Multiobjective optimization Test Instances for the CEC 2009 Special Session and Competition

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I. INTRODUCTION

Due largely to the nature of multiobjective evolutionary algorithms (MOEAs), their behaviors and performances are mainly studied experimentally. In the past 20 years, Several continuous multiobjective optimization problem (MOP) test suites have been proposed the evolutionary computation community [1]-[9], which have played an crucial role in developing and studying MOEAs. However, more test instances are needed to resemble complicated real-life problems and thus stimulate the MOEA research. This report suggest a set of unconstrained (bound constrained) MOP test instances and a set of general constrained test instances for the CEC09 algorithm contest. It also provides performance assessment guidelines.

II. UNCONSTRAINED (BOUND CONSTRAINED) MOP TEST PROBLEMS

Unconstrained Problem 1 (F2 in [9])

The two objectives to be minimized:

$$\begin{aligned} f_1 &= \quad x_1 \quad &+\frac{2}{|J_1|}\sum_{j\in J_1}[x_j-\sin(6\pi x_1+\frac{j\pi}{n})]^2 \\ f_2 &= 1-\sqrt{x_1} &+\frac{2}{|J_2|}\sum_{j\in J_2}[x_j-\sin(6\pi x_1+\frac{j\pi}{n})]^2 \end{aligned}$$

where $J_1 = \{j|j \text{ is odd and } 2 \le j \le n\}$ and $J_2 = \{j|j \text{ is even and } 2 \le j \le n\}$.

The search space is $[0,1]\times[-1,1]^{n-1}$.

Its PF is

$$f_2 = 1-\sqrt{f_1}, \quad 0 \le f_1 \le 1.$$

Its PS is

$$x_j = \sin(6\pi x_1+\frac{j\pi}{n}), j = 2,\ldots,n, \quad 0 \le x_1 \le 1.$$

$n = 30$ in the CEC 09 algorithm contest.

Its PF and PS are illustrated in Fig. 1.

Unconstrained Problem 2 (F5 in [9])

The two objectives to be minimized:

$$\begin{aligned} f_1 &= \quad x_1 \quad &+\frac{2}{|J_1|}\sum_{j\in J_1} y_j^2 \\ f_2 &= 1-\sqrt{x_1} &+\frac{2}{|J_2|}\sum_{j\in J_2} y_j^2 \end{aligned}$$

where $J_1 = \{j|j \text{ is odd and } 2 \le j \le n\}$, $J_2 = \{j|j \text{ is even and } 2 \le j \le n\}$, and

$$y_j = \begin{cases} x_j - [0.3x_1^2\cos(24\pi x_1+\frac{4j\pi}{n})+0.6x_1]\cos(6\pi x_1+\frac{j\pi}{n}) & j \in J_1 \\ x_j - [0.3x_1^2\cos(24\pi x_1+\frac{4j\pi}{n})+0.6x_1]\sin(6\pi x_1+\frac{j\pi}{n}) & j \in J_2 \end{cases}$$

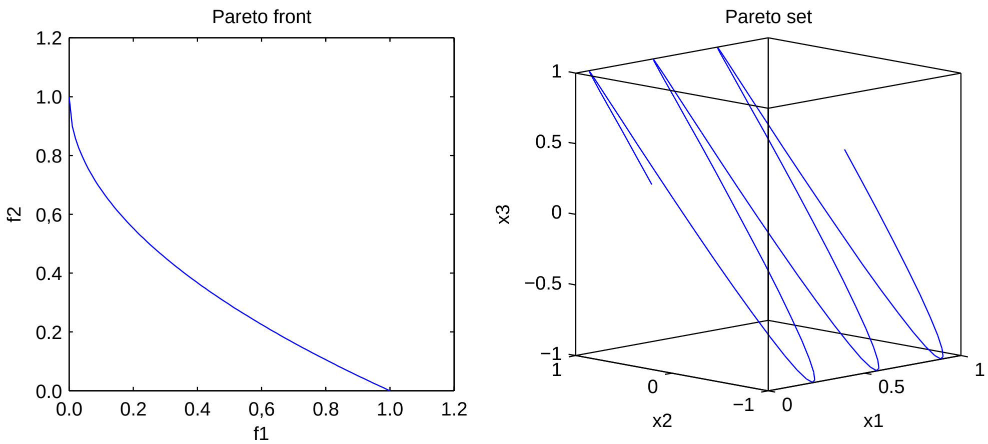

Fig. 1. Illustration of the PF and the PS of UF1.

Its search space is $[0,1] \times [-1,1]^{n-1}$.

Its PF is

$$f_2 = 1 - \sqrt{f_1}, \quad 0 \leq f_1 \leq 1.$$

Its PS is

$$x_j = \begin{cases} \{0.3x_1^2 \cos(24\pi x_1 + \frac{4j\pi}{n}) + 0.6x_1\} \cos(6\pi x_1 + \frac{j\pi}{n}) & j \in J_1 \\ \{0.3x_1^2 \cos(24\pi x_1 + \frac{4j\pi}{n}) + 0.6x_1\} \sin(6\pi x_1 + \frac{j\pi}{n}) & j \in J_2 \end{cases}$$
$$0 \leq x_1 \leq 1.$$

$n = 30$ in the CEC 09 algorithm contest.

Its PF and PS are illustrated in Fig. 2.

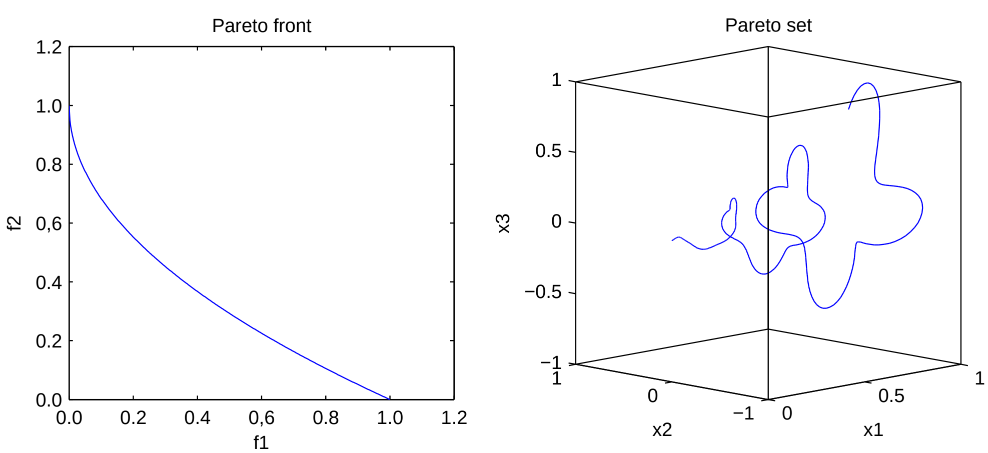

Fig. 2. Illustration of the PF and the PS of UF2.

Unconstrained Problem 3 (F8 in [9])

The two objectives to be minimized:

$$\begin{aligned} f_1 = & \quad x_1 & + \frac{2}{|J_1|}(4\sum_{j\in J_1} y_j^2 - 2\prod_{j\in J_1}\cos(\frac{20y_j\pi}{\sqrt{j}}) + 2) \\ f_2 = & \quad 1-\sqrt{x_1} & + \frac{2}{|J_2|}(4\sum_{j\in J_2} y_j^2 - 2\prod_{j\in J_2}\cos(\frac{20y_j\pi}{\sqrt{j}}) + 2) \end{aligned}$$

where J_1 and J_2 are the same as those of F1, and

$$y_j = x_j - x_1^{0.5(1.0+\frac{3(j-2)}{n-2})}, j = 2, \ldots, n,$$

The search space is $[0,1]^n$

Its PF is

$$f_2 = 1 - \sqrt{f_1}, \quad 0 \le f_1 \le 1.$$

Its PS is

$$x_j = x_1^{0.5(1.0+\frac{3(j-2)}{n-2})}, j = 2, \ldots, n, \quad 0 \le x_1 \le 1.$$

$n = 30$ in the CEC 09 algorithm contest.

Its PF and PS are illustrated in Fig. 3.

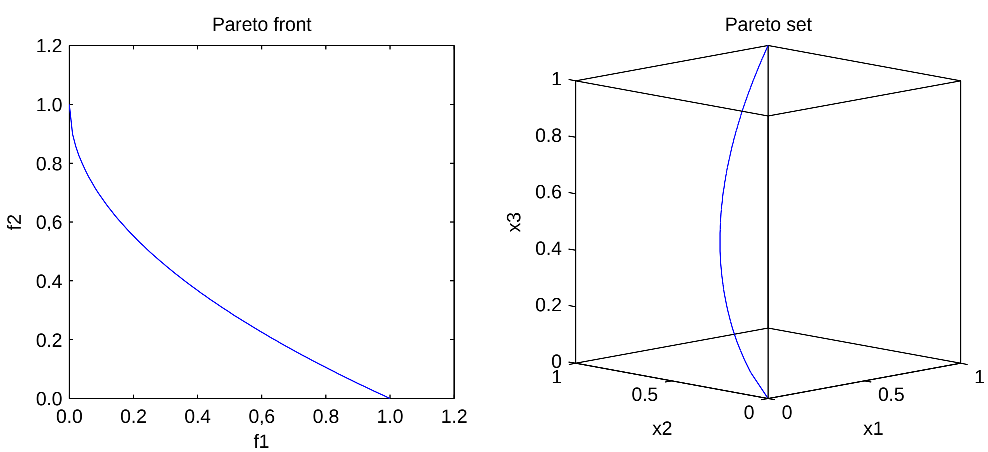

Fig. 3. Illustration of the PF and the PS of UF3.

