CHAPTER 9

USE OF THE ROBUST CONCEPT EXPLORATION METHOD TO FACILITATE THE DESIGN OF A FAMILY OF PRODUCTS

Timothy W. Simpson, Wei Chen, Janet K. Allen, and Farrokh Mistree

1. FRAME OF REFERENCE

The United States, despite possessing abundant resources of all kinds and having at one time, "made half the manufactured products sold anywhere in the world,"[5] now faces an agile and unforgiving global marketplace in which the formerly all-important concept of economies of scale is now a thing of the past. Producers must offer enhanced system flexibility and a wide variety of products in order to be competitive in today's markets. This raises the following question, *how can product realization teams provide the increased product variety and enhanced flexibility necessary to be competitive in today's and tomorrow's markets?*

Answers to this question vary in the literature. Wheelwright and Clark suggest designing "platform projects" which are capable of meeting the needs of a core group of customers but are easily modified into derivatives through addition, substitution, or removal of features.[23] Uzumeri and Sanderson emphasize standardization and flexibility as a means for enhancing *product flexibility*.[22] Ishii and his coauthors have developed the variety importance-cost map to help minimize the life-cycle cost associated with offering product variety.[7] This work has been expanded to include metrics for assessing the costs of providing variety.[10] Chen and her coauthors suggest designing *flexible products* which can be readily adapted in response to large changes in customer requirements by changing a small number of components or modules.[3] This idea is elaborated in later work in which we introduce *open engineering systems*:[20] systems which are readily adaptable to changes in their environment and enable producers to remain competitive through indefinite growth and continuous improvement of an existing technological base. Finally, Rothwell and Gardiner advocate *robust designs* as a means to improve system flexibility; they assert that robust designs have sufficient inherent *design flexibility* or "technological slack" to enable them to evolve into a *design family of variants* which meet a variety of changing market requirements.[17]

It is this notion of developing a robust design we exploit to design a family of products. Specifically, we use the Robust Concept Exploration Method (RCEM) to facilitate the design of a family of products. The example problem that we have chosen to demonstrate our method is the conceptual design of a family of General Aviation aircraft -- a family of three aircraft consisting of a two, a four, and a six seater models. The RCEM and its components are introduced in the next section. The example problem is presented in Section 3.

2. DESIGNING A FAMILY OF PRODUCTS: OUR APPROACH

2.1. Overview of the Robust Concept Exploration Method

The RCEM has been developed to facilitate quick evaluation of different design alternatives and to generate robust top-level design specifications in the early stages of design.[1] It is primarily useful for designing complex systems and for computationally expensive design analysis. The RCEM has been created by integrating robust design methods[16] and response surface methodology[13] within one mathematical construct, namely, the compromise Decision Support Problem (DSP).[11] Each of these will be discussed in turn, but first we provide an overview of the RCEM.

The RCEM is a four step process; the computer infrastructure for implementing the RCEM is illustrated in Figure 1. The steps of the RCEM are as follows.

Step 1: Classify Design Parameters - Given the overall design requirements, this step involves the use of Processor A (Figure 1) from the RCEM computer infrastructure to (a) classify design parameters as either control factors, noise factors, or responses following the terminology used in robust design and (b) define the concept exploration space.

Step 2: Screening Experiments - This step requires the use of the point generator (Processor B in Figure 1), simulation programs (Processor C), and an experiment analyzer (Processor D) to set up and perform *initial screening experiments* and analyze the results. The results of the screening experiments are used to (a) fit low order response surface models, (b) identify the significance of main effects and omit those factors which have little or no influence, and (c) reduce the design region.

Step 3: Elaborate the Response Surface Model - This step also requires the use of the point generator (Processor B), simulation programs (Processor C), and experiment analyzer (Processor D) from Figure 1 to set up and perform *secondary experiments* and analyze the results. The results from the secondary experiments are used to (a) fit high order response surface models (using Processor E) which replace the original (computationally expensive) analysis programs, (b) identify key design drivers and the significance of different design factors and their interactions, and (c) quickly evaluate different design alternatives and answer "what-if" questions in Step 4.

Step 4: Generate Top-Level Design Specifications with Quality Considerations - Once an accurate response surface model has been created, Step 4 involves the use of the compromise DSP (Processor F in Figure 1) to determine robust top-level design specifications. In Step 4, the original analysis or simulation program(s) is replaced by response surfaces which are functions of both control and noise factors. The multiobjective compromise DSP is then solved to determine robust top-level design specifications.

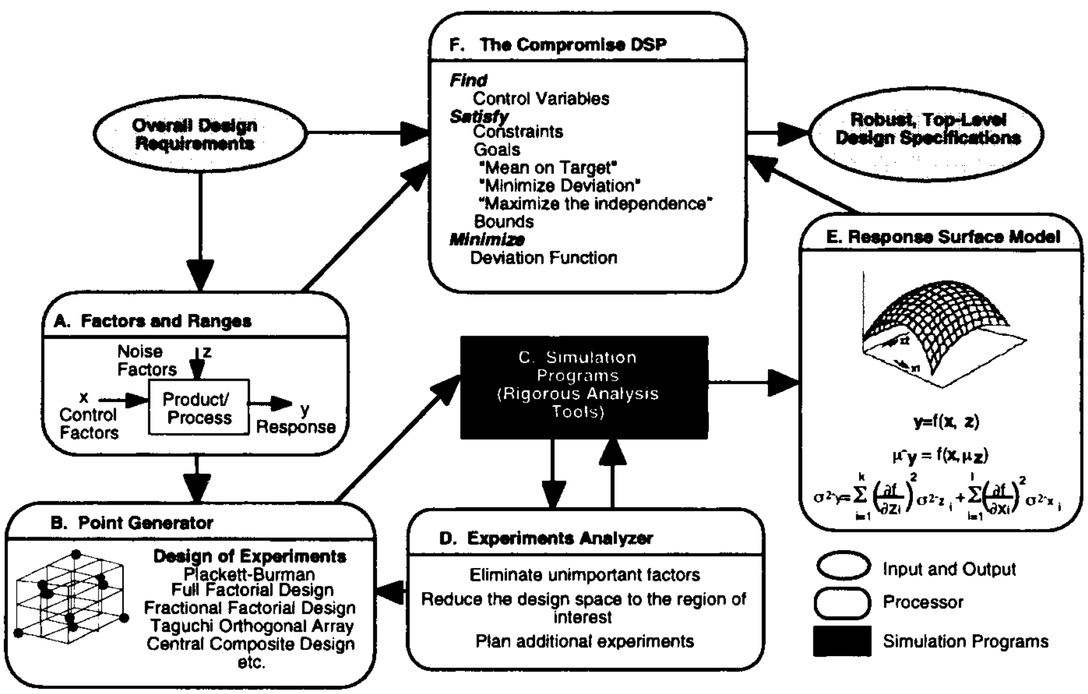

FIGURE 1. Computer infrastructure of the RCEM.

2.2. Overview of Robust Design Principles

One of the main components of the RCEM is an *adaptation* of robust design methods which relies on modeling the robust design objectives of "bringing the mean on target" and "minimizing the deviation" as separate goals in the compromise DSP.[2] In robust design, a designer adjusts control factors, x, to dampen variations caused by the noise factor, z, as illustrated in Figure 2.

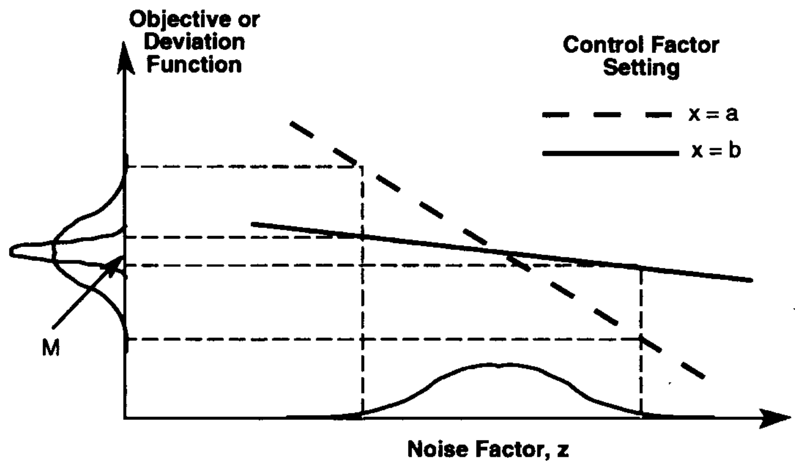

FIGURE 2. Principles of robust design.

If the objective is to achieve a performance as close as possible to the target, M, the designs at levels x = a and x = b are both acceptable because their means are on target. However, introducing *robustness*, when x = a the performance varies significantly with the deviation of the noise factor, z; however, when x = b the performance deviates considerable less. Therefore, x = b is a more robust solution than x = a because x = b dampens the effect of the noise more than x = a. By selecting the control factor settings which best minimize the effects of noise on the system performance, we can *develop a set of specifications which is robust with respect to a given noise factor.* It is in this manner that we can design a family of products simultaneously around a common noise factor as we demonstrate in Section 3.

2.3. Overview of Response Surface Methodology

In the RCEM, response surface models are created based on computer simulations and used to approximate the design space; they can also be used to facilitate the implementation of robust design. The major elements of the response surface model approach[13] are:

- combining control and noise factors in a single array instead of using Taguchi's inner- and outer-array approach,
- modeling the response itself rather than the expected loss, and
- approximating a prediction model for loss based on the fitted-response model.

Central composite designs[13] are frequently employed to fit second-order response surface models based on design simulations. In our work, this process is facilitated through the use of experimental sequencing and analysis software package entitled NORMAN®. In general, the response surface model postulates a single, formal model of the type:

$$\hat{y} = f(\mathbf{x}, \mathbf{z}) \tag{1}$$

where $\hat{y}$ is the estimated response and $\mathbf{x}$ and $\mathbf{z}$ represent the settings of the control and noise variables, respectively. From the response surface model it is possible to estimate the mean and variance of the response. For applications in which the deviations of noise factors are the source of variation, the mean and variance of the response can be approximated by:

- Mean of the response: $$\mu_{\hat{y}} = f(\mathbf{x}, \mu_{\mathbf{z}}) \tag{2}$$

- Variance of the response: $$\sigma_{\hat{y}}^{2} = \sum_{i=1}^{k} \left(\frac{\partial f}{\partial \mathbf{z}_i}\right)^2 \sigma_{\mathbf{z}_i}^2 \tag{3}$$

where μ represents the mean, k is the number of noise factors, and σ_{z_i} is the standard deviation associated with each noise factor.

