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Vector Calculus



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1. **Vectors in Euclidean Space**
   1. **Vector and its Properties**

This section defines a vector and describes its common properties. Also, defines a scalar quantity.

* + 1. **About Vector**

A vector, **v**, is defined as an element in *vector space* (a Euclidean dimension). A vector is a directed line segment from **initial point** P to **terminal point** Q, where P ≠ Q. It is denoted by .

A vector consists of the two components:

* Magnitude - The norm of **v**, denoted by |**v**|
* Direction - The direction of **v**, denoted by 

**Example**

A vector **v** = (a, b) in 2-D Cartesian Coordinate System R2 have origin at (0, 0).

* + 1. **Vector Magnitude**

Let there is a vector **v** in Euclidean n-space, **Rn** (vector space), where

**v** = (**v1**, **v2**, …, **vn**)

The magnitude of the vector in Euclidean space is the length of the line segment and is defined as

|**v**| = 

The magnitude of a vector where P = (a1, a2, …, an) and Q = (b1, b2, …, bn) in n-space Euclidean dimension is,

|| = 

The magnitude is never negative.

* + 1. **Vector Direction (Unit Vector)**

A unit vector is a vector of length 1, sometimes also called a **direction vector**. The direction is the same as that of the directed line segment. The **unit vector**  having the same direction as a given (non-zero) vector v is defined by

 where **v** is not Zero vector

* + 1. **Zero Vector (Null Vector)**

A zero vector, denoted 0, is a vector of length 0, and thus has all components equal to zero. It is just a point in Euclidean space.

For Zero vector, initial and terminal points coincide. Zero vector can not be assigned a definite direction as it has zero magnitude.

**Example**

The vectors ,  represent the zero vector.

A zero vector in 2-D Cartesian Coordinate R2 is represented as (0, 0).

A zero vector in 2-D Cartesian Coordinate R3 is represented as (0, 0, 0).

* + 1. **Coinitial Vector**

Two or more vectors having the same **initial point** are called coinitial vectors.

**Example**

Vectors  and are coinitial vectors.

* + 1. **Basis Vector**

A vector basis of a vector space V is defined as a subset v1,...,vn of vectors in V that are linearly independent and span V.

Consequently, if IMG_256 is a list of vectors in IMG_257, then these vectors form a vector basis if and only if every IMG_258 can be uniquely written as

IMG_256

**Example**

Mass, Electric Charge and speed

* + 1. **Direction Cosines**

The cosines of angles that the vector forms with the coordinate axes.

Let **v** is the vector in an n-dimensional Euclidean space where

**v** = (**v1**, **v2**, …, **vn**)

Then the direction cosines are defined as,

**D**j = cos(θj)=****, where j = 1,2, …,n

* 1. **Properties of Vector**

This section defines the algebraic operations that can be performed on vectors.

* + 1. **Equality**

Two vectors  and are equal if and only if

* They are collinear (lie on the same line or on a parallel line to this),
* They are co-directed, , and
* Have same magnitude, 
  + 1. **Negative vector**

A vector whose magnitude is the same as that of a given vector (say, ), but direction is opposite to that of it, is called negative of the given vector.

**Example**

Vector is negative of the vector , and written as  = -

* 1. **Scalar**

A quantity that can be defined by only Magnitude. It doesn’t have any direction.

**Example**

Mass, Electric Charge and speed

* + 1. **Scalar Multiple**

Let there is a scalar k, and a vector .

The scalar multiple of  by k is

 = k

**Magnitude** of Scalar Multiple is, || = |k| = |k|||

**Direction**

* Same if k>0
* Opposite if k<0
* Zero vector if same if k=0

**Nomenclature**  
The term *scalar* was invented by 19th century Irish mathematician, physicist and astronomer William Rowan Hamilton, to convey the sense of something that could be represented by a point on a scale or graduated ruler. The word vector comes from Latin, where it means “carrier”.

* + 1. **Parallel (Co-linear) vectors**

Two vectors  and  are parallel, if and only if one is a scalar multiple of other

 = k

Where k is a scalar quantity

* + 1. **Associative Property**

For is a vector , and two scalars k and l, then

*k*(*l***v**) = (*kl*)**v**

* + 1. **Distributive Property**

For vectors and , and a scalar k,

k(+)=k+k

* + 1. **Scalar Distributive Property**

For vectors , and a scalar k and l

(k + l)=k+l

* 1. **Addition**

The addition of two vectors  and is defined as

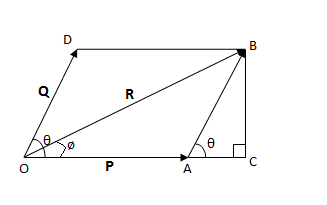
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* + 1. **Parallelogram law of vector addition**

**Reference: <https://www.mathstopia.net/vectors/parallelogram-law-vector-addition>**

If two vectors acting simultaneously at a point can be represented both in magnitude and direction by the adjacent sides of a parallelogram drawn from a point, then the resultant vector is represented both in magnitude and direction by the diagonal of the parallelogram passing through that point.

Let P and Q be two vectors acting simultaneously at a point and represented both in magnitude and direction by two adjacent sides OA and OD of a parallelogram OABD as shown in figure. Let θ be the angle between P and Q and R be the resultant vector. Then, according to parallelogram law of vector addition, diagonal OB represents the resultant of P and Q.



So, we have

****R****=****P****+****Q****

* + 1. **Triangle Law of Vector Addition**

If 2 vectors acting simultaneously on a body are represented both in magnitude and direction by 2 sides of a triangle taken in an order then the resultant(both magnitude and direction) of these vectors is given by 3rd side of that triangle taken in opposite order.

Both the laws of vector are equivalent to each other.

* + 1. **Magnitude of the resultant**



* + 1. **Direction of the resultant**

Let ø be the angle made by resultant R with P.



* + 1. **Commutative Property**

**Reference: <http://ncert.nic.in/ncerts/l/lemh204.pdf>**

For any two vectors  and ,

+ = +

**Proof**

* + 1. **Associative Property**

For any three vectors ,  and ,

(+)+  = +(+)

* + 1. **Additive Identity**

For a vector and zero vector ,

+=+=

Then the zero vector is called the additive identity for the vector addition.

* + 1. **Additive Inverse**

For a vector and the negative vector -,

+(-)=(-)+=

Then the zero vector is called the additive identity for the vector addition.

* + 1. **Final Points**

The sum of vectors v and w, denoted by v+w,The sum of vectors v and w, denoted by v+w, has the initial point of v+w is the initial point of v, and its terminal point is the new terminal point of w.

* 1. **Subtraction**

The subtraction of two vectors  and is defined as

 - 

* + 1. **Distance between Points**

The distance between two points = (*x*1, *y*1, *z*1) and *Q* = (*x*2, *y*2, *z*2) in R3 is the same as the length of the vector **w**−**v**,

where the vectors **v** and **w** are defined as **v** = (*x*1, *y*1, *z*1) and **w**= (*x*2, *y*2, *z*2) (see Figure 1.2.8).

So since **w**−**v** = (*x*2−*x*1, *y*2− *y*1, *z*2−*z*1), then *d* = k**w**−**v**k =

p

(*x*2 −*x*1)2 +(*y*2 − *y*1)2 +(*z*2 − *z*1)2