Unconstrained Problem 4

The two objectives to be minimized:

$$\begin{aligned} f_1 = & \quad x_1 & + \frac{2}{|J_1|}\sum_{j\in J_1} h(y_j) \\ f_2 = & \quad 1-x_1^2 & + \frac{2}{|J_2|}\sum_{j\in J_2} h(y_j) \end{aligned}$$

where $J_1 = \{j|j$ is odd and $2 \leq j \leq n\}$ and $J_2 = \{j|j$ is even and $2 \leq j \leq n\}$,

$$y_i = x_j - \sin(6\pi x_1 + \frac{j\pi}{n}), j = 2, \ldots, n.$$

and

$$h(t) = \frac{|t|}{1 + e^{2|t|}}.$$

Its search space $[0, 1] \times [-2, 2]^{n-1}$

Its PF is

$$f_2 = 1 - f_2^2, \quad 0 \leq f_1 \leq 1.$$

Its PS is

$$x_j = \sin(6\pi x_1 + \frac{j\pi}{n}), j = 2, \ldots, n. \quad 0 \leq x_1 \leq 1.$$

$n = 30$ in the CEC 09 algorithm contest.

Its PF and PS are illustrated in Fig. 4.

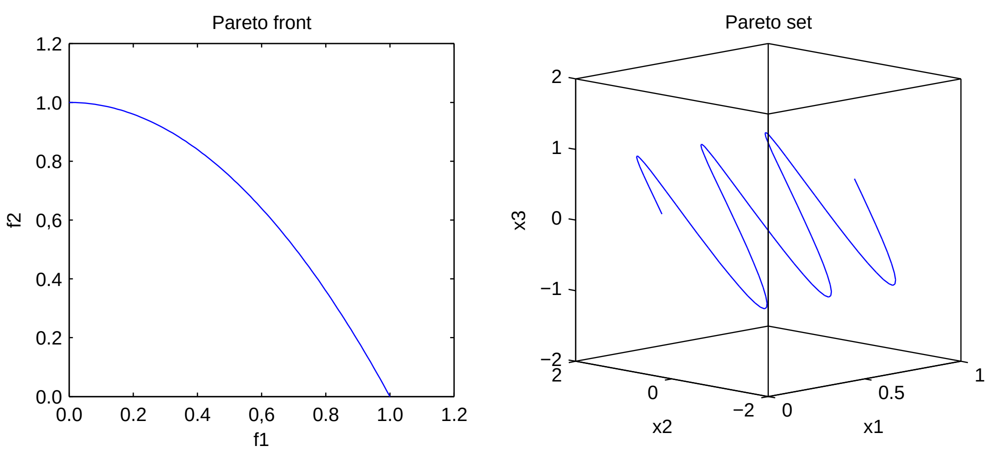

Fig. 4. Illustration of the PF and the PS of UF4.

Unconstrained Problem 5

The two objectives to be minimized:

$$\begin{aligned} f_1 = & \quad x_1 & +(\frac{1}{2N} + \varepsilon)|\sin(2N\pi x_1)| + \frac{2}{|J_1|}\sum_{j\in J_1} h(y_j) \\ f_2 = & \quad 1 - x_1 & +(\frac{1}{2N} + \varepsilon)|\sin(2N\pi x_1)| + \frac{2}{|J_2|}\sum_{j\in J_2} h(y_j) \end{aligned}$$

where $J_1 = \{j|j$ is odd and $2 \le j \le n\}$ and $J_2 = \{j|j$ is even and $2 \le j \le n\}$. N is an integer, $\varepsilon > 0$,

$$y_j = x_j - \sin(6\pi x_1 + \frac{j\pi}{n}), j = 2, \ldots, n$$

and

$$h(t) = 2t^2 - cos(4\pi t) + 1.$$

The search space is $[0, 1] \times [-1, 1]^{n-1}$.

Its PF has $2N + 1$ Pareto Optimal solutions:

$$(\frac{i}{2N}, 1 - \frac{i}{2N})$$

for $i = 0, 1, \ldots, 2N$.

$N = 10, \varepsilon = 0.1$ and $n = 30$ in the CEC 09 algorithm contest.

Its PF and PS are illustrated in Fig. 5.

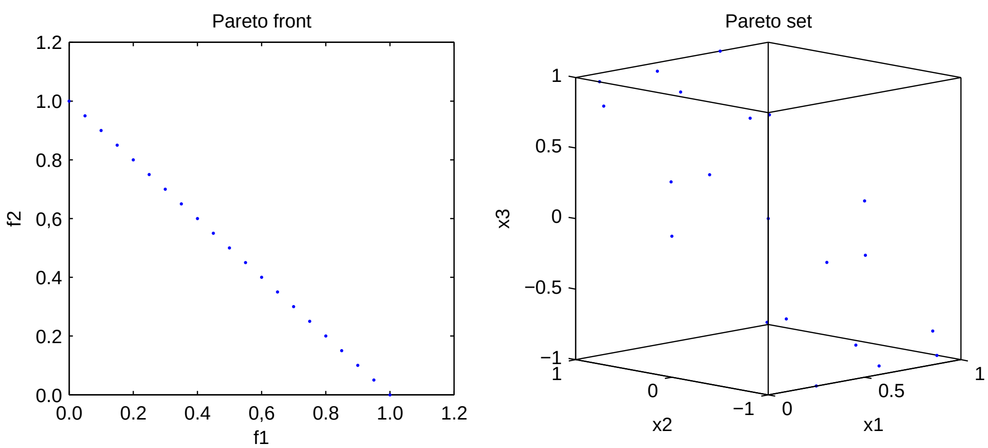

Fig. 5. Illustration of the PF and the PS of UF5.

Unconstrained Problem 6

The two objectives to be minimized:

$$f_1 = \quad x_1 \quad + \max\{0, 2(\frac{1}{2N} + \varepsilon)\sin(2N\pi x_1)\} + \frac{2}{|J_1|}(4\sum_{j\in J_1} y_j^2 - 2\prod_{j\in J_1}\cos(\frac{20y_j\pi}{\sqrt{j}}) + 2)$$

$$f_2 = \quad 1 - x_1 \quad + \max\{0, 2(\frac{1}{2N} + \varepsilon)\sin(2N\pi x_1)\} + \frac{2}{|J_2|}(4\sum_{j\in J_2} y_j^2 - 2\prod_{j\in J_2}\cos(\frac{20y_j\pi}{\sqrt{j}}) + 2)$$

where $J_1 = \{j|j$ is odd and $2 \le j \le n\}$, $J_2 = \{j|j$ is even and $2 \le j \le n\}$, and

$$y_j = x_j - \sin(6\pi x_1 + \frac{j\pi}{n}), j = 2, \ldots, n.$$

The search space is $[0,1]\times[-1,1]^{n-1}$.

Its PF consists of

- one isolated point, $(0,1)$, and
- N disconnected parts:

$$f_2 = 1 - f_1, f_1 \in \bigcup_{i=1}^{N}[\frac{2i-1}{2N}, \frac{2i}{2N}].$$

$N=2$, $\varepsilon = 0.1$ and $n=30$ in the CEC 09 algorithm contest.

Its PF and PS are illustrated in Fig. 6.

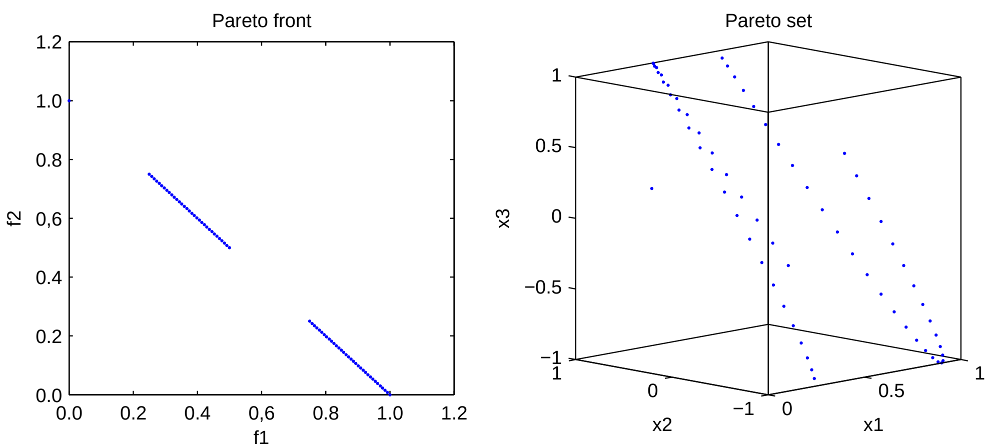

Fig. 6. Illustration of the PF and the PS of UF6.

Unconstrained Problem 7

The two objectives to be minimized:

$$\begin{aligned} f_1 &= \quad \sqrt[5]{x_1} &+ \frac{2}{|J_1|}\sum_{j\in J_1} y_j^2 \\ f_2 &= 1 - \sqrt[5]{x_1} &+ \frac{2}{|J_2|}\sum_{j\in J_2} y_j^2 \end{aligned}$$

where $J_1 = \{j|j \text{ is odd and } 2 \le j \le n\}$, $J_2 = \{j|j \text{ is even and } 2 \le j \le n\}$ and

$$y_j = x_j - \sin(6\pi x_1 + \frac{j\pi}{n}), j = 2, \ldots, n$$

The search space is $[0,1]\times[-1,1]^{n-1}$.

