THE INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS INC.

SIGNAL PROCESSING
ALGORITHMS, ARCHITECTURES, ARRANGEMENTS, AND APPLICATIONS
SPA 2014
September 22-24th, 2014, Poznań, POLAND

Smart Virtual Bass Synthesis Algorithm Based on Music Genre Classification

Piotr Hoffmann[1], Tomasz Sanner[2]
[1]Audio Acoustics Laboratory,
[2]Multimedia Systems Department
Gdansk University of Technology
Gdansk, Poland
{phoff, sanner}@sound.eti.pg.gda.pl

Bożena Kostek[1]
[1]Audio Acoustics Laboratory
Gdansk University of Technology
Gdansk, Poland
bokostek@audioacoustics.org

Abstract—**The aim of this paper is to present a novel approach to the Virtual Bass Synthesis (VBS) algorithms applied to portable computers. The proposed algorithm employed automatic music genre recognition to determine the optimum parameters for the synthesis of additional frequencies. The synthesis was carried out using the non-linear device (NLD) and phase vocoder (PV) methods depending on the music excerpt genre. Classification of musical genres was performed utilizing the *k*-Nearest Neighbor algorithm and the extracted MPEG 7-based feature vectors. To confirm the relationship between the presented music excerpt genre and the listener's preferences, subjective tests were carried out. The pairwise comparison test was performed. Test material consisted of 18 pair samples belonging to six music genres: classical, pop, rock, rap, jazz, electronic. For comparison purposes music samples were prepared with the benchmark MaxxBass system and the Smart VBS algorithm proposed by the authors. On the basis of the listeners' opinions statistical tests were carried out to confirm the validity of adjusting low frequency synthesis settings according to the music content of audio files.**

Keywords - Digital Signal Processing, Virtual Bass Algorithm, Low Frequency Enhancement, Automatic Music Genre Classification.

I. INTRODUCTION

In recent years rapid development of mobile technologies is observed. They are growing in functionality and computing power. This results in wider availability of mobile services and improvement of performance in commonly used handheld devices. Moreover the users, attracted to high-definition standards, would like to obtain high quality of the multimedia content, which cannot always be achieved even if computational capabilities of mobile devices enable this. Due to the physical constraints of such devices which are hard to overcome, i.e. reproduction of low frequencies in audio content, this is not always possible. What's more, mobile devices are very widely used, but not every user does have sufficient technical knowledge to utilize whole device capabilities manually. Those users expect achieving best performance of the devices without any manual configuration. In this paper we present an intelligent virtual bass synthesis (Smart VBS) algorithm adapting automatically its parameters according to the presented audio content. This is advantageous because using Smart VBS does not require any involvement of the end-user [17].

First approaches for improving low frequency reproduction capabilities of loudspeakers were bass-boosting techniques. They were based on the idea of a simple amplification of frequency range that was not presented properly. Despite the commercial success of bass-boosts algorithms they do not resolve the main problem connected with reproduction of low frequencies. In small loudspeakers in which reproduction of low frequencies is physically impossible these algorithms make barely no effect and what is worse using them can lead to risk of damaging the hardware due to the fact that bass boosting techniques may cause excessive strain of loudspeakers and amplifiers, especially in the case when the amplified frequencies are below the device cutoff frequency.

To overcome these problems the VBS techniques were proposed. They are based on the well-known psychoacoustic phenomenon of the missing fundamental. According to this phenomenon the fundamental frequency can be reconstructed by the human central auditory system on the basis of higher order harmonics even if the fundamental partial is missing in the signal spectrum. Therefore, generation of the higher order partials can lead to recreation and amplification of the perceived low frequencies.

There are several advantages of VBS in comparison to standard bass boost techniques. Firstly, it enables to perceive low frequencies even if loudspeakers have no capabilities to reproduce them. Moreover, the effect of utilizing such an algorithm in small loudspeakers is generally better than using bass boost techniques. And finally, by using the VBS technique the risk of damaging the hardware can be avoided.

