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Reliability-based design optimization for crashworthiness of vehicle side impact

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Abstract With the advent of powerful computers, vehicle safety issues have recently been addressed using computational methods of vehicle crashworthiness, resulting in reductions in cost and time for new vehicle development. Vehicle design demands multidisciplinary optimization coupled with a computational crashworthiness analysis. However, simulation-based optimization generates deterministic optimum designs, which are frequently pushed to the limits of design constraint boundaries, leaving little or no room for tolerances (uncertainty) in modeling, simulation uncertainties, and/or manufacturing imperfections. Consequently, deterministic optimum designs that are obtained without consideration of uncertainty may result in unreliable designs, indicating the need for Reliability-Based Design Optimization (RBDO).

Recent development in RBDO allows evaluations of probabilistic constraints in two alternative ways: using the Reliability Index Approach (RIA) and the Performance Measure Approach (PMA). The PMA using the Hybrid Mean Value (HMV) method is shown to be robust and efficient in the RBDO process, whereas RIA yields instability for some problems. This paper presents an application of PMA and HMV for RBDO for the crashworthiness of a large-scale vehicle side impact. It is shown that the proposed RBDO approach is very effective in obtaining a reliability-based optimum design.

Key words uncertainty, reliability-based optimization, crashworthiness, performance measure approach, hybrid mean value method

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Nomenclature

$\mathbf{X}$	Random parameter; $\mathbf{X}=[X_1,X_2,...,X_n]^T$
$\mathbf{x}$	Realization of $\mathbf{X}$; $\mathbf{x}=[x_1,x_2,...,x_n]^T$
$\mathbf{U}$	Independent and standard normal random parameter; $\mathbf{U}=[U_1,U_2,...,U_n]^T$
$\mathbf{u}$	Realization of $\mathbf{U}$; $\mathbf{u}=[u_1,u_2,...,u_n]^T$
$\boldsymbol{\mu}$	Mean of random parameter $\mathbf{X}$; $\mu=[\mu_1,\mu_2,...,\mu_n]^T$
$\mathbf{d}$	Design parameter; $\mathbf{d}=[d_1,d_2,...,d_n]^T$
$\mathbf{d}^L,\mathbf{d}^U$	Lower and upper bounds of design parameter $\mathbf{d}$
$P(\bullet)$	Probability function
$f_{\mathbf{X}}(\mathbf{x})$	Joint Probability Density Function (JPDF) of the random parameter
$\Phi(\bullet)$	Standard normal Cumulative Distribution Function (CDF) $\mathbf{\Phi}(\bullet)$
$F_G(\bullet)$	CDF of the performance function $G(\mathrm{X})$; $F_G(g)=\mathrm{P}(G(\mathbf{X})<g)$
β_s	Safety reliability index; $\beta_s=-\mathbf{\Phi}^{-1}(P_f)=-\mathbf{\Phi}^{-1}\,(F_G(g))$
β_t	Target reliability index
$G(\mathbf{X})$	Performance function; the design is considered “fail” if $G(\mathrm{X})<0$
G_p^{RIA}	Probabilistic constraint in the Reliability Index Approach (RIA)
G_p^{PMA}	Probabilistic constraint in the Performance Measure Approach (PMA)
$\mathbf{u}^*_{\beta=\beta_t}$	Most Probable Point (MPP) in first-order inverse reliability analysis
$\mathbf{u}^*_{\mathrm{(A)MV}}$	MPP using (advanced) mean value method in PMA
$\mathbf{u}^*_{\mathrm{CMV}}$	MPP using conjugate mean value method in PMA
$\mathbf{u}^*_{\mathrm{HMV}}$	MPP using hybrid mean value method in PMA
$\mathbf{n}$	Normalized steepest descent direction of performance function
ζ	Criteria for the type of performance function $G(\mathbf{X})$
DSA	Design sensitivity analysis

1 Introduction

The computational analysis of crashworthiness for vehicle impact has become a powerful and efficient tool in reducing the cost and development time for a new product that meets corporate and government crash safety requirements. Today, nonlinear finite element (FE)-based crash analysis codes successfully simulate many laboratory vehicle crash events, and are able to design vehicles that meet safety guidelines for frontal impact, side impact, roof crush, interior head impact, rear impact, and rollover.

However, due to the increasingly competitive global market, optimum designs are pushed to the limits of system failure boundaries using multidisciplinary design optimization techniques, and leave very little or no room for tolerances in modeling, simulation uncertainties, and/or manufacturing imperfections. Consequently, deterministic optimum designs obtained without any consideration of these uncertainties could lead to unreliable designs. Given the increased computational capabilities that have been developed recently, the existence of uncertainty can be addressed in Reliability-Based Design Optimization (RBDO).

In the RBDO model, a probabilistic constraint can be identified in two different approaches: the Reliability Index Approach (RIA) and the Performance Measure Approach (PMA). During the initial efforts in RBDO development, RIA was implemented by defining a probabilistic constraint as a reliability index (Enevoldsen and Sorensen 1994; Yu *et al.* 1997). However, it was found that RIA converges very slowly, or even fails to converge, for a number of problems having largely inactive or violate constraints. To alleviate this difficulty in RBDO, PMA has been introduced by solving an inverse problem in the First-Order Reliability Method (FORM) (Tu and Choi 1999; Choi and Youn 2001). It was demonstrated that PMA is inherently robust and more effective, since it is easier to minimize a complicated cost function subject to a simple spherical constraint with a known distance (in other words, the reliability index) than to minimize a simple cost function subject to a complicated constraint (Choi and Youn 2001; Youn *et al.* 2002). Subsequently, a robust and efficient Hybrid Mean-Value (HMV) method has been developed as a numerical solution to the inverse PMA problem (Choi and Youn 2001; Youn *et al.* 2003). It has been shown that for PMA the HMV method is very efficient and robust, which adaptively utilizes the Conjugate Mean-Value (CMV) method (Choi and Youn 2001; Youn *et al.* 2003) or the Advanced Mean-Value (AMV) method (Wu *et al.* 1990; Wu 1994) in probabilistic constraint assessment during the RBDO process.