Its PF is

$$f_2 = 1 - f_1, \quad 0 \le f_1 \le 1.$$

Its PS is

$$x_j = \sin(6\pi x_1 + \frac{j\pi}{n}), j = 2, \ldots, n, \quad 0 \le x_1 \le 1.$$

$n = 30$ in the CEC 09 algorithm contest.

Its PF and PS are illustrated in Fig. 7.

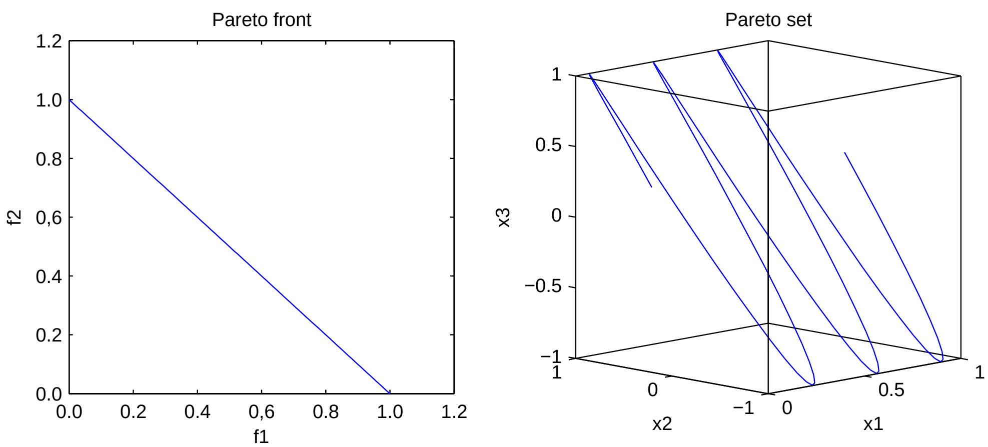

Fig. 7. Illustration of the PF and the PS of UF7.

Unconstrained Problem 8 (F6 in [9])

The three objectives to be minimized:

$$\begin{aligned}
f_1 = &\quad \cos(0.5x_1\pi)\cos(0.5x_2\pi) &+\frac{2}{|J_1|}\sum_{j\in J_1}(x_j - 2x_2\sin(2\pi x_1 + \frac{j\pi}{n}))^2 \\
f_2 = &\quad \cos(0.5x_1\pi)\sin(0.5x_2\pi) &+\frac{2}{|J_2|}\sum_{j\in J_2}(x_j - 2x_2\sin(2\pi x_1 + \frac{j\pi}{n}))^2 \\
f_3 = &\quad \sin(0.5x_1\pi) &+\frac{2}{|J_3|}\sum_{j\in J_3}(x_j - 2x_2\sin(2\pi x_1 + \frac{j\pi}{n}))^2
\end{aligned}$$

where

$J_1 = \{j|3 \le j \le n$, and $j-1$ is a multiplication of $3\}$,

$J_2 = \{j|3 \le j \le n$, and $j-2$ is a multiplication of $3\}$,

$J_3 = \{j|3 \le j \le n$, and j is a multiplication of $3\}$.

The search space is $[0,1]^2 \times [-2,2]^{n-2}$.

Its PF is $f_1^2 + f_2^2 + f_3^3 = 1, 0 \le f_1, f_2, f_3 \le 1$.

Its PS is $x_j = 2x_2\sin(2\pi x_1 + \frac{j\pi}{n}), j = 3, \ldots, n$.

$n = 30$ in the CEC 09 algorithm contest.

Its PF and PS are illustrated in Fig. 8.

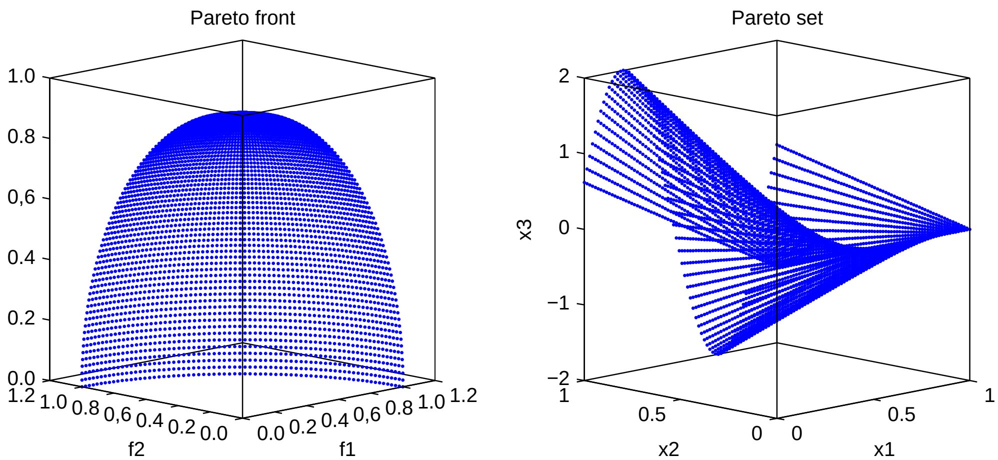

Fig. 8. Illustration of the PF and the PS of UF8.

Unconstrained Problem 9

The three objectives to be minimized:

$$f_1 = 0.5[max\{0,(1+\varepsilon)(1-4(2x_1-1)^2)\}+2x_1]x_2 \quad +\frac{2}{|J_1|}\sum_{j\in J_1}(x_j-2x_2\sin(2\pi x_1+\frac{j\pi}{n}))^2$$

$$f_2 = 0.5[max\{0,(1+\varepsilon)(1-4(2x_1-1)^2)\}-2x_1+2]x_2 \quad +\frac{2}{|J_2|}\sum_{j\in J_2}(x_j-2x_2\sin(2\pi x_1+\frac{j\pi}{n}))^2$$

$$f_3 = 1-x_2 \quad +\frac{2}{|J_3|}\sum_{j\in J_3}(x_j-2x_2\sin(2\pi x_1+\frac{j\pi}{n}))^2$$

where

$J_1 = \{j|3 \le j \le n$, and $j-1$ is a multiplication of $3\}$,

$J_2 = \{j|3 \le j \le n$, and $j-2$ is a multiplication of $3\}$,

$J_3 = \{j|3 \le j \le n$, and j is a multiplication of $3\}$,

and

$$\varepsilon = 0.1$$

ε can take any other positive values.

The search space is $[0,1]^2 \times [-2,2]^{n-2}$.

The PF has two parts. The first part is

$$0 \le f_3 \le 1,$$
$$0 \le f_1 \le \frac{1}{4}(1-f_3),$$
$$f_2 = 1-f_1-f_3;$$

and the second one is

$$
\begin{aligned}
0 \leq f_3 &\leq 1, \\
\frac{3}{4}(1 - f_3) \leq f_1 &\leq 1, \\
f_2 &= 1 - f_1 - f_3.
\end{aligned}
$$

The PS also has two disconnected parts:

$$
\begin{aligned}
x_1 \in [0, 0.25] \cup [0.75, 1], 0 \leq x_2 \leq 1, \\
x_j = 2x_2 \sin(2\pi x_1 + \frac{j\pi}{n}), j = 3, \ldots, n.
\end{aligned}
$$

$n = 30$ in the CEC 09 algorithm contest.

Its PF and PS are illustrated in Fig. 9.

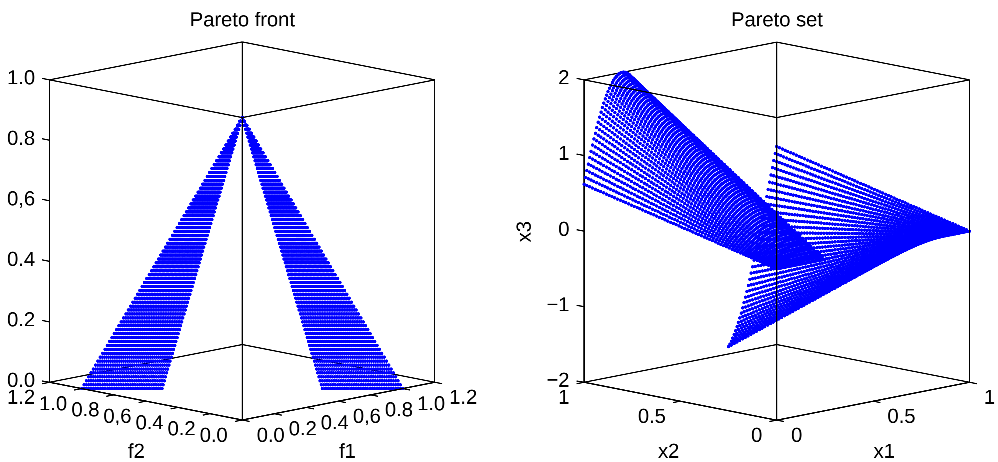

Fig. 9. Illustration of the PF and the PS of UF9.

