Tidal Forces on Earth

Morris Blaustein
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Abstract
Two high tides bulge on opposite sides of the Earth. This happens as a result of sinusoidal acceleration of ocean water molecules provided by tidal forces, the varying gravitational field throughout Earth caused by the mass of the Moon and Sun. This project focuses on the tidal bugle closer to the Moon, which slightly leads the Moon in its orbit. Because more mass is located in front of the Moon's orbit, the Moon pulls back on the Earth, causing a torque in the opposite direction of the Earth's rotation. As a result, the Earth is slowing down. When the Earth was born 4.5 billion years ago, one day was approximately 6.5 hours [1]. The changing rate of Earth's rotation has had interesting geophysical effects throughout Earth's history. This project simulated the Earth-Moon system to determine how many years it will take until the torque slows the Earth such that one day equals one month. Using VPython, 800 water molecules were placed on Earth's surface. Each molecule felt a different force from the moon. The simulation showed that a tidal bulge leads the Moon when the Earth is rotating faster than the Moon's orbit. It determined that the torque would continue for 2.5 billion years, a significantly smaller amount of time than the scientifically observed 1.9 trillion years [2]. This means that the net torque for each time step in this simulation was larger than the real torque in the Earth-Moon system. The deviation in torques could have resulted from several reasons: the model of the ocean was very basic, the net torque decreases as a function of time and Earth's angular velocity, and lastly the code used showed several small errors. Considering the possible sources of error and the relatively similar magnitude of both scientific and simulated results, this program provided a good basic model of the movement of the ocean tides.

Introduction
The waters on Earth are known to follow cycles in which water level, for a given location, rises and falls daily. These changes are most commonly known to be the result of gravitation forces upon the Earth by the Sun and the Moon. An interesting observation is that although the sun is 27 million times more massive than the moon (and therefore generates a much larger gravitational pull), it only accounts for roughly 30% of the change in Earth's tides. This is because changes in Earth's tides are due to tidal forces, which are inversely proportional to the cube of the distance of the objects being considered. The moon is much closer to the Earth and therefore generates a much larger tidal force. Tidal forces are gravitational forces that account for changes in mass location throughout a body of mass. Each point inside Earth has a different distance from the moon, and therefore feels a different strength of gravitational pull. As a result, tidal forces from the moon stretch the Earth horizontally and squeeze the Earth vertically.

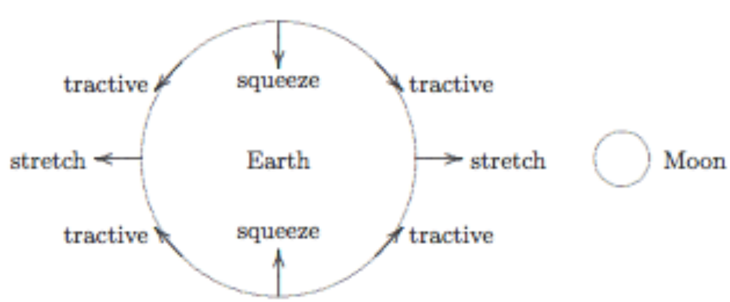

Figure 1. Effect of tidal forces of the Moon on Earth

Effects of Tidal Forces

The Earth spins at about 14.5 degrees per hour and drags ocean water molecules with it. Each molecule therefore is at a different distance from the moon every instant and feels a varying gravitational force. At the closest and furthest point from the moon, the magnitude of the tidal acceleration for one water molecule is about 175nano-g [3]. This is not enough acceleration to cause the tidal changes seen on Earth. In the Bay of Fundy in New Brunswick, the peak tidal range is 15m. The reason that water levels change is a combination of tidal forces and Earth's rotational speed. This system creates an equilibrium in which acceleration of tidal forces create a noticeable effect. A view of Earth from the North Pole can be split into four quadrants such that water molecules in each quadrant either feel an acceleration or deceleration. This is a result of tidal forces and in Figure 3 explains the differences in water levels. Each water molecule on Earth feels varying acceleration based on its position, producing a sinusoidal force cycle per molecule with a period of half a day.

