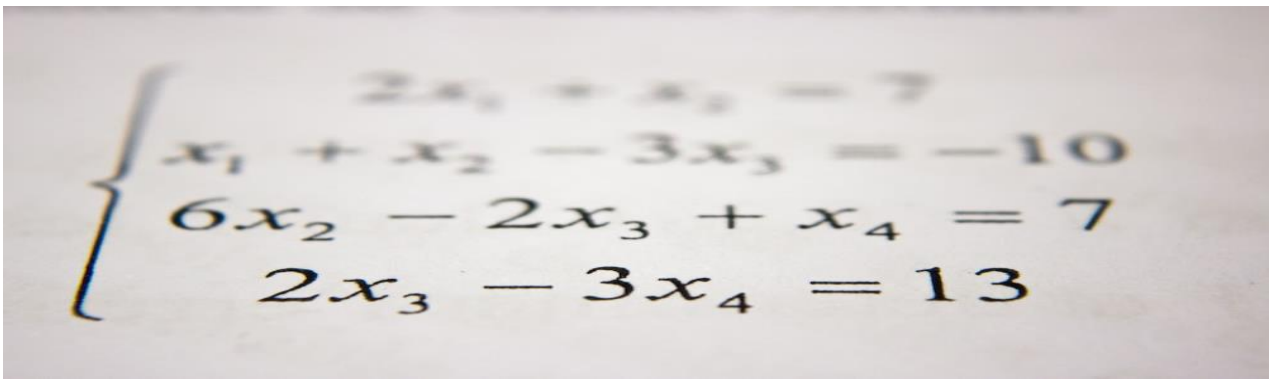
How to Study Mathematics

Here is my brief guide on how to study. For more information on a variety of topics related to learning see my website: http://wolczuk.com/learning.html or the Student Success Office (SSO): https://uwaterloo.ca/student-success/.



The Basics

- Make a List of Important Concepts/Formulas. Review your notes and make a concise list of important concepts and formulas. Make sure you know these formulas and more importantly how to use them! Using flashcards is a good method for helping you remember things.

- Space out your studying. Studies in how the brain works suggests that our brain has difficulty processing more than about an hour of new information. So, long study sessions are largely a waste of time. Experts say that after an hours worth of solid studying, you should take at least a 10 minute break. If you want to keep studying after that, you should switch courses. If you wanted to study for a course 3 hours in one day, then it should be 1 hour in the morning, 1 hour in the afternoon, and 1 hour in the evening.

- Remove as many distractions as possible when studying. Definitely turn off your cell phone, TV, and even no music. You might not think music is a distraction, but anything that your brain is paying attention to other than math is affecting how quickly/deeply you will learn.

- Try to spend most of your time (not all) working on the things you don't know/find difficult. It doesn't feel good, but spending your time practicing what you already know is not an effective/efficient use of time. Part of the key here is figuring out what you know and what you don't know. I'll talk about this in the next section.

- Focus on methods, not answers. It is not about getting the correct final answer. If your method is wrong, we don't get what your final answer is.

The Levels of Learning

Knowing something isn't really an all or nothing kind of thing. It isn't, you either know it or you don't. There are a variety of levels of learning which extend from beginner to expert. The two main keys are being able to accurately identify where your level of learning is and knowing how to move to the next level.

The challenging part about the levels of learning is that you can be at significantly different levels for even extremely closely related topics. For example, you might be at level 4 for taking derivatives of polynomials, but a level 2 for taking derivatives of trigonometric functions.

I'm going to phrase the levels of learning in terms of solving problems. These would be worded a little different for other things such a understanding of concepts, or something like swinging a bat. However, the general principle is the same.

Level 0: You need someone to show you how to solve the problem.

Level 1: You can't solve the problem fully on your own, but you can do some parts of it. With someone assisting you and giving you hints/checking your work, you would be able to get a answer.

Level 2: With enough time and thought you can fully solve the problem on your own. However, if you don't spend enough time and/or thought you certainly may make a mistake. You are not capable of checking your answer on your own. You need to read a solution to determine if you are correct or not.

