Session 1

1.0 Motivation why to use tensor networks
1.1. Building block vectors matrices tensors and their contractions in Penrose notation
1.2 The idea of data compression
- Counting strings of bits
- Counting strings of bits with constraints

1.3 From strings of bits to probability distributions of classical spins
1.4 Tensor networks for the many body problem
1.5 Defining many body systems
- Without interaction, the product states
- With interactions, strongly interacting problems in physics
- The exponential scaling

1.6 Classes of states that can be represented efficiently

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Session 1

1.0 Introduction

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Tensor networks

tensors networks represent set of correlated data, they are mostly used for

- quantum many body systems, wave-functions operators
- path integrals in field theories (space-time?)
- statistical mechanics, probabilities of configurations (canonical distributions, partition functions)
- big data, any field in which data have some correlations, or specific origin that allow to compress them.

Why do we use them

- provide a compressed data-set
- provide clear characterization of the structure, visually and computationally of correlation, bunching together more correlated local structure
- provide a distributed representation in terms of individual tensors (subset of the data)
- provide a unified framework
- robust to noise and missing data

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1.1 Introduction

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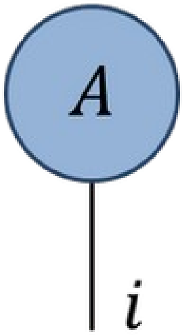
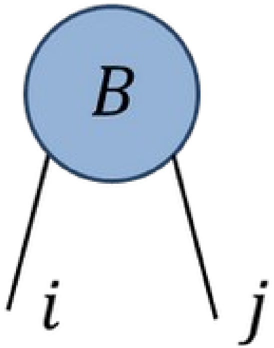
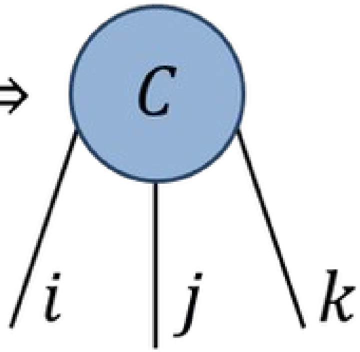
Building blocks

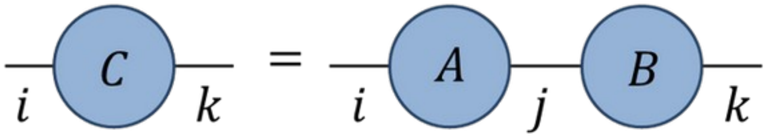
$$A = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & \cdots & B_{1n} \\ \vdots & \ddots & \vdots \\ B_{m1} & \cdots & B_{mn} \end{bmatrix} \qquad C = \left[\left[\begin{bmatrix} C_{111} & \cdots & C_{1n1} \\ \vdots & \ddots & \vdots \\ C_{m11} & \cdots & C_{mn1} \end{bmatrix}_1\right]_2\right]_3$$

$A_i \Leftrightarrow$ 

$B_{ij} \Leftrightarrow$ 

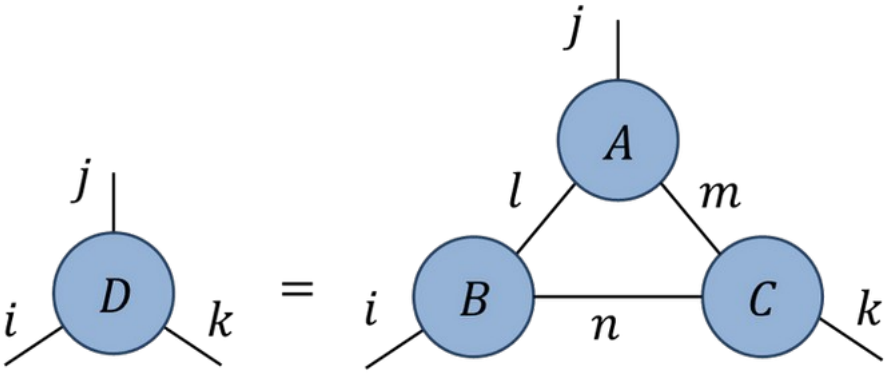
$C_{ijk} \Leftrightarrow$ 

Operations



$\Updownarrow$

$$C_{ik} = \sum_j A_{ij} B_{jk}$$



$\Updownarrow$

$$D_{ijk} = \sum_{lmn} A_{ljm} B_{iln} C_{nmk}$$

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1.2 Data compression

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Counting

- Counting the number of the sequences of L bits that do not contain two consecutive ones
- How many are they?
- In order to create them sequentially what is the relevant information?