2.4. Overview of the Compromise DSP

The compromise DSP is the mathematical construct through which the various metrics and tools of the RCEM are integrated. The compromise DSP is a multiobjective decision model which is a hybrid formulation based on Mathematical Programming and Goal Programming.[11] The compromise DSP has been successfully used in designing aircraft, thermal energy systems, mechanisms, damage tolerant structural systems, ships, aircraft engines, and satellite trajectories. The RCEM represents an extension of the compromise DSP and our earlier work[12] from which the following description is taken.

In the compromise DSP, the set of system constraints and bounds define the feasible design space and the set of system goals define the aspiration space as shown in Figure 3. For feasibility the system constraints and bounds must be satisfied. A solution then is that feasible point which achieves the system goals as best as possible. The solution to this problem represents a tradeoff between that which is desired (as modeled by the aspiration space) and that which can be achieved (as modeled by the design space); the solution in Figure 3 is indicated by point **A**.

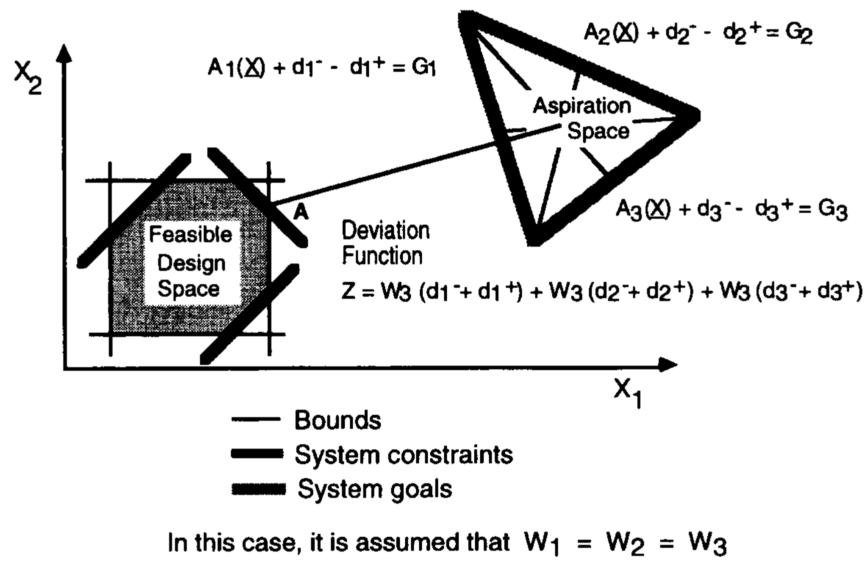

Figure 3. Graphical representation of a two dimensional compromise DSP, Archimedean formulation.[12]

In general, compromise DSPs are written in terms of n **system variables**. The vector of variables, $\underline{X}$, may include continuous variables and Boolean (1 if TRUE, 0 if FALSE) variables. System variables are independent of the other descriptors and can be changed to alter the state of the system. System variables that define the physical attributes of an artifact must be positive.

A **system constraint** models a limit that is placed on the design. The set of system constraints must be satisfied for the feasibility of the design. Mathematically, system constraints are functions of system variables only. They are rigid and no violations are allowed. They relate the demand placed on the system, $D(\underline{X})$, to the capability of the system, $C(\underline{X})$. The set of system constraints may be a mix of linear and nonlinear functions. In engineering problems the system constraints are invariably inequalities. However, occasions requiring equality system constraints may arise. The region of feasibility defined by the system constraints is called the **feasible design space**.

A set of **system goals** is used to model the aspiration a designer has for the design. It relates the goal, G_i, of the designer to the actual performance, $A_i(\underline{X})$, of the system with respect to the goal. The **deviation**

variable is introduced as a measure of achievement since we would like the value of $A_i(\underline{X})$ to equal G_i. Constraining the deviation variables to be non-negative, the system goal becomes:

$$A_i(\underline{X}) + d_i^- - d_i^+ = G_i \,;\, i = 1, \ldots, m$$

$$\text{where } d_i^- . d_i^+ = 0 \text{ and } d_i^- , d_i^+ \geq 0.$$

The product constraint ($d_i^- . d_i^+ = 0$) ensures that at least one of the deviation variables for a particular goal will always be zero. If the problem is solved using a vertex solution scheme, then this condition is automatically satisfied.

Bounds are specific limits placed on the magnitude of each of the system and deviation variables. Each variable has associated with it a lower and an upper bound. Bounds are important for modeling real-world problems because they provide a means to include the experience-based judgment of a designer in the mathematical formulation.

In the compromise DSP formulation the aim is to minimize the difference between that which is desired and that which can be achieved. This is done by minimizing the **deviation function**, $Z(\underline{d}^-, \underline{d}^+)$. This function is always written in terms of the deviation variables. All goals may not be equally important to a designer and the formulations are classified as **Archimedean** or **Preemptive** -- based on the manner in which importance is assigned to satisfying the goals. The general form of the deviation function for m goals in the Archimedean formulation is

$$Z(\underline{d}^-, \underline{d}^+) = \sum_{i=1}^{m} w_i^- \bullet d_i^- + w_i^+ \bullet d_i^+$$

where the weights, w_i^- and w_i^+, reflect the level of desire to achieve each of the goals. Generally, these weights would be chosen to sum to unity.

The most general approach for assigning priority is a Preemptive one where the goals are rank ordered. Multiple goals can be assigned the same rank or level, in which case, Archimedean styled weights may be used within a level. This assignment of priority is probably easier in an industrial environment or in the earlier stages of design. The measure of achievement is then obtained in terms of the lexicographic minimization of an ordered set of goal deviations. Ranked lexicographically, an attempt is made to achieve a more important goal (or set of goals) before other goals are considered. The mathematical definition of lexicographic minimum[6] is as follows.

LEXICOGRAPHIC MINIMUM

Given an ordered array $\underline{f}$ of nonnegative elements f_k's, the solution given by $\underline{f}^{(1)}$ is preferred to $\underline{f}^{(2)}$ if $f_k^{(1)} < f_k^{(2)}$ and all higher ordered elements (i.e., $f_1, \ldots, f_{k-1}$) are equal. If no other solution is preferred to $\underline{f}$, then $\underline{f}$ is the lexicographic minimum.

If we consider $\underline{f}^{(r)}$ and $\underline{f}^{(s)}$, where $\underline{f}^{(r)} = (0, 10, 400, 56)$ and $\underline{f}^{(s)} = (0, 11, 12, 20)$, then $\underline{f}^{(r)}$ is preferred to $\underline{f}^{(s)}$. Hence, the deviation function for the Preemptive formulation is written as

$$Z = \{ f_1(d_i^-,d_i^+), \ldots, f_k(d_i^-,d_i^+) \}.$$

For a problem with four goals, the deviation function might look like

$$Z(\underline{d}^-,\underline{d}^+) = \{ (d_1^- + d_2^+), (d_3^-), (d_4^+) \}.$$

The compromise DSP is solved using the ALP algorithm[11] which is a part of DSIDES (Decision Support in Designing Engineering Systems). The Mathematical Formulation of the Compromise DSP is as follows.

GIVEN

An alternative to be improved; assumptions used to model the domain of interest.

System Parameters

n	number of system variables,	q	inequality constraints
$p + q$	number of system constraints,	m	number of system goals
$g_i(\underline{X})$	system constraint function		
$f_k(d_i)$	function of deviation variables to be minimized at priority level k for the preemptive case.		

FIND

$X_i \quad i = 1, \ldots, n; \qquad d_i^-, d_i^+ \quad i = 1, \ldots, m$

SATISFY

System constraints (linear, nonlinear)

$g_i(\underline{X}) = 0 ; \quad i = 1, \ldots, p \qquad g_i(\underline{X}) \geq 0 ; \quad i = p+1, \ldots, p+q$

System goals (linear, nonlinear)

$A_i(\underline{X}) + d_i^- - d_i^+ = G_i; \quad i = 1, \ldots, m$

Bounds

$X_i^{min} \leq X_i \leq X_i^{max}; \quad i = 1, \ldots, n$

$d_i^-, d_i^+ \geq 0$ and $d_i^+ \bullet d_i^- = 0 ; \quad i = 1, \ldots, m$

MINIMIZE: *deviation function*

$\mathbf{Z} = [f_1(d_i^-, d_i^+), \ldots, f_k(d_i^-, d_i^+)]$

A solution to the compromise DSP is called a *satisficing*[18] solution since it is a *feasible point that achieves the system goals to the "best" extent that is possible*. This notion of *satisficing solutions* is philosophically in harmony with the notion of developing a *broad and* *robust set of top-level design specifications*. By finding ranged sets of top-level design specifications rather than point solutions, we can maintain design flexibility which enables a system to be adapted to changing design requirements.

Consider for example the "system" shown in Figure 4. In the top half of Figure 4, the "system" sits atop a flat mesa representing a ranged solution in the design space. After a problem "shift", e.g., a change in a design requirement, the system is more likely to remain a good solution because we have identified ranges of good design which enhance system flexibility. Shown in the bottom half of Figure 4 is an optimum or point solution. Optimizers seek to find the "best" point solution for a given set of design requirements; thus, the solution sits atop a sharp peak rather than a flat mesa. But look what happens if the problem experiences a "shift" as it does in the top half of the figure. A shift in the problem is liable to make the point solution no longer a good one, and the problem requires rework in order to obtain a new solution. The efficacy of the compromise DSP in creating ranged sets of top-level design specifications has been demonstrated in both aircraft design[9] and ship design.[21] Use of the compromise DSP in the RCEM to develop a ranged set of design specifications for a family of products is demonstrated in the next section.