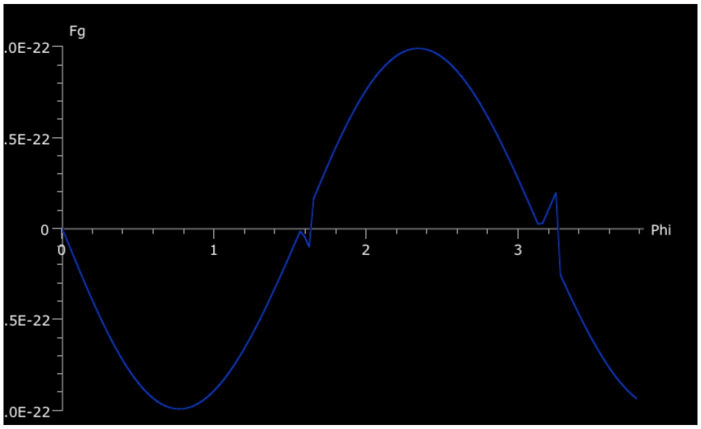

Figure 2. Gravitational Force for one Water Molecule

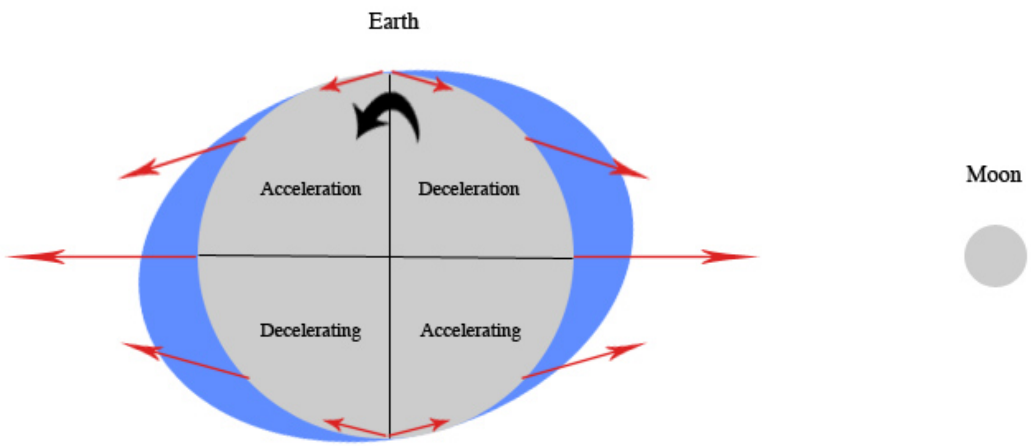

Figure 3. North Pole view of Earth. Red arrows show tidal forces. Four quadrants represent accelerations of water molecules throughout Earth. Blue shows resulting tides.

The tides shown in Figure 2 are not aligned with the Moon because the nature of the accelerating molecules gives them the highest current speed at 0 and 180 degrees, resulting in the tidal bulges shown. This offset bulge on Earth causes a torque from the Moon's gravitation onto Earth. As a result, Earth's rotational speed slows down at a rate of 1.7 milliseconds per century [2]. Earth's lost rotational energy is gained by the Moon in kinetic energy, explaining why the Moon is slowly receding from Earth at a rate of about 4cm per year. This will continue until one day on Earth equals the orbital period of the Moon. In this scenario, the moon will always be above the same location and so the Earth-Moon system sees no change in tidal forces.

