



INTEGERS

What is an integer?

- Integers are whole numbers, positive or negative including zero.
- All numbers with a plus sign before them are positive integers. While those with a minus sign before them are negative integers.
- Zero is neither negative nor positive.

Note :

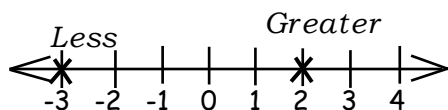
- * The bigger the negative integer, the less the value e.g. -9 is less than -2 and -2 is greater than -9 .
- * Zero is greater than all negative integers. Therefore -6 is less than 0 .
- * Any positive integer is greater than all negative integers e.g. $+2$ is greater than -8 and -8 is less than $+2$.

Comparing integers

Example

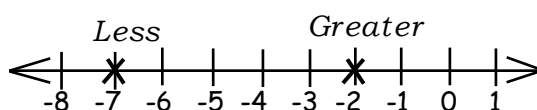
Use $<$, $>$ or $=$ to complete the statements below.

a) -3 _____ $+2$



$-3 < +2$

b) -2 _____ -7



$-2 > -7$

Exercise

1. Use $<$, $>$ or $=$ to complete the statements below.

i) 0 _____ -7

iv) $+1$ _____ -9

vii) $+4$ _____ -10

ii) $+8$ _____ 8

v) -23 _____ -13

viii) 0 _____ $+5$

iii) -2 _____ -15

vi) $+3$ _____ -1

ix) -9 _____ 0

2. Which integer is less than the other?

a) $+3$ or $+6$

c) -8 or -10

e) -6 or -13

b) -12 or $+5$

d) -4 or 0

f) 0 or -6

3. Which integer is greater than the other?

a) $+6$ or -4

c) 0 or $+5$

e) -6 or -13

b) -8 or -2

d) -4 or 0

f) $+3$ or -9

4. The temperature on Mt. Rwenzori and Mt. Elgon was -10°C and -7°C respectively. Which of the two mountains had the higher temperature?

TOPIC 8: INTEGERS AND FINITE SYSTEMS



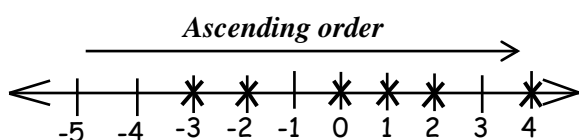
Ordering integers

Ascending order (Increasing order) means from lowest to greatest.

Descending order (decreasing order) means from greatest to lowest.

Example 1

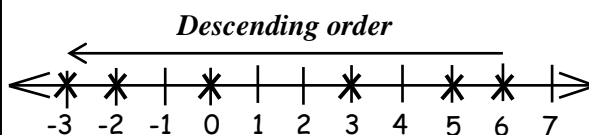
Arrange $+4, -2, 0, -3, +1, +2$ in ascending order.



$-3, -2, 0, +1, +2, +4$

Example 2

Arrange $-2, +5, +6, -3, 0, +3$ in descending order.



$+6, +5, +3, 0, -2, -3$

Exercise

1. Arrange the following in ascending order.

- | | | |
|------------------------------|-----------------------------|-----------------------------|
| a) $-1, +5, -4, -2, 0, +2$ | d) $+5, -1, -4, -2, +3, +1$ | g) $-4, +3, -10, +4$ |
| b) $-8, -1, +3, +2, -4, -7$ | e) $-4, +5, -1, 0, 2$ | h) $+1, -1, 0, +4, -2$ |
| c) $4, 0, -1, 3, -7, -5, -8$ | f) $+7, -5, -3, 0, +3$ | i) $-6, -1, +3, +7, -4, -7$ |

2. Arrange these from the lowest to the greatest.

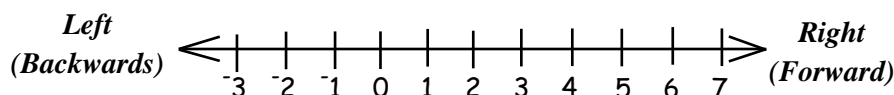
- | | |
|--------------------|------------------------|
| a) $-3, -6, +6, 0$ | b) $-9, -4, 0, +4, -1$ |
|--------------------|------------------------|

3. Arrange the following integers in descending order.

- | | | |
|-----------------------------|-----------------------------|-------------------------------|
| a) $-2, +5, -4, -3, 0, +2$ | d) $+6, -1, -7, -2, +3, +4$ | g) $-4, -5, +1, 0, +3$ |
| b) $-6, -1, +3, +7, -4, -7$ | e) $4, -8, 6, -5, -1$ | h) $-6, -7, +1, 0$ |
| c) $+5, 0, -2, +3, -5$ | f) $+3, 0, +2, -5, -2$ | i) $-4, 0, -1, 3, -7, -5, -8$ |

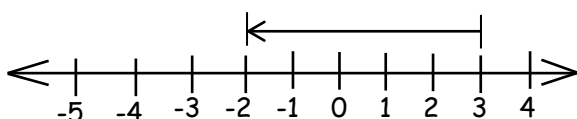
Positing integers on a number line

Study the number line below.



Example 1

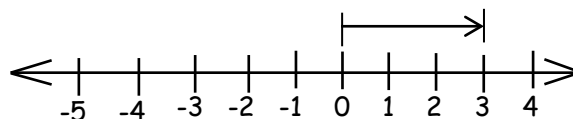
Which integer is 5 steps to the left of 3?



The integer is -2

Example 2

Which integer is 3 steps to the right of 0?



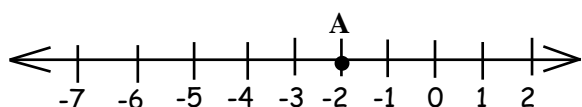
The integer is $+3$

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Exercise

- What name is given to integers which are to the left of zero?
- Name the integers to the right of zero.
- Which integer is 3 steps to the left of 0?
- Which integer is 5 steps to the left of;
 - 2
 - 0
 - +3
 - 1
 - 3
 - +5
- Write the integers between -1 and -6.
- Which integer is 6 steps to the right of +3?
- Find the integer which is 7 steps to the right of -4
- Write the first 4 positive integers in decreasing order.
- How many integers are there between +1 and +6.
- Write the first 5 negative integers in ascending order.
- A frog jumped 3 steps to the left from point A to point B.



Locate point B on the number line.

- In which position is -4 from +2 on a number line?

Addition of integers

Example 1

Workout : $+3 + +2$

$$+3 + +2$$

$$+3 + (+2)$$

$$+3 + 2$$

$$+5$$

Example 2

Simplify : $+4 + -5$

$$+4 + -5$$

$$+4 + (-5)$$

$$+4 - 5$$

$$-1$$

Example 3

Add -2 to -6

$$-6 + -2$$

$$-6 + (-2)$$

$$-6 - 2$$

$$-8$$

Exercise

- Simplify the following;
 - $+4 + 3$
 - $-6 + 8$
 - $+6 + +2$
 - $-8 + +8$
 - $+3 + -4$
 - $-9 + (-6)$
 - $-12 + +3$
 - $-5 + -8$
 - $-7 + -6$
 - $-2 + -3$
 - $+3 + -6$
 - $+9 + -7$
- Find the sum of -6 and 10
- Add +4 to -13
- What integer must be reduced by -3 to get +2?
- The sum of 4 consecutive integers is 2. Find these integers.
- The range of two integers is +7. If the smaller integer is -4, find the other integer.

TOPIC 8: INTEGERS AND FINITE SYSTEMS

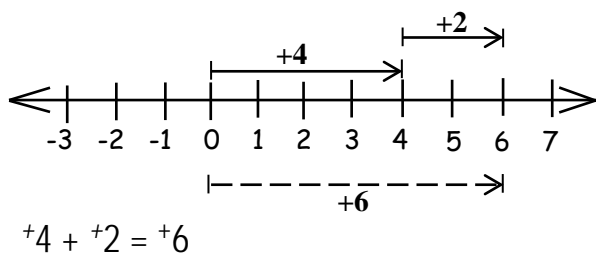


7. The median of 5 consecutive integers is -1 . Find these integers.
8. The mean of four consecutive integers is -5 .
 - a) Find these integers
 - b) Find the sum of the least and the greatest integer.
9. Simplify: $-4 + -2 + +8$.

Addition of integers using a number line

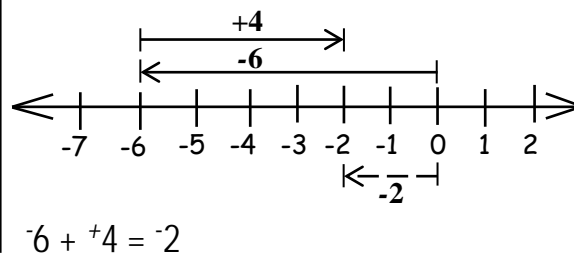
Example 1

Workout $+4 + +2$ using a number line.



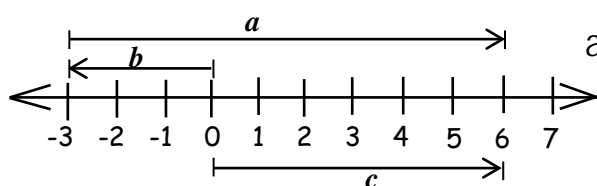
Example 2

Workout $-6 + +4$ using a number line.