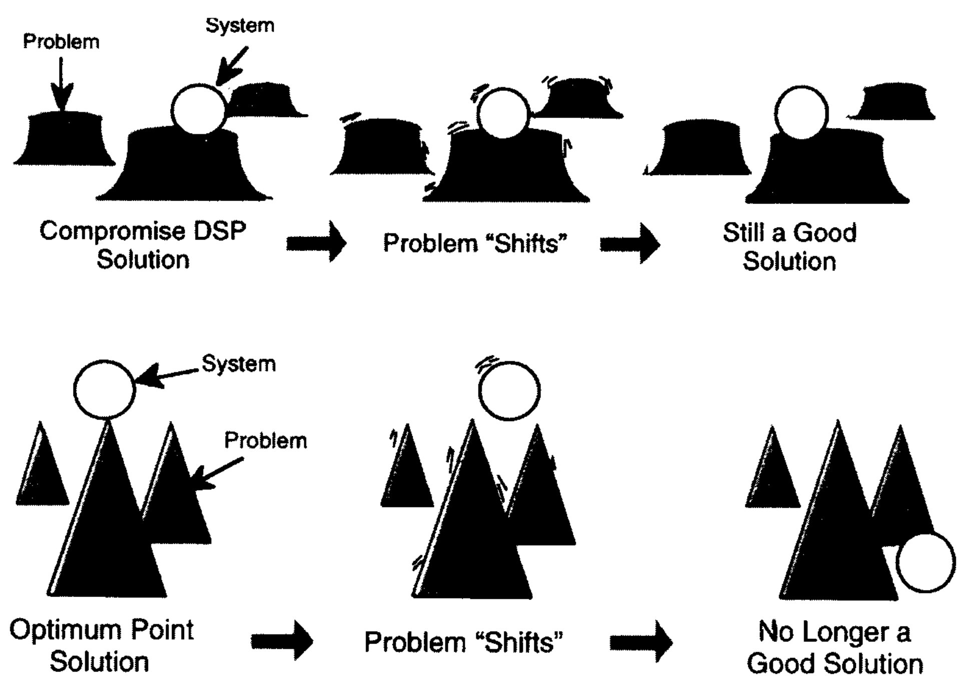

FIGURE 4. Ranged solutions and optimum (point) solutions.

3. EXAMPLE: CONCEPTUAL DESIGN OF A FAMILY OF AIRCRAFT

To demonstrate the use of the RCEM to design a family of products, we now present an example, namely, the conceptual design of a family of General Aviation aircraft. The term General Aviation encompasses all flights except military operations and commercial carriers. Currently, there are approximately 212,000 General Aviation aircraft in use in the United States. General Aviation aircraft in the U.S. account for approximately 62% of all flight hours, 37% of all miles flown, and 78% of all departures. Its potential buyers form a diverse group that include weekend and recreational pilots, instructors and training pilots, traveling business executives, and small commercial operators.

Satisfying a group with such diverse needs and economic potential poses a constant challenge for the General Aviation industry since it is impossible to satisfy all the market needs with a single aircraft. The present financial and legal pressure endured by the general aviation sector makes small production runs of specialized models unprofitable. As a result, many General Aviation aircraft are no longer being produced, and the few remaining models are beyond the financial capability of all but the wealthiest buyers. Combined with the harsh legal environment to which General Aviation airplanes and component producers have been subjected, the General Aviation sector is in a deep recession. In 1979 nearly 18,000 General Aviation aircraft were produced in the U.S., while in 1993 barely over 800 units were sold in this country.

A reasonable solution to the General Aviation aircraft (GAA) crisis is to develop a family of aircraft which can be easily modified to satisfy distinct groups of customers. Toward this end, we seek to design a family of GAA around the 2, 4, and 6 seater aircraft configurations; the general dimensions of each aircraft are to be developed such that a significant number of top-level design specifications* will be shared by the three aircraft to facilitate their development. Our objective then is to:

> *develop a ranged set of top-level design specifications for a family of General Aviation aircraft capable of satisfying the diverse demands of the General Aviation public at an affordable price and operating cost while meeting desired technical and economic performance considerations.*

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* Top-level design specifications are used to define the overall system configuration, e.g., wing area, aspect ratio, sweep angle, etc., and subsystems at an abstract level. They can be either continuous, e.g., aspect ratio = 7 - 11, sweep angle = 0° - 6°, etc., or they can be discrete, e.g., single- or twin-engine, high or low wing, etc.

Given that we seek to find a ranged set of top-level design specifications for a family of aircraft and not just a single model, there are several approaches we could take, see Figure 5. We can either design all three aircraft individually and look for commonalties between them, as shown in the top half of Figure 5, or use one of the approaches mentioned in Section 1 (e.g., standardization, modularity, robustness, etc. as shown on the bottom left hand side of Figure 5) to facilitate the design of the product family. As stated previously, our interest is on the use of robust design to develop a family of products, and in the dashed box in the bottom center of Figure 5 we illustrate our implementation of robust design principles embodied in the RCEM to develop *a family of products around a common noise factor*. Specifically, we invoke the RCEM in this example to develop a *ranged* set of top-level design specifications for a *family* of aircraft.

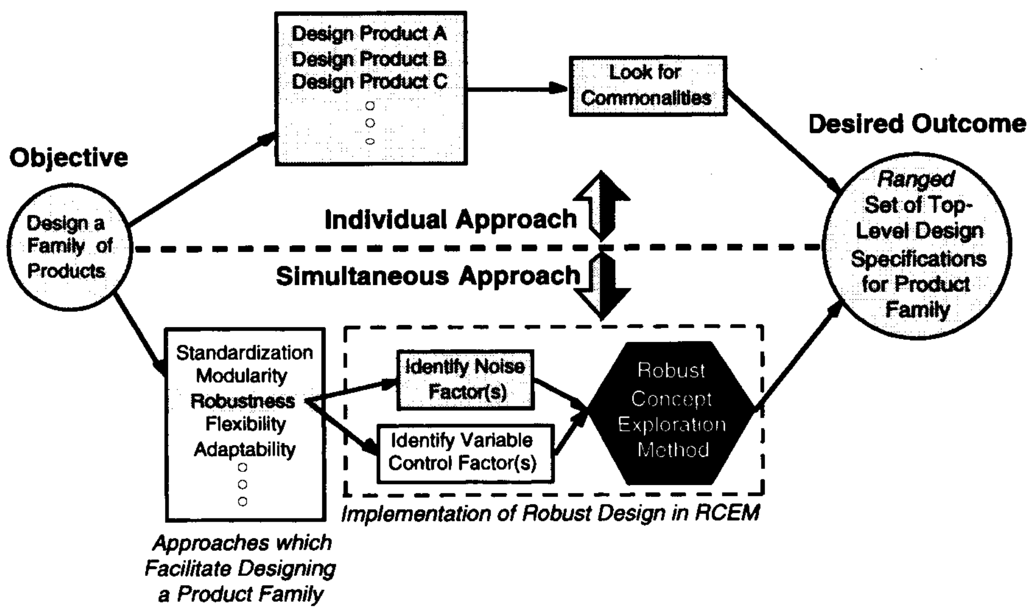

FIGURE 5. Pictorial overview of approach for designing a family of products using the RCEM.

We implement both approaches for this example. In Section 3.1, we design all three aircraft individually (for benchmarking) and look for a "common and good" ranged set of top-level design specifications for the family of aircraft. Then in Section 3.2 we design all three aircraft simultaneously using the RCEM. The results from these sections are then compared and contrasted in Section 4.

3.1. Designing the 2, 4, and 6 Seater Benchmark Aircraft

As highlighted in the top half of Figure 5, our first approach for designing the family of aircraft is to design all three aircraft individually and look for a "common and good" set of top-level design specifications for all three aircraft which constitute the family of aircraft. To obtain the designs for the benchmark aircraft, we perform the following steps..

1) Formulate a compromise DSP for *each aircraft.*
2) Create *three* sets of response surface models, one set for each aircraft, which relates the desired system responses (system constraints and goals) to the top-level design specifications (design variables for use in each compromise DSP.
3) Solve each compromise DSP for eight (8) different design scenarios (tradeoff studies) to examine the extent to which the design variables change from one scenario to the next.
4) Identify trends for each design variable for each aircraft for the eight design scenarios and establish a ranged set of specifications for the family of aircraft based on these results.

3.1.1. Compromise DSP formulation. Step (1) We are interested in finding the appropriate *ranged* values for the following top-level design specifications for the family of GAA. The initial ranges of interest are defined by the upper and lower bounds given in parentheses; these bounds define our design space.

1. Cruise speed, CSPD (Mach 0.24 ≤ CSPD ≤ Mach 0.48)
2. Aspect ratio, AR (7 ≤ AR ≤ 11)
3. Propeller diameter, DPROP (5.5 ft ≤ DPROP ≤ 6.8 ft)
4. Wing loading, WL (19 lb/ft^2 ≤ WL ≤ 25 lb/ft^2)
5. Wing sweep angle, SW (0° ≤ SW ≤ 6.0°)
6. Engine activity factor, AF (85 ≤ AF ≤ 110)
7. Seat width, WS (10 in ≤ WS ≤ 20.0 in)
8. Tail length to diameter ratio, ELODT (3.0 ≤ ELODT ≤ 3.75)

The constraints and goals for each aircraft are summarized in the compromise DSP in Figure 6. Notice that some constraints and goal targets apply to all three aircraft while others differ. For instance, constraints on takeoff noise, propeller tip speed, direct operating cost, empty weight, and ride roughness are the same for all three aircraft, but the fuel weight constraint differs for each aircraft. Targets for maximum lift/drag, cruise speed, range and direct operating cost are the same for all three aircraft, but the targets for fuel weight, empty weight, and purchase price vary from one aircraft to another.