The paper presents a new intelligent method for adapting the main parameters of low frequency synthesis algorithms according to the content of the reproduced audio material. Section 2 recalls shortly the theoretical background of the missing fundamental phenomenon, principles of the VBS algorithms and the fuzzy logic basics. In Section 3 the main assumptions of the proposed smart VBS method are presented. Section 4 contains description of the conducted subjective listening tests and the statistical analysis of results. Finally, in Section 5 brief summary of the proposed idea and plans for future work are included.

II. THEORETICAL BACKGROUND

A. Virtual Pitch Phenomenon

The psychoacoustical phenomenon responsible for perception of virtual pitch is the missing fundamental phenomenon. It occurs in the situation when harmonics create the fundamental frequency which is not present in the spectrum of the signal. It is explained by the fact that the human brain perceives the sound pitch based not only on the fundamental partial but also on the relation between the spectral components. As a result, we are able to perceive two sounds, when the basic frequency exists or not, as identical in pitch [3, 10, 19]. The missing fundamental phenomenon is illustrated in Fig. 1.

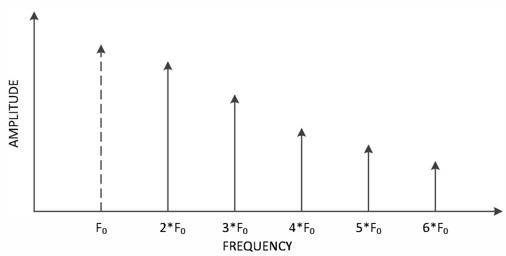

Figure 1. Example of the virtual pitch effect. The dotted frequency component denotes the perceived frequency which is not acoustically reproduced

B. Virtual Bass Synthesis Algorithm

On the basis of the missing fundamental phenomena described in the literature a family [3, 10, 19] of algorithms was designed to enhance low-frequency response by adding harmonics to sound in the mid-range bass bandwidth. In the case where the fundamental frequency of the low frequency cannot be reproduced due to the physical limitations of the speakers, it is compensated by the perceived fundamental frequency formed as a result of a virtual pitch [25].

Virtual bass synthesis algorithms, described in the literature, use non-linear element (NLD), phase vocoder (PV) and hybrid methods [1, 10]. The NLD method operates in the time domain and generates signal harmonics distortion according to the nonlinear function. A variety of mathematical functions are used for this purpose [15, 16, 22]. A major problem of this process are the artifacts due to intermodulation distortion, i.e. caused by two closely-spaced spectral components in the input signal. This is a very undesirable characteristic. The NLD method achieves best results for the transient states, since it has a high temporal resolution. The second well know method is phase vocoder. It is based on the signal processing in the frequency domain and allows for precise control of the various harmonics in both amplitude and the frequency domain. It should be stressed that this method is insensitive to changes of the amplitude of the input signal. The disadvantage of the PV method is that to achieve the proper frequency analysis resolution it needs a relatively wide window of the time-domain analysis and a relatively long time duration of low frequency sound input (100 ms or more). Hence, the method is unsuitable for fast changes, such as transients, but gives good results in the case of long steady state sounds.

To exploit the advantages of both methods, hybrid systems were proposed by several researchers [5, 7]. In the proposed algorithm, the detection of the transient states plays an important role. Thus, the signal is subjected to processing either by NLD or the PV method. If the input signal is of transient character, the signal is processed by the NLD method, and when the signal is in a steady state, the preferred is the PV method. The method uses the transient detector (TCD called - Transient Content Detector), which operates in the field of CQT transform (called Constant - Q Transform). Based on the analysis appropriate weights are selected and applied to the signals processed by the NLD method or PV [7].

C. Fuzzy Logic

Fuzzy sets were proposed by L. A. Zadeh in 1965 in [24]. Zadeh's paper laid the foundation for fuzzy logic that was followed by mathematically defining fuzzy sets and their properties. Fuzzy logic is a form of many-valued logic and deals with reasoning in an approximate way. Compared to traditional binary sets, where variables may take on true or false values, fuzzy logic variables may have a truth value that ranges in degree between 0 and 1. Membership functions for fuzzy sets can be defined in any number of ways as long as they follow the rules of the definition of a fuzzy set. The shape of the membership function used defines the fuzzy set and so the decision on which type should be used is dependent on the purpose. The membership function choice is a subjective aspect of fuzzy logic, it allows the desired values to be interpreted appropriately.