Example 3

Study the number line below and use it to answer questions that follow.



a) Write the integers represented by the arrows.

- i) $a = +9$ ii) $b = -3$ iii) $c = +6$

b) Write down the mathematical sentence shown on the number line.

$$b + a = c$$

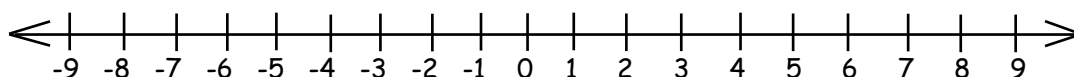
$$-3 + +9 = +6$$

Exercise

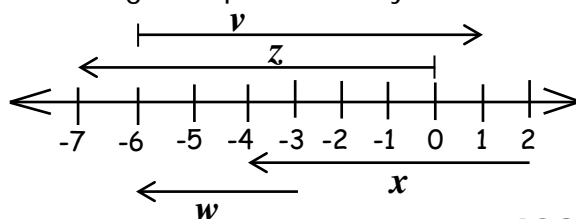
1. Work out the following using a number line.

- | | | | |
|--------------|--------------|--------------|--------------|
| a) $+4 + +3$ | c) $+1 + +4$ | e) $+6 + -8$ | g) $-4 + -5$ |
| b) $+2 + +5$ | d) $+7 + -3$ | f) $-3 + -2$ | h) $-3 + +7$ |

2. Use the number line below to work out: $-8 + +3$



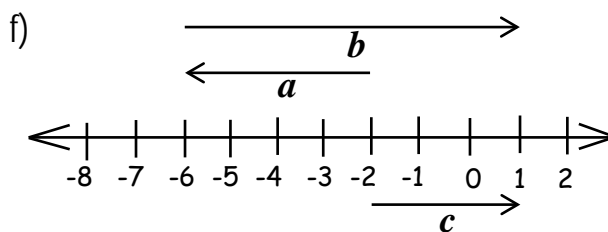
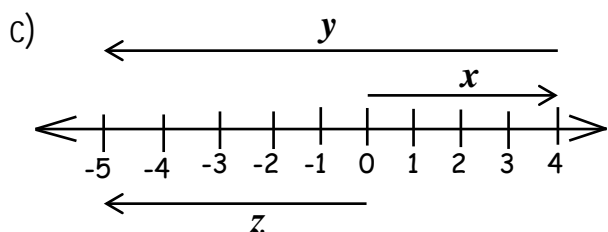
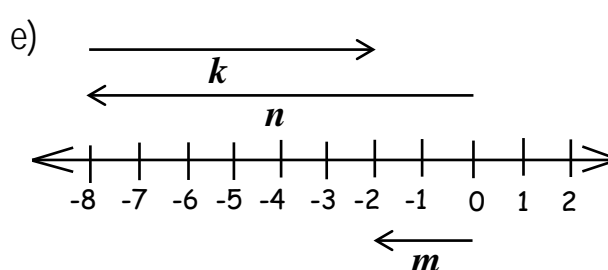
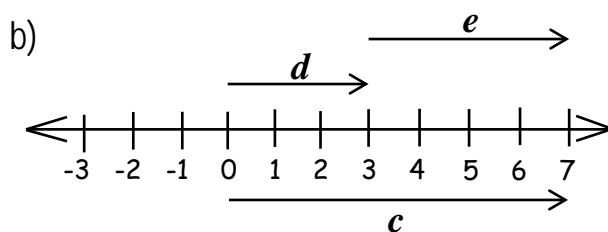
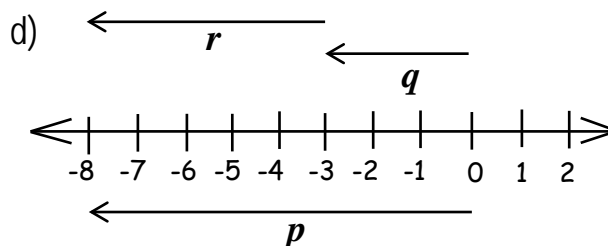
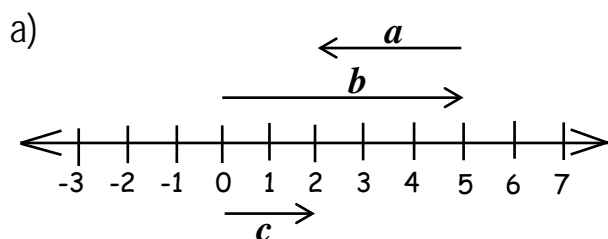
3. Write the integers represented by the arrows on the number line below.



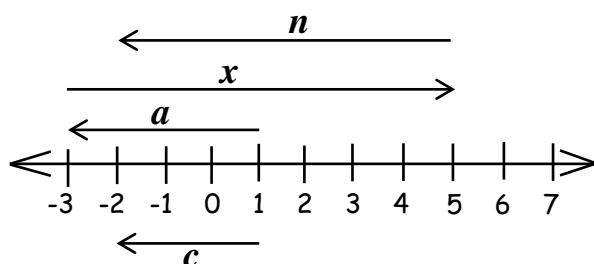
TOPIC 8: INTEGERS AND FINITE SYSTEMS



4. Write the integers represented by the arrows on each of the number line then write the mathematical sentence.



5. Study the number line below and use it to answer questions that follow:



a) Write the integers represented by

i) n =

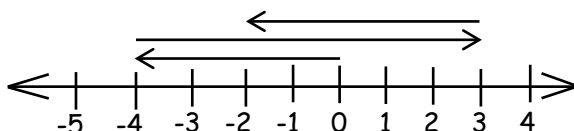
ii) a =

iii) x =

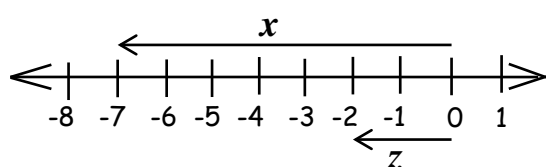
iv) c =

b) Write down the mathematical sentence shown on the number line above.

6. Write the mathematical statement shown on the number line below.



7. A candidate added two integers x and y using a number line to get the answer " z ".



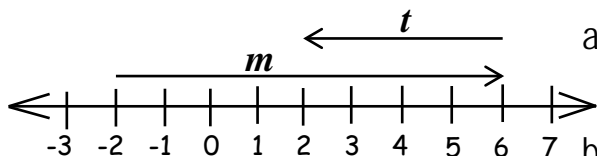
a) Draw arrow y on the number line to make the sentence true.

b) Write the integers represented by x , y and z .

c) Write down the additional mathematical sentence.



8. Two integers were added using the number line below.



a) On the number line above, draw arrow n to represent the answer.

b) Write the value of the integers represent by:

i) m ii) t iii) n

c) Write the mathematical statement shown on the number line.

Additive inverse

Additive inverse is the number when added to the original number will result in a sum zero.

The additive inverse of $+7$ is -7

Example 1

Find the additive inverse of -6 .

Let the additive inverse be y

$$-6 + y = 0$$

$$-6 + 6 + y = 0 + 6$$

$$y = +6$$

The additive inverse of -6 is $+6$

Example 2

Find the additive inverse of 4 .

Let the additive inverse be y

$$4 + y = 0$$

$$4 - 4 + y = 0 - 4$$

$$y = -4$$

The additive inverse of $+4$ is -4

Exercise

1. Find the additive inverse of each of the following.

a) 5

b) $+19$

e) -6

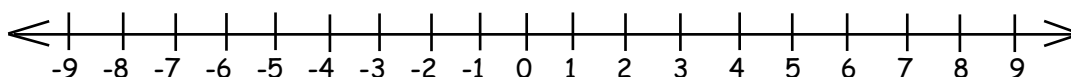
b) $+8$

e) -12

f) -78

2. Find the additive inverse of the sum -3 and $+5$.

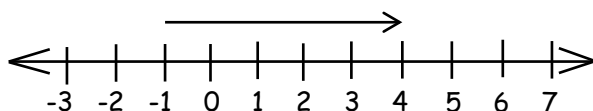
3. On the number line below, draw an arrow to show the additive inverse of $+4$.



4. Add the additive inverse of -3 to the additive inverse of $+4$.

5. What must be added to -102 to get zero?

6. Study the number line below carefully and use it to answer the question that follows



Find the additive inverse of the integer represented by the arrow.

