m

2_3, b, 2_4, 2_1, 3_1
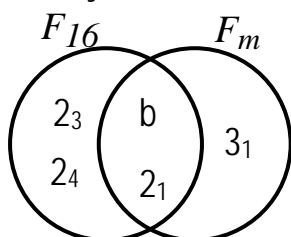
a) Find the value of b
b) Find the value of m
c) Find the least common multiple of 16 and m
d) Work out the GCF of 16 and m

18. Study the Venn diagram below and use it to answer the questions that follow.

F_{6y} F_{100}

p, 2_1 5_1, 2_2 y
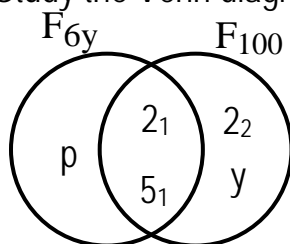
a) Find the value of y
b) Find the value of p
c) Find the least common multiple of 6y and 100
d) Work out the GCF of 6y and 100

19. p and g are two numbers prime factorized such that $F(p - g) = \{2_3\}$, $Fp \cap Fg = \{2_1, 2_2, 5_1\}$ and $F(g - p) = \{3_1, 3_2\}$
   a) Find the value of
      i) p
      ii) g
   b) Work out the;
      i) LCM of g and p
      ii) GCF of g and p

20. Fill in the missing numbers:

a)

b)

c)

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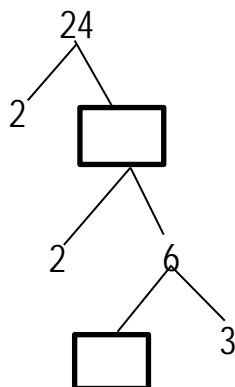
Multiples of numbers

The multiple of a number is the product of a number and another whole number.

Example 1

Find the multiples of 6

$1 \times 6 = 6$
$2 \times 6 = 12$
$3 \times 6 = 18$
$4 \times 6 = 24$

Multiples of 6 (M_6) = {6, 12, 18, 24 …}

Example 2

Find the sum of the first 5 multiples of 4

$1 \times 4 = 4$
$2 \times 4 = 8$
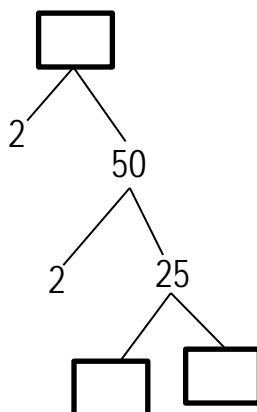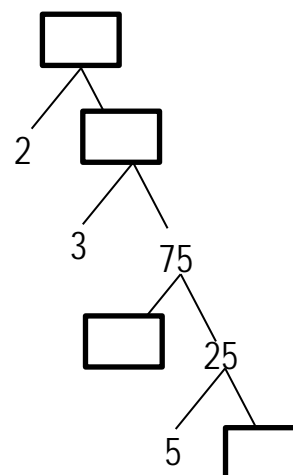$3 \times 4 = 12$
$4 \times 4 = 16$
$5 \times 4 = 20$

Sum = 4 + 8 + 12 + 16 + 20
$\quad\quad$ = 60

Example 3

The sum of three consecutive multiples of 6 is 90. Find these multiples.

Let the least / first multiple be m

1st	2nd	3rd
m	m + 6	m + 12

Value of m

$m + (m + 6) + (m + 12) = 90$
$\quad m + m + 6 + m + 12 = 90$
$\quad m + m + m + 6 + 12 = 90$
$\quad\quad\quad\quad\quad 3m + 18 = 90$
$\quad\quad\quad 3m + 18 - 18 = 90 - 18$
$\quad\quad\quad\quad\quad\quad 3m = 72$
$\quad\quad\quad\quad\quad\quad \dfrac{3m}{3} = \dfrac{72}{3}$
$\quad\quad\quad\quad\quad\quad m = 24$

The least multiple is m $\quad\quad\quad\quad$ = 24
The second multiple is m+6 = 24+6 $\quad$ = 30
The third multiple is m+12 = 24+12 $\quad$ = 36
The multiples are 24, 30 and 36

Exercise 4:18

1. Write down the multiples of each of the following:

a) 8	c) 3	e) 7	g) 12	i) 15	k) 18	m) 40
b) 9	d) 6	f) 10	h) 11	j) 45	l) 24	n) 75

2. Given that W = {Multiples of 7 between 30 and 80}. Find n(W)
3. Find the sum of the first four multiples of 8
4. Circle the numbers which are not multiples of 8: 8, 24, 48, 18, 16, 78, and 64.
5. The sum of three consecutive multiples of 7 is 84. Find the multiples.
6. The sum of four consecutive multiples of 3 is 90. Find the multiples.
7. The total of two consecutive multiples of 9 is 81. Find their product.
8. Find the four consecutive multiples of 5 whose sum is 170.
9. A business woman sold 120 bags of cement in 3 consecutive days. A day, she sold 6 bags more than the previous day. How many bags were sold on the third day?