Crash simulation gives crash-based safety design engineers the opportunity to explore many more alternative designs than they could with hardware. A feasibility study of vehicle safety CAE optimization has been done (Yang *et al.* 1994). Based on the optimal Latin Hyper Cube sampling (LHS), Sudjianto *et al.* demonstrated an efficient computational procedure to emulate a computationally intensive CAE model by using Multivariate Adaptive Regression Splines (MARS) (Sudjianto *et al.* 1998). Stander investigated the crashworthiness problem using both the response surface method and massively parallel programming (Stander 1999). Approximate methods for safety optimization of large systems were compared in terms of accuracy and effectiveness (Yang *et al.* 2000). RBDO for vehicle crashworthiness was attempted using the Monte Carlo Simulation (MCS) method on a global response surface (Yang *et al.* 2000; Gu *et al.* 2001).

This paper presents RBDO using PMA for vehicle crashworthiness, dealing with vehicle safety-rating scores related to human safety issues. Since crash analysis is computationally intensive, a global response surface generated using Stepwise Regression (SR) (Myers and Montgomery 1995), coupled with the optimal LHS Design of Experiment (DOE) (Currin *et al.* 1991), is treated as the original problem for RBDO. The rationale for this approach is that the global response surface may have the characteristic of the design dependency and the main purpose is to show the proposed RBDO methodology for a large-scale problem.

2 Review of reliability-based design optimization

2.1 General definition of RBDO model

In system parameter design, the RBDO model (Enevoldsen and Sorensen 1994; Yu *et al.* 1997; Tu and Choi 1999; Choi and Youn 2001; Youn *et al.* 2003) can generally be defined as

$$
\begin{aligned}
&\text{minimize} && \mathrm{Cost}(\mathbf{d}) \\
&\text{subject to} && P(G_i(\mathbf{X} \le 0) - \varPhi(-\beta_t) \le 0\,, \\
& && i = 1, 2, \ldots, np \\
& && \mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \mathbf{d} \in R^{ndv} \text{ and } \mathbf{X} \in R^{nrv}
\end{aligned} \tag{1}
$$

where $\mathbf{d} = \boldsymbol{\mu}(\mathbf{X})$ is the design vector, which is the mean value of $\mathbf{X}$; $\mathbf{X}$ is the random vector; and the probabilistic constraints are described by the performance function $G_i(\mathbf{X})$, their probability distributions, and their prescribed reliability target β_t.

The statistical description of the failure of the performance function $G_i(\mathbf{X})$ is characterized by the Cumulative Distribution Function (CDF) $F_{G_i}(0)$ as

$$
P(G_i(\mathbf{X}) \le 0) = F_{G_i}(0) \le \varPhi(-\beta_t) \tag{2}
$$

where the CDF is described as

$$F_{G_i}(0) = \int\limits_{G_i(\mathbf{x})\leq 0} \cdots \int f_{\mathbf{X}}(\mathbf{x})\,\mathrm{d}\mathbf{x} \qquad (3)$$

In (3), $f_{\mathbf{X}}(\mathbf{x})$ is the Joint Probability Density Function (JPDF) of all random parameters. The evaluation of probabilistic constraints in (2) requires a reliability analysis where multiple integration is involved, as shown in (3). Some approximate probability integration methods have been developed to provide efficient solutions, such as the First-Order Reliability Method (FORM) or the asymptotic Second-Order Reliability Method (SORM), which has a rotationally invariant measure as the reliability (Madsen *et al.* 1986; Palle and Michael 1982). FORM often provides adequate accuracy and is widely used for RBDO applications. In FORM, reliability analysis requires transformation $\mathbf{T}$ (Rackwitz and Fiessler 1978; Hohenbichler and Rackwitz 1981) from the original random parameter $\mathbf{X}$ to the independent and standard normal random parameter $\mathbf{U}$. The performance function $G(\mathbf{X})$ in X-space can then be mapped onto $G(\mathbf{T}(\mathbf{X})) \equiv G(\mathbf{U})$ in U-space.

As described in Sect. 1, the probabilistic constraint in (2) can be further expressed in two ways through inverse transformations as (Tu and Choi 1999; Choi and Youn 2001; Youn *et al.* 2003)

$$G_{pi}^{\mathrm{RIA}} = \beta_{s_i} - \beta_t = -\Phi^{-1}(F_{G_i}(0)) - \beta_t \geq 0 \qquad (4)$$

$$G_{pi}^{\mathrm{PMA}} = F_{Gi}^{-1}(\Phi(-\beta t)) \geq 0 \qquad (5)$$

where $G_{p_i}^{\mathrm{RIA}}$ with the safety reliability index β_{S_i} and $G_{p_i}^{\mathrm{RIA}}$ with the probabilistic performance measure G_{p_i} are called the i^{th} probabilistic constraints in the Reliability Index Approach (RIA) and Performance Measure Approach (PMA), respectively. Equation (4) is employed to describe the probabilistic constraint in (1) using the reliability index; in other words RIA. However, RIA either converges slowly