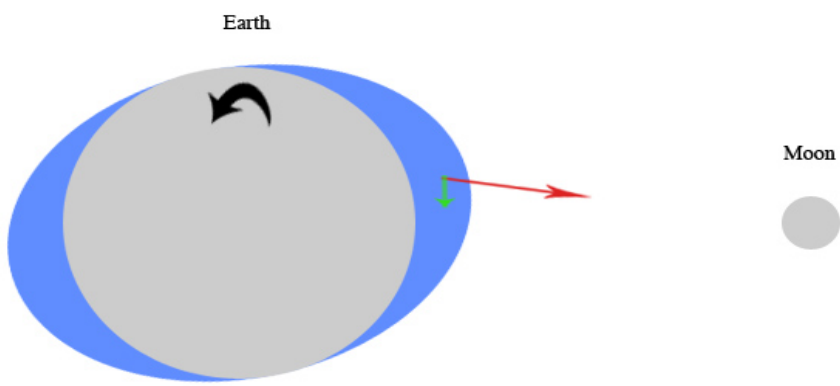

Figure 4. Red Arrow shows net gravitational pull of Moon of Earth from bulged region. Green arrow shows resulting torque.

Geophysical Effects of Tidal Forces

Earth's rotational period when it was born 4.5 billion years ago has been estimated to be 6.5 hours [1]. Because it had such a high angular momentum, scientists theorize that Earth could have had rings similar to Saturn's rings in its early days. It has also been estimated that Earth originally had a 2500-mile bulge at the equator, compared to the 17-mile bulge it has today. This enormous shift in dimensions has had many effects on the dynamics of the Earth's geophysical system.

One of the reasons Earth is not perfectly spherical is because its angular momentum and tidal forces stretch it horizontally and squeeze it vertically. However, as the Earth slows and its angular momentum decreases, it takes a more spherical shape. The Lageos satellite monitored by JPL has confirmed this gradual change of dimensions [1]. The change has had dramatic effects on Earth's crust creating increased pressure inside Earth. The Pacific Ring Of Fire, an area in the Pacific Ocean where there are many volcanoes, is partly active as a result of Earth's squeezed plates.

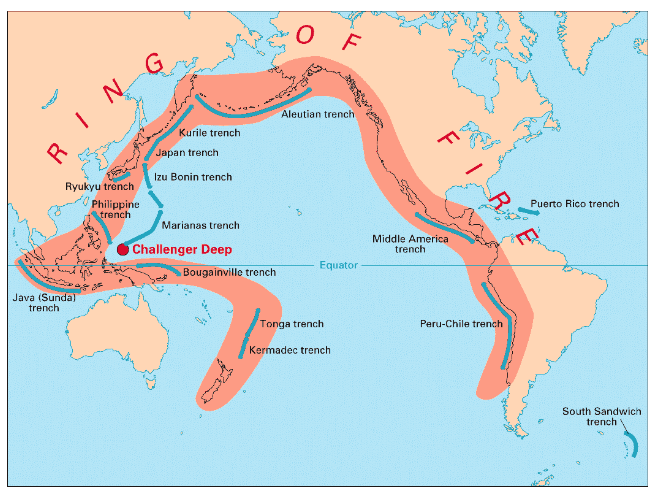

Figure 5. Pacific Ring of Fire

When Earth was spinning much faster, more of the ocean water was concentrated towards the equator. The atmospheric layers were also concentrated more towards the equator. These differences indicate that the climate of Earth's early days has vastly changed compared to its present climate.

Purpose of Program

The program will simulate the Earth-Moon system, the tidal gravitational effects on ocean water, and the resulting high and low tides. The torque caused by the leading high tide is slowing down the Earth's rotation at approximately 1.7ms per century [2]. This program will determine the how many years it will take for the torque caused by the high tide until the Earth-Moon system reaches equilibrium such that one day equals one month.

Program Structure

The program consists of five main components shown in Table 1. The Earth is a sphere located at position (0,0,0). 800 Water molecules are placed on the surface of Earth. The water molecules are contained in a list called Ocean. The Ocean list corresponds to the Ocean Frame, which rotates along with Earth. The Moon is a sphere placed initially at (50,000,0,0). Its position is updated using trigonometric functions.

