Introduction to tensor networks session 2, Entanglement

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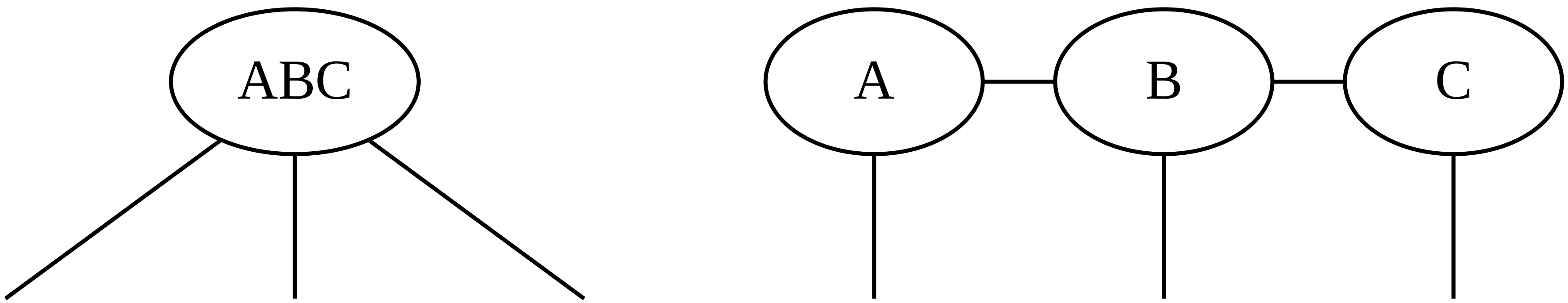
Specific set of states

$$|\psi_{ABC}\rangle$$



Correlations and entanglemet

The main idea is that either we saw

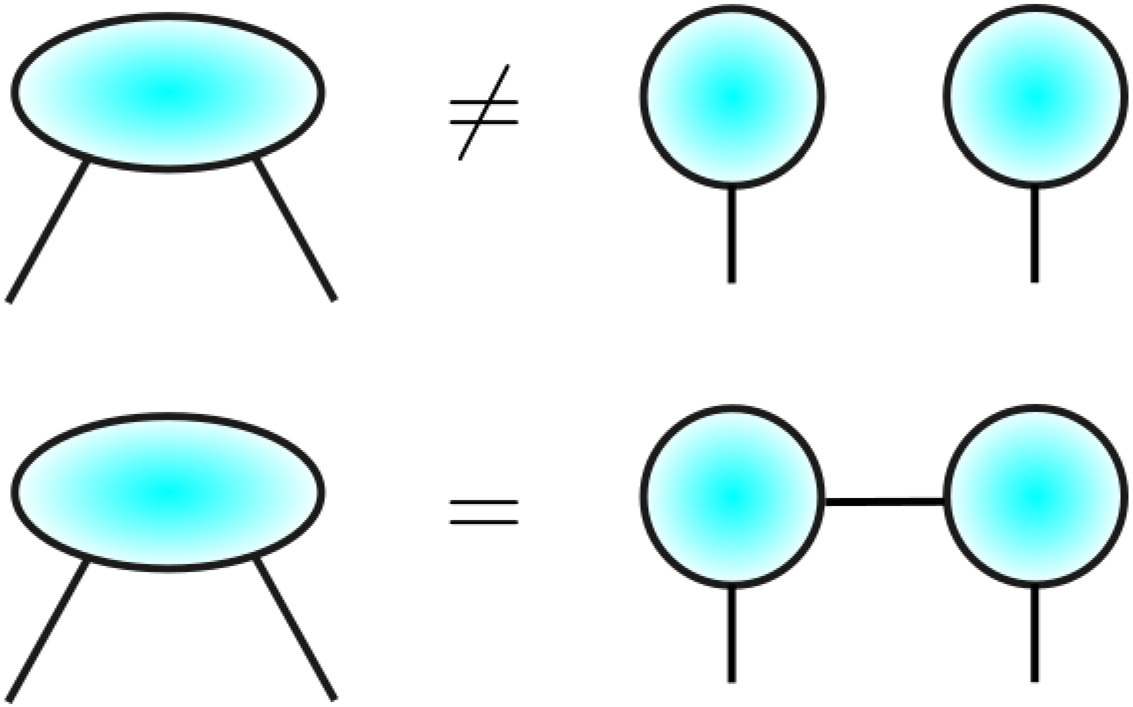
- uncorrelated states
- states and prob. dist. with very precise *short range* correlations $...010101...$