Given:

- Response surfaces for 2, 4, and 6 seater aircraft based on GASP[14] simulations
- Mission (altitude, range, etc.), low wing position, 3 blades, retractable landing gear

Find:

- *The system variables,* $\underline{X}$:
  - the cruise speed, CSPD
  - the wing aspect ratio, AR
  - the sweep angle, SW
  - the propeller diameter, DPROP
  - the wing loading, WL
  - the engine activity factor, AF
  - the tail length to diameter ratio, ELODT
  - the seat width, WS

Satisfy:

- *The system constraints,* C($\underline{X}$):
  - takeoff noise ≤ 75 dbA
  - propeller tip speed ≤ 850 ft/sec
  - ride roughness factor ≤ 2.0 nmu
  - direct operating cost ≤ 80 $/hr
  - empty weight ≤ 2200 lbs
  - fuel weight ≤ target (see Table 1)
- *The system goals,* G($\underline{X}$), *for brining mean on target*
  - minimize fuel weight: WFUEL($\underline{X}$)/target (see Table 1) + $d_1^- - d_1^+ = 1.0$
  - minimize empty weight: WEMP($\underline{X}$)/target (see Table 1) + $d_2^- - d_2^+ = 1.0$
  - minimize direct operating cost: DOC($\underline{X}$)/60 + $d_3^- - d_3^+ = 1.0$
  - minimize purchase price: PURCH($\underline{X}$)/target (see Table 1) + $d_4^- - d_4^+ = 1.0$
  - maximize max. lift/drag: LDMAX($\underline{X}$)/17 + $d_5^- - d_5^+ = 1.0$
  - maximize max. cruise speed: VCRMX($\underline{X}$)/200 + $d_6^- - d_6^+ = 1.0$
  - maximize max. range: RANGE($\underline{X}$)/2500 + $d_7^- - d_7^+ = 1.0$

  subject to: $d_i^- \bullet d_i^+ = 0$ and $d_i^-, d_i^+ \geq 0$, for i = 1, 2, ..., 7.

Minimize:

- *The sum of the deviation variables associated with:*
  - fuel weight, d_1^+
  - empty weight, d_2^+
  - direct operating cost, d_3^+
  - purchase price, d_4^+
  - maximum lift to drag ratio, d_5^-
  - maximum cruise speed, d_6^-
  - maximum range, d_7^-

  $Z = \{f_1(d_1^+), f_2(d_2^+), f_3(d_3^+), f_4(d_4^+), f_5(d_5^-), f_6(d_6^-), f_7(d_7^-)\}$

FIGURE 6. Compromise DSP for the benchmark aircraft.

The individual goal targets and constraints which are unique for each aircraft are summarized in Table 1. For example, the target purchase price for each aircraft is $41,000, $42,000, and $43,000 for the 2, 4, and 6 seater aircraft, respectively; the target empty weight for each aircraft is 1900 lbs, 1950 lbs, and 2000 lbs for the 2, 4, and 6 seater aircraft, respectively.

TABLE 1. Goal targets and constraints for benchmark aircraft.

Targets for Goals	Acronym	2 Seater GAA	4 Seater GAA	6 Seater GAA
Aircraft fuel weight	WFUEL	450 lbs	400 lbs	350 lbs
Aircraft empty weight	WEMP	1900 lbs	1950 lbs	2000 lbs
Purchase price	PURCH	$41,000	$42,000	$43,000
Constraints				
Aircraft fuel weight	WFUEL	450 lbs	475 lbs	500 lbs

Step (2) Second-order response surface models are developed to serve as the fast analysis module when solving the compromise DSP in DSIDES. The second-order response surface for direct operating cost (DOC), for example, is:

$$\begin{aligned}\mathrm{DOC} = {} & 83.17+12.53x_1-0.0477x_2-0.0215x_3+3.597x_4-0.7367x_5+0.7481x_6+0.733x_7\\ & -0.2029x_8+0.6526x_1x_2+0.0481x_1x_3+1.208x_1x_4+0.6802x_1x_5+0.0992x_1x_6\\ & -0.7074x_1x_7+0.2768x_1x_8+0.0031x_2x_3+0.2146x_2x_4-0.0721x_2x_5-0.2445x_2x_6\\ & -0.0172x_2x_7+0.0127x_2x_8+0.0169x_3x_4+0.0151x_3x_5-0.0063x_3x_6-0.0001x_3x_7\\ & -0.0042x_3x_8+0.0789x_4x_5+0.676x_4x_6-0.1912x_4x_7+0.0519x_4x_8+0.0136x_5x_6\\ & +0.0804x_5x_7+0.0577x_5x_8-0.0617x_6x_7+0.0058x_6x_8+0.0901x_7x_8-11.37x_1^2\\ & -0.2836x_2^2-0.314900x_3^2+5.337x_4^2-0.3711x_5^2-0.071x_6^2-0.2177x_7^2-0.2354x_8^2\end{aligned}$$

where all x_i are normalized between [-1,1], and x_1 is cruise speed; x_2 is aspect ratio; etc. These second-order response surfaces are created using the experiment sequencer and analyzer, NORMAN®, and are based on simulations of GASP; GASP is the General Aviation Synthesis Program which performs parametric analysis necessary for preliminary aircraft design.[14] Development and validation of the response surfaces are detailed elsewhere and are not included here due to the length.[19]

3.1.2. Top-level design specifications for the family of general aviation aircraft. Step (3) A total of eight different design scenarios, see Table 2, are used when solving each compromise DSP to develop a common and good ranged set of top-level design specifications for the family of aircraft. The design scenarios consist of equally weighted deviation functions at each priority level (Preemptive formulation). A summary of each group of design scenarios follows.

Group I - Tradeoff studies - includes Scenarios 1, 2 and 3. An overall tradeoff study with all goals weighted equally is performed in the first scenario. The second scenario is an economic tradeoff study with WEMP, PURCH, and DOC given first priority. A performance tradeoff study with WFUEL, VCRMX, RANGE, and LDMAX ranked highest is the third scenario. The deviation function formulations for Scenarios 1-3 are summarized in Table 2.

Group II - Economic tradeoff studies - includes Scenarios 4 and 5. In these two scenarios, economics plays the lead role while performance takes a back seat; in other words, we first worry about achieving the economic targets, and then we worry about achieving performance targets. Two scenarios are considered, one with manufacturing considerations (WEMP and PURCH) as the main driver, Scenario 4, and one with operating cost (DOC) as the main driver, Scenario 5. The deviation functions formulations for these scenarios are summarized in Table 2.

Group III - Performance studies - includes Scenarios 6, 7, and 8. In these scenarios performance takes the lead rather than economics. Scenario 6 is based on speed considerations with VCRMX and WFUEL ranked first, Scenario 7 is based on aerodynamic considerations with LDMAX ranked first, and Scenario 8 is based on the flight distance with RANGE ranked first. The deviation function formulations for these scenarios are listed in Table 2.

TABLE 2. GAA compromise DSP design scenarios for benchmark aircraft.

Scenario	Deviation Function			
	Priority Level 1	*Priority Level 2*	*Priority Level 3*	*Priority Level 4*
Group I: Tradeoff Studies				
1. **Overall tradeoff** (All goals equally weighted)	$1/7*(d_1^+ + d_2^+ + d_3^+ + d_4^+ + d_5^- + d_6^- + d_7^-)$			
2. **Economic tradeoffs** (DOC, WEMP, & PURCH ranked first)	$1/3*(d_2^+ + d_3^+ + d_4^+)$	$1/4*(d_1^+ + d_5^- + d_6^- + d_7^-)$		
3. **Performance tradeoffs** (WFUEL, VCRMX, RANGE, & LDMAX ranked first)	$1/4*(d_1^+ + d_5^- + d_6^- + d_7^-)$	$1/3*(d_2^+ + d_3^+ + d_4^+)$		
Group II: Economics				
4. **Economic driver:** WEMP & PURCH ranked first	$1/2*(d_2^+ + d_9^+ + d_4^+ + d_{11}^+)$	d_3^+	$1/4*(d_1^+ + d_5^- + d_6^- + d_7^-)$	
5. **Economic driver:** DOC ranked first	d_3^+	$1/2*(d_2^+ + d_4^+)$	$1/4*(d_1^+ + d_5^- + d_6^- + d_7^-)$	
Group III: Performance				
6. **Performance driver**: VCRMX & WFUEL ranked first	$1/2*(d_1^+ + d_6)$	$1/2*(d_5^- + d_7^-)$	$1/3*(d_2^+ + d_3^+ + d_4^+)$	
7. **Performance driver**: LDMAX ranked first	d_5^-	d_7^-	$1/2*(d_1^+ + d_6^-)$	$1/3*(d_2^+ + d_3^+ + d_4^+)$
8. **Performance driver**: RANGE ranked first	d_7^-	d_5^-	$1/2*(d_1^+ + d_6^-)$	$1/3*(d_2^+ + d_3^+ + d_4^+)$

d_1^+ is for WFUEL
d_2^+ is for WEMP
d_3^+ is for DOC
d_4^+ is for PURCH
d_5^- is for LDMAX
d_6^- is for VCRMX
d_7^- is for RANGE

Each compromise DSP for each of the three aircraft is solved under the aforementioned eight design scenarios. The resulting top-level design specifications and system performance characteristics for each aircraft are plotted in Figures 7 and 8, respectively. All final solutions are feasible; therefore, final constraint values are not included since all of the constraints have been met.

Step (4) is to determine the ranged set of top-level design specifications for the family of aircraft based on the trends in Figure 7. As shown in the key, the x's are for the 2 seater aircraft; the circles, the 4seater; and the squares, the 6 seater.

- *Cruise speed* = [Mach 0.24-0.35]: consistently higher for all three aircraft in Scenarios 3, 6, 7, and 8 when performance considerations are most important, stays nears its lower bound otherwise.
- *Aspect ratio* = [7-10]: fluctuates between a low of 7 and a high of 10; high values occur primarily in Scenarios 6-8 when performance considerations are most important.
- *Sweep angle* = [0°-6°]: is steady for the 6 seater aircraft, but fluctuates for the 2 or 4 seater aircraft.
- *Seat width* = [14-20 in.]: fluctuates from a low of 14 to a high of 20 in.
- *Wing loading* = [19-25 lb/ft^2]: constantly increasing for all three aircraft.
- *Engine activity factor* = [85-105]: steady for the 6 seater aircraft at 85, but exhibits erratic behavior for the 2 and 4 seater aircraft.
- *Propeller diameter* = [5.5-5.96 ft]: varies between a low of 5.5 ft and a high of 5.96 ft; values above 5.96 ft violate the noise constraint.
- *Tail length to diameter ratio* = [3.6-3.75]: converges at or near its upper bound in all 8 scenarios.