TOPIC 8: INTEGERS AND FINITE SYSTEMS



Subtraction of integers

Example 1

Simplify: $+3 - +2$

$$+3 - +2$$

$$+3 - (+2) \quad -x+ = -$$

$$+3 - 2$$

$$+1$$

Example 2

Simplify: $+5 - +8$

$$+5 - +8$$

$$+5 - (+8) \quad -x+ = -$$

$$+5 - 8$$

$$-3$$

Example 3

Workout: $+4 - -2$

$$+4 - -2$$

$$+4 - (-2) \quad -x- = +$$

$$+4 + 2$$

$$+6$$

Example 4

Take away $+4$ from -5

$$-5 - +4$$

$$-5 - (+4) \quad -x+ = -$$

$$-5 - 4$$

$$-9$$

Exercise

1. Simplify:

a) $+3 - +1$

d) $+10 - +7$

g) $-5 - -3$

j) $-5 - +3$

b) $+6 - +9$

e) $+9 - -1$

h) $-10 - -23$

k) $-2 - -7$

c) $+4 - +3$

f) $+2 - -5$

i) $-4 - +1$

l) $-7 - +10$

2. Take away -4 from $+6$

3. Subtract $+2$ from -5

4. What must be added to -2 to become -6 .

5. Find the integer which is 3 less than -4

6. The sum of two integers is -5 . If one of the integers is $+4$, find the other integer.

7. Decrease -4 by 3.

8. Reduce $+1$ by $+6$

9. The range of two integers is $+4$. If the larger integer is -3 , find the smaller integer.

10. Solve: $-4 - -m = -3$

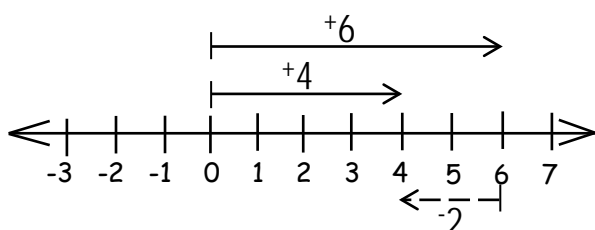
Subtracting integers using a number line

Note;

- When subtracting integers, both arrows start from zero (same point)
- The gaps between the arrow heads represents the answer
- The gap should begin from the last to the first arrow head.

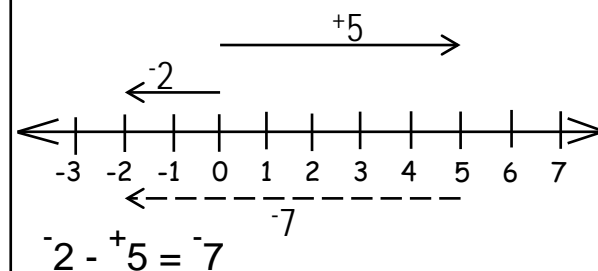
Example 1

Work out $+4 - +6$ using a number line.



Example 2

Workout $-2 - +5$ using a number line.

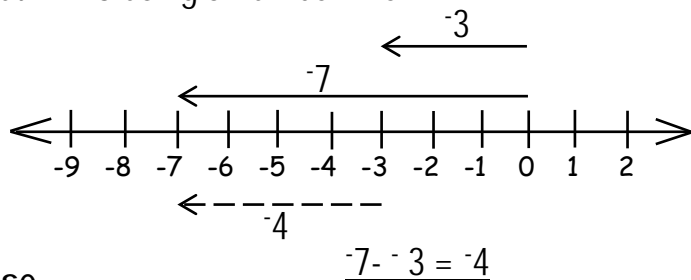


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Example 3

Workout $-7 - 3$ using a number line.

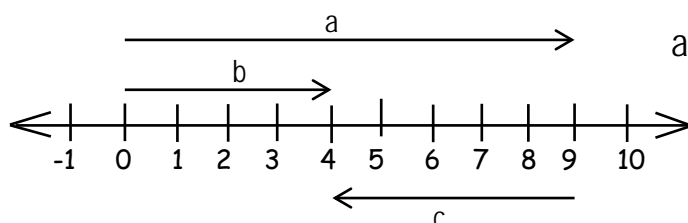


Exercise

1. Work out the following using a number line.

- | | | | |
|--------------|--------------|--------------|--------------|
| a) $+3 - +7$ | d) $-4 - -2$ | g) $-6 - -1$ | j) $-2 - +5$ |
| b) $+5 - +4$ | e) $-2 - -7$ | h) $+3 - -6$ | k) $-4 - +1$ |
| c) $+8 - +3$ | f) $-3 - -5$ | i) $+2 - -6$ | l) $+7 - -5$ |

2. Study the number line below and use it to answer questions that follow.

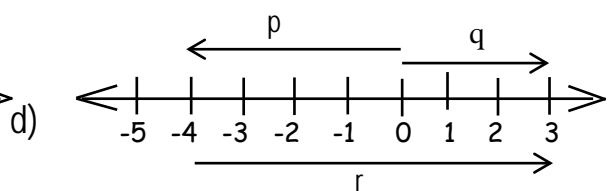
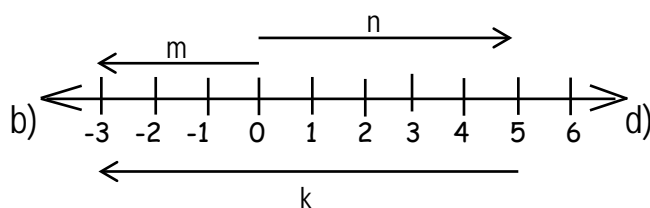
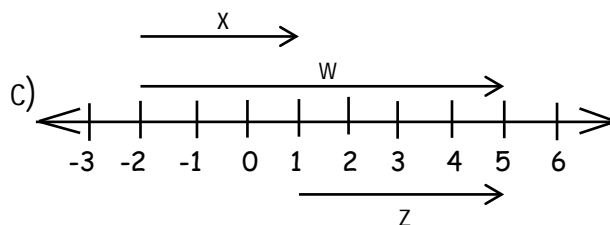
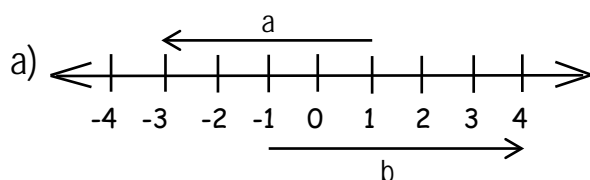


a) Write the integers represented by

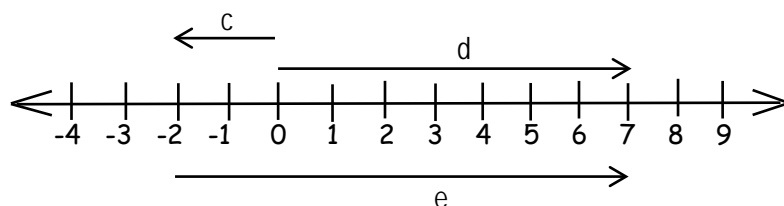
- i) $a = \dots\dots\dots$
 ii) $b = \dots\dots\dots$
 iii) $c = \dots\dots\dots$

b) Write the mathematical sentence shown on the number line.

3. Write the integers represented by arrows.



4. Study the number line below and use it to answer questions that follow.



a) Write the in integers represented by:

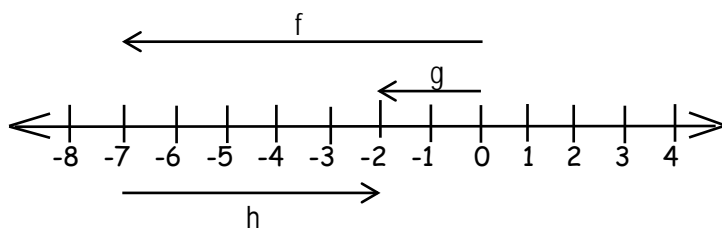
- i) $e = \dots\dots\dots$ ii) $d = \dots\dots\dots$ iii) $c = \dots\dots\dots$

b) Write the mathematical statement shown on the number line above.

TOPIC 8: INTEGERS AND FINITE SYSTEMS

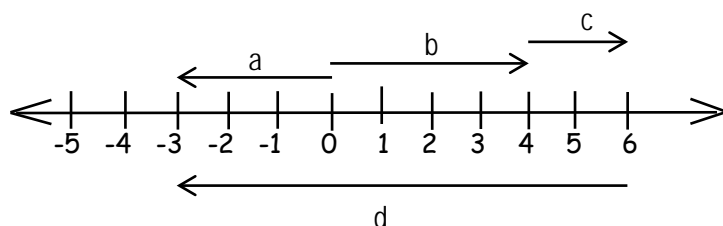


5. Study the number line below and use it to answer questions that follow.



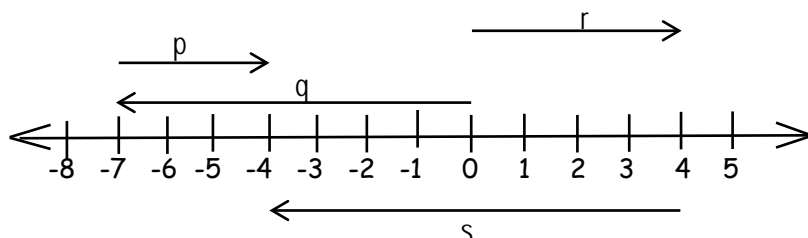
Write the mathematical sentence shown on the number line above.