Unconstrained Problem 10

The three objectives to be minimized:

$$
\begin{aligned}
f_1 = &\quad \cos(0.5x_1\pi)\cos(0.5x_2\pi) &+\frac{2}{|J_1|}\sum_{j\in J_1}[4y_j^2 - \cos(8\pi y_j) + 1] \\
f_2 = &\quad \cos(0.5x_1\pi)\sin(0.5x_2\pi) &+\frac{2}{|J_2|}\sum_{j\in J_1}[4y_j^2 - \cos(8\pi y_j) + 1] \\
f_3 = &\quad \sin(0.5x_1\pi) &+\frac{2}{|J_3|}\sum_{j\in J_1}[4y_j^2 - \cos(8\pi y_j) + 1]
\end{aligned}
$$

where

$J_1 = \{j | 3 \leq j \leq n$, and $j - 1$ is a multiplication of $3\}$,

$J_2 = \{j|3 \leq j \leq n$, and $j-2$ is a multiplication of $3\}$,

$J_3 = \{j|3 \leq j \leq n$, and j is a multiplication of $3\}$,

and

$$y_j = x_j - 2x_2 \sin(2\pi x_1 + \frac{j\pi}{n}), j = 3, \ldots, n$$

The search space is $[0,1]^2 \times [-2,2]^{n-2}$.

Its PF is $f_1^2 + f_2^2 + f_3^3 = 1, 0 \leq f_1, f_2, f_3 \leq 1$

Its PS is $x_j = 2x_2 \sin(2\pi x_1 + \frac{j\pi}{n}), j = 3, \ldots, n$.

$n = 30$ in the CEC 09 algorithm contest.

Its PF and PS are illustrated in Fig. 10.

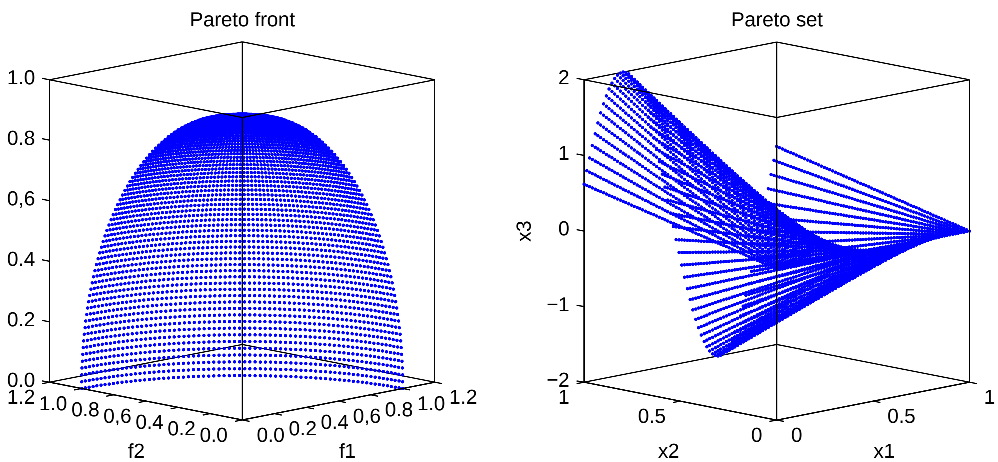

Fig. 10. Illustration of the PF and the PS of UF10.

In this report, we also include 3 five-objective optimization problems. We assume that the optimization problems under consideration involve 5 objective functions $f_1,..f_5$ that are all to be minimized.
We select two problems from two immensely popular test suites, DTLZ [2, 3], as well as one test functions of the proposed WFG test suite [5].
However, the original DTLZ test suites have the following problems:

- For all problems, the global optimum has the same parameter values for different variables/dimensions
- The global optimum lies in the center of the search range
- The global optimum lies on the bounds
- All of these problems are separable

To overcome these shortcomings, we rotated the original DTLZ problem.
*f(***z***)*: original function. Search range [**zmin** , **zmax**]
*F(***x***)*: new extended function. Search range [**xmin**, x**max**]
D: dimension
$\mathbf{d} = [d_1, d_2, ...d_D]$: the extended length of the lower bound
$\boldsymbol{\lambda} = [\lambda_1, \lambda_2, ...\lambda_D]$: the scale factor
$\mathbf{p} = [p_1, p_1, ...p_D]$: the penalty value

To overcome the shortcomings of the DTLZ test functions for which the global optimum lies on the lower bound, or in the center of the search range, we extend the lower bound **zmin** by **d**. Then, for the solution in the extended region, the function value is obtained by mapping and stretching.

$$f'(\mathbf{z}') = S(psum)\left(f(\mathbf{z}') + f\_bias\right)$$

where $z_i' = \begin{cases} z_i, & \mathrm{z}_i \geq z\min_i \\ z\min_i + \lambda_i (z\min_i - z_i), & \mathrm{z}_i < z\min_i \end{cases}$,

$S(psum) = \dfrac{2}{1+\exp(-psum)}$, $psum = \sqrt{\sum_{i \in \mathrm{I}} p_i^2}, I \subseteq \{1, 2, ...D\}$ (*I* is a set of all variables included in the objective function *f(x)*)
Here the constant parameter vector λ is used to make the searching region not symmetric with respect to the variable.

Here the stretching function **S** is used to guarantee that the objective function values of solutions in the extended region are always worse than those in the original region, i.e., the Pareto Optimal front remains unchanged. This assumption holds true on the condition that *f>0*. Therefore we shift *f* to $f + f\_{bias}$ to make sure that all function values are positive. The range of the function S is [2, 3]. When one solution in the extended region is near the mapping center, there will be $psum \to 0$ and $S \to 1$. On the contrary, if the solution is far from the mapping center, $S \to 2$. Thus we enlarge the objective value in the extended region whilst at the same time keeping the function connected.
The penalty value p_i in each variable is calculated as:

$$p_i = \begin{cases} 0, & \mathrm{z}_i \geq z\min_i \\ \left| z\min_i - z_i \right| / d_i, & \mathrm{z}_i < z\min_i \end{cases}, \quad i = 1, 2, ..., D$$

After extending the region, rotated the parameter space by vector matrix **M**, and then the new function:

$$F_m(\mathbf{x}) = \begin{cases} f_m(\mathbf{z}')+1 & \text{for all } x\min_i \le x_i \le x\max_i \\ S(psum_m)\left(f_m(\mathbf{z}')+1\right) & \text{otherwise} \end{cases},\quad m=1,2,\ldots M,\quad \mathbf{z}=\mathbf{Mx}$$

where $S(psum) = \dfrac{2}{1+\exp(-psum)}$, $psum = \sqrt{\sum_{i\in I} p_i^2}, I \subseteq \{1,2,\ldots D\}$

$$z_i' = \begin{cases} z\min_i + \lambda_i (z\min_i - z_i), & z_i < z\min_i \\ z_i, & z\min_i \le z_i \le z\max_i \\ z\max_i - \lambda_i (z_i - z\max_i), & z_i > z\max_i \end{cases}$$

$$p_i = \begin{cases} z\min_i - z_i, & z_i < z\min_i \\ 0, & z\min_i \le z_i \le z\max_i \\ z_i - z\max_i, & z_i > z\max_i \end{cases}$$

$$i = 1,2,\ldots,D$$

M: linear transformation orthogonal matrix, with condition number=1.

According to the above description, we extended and rotated DTLZ2 and DTLZ3, obtaining R2_DTLZ2_M5 and R2_DTLZ3_M5.

Unconstrained Problem 11

New Extended Rotated DTLZ2 (R2_DTLZ2_M5)

$$f_1(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\cos\left(z_1'\pi/2\right)\cos\left(z_2'\pi/2\right)\ldots\cos\left(z_{M-2}'\pi/2\right)\cos\left(z_{M-1}'\pi/2\right)+1, & z_i \ge 0 \\ S(psum_1)\left((1+g(\mathbf{x}_M))\cos\left(z_1'\pi/2\right)\cos\left(z_2'\pi/2\right)\ldots\cos\left(z_{M-2}'\pi/2\right)\cos\left(z_{M-1}'\pi/2\right)+1\right), & \text{otherwise} \end{cases}$$

$$f_2(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\cos\left(z_1'\pi/2\right)\cos\left(z_2'\pi/2\right)\ldots\cos\left(z_{M-2}'\pi/2\right)\sin\left(z_{M-1}'\pi/2\right)+1, & z_i \ge 0 \\ S(psum_2)\left((1+g(\mathbf{x}_M))\cos\left(z_1'\pi/2\right)\cos\left(z_2'\pi/2\right)\ldots\cos\left(z_{M-2}'\pi/2\right)\sin\left(z_{M-1}'\pi/2\right)+1\right), & \text{otherwise} \end{cases}$$

$$f_3(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\cos\left(z_1'\pi/2\right)\cos\left(z_2'\pi/2\right)\ldots\sin\left(z_{M-2}'\pi/2\right)+1, & z_i \ge 0 \\ S(psum_3)\left((1+g(\mathbf{x}_M))\cos\left(z_1'\pi/2\right)\cos\left(z_2'\pi/2\right)\ldots\sin\left(z_{M-2}'\pi/2\right)+1\right), & \text{otherwise} \end{cases}$$

$$\vdots$$

$$f_{M-1}(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\cos\left(z_1'\pi/2\right)\sin\left(z_2'\pi/2\right)+1, & z_i \ge 0 \\ S(psum_{M-1})\left((1+g(\mathbf{x}_M))\cos\left(z_1'\pi/2\right)\sin\left(z_2'\pi/2\right)+1\right), & \text{otherwise} \end{cases}$$

$$f_M(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\sin\left(z_1'\pi/2\right)+1, & z_i \ge 0 \\ S(psum_M)\left((1+g(\mathbf{x}_M))\sin\left(z_1'\pi/2\right)+1\right), & \text{otherwise} \end{cases}$$

$$g(\mathbf{x}_M) = \sum_{x_i\in \mathbf{x}_M} \left(z_i' - 0.5\right)^2$$

where $z_i' = \begin{cases} -\lambda_i z_i, & z_i < 0 \\ z_i, & 0 \le z_i \le 1 \\ \lambda_i z_i, & z_i > 1 \end{cases}$, $p_i = \begin{cases} -z_i, & z_i < 0 \\ 0, & 0 \le z_i \le 1, \\ z_i - 1, & z_i > 1 \end{cases}$ $i = 1,2,\ldots,D$

$$\mathbf{z} = \mathbf{M}\mathbf{x}, \quad \mathbf{x} = [x_1, x_2, \ldots x_D], \mathbf{z} = [z_1, z_2, \ldots z_D]$$

The Pareto-optimal solutions correspond to $x_M^* = 0.5$ and all the objective function values must satisfy the following condition: $\sum_{i=1}^{M} (f_i^*)^2 = 1$, and we include the Pareto-optimal front data in the folder.