Table 1. Components of the Program

Object	Type	Extensions	Rationale
Earth	Sphere	Earth.i Earth.o Earth.p	Focus of Study
Moon	Sphere	Moon.rpos Moon.opp	Provides Gravitational Force
WaterMol	Sphere	WaterMol.r WaterMol.beta WaterMol.Fg WaterMol.rpos	Represents part of the Ocean
Ocean	List	Ocean_Frame	Represents the Ocean
Tide Ratio	Int		Calculate Net Torque

Time and Angle Parameters

The program uses one time and two angle parameters to keep the system equalized. Theta is used to control the Moon's orbit and dTheta = 0.001. T is the time variable, which updates along with dTheta. Phi is used to control Earth's rotation and is updated with dPhi = Earth.Omega*dt.

Program Functions

The program uses two separately defined functions in the main while loop. The functions are shown in Table 2

Table 2. Functions Used in the Program

Function	Parameters	Action/Output
Update_pos	Sphere Alpha Radius	Update position of sphere
World_space_pos	Frame Local	Return real space position of object in frame

While Loop
A while loop is used to control all the action of the program. For each iteration of the while loop, the moon position is updated, the earth and ocean frame are rotated, the gravitational force for each water molecule is calculated, and the position of each water molecule is updated.

Geometry
In order to accurately and efficiently simulate the model of the Earth-Moon system, three angle parameters are used. Figures 5 and 6 indicate the assignment of each angle parameter. All angles shown are measured from the positive x-axis. Theta is the angle of the moons orbit, phi is the angle of Earth's rotation, and beta is the angle of a water molecule with respect to the ocean frame it is in. The square shows the movement of the ocean frame.