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1.3 Probability distributions

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Probablility distribution

- Encoding the probability distribution for a sequence of L bits that is uniform if there are no two adjacent ones 0 otherwise

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1.4 Many-body problem

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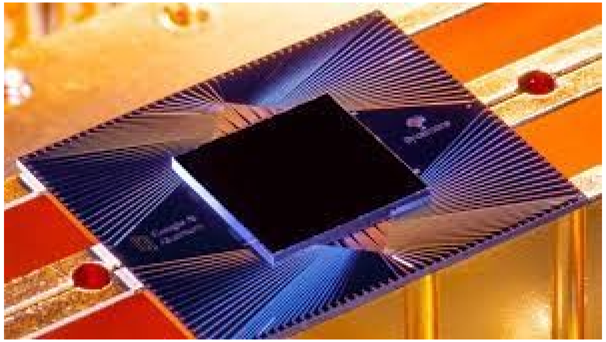
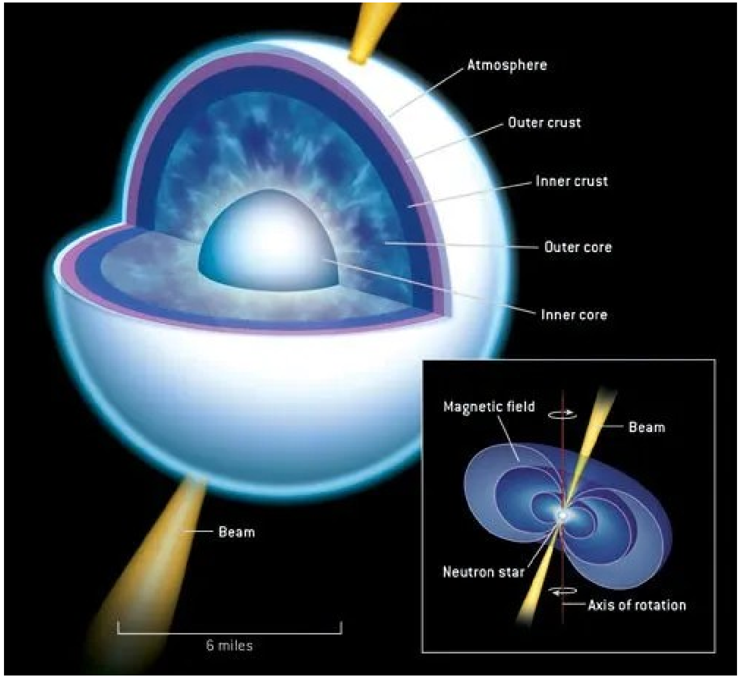
Tensor Network for many body

