STAT 4510J Assignment 4

Due 11/27/2022

Problem 1

Widgets cost \$2 each to manufacture and you can sell them for \$3. Your forecast for the market for widgets is (approximately) normally distributed with mean 10,000 and standard deviation 5,000. How many widgets should you manufacture in order to maximize your expected net profit?

Problem 2

Let X follow Poisson(θ) distribution, with $\Theta = (0, \infty)$, and Action space $\mathcal{A} = [0, \infty)$. The loss function is $L(\theta, \delta) = (\theta - \delta)^2$. Assume prior density $\pi(\theta) = e^{-\theta}$

  (a) Assume X is not observed. Calculate the prior risk $\rho(\pi, \delta) = E_{\pi(\theta)}L(\theta, \delta)$ of a decision δ and find the Bayes decision.

  (b) Now, suppose X is observed, and from now on, we use decision rules of the form $\delta_c(x) = cx$. Find the frequentist risk $R(\theta, \delta_c)$.

  (c) Find the Bayes risk of δ_c.

  (d) Find the value of c which minimizes the Bayes risk.

Problem 3

A farmer has to decide whether or not to plant his crop early. If he plants early and no late frost occurs, he will gain \$5000 in extra harvest. If he plants early and a late frost does occur, he will lose \$2000 as the cost of reseeding. If he doesn't plant early, his gain will be \$0. Consulting the weather service, he finds that the chance of late frost is 0.6.

  (a) Describe $\mathcal{A}, \Theta$, the loss function, and the prior distribution.

  (b) What is the Bayes decision? (Show all of your work)