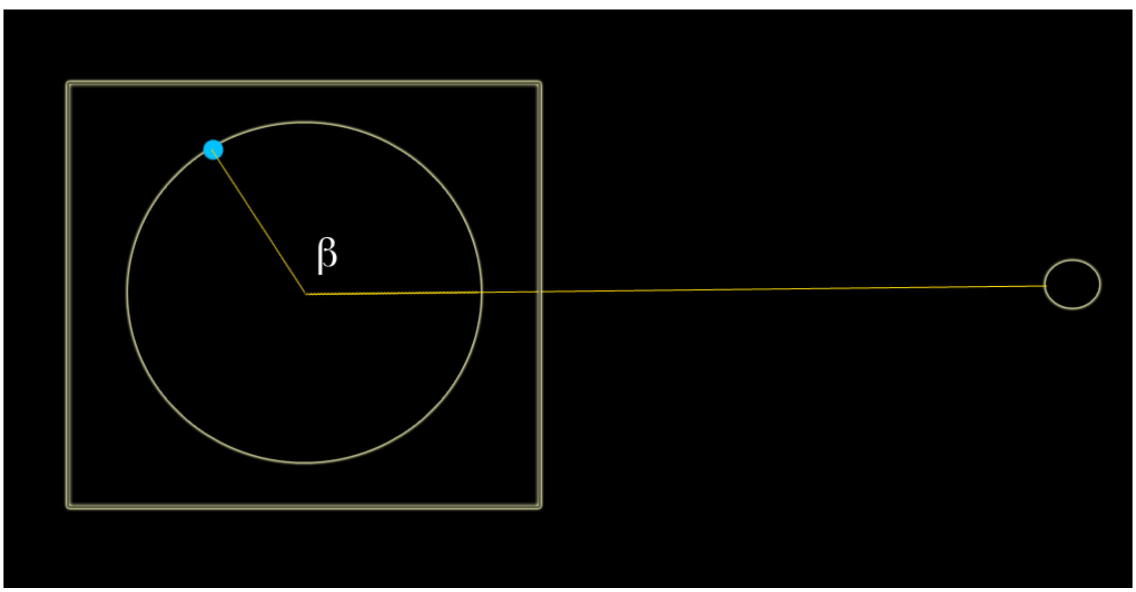

Figure 5. Initial Frame of Earth-Moon System

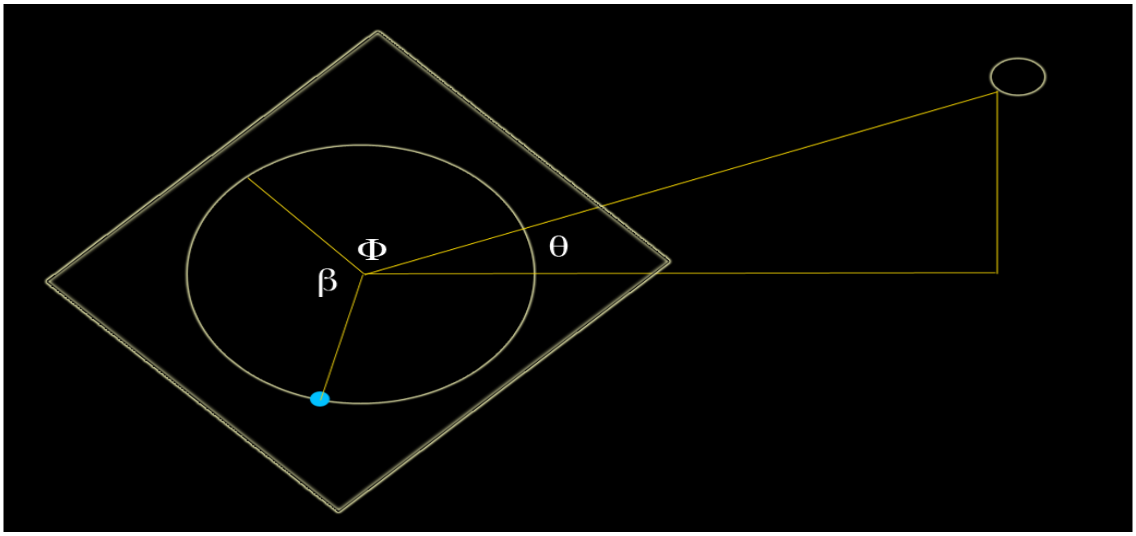

Figure 6. Frame of Earth-Moon System After Several Time Steps

```
for object in Ocean:
    object.rpos = world_space_pos(Ocean_Frame,object.pos)
    while object.beta >= 2*pi:
        object.beta = object.beta - 2*pi
    if object.beta > Theta and object.beta < (Theta + pi/2):
        object.Fg = -GMm/mag2(Moon.rpos-object.rpos)
    elif object.beta > (Theta + pi/2) and object.beta < (Theta + pi):
        object.Fg = GMm/mag2(Moon.opp-object.rpos)
    elif object.beta > (Theta + pi) and object.beta < (Theta + 3*pi/2):
        object.Fg = -GMm/mag2(Moon.opp-object.rpos)
    else:
        object.Fg = GMm/mag2(Moon.rpos-object.rpos)
    object.Fg = object.Fg*abs(sin(object.beta)*cos(object.beta))
    object.beta = object.beta + object.Fg/Water_Mass/Earth_Radius + dPhi
    update_pos(object,object.beta,object.r)
```

Figure 7. Code Used to Determine Force of Gravity for each Water Molecule

Calculation of Net Torque

After the program runs several time steps and a tidal bulge emerges that leads the moon, the while loop stops. A for loop iterates through the ocean list and calculates the ratio of water molecules in high tide and in low tide. Shown in Table 1, this is the tide ratio. The tidal bulge is assumed to lead the moon by an angle of pi/4. The mass of ocean water is about 6e19 kg and the average tide change is 1.3 meters [4]. Not all of the ocean water mass can be considered effective in the tide ratio because only 1.3 meters of water is moving back and forth between high and low tide. Using the tide ratio, the force of gravity between the moon and the high and low tides is determined. The Moon orbits in the x-z plane and therefore the difference in the z components of the two forces is the resulting net force of tidal gravity. This force is applied at one Earth radius from the axis of rotation, so the net torque is the net force multiplied by the Earth radius.