- Many body systems are systems described by several constituents

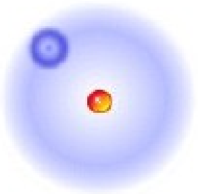


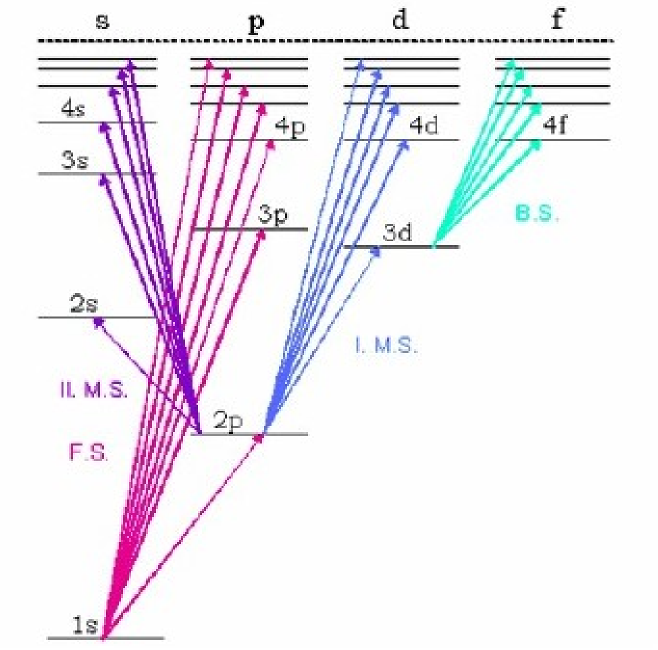


Quantum Systems

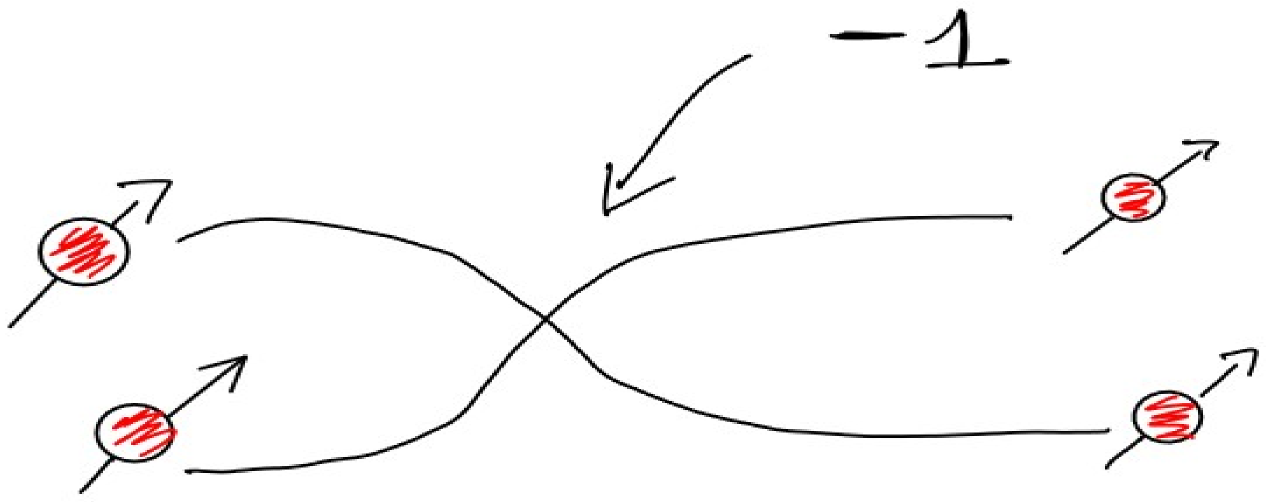
- Hamiltonian, wave-functions

Hydrogen atom



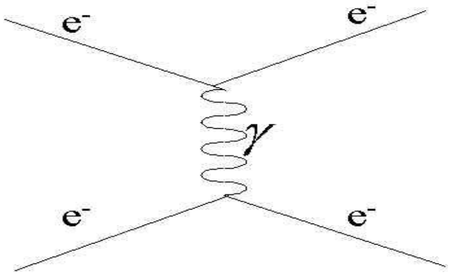


Free fermions



$$\left\{c_i^\dagger, c_j^\dagger\right\} = 0$$

