

1.1) 设 1234 的 2 乘积为 M_2 . 则

$$M_2 = 1 \times 2 \times 4 + 12 \times 3 \times 4 + 123 \times 4 \\ = 1134$$

2) 设 $C[i][j]$ 表示从第 i 位到第 j 位组成的十进制数

$M[i][j]$ 表示前 i 位分成 j 段的最大乘积, 则前 i 位分成 j 段且 $n-i$ 位分成 $k-j$ 段且 $1 \leq j \leq i$

$$\text{则 } M[i][j] = \max_{1 \leq r < j} \{ M[i][r][j], M[r][j-1] \times C[r+1][i] \}$$

特别地 $M[i][1] = C[i][1]$

伪代码如下:

Input: P, n, k

Output: $M[n][k]$, P 的最大 k 乘积

for $i = 1$ to n

for $j = 1$ to n

$C[i][j] = \text{the number of } i \text{ th } i2 \dots j$

for $i = 1$ to n

$t_0 = P \bmod 10$

$C[i] = t_0 \times 10^{i-1}$

$P = P \% 10$

for $i = 1$ to n

$M[i][1] = C[i][1]$

for $j = 1$ to n

for $j = k - n + i$ to i $\leftarrow M[i][j] = -\infty$

for $r = 1$ to $i-1$

$$M[l][j] = \max\{M[l][j], M[l][j-1] \cdot C[r+1][j]\}$$

endfor
endfor
endfor

return $M[l][k]$.

2. 将这些区间看成是点的坐标.

① 将点按 x 坐标排序

② 找出'极大点', 即从左往右扫描时没有点好该点的点

③ 排除极大点取的点即为所求区间.

Input: a set of intervals on a number axis. $S = \{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$

Output: a set of intervals ^P contained by other intervals in S .

sort all intervals in S , x in ascending order and y in descending order

$y_{\max} = -\infty$

for $i = 1$ to n .

if $y_i > y_{\max} : y_{\max} = y_i$

// means (x_i, y_i) is a maximum point.

else put (x_i, y_i) in P .

~~Output~~ return $n - |P|$.

3. 证明: 对于所有 $A_{n \times n} * B_{n \times n}$ - 所以构造

$$\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} \times \begin{pmatrix} B & 0 \\ 0 & A \end{pmatrix} \\ = \begin{pmatrix} \textcircled{AB} & 0 \\ 0 & BA \end{pmatrix} \quad \text{Q.E.D.}$$