Calculation of Days Remaining which Day < Month

This program assumes a linear torque relationship versus time. To find the final answer, Earth's total momentum is divided by the net torque to find the number of time steps it will take to slow the Earth such that one day equals one month. The number of cycles is multiplied by the length dt of each cycle.

```
Tidal_Force = Quad_1_Pull.z + Quad_4_Pull.z
Tidal_Torque = Tidal_Force*(Earth_Radius-Ocean_Depth/2)
cycles = Earth.p/Tidal_Torque
seconds = cycles*dt
years = seconds/(60*60*24*365)
```

Figure 8. Code Used to Determine Net Torque and Final Answer

Tests

There were two tests in writing this project that confirmed successful progress. The first test is shown in Figure 2. This test showed that the force of gravity for one water molecule varied with a period of pi. This correctly matches the expected force from the four quadrants shown in Figure 9.

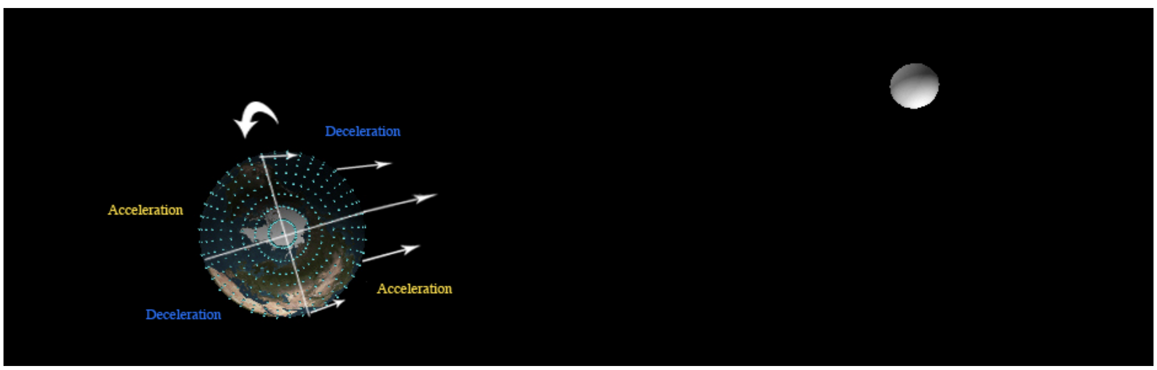

Figure 9. Aerial View of Earth-Moon System. Four quadrants show varying acceleration of water molecules.

The second test confirmed that all the time and angle parameters matched when each were updated in the while loop. This was an important test and was performed early in the program development.

```
Theta=  0.212 T=  0.995859872611 Phi=  6.25527470064
Theta=  0.213 T=  1.00055732484 Phi=  6.28478071338
Theta=  0.214 T=  1.00525477707 Phi=  6.31428672611
Theta=  0.215 T=  1.0099522293 Phi=  6.34379273885
Theta=  0.216 T=  1.01464968153 Phi=  6.37329875159
```

Figure 10. Time and Angle Parameters After One Day

Results

The program runs successfully until theta moves past pi/6. At this point the tidal bulges are still opposite from each other, but not aligned with their real position. Figure 11 shows the result of the simulation. The tidal bulges lead the Moon, as they should. The net torque determined by the program was 2.58e19 Nm. Considering that Earth has a moment of inertia of 8.034e37kgm^2 and it spins at 7.27e-5 rad/sec, the program computed that it will take 2.5 billion years until the Earth-Moon system reaches equilibrium [5]. According to scientific observation however, it will actually take near 1.9 trillion years to reach equilibrium.

Figure 11. Screen Shot of Simulation after Several Time Steps. High tide leads the moons orbit.