Finding the Lowest Common Multiple (LCM) of two or more numbers

Example 1

Find the Lowest Common Multiple (LCM) of 6 and 8

Approach a

$M_6 = \{6,12,18,\mathbf{24},30,36,42 \ldots\}$

$M_8 = \{8,16,\mathbf{24},32,40 \ldots\}$

$LCM = 24$

Approach b

2	6	8
2	3	4
2	3	2
3	3	1
	1	1

$LCM = 2x2x2x3$

$LCM = 24$

Example 2

Work out the LCM of 8, 12 and 15

2	8	12	15
2	4	6	15
2	2	3	15
3	1	3	15
5	1	1	5
	1	1	1

$LCM = 2x2x2x3x5$

$LCM = 120$

Exercise 4:19

Find the lowest common multiple (LCM) of;

a) 8 and 4	g) 7 and 8	m) 24 and 30	s) 3, 2, and 4
b) 5 and 10	h) 4 and 5	n) 40 and 30	t) 2, 3 and 6
c) 3 and 6	i) 6 and 12	o) 18 and 12	u) 6, 8 and 10
d) 3 and 9	j) 8 and 12	p) 24 and 36	v) 10,12 and 15
e) 4 and 6	k) 5 and 9	q) 48 and 30	w) 60,50 and 45
f) 15 and 30	l) 12 and 15	r) 90 and 72	x) 45,30 and 40

Application of LCM in our daily life

Example 1

Find the least number of sweets when shared by either 8 boys or 6 girls equally, leaves 2 sweets as remainder.

First work out the LCM of 8 and 6 after which add the remaining two sweets to the LCM to get the least number of sweets shared.

LCM of 8 and 6

$M_8 = \{8, 16, 24, 32, 40 \ldots\}$

$M_6 = \{6, 12, 18, 24, 30 \ldots\}$

$LCM = 24$

While finding the LCM, you can also use the other alternative of prime factorization.

Least number of sweets shared

24 + 2 = 26 sweets

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Example 2

What number is when divided by 10, 15 or 20 leaves no remainder

$M_{10} = \{10,20,30,40,50,60,70,...\}$
$M_{15} = \{15, 30, 45, 60, 75 ...\}$
$M_{20} = \{20, 40, 60, 80, 100 ...\}$
$LCM = 60$
  The number is 60

Example 3

Two bells ring at intervals of 30 and 50 minutes for lower and upper primary respectively.

a) After how many minutes will they ring together at the same time?

M_{30} = {30,60,90,120,150,180, ...}

M_{50} = {50,100,150,200,250...}

LCM = 150

After 150 minutes

b) If they ring together at 9:00am. When will they ring together again?

60 min = 1hr

150 min = $\underline{150}$ hrs
                50

150 min = 2 hour 30 minutes

  9 : 00
+ 2 : 30
 11 : 30 a.m

Note

"If a word like least, smallest, lowest is used, the problem is on LCM"

Exercise 4:20

1. What is the smallest number when divided by either 9 or 12 leaves no remainder?
2. Find the smallest number that can be divided by either 8 or 12 and 5 remains
3. What number is divisible by 7, 15 and 18 and leaves 4 as a remainder?
4. Soldiers were firing a target at intervals of 15, 20 and 25 minutes respectively.
   i) After how many minutes will they fire together?
   ii) If they fire together at 8:00a.m, at what time will they fire together again?
5. At Kampala bus park, buses travelling to Masaka and Mbale leave after every 40 minutes and 50 minutes respectively. The first buses to the two towns leave together at 7:00a.m. At what time will buses to the two towns leave Kampala together again?
6. Alex and Allen was told to visit the doctor every after 6 hours and 8 hours respectively. After how long will the two have to visit the doctor at the same time?
7. In Ndeku P/S, two bells ring at intervals of 40 minutes and 50 minutes for lower and upper primary section respectively. If they are rung together at 1:40p.m,
   a) After how long in hours will they take to ring together?
   b) At what time will they ring together again?
8. Three buckets have capacities are 10, 15 and 24 litres. Determine the capacity of the smallest tank that can exactly be filled using any of the buckets