D : dimension

$\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \ldots \lambda_D]$: the scale factor

$\mathbf{p} = [p_1, p_1, \ldots p_D]$: the penalty value

$x_i \in [x\min_i, x\max_i]$, $\mathbf{xmin} = [x\min_1, x\min_2, \ldots x\min_D]$ and

$\mathbf{x}\max = [x\max_1, x\max_2, \ldots x\max_D]$

Data file:

30D

Name	Variable
R2_ DTLZ2_M_30D.dat	**M** 30*30 matrix
R2_ DTLZ2_bound_30D.dat	2*30 matrix 1st row: **xmin** 2nd row: **xmax**
R2_ DTLZ2_lamda_30D.dat	$\boldsymbol{\lambda}$ 1*30D vector

Unconstrained Problem 12

New Extended Rotated DTLZ3 (R2_DTLZ3_M5)

$$f_1(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\cos(z_1'\,\pi/2)\cos(z_2'\,\pi/2)\ldots\cos(z_{M-2}'\,\pi/2)\cos(z_{M-1}'\,\pi/2)+1, & z_i \geq 0 \\ S(psum_1)\left((1+g(\mathbf{x}_M))\cos(z_1'\,\pi/2)\cos(z_2'\,\pi/2)\ldots\cos(z_{M-2}'\,\pi/2)\cos(z_{M-1}'\,\pi/2)+1\right), & \text{otherwise} \end{cases}$$

$$f_2(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\cos(z_1'\,\pi/2)\cos(z_2'\,\pi/2)\ldots\cos(z_{M-2}'\,\pi/2)\sin(z_{M-1}'\,\pi/2)+1, & z_i \geq 0 \\ S(psum_2)\left((1+g(\mathbf{x}_M))\cos(z_1'\,\pi/2)\cos(z_2'\,\pi/2)\ldots\cos(z_{M-2}'\,\pi/2)\sin(z_{M-1}'\,\pi/2)+1\right), & \text{otherwise} \end{cases}$$

$$f_3(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\cos(z_1'\,\pi/2)\cos(z_2'\,\pi/2)\ldots\sin(z_{M-2}'\,\pi/2)+1, & z_i \geq 0 \\ S(psum_3)\left((1+g(\mathbf{x}_M))\cos(z_1'\,\pi/2)\cos(z_2'\,\pi/2)\ldots\sin(z_{M-2}'\,\pi/2)+1\right), & \text{otherwise} \end{cases}$$

$$\vdots$$

$$f_{M-1}(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\cos(z_1'\,\pi/2)\sin(z_2'\,\pi/2)+1, & z_i \geq 0 \\ S(psum_{M-1})\left((1+g(\mathbf{x}_M))\cos(z_1'\,\pi/2)\sin(z_2'\,\pi/2)+1\right), & \text{otherwise} \end{cases}$$

$$f_M(\mathbf{x}) = \begin{cases} (1+g(\mathbf{x}_M))\sin(z_1'\,\pi/2)+1, & z_i \geq 0 \\ S(psum_M)\left((1+g(\mathbf{x}_M))\sin(z_1'\,\pi/2)+1\right), & \text{otherwise} \end{cases}$$

where $g(\mathbf{x}_M) = 100\left(|\mathbf{x}_M| + \sum_{x_i \in \mathbf{x}_M} (z_i - 0.5)^2 - \cos(20\pi(z_i - 0.5))\right)$

where $z_i' = \begin{cases} -\lambda_i z_i, & z_i < 0 \\ z_i, & 0 \le z_i \le 1 \\ \lambda_i z_i, & z_i > 1 \end{cases}$, $p_i = \begin{cases} -z_i, & z_i < 0 \\ 0, & 0 \le z_i \le 1, \\ z_i - 1, & z_i > 1 \end{cases}$ $i = 1, 2, ..., D$

$$\mathbf{z} = \mathbf{M}\mathbf{x}, \quad \mathbf{x} = [x_1, x_2, \ldots x_D], \mathbf{z} = [z_1, z_2, \ldots z_D]$$

The Pareto-optimal solutions correspond to $x_M^* = 0.5$ and all the objective function values must satisfy the following condition: $\sum_{i=1}^{M} (f_i^*)^2 = 1$ (at $g^* = 0$) and we include the Pareto-optimal front data in the folder.

D : dimension

$\boldsymbol{\lambda} = [\lambda_1, \lambda_2, ...\lambda_D]$: the scale factor

$\mathbf{p} = [p_1, p_1, ...p_D]$: the penalty value

$x_i \in [x\min_i, x\max_i]$, $\mathbf{xmin} = [x\min_1, x\min_2, ...x\min_D]$ and

$\mathbf{x}\max = [x\max_1, x\max_2, ...x\max_D]$

Data file:

30D

Name	Variable
R2_ DTLZ3_M_30D.dat	**M** 30*30 matrix
R2_ DTLZ3_bound_30D.dat	2*30 matrix 1st row: **xmin** 2nd row: **xmax**
R2_ DTLZ3_lamda_30D.dat	λ 1*30D vector

Unconstrained Problem 13

WFG1_M5

WFG [5]

Given $\mathbf{z} = \{z_1, ..., z_k, z_{k+1}, ..., z_n\}$

Minimize $f_{m=1:M}(\mathbf{x}) = Dx_M + S_m h_m(x_1, ..., x_{M-1})$

Where

$$\mathbf{x} = \{x_1, ..., x_M\}$$
$$= \{\max(t_M^p, A_1)(t_1^p - 0.5) + 0.5, ..., \max(t_M^p, A_{M-1})(t_{M-1}^p - 0.5) + 0.5, t_M^p\}$$
$$\mathbf{t}^p = \{t_1^p, ..., t_M^p\} \leftarrow\mid \mathbf{t}^{p-1} \leftarrow\mid \cdots \leftarrow\mid \mathbf{t}^1 \leftarrow\mid \mathbf{z}_{[0,1]}$$
$$\mathbf{z}_{[0,1]} = \{z_{1,[0,1]}, ..., z_{n,[0,1]}\}$$
$$= \{z_1 / z_{1,\max}, ..., z_n / z_{n,\max}\}$$

where M is the number of objectives, $\mathbf{x}$ is a set of M underlying parameters (where x_M is an underlying distance parameter and $x_{1:M-1}$ are underlying position parameters), z is a set of $k + l = n \ge M$ working parameters (the first k and the last l working parameters are position-and distance-related parameters, respectively).

$D>0$ is a distance scaling constant, $A_{1:M-1} \in \{0,1\}$ are degeneracy constants (for each $A_i = 0$, the dimensionality of the Pareto optimal front is reduced by one), $h_{1:M}$ are shape functions, $S_{1:M}>0$ are scaling constants, and $\mathbf{t}^{1:p}$ are transition vectors, where "←|" indicates that each transition vector is created from another vector via transformation functions. The domain of all $z_i \in \mathbf{z}$ is [0, 2i], $i=1,...,n$. Note that all $x_i \in \mathbf{x}$ will have domain [0,1].

Constants $S_{m=1:M} = 2m, \quad D = 1, \quad A_1 = 1, \; A_{2:M-1} = 1$

WFG1_M5

Shape $h_{m=1:M-1}$=convex$_m$

h_M = mixed$_M$(with α=1 and A=5)

t^1 $t^1_{i=1:k} = y_i$

$t^1_{i=k+1:n}$ = s_linear(y_i,0.35)

t^2 $t^2_{i=1:k} = y_i$

$t^2_{i=k+1:n}$ = b_flat (y_i, 0.8,0.75,0.85)

t^3 $t^3_{i=1:n}$ = b_poly(y_i,0.02)

t^4 $t^4_{i=1:M-1}$ = r_sum ($\{y_{(i-1)k/(M-1)+1},\ldots, y_{ik/(M-1)}\}$,

$\{2((i-1)k/(M-1)+1),\ldots,2ik/(M-1)\}$)

t^4_M = r_sum ($\{y_{k+1},\ldots, y_n\},\{2(k+1),\ldots,2n\}$)

The Pareto-optimal solutions correspond to: $z_{i=1:k}$: any combination of values in the range [0, 2i] and $z_{i=k+1:n} = 2i \times 0.35$. We include the Pareto-optimal front data in the folder.

Table 1: Properties of the test functions [5]

Test functions	Objective	# Parameters	Separability	Modality	No Extremal	No Medial	Optima Known	Geometry	Pareto many-to-one	Flat Regions
1.R2_DTLZ2_M5	$f_{1:M}$	√	NS	M	√	√	√	concave	√	X
2.R2_DTLZ3_M5	$f_{1:M}$	√	NS	M	√	√	√	concave	√	X
3.WFG1_M5	$f_{1:M}$	√	S	U	√	√	√	convex,mixed	√	√

S: Separable; NS: nonseparable; U: Uni-modal; M: Multi-modal;

III. Constrained Multiobjective Test Problems

The construction of problems 1-3 and 8-10 is inspired by the method used in [6].