The Coulomb potential



At low energies interaction between photons and electrons

$$\alpha = 1/137$$

$$V \propto \frac{1}{r}$$

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1.5 Many-body systems

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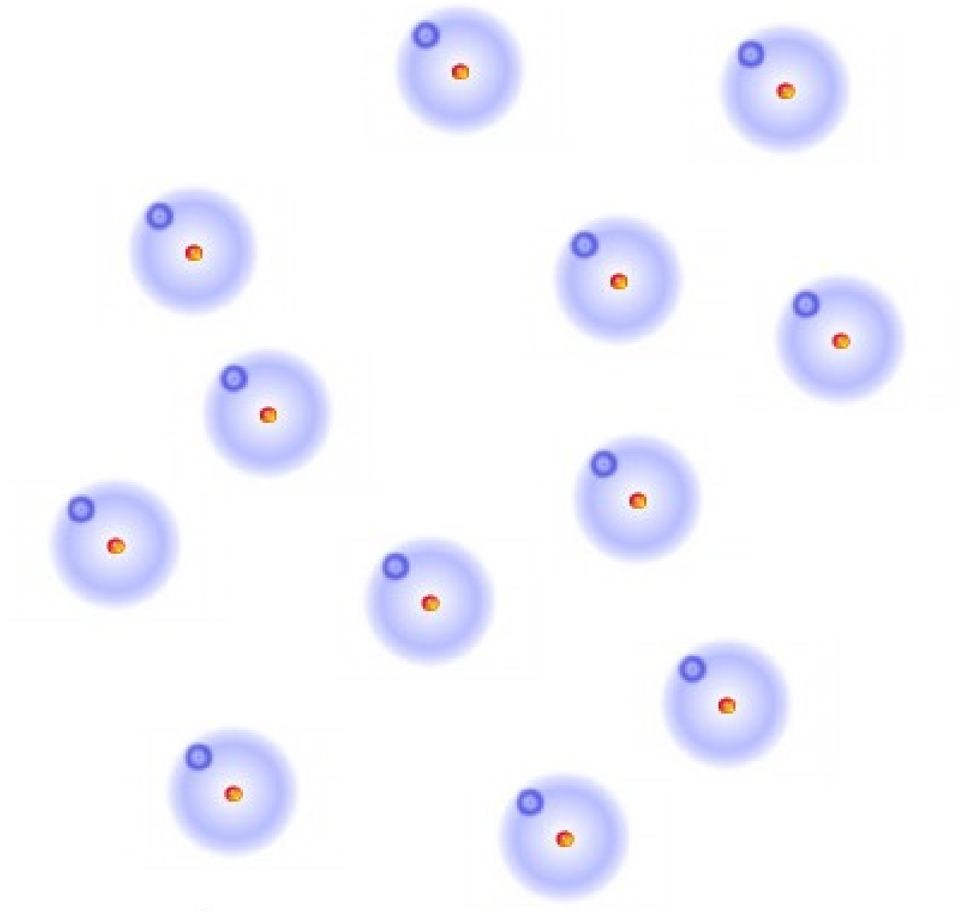
Many body non-interacting

