closely. The updated model is considerably improved compared to the original finite element model.

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A. Berman
Associate Editor

Golinski's Speed Reducer Problem Revisited

Tapabrata Ray*
National University of Singapore, Singapore 119260, Republic of Singapore

Introduction

GOLINSKI'S[1,2] speed reducer problem is one of the most well-studied problems of the NASA Langley Multidisciplinary Design Optimization (MDO) Test Suite. Golinski modeled the speed reducer with an aim to minimize its weight while satisfying a number of constraints imposed by gear and shaft design practices. Since then, many researchers, for example, Rao,[3] Li and Papalambros,[4] Kuang et al.,[5] and Azarm and Li[6] have reported solutions of this problem. However, the solutions reported by all of the researchers just mentioned are not feasible, including the one that appears in the MDO Test Suite itself, obtained using the constrained minimizer CONMIN.[7] The present Note presents the best-known feasible solution to this problem and provides a comparison of this solution with those of Refs. 3–7. The details of the swarm algorithm are described in the Appendix. The notation and sequence of constraints used in the present Note are the same as the problem description in the NASA Langley MDO Test Suite.

Problem Statement

The objective is to find the minimum gearbox volume f (and hence, its minimum weight), subject to several constraints. There are seven design variables: width of the gear face x_1, teeth module x_2, number of pinion teeth x_3, shaft 1 length between bearings x_4, shaft 2 length between bearings x_5, diameter of shaft 1 x_6, diameter of shaft 2 x_7. These objectives lead to the following constrained optimization problem.

Minimize:

$$
\begin{aligned}
f(\boldsymbol{x}) = {} & 0.7854x_1x_2^2\left(3.3333x_3^2 + 14.9334x_3 - 43.0934\right) \\
& - 1.5079x_1\left(x_6^2 + x_7^2\right) + 7.477\left(x_6^3 + x_7^3\right) \\
& + 0.7854\left(x_4x_6^2 + x_5x_7^2\right)
\end{aligned}
$$

Subject to (constraints in right-hand column)

$$27x_1^{-1}x_2^{-2}x_3^{-1} \le 1 \qquad G_1$$

$$397.5x_1^{-1}x_2^{-2}x_3^{-2} \le 1 \qquad G_2$$

$$1.93x_2^{-1}x_3^{-1}x_4^3x_6^{-4} \le 1 \qquad G_3$$

$$1.93x_2^{-1}x_3^{-1}x_5^3x_7^{-4} \le 1 \qquad G_4$$

$$\left[\left(745x_4x_2^{-1}x_3^{-1}\right)^2 + 16.9\times 10^6\right]^{\frac{1}{2}} \Big/ \left[110.0x_6^3\right] \le 1 \qquad G_5$$

$$\left[\left(745x_5x_2^{-1}x_3^{-1}\right)^2 + 157.5\times 10^6\right]^{\frac{1}{2}} \Big/ \left[85.0x_7^3\right] \le 1 \qquad G_6$$

$$x_2x_3/40 \le 1.0 \qquad G_7$$

$$5x_2/x_1 \le 1.0 \qquad G_8$$

$$x_1/12x_2 \le 1.0 \qquad G_9$$

$$(1.5x_6 + 1.9)x_4^{-1} \le 1 \qquad G_{24}$$

$$(1.1x_7 + 1.9)x_5^{-1} \le 1 \qquad G_{25}$$

Constraints G_{10} and G_{11} are the side constraints of x_1, constraints G_{12} and G_{13} are the side constraints of x_2, constraints G_{14} and G_{15} are the side constraints of x_3, constraints G_{16} and G_{17} are the side constraints of x_4, constraints G_{18} and G_{19} are the side constraints of x_5, constraints G_{20} and G_{21} are the side constraints of x_6, and constraints G_{22} and G_{23} are the side constraints of x_7. The variable bounds for the problem are as follows: $2.6 \le x_1 \le 3.6$, $0.7 \le x_2 \le 0.8$, $17 \le x_3 \le 28$ (integer values), $7.3 \le x_4 \le 8.3$, $7.3 \le x_5 \le 8.3$, $2.9 \le x_6 \le 3.9$, and $5.0 \le x_7 \le 5.5$.

Discussion

From Table 1 it is clear that the solutions reported by Rao,[3] Li and Papalambros,[4] and Azarm and Li[6] violate constraint G_5 and G_{25}. The result reported by Kuang et al.[5] violates G_6, and that using CONMIN at the MDO test suite violates G_5, G_6, and G_{25}. Although for some solutions the amount of constraint violation is marginal, the solutions are still infeasible form a mathematical point of view.