In mathematics a set, by definition, is a collection of objects that belong to it. Any item either belongs to that set or does not belong to that set. When we look at the example of classifying music genres, then from the perception point of view it is difficult to define them as separate sets. Many songs may be assigned to two or more music genres, i.e. pop and rock, because sometimes it is hard to differentiate between the given genres. In the traditional binary set the song is either in the set or it is not. And, in disjointed sets, it cannot belong to another set. Such sharp edged membership function works for binary operations and mathematics, but it does not work in the real world, especially one that is connected to human perception.

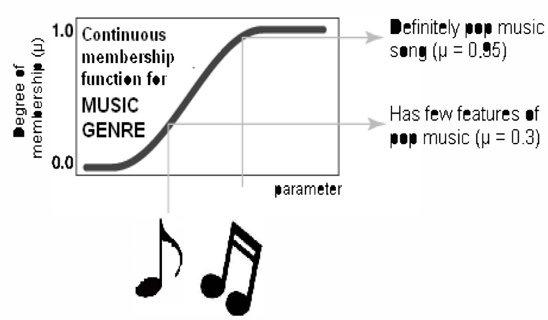

Figure 2. Example of fuzzy logic

When it is difficult to discern between two or more music genres, the only solution seems an application of fuzzy sets.

The fuzzy set approach to pop music provides a much better representation of songs belonging to some degree to the given music genre. The set, shown in Fig. 2., is defined by a continuously inclining function.

The membership function defines the fuzzy set for the possible values of the given parameter on the horizontal axis. The vertical axis, defined between 0 and 1, provides the membership value of belonging to the fuzzy set. Thus for the two songs shown above the first has a membership of 0.3, meaning that it has few features characteristic for pop music. The second song has a membership of 0.95 and it definitely belongs to pop music.

III. SMART VIRTUAL BASS SYNTHESIS ALGORITHM

The enhancement of the low-frequencies in audio material in the proposed method is done by using an intelligent algorithm that adapts the main parameters of low frequency synthesis depending on the music content of the presented audio material. The enhancement processing is carried out automatically without the user involved in the synthesis process. This means that no specialized skills and knowledge are required to fully exploit the potential provided by the proposed algorithm.

In Figure 3 the block diagram of the proposed VBS Smart algorithm is presented. The signal is processed in three main parts that are used for different purposes. One refers to the VBS control parameter computation path, the second is the main signal enhancement path, with the algorithm adapted due to the parameters calculated in the first path and the third one is a delayed input signal. The input signal comprises a frame of the audio file, the length of which is 2048 samples, and the value is fixed during the algorithm processing. Each frame is windowed by a Hamming window.

The classification part of processing uses the input signal. It is parameterized to carry out the process of genre classification of the audio content. The parameterization is performed in the frequency sub-bands and then a set of parameters based on the MPEG7 [9, 14] standard are calculated. The originally calculated Feature Vector contains 173 descriptors. Such a large number of parameters allows for an effective classification of musical genres, but at the same time it leads to a very high data redundancy, what results in a reduced classification effectiveness in terms of time consumption. Therefore the Principal Component Analysis (PCA) algorithm is employed. The advantage of using the PCA method is a significant reduction of parameter number by retaining only the most relevant in the context of the transmitted information about the music genre [8, 23]. This results in a considerable time reduction of the classification process. In fact, it allows for classifying music genres in real time based on buffered parts of the processed signals of the presented audio material. The next step is the assignment of the audio content to the one of the six predefined music genres: Classical, Electronic, Jazz, Pop, Rap, Rock. The classification of music genres is performed using the k-Nearest Neighbor (k-NN) algorithm with k set to 1 [4]. Information about the content of an audio excerpt belonging to a particular music genre constitutes the basis for the intelligent control algorithm of the low-frequency sound enhancement in a mobile device. The intelligent control of sound enhancement is conducted through the VBS algorithm parameter adjustment module. The system performs an automatic assignment of the predefined parameters of low frequency synthesis depending on the classified musical genre. The editable parameters are: NLD gain, PV gain, NLD function, PV harmonic range and PV to NLD ratio. The values of the synthesis parameters were obtained in subjective tests on a group of 25 persons with no hearing problems reported. Each subject was asked to choose the best sounding audio file from the set of 10-second long audio excerpts belonging to the six above mentioned genres. Each file was synthesized with a different value of the tested