Constrained Problem 1

The two objectives to be minimized:

$$\begin{aligned} f_1(x) = & \quad x_1 & + \frac{2}{|J_1|} \sum_{j \in J_1} (x_j - x_1^{0.5(1.0+\frac{3(j-2)}{n-2})})^2 \\ f_2(x) = & \quad 1 - x_1 & + \frac{2}{|J_2|} \sum_{j \in J_2} (x_j - x_1^{0.5(1.0+\frac{3(j-2)}{n-2})})^2 \end{aligned}$$

where $J_1 = \{j|j$ is odd and $2 \leq j \leq n\}$ and $J_2 = \{j|j$ is even and $2 \leq j \leq n\}$.

The constraint is

$$f_1 + f_2 - a|\sin[N\pi(f_1 - f_2 + 1)]| - 1 \geq 0$$

where N is an integer and $a \geq \frac{1}{2N}$.

The search space is $[0, 1]^n$.

The Pareto Front (PF) in the objective space consists of $2N + 1$ points:

$$(i/2N, 1 - i/2N), i = 0, 1, \ldots, 2N.$$

$N = 10, a = 1$ and $n = 10$ in the CEC 09 algorithm contest.

Its PF is illustrated in Fig. 11.

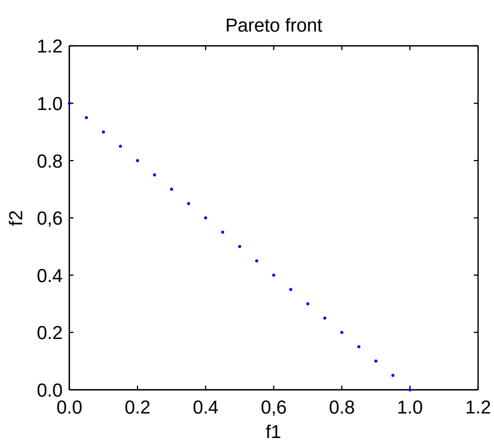

Fig. 11. Illustration of the PF of CF1.

Constrained Problem 2

The two objectives to be minimized:

$$
\begin{aligned}
f_1 = & \quad x_1 & + \frac{2}{|J_1|}\sum_{j \in J_1}(x_j - \sin(6\pi x_1 + \frac{j\pi}{n}))^2 \\
f_2 = & \quad 1 - \sqrt{x_1} & + \frac{2}{|J_2|}\sum_{j \in J_2}(x_j - \cos(6\pi x_1 + \frac{j\pi}{n}))^2
\end{aligned}
$$

where $J_1 = \{j | j \text{ is odd and } 2 \le j \le n\}$ and $J_2 = \{j | j \text{ is even and } 2 \le j \le n\}$.

The search space is $[0, 1] \times [-1, 1]^{n-1}$.

The constraint is:

$$\frac{t}{1 + e^{4|t|}} \ge 0$$

where

$$t = f_2 + \sqrt{f_1} - a\sin[N\pi(\sqrt{f_1} - f_2 + 1)] - 1.$$

Its PF in the objective space consists of

- an isolated Pareto optimal solution $(0, 1)$ in the objective space, and
- N disconnected parts, the i-th part is

$$f_2 = 1 - \sqrt{f_1}, \quad (\frac{2i-1}{2N})^2 \le f_1 \le (\frac{2i}{2N})^2, i = 1, \dots, N.$$

$N = 2, a = 1$ and $n = 10$ in the CEC 09 algorithm contest.

Its PF is illustrated in Fig. 12.

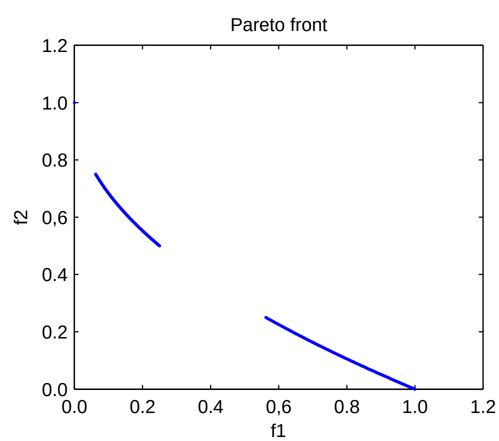

Fig. 12. Illustration of the PF of CF2.

Constrained Problem 3

The two objectives to be minimized:

$$\begin{aligned} f_1 = &\quad x_1 &+ \frac{2}{|J_1|}(4\sum_{j\in J_1} y_j^2 - 2\prod_{j\in J_1}\cos(\frac{20y_j\pi}{\sqrt{j}}) + 2) \\ f_2 = &\quad 1 - x_1^2 &+ \frac{2}{|J_2|}(4\sum_{j\in J_2} y_j^2 - 2\prod_{j\in J_2}\cos(\frac{20y_j\pi}{\sqrt{j}}) + 2) \end{aligned}$$

where $J_1 = \{j|j \text{ is odd and } 2 \le j \le n\}$ and $J_2 = \{j|j \text{ is even and } 2 \le j \le n\}$, and

$$y_j = x_j - \sin(6\pi x_1 + \frac{j\pi}{n}), j = 2, \ldots, n.$$

The constraint is:

$$f_2 + f_1^2 - a\sin[N\pi(f_1^2 - f_2 + 1)] - 1 \ge 0.$$

The search space is $[0,1] \times [-2,2]^{n-1}$.

Its PF in the objective space consists of

- an isolated Pareto optimal solution $(0,1)$ in the objective space, and
- N disconnected parts, the i-th part is

$$f_2 = 1 - f_1^2, \quad \sqrt{\frac{2i-1}{2N}} \le f_1 \le \sqrt{\frac{2i}{2N}}, i = 1, \ldots, N.$$

$N = 2, a = 1$ and $n = 10$ in the CEC 09 algorithm contest.

Its PF is illustrated in Fig. 13.

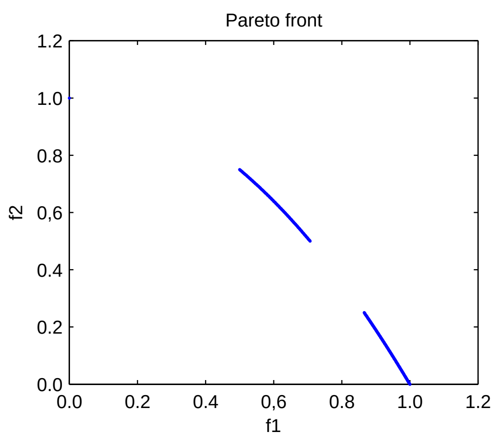

Fig. 13. Illustration of the PF of CF3.

Constrained Problem 4

The two objectives to be minimized:

$$\begin{aligned} f_1 &= \quad x_1 \quad + \sum_{j \in J_1} h_j(y_j) \\ f_2 &= \quad 1 - x_1 \quad + \sum_{j \in J_2} h_j(y_j) \end{aligned}$$

where $J_1 = \{j|j$ is odd and $2 \leq j \leq n\}$ and $J_2 = \{j|j$ is even and $2 \leq j \leq n\}$.

$$y_j = x_j - \sin(6\pi x_1 + \frac{j\pi}{n}), j = 2, \ldots, n.$$

The search space is $[0, 1] \times [-2, 2]^{n-1}$.

$$h_2(t) = \begin{cases} |t| & \text{if } t < \frac{3}{2}(1 - \frac{\sqrt{2}}{2}) \\ 0.125 + (t-1)^2 & \text{otherwise} \end{cases}$$

and

$$h_j(t) = t^2$$

for $j = 3, 4, \ldots, n$.

The constraint is:

$$\frac{t}{1 + e^{4|t|}} \geq 0$$

where

$$t = x_2 - sin(6\pi x_1 + \frac{2\pi}{n}) - 0.5x_1 + 0.25.$$

The PF in the objective space is:

$$f_2 = \begin{cases} 1 - f_1 & \text{if } 0 \leq f_1 \leq 0.5 \\ -0.5f_1 + \frac{3}{4} & \text{if } 0.5 < f_1 \leq 0.75 \\ 1 - f_1 + 0.125 & \text{if } 0.75 < f_1 \leq 1. \end{cases}$$

$n = 10$ in the CEC 09 algorithm contest.

Its PF is illustrated in Fig. 14.