Swarm algorithms have been applied successfully to single-objective unconstrained and constrained optimization problems by Ray and Liew[8] and to engineering design optimization problems by Ray and Saini.[9] A swarm size of 70 has been used in this study, and the results are after 70,000 function evaluations. Results of five successive runs are presented in Table 1 along with their objective function values. The best solution obtained using the swarm algorithm is compared with other reported results in Table 2.

Conclusions

This Note provides a comparison between the results reported by various sources to Golinski's speed reducer problem. It is interesting to take note that several of these reported solutions are infeasible. A swarm algorithm has been used in this study to solve the nonlinear constrained minimization problem and arrive at the best known feasible solution.

Appendix: Swarm Algorithm

A general constrained optimization problem (in the minimization sense) is presented as follows.

Minimize:

$$f(x)$$

Subject to:

$$g_i(\boldsymbol{x}) \ge a_i, \qquad i = 1, 2, \ldots, q$$

$$h_j(\mathbf{x}) = b_j, \qquad j = 1, 2, \ldots, r$$

where there are q inequality and r equality constraints, $\boldsymbol{x} = [x_1\ x_2\ \ldots\ x_n]$ is the vector of n design variables, and a_i and b_j

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*Senior Research Scientist, Temasek Laboratories, 10 Kent Ridge Crescent; tsltray@nus.edu.sg.

Table 1 Results of five trials using swarm

Solution	Run 1	Run 2	Run 3	Run 4	Run 5
x_1	3.50000002	3.50000004	3.50000004	3.50000002	3.50000004
x_2	0.70000000	0.70000000	0.70000000	0.70000000	0.70000000
x_3	17	17	17	17	17
x_4	7.30000083	7.30000014	7.30000083	7.30000009	7.30000021
x_5	7.80000058	7.80000024	7.80000011	7.80000000	7.80000007
x_6	3.35021468	3.35021472	3.35021472	3.35021468	3.35021467
x_7	5.28668324	5.28668324	5.28668323	5.28668325	5.28668325
Objective	2996.232165	2996.232169	2996.232165	2996.232157	2996.232160

Table 2 Results of speed reducer design[a]

Solution	Rao[3]	Li and Papalambros[4]	Kuang et al.[5]	Azarm and Li[6]	MDO Test Suite (NASA) using CONMIN[7]	Present
x_1	3.50000000	3.50000000	3.60000000	3.50000000	3.50000000	3.50000002
x_2	0.70000000	0.70000000	0.70000000	0.70000000	0.70000000	0.70000000
x_3	17.00000000	17.00000000	17.00000000	17.00000000	17.00000000	17
x_4	7.30000000	7.30000000	7.30000000	7.30000000	7.30000020	7.30000009
x_5	7.30000000	7.71000000	7.80000000	7.71000000	7.30000020	7.80000000
x_6	3.35000000	3.35000000	3.40000000	3.35000000	3.35021450	3.35021468
x_7	5.29000000	5.29000000	5.00000000	5.29000000	5.28651760	5.28668325
G_1	0.9260847	0.9260847	0.9003601	0.9260847	0.9260847	0.9260847
G_2	0.8020015	0.8020015	0.7797237	0.8020015	0.8020015	0.8020015
G_3	0.5009561	0.5009561	0.4721318	0.5009561	0.5008279	0.5008279
G_4	0.0805668	0.0949185	0.1231443	0.0949185	0.0807793	0.0985283
G_5	**1.0001923**	**1.0001923**	0.9567119	**1.0001923**	**1.0000001**	1.0000000
G_6	0.9980266	0.9981029	**1.1820609**	0.9981029	**1.0000002**	1.0000000
G_7	0.2975000	0.2975000	0.2975000	0.2975000	0.2975000	0.2975000
G_8	1.0000000	1.0000000	0.9722222	1.0000000	1.0000000	1.0000000
G_9	0.4166667	0.4166667	0.4285714	0.4166667	0.4166667	0.4166667
G_{10}	0.7428571	0.7428571	0.7222222	0.7428571	0.7428571	0.7428571
G_{11}	0.9722222	0.9722222	1.0000000	0.9722222	0.9722222	0.9722222
G_{12}	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
G_{13}	0.8750000	0.8750000	0.8750000	0.8750000	0.8750000	0.8750000
G_{14}	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
G_{15}	0.6071429	0.6071429	0.6071429	0.6071429	0.6071429	0.6071429
G_{16}	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	0.9999999
G_{17}	0.8795181	0.8795181	0.8795181	0.8795181	0.8795181	0.8795182
G_{18}	1.0000000	0.9468223	0.9358974	0.9468223	1.0000000	0.9358974
G_{19}	0.8795181	0.9289157	0.9397590	0.9289157	0.8795181	0.9397591
G_{20}	0.8656716	0.8656716	0.8529412	0.8656716	0.8656162	0.8656162
G_{21}	0.8589744	0.8589744	0.8717949	0.8589744	0.8590294	0.8590294
G_{22}	0.9451796	0.9451796	1.0000000	0.9451796	0.9458022	0.9457726
G_{23}	0.96181818	0.96181818	0.90909091	0.96181818	0.96118502	0.96121513
G_{24}	0.94863014	0.94863014	0.95890411	0.94863014	0.94867419	0.94867413
G_{25}	**1.05739726**	**1.00116732**	0.94871795	**1.00116732**	**1.05687249**	0.98914762
Objective	2987.298504	2996.309776	2876.117623	2996.309776	2985.151875	2996.232157