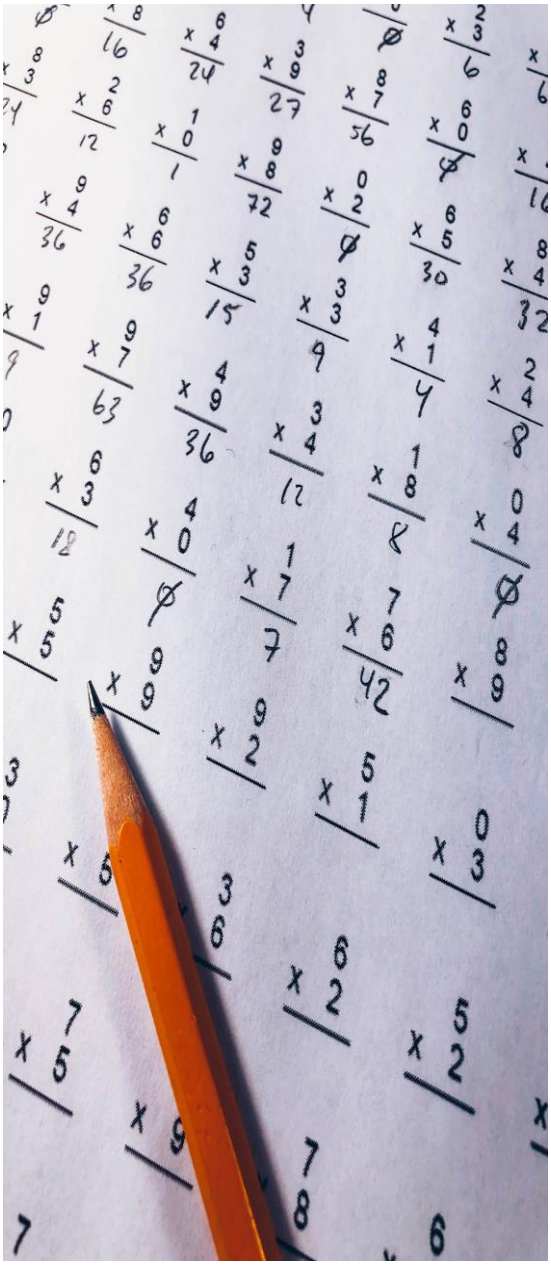
Level 3: It may take a little thought, but you can fully solve the problem on your own although you may not always do it correctly on the first try. However, you are able to check and correct any mistakes you've made without having to look at a solution.

Level 4: You immediately know how to solve the problem. Reading a solution is a waste of time. It is only worth checking your own answer if you were rushed and/or were under pressure (like on an exam).

There are higher levels, but this is all we need to discuss here.

NOTE: The most dangerous thing, which is unfortunately, quite common, is thinking you are at a higher level than you actually are. For more information Google "Illusion of Competence".

To help you understand these, let's look at an example.



Example: Single Digit Multiplication

Level 0: When we were very young, we hadn't been taught multiplication yet. If someone asked what is 3 times 3 we wouldn't even know what they were talking about. Of course, someone started teaching us how to do it and understand it.

Level 1: When asked what a number times a number was, we had a rough idea of what they were asking but wouldn't always know the answer or how to figure it out.

Level 2: We have quite a few multiplications memorized, but don't really understand them. We have learned some tricks of figuring out some of the others we don't know. Even many of the ones we 'know' or can figure out, we are not confident in our answer.

Level 3: We know our multiplication tables. There are a few of them that we need to think about, but we can figure them out. We are rarely wrong and when we are we can correct ourselves (if we think about it).

Level 4: This is where you are now. You don't even have to think about it and wouldn't think about having to check it.

Moving Between the Levels of Learning

The key questions are: ``How do you know which level you are in?'' and ``How do you move from one level to the next?''

Moving from Level 0 to Level 1: Read/watch pre-lecture material. Go to class! Read the textbook. Ask questions about things you don't understand. After taking time to try to understand the methods/concepts you were taught, start doing practice problems.

NOTE: There are two key points here.

1. Get help whenever you need it. By getting help (not answers) and by asking lots of questions, you will make your learning much more efficient. I have never met a student who has a lot of free time. So, making your learning more efficient is very important.
2. You should be studying before you start doing practice problems.

Moving from Level 1 to Level 2: To move from level 1 to level 2, you need to identify the places that you get stuck/don't understand and get help fixing those.

When working on a practice problem:

(a) If you cannot even start it, you are in level 0. Stop working on the problem and get more instruction… not a solution to that particular problem… instruction as if you were a beginner (i.e. read the lecture notes/textbook, watch an instructional video).

(b) If you can start, but get stuck, then start comparing the solution to the start of your answer. The first time you notice a difference, stop reading the solution. If that difference is at the first sentence of the solution, go to (a). Otherwise, read that first sentence that is different than yours and no further. Think about why the solution did that. Focus on trying to remember that step for future problems. Return back to your solution, write that line in … again thinking about how it flows from what you had already written and then continue trying to make the solution. Repeat (b) as many times as necessary.

NOTES:

1. Many students frequently move in the wrong direction. That is when they get to level 1, rather than trying to proceed to level 2, they try to take the easy route and return to level 0. That is, rather than trying to figure out the places they don't understand, they just get someone to show them how to solve the problem (i.e., they read the solution/have someone else solve the problem for them). The only way to move to level 2 is to build on what they know (which is challenging) rather than taking the easy route (looking at someone else's solution).