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9. At Mpape Primary School, two bells are rung during intervals of 30 minutes and 40 minutes respectively to change lessons.
   a) After how long will the two bells take to ring?
   b) If they first rung together at 8:30a.m, at what time will they ring together again?
10. Three bells at Hozambe Primary School ring at intervals of 20 minutes, 15 minutes and 12 minutes. If they rang together the third time at 1:00p.m, when did they first ring together?
11. Two bells ring at intervals of 20 minutes and 30 minutes respectively. At what time will they ring together for the second time if they were first rung at 8:00a.m?
12. At MMM Primary School, two bells are rung during intervals of 30 minutes and 50 minutes respectively to change lessons.
    i) After how many hours will the two bells take to ring together again?
    ii) If they were first rung together at 7:40a.m, at what time will they ring together again?.
13. Machines A, B and C give out signals at intervals of 30, 50 and 24 seconds respectively. At 11:58am, they all gave a signal together. At what time will the three machines gave out signals at the same time?
14. Kiptegei, Kiprotich and Nzikuru can run round a circular field in 15 minutes, 30 minutes and 45 minutes respectively. They set off at 11:55a.m
    a) At what time will they be on the same line again?
    b) How many rounds will Kiptegei make?
15. Find the least number of pens that can be shared by 15 girls or 12 boys such that 5 pens remain.
16. On Thursday, Mukasa and Mukiibi were released from jail. They were told to report back after every 18 days and 12 days respectively. What day of the week will Mukasa and Mukiibi report themselves back to court at the same time?

Greatest common factor (GCF)
* A factor is a number that can divide another number without leaving a remainder.
* 1 is a factor of every number. Any number is a factor of itself.
* The Greatest Common Factor / Divisor (GCF/GCD) is the largest number that can divide two or more numbers without leaving a remainder.

Example

Find the GCF of 18 and 12.

Approach 1

F18 = {1,2,3,**6**,9,18}
F12 = {1,2,3,4,**6**,12}
GCF = 6

Approach 2

2	18	12
3	9	6
	3	2

GCF = 2 x 3
GCF = 6

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Exercise 4:21

1. Find the greatest common factor of;

a) 12 and 8	e) 12 and 15	i) 20, 30 and 40
b) 16 and 20	f) 18 and 30	j) 25, 30 and 35
c) 45 and 35	g) 84 and 90	k) 12, 18 and 30
d) 60 and 24	h) 24, 54 and 90	l) 15, 27 and 36

2. Find the highest number that can exactly divide 48 and 54.

3. Work out the highest common divisor of 72 and 96.

Application of GCF in our daily life.

Example 1

Alex bought animals feeds in sacks of 90kg, 120kg and 75kg. He sells the feeds in small sachets. What is the largest weight of the sachets that can be measured out of these sacks without leaving a remainder?

3	90	120	75
5	30	40	25
	6	8	5

GCF = 3x5
GCF = 15
    = 15kg

Example 2

Find the highest number of girls that can share 30 pens or 42 pens leaving no remainder.

2	30	42
3	15	21
	5	7

GCF = 2x3
GCF = 6
    = 6 girls

Example 3

Tracy is making balloon arrangement for a class party. She has 126 pink balloons, 105 white balloons and 84 orange balloons. She wants to group the balloons such that each group has the same number of balloons.

(a) Find the largest number of balloons in each group.

3	126	105	84
7	42	35	28
	6	5	4

    3 × 7 = 21 balloons

(b) How many groups were formed from pink balloons?

    126 ÷ 21 = 6 groups

Example 4

Find the highest number of pupils who can share 27 sweets, 43 sweets and 16 sweets such that 3 sweets, 1 sweet and 4 sweets remain respectively

First take away the respective remainders from the given numbers, then work out the GCF/HCF

27 – 3 = 24
43 – 1 = 42
16 – 4 = 12

GCF of 24, 42 and 12

2	24	42	12
3	12	21	6
	4	7	2

2x3
6 pupils

Exercise 4:22

1. Three wires of lengths 28m, 42m and 56m. They are to be cut into shorter pieces of equal length. Find the largest length of wire that can be cut from the wires.
2. Find the greatest number that can divide 24 and 18 and leaves no remainder
3. Find the highest number that can divide 14 and 20 leaving 2 as a remainder.
4. Work out the largest number that can divide either 40 or 34 leaving 4 as a remainder.
5. Find the highest number of pupils that can share either 21 mangoes or 27 oranges such that 3 mangoes and 3 oranges remain.
6. What is the greatest number that can divide 11, 13 and 18 to leave remainders 3, 1 and 2 respectively.
7. Find the greatest number of pupils that can share 51 pens, 31 pens and 41 pens such that 3 pens, 7 pens and 5 pens remain respectively.
8. Find the largest number of pupils such that they share 30 pens, 6 pens remain and when they share 21 pens, 3 pens remain.