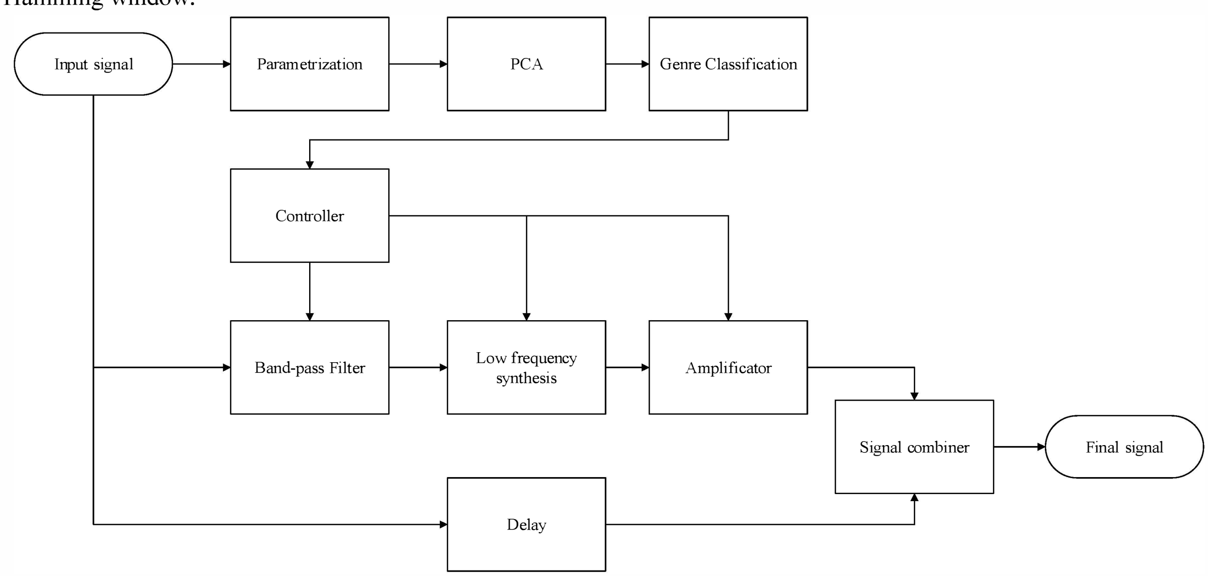

Figure 3. Block diagram proposed solution Smart VBS

parameter. On the basis of the test results the correlation analysis for relationship between users' answers and parameter values were performed. The chi-squared test and V-Cramer factor were used. As a result optimum values of parameters were chosen.

The key source of information is the type of music genre and spectral characteristics of the mobile device. In the real world usually the border between classification groups of objects are not very strict. As mentioned before, in terms of music genre classification it could results in the situation when the music excerpt could not be strictly classified into only one genre, i.e. it has properties that are characteristic for example for both pop and electronic music genre. In such cases even classification performed by two persons could be different according to what the listeners perceive. To avoid a sharp distinction between music genres and thus sharp changes in the synthesis algorithm parameters, fuzzy logic is employed. According to the music genre classification two best fitting genres are chosen and the final synthesis parameters are calculated based on the parameters for each of those genres.