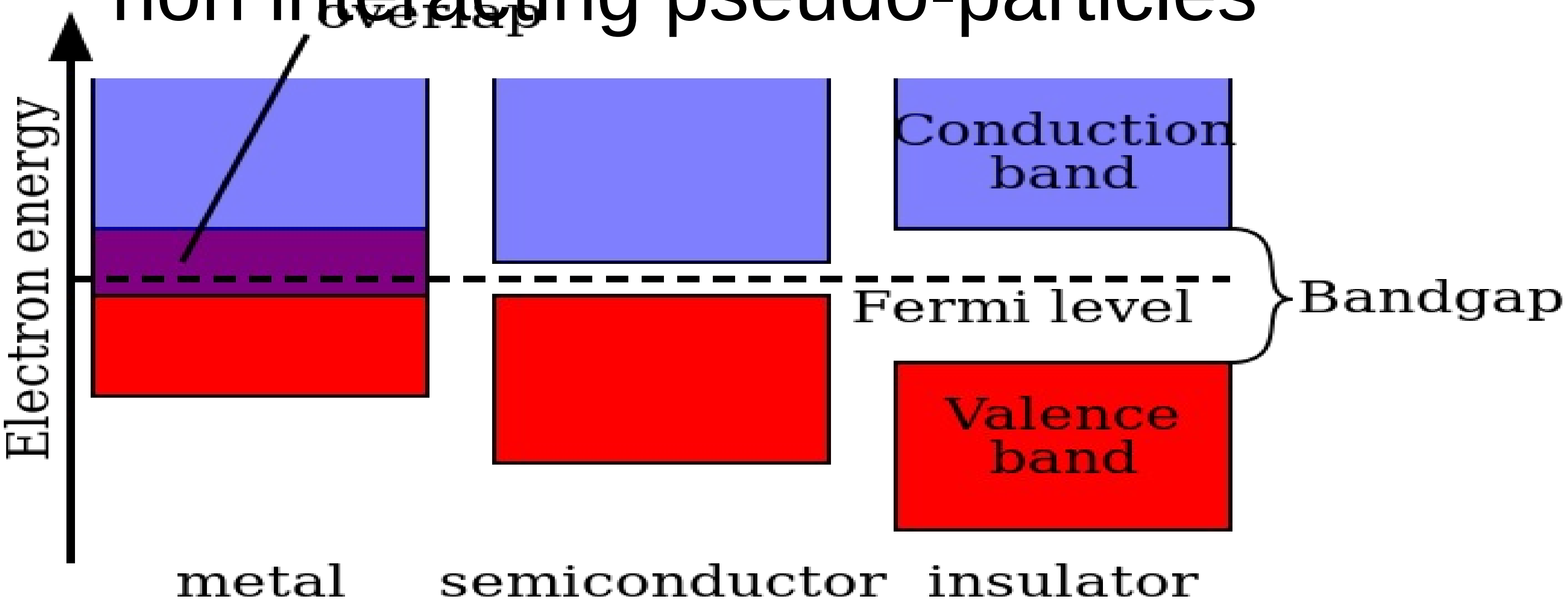
$$H = \sum_i H_i$$

$$|\psi\rangle = \prod_i |\psi_i\rangle$$

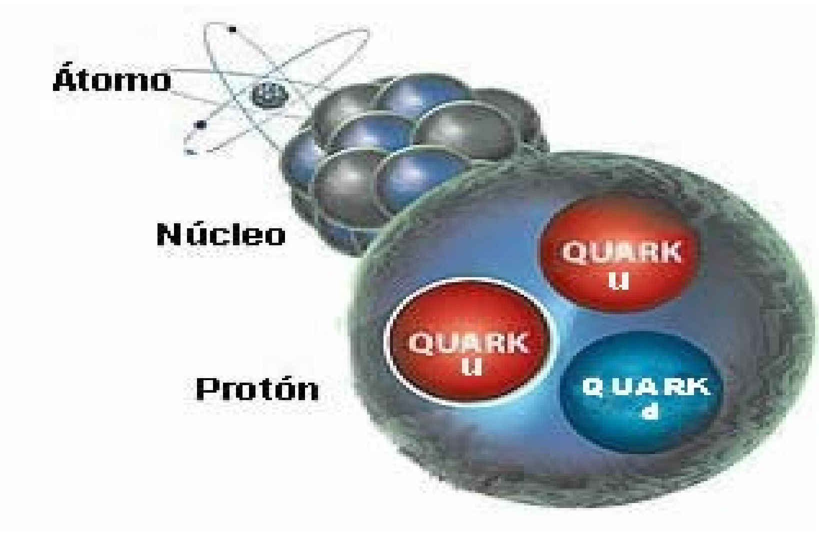


E.g. Band theory

Interacting systems
close the wave function of
non interacting pseudo-particles



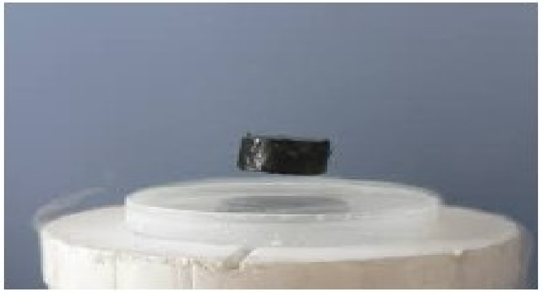
Not all fermions are “free”