6. Below is a number line. Use it to answer questions that follow.



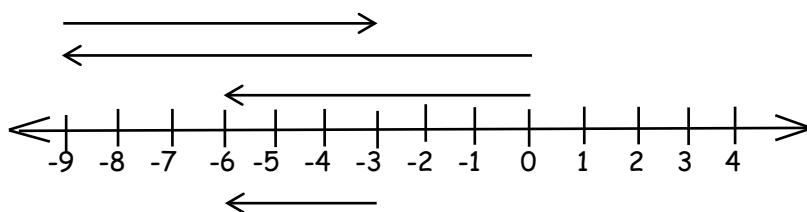
- Write the integers represented by each arrow.
- Write the mathematical sentence on the number line.

7. Study the number line below carefully and use it to answer the question that follows.



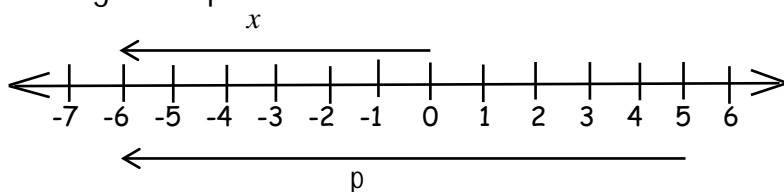
Write the mathematical statement on the number line above.

8. Write the mathematical sentence on the number line below.



9. Work out $-3 - (-4 + +7)$ using a number line.

10. A pupil subtracted two integers x and y using a number line. He represented his answer using arrow p as shown below.



- On the number line above, draw arrow y .
- Write the integers.

TOPIC 8: INTEGERS AND FINITE SYSTEMS



Multiplying integers

Below is a quick summary for the rules of multiplying integer.

| Signs of integers | Answer sign | Example |
|-------------------|-------------|--------------------------|
| $+ \times +$ | $+$ | $(+3) \times (+5) = +15$ |
| $- \times -$ | $+$ | $(-4) \times (-2) = +8$ |
| $+ \times -$ | $-$ | $(+6) \times (-3) = -18$ |
| $- \times +$ | $-$ | $(-5) \times (+2) = -10$ |

Example

Work out the following.

a) $+2 \times +9$

$+2 \times +9 = +18$

c) -7×-6

$-7 \times -6 = +42$

e) $+8 \times -4$

$+8 \times -4 = -32$

b) $-5 \times +4$

$-5 \times +4 = -20$

d) $-2 \times -3 \times -4$

$-2 \times -3 \times -4 = -24$

f) $-3 \times +4 \times -1$

$-3 \times +4 \times -1 = +12$

Exercise

1. Workout the following;

a) $+8 \times +3$

e) -10×-7

i) $-28 \times +2$

b) $+6 \times +2$

f) $+1 \times -13$

j) $-5 \times +5$

c) $4 \times +5$

g) $+9 \times -8$

k) $6 \times -1 \times -10$

d) -1×-8

h) $-7 \times +2$

l) $+8 \times -12 \times -15$

2. Work out the product of -6 and -4 .

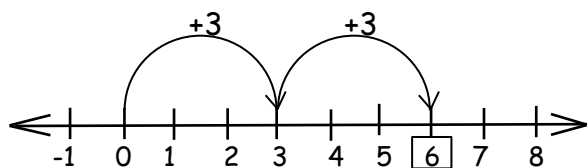
3. Simplify: $-4 \times +5 \times -2$

Multiplying integers using a number line

Example 1

Work out 2×3 using a number line.

2×3 means 2 groups of 3

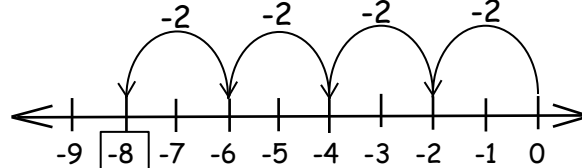


$2 \times 3 = 6$

Example 2

Work out 4×-2 using a number line.

4×-2 means 4 groups of -2



$4 \times -2 = -8$

Exercise

1. Workout the following using a number line.

a) 2×4

e) 3×5

i) 3×-2

b) 3×2

f) 2×-4

j) 5×-3

c) 4×2

g) 1×-8

k) -2×5

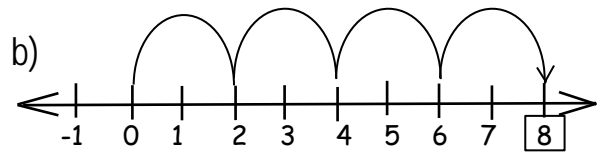
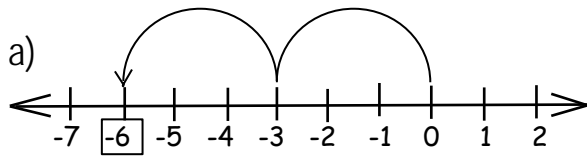
d) 5×2

h) 2×-5

l) -4×3



2. Write the mathematical sentence on the number lines below.



Division of integers

Remember

$$+ \div + = +$$

$$- \div - = +$$

$$+ \div - = -$$

$$- \div + = -$$

Examples

Work out the following.

a) $-15 \div +3 = -5$

b) $-24 \div -6 = 4$

c) $51 \div -17 = -3$

Exercise

Work out the following.

a) $120 \div -3$

e) $132 \div -11$

i) $-32 \div 4$

b) $-42 \div +7$

f) $-72 \div 12$

j) $54 \div -6$

c) $8 \div -1$

g) $-18 \div -6$

k) $-100 \div 4 \div -5$

d) $+60 \div +15$

h) $-27 \div -9$

l) $-144 \div -4 \div -9$

Application of integers in real life situation

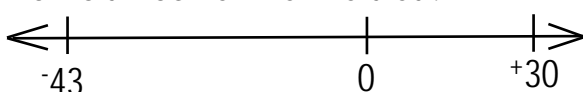
Words that may mean positive and negatives in real life situation.

| Positive (+) | Negative (-) |
|------------------------------|--------------------------------|
| receive / make / get (money) | spend / pay / withdraw (money) |
| profit / gain / win | loss |
| increment | deduct |
| bonus | drop |
| later | earlier |
| high | lower |

| Positive (+) | Negative (-) |
|-----------------|------------------|
| increase / rise | decrease / drop |
| above | below |
| load | offload / remove |
| over | under |
| gain | loss |
| ascend | descend |
| birth | death |

Example 1

John was born in 43BC and died in 30AD. How old was he when he died?



$$+30 - -43$$

$$+30 - (-43)$$

$$+30 + 43$$

$$+73$$

He was 73 years

Example 2

Mayiga arrived at Masaka recreation ground for a match between Buddu and Gomba 35 minutes before the kickoff but match delayed by 15 minutes. For how long did Mayiga wait for the match?

$$+15 - (-35) = +15 + 35$$

$$= +50$$

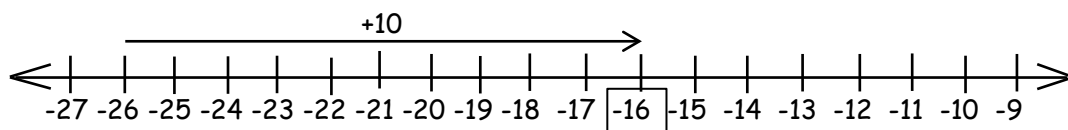
He waited for 50 minutes

TOPIC 8: INTEGERS AND FINITE SYSTEMS



Example 3

The temperature on mountain Rwenzori was -26°C at midnight. It has rose by 10°C . Find the new temperature on mountain Rwenzori.



Original temperature + Rise in temperature = New temperature

$$-26^{\circ}\text{C} + 10^{\circ}\text{C} = -16^{\circ}\text{C}$$

Example 4

A mountain climber recorded the temperature at the foot of the mountain as 12°C . On reaching its peak, the temperature had dropped to -9°C . What was the temperature range.

$$12^{\circ}\text{C} - -9^{\circ}\text{C}$$

$$12^{\circ}\text{C} - (-9^{\circ}\text{C})$$

$$12^{\circ}\text{C} + 9^{\circ}\text{C} = 21^{\circ}\text{C}$$

Remember

Range = highest value – lowest value

Example 5

In a quiz contest, 5 marks were awarded for any correct response and 2 marks deducted for any wrong response given by a contestant. Given that 20 questions were asked by the panel.

- a) Find the score for a contestant who gave 15 correct responses.

Correct responses $\rightarrow 15$

Wrong responses $\rightarrow 20 - 15 = 5$

$$\text{Score} = (15 \times +5) + (5 \times -2)$$

$$= +75 + (-10)$$

$$= +75 - 10$$

$$= 65 \text{ marks}$$

- b) Calculate the score for a contestant who gave 7 wrong responses.

Wrong responses $\rightarrow 7$

Correct responses $\rightarrow 20 - 7 = 13$

$$\text{Score} = (13 \times +5) + (7 \times -2)$$

$$= +65 + (-14)$$

$$= +65 - 14$$

$$= 51 \text{ marks}$$

- c) Find the score for the contestant who gave 17 correct responses.

Correct responses $\rightarrow 17$

Wrong responses $\rightarrow 20 - 17 = 3$

$$\text{Score} = (17 \times +5) + (3 \times -2)$$

$$= +85 + (-6)$$

$$= +85 - 6$$

$$= 79 \text{ marks}$$

TOPIC 8: INTEGERS AND FINITE SYSTEMS



d) If a contestant scored 72 marks. How many correct responses were given?