Possible Sources of Error

There are several possible sources of errors that could have led to the deviation in the program's and the observed number of years. First, the model of the ocean was very general. It only modeled the surface of the ocean and did not consider that ocean molecules at deeper levels are further away from the moon. This would have decreased the net torque calculated by the program and increased the number of years. Second, torque should decrease as a function of omega and time because as the earth slows, the tidal bulge begins to align with the moon. This would decrease the angle of the center of gravity between the tidal bulge and the moon, therefore decreasing the force component that opposes the Earth's rotation. Figure 12 shows how the number of years varies as a function of the angle between the center of mass of the tidal bulge and the moon. To expand on this project, the net torque could be integrated with respect to the changing center of mass angle. The torque would be a function of omega, rather than constant, which would calculate a larger and more accurate answer.

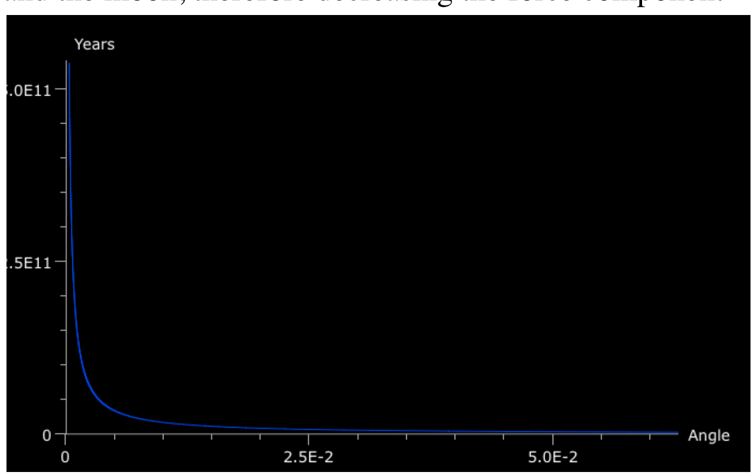

Figure 12. Final answer as a function of center of mass angle of leading tidal bulge

Conclusion

Although the program and the scientifically observed answer were different, they had relatively similar magnitude. Considering the possible sources of error, the program provided a good simulation of the Earth-Moon system and the tidal gravity effects on the ocean. It was simple and clear demonstration of varying gravitational through a body of mass. As computer systems become more complex and research finds more answers to the fundamental questions of life and the universe, programs will be able to completely and accurately complex physical interactions.

Tidal gravity lies at the core of physics and interactions of matter. Gravity is an attractive force between all objects, and each object experiences the variation of gravitational fields. Though effects of tidal gravity are only observable on the macroscopic scale relative to humans, every massed object experiences some sort of squeezing and stretching due to tidal gravity. Within reference frames of relativistic space, tidal gravity explains many phenomena seen in the universe. In the relatively small space curvature in the solar system, the effects of tidal gravity can be simply modeled. However in areas of large space curvature, such as near black holes and neuron stars, tidal gravity cannot be simply modeled. In these areas, objects are stretched beyond imagination.

References

[1] "The Geophysical Effects of the Earth's Slowing Rotation!" "Human Imagination, Mankind's Seventh Sense". Web. 10 Apr. 2012. <http://novan.com/earth.htm>

[2] Moore, Patrick, Robin Rees, and Patrick Moore. Patrick Moore's Data Book of Astronomy. Cambridge: Cambridge UP, 2011. Print.

[3] Jordaan, Burt A. Einstein's Relativity For Engineers.

[4] "How Much Water Is There on Earth?" HowStuffWorks. Web. 16 Apr. 2012. <http://science.howstuffworks.com/environmental/earth/geophysics/question157.htm>.

[5] "Moment of Inertia--Earth -- from Eric Weisstein's World of Physics." ScienceWorld. Web. 16 Apr. 2012. <http://scienceworld.wolfram.com/physics/MomentofInertiaEarth.html>.