How do we formalize this ?

ENTANGLEMENT

$$|\psi_{AB}\rangle \neq |\psi\rangle_A \otimes |\psi\rangle_B$$

Entanglement



Examples

- Singlet

$$|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |11\rangle_{AB}\right)$$

- Reduced state becomes

$$\rho_A = \mathrm{tr}_B\,|\psi\rangle\langle\psi| = \frac{1}{2}\left(|0\rangle\langle 0|_A + |1\rangle\langle 1|_A\right)$$

Entanglement measures

Entanlgement entropy, Von Neuman entropy of the prob. distribution of the eigenvalues of the rdm

$$S(\rho_A) = \mathrm{tr}\left(-\rho_A \log(\rho_A)\right)$$

Examples

- Product state:

$|00\rangle_{AB} \to \rho_A = |0\rangle\langle 0|_A$ implies $S_A = 0$

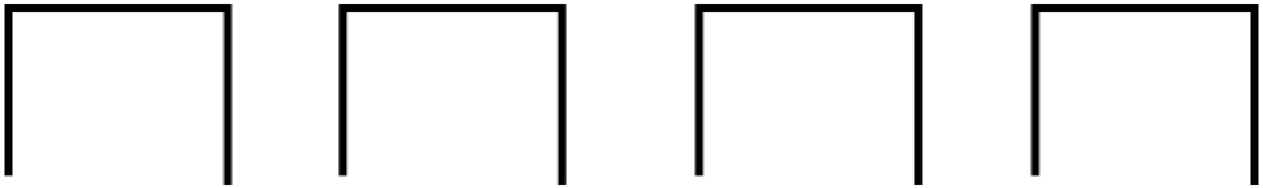
- Singlet:

$$|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |11\rangle_{AB}\right) \to$$

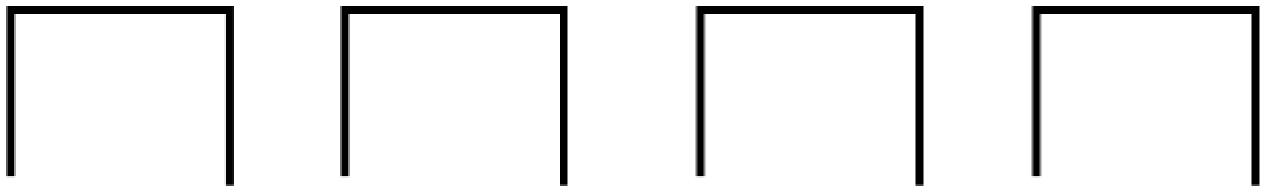
$$\rho_A = \frac{1}{2}\left(|0\rangle\langle 0|_A + |1\rangle\langle 1|_B\right)$$

implies $S_A = 1$

Chain of singlets



$\frac{1}{\sqrt{2}}\Big($



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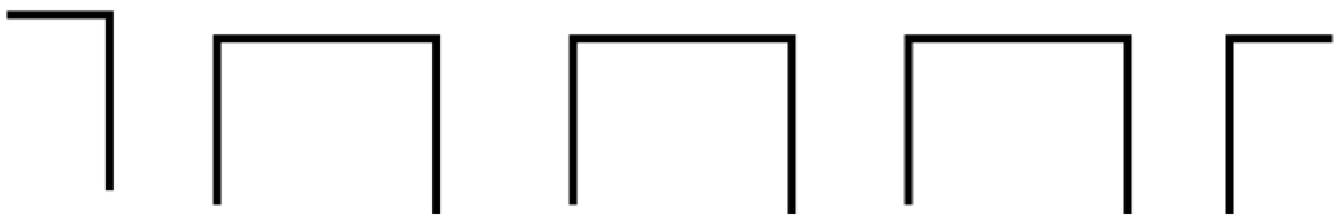