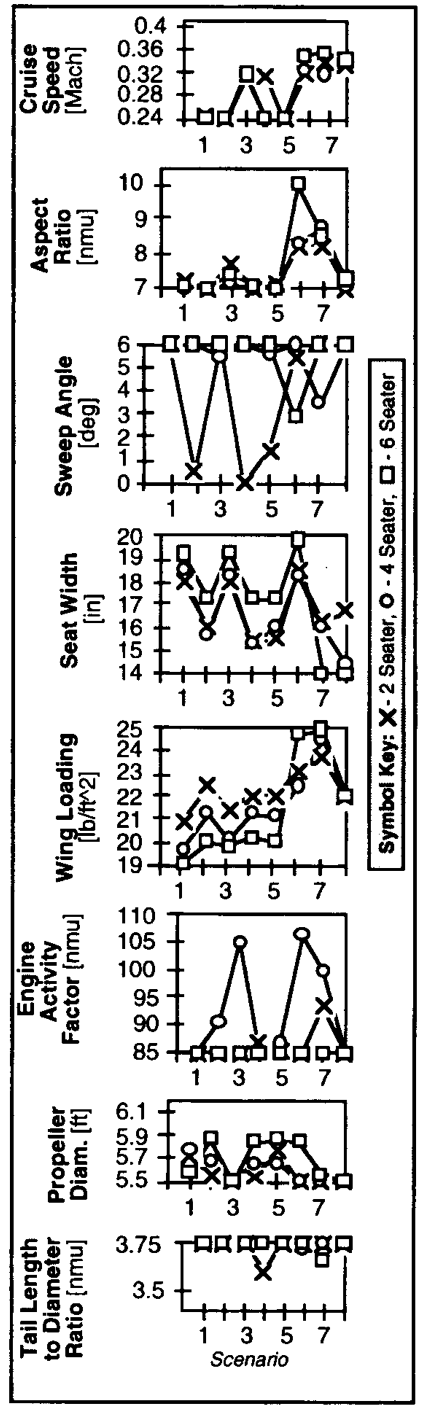

FIGURE 7. System variables for the 3 benchmark aircraft.

The corresponding system responses for each aircraft are plotted in Figure 8 for each scenario. The x's mark the values for the 2 seater aircraft; the circles, the 4 seater; and the squares, the 6 seater. Desired target values are indicated by a dashed line and numbered when different for each aircraft, e.g., the cruise speed target is 200 kts for all three aircraft while the target for empty weight is 2000 lbs, 1950 lbs, and 1900 lbs for the 6, 4, and 2 seater aircraft, respectively. The arrow in each plot indicates on which side of the target we wish to be; fuel weight, empty weight, direct operating cost, and purchase price are to be minimized while maximum lift to drag ratio, maximum cruise speed, and maximum range are to be maximized. For a complete listing of the target values for each aircraft refer to Table 1 and the compromise DSP formulation in Figure 6.

Looking at Figure 8, we see that fuel weight and empty weight mirror each other; when one increases the other decreases and vice versa. This is because in the simulation program GASP, an aircraft is sized for a fixed take-off weight (take-off weight being the sum of the empty weight and fuel weight). The targets for purchase price and direct operating cost are only achieved by each aircraft when economics is ranked at the top priority level (Scenarios 2, 4, and 5), and do not come close otherwise. The target for lift to drag ratio is only reached when it is ranked highest (Scenario 7), and the cruise speed only

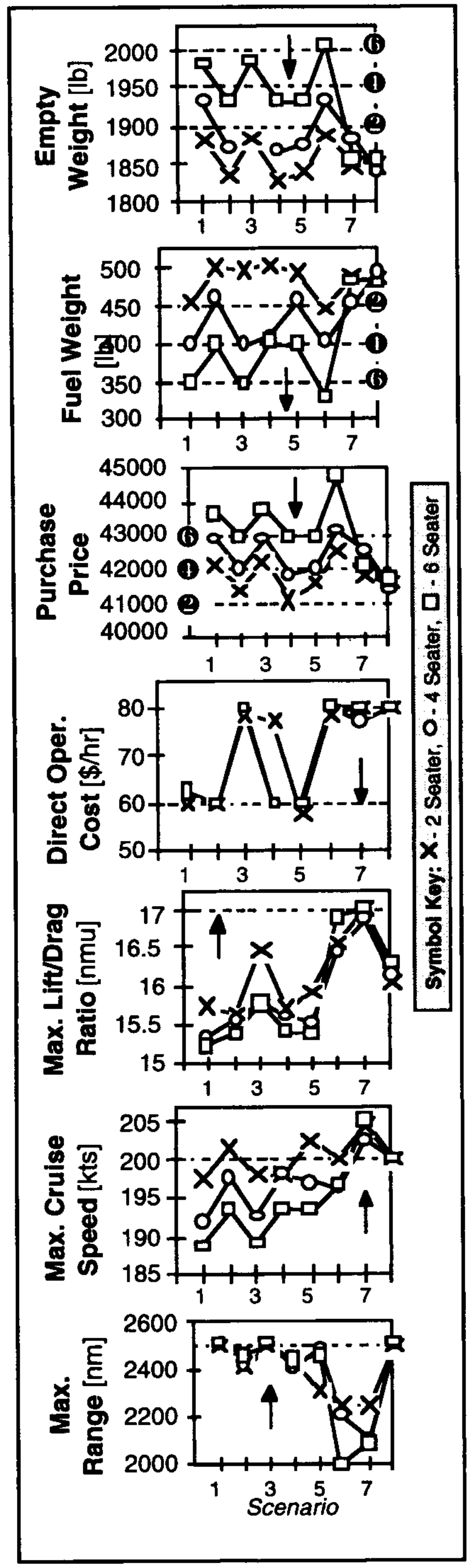

FIGURE 8. System goals for the 3 benchmark aircraft

reaches its target for all three aircraft when performance is placed at the highest priority level (Scenarios 6, 7, and 8). Maximum range reaches its target in Scenarios 1, 3, 5 and 8 when it is placed at a high priority level; otherwise, it varies widely from aircraft to aircraft.

Now that we have designed the benchmark aircraft and found a "common and good" ranged set of top-level design specifications for all three aircraft, we can implement the Robust Concept Exploration Method and design all three aircraft simultaneously. Afterwards, we can compare and contrast the results and also examine how much more efficient we are using the RCEM to design all three aircraft simultaneously as opposed to designing all three aircraft individually as we have just done.

3.2. Designing the 2, 4, and 6 Seater Aircraft Simultaneously Using the RCEM

To implement the RCEM, we use the approach highlighted in the bottom of Figure 5 and perform the following steps.

1) Formulate a compromise DSP for the *family of aircraft*.
2) Create *one* set of response surface models which includes the number of passengers as a noise factor. This is possible because in the simulation program, GASP[14], an aircraft is sized for passenger weight, each passenger weighing 200 lbs. Seats must be specified in pairs, and the number of seats is equal to the number of passengers plus one pilot. Using the number of passengers (PAX) as a noise factor ranging from 1 to 5 allows us to size a *family* of aircraft based on the 2, 4, and 6 seater configurations for a total passenger weight varying from 200-1000 lbs which we consider as noise during the design process.
3) Solve the new compromise DSP for the same eight (8) design scenarios used previously to examine the extent to which the design variables change for the entire family of aircraft.
4) Identify trends for each design variable for the family of aircraft for the eight design scenarios and establish a ranged set of specifications for the family of aircraft based on these results.

3.2.1. Compromise DSP for the family of aircraft. Step (1) The design variables, ranges, constraints and goals for the family of aircraft are the same as before; the only difference is how the compromise DSP is formulated. In order to design all three aircraft simultaneously, the number of passengers is introduced as a noise variable into the design process as explained previously. Using the number of passengers as a noise factor, robust design principles embodied in the RCEM can be used to design the aircraft by having separate goals for "bringing the mean on target" and "minimizing the variance" of the performance of the family of aircraft. It is in this manner that all three aircraft can be designed simultaneously. The compromise DSP for designing all three aircraft simultaneously using the RCEM is given in Figure 9.

Given:
- Response surface models from GASP which includes # of passengers
- Mission (altitude, range, etc.), low wing position, retractable landing gear, 3 blades

Find:
- *The system variables,* $\underline{X}$:
  - the cruise speed, CSPD
  - the wing aspect ratio, AR
  - the sweep angle, SW
  - the propeller diameter, DPROP
  - the wing loading, WL
  - the engine activity factor, AF
  - the tail length to diameter ratio, ELODT
  - the seat width, WS

Satisfy:
- *The system constraints,* $C(\underline{X})$:
  - takeoff noise ≤ 75 dbA
  - propeller tip speed ≤ 850 ft/sec
  - ride roughness factor ≤ 2.0 nmu
  - direct operating cost ≤ 80 $/hr
  - empty weight ≤ 2200 lbs
  - fuel weight ≤ 500 lbs
- *The system goals,* $G(\underline{X})$, *for brining mean on target*
  - minimize fuel weight mean: $\mu_{WFUEL}(\underline{X})/400 + d_1^- - d_1^+ = 1.0$
  - minimize empty weight mean: $\mu_{WEMP}(\underline{X})/1950 + d_2^- - d_2^+ = 1.0$
  - minimize direct operating cost mean: $\mu_{DOC}(\underline{X})/60 + d_3^- - d_3^+ = 1.0$
  - minimize purchase price mean: $\mu_{PURCH}(\underline{X})/42000 + d_4^- - d_4^+ = 1.0$
  - maximize max. lift/drag mean: $\mu_{LDMAX}(\underline{X})/17 + d_5^- - d_5^+ = 1.0$
  - maximize max. cruise speed mean: $\mu_{VCRMX}(\underline{X})/200 + d_6^- - d_6^+ = 1.0$
  - maximize max. range mean: $\mu_{RANGE}(\underline{X})/2500 + d_7^- - d_7^+ = 1.0$
- *The system goals,* $G(\underline{X})$, *for minimizing deviation*
  - minimize deviation of fuel weight: $3*\sigma_{WFUEL}(\underline{X})/50 + d_8^- - d_8^+ = 1.0$
  - minimize deviation of empty weight: $3*\sigma_{WEMP}(\underline{X})/50 + d_9^- - d_9^+ = 1.0$
  - minimize deviation of operating cost: $3*\sigma_{DOC}(\underline{X})/5 + d_{10}^- - d_{10}^+ = 1.0$
  - minimize deviation of purchase price: $3*\sigma_{PURCH}(\underline{X})/1000 + d_{11}^- - d_{11}^+ = 1.0$
  - minimize deviation of max. lift/drag: $3*\sigma_{LDMAX}(\underline{X})/1 + d_{12}^- - d_{12}^+ = 1.0$
  - minimize deviation of max. speed: $3*\sigma_{VCRMX}(\underline{X})/10 + d_{13}^- - d_{13}^+ = 1.0$
  - minimize deviation of max. range: $3*\sigma_{RANGE}(\underline{X})/225 + d_{14}^- - d_{14}^+ = 1.0$

  subject to: $d_i^- \bullet d_i^+ = 0$ and $d_i^-, d_i^+ \geq 0$, for $i = 1, 2, ..., 14$.