Relationship between numbers, their LCM and GCF

Note

The product of two numbers of two numbers is equal to the product of their LCM and GCF

* Product = LCM x GCF
* 1st No. = LCM x GCF
    2nd No.
* 2nd No. = LCM x GCF
    1st No.

* LCM = Product of numbers ÷ GCF
* GCF = Product of numbers ÷ LCM
* LCM = GCF × Product of the ratio of the numbers

Example 1

The LCM of two numbers is 72 and their GCF is 6 if one of the numbers is 24, find the other number.

Approach a
Let the number be m
   Product = LCM x GCF
   24 x m = 72 x 6
   $\frac{24 \times m}{24}$ = $\frac{72 \times 6}{24}$
      m = 18

Approach b
Second number = $\frac{LCM \times GCF}{1st\ No.}$

= $\frac{72 \times 6}{24}$

= 18

Example 2

The least number that can be divided by either 18 or g leaving 4 as the remainder is 40. The greatest common factor (GCF) of 18 and g is 6. Find the value of g.

LCM of 18 and g
LCM = 40 – 4
LCM = 36

Value of g
Second number. = $\frac{LCM \times GCF}{1st\ No.}$

g = $\frac{36 \times 6}{18}$

g = 2x6

g = 12

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Exercise 4:23

1. The LCM of two numbers is 48 and their GCF is 4. If one of the numbers is 16, find the second number
2. The Lowest Common Multiple (LCM) of two numbers is 144 and their Greatest Common Factor (GCF) is 6. If one of the numbers is 36, find the second number.
3. The LCM of m and p is 72 and their GCF is 6. If m = 18, find the value of p.
4. The GCF of two numbers is 6. Their LCM is $2^2 \times 3^2 \times 5$. One of the numbers is 72. Find the other number
5. The product of two numbers is 144. If one of the numbers is 18, find the HCF of the numbers.
6. The least number that can be divided by either 12 or g leaving 3 as the remainder is 39. The greatest common factor (GCF) of 12 and t is 6.
    (a) Find the LCM of g and 12.
    (b) Find the value of g.
7. The smallest number that can be divided by two numbers leaving 13 as the remainder is 85. The greatest common factor (GCF) of the two numbers is 6. One of the numbers is 24. Find the other number.
8. Given that Fx = $\{2_1, 2_2, 3_1, 5_1\}$, Fx∩Fy = $\{2_1, 2_2, 3_1\}$ and the lowest common multiple (LCM) of x and y is 180. Find the value of y
9. Two numbers are in the ratio of 2:3 respectively. Their LCM is 24.
    (a) Find the GCF of the two numbers
    (b) Work out the range of the two numbers,
10. The product of two numbers is 1728. Their GCF is 18. Find their LCM
11. Two numbers are in the ratio of 5:6 respectively. Their GCF is 12.
    a) Find the two numbers.
    b) Work out their LCM.

"Think as a mathematician"

1. The prime factors of h and 630 are shown in Venn diagram below. The GCF of 630 and h is 14. Use the Venn diagram to answer questions that follow.

F_{630} F_h

3_1 2_1 2_2
k 2_3
3_2 m 13_1
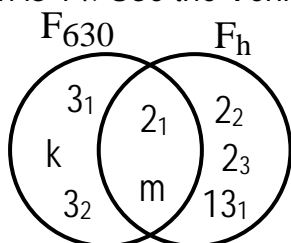
a) Find the value of
    i) m   iii) k
    ii) h
b) Work out the LCM of 630 and h

2. A man had sh. 62,500 and received more money in three consecutive days. A day, he received sh. 2,300 less than the previous day. By the end of the third day, he had sh.101,600 altogether. How much money did he have by the end of the second day?
3. In a class, pupils shared sweets such that each gets 5 sweets. The least number of sweets that could be shared by either girls or boys such that 3 sweets remain was 217. There are 24 girls in the class. How many sweets did boys share?