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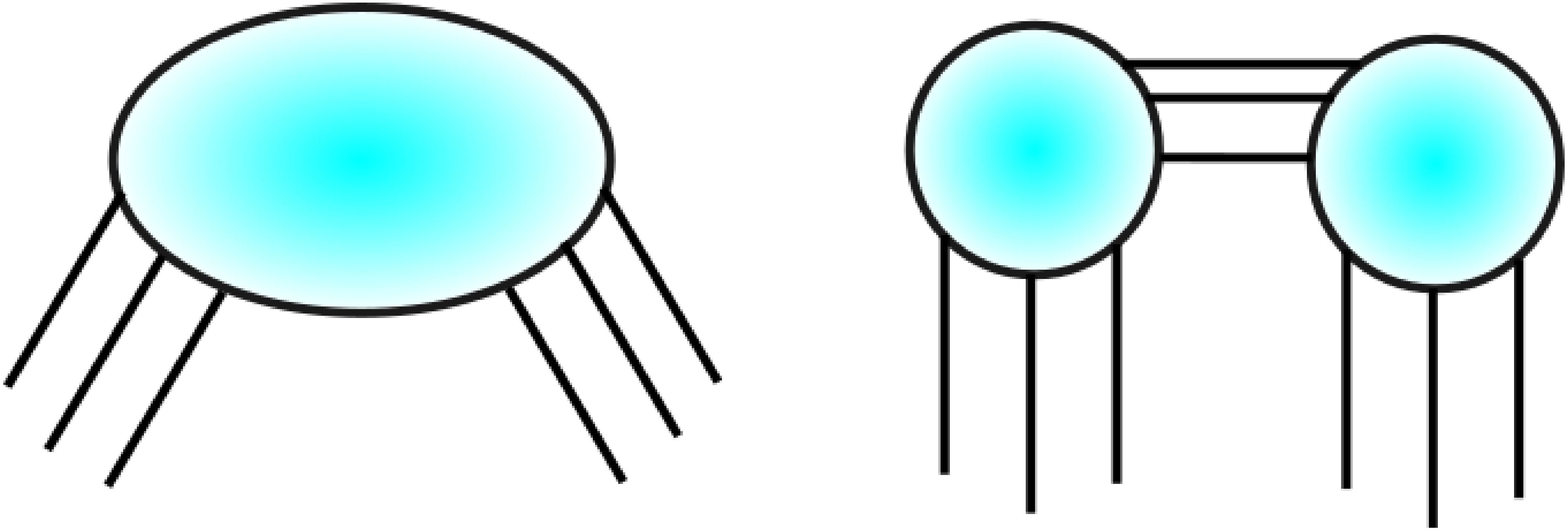
Longer distance singlets

We see that if A and B are connected by more singlets,



The bond dimension increases

How much entangled are random states

$$|\psi\rangle = \exp(-iHt)\,|0...0\rangle$$

```

import numpy as np
import scipy.linalg as LA
import matplotlib.pyplot as plt
mean_ent=[]
std_ent=[]
for N in range(2,10,2):
    #print(N)
    dim_h =2**N
    ent_entropies=[]
    for _ in range(0,100):
        init_state = np.zeros([dim_h,1])
        init_state[0]=1.
        random_h = np.array(np.random.rand(dim_h,dim_h)+1j*np.random.rand(d
        random_h = random_h+random_h.T.conj()
```

The result