Method 1

Let number of correct responses be n

| Correct | Wrong | Total |
|---------|----------|-------|
| n | $20 - n$ | 20 |

$$(5 \times n) + -2(20 - n) = 72$$

$$5n - 2(20 - n) = 72$$

$$5n - 40 + 2n = 72$$

$$5n + 2n - 40 = 72$$

$$7n - 40 = 72$$

$$7n - 40 + 40 = 72 + 40$$

$$7n = 112$$

$$\frac{7n}{7} = \frac{112}{7}$$

$$n = 16$$

16 correct responses

Method 2

Total score

$$20 \times 5 = 100 \text{ marks}$$

Total marks lost

$$100 - 72 = 28 \text{ marks}$$

Marks lost per question

$$5 + 2 = 7 \text{ marks}$$

Number of wrong responses

$$28 \div 7 = 4$$

Number of correct responses

$$20 - 4 = 16$$

16 correct responses

Exercise

1. Cate had sh. 2000 and gave out sh. 1250. How much money did she remain with?
2. A trader made a profit of sh. 18000 on the first day and a loss of sh. 5000 on the second day. What did he end up with?
3. A boy has a debt of sh. 1500. If he has sh. 4000, how much will he remain with?
4. Kalungi borrowed sh. 3400 from his friend. His mother gave him sh. 6200. If he pays the debt, how much money will he remain with?
5. A shopkeeper made a loss of sh. 7000 and another loss of sh. 6500. Find the total loss which was incurred.
6. Tom made 80 bricks in one day. 18 bricks got damaged by rain and 15 got broken. How many good bricks was he left with?
7. Luke was born in 30BC and died in 15AD. How old was he when he died?
8. Nambi was born in 7BC and died in 54AD. Find Nambi's age then.
9. A man was born in 12BC and died at the age of 38 years when did he die?
10. A boy died in 8AC at the age of 13 years. In which year was he born?
11. Pythagoras who was born in 580BC and died at the age of 80 years. In which year did he die?
12. The temperature on a mountain was -17°C at midnight. It is now -2°C . Find the temperature range.
13. The temperature of the patient was 41°C . It dropped by 3°C . Find the new temperature of the patient.

TOPIC 8: INTEGERS AND FINITE SYSTEMS



14. The temperature of ice was -6°C . When put a refrigerator, the temperature dropped by 4°C . Find the final temperature of ice.
15. The original temperature of water in a kettle was 18°C . The temperature rose by 5°C per minute when heated. Find the temperature of water in the kettle after 14 minutes.
16. The temperature of an object dropped by 4°C per minute to -15°C . Find the temperature of the object 5 minutes before.
17. Kisa arrived at the bus station 18 minutes before the normal departure time of the bus to Arua. If the bus was 6 minutes late, how long did he wait at the station.
18. Akiki misses the train by 7 minutes. If the next train arrives after 15 minutes, how long does she wait for the next train.
19. Messi played a video game and won 120 points and another 40 points then lost 55 points. What was his final score?
20. Kato won 60 points, then another 23 points but lost 35 points and another 40 points. What was his final score?
21. While marking a test of 25 questions, a teacher awards 4 marks for every correct answer and deducts a mark for each wrong answer.
 - a) Find the score for a pupil who got 16 correct answers.
 - b) If a pupils scored 35 marks, how many answer did get correct?
22. In an interview, 3 marks are awarded for any correct response and 2 marks are deducted for any wrong response. The interview had 20 questions.
 - a) Find the score for a candidates who failed 3 questions.
 - b) If a candidates got 35 marks, how many response were correct?
23. A famer had 30 eggs. He was given sh. 300 for every good egg and fined sh. 200 for any egg found rotten. After giving out all the eggs, he ended up with sh. 7000/=. Find number of rotten eggs.
24. Okiring had a box containing 50 pens. Some were blue and the rest were green. By selling the blue pens, he realised a profit of sh.150 per pen and a loss of sh. 50 per pen after selling the green pens. He ended up with a profit of sh. 5500. Find the number of blue pens in the box.
25. My father bought a duck and a cock. He later sold a cock at sh. 5500 less than a duck which was sold at loss of sh. 2500. If they were sold at sh. 52000 altogether, how much money did he sell the cock?
26. Mudondo bought a tray of 30 eggs at sh. 400 each .On her way back home, some eggs got broken. She sold the remaining eggs each at sh.450 making a loss of sh. 3000. How many eggs got broken ?
27. A man was 15 metres away from to mango tree. He moved 7 metres towards it and then 4 metres away. How far from the tree was the man at last?
28. A cat fell into a pit 30 metres deep. For every 4m it climbed, it slid back 1m. How many times did the cat do this before it got out?
29. A business woman recorded losses of sh. 45,000 each day for three days. What was the loss for the three days?

TOPIC 8: INTEGERS AND FINITE SYSTEMS



30. The freezing point of water is 32°F . Okia added some chemicals and found out that the freezing point went down by 12°F . What was the freezing point of water with chemicals?
31. Pythagoras was born in 582BC. Isaac Newton was born in 1643AD. Find the range of their years of birth.
32. The average temperature at the South Pole is -45°F . The average temperature on the equator is 92°F . How much warmer is the average temperature on the Equator than at the South Pole?
33. It will be -3°C tonight. The weatherman predicts it will be 22°C warmer by noon tomorrow. What will be the temperature by noon tomorrow?
34. A plane is flying at a height of 4500 metres above the sea level. At a particular point, it is exactly above a submarine floating 1800 metres below the sea level. What is the vertical distance between them?
35. To break the record, a runner had to complete a 100 metre race in 30 seconds. The winner broke it by seconds. The last runner ran for 42 seconds. Find the range of the time taken.
36. A watch loses 4 seconds every one hour. How many minutes will it lose in five days?
37. A clock face shows 7 minutes to 12:00 noon but the clock gains time by 4 minutes. What is the exact time?

FINITE SYSTEMS / CLOCK ARITHMETIC

Addition in finite system

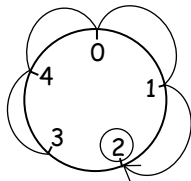
Example 1

Add: $3 + 4 = \underline{\quad}$ (finite 5)

Method 1

$3 + 4 = \underline{\quad}$ (finite 5)
 $7 = \underline{\quad}$ (finite 5)
 $7 \div 5 = 1 \text{ remainder } 2$
 $3 + 4 = \underline{2}$ (finite 5)

Method 2



$3 + 4 = \underline{2}$ (finite 5)

Example 2

Workout: $5 + 26 = \underline{\quad}$ (finite 7)

$5 + 26 = \underline{\quad}$ (finite 7)
 $31 = \underline{\quad}$ (finite 7)
 $31 \div 7 = 7 \text{ remainder } 3$
 $5 + 26 = \underline{3}$ (finite 7)

Exercise

1. Work out the following.

a) $3 + 2 = \underline{\quad}$ (finite 6)

b) $4 + 3 = \underline{\quad}$ (finite 7)

c) $2 + 5 = \underline{\quad}$ (finite 5)

d) $1 + 12 = \underline{\quad}$ (finite 4)

e) $7 + 43 = \underline{\quad}$ (finite 7)

f) $8 + 6 = \underline{\quad}$ (finite 7)

f) $5 + 16 = \underline{\quad}$ (mod 7)

g) $2 + 48 = \underline{\quad}$ (finite 7)

h) $9 + 23 = \underline{\quad}$ (finite 12)

2. Work out the following using a dial.

a) $2 + 5 = \underline{\quad}$ (finite 6)

b) $4 + 3 = \underline{\quad}$ (finite 7)

c) $1 + 5 = \underline{\quad}$ (finite 4)

d) $3 + 6 = \underline{\quad}$ (finite 7)

f) $6 + 23 = \underline{\quad}$ (mod 7)

e) $3 + 19 = \underline{\quad}$ (finite 5)

TOPIC 8: INTEGERS AND FINITE SYSTEMS



3. Complete the tables below.

a)

Finite 7

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | |
| 2 | | | | | | | |
| 3 | | | | | | | |
| 4 | | | | | | | |
| 5 | | | | | | | |
| 6 | | | | | | | |

b)

Finite 5

| + | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | | | | | |
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |

c)

Finite 4

| + | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | | | | |
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |

4. Simplify:

a) $2 + 6 + 1 = \underline{\hspace{1cm}}$ (finite 7)

b) $12 + 7 + 13 = \underline{\hspace{1cm}}$ (finite 10)

c) $4 + 5 + 92$ (finite 5)

5. Keep on adding the given finite to find the equivalent whole numbers.

a) 4 (finite 5) = 4, 9, , ,

f) 5 (finite 7) = 5, 12, 19, , ,

b) 1 (finite 7) = 1, 8, 15, , ,

g) 10 (finite 11) = 10, 21, 32, , ,

c) 2 (finite 12) = 2, 14, , ,

h) $9 \pmod{12} = 9, 21, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$

d) 7 (finite 10) = 7, 17, 27, , ,

e) $3 \pmod{6} = 3, 9, 15, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$

Subtraction in finite systems

Example 1

Work out: $2 - 5 =$ (finite 7)

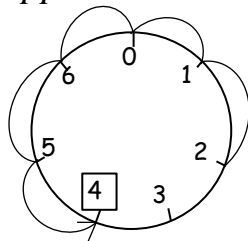
Approach 1

$2 - 5 = \underline{\hspace{1cm}}$ (finite 7)

$(2+7) - 5 = \underline{\hspace{1cm}}$ (finite 7)

$9 - 5 = 4$ (finite 7)

Approach 2



$2 - 5 = \underline{4}$ (finite 7)

Example 2

Simplify: $2 - 13$ (finite 5)

Approach 1

$2 - 13$ (finite 5)

$(2+5) - 13$ (finite 5)

$7 - 13$ (finite 5)

$(7+5) - 13$ (finite 5)

$12 - 13$ (finite 5)

$(12+5) - 13$ (finite 5)

$17 - 13$ (finite 5)

4 (finite 5)

$2 - 13 = 4$ (finite 5)

Approach 2

$2 - 13$ (finite 5)

2 (finite 5) = 2, 7, 12, 17 ...