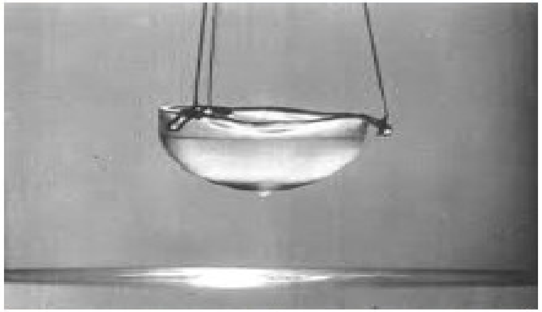


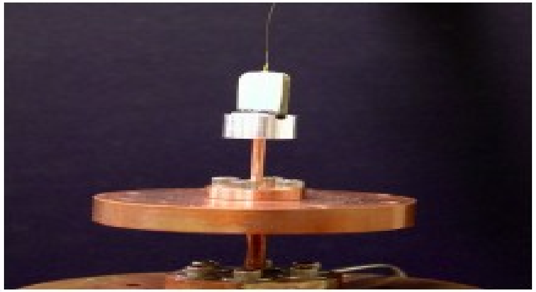
Quarks, are confined

Exotic Emerging phenoma

- Superconductor
- Super-fluid
- Super-solid



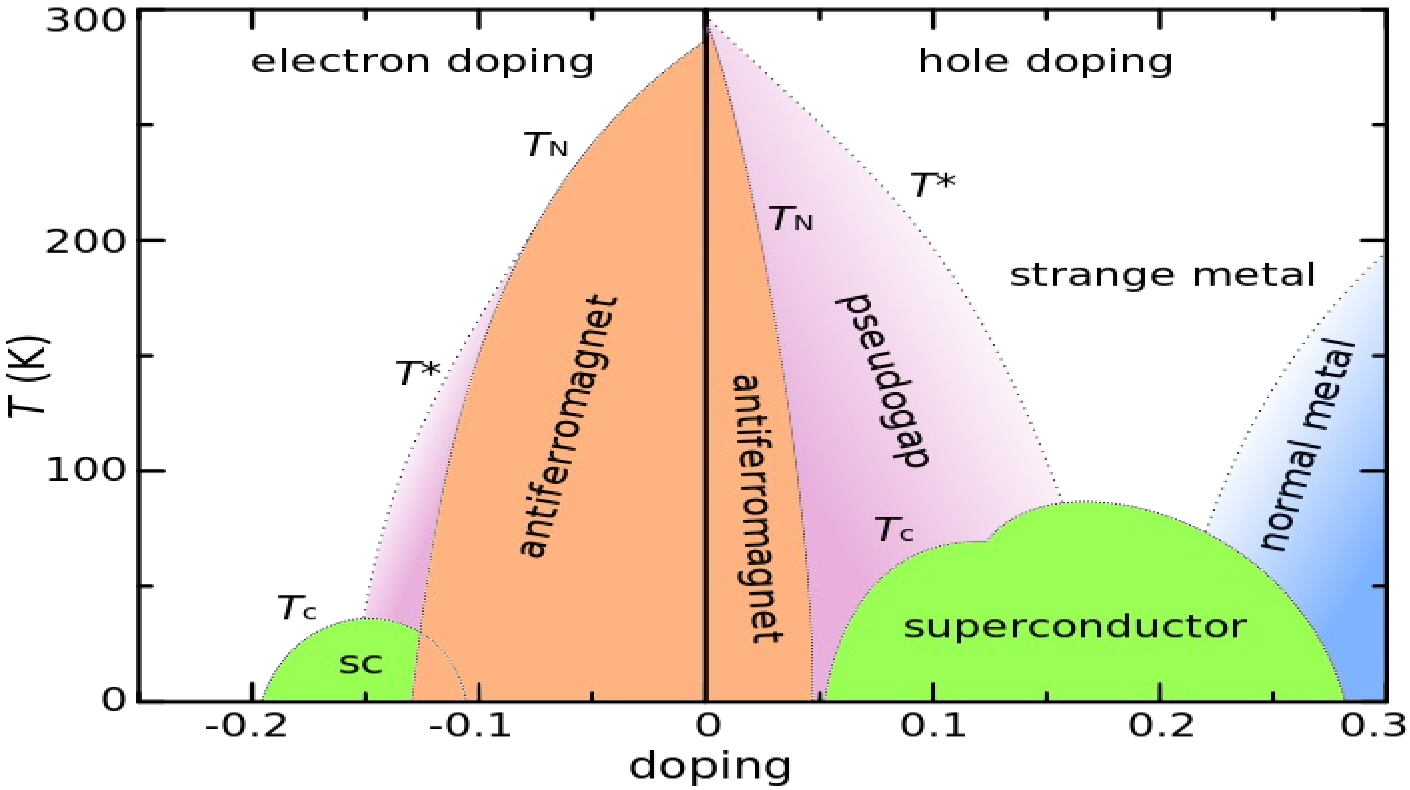




E.g. compounds of transition metals

21 44.956 Sc Scandium $[Ar]3d^{1}4s^{2}$	22 47.88 Ti Titanium $[Ar]3d^{2}4s^{2}$	23 50.942 V Vanadium $[Ar]3d^{3}4s^{2}$	24 51.996 Cr Chromium $[Ar]3d^{5}4s^{1}$	25 54.938 Mn Manganese $[Ar]3d^{5}4s^{2}$	26 55.933 Fe Iron $[Ar]3d^{6}4s^{2}$	27 58.933 Co Cobalt $[Ar]3d^{7}4s^{2}$	28 58.693 Ni Nickel $[Ar]3d^{8}4s^{2}$	29 63.546 Cu Copper $[Ar]3d^{10}4s^{1}$	30 65.39 Zn Zinc $[Ar]3d^{10}4s^{2}$
39 88.906 Y Yttrium $[Kr]4d^{1}5s^{2}$	40 91.224 Zr Zirconium $[Kr]4d^{2}5s^{2}$	41 92.906 Nb Niobium $[Kr]4d^{4}5s^{1}$	42 95.95 Mo Molybdenum $[Kr]4d^{5}5s^{1}$	43 98.907 Tc Technetium $[Kr]4d^{5}5s^{2}$	44 101.07 Ru Ruthenium $[Kr]4d^{7}5s^{1}$	45 102.906 Rh Rhodium $[Kr]4d^{8}5s^{1}$	46 106.42 Pd Palladium $[Kr]4d^{10}$	47 107.868 Ag Silver $[Kr]4d^{10}5s^{1}$	48 112.411 Cd Cadmium $[Kr]4d^{10}5s^{2}$