2. While in Level 1, you will be still getting some instruction/help. That is expected and I would even say required. Not getting that instruction/help you need could slow your learning progress.
3. An excellent source of practice problems with solutions is posted lecture slides and the textbook… what they call examples, you can use as exercises with solutions.

Moving from Level 2 to Level 3: This is mostly just time and practice. The main key here is to allow yourself the time and thought needed. If you give up after a few minutes of looking at a problem and look at a solution… or even get a hint… you are not deepening your understanding of the method. You should allow yourself to struggle. Try to think about other problems you have solved (don't go and look at them… think about them). The goal here is to reinforce the neural pathways needed. Don't feel you have to solve the problem in one sitting. Sometimes it is necessary to move onto another problem or even wait several hours before returning to the problem.

If you can't actually solve the problem without getting a hint/help, then you are actually still in Level 1. Of course, that is okay. You just need to keep applying the level 1 to level 2 strategy.

In this stage you should never ever look at a solution until you have fully completed the problem. Like in (b) above, you should compare your answer to the solution or to your classmates' solutions looking for any differences. If you find you've made a mistake, think carefully about why you made the mistake so that you are less likely to make it again.

Moving from Level 3 to Level 4: This is also just time and practice. But, not only about practicing getting the answer, but practicing checking your answer. You should be trying to work on harder problems and thinking about how the method could be adjusted to solve slightly different problems. If you ever feel you need to look at a solution, you are still in Level 2. The only reason to ever look at solution is to see if there was another way of solving the problem.

What Level Should You Aim For? As mentioned above, you are going to be at different levels for everything. You really should be aiming for at least a Level 3 is most things. If you are a Level 2 at too many things, then you risk either getting questions wrong or not being able to finish the test in the allotted time. If you are at a Level 4 for some things, then that may actually give you some extra time on a test to correctly solve things you are at a Level 2 at. If your goal is a very high mark on the test, then you should be aiming for Level 4 on most things.

Again, the danger is thinking you are at a higher level than you actually are.

The general rules are:

- If you feel you need to read the entire solution to solve the problem, you are at level 0.
- If you only need a few hints to solve the problem (this includes looking at similar examples right before), you are at Level 1.
- You need to be able to solve the problem correctly most of the time without any help (i.e. under the condition of a test) even if it takes you a long time, you are probably at Level 2.
- If you can find mistakes in solutions (yours or someone else's) and can solve the problems correctly almost all the time (and can fix your mistake when you make them), then you are at least a Level 3.

Final Thoughts

Getting Help: Getting help when you need it is absolutely required for moving up from the lower levels of any topic (remembering that whenever you start a new topic you are at level 0, so you should be getting help throughout your undergraduate degree). Remember to 'get help' not have someone give you answers. I also highly recommend working in groups. However, ensure that the group is helping you achieve a higher level of learning.

Two of the Biggest Mistakes:

1. Misusing Solutions. At the beginning, solutions should be used as a source of hints, not as instruction. Imagine if you just started learning to play the piano and every time you made a mistake in a song, the teacher would stop you and play the entire song for you. That would not be helpful, just as reading the full solution is not helpful. As you proceed in your learning, solutions become a way of double checking that you've done it correctly or to see if there was another way of solving the problem. This is like the piano teacher playing the song after you have played it to show you some variations in pitch or tempo. Eventually, you should be able to judge the quality of a solution on your own and so should no longer need to look at solutions (expect possibly to see if there is another way of solving the problem).
2. Illusion of Competence. This is when students think they are at a higher level than they actually are. This generally happens from students reading solutions and thinking "Yeah, I knew how to do that" when if they had to read the solution, they definitely didn't. It also happens from them not practicing under the conditions of a test. That is, they get lots of help/hints while doing practice problems (looking up formulas, looking at similar examples, etc.).

Top Two Recommendations:

1. Mistakes are the best teacher. Don't be afraid of making mistakes and/or getting things wrong. These are the best opportunities to learn… as long as you use your mistakes to learn from. The key to success in anything is knowing how to learn from your mistakes.
2. Build your confidence. Many students tend to overthink problems as they are not confident. This largely comes from always checking solutions. If you build your confidence and *know* how to solve the problems, then you are much less likely to overthink them.

Learning takes time and is supposed to be difficult. If you try to avoid difficulty, you are not learning as much as you could. Eating healthily, drinking plenty of water, and getting enough sleep are also all extremely important in learning (I really should add in getting some exercise).

I hope this helps! Note that changing study/learning habits is not so easy. It will also take you effort to adjust to these methods. That is okay. Be patient with yourself.

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