Minimize:
- *The sum of the deviation variables associated with:*
  - μ_{WFUEL} and σ_{WFUEL}, d_1^+ and d_8^+
  - μ_{WEMP} and σ_{WEMP}, d_2^+ and d_9^+
  - μ_{DOC} and σ_{DOC}, d_3^+ and d_{10}^+
  - μ_{PURCH} and σ_{PURCH}, d_4^+ and d_{11}^+
  - μ_{LDMAX} and σ_{LDMAX}, d_5^- and d_{12}^+
  - μ_{VCRMX} and σ_{VCRMX}, d_6^- and d_{13}^+
  - μ_{RANGE} and σ_{RANGE}, d_7^- and d_{14}^+

  $Z = \{f_1(d_1^+), f_2(d_2^+), f_3(d_3^+), f_4(d_4^+), f_5(d_5^-), f_6(d_6^-), f_7(d_7^-),$
  $f_8(d_8^+), f_9(d_9^+), f_{10}(d_{10}^+), f_{11}(d_{11}^+), f_{12}(d_{12}^+), f_{13}(d_{13}^+), f_{14}(d_{14}^+)\}$

FIGURE 9. Compromise DSP for implementing the RCEM.

The means and deviation targets for the family of aircraft are based on those values listed previously in Table 1. For example, the target for the mean of the fuel weight (WFUEL) goal is simply

$$\mu_{\text{WFUEL, Target}} = \frac{450 + 400 + 350}{3} = 400 \text{ lb} \tag{4}$$

while the target for its standard deviation is

$$\sigma_{\text{WFUEL, Target}} = \frac{450 - 350}{6} = 16.65 \text{ lb} \tag{5}$$

which is the permissible range for WFUEL divided by $\pm 3\sigma$ (i.e., six standard deviations). The remaining mean and deviation targets are computed in a similar manner. In cases where the target is the same for all three aircraft, the mean is taken as the target value itself, and 3σ is set to $\pm 10\%$ of the mean. For example, direct operating cost (DOC) has a target value for all three aircraft of \$60/hr. This value, \$60/hr, serves as the target for μ_{DOC} in the compromise DSP for the family of aircraft (see Figure 9) and the target for the standard deviation is computed as 3σ = \$5/hr (which is about 10% of the mean target) or $\sigma = 5/3$. When checking constraints, worst case scenarios are used.

Step (2) To facilitate the implementation of robust design, second-order response surface models which include the number of passengers as a noise factor are created using the experiment sequencer/analyzer, NORMAN®. Development and validation of these response surface models are detailed elsewhere[19] and are not included here because of length. These second-order response surface models are used as the fast analysis module for solving the GAA compromise DSP in DSIDES as discussed in Section 2. Use of the second-order response surface models facilitates estimating system response means and standard deviations, refer to Eqns. 2 and 3. For instance, the mean of fuel weight (μ_{WFUEL}) is the average of the fuel weights for the 2, 4, and 6 seater GAA, and the standard deviation is approximated using a first order Taylor series expansion, i.e.,

$$\sigma_{\text{WFUEL}} \approx \sqrt{\sum_i \left(\frac{\partial f}{\partial z_i}\right)^2 \sigma_{z_i}^{\,2}} = \left| \left(\frac{\partial(\text{WFUEL})}{\partial(\text{PAX})}\right) \sigma_{\text{PAX}} \right| \tag{6}$$

The first part of Eqn. 6 is the basic Taylor expansion for $f(\mathbf{x})$ with noise factors, z_i, each with a standard deviation, σ_{z_i}. Since the number of passengers, PAX, is the only noise factor in our example problem, there is only one partial derivative, ∂PAX, and the square root can be dropped. In this manner, we can approximate the means and standard deviations for all of the system responses and find a ranged set of top-level design specifications for our family of aircraft.

3.2.2. Top-level design specifications for the family of general aviation aircraft. Step (3) The same eight design scenarios used in Section 3.1.2 are again used when solving the new compromise DSP (Figure 9) to develop a ranged set of top-level design specifications for the family of aircraft, see Table 3.

TABLE 3. GAA compromise DSP design scenarios.

Scenario	Deviation Function			
	Priority Level 1	*Priority Level 2*	*Priority Level 3*	*Priority Level 4*
Group I: Tradeoff Studies				
1. **Overall tradeoff** (All goals equally weighted)	$1/14*(d_1^+ + d_2^+ + d_3^+ + d_4^+ + d_5^- + d_6^- + d_7^- + d_8^+ + d_9^+ + d_{10}^+ + d_{11}^+ + d_{12}^+ + d_{13}^+ + d_{14}^+)$			
2. **Economic tradeoff** (DOC, WEMP, & PURCH ranked first)	$1/6*(d_2^+ + d_3^+ + d_4^+ + d_9^+ + d_{10}^+ + d_{11}^+)$	$1/8*(d_1^+ + d_5^- + d_6^- + d_7^- + d_8^+ + d_{12}^+ + d_{13}^+ + d_{14}^+)$		
3. **Performance tradeoff** (WFUEL, VCRMX, WFUEL, & LDMAX ranked first)	$1/8*(d_1^+ + d_5^- + d_6^- + d_7^- + d_8^+ + d_{12}^+ + d_{13}^+ + d_{14}^+)$	$1/6*(d_2^+ + d_3^+ + d_4^+ + d_9^+ + d_{10}^+ + d_{11}^+)$		
Group II: Economics				
4. **Economic driver:** (WEMP & PURCH ranked first)	$1/4*(d_2^+ + d_9^+ + d_4^+ + d_{11}^+)$	$1/2*(d_3^+ + d_{10}^+)$	$1/8*(d_1^+ + d_5^- + d_6^- + d_7^- + d_8^+ + d_{12}^+ + d_{13}^+ + d_{14}^+)$	
5. **Economic driver:** (DOC ranked first)	$1/2*(d_3^+ + d_{10}^+)$	$1/4*(d_2^+ + d_9^+ + d_4^+ + d_{11}^+)$	$1/8*(d_1^+ + d_5^- + d_6^- + d_7^- + d_8^+ + d_{12}^+ + d_{13}^+ + d_{14}^+)$	
Group III: Performance				
6. **Performance driver:** (VCRMX & WFUEL ranked first)	$1/4*(d_1^+ + d_6^- + d_8^+ + d_{13}^+)$	$1/4*(d_5^- + d_7^- + d_{12}^+ + d_{14}^+)$	$1/6*(d_2^+ + d_3^+ + d_4^+ + d_9^+ + d_{10}^+ + d_{11}^+)$	
7. **Performance driver:** (LDMAX ranked first)	$1/2*(d_5^- + d_{12}^+)$	$1/2*(d_7^- + d_{14}^+)$	$1/4*(d_1^+ + d_6^- + d_8^+ + d_{13}^+)$	$1/6*(d_2^+ + d_3^+ + d_4^+ + d_9^+ + d_{10}^+ + d_{11}^+)$
8. **Performance driver:** (RANGE ranked first)	$1/2*(d_7^- + d_{14}^+)$	$1/2*(d_5^- + d_{12}^+)$	$1/4*(d_1^+ + d_6^- + d_8^+ + d_{13}^+)$	$1/6*(d_2^+ + d_3^+ + d_4^+ + d_9^+ + d_{10}^+ + d_{11}^+)$

d_1^+ and d_8^+ are for μ_{WFUEL} and σ_{WFUEL}
d_2^+ and d_9^+ are for μ_{WEMP} and σ_{WEMP}
d_3^+ and d_{10}^+ are for μ_{DOC} and σ_{DOC}
d_4^+ and d_{11}^+ are for μ_{PURCH} and σ_{PURCH}
d_5^- and d_{12}^+ are for μ_{LDMAX} and σ_{LDMAX}
d_6^- and d_{13}^+ are for μ_{VCRMX} and σ_{VCRMX}
d_7^- and d_{14}^+ are for μ_{RANGE} and σ_{RANGE}

The eight design scenarios are grouped the same as before with **Group I** being the tradeoff studies (Scenarios 1-3), **Group II** being the economic studies (Scenarios 4-5), and **Group III** being the performance studies (Scenarios 6-8). Refer to Section 3.1.2 for a more detailed description of each scenario.

The deviation function formulations for each scenario are listed in Table 3. For example, in Scenario 2 the economic goals (d_2^+ and d_9^+ for μ_{WEMP} and σ_{WEMP}, d_3^+ and d_{10}^+ for μ_{DOC} and σ_{DOC}, and d_4^+ and d_{11}^+ for μ_{PURCH} and σ_{PURCH}) are equally weighted at the first priority level while the performance goals (d_1^+ and d_8^+ for μ_{WFUEL} and σ_{WFUEL}, d_5^- and d_{12}^+ for μ_{LDMAX} and σ_{LDMAX}, d_6^- and d_{13}^+ for μ_{VCRMX} and σ_{VCRMX} and d_7^- and d_{14}^+ for μ_{RANGE} and σ_{RANGE}) are equally weighted at the second priority level. For a more complicated deviation function formulation, consider Scenario 7 which is a performance study in which the maximum lift to drag ratio (LDMAX) is given the highest priority. In Scenario 7,

- d_5^- and d_{12}^+ are placed at the first priority level, indicating that satisfying the targets for the mean and standard deviation of LDMAX (μ_{LDMAX} and σ_{LDMAX}) is the first priority;
- d_7^- and d_{14}^+ are placed at the second priority level, indicating that satisfying the targets for the mean and standard deviation of RANGE (μ_{RANGE} and σ_{RANGE}) is the second priority provided the targets for μ_{LDMAX} and σ_{LDMAX} have been satisfied;
- d_1^+ and d_8^+ and d_6^- and d_{13}^+ are placed at the third priority level, indicating that satisfying the targets for the mean and standard deviation of WFUEL (μ_{WFUEL} and σ_{WFUEL}) and VCRMX (μ_{VCRMX} and σ_{VCRMX}) is the third priority provided the targets for the first *and* second priority levels have been met; and
- all deviation variables associated with economic considerations (d_2^+ and d_9^+ for μ_{WEMP} and σ_{WEMP}, d_3^+ and d_{10}^+ for μ_{DOC} and σ_{DOC}, and d_4^+ and d_{11}^+ for μ_{PURCH} and σ_{PURCH}) are placed at the fourth priority level which is only considered if the first, second, *and* third priority level targets have been achieved.