Constrained Problem 5

The two objectives to be minimized:

$$\begin{aligned} f_1 &= \quad x_1 \quad + \sum_{j \in J_1} h_j(y_j) \\ f_2 &= \quad 1 - x_1 \quad + \sum_{j \in J_2} h_j(y_j) \end{aligned}$$

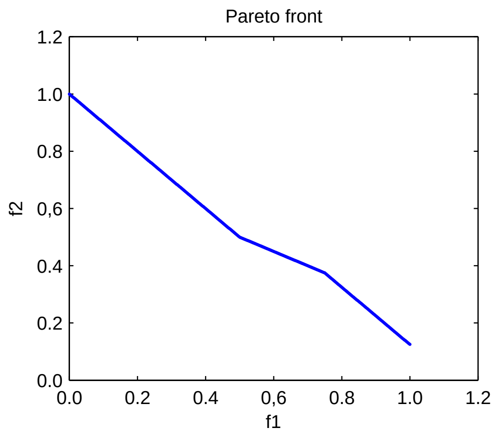

Fig. 14. Illustration of the PF of CF4.

where $J_1 = \{j|j \text{ is odd and } 2 \leq j \leq n\}$ and $J_2 = \{j|j \text{ is even and } 2 \leq j \leq n\}$.

$$y_j = \begin{cases} x_j - 0.8x_1 \cos(6\pi x_1 + \frac{j\pi}{n}) & \text{if } j \in J_1 \\ x_j - 0.8x_1 \sin(6\pi x_1 + \frac{j\pi}{n}) & \text{if } j \in J_2, \end{cases}$$

$$h_2(t) = \begin{cases} |t| & \text{if } t < \frac{3}{2}(1 - \frac{\sqrt{2}}{2}) \\ 0.125 + (t-1)^2 & \text{otherwise,} \end{cases}$$

and

$$h_j(t) = 2t^2 - cos(4\pi t) + 1$$

for $j = 3, 4, \ldots, n$.

The search space is $[0,1] \times [-2,2]^{n-1}$.

The constraint is:

$$x_2 - 0.8x_1 \sin(6\pi x_1 + \frac{2\pi}{n}) - 0.5x_1 + 0.25 \geq 0.$$

The PF in the objective space is:

$$f_2 = \begin{cases} 1 - f_1 & \text{if } 0 \leq f_1 \leq 0.5 \\ -0.5f_1 + \frac{3}{4} & \text{if } 0.5 < f_1 \leq 0.75 \\ 1 - f_1 + 0.125 & \text{if } 0.75 < f_1 \leq 1. \end{cases}$$

$n = 10$ in the CEC 09 algorithm contest.

Its PF is illustrated in Fig. 15.

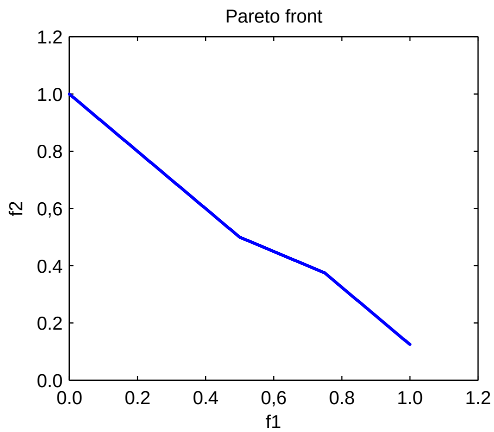

Fig. 15. Illustration of the PF of CF5.

Constrained Problem 6

The two objectives to be minimized:

$$\begin{aligned} f_1 = &\quad x_1 &+ \sum_{j\in J_1} y_j^2 \\ f_2 = &\quad (1-x_1)^2 &+ \sum_{j\in J_2} y_j^2 \end{aligned}$$

where $J_1 = \{j|j \text{ is odd and } 2 \leq j \leq n\}$ and $J_2 = \{j|j \text{ is even and } 2 \leq j \leq n\}$, and

$$y_j = \begin{cases} x_j - 0.8x_1\cos(6\pi x_1 + \frac{j\pi}{n}) & \text{if } j \in J_1 \\ x_j - 0.8x_1\sin(6\pi x_1 + \frac{j\pi}{n}) & \text{if } j \in J_2 \end{cases}.$$

The search space is $[0,1] \times [-2,2]^{n-1}$.

The contraints are

$$x_2 - 0.8x_1\sin(6\pi x_1 + \frac{2\pi}{n}) - sign(0.5(1-x_1) - (1-x_1)^2)\sqrt{|0.5(1-x_1) - (1-x_1)^2|} \geq 0$$

and

$$x_4 - 0.8x_1\sin(6\pi x_1 + \frac{4\pi}{n}) - sign(0.25\sqrt{1-x_1} - 0.5(1-x_1))\sqrt{|0.25\sqrt{1-x_1} - 0.5(1-x_1)|} \geq 0.$$

The PF is:

$$f_2 = \begin{cases} (1-f_1)^2 & \text{if } 0 \leq f_1 \leq 0.5 \\ 0.5(1-f_1) & \text{if } 0.5 < f_1 \leq 0.75 \\ 0.25\sqrt{(1-f_1)} & \text{if } 0.75 < f_1 \leq 1. \end{cases}$$

$n = 10$ in the CEC 09 algorithm contest.

Its PF is illustrated in Fig. 16.

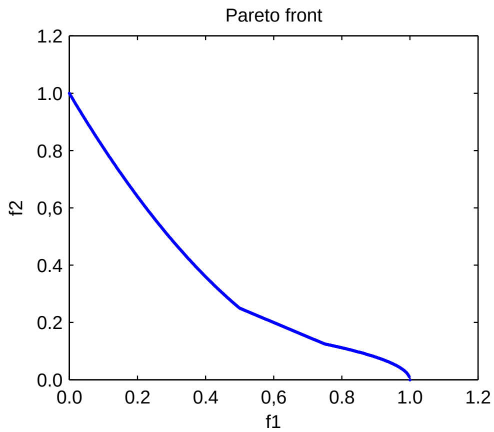

Fig. 16. Illustration of the PF of CF6.

Constrained Problem 7

The two objectives to be minimized:

$$\begin{aligned} f_1 = & \quad x_1 & + \sum_{j \in J_1} h_j(y_j) \\ f_2 = & \quad (1 - x_1)^2 & + \sum_{j \in J_2} h_j(y_j) \end{aligned}$$

where $J_1 = \{j | j \text{ is odd and } 2 \le j \le n\}$ and $J_2 = \{j | j \text{ is even and } 2 \le j \le n\}$,

$$y_j = \begin{cases} x_j - \cos(6\pi x_1 + \frac{j\pi}{n}) & \text{if } j \in J_1 \\ x_j - \sin(6\pi x_1 + \frac{j\pi}{n}) & \text{if } j \in J_2 \end{cases},$$

$$h_2(t) = h_4(t) = t^2,$$

and

$$h_j(t) = 2t^2 - cos(4\pi t) + 1$$

for $j = 3, 5, 6, \dots, n$.

The search space is $[0, 1] \times [-2, 2]^{n-1}$.

The constraints are:

$$x_2 - \sin(6\pi x_1 + \frac{2\pi}{n}) - sign(0.5(1 - x_1) - (1 - x_1)^2)\sqrt{|0.5(1 - x_1) - (1 - x_1)^2|} \ge 0$$

and

$$x_4 - \sin(6\pi x_1 + \frac{4\pi}{n}) - sign(0.25\sqrt{1-x_1} - 0.5(1-x_1))\sqrt{|0.25\sqrt{1-x_1} - 0.5(1-x_1)|} \geq 0.$$

The PF is:

$$f_2 = \begin{cases} (1-f_1)^2 & \text{if } 0 \leq f_1 \leq 0.5 \\ 0.5(1-f_1) & \text{if } 0.5 < f_1 \leq 0.75 \\ 0.25\sqrt{(1-f_1)} & \text{if } 0.75 < f_1 \leq 1 \end{cases} \cdot$$

$n = 10$ in the CEC 09 algorithm contest.

Its PF is illustrated in Fig. 17.

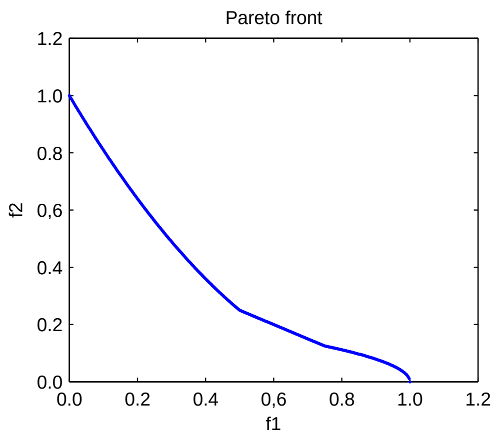

Fig. 17. Illustration of the PF of CF7.

Constrained Problem 8

The three objectives to be minimized:

$$\begin{aligned} f_1 = & \quad \cos(0.5x_1\pi)\cos(0.5x_2\pi) & +\frac{2}{|J_1|}\sum_{j\in J_1}(x_j - 2x_2\sin(2\pi x_1 + \frac{j\pi}{n}))^2 \\ f_2 = & \quad \cos(0.5x_1\pi)\sin(0.5x_2\pi) & +\frac{2}{|J_2|}\sum_{j\in J_2}(x_j - 2x_2\sin(2\pi x_1 + \frac{j\pi}{n}))^2 \\ f_3 = & \quad \sin(0.5x_1\pi) & +\frac{2}{|J_3|}\sum_{j\in J_3}(x_j - 2x_2\sin(2\pi x_1 + \frac{j\pi}{n}))^2 \end{aligned}$$

where

$J_1 = \{j|3 \leq j \leq n$, and $j-1$ is a multiplication of $3\}$,

$J_2 = \{j|3 \leq j \leq n$, and $j-2$ is a multiplication of $3\}$,

$J_3 = \{j|3 \leq j \leq n$, and j is a multiplication of $3\}$.

The search space is $[0,1]^2 \times [-4,4]^{n-2}$.

The constraint is

$$\frac{f_1^2+f_2^2}{1-f_3^2} - a|\sin[N\pi(\frac{f_1^2-f_2^2}{1-f_3^2}+1)]| - 1 \geq 0.$$

Its PF will have $2N+1$ disconnected parts:

$$f_1 = [\tfrac{i}{2N}(1-f_3^2)]^{\frac{1}{2}}$$
$$f_2 = [1-f_1^2-f_3^2]^{\frac{1}{2}}$$
$$0 \leq f_3 \leq 1$$

for $i = 0, 1, \ldots, 2N$.