[a]Boldfaced type indicates the violated constraints.

are constants. The equality constraints are transformed to a pair of inequalities $h_j(\boldsymbol{x}) \le b_j + \delta$ and $h_j(\boldsymbol{x}) \ge b_j - \delta$. Thus r equality constraints will give rise to $2r$ inequalities, and the total number of inequalities for the problem is denoted by s, where $s = q + 2r$. For each individual $\boldsymbol{c}$ denotes the constraint satisfaction vector given by $\boldsymbol{c} = [c_1 \; c_2 \; \ldots \; c_s]$, where

$$c_i = \begin{cases} 0 & \text{if } i\text{th constraint is satisfied}, \quad i = 1, 2, \ldots, s \\ a_i - g_i(\boldsymbol{x}) & \text{if } i\text{th constraint is violated}, \quad i = 1, 2, \ldots, q \\ b_i - \delta - h_i(\boldsymbol{x}) & \text{if } i\text{th constraint is violated}, \quad i = q+1, q+2, \ldots, q+r \\ -b_i - \delta + h_i(\boldsymbol{x}) & \text{if } i\text{th constraint is violated}, \quad i = q+r+1, q+r+2, \ldots, s \end{cases}$$

For the preceding c_i, $c_i = 0$ indicates the ith constraint is satisfied, whereas $c_i > 0$ indicates the violation of the constraint. The CONSTRAINT matrix for a swarm of M individuals assumes the form

$$\text{CONSTRAINT} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1s} \\ c_{21} & c_{22} & \cdots & c_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ c_{M1} & c_{M2} & \cdots & c_{Ms} \end{bmatrix}$$

A nondominated sorting is used to ranks the individuals based on the preceding constraint matrix. One can observe that in absence of feasible solutions the nondominated solutions are the ones with minimal constraint violation. Whenever there is one or more feasible individuals in the swarm, the feasible solutions will take over as rank 1.

The initial swarm consists of a collection of random individuals. Over time, the individuals communicate with their neighboring better performers and derive information from them to look for optimum in smaller groups. The pseudocode of the algorithm is presented as follows:

Algorithm

Initialize M unique individuals in the Swarm.

Unconstrained Problem Strategy

```
Do{
  Compute Objective Values of each Individual in the Swarm;
  Compute Average Objective Value for the Swarm;
  For (each Individual){
    If (Objective Value of an Individual ≤ Average Objective
    Value for the Swarm): Assign Individual to
    Better Performer List (BPL);
  }
  For (each Individual not in BPL){
    Do {
      Select a Leader from BPL to derive information;
      Acquire information from the Leader and move to a
      new point in the search space;
      } while (all individuals are not unique)
  }
} while (termination condition = false)
```

Constrained Problem Strategy

```
Do {
  Compute Objective values for each Individual in the Swarm;
  Compute Constraint values for each Individual in the Swarm;
  Use Nondominated Sorting to Rank Individuals based
  on Constraint Matrix;
  Assign all Individuals with Constraint Rank = 1 to BPL;
  If (size of BPL > M/2){
    Compute Average Objective Value for the Individuals
    in the BPL;
    Shrink the BPL to contain Individuals with Objective
    value ≤ Average Objective Value;
  }
  For (each Individual not in BPL){
    Do {
      Select a Leader from the BPL that is closest
      in parametric space to derive information;
      Acquire information from the Leader and
      move to a new point in the search space;
    }while (all individuals are not unique)
  }
}while (termination condition = false)
```

Information Acquisition

The pseudocode of the information acquisition operator is as follows:

```
Select the leader and assign it as P;
Compute the Euclidean distance (D) between the follower
and the leader in the parametric space;
For i = 1 to Number of Variables
  Generate a random number (R) using a Gaussian
  distribution with μ = 0 and σ = 1;
  C(i) = P(i) + R.D.
```

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G. M. Faeth
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