57-71	72 178.49 Hf Hafnium $[Xe]4f^{14}5d^{2}6s^{2}$	73 180.948 Ta Tantalum $[Xe]4f^{14}5d^{3}6s^{2}$	74 183.85 W Tungsten $[Xe]4f^{14}5d^{4}6s^{2}$	75 186.207 Re Rhenium $[Xe]4f^{14}5d^{5}6s^{2}$	76 190.23 Os Osmium $[Xe]4f^{14}5d^{6}6s^{2}$	77 192.22 Ir Iridium $[Xe]4f^{14}5d^{7}6s^{2}$	78 195.08 Pt Platinum $[Xe]4f^{14}5d^{9}6s^{1}$	79 196.967 Au Gold $[Xe]4f^{14}5d^{10}6s^{1}$	80 200.59 Hg Mercury $[Xe]4f^{14}5d^{10}6s^{2}$
89-103	104 [261] Rf Rutherfordium $[Rn]5f^{14}6d^{2}7s^{2*}$	105 [262] Db Dubnium $[Rn]5f^{14}6d^{3}7s^{2*}$	106 [266] Sg Seaborgium $[Rn]5f^{14}6d^{4}7s^{2*}$	107 [264] Bh Bohrium $[Rn]5f^{14}6d^{5}7s^{2*}$	108 [269] Hs Hassium $[Rn]5f^{14}6d^{6}7s^{2*}$	109 [268] Mt Meitnerium $[Rn]5f^{14}6d^{7}7s^{2*}$	110 [269] Ds Darmstadtium $[Rn]5f^{14}6d^{8}7s^{2*}$	111 [272] Rg Roentgenium $[Rn]5f^{14}6d^{9}7s^{2*}$	112 [277] Cn Copernicium $[Rn]5f^{14}6d^{10}7s^{2*}$