In general, the deviation variables d_1 - d_7 are for "bringing the mean on target" whereas the deviation variables d_8 - d_{14} are for "minimizing the variance." In this example, "bringing the mean on target" and "minimizing the variance" for each goal are equally important and placed at the same priority level whereas they could be placed at different levels depending on the preference of the designer.

As we did in Section 3.1.2, the compromise DSP is solved under these eight design scenarios, and the resulting top-level design specifications and system performance means (averages) are plotted in Figures 10 and 11, respectively. All final solutions are feasible; therefore, final constraint values are not included since all of the constraints have been met by all three aircraft.

Step (4) is to determine the ranged set of top-level design specifications for the family of aircraft based on the trends in Figure 10.

- *Cruise speed* = [Mach 0.24-0.34]: jumps to higher values in Scenarios 3, 6-8 when performance is most important; it stays at its lower bound otherwise.
- *Aspect ratio* = [7-8.8]: fluctuates between a low of 7 and a high of 8.8; high values occur in Scenarios 3, 6-8 when performance considerations are most important.
- *Sweep angle* = [6°]: converges at or near its upper bound in seven out of 8 scenarios; fix value at the upper bound.
- *Seat width* = [15.5-19 in.]: fluctuates between 15.5 in. and 19 in. from one scenario to the next.
- *Wing loading* = [21-25 lb/ft^2]: remains steady for Scenarios 1-5 but then fluctuates when different performance considerations are weighted more importantly in Scenarios 6-8.
- *Engine activity factor* = [85-95]: shows no apparent trend, fluctuating between 85 in Scenarios 6 and 7 and 95 in Scenarios 1 and 8.
- *Propeller diameter* = [5.5-5.96 ft]: varies between a low of 5.5 ft and a high of 5.96 ft; values above 5.96 ft violate the takeoff noise constraint.
- *Tail length to diameter ratio* = [3.75]: converges at or near its upper bound in 7 out of 8 scenarios; fix at its upper bound. It would be worthwhile to investigate further sweep angle and ELODT at their lower bounds since this may represent a bifurcation in the design space indicating a second distinct family of aircraft.

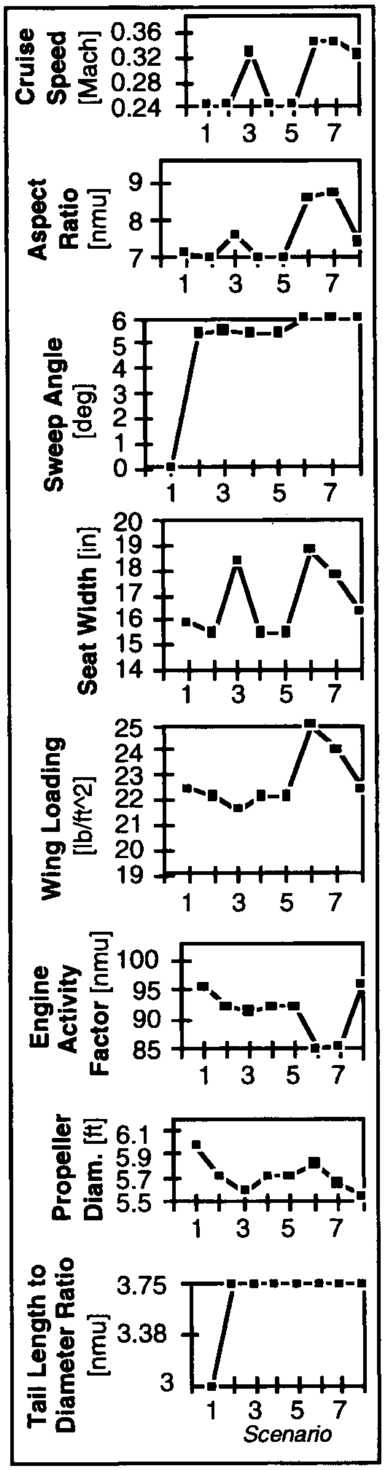

FIGURE 10. System variables using RCEM.

The final achieved means of the system performance responses and their target values (indicated by dashed lines) are plotted in Figure 11 for each scenario. The arrow in each plot indicated on which side of the target we wish to be; the fuel weight, empty weight, direct operating cost, and purchase price means are to be minimized while the maximum lift to drag ratio, maximum cruise speed, and maximum range means are to be maximized. The standard deviations are not included since all of the targets for the standard deviations are met.

The average empty weight achieves its target in all scenarios whereas average fuel weight only reaches its target in Scenario 3. The average purchase price and the average direct operating cost meet their targets in the three scenarios when economics is the first priority (Scenarios 2, 4, and 5); they do not achieve their targets otherwise. The average maximum lift/drag ratio exhibits poor performance except in Scenario 7; in Scenario 7 it achieves its target of 17 only because it is at the first priority level. The mean of maximum cruise speed achieves its target in all but Scenarios 1 and 3, the overall and performance tradeoff studies. The average maximum range reaches its target in 5 out of 8 cases. During the economic studies (Scenarios 2-5) it meets its target, but when other performance factors are ranked higher in Scenarios 6 and 7, the average maximum range does not reach its target.

Now that we have both sets of results, we can compare and contrast them. These comparisons are discussed in detail in the next section.

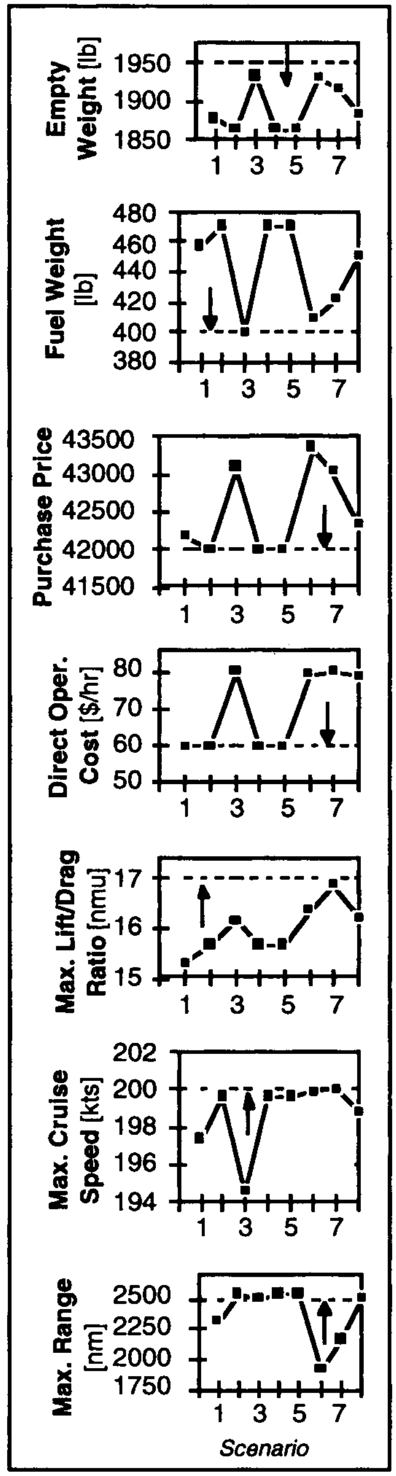

FIGURE 11. System performance means found using RCEM.

4. COMPARING THE RESULTS: BENCHMARK AIRCRAFT VS. RCEM SOLUTIONS

When comparing the two sets of results found in Sections 3.1.2 and 3.2.2, we consider:

- *how do the final ranged sets for the design variables compare with one another?* and
- *how does the system performance for the family of aircraft compare?*

Each of these is addressed in turn in the subsequent sections.

4.1. Comparing the Ranged Sets of Top-Level Design Specifications

The ranged sets of top-level design specifications found in Sections 3.1.2 and 3.2.2 are summarized in Table 4. Notice that similar trends show up in both sets of results, refer to Figures 7 and 8 in Section 3.1.2 and Figures 10 and 11 in Section 3.2.2. Additional observations regarding the two sets of solutions in Table 4 are the following.

TABLE 4. Ranged sets of top-level design specifications.

Top-Level Design Specification	Acronym	*Benchmark Aircraft* from Section 3.1	*RCEM Solutions* from Section 3.2
Cruise Speed	CSPD	Mach 0.24-0.35	Mach 0.24-0.34
Aspect Ratio	AR	7-10	7-8.8
Propeller Diameter	DPROP	5.5-5.96 ft	5.5-5.96 ft
Wing Loading	WL	20-25 lb/ft^2	21-25 lb/ft^2
Sweep Angle	SW	0-6°	6°
Engine Activity Factor	AF	85-105	85-95
Seat Width	WS	14-20 in	15.5-19 in
Tail Length/Diameter	ELODT	3.6-3.75	3.75

- For all eight design variables, there is considerable overlap between each specified range, e.g., cruise speed (CSPD) ranges from Mach 0.24-0.35 for the benchmark aircraft and ranges from Mach 0.24-0.34 for the RCEM solutions, and aspect ratio (AR) ranges from 7-10 and 7-8.8 in the two sets of top-level design specifications.
- The specified ranges for propeller diameter (DPROP), wing loading (WL), and seat width (WS) are identical for both sets.
- For the aircraft designed individually, sweep angle (SW) and tail length/diameter (ELODT) remain specified by a range because the results varied so much from one scenario to the next and from one aircraft to the next. When using the RCEM, the values for SW and ELODT are essentially constant from one scenario to the next and are

thus fixed to point values. The point values found using the RCEM are contained within the ranges of the benchmark designs.