The modification of the proposed intelligent algorithm was performed in the VBS synthesis path taking into account the computed parameters. The modification part contains three blocks: band-pass filter, low frequency synthesis and amplification blocks. Computation in the whole algorithm is carried out to normalize data to <-1,1> range. The band-pass filter prepares the bandwidth of the input signal, which is to be processed. The cut-off frequency of the filter corresponds with the resonance frequency of the device used. Mobile device speakers usually have the resonance frequency at 300 Hz. The filter is projected in the frequency domain and it is automatically tuned to the right cut-off frequency of the device tested. Adding harmonics only to the filtered bandwidth strongly reduces distortion to the signal. Signal filtering is performed in the frequency domain using the Overlap-Add method [6], therefore increasing the processing speed of the whole algorithm. Filter is automatically set to the appropriate band according to device used. Subsequently sound quality modification takes place only on the filtered signal.

The block of the synthesis of low frequencies is the main part leading to the amplification of low frequency. The method of perceptual amplification of low frequency consists in adding additional harmonics to the signal. It is a well known method extensively described in the literature [1, 5, 7, 19, 20, 22, 26]. The distinctive feature of the proposed solution in relation to the current ones is the intelligent control of the synthesis parameters. The authors proposed a modification of parameters: amplification of NLD harmonics, amplification of PV components, nonlinear function of the NLD, ratio of using NLD/PV method. Optimum values of the parameters in the algorithm are selected based on the determined music genre. The filtered signal is summed with the NLD function or PV harmonics. In the method presented the NLD signal is multiplied by one of the chosen functions. In Table 1 a list of typically used NLD functions is presented. The PV method operates in the frequency domain, i.e. "extrapolates" the missing fundamental in the low-frequency range based on higher harmonics. The PV method synthesizes components from second through fifth harmonics. Next, an appropriate harmonic weighting operation is performed on the signal to set the desired shape. The harmonic weighting operation is perform to reflect proper timbre and loudness. In the Smart VBS algorithm the loudness method shown in Eq. 1 is used for this purpose. The amount of attenuation needed can be determined from the difference between the two SPLs. On the output of the synthesis block the signal is band-pass modified.

$$W_i^{Loudness} = 10^{\frac{SPL(if_o)-SPL(f_o)}{10}} \tag{1}$$

where:

$SPL(if_o)$ – sound pressure level of the *i*th harmonic

$SPL(f_o)$ – sound pressure level of the fundamental frequency.

The signal after passing through the synthesis block has an improper level due to added harmonics. It is therefore necessary to add an additional block to the algorithm which adjusts the appropriate amplification of signal, comparable to the original one. Amplification takes into account the level of the reference (original) signal. The level of the reference signal is transmitted from the controller. The output signal is then ready to be summed with the delayed input signal.

TABLE 1 EXAMPLES OF NLD FUNCTIONS.

Exp1	$y = sign(x) \cdot \frac{(1-e^{-\lvert x \rvert})}{1-e^{-1}}$
Exp2	$y = sign(-x) \cdot \frac{(1-e^{\lvert x \rvert})}{e-1}$
Exp3	$y = \frac{e-e^{1-x})}{e-1}$
Arctg	$y = 2{,}5 \cdot arctg(0{,}9x) + 2{,}5\sqrt{(1-0{,}9x)^2} - 2{,}5$

The last part of the algorithm consists of two blocks: the delay and summing ones. In the delay block the not processed signal is delayed for a specified number of samples. This number is connected with time performance that the second part of the algorithm takes. The delayed signal is sent into the combiner where the operation of summing the modified and the delayed signals is performed. At the output the signal frame is provided with the amplification of low frequencies. Therefore the enhancement of the audio signal is performed in an automatic way with regard to the original signal and its assigned music genre.

IV. EXPERIMENTS

A. Preparations

Subjective listening tests were conducted in auditory conditions that refer to realistic conditions of using a portable computer on the daily basis. The listener was situated directly in front of the monitor of the computer which was placed on a table. A computer ASUS Eee PC 1201N was used in tests.

Test samples consisted in two different groups of signals:
- samples processed with the VBS algorithm proposed,
- samples processed with a commercial Bass Boost (BB) algorithm [11].