$17 - 13$ (finite 5)

4 (finite 5)

$2 - 13 = 4$ (finite 5)

Approach 3

$2 - 13$ (finite 5)

$13 \div 5 = 2$ remainder 3

$2 - 3$ (finite 5)

$(2+5) - 3$ (finite 5)

$7 - 3$ (finite 5)

4 (finite 5)

$2 - 13 = 4$ (finite 5)

TOPIC 8: INTEGERS AND FINITE SYSTEMS



Example 3

Find the value of y : $y - 5 = 6$ (finite 7)

$$y - 5 = 6 \text{ (finite 7)}$$

$$y - 5 + 5 = 6 + 5 \text{ (finite 7)}$$

$$11 \div 7 = 1 \text{ remainder } 4$$

$$y = 4 \text{ (finite 7)}$$

Example 4

Find the value of m : $5 + m = 3$ (finite 7)

$$5 + m = 3 \text{ (finite 7)}$$

$$5 - 5 + m = 3 - 5 \text{ (finite 7)}$$

$$m = (3 + 7) - 5 \text{ (finite 7)}$$

$$m = 10 - 5 \text{ (finite 7)}$$

$$m = 5 \text{ (finite 7)}$$

Exercise

1. Workout the following.

a) $3 - 4$ (finite 5)

b) $1 - 6$ (finite 8)

c) $2 - 10$ (finite 12)

d) $3 - 5$ (finite 7)

e) $2 - 8$ (finite 4)

f) $5 - 6 = \underline{\hspace{1cm}}$ (finite 7)

g) $(2 + 7) - 4 = \underline{\hspace{1cm}}$ (finite 7)

h) $3 - 18$ (finite 5)

i) $4 - 34$ (finite 7)

j) $11 - 13$ (finite 12)

k) $6 - 50 = \underline{\hspace{1cm}}$ (finite 7)

l) $4 - 85 = \underline{\hspace{1cm}}$ (mod 7)

2. Find the value of the unknowns.

a) $g - 2 = 4 \pmod{6}$

b) $m - 3 = 2 \pmod{5}$

c) $y - 4 = 3$ (finite 4)

d) $k - 6 = 5$ (finite 7)

e) $y - 7 = 4$ (finite 8)

f) $r - 4 = 3$ (finite 7)

g) $m + 3 = 6$ (finite 8)

h) $n + 7 = 8$ (finite 9)

i) $p + 3 = 2$ (finite 5)

j) $a + 4 = 5$ (finite 7)

k) $7 + m = 4$ (finite 5)

l) $23 + y = 2$ (finite 3)

3. Subtract the following using a dial.

a) $1 - 3 =$ (finite 6)

b) $5 - 8 =$ (finite 9)

c) $2 - 3 = \underline{\hspace{1cm}}$ (finite 7)

c) $4 - 5$ (finite 8)

d) $2 - 5$ (finite 7)

e) $3 - 4$ (finite 7)

4. Workout the following.

a) $3 + 9 - 15$ (finite 7) b) $3 - 4^2 \pmod{5}$ c) $2 - (8 + 8 + 8) \pmod{6}$

Multiplication in finite system

Example

Work out: 2×4 (finite 6)

Approach 1

$$2 \times 4 \text{ (finite 6)}$$

$$8 \text{ (finite 6)}$$

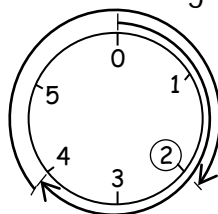
$$8 \div 7 = 1 \text{ remainder } 2$$

$$2 \text{ (finite 6)}$$

$$2 \times 4 = 2 \text{ (finite 6)}$$

Approach 2

2×4 means 2 groups of 4



$$2 \times 4 = 2 \text{ (finite 6)}$$

TOPIC 8: INTEGERS AND FINITE SYSTEMS



Exercise

1. Work out the following.

- | | | |
|--|--|--|
| a) 3×3 (finite 7) | e) $2 \times 8 = \underline{\hspace{1cm}}$ (finite 6) | i) $4 \times 7 = \underline{\hspace{1cm}}$ (finite 5) |
| b) 3×2 (finite 5) | f) $2 \times 3 \times 4 \pmod{5}$ | j) $9 \times 2 \times 6 = \underline{\hspace{1cm}} \pmod{4}$ |
| c) $2 \times 7 = \underline{\hspace{1cm}}$ (finite 12) | g) $9 \times 7 = \underline{\hspace{1cm}}$ (finite 10) | k) $12 \times 6 \pmod{6}$ |
| d) $3 \times 6 = \underline{\hspace{1cm}}$ (finite 5) | h) $5 \times 13 = \underline{\hspace{1cm}}$ (finite 7) | l) $2^2 \times 3^2 = \underline{\hspace{1cm}} \pmod{7}$ |

2. Work out the following using a dial.

- | | |
|----------------------------|---|
| a) 2×3 (finite 7) | e) $4 \times 3 = \underline{\hspace{1cm}}$ (finite 7) |
| b) 4×2 (finite 5) | f) $2 \times 4 = \underline{\hspace{1cm}}$ (finite 5) |
| c) 3×6 (finite 7) | g) $3 \times 2 = \underline{\hspace{1cm}} \pmod{5}$ |
| d) 2×5 (finite 6) | h) $3 \times 4 \pmod{10}$ |

3. Find the value of the unknown.

- | | |
|---|--|
| a) $2 \times 4 = \mathbf{y}$ (finite 7) | d) $6 \times 7 = \mathbf{a}$ (finite 8) |
| b) $\mathbf{k} = 6 \times 4$ (finite 12) | e) $2^3 \times 3 \times 4 = \mathbf{e}$ (finite 5) |
| c) $\mathbf{e} = 12 \times 4$ (finite 11) | f) $2^2 + 3^2 = \mathbf{y} \pmod{7}$ |

4. Complete the table below.

a)

Finite 3

| \times | 0 | 1 | 2 |
|----------|---|---|---|
| 0 | | | |
| 1 | | | |
| 2 | | | |

b)

Finite 6

| \times | 0 | 1 | 2 | 3 | 4 | 5 |
|----------|---|---|---|---|---|---|
| 0 | | | | | | |
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |

c)

Finite 7

| \times | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|----------|---|---|---|---|---|---|---|
| 3 | | | | | | | |
| 4 | | | | | | | |
| 5 | | | | | | | |
| 6 | | | | | | | |

Division in finite systems

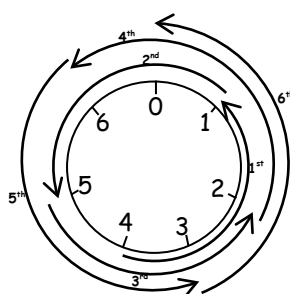
Example

Workout $4 \div 3$ (finite 7)

Approach 1

- $4 \div 3$ (finite 7)
 $(4 + 7) \div 3$ (finite 7)
 $11 \div 3$ (finite 7)
 $(11 + 7) \div 3$ (finite 7)
 $18 \div 3$ (finite 7)
 6 (finite 7)

Approach 2



$$4 \div 3 = 6 \text{ (finite 7)}$$

Approach 3

- $4 \div 3$ (finite 7)
 $4 \pmod{7} = 4, 11, 18 \dots$
 $18 \div 3$ (finite 7)
 6 (finite 7)
 $4 \div 3 = 6$ (finite 7)

TOPIC 8: INTEGERS AND FINITE SYSTEMS



Exercise

- Work out the following.

| | | |
|--------------------------|--|---|
| a) $1 \div 5$ (finite 6) | e) $5 \div 2$ (finite 5) | i) $3 \div 4 = \underline{\hspace{1cm}}$ (finite 7) |
| b) $3 \div 2$ (finite 5) | f) $3 \div 5$ (finite 9) | j) $6 \div 9$ (finite 12) |
| c) $5 \div 3$ (finite 7) | g) $3 \div 5 = \underline{\hspace{1cm}}$ (finite 12) | k) $11 \div 7$ (finite 12) |
| d) $4 \div 3$ (finite 7) | h) $4 \div 3 = \underline{\hspace{1cm}}$ (finite 5) | l) $5 \div 7$ (finite 12) |
- Work out $2 \div 3 \pmod{5}$ using a dial.
- Find the unknown value.