$a = 4, N = 2$, $n = 10$ in the CEC 09 algorithm contest.

Its PF is illustrated in Fig. 18.

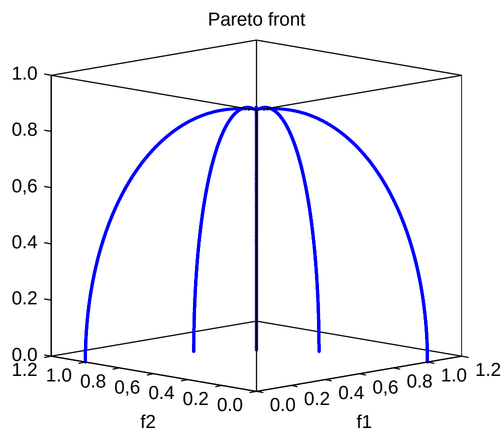

Fig. 18. Illustration of the PF of CF8.

Constrained Problem 9

The three objectives to be minimized:

$$\begin{aligned}
f_1 = &\quad \cos(0.5x_1\pi)\cos(0.5x_2\pi) &+\frac{2}{|J_1|}\sum_{j\in J_1}(x_j - 2x_2\sin(2\pi x_1 + \frac{j\pi}{n}))^2 \\
f_2 = &\quad \cos(0.5x_1\pi)\sin(0.5x_2\pi) &+\frac{2}{|J_2|}\sum_{j\in J_2}(x_j - 2x_2\sin(2\pi x_1 + \frac{j\pi}{n}))^2 \\
f_3 = &\quad \sin(0.5x_1\pi) &+\frac{2}{|J_3|}\sum_{j\in J_3}(x_j - 2x_2\sin(2\pi x_1 + \frac{j\pi}{n}))^2
\end{aligned}$$

where

$J_1 = \{j|3 \leq j \leq n$, and $j-1$ is a multiplication of $3\}$,

$J_2 = \{j|3 \leq j \leq n$, and $j-2$ is a multiplication of $3\}$,

$J_3 = \{j|3 \leq j \leq n$, and j is a multiplication of $3\}$.

The search space is $[0,1]^2 \times [-2,2]^{n-2}$.

The constraint is

$$\frac{f_1^2+f_2^2}{1-f_3^2} - a\sin[N\pi(\frac{f_1^2-f_2^2}{1-f_3^2}+1)] - 1 \geq 0.$$

Its PF consists of:

- a curve:

$$f_1 = 0$$
$$0 \leq f_2 \leq 1$$
$$f_3 = (1-f_2^2)^{1/2}$$

- N disconnected nonlinear 2-D surfaces, the i-th one is:

$$\begin{aligned} 0 \leq f_3 &\leq 1 \\ \{\frac{2i-1}{2N}(1-f_3^2)\}^{\frac{1}{2}} &\leq f_1 \leq \{\frac{2i}{2N}(1-f_3^2)\}^{\frac{1}{2}} \\ f_2 &= [1-f_1^2-f_2^2]^{\frac{1}{2}}. \end{aligned}$$

$N = 2$ and $a = 3$, $n = 10$ in the CEC 09 algorithm contest.

Its PF is illustrated in Fig. 19.

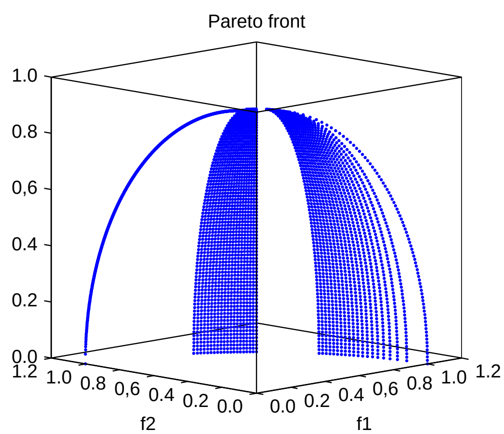

Fig. 19. Illustration of the PF of CF9.

Constrained Problem 10

The three objectives to be minimized:

$$\begin{aligned} f_1 = & \cos(0.5x_1\pi)\cos(0.5x_2\pi) & +\frac{2}{|J_1|}\sum_{j\in J_1}[4y_j^2 - cos(8\pi y_j) + 1] \\ f_2 = & \cos(0.5x_1\pi)\sin(0.5x_2\pi) & +\frac{2}{|J_2|}\sum_{j\in J_1}[4y_j^2 - cos(8\pi y_j) + 1] \\ f_3 = & \sin(0.5x_1\pi) & +\frac{2}{|J_3|}\sum_{j\in J_1}[4y_j^2 - cos(8\pi y_j) + 1] \end{aligned}$$

where

$J_1 = \{j|3 \leq j \leq n$, and $j-1$ is a multiplication of $3\}$,

$J_2 = \{j|3 \leq j \leq n$, and $j-2$ is a multiplication of $3\}$,

$J_3 = \{j|3 \leq j \leq n$, and j is a multiplication of $3\}$,

and

$$y_j = x_j - 2x_2\sin(2\pi x_1 + \frac{j\pi}{n})$$

for $j = 3, \ldots, n$.

The search space is $[0,1]^2 \times [-2,2]^{n-2}$.

The constraint is

$$\frac{f_1^2 + f_2^2}{1 - f_3^2} - a\sin[N\pi(\frac{f_1^2 - f_2^2}{1 - f_3^2} + 1)] - 1 \geq 0.$$

Its PF consists of:

- a curve:

$$f_1 = 0$$
$$0 \leq f_2 \leq 1$$
$$f_3 = (1 - f_2^2)^{1/2}$$

- N disconnected nonlinear 2-D surfaces, the i-th one is:

$$\begin{aligned} 0 \leq f_3 & \leq 1 \\ \{\frac{2i-1}{2N}(1-f_3^2)\}^{\frac{1}{2}} & \leq f_1 \leq \{\frac{2i}{2N}(1-f_3^2)\}^{\frac{1}{2}} \\ f_2 & = [1 - f_1^2 - f_2^2]^{\frac{1}{2}}. \end{aligned}$$

$N = 2$ and $a = 1$, $n = 10$ in the CEC 09 algorithm contest.

Its PF is illustrated in Fig. 20.

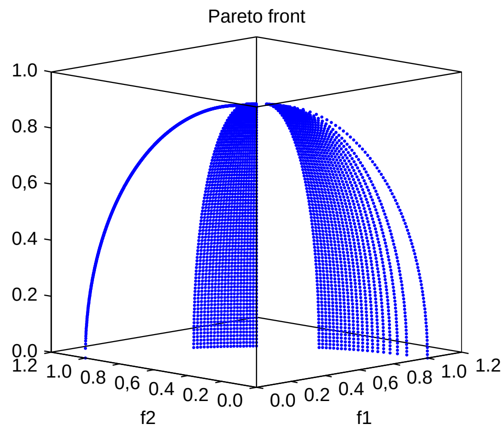

Fig. 20. Illustration of the PF of CF10.

IV. Performance Assessments

There will be two competitions in CEC 09: one is on unconstrained problems and the other is on constrained ones. All the test problems should be treated as black-box problems, i.e., the mathematical formulations of these problems could not be used in the algorithms.

A. Performance Metric (IGD)

Let P^* be a set of uniformly distributed points along the PF (in the objective space). Let A be an approximate set to the PF, the average distance from P^* to A is defined as:

$$IGD(A, P^*) = \frac{\sum_{v \in P^*} d(v, A)}{|P^*|}$$

where $d(v, A)$ is the minimum Euclidean distance between v and the points in A. If $|P^*|$ is large enough to represent the PF very well, $IGD(A, P^*)$ could measure both the diversity and convergence of A in a sense. To have a low value of $D(A, P^*)$, The set A must be very close to the PF and cannot miss any part of the whole PF.

The data file and source code of computing IGD can be downloaded from:
dces.essex.ac.uk/staff/qzhang or
http://www.ntu.edu.sg/home/EPNSugan

B. Constraints

For each constraint:

$$g_i(x) \geq 0.$$

all the solutions in the approximate set for computing the IGD should satisfy:

$$g_i(x) \geq -10^{-6}.$$

C. The Maximal Number of Approximate Solutions

The maximal number of the solutions in the approximate set produced by each algorithm for computing the IGD should be:

- 100 for two objective problems.
- 150 for three objective problems.
- 800 for five objective problems.

D. The maximal Number of Function Evaluations

It is set to be 300,000 for all the problems.

E. The Number of Independent Runs

Each algorithm should be run independently 30 times for each test problem.

F. Algorithmic Parameter Setting

The parameter setting should be the same for the test problems with the same number of objectives.

G. Results Format

Participants should present in their submission:

- **PC Configuration:**
  - System
  - RAM
  - CPU
  - Computer Language
- **Algorithmic Parameter Setting:**
  - the list of all the parameters,
  - guidelines on how to set these parameters,
  - the values of these parameters used in this competition.
- **Experimental Results**
  - the average IGD value of the 30 final approximation sets obtained for each test problem, which is the only merit of figure for comparing different algorithms for competition purpose.
  - the average CPU time used for each test problem.
  - the figure showing the evolution of the means/standard deviations of IGD values of the approximate solution sets obtained with the number of function evaluations for each test instances.
  - any other statistics which you think are useful for other researchers to understand your algorithms.

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