The Bass Boost algorithm used is the benchmark MaxxBass algorithm, which exploits the psychoacoustics phenomenon of the missing fundamental frequency. The algorithm was used as the plugin to the commercial audio editing software. The enhancement was performed in the bandwidth of 50-300 Hz. Amplification resulted in 8dB, this was the highest value which did not overdrive the signal. The cut-off frequency of the used netbook Asus was at 400 Hz, so in our experiments we enhanced the low frequencies in the device that cannot physically reproduce it. To illustrate the process of the enhancement of the low frequencies by the Bass Boost algorithm, the enhancement curve is shown in Figure 4.

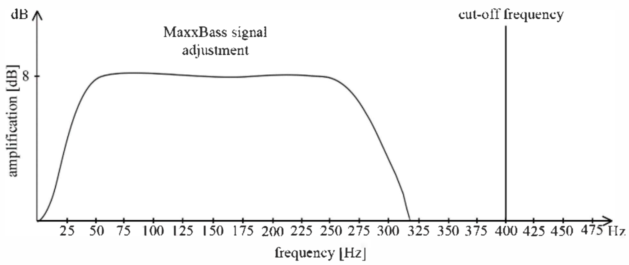

Figure 4. *Low frequency adjustment using Maxx Bass algorithm.*

There were 30 pairs of samples presented in a single test series. The duration of samples was 10 seconds, breaks between pairs of samples lasted 2 seconds. Finally, the duration of the whole test was equal to 20 minutes. The order of testing pairs was random and users could listen to the samples any number of times. Each test pair of samples was categorized as one of the six genres: Classical, Jazz, Pop, Rock, Rap, Electronics. The listeners were asked to determine which of the samples contained more bass than the other. The 7-point grading scale was used in the evaluation process (Fig. 5).

In the test 25 untrained subjects of ages ranging from 20 to 60 were involved. All of the subjects reported no hearing problems.

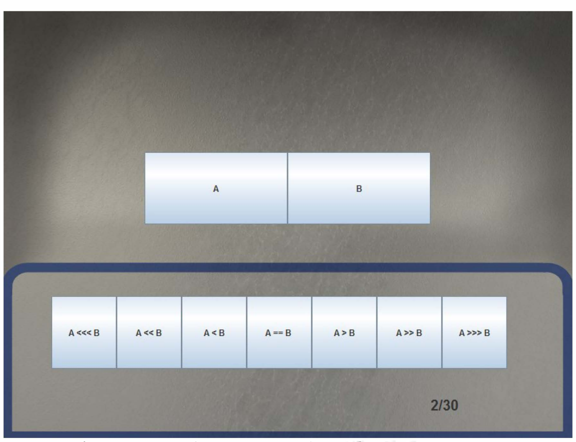

Figure 5. *Graphic User Interface of pairwise comparison test.*

B. *Results and analysis*

Data were checked for the reliability of listeners. The procedure was preceded by verification of the way of returning answers by the respondents. This procedure was prepared to check the reliability of the answers given by the person during the test. To find out whether the answers given by a person were valid, the analysis of variance of answers and mean difference of answers given by listeners in the first and second series were performed. If the listener's answers were not random, the sequence of differences and variance should be zero. These requirements form the hypothesis that need to be verified during the statistical tests. [12, 13]. The statistical analysis was performed with the Statistica software [20].

The results obtained in the statistical analysis are presented in graphical form using the box-plot chart. Figure 6 shows results of the tests performed. The chart contains a collection of statistical data: lower, upper quartile, median, minimum, average, maximum values. Variance tests conducted showed that 14 listeners can be considered as reliable ones. Their answers had variance bellow 1.7. The positive values indicated that the listeners chosen the VBS algorithm-processed excerpts as making better impression in comparison to the Bass Boost one. The biggest difference was observed in the case of rock genre. Most of listeners chose the Smart VBS algorithm as a better sounding sample in rock genre. The smallest difference was observed in the case of classical genre, which could be predicted due to the small low frequency spectrum components present in the samples belonging to this genre. Based on the analysis of the average values and distribution of answers, it can be concluded that most of the listeners have confirmed that the proposed Smart VBS algorithm sounds better in comparison to the MaxxBass algorithm.