| | |
|------------------------------|------------------------------|
| a) $y = 3 \div 4$ (finite 5) | c) $2 \div 5 = m$ (finite 7) |
| b) $x = 3 \div 2$ (finite 5) | d) $k = 4 \div 3 \pmod{7}$ |

Finding the values of unknowns in finite system

Example 1

Find the value of x if $2x + 1 = 4$ (finite 5)

$$\begin{aligned}
 2x + 1 &= 4 \text{ (finite 5)} \\
 2x + 1 - 1 &= 4 - 1 \text{ (finite 5)} \\
 2x &= 3 \text{ (finite 5)} \\
 2x &= 3 + 5 \text{ (finite 5)} \\
 \frac{2x}{2} &= \frac{8}{2} \text{ (finite 5)} \\
 x &= 4 \text{ (finite 5)}
 \end{aligned}$$

Example 2

Find the value of m : $2(m - 2) = 1$ (finite 9)

$$\begin{aligned}
 2(m - 2) &= 1 \text{ (finite 9)} \\
 2m - 4 &= 1 \text{ (finite 9)} \\
 2m - 4 + 4 &= 1 + 4 \text{ (finite 9)} \\
 2m &= 5 \text{ (finite 9)} \\
 2m &= 5 + 9 \text{ (finite 9)} \\
 \frac{2m}{2} &= \frac{14}{2} \text{ (finite 9)} \\
 m &= 7 \text{ (finite 9)}
 \end{aligned}$$

Example 3

Given that $4(p + 2) = 4$ (finite 7).
Find the value of p .

$$\begin{aligned}
 4(p + 2) &= 4 \text{ (finite 7)} \\
 4p + 8 &= 4 \text{ (finite 7)} \\
 4p + 8 - 8 &= 4 - 8 \text{ (finite 7)} \\
 4p &= (4 + 7) - 8 \text{ (finite 7)} \\
 4p &= 11 - 8 \text{ (finite 7)} \\
 4p &= 3 \text{ (finite 7)} \\
 3 \text{ (finite 7)} &= 3, 10, 17, 24 \dots \\
 \frac{4p}{4} &= \frac{24}{4} \text{ (finite 7)} \\
 p &= 6 \text{ (finite 7)}
 \end{aligned}$$

Example 4

Work out: $\frac{1}{5} + 2m = 3m$ (finite 6)

$$\begin{aligned}
 \left(\frac{1}{5} \times 5\right) + (2m \times 5) &= 3m \times 5 \text{ (finite 6)} \\
 1 + 10m &= 15m \text{ (finite 6)} \\
 1 + 10m - 10m &= 15m - 10m \text{ (finite 6)} \\
 1 &= 5m \text{ (finite 6)} \\
 \frac{1}{5} &= \frac{5m}{5} \text{ (finite 6)} \\
 \frac{1}{5} &= m \text{ (finite 6)} \\
 1 \div 5 &= m \text{ (finite 6)} \\
 1 \text{ (finite 6)} &= 7, 13, 19, 25, 31, \dots \\
 25 \div 5 &= 5 \text{ (finite 6)} \\
 m &= 5 \text{ (finite 6)}
 \end{aligned}$$

TOPIC 8: INTEGERS AND FINITE SYSTEMS



Exercise

1. Find the value of the unknown.

- | | | |
|----------------------------|----------------------------|-----------------------------|
| a) $3p = 4$ (finite 7) | g) $4m + 3 = 8$ (finite 9) | m) $2a - 4 = 2$ (finite 5) |
| b) $2k = 3$ (finite 5) | h) $3x - 6 = 2$ (finite 8) | n) $3n - 6 = 4$ (finite 7) |
| c) $2y = 5$ (finite 7) | i) $3p - 3 = 4$ (finite 7) | o) $6p - 9 = 5$ (finite 11) |
| d) $3g = 5$ (finite 8) | j) $2a - 2 = 3$ (finite 7) | p) $3w - 4 = 5$ (mod 6) |
| e) $2n - 1 = 4$ (finite 5) | k) $2x - 6 = 3$ (finite 6) | q) $3m - 6 = 2$ (mod 8) |
| f) $3b - 2 = 2$ (finite 7) | l) $4y - 8 = 1$ (finite 9) | r) $4p - 2 = 5$ (mod 7) |

2. Find the value of the unknown.

- | | | |
|------------------------------|------------------------------|--------------------------------|
| a) $2(p - 3) = 3$ (finite 6) | d) $3(x - 1) = 4$ (finite 5) | e) $2(m - 4) = 10$ (finite 12) |
| c) $3(y - 1) = 4$ (finite 7) | h) $4(p - 2) = 3$ (mod 5) | g) $5(a - 1) = 2$ (finite 7) |

Application of the finite systems (finite 7)

Example 1

If today is Thursday, what day of the week will it be 64 days to come.

| S | M | T | W | T | F | S |
|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |

$$\text{Day} + 64 = \underline{\hspace{2cm}} \text{ (finite 7)}$$

$$4 + 64 = \underline{\hspace{2cm}} \text{ (finite 7)}$$

$$68 = \underline{\hspace{2cm}} \text{ (finite 7)}$$

$$68 \div 7 = 9 \text{ rem } 5$$

$$= 5 \text{ (finite 7)}$$

5 stands for Friday.

The day will be Friday

Example 2

If today is Monday, what day of the week was it 103 days ago?

| S | M | T | W | T | F | S |
|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |

$$\text{Day} - 103 = \underline{\hspace{2cm}} \text{ (finite 7)}$$

$$1 - 103 = \underline{\hspace{2cm}} \text{ (finite 7)}$$

$$103 \div 7 = 14 \text{ rem } 5$$

$$1 - 5 = \underline{\hspace{2cm}} \text{ (finite 7)}$$

$$(1 + 7) - 5 = \underline{\hspace{2cm}} \text{ (finite 7)}$$

$$8 - 5 = 3 \text{ (finite 7)}$$

3 stands for Wednesday.

The day will be Wednesday

Example 3

Eddy was born 368 days ago. If today is Sunday, what day of the week was he born?

| S | M | T | W | T | F | S |
|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |

$$\text{Day} - 368 = \underline{\hspace{2cm}} \text{ (finite 7)}$$

$$0 - 368 = \underline{\hspace{2cm}} \text{ (finite 7)}$$

$$368 \div 7 = 52 \text{ remainder } 4$$

$$0 - 4 = \underline{\hspace{2cm}} \text{ (finite 7)}$$

$$(0 + 7) - 4 = \underline{\hspace{2cm}} \text{ (finite 7)}$$

$$7 - 4 = 3 \text{ (finite 7)}$$

3 stands for Wednesday.

The day will be Wednesday

Example 4

Muslims started fasting on Tuesday. The fasting took 30 days. On which day of the week did it end?

| S | M | T | W | T | F | S |
|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |

$$30 - 1 = 29 \text{ days}$$

$$\text{Day} + 29 = \underline{\hspace{2cm}} \text{ (finite 7)}$$

$$2 + 29 = \underline{\hspace{2cm}} \text{ (finite 7)}$$

$$31 = \underline{\hspace{2cm}} \text{ (finite 7)}$$

$$31 \div 7 = 4 \text{ rem } 3$$

$$= 3 \text{ (finite 7)}$$

The fasting ended of Wednesday.

TOPIC 8: INTEGERS AND FINITE SYSTEMS



Exercise

- If today is Friday, what day of the week will it be 8 days from now?
- Today is Tuesday what day of the week will it be after;
 - 10 days
 - 15 days
 - 48 days
 - 35 days
 - 24 days
 - 28 days
 - 59 days
 - 90 days
 - 106 days
 - 124 days
 - 200 days
 - 578 days
- Workers receive their monthly salary every after 30 days. They last received their salary on Saturday. What day of the week will they receive their next salary?
- Ivan is planning to buy a car 69 days from now. If today is Wednesday, what day of the week will it be?
- P.7 candidates are to sit for PLE 78 days from today. Today is Saturday, what day of the week will it be?
- If yesterday was Monday, what day of the week will it be 100 days from today?
- A crusade which started on a Friday took 23 days. On what day of the week did it end?
- If today is Sunday, what day of the week was it 12 days ago?
- Today is Monday. What day of the week was it;
 - 20 days ago
 - 48 days ago
 - 32 days ago
 - 29 days ago
 - 71 days ago
 - 106 days ago
 - 133 days ago
 - 496 days ago
 - 60 days from today
- It is 44 days since Mr. Kataala went to Kampala. If today is Monday, on what day of the week did he go to Kampala?
- A training which took 68 days ended on a Monday. On which day of the week did the training start?
- Muzimbi has taken 78 days to build a house. If today is Sunday, on what day of the week did he start his work?
- A man took 40 days to do a job which he started on Thursday. What day of the week did he finish the job?
- Okwi and Ogalal were told to visit the hospital every after 12 days and 16 days respectively. They first visited the hospital together on Tuesday. What day of the week will they visit the hospital together again?