The phase diagram of cuprates

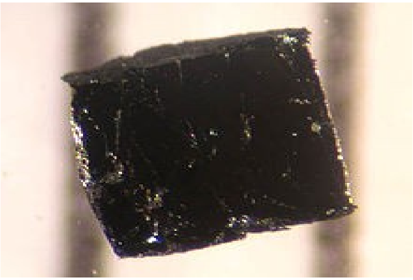


Exotic emerging phenomena

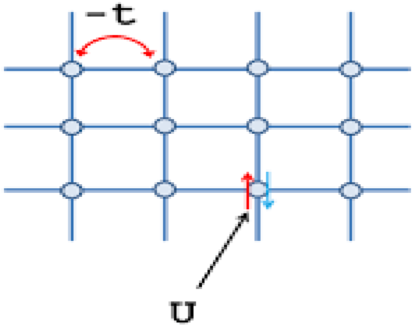
- Spin Liquid
- Fractional quantum Hall
- High temperature superconductors
- Quark confinement
- …

Model Hamiltonians, Hubbard

- Tight binding...
- Single band...
- Screening...



$$H = -t \sum_{\langle i,j \rangle,\sigma} (c_{i,\sigma}^{\dagger} c_{j,\sigma} + c_{j,\sigma}^{\dagger} c_{i,\sigma}) + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow},$$



$$H = J \sum_{<i,j>} \sigma_i \sigma_j$$

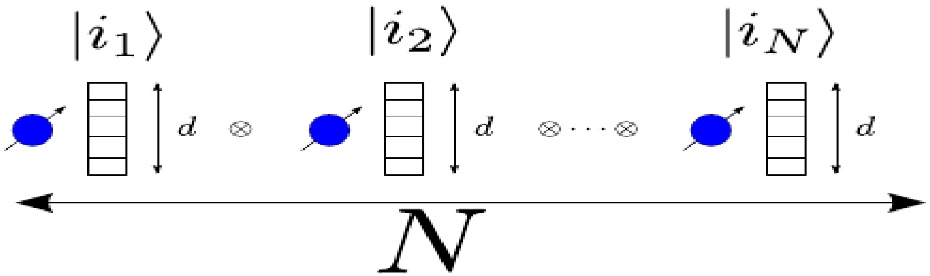
$$|0\rangle, |\uparrow\rangle, |\downarrow\rangle$$

Quantum many body problem

A state of a many body systems

$$|\psi\rangle = \sum_{i_1 \cdots i_N} c^{i_1 \cdots i_N} |i_1 \cdots i_N\rangle$$

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \cdots \otimes \mathcal{H}_N$$



$c^{i_1 \cdots i_N}$ contains d^N parameters

Exponential complexity

- $2^2 = 4$
- $2^3 = 8$
- $2^4 = 16$
- $2^5 = 32$
- $2^6 = 64$
- …
- $2^{40} = 1.0995116e+12$

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1.6 Classes of states that can be represented

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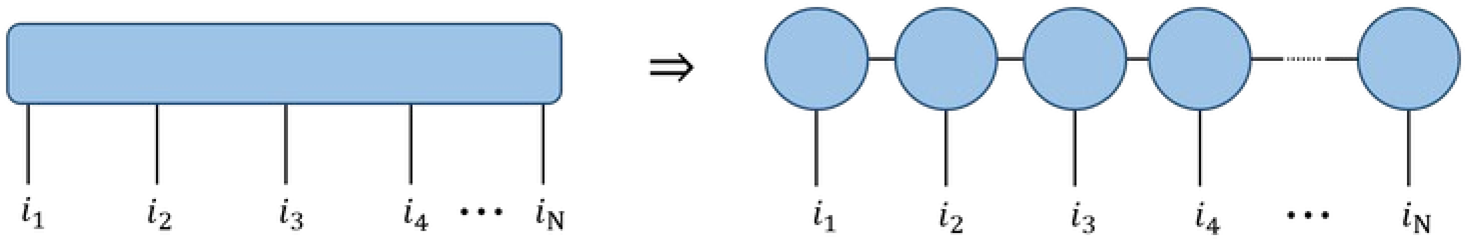
Special states



- Special case, product sates.

Summary

- Tensor networks try to extract structure from data and put it into structure
- They provide a computational representation of data
- There are special classes of counting problems, probabilities distributions, quantum states that can be represented