Based on these observations regarding the data in Table 4, the ranged sets of top-level design specifications found using the different approaches are essentially the same. Whether we individually design all three aircraft and look for commonalties or whether we design all three aircraft simultaneously using the RCEM, the design specifications turn out to be essentially the same. However, a point worth noting the fact that by invoking the RCEM, we have been able to design the family of aircraft in about 1/3 the time it took to design all three aircraft individually. By incorporating the number of passengers as a noise factor in the design process and using the RCEM to find a ranged set of top-level design specifications which is robust with regards to this noise, *we have been able to design a family of products as efficiently as a single product.*

4.2. Comparing System Performance

4.2.1. Comparison of individual aircraft performances. Having found that the ranged sets of top-level design specifications found in Sections 3.1 and 3.2 are quite similar, in this section we examine the extent to which individual system performance is (or is not) sacrificed when designing all three aircraft simultaneously as opposed to individually. For the sake of comparison, we consider solutions from three different starting points where all goals are weighted equally (these results correspond to those found in Scenario 1, see Tables 2 and 3.) The corresponding system responses are listed in Table 5, and some observations follow.

- In general, the empty weight, direct operating cost, maximum lift to drag ratio, and maximum cruise speed characteristics for the benchmark aircraft and the RCEM solutions are equivalent.
- The RCEM solutions have slightly lower purchase prices and slightly higher maximum cruise speeds; however, the benchmark aircraft perform better technically than the RCEM solutions; they have slightly higher maximum lift to drag ratios, longer flight ranges using less fuel.
- Although the target value of 2500 nautical miles is not met by either family of aircraft, the benchmark aircraft can fly 100 - 150 nm farther (with less fuel) than the solutions found using the RCEM. This is a substantial loss that occurs when using the RCEM to design all three aircraft simultaneously in this example.
- The final deviation function values of the benchmark aircraft from Section 3.1 are much better than the deviation function values for the RCEM solutions from Section 3.2. For the 2 seater GAA for instance,

the deviation function of the baseline model is approximately 5 times better than the 2 seater GAA found using the RCEM.

TABLE 5. Comparison of system responses.

Run #	Goal	*Benchmark Aircraft* from Section 3.1			*RCEM Solutions* from Section 3.2		
		2 Seater	4 Seater	6 Seater	2 Seater	4 Seater	6 Seater
1. upper bounds	fuel wt, lb	442.8	400.1	349.1	483.9	460.4	441.6
	empty wt, lb	1892.1	1931.7	1983.0	1847.7	1870.5	1889.8
	DOC, $	60.0	60.0	61.6	59.2	60.0	60.4
	purchase, $	42418.8	42972.6	43952.1	41767.0	42116.0	42518.0
	max. lift/drag	15.9	15.3	15.5	15.4	15.2	15.2
	max. speed, kts	197.6	193.1	191.6	199.4	197.5	197.6
	max. range, nm	2473.2	2473.3	2367.4	2318.3	2281.0	2261.9
	DEV FCN, Z	0.0178	0.0245	0.0334	0.0788	0.0747	0.0685
2. mid-points	fuel wt, lb	450.1	401.9	350.0	487.2	464.0	445.5
	empty wt, lb	1885.4	1929.9	1982.0	1845.5	1868.0	1887.0
	DOC, $	60.0	61.2	61.6	58.1	59.0	59.3
	purchase, $	42310.2	43046.0	43830.9	41665.0	42006.0	42401.0
	max. lift/drag	16.0	15.5	15.4	15.5	15.3	15.3
	max. speed, kts	198.6	194.2	190.5	199.4	197.5	197.6
	max. range, nm	2492.2	2432.8	2439.1	2370.0	2331.7	2311.5
	DEV FCN, Z	0.0148	0.0278	0.0304	0.0766	0.0725	0.0657
3. lower bounds	fuel wt, lb	449.7	400.2	350.1	477.9	454.4	435.6
	empty wt, lb	1885.5	1931.4	1983.5	1854.4	1877.2	1896.5
	DOC, $	60.2	60.2	61.4	59.1	60.0	60.4
	purchase, $	42322.1	42940.8	43956.8	41806.0	42154.0	42556.0
	max. lift/drag	16.0	15.2	15.6	15.4	15.2	15.2
	max. speed, kts	198.5	192.7	192.1	198.6	196.7	196.8
	max. range, nm	2494.9	2496.4	2359.0	2340.8	2301.7	2280.8
	DEV FCN, Z	0.0151	0.0240	0.0319	0.0757	0.0715	0.0654

Based on these observations, it appears that individual aircraft performance suffers in many instances having designed all three aircraft in the family simultaneously using the RCEM. *How is overall system performance affected?*

4.2.2. Comparison of overall system performance for the families of aircraft. To compare the overall system performance of the families of aircraft consider the results listed in Table 6. In Table 6, we list the means and standard deviations for each of the system responses for solutions from three different starting points and for all goals weighted equally (again, these are results from Scenario 1 in Tables 2 and 3). The mean for a given response is simply the average of the values of that response for each of the three aircraft, and the standard deviation is simply the spread of that response divided by $\pm 3\sigma$, refer to Eqns. 4 and 5 for examples of each of these calculations. The deviation function is computed as the sum of the normalized difference between the achieved response and its target value, the difference is zero if the target has been achieved.

TABLE 6. Comparison of overall system performance.

Run #	Goal	*Benchmark Aircraft from Section 3.1.2*		*RCEM Solutions from Section 3.2.2*		*Compromise DSP Goal Targets*	
		Mean	**Deviation**	**Mean**	**Deviation**	**Mean**	**Deviation**
1. high	fuel wt	397.3	53.4	462.0	7.1	400.0	16.67
	empty wt	1935.6	51.6	1869.4	7.0	1950.0	16.67
	DOC $	60.5	1.0	59.9	0.2	60.0	1.67
	purchase $	43114.5	859.1	42134.0	125.1	42000.0	333.3
	max. l/d	15.5	0.4	15.3	0.0	17.0	0.33
	max. speed	194.1	3.9	198.2	0.3	200.0	3.33
	max. range	2438.0	64.4	2287.1	9.4	2500.0	75.0
	DEV FCN, Z =		**0.467**		**0.0254**		
2. mid	fuel wt	400.6	57.4	465.6	7.0	400.0	16.67
	empty wt	1932.4	39.4	1866.8	6.9	1950.0	16.67
	DOC $	60.9	0.7	58.8	0.2	60.0	1.67
	purchase $	43062.4	620.9	42024.0	122.7	42000.0	333.3
	max. l/d	15.6	0.3	15.4	0.0	17.0	0.33
	max. speed	194.4	3.3	198.2	0.3	200.0	3.33
	max. range	2454.7	26.6	2337.7	9.8	2500.0	75.0
	DEV FCN, Z =		**0.467**		**0.02399**		
3. low	fuel wt	400.0	57.5	456.0	7.1	400.0	16.67
	empty wt	1933.5	56.0	1876.0	7.0	1950.0	16.67
	DOC $	60.6	0.8	59.8	0.2	60.0	1.67
	purchase $	43073.2	917.6	42172.0	125.0	42000.0	333.3
	max. l/d	15.6	0.5	15.3	0.0	17.0	0.33
	max. speed	194.4	4.7	197.4	0.3	200.0	3.33
	max. range	2450.1	83.1	2307.8	10.0	2500.0	75.0
	DEV FCN, Z =		**0.467**		**0.02397**		

Based on the data in Table 6, we see that the family of aircraft designed simultaneously using the RCEM has a lower deviation function for each starting point than do the three benchmark aircraft taken as a group. Since the deviation function measures how well the targets are achieved by the system, the overall performance of the RCEM solutions is better than that of the three benchmark aircraft when taken as a group. Furthermore, the family of aircraft found using the RCEM meet all of the targets for the "minimize the deviation" goals, meaning that there is less variation between each aircraft. This is as expected because we used robust design principles to minimize the effects of noise (i.e., the number of passengers) on the system and develop a ranged set of top-level design specifications which is "common and good" for all three aircraft. Not only have we saved time by designing all three aircraft simultaneously, but we have also achieved better *overall* system performance (while sacrificing individual performance to some extent).

5. CLOSURE

In the GAA example problem in Section 3, we have demonstrated how to *design a family of products around a common noise factor as efficiently as a single product* through the incorporation of robust design principles in the RCEM. In our example problem, we have been able to design the entire family of aircraft using the RCEM in 1/3 the time it would take to design all three aircraft individually. Within the RCEM, implementation of robust design is facilitated through the use of second-order response surface models which allow us to approximate system performance means and standard deviations for modeling the robust design objectives of "bringing the mean on target" and "minimizing the deviation" for a family of products. By specifying a ranged set of top-level design specifications which are common and good for a variety of design scenarios, we have created a robust and flexible family of designs which can be quickly adapted to meet changing customer demands. Thus, *the RCEM enables us to improve system flexibility and provides a means for designing families of products.*

In the situation where a family of products can be designed around a common noise factor(s), the RCEM provides a suitable starting point for *developing a ranged set of top-level design specifications for a family of products*. Although not shown here, the RCEM could also be used to design a family of products around variable control factors as well.[2] The RCEM has also been used to integrate product and process design[15] and design an advanced short take-off and vertical landing lift engine.[8] We have also investigated alternate robust design approaches for use in the RCEM such as design capability indices[4] which can be used to measure the capability of a family of designs to satisfy a *ranged* set of design requirements.

This work represents only an initial step toward developing a foundation for designing families of products; there is still much to be done. Additional questions worth investigating include: *what constitutes a product family (in a more general sense) and what are the characteristics of products which all us to consider it a family?, how can a family of products be mathematically modeled, analyzed, and synthesized?*, and *how can we evaluate alternative families of products, i.e., what are suitable metrics for designing good product families?* We invite the reader to contemplate these and other questions that our work may have inspired.

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