More problems involving application of finite 7 in daily life situation.

Example 1

Today is Wednesday 18th, October. What day of the week will it be 23rd, December the this year?

| Month | Days |
|------------------------|-------------------|
| October | 31 – 18 = 13 days |
| November | 30 days |
| December | 23 days |
| 13 + 30 + 23 = 66 days | |

Day + 66 = ____ (finite 7)
 $3 + 66 = \underline{\quad}$ (finite 7)
 $69 = \underline{\quad}$ (finite 7)
 $69 \div 7 = 9 \text{ rem } 6$
 $= 6$ (finite 7)
 6 stands for Saturday.
The day will be Saturday.

TOPIC 8: INTEGERS AND FINITE SYSTEMS



Example 2

Today is Thursday, 9th September. What day of the week was it 17th, March the same year?

| Month | Days |
|-----------|---------------------------------|
| March | 31 – 17 = 14 days |
| April | 30 days |
| May | 31 days |
| June | 30 days |
| July | 31 days |
| August | 31 days |
| September | 9 days |
| Total | |
| | 14 + 30 + 31 + 30 + 31 + 31 + 9 |
| | = 176 days |

| | | | | | | |
|---|---|---|---|---|---|---|
| S | M | T | W | T | F | S |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |

Day – 176 = ____ (finite 7)
 4 – 176 = ____ (finite 7)
 $176 \div 7 = 25$ remainder 1
 $4 - 1 = 3$ (finite 7)
 3 stands for Wednesday
The day was Wednesday.

Exercise

1. If today is Wednesday, 8th August, what day of the week will it be on 29th August the same year?
2. If today is Monday, 12th April. What day of the week will it be on 23rd July the same year?
3. Nsimbi deposited money on his bank account on Thursday 12th May. He withdrew the money on 17th July the same year. On what day of the week did he withdraw the money?
4. If today is Wednesday 12th August, what day of the week was it on 5th June the same year.
5. Kitoke went to Sudan on 15th March. If today is Thursday 13th July, what day of the week did Kitoke go to Sudan?
6. Today is Monday, 23rd August. Jaja Ichuli died on 4th May the same year. What day of the week did he die?

Finite 12

Example 1

It is March now, what month of the year will it be 53 months to come?

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|----|----|---|
| J | F | M | A | M | J | J | A | S | O | N | D |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 0 |

Month + 53 = ____ (finite 12)
 $3 + 53 =$ ____ (finite 12)
 $56 =$ ____ (finite 12)
 $56 \div 12 = 4$ remainder 8
 $3 + 53 = 8$ (finite 12)

8 stands for August.
The month will be August.

TOPIC 8: INTEGERS AND FINITE SYSTEMS



Example 2

We are in February now. Which month of the year was it 90 months ago?

| J | F | M | A | M | J | J | A | S | O | N | D |
|---|---|---|---|---|---|---|---|---|----|----|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 0 |

$$\text{Month} - 90 = \underline{\hspace{1cm}} \text{ (finite 12)}$$

$$2 - 90 = \underline{\hspace{1cm}} \text{ (finite 12)}$$

$$90 \div 12 = 7 \text{ rem } 6$$

$$2 - 6 = \underline{\hspace{1cm}} \text{ (finite 12)}$$

$$(2 + 12) - 6 = \underline{\hspace{1cm}} \text{ (finite 12)}$$

$$14 - 6 = 8 \text{ (finite 12)}$$

8 stands for August.

The month will be August.

Exercise

- If this month is July, which month of the year will it be;
 - 14 months to come
 - 28 months to come
 - 34 months to come
 - 55 months to come
 - 96 months to come
 - 169 months to come.
- I will travel to London 93 months from now. If this month is October, which month of the year will it be?
- It is December now, which month of the year will it be after;
 - 121 months
 - 260 months
 - 572 months
 - 1000 months
 - 804 months
 - 3248 months.
- A house was built 50 months ago. This month is March. Which month of the year was the house built?
- It is January now, what month of the year was it 145 months ago?
- Mukozi has worked in that company for 347 months. It is February now, in which month of the year did he join the company?
- It is November now, which month of the year was it;
 - 70 months ago
 - 28 months ago
 - 102 months ago
 - 240 months ago
 - 392 months ago
 - 1234 months ago.
- Uganda got her independence 735 months ago. It is January now, find the month in which Uganda got her independence.
- Nubian is 12 years 8 months old. It is April now, in which month was he born?
- Bongole left Uganda for South Africa where he spent 173 months, he later went to America where he has spent 249 months. If it is May now, which month of the year did Bongole leave Uganda?

TOPIC 8: INTEGERS AND FINITE SYSTEMS



Finite 12

Note: - An odd quotient changes a.m to p.m or p.m to a.m
 - An even quotient leaves a.m as a.m and p.m as p.m

Example 1

It is now 8:00 a.m. What time of the day will it be 7 hours from now?

$$\begin{aligned} 8 + 7 &= \underline{\quad} \text{ (finite 12)} \\ 15 &= \underline{\quad} \text{ (finite 12)} \\ 15 \div 12 &= 1 \text{ remainder } 3 \\ &= 3 \text{ (finite 12)} \end{aligned}$$

The time will be 3:00 p.m.

Example 2

It is now 3:00p.m. What time of the day will it be after 73 hours?

$$\begin{aligned} 3 + 73 &= \underline{\quad} \text{ (finite 12)} \\ 76 &= \underline{\quad} \text{ (finite 12)} \\ 76 \div 12 &= 6 \text{ remainder } 4 \\ &= 4 \text{ (finite 12)} \end{aligned}$$

The time will be 4:00p.m.

Exercise

- It is now 8:00 a.m. What time of the day will it be after;
 - 4 hours
 - 10 hours
 - 36 hours
 - 49 hours
 - 127 hours
 - 99 hours
 - 70 hours
 - 267 hours
- It is now 5:00 p.m. What time of the day will it be after;
 - 7 hours
 - 13 hours
 - 25 hours
 - 16 hours
 - 50 hours
 - 39 hours
 - 77 hours
 - 98 hours
- It is 7:00 a.m, what time of the day will it be 11 hours from now?
- Obed left Kampala for Yumbe at 7:00 p.m. He reached Yumbe after 13 hours. At what time did he reach Yumbe?
- It took 16 hours for a man to travel from Mutukula to Koboko. If he left Mutukula at 11:00 a.m, at what time of the day did he reach Koboko?
- Teddy walked from Kassebwera to Namugongo. It took her 42 hours. If she left Kassebwera at 8:00 a.m, at what time did she arrive at Namugongo?
- A party which took 13 hours started at 12 noon. At what time of the day did it end?
- It took a bird 78 hours to fly from Island X to Island Y. If it left Island X at 2 p.m, at what time of the day did it arrive at Island Y?

TOPIC 8: INTEGERS AND FINITE SYSTEMS



More application of finite system.

Example 1

Find the least number such that when divided by 5, 4 remains and when divided by 6, 1 remains.

4(finite 5) = 4, 9, 14, **19**, 24, 29, 34, 39, 44, 49, 54

1(finite 6) = 1, 7, 13, **19**, 25, 31

The number is 19.

Example 2

A farmer had some mangoes, when he grouped them in nines, 2 mangoes remained, in tens, 5 mangoes remained and in eights, only 1 mango remained. Find the least number of mangoes the farmer had.

2(finite 9) = 2, 11, 20, 29, 38, 47, 56, **65**, 74,

5(finite 10) = 5, 15, 25, 35, 45, 55, **65**, 75, 85

1(finite 8) = 1, 9, 17, 25, 33, 41, 49, 57, **65**,

The farmer had 65 mangoes.

Exercise

1. Find the least number which when divided by 7, 4 remains and by 6, 1 remains.
2. Find the least number that such that when divided by 3, 1 remains, by 5, 4 remains and by 4, 3 remains.
3. A farmer had some eggs. When he grouped them in threes, 1 egg remained, in sevens, 2 eggs remained and in sixs, 4 eggs remained. Find the least number of eggs the farmer had.
4. Assumpta sells mangoes in a market. When she groups them in 6s, 3 mangoes remain in 7s, 2 mangoes remain and in 4s, 3 mangoes remain. Find the least number of mangoes Assumpta had.
5. Find the least number of pencils which when shared by 7 boys, 1 pencil remains and by 5 girls, 4 pencils.
6. Find the least number that when divided by 9, 7 remain, when divided by 8, 4 remain and by 3, 1 remains.
7. Mbaga was to seat his invited guests. In seating fours, 2 could remain, in seating fives, 2 could remain but in seating sixs, 4 could remain. Find the least number of the invited guests.
8. K is a least number which when divided by 12, the remainder is 7, when divided by 9, the remainder is 1 and when divided by 7, 6 remains. Find the value of k.
9. When a famer grouped his goats on the farm in 12s, 3 remained, in 11s, 9 remained and in 10s, 5 remained. Find the least number of goats on the farm.
10. P.7 pupils at Tuyita P/S were told to form discussion groups. When they formed groups of nines, 3 remained, when they formed groups of tens, 8 remained but when they formed groups of eights, no one remained. There were 19 boys in the class. Find the number of girls in the class.