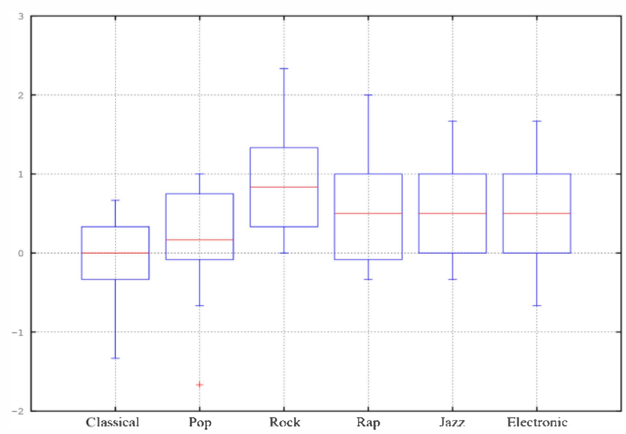

Figure 6. Statistical box-plot chart with information about test answers given by the listeners.

For further analysis the statistical verification of collected data was carried out. For this purpose t-student test with significance level of 0.1 with the threshold value of the calculated t-statistics equaled 1.7613 was used. The t-student formula is shown below [2]:

$$T = \frac{\bar{D}}{\frac{S_D}{\sqrt{n}}} \tag{2}$$

where:

$$S_d = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{n} (x_i - y_i - \bar{D})^2} \tag{3}$$

where:

$\overline{D}$ - the average difference between the first and second classes

x_i, y_i - classes of analyzed objects

n - number of objects

The results of the t-student statistic analysis for tested genres are presented in Table 2.

TABLE 2 SUMMARY OF THE RESULTS OF T-STUDENT TEST

Music Genre	***T***
Classical	-0.59047
Pop	0.46120
Rock	2.9333
Rap	1.5626
Jazz	2.5459
Electronic	1.9338

The highest value of t-student statistics was achieved for rock genre, and the lowest one for classical genre. From the obtained results it can be concluded that in most tested genres, listeners chose the Smart VBS over the Bass Boost algorithm. The only exception was classical music genre where listeners probably might not hear much difference between samples. This could be caused by a small share of low-frequency samples in music excerpts presented to the listeners. The above conclusion was confirmed by results obtained with statistics showed in Figure 6 and in Table 2.

V. SUMMARY

A new method for smart VBS algorithm was presented. It can adjust the synthesis parameters automatically without any manual configuration. To prove the influence of music genre on the listeners' preferences for VBS, the subjective pairwise comparison test was proposed. Two algorithms were compared: Bass Boost – a commercial software and the authors' implementation of the Virtual Bass Synthesis algorithm. A group of listeners was employed to evaluate music samples processed with both algorithms. The following conclusions can be drawn on the basis of the obtained results:

1. In general, the VBS algorithm performed better than the Bass-Boost algorithm.
2. The VBS algorithm improved perception of low frequencies but the improvement was not noticeable in all tested samples and genres of music (i.e. classical genre). Therefore, an observation can be made that VBS should be used when it is desired.
3. The effect achieved with the VBS algorithm depended strongly on the music content of the signal. Thus classification based on the content of the signal and adjustment of the synthesis parameters based on these results seemed to be a very promising approach.

Further investigation on the smart VBS algorithm will include implementation of the algorithm working in real-time and some additional comparison tests with the traditional VBS algorithms implementations.

ACKNOWLEDGMENT

This research was funded by the grant no. PBS1/B3/16/2012 entitled „Multimodal system supporting acoustic communication with computers" financed by the Polish National Centre for R&D.

REFERENCES

[1] Aarts R. M., Larsen E., Schobben D., Improving Perceived Bass And Reconstruction Of High Frequencies for Band Limited Signals, 1st IEEE Beneloux Workshop on Model based Processing and Coding of Audio (MPC 2002), Leuven, Belgium, November 15, 2002

[2] Bedyńska S., Brzezicka A., Statystyczny drogowskaz, SWPS Academica, Warszawa, 2007

[3] Bekesy G., The missing fundamental and periodicity detection in hearing, J. Acoust. Soc. Am. Volume 51, Issue 2B, pp. 631-637, 1972

[4] Hall P, Park BU, Samworth RJ, Choice of neighbor order in nearest-neighbor classification". Annals of Statistics 36, 2008

[5] Hao M., Woon-Seng G., Ee-Leng T., A psychoacoustic bass enhancement system with improved transient and steady-state performance, ICASSP 2012 IEEE, 141-144, 2012.

[6] Hayes, M. Horace, Digital Signal Processing, Schaum's Outline Series, New York, McGraw Hill, 1999

[7] Hill A. J., Hawksford M. O. J., A Hybrid Virtual Bass System for Optimized Steady State and Transient Performance, CEEC Conf. 8-9.09. 2010.

[8] Hoffmann P., Kostek B., Kaczmarek A., Spaleniak P., Music Recommendation System, Journal of Telecommunication and Information Technology, Warsaw 2013.

[9] Kostek B., Hoffmann P., Kaczmarek A., Spaleniak P., Creating a Reliable Music Discovery and Recommendation System, Springer Verlag, 107-130, XIII, 2013.

[10] Larsen E., Aarts R.M., Perceiving Low Pitch Through Small Loudspeakers, 108th AES Convention, 19-22 Feb 2000, Paris, France.

[11] Maxx Base algorithm, http://www.waves.com/plugins/maxxbass

[12] Method for the subjective assessment of intermediate quality level of coding systems, ITU-R, BS.1116-1, http://www.itu.int/rec/R-REC-BS.1116-1-199710-I/e

[13] Montgomery Douglas C., Design and Analysis of Experiments, 5th Edition, Wiley, 2000

[14] MPEG 7 standard, http://mpeg.chiariglione.org/standards/mpeg-7

[15] Nay Oo, Woon-Seng Gan, Wee-Tong Lim, Generalized Harmonic Analysis of Arc-tangent Square Root (ATSR) Nonlinear Device for Virtual Bass System, IEEE International Conference on Acoustics Speech and Signal Processing (ICASSP), 14-19 March 2010

[16] Nay Oo, Woon-Seng Gan, Harmonic Analysis of Nonlinear Devices for Virtual Bass System, IEEE International Conference on Audio, Language and Image Processing (ICALIP), 7-9 July 2008.

[17] Patent Pending, Kostek B., Hoffmann P., Sanner T., Sposób poprawy jakości brzmienia dźwięku w przenośnych urządzeniach elektronicznych i układ do realizacji tego sposobu, Polski Urząd Patentowy, May 2014

[18] Sanner T., Łopatka K., Czyżewski A., Evaluation of Sound Enhancement in Mobile Device using Virtual Bass Algorithm, 1-12, ISSET Conf., Krakow, 2013.

[19] Schouten, J.F., Ritsma R.J., and Cardozo B.L., "Pitch of the residue.", Journal of the Acoustical Society of America: 825-834, 1962

[20] Statistical software Statistica, http://www.statsoft.pl/

[21] Terhardt E., Zur Tonhöhenwahrnehmung von Klängen. I. Psychoakustische Grundlagen. Acustica 26, 173-186, 1972

[22] Wee-Tong Lim, Nay Oo, Woon-Seng Gan, Synthesis of Polynomial-Based Nonlinear Device and Harmonic Shifting Technique for Virtual Bass System, ISCAS IEEE, 24-27 May 2009.

[23] Williams L.J., Abdi H., Principal Component Analysis, Wiley Interdisciplinary Reviews: Computational Statistics, 2, 2010.

[24] Zadeh L.A., Fuzzy Sets, Information and Control, Vol. 8, Issue 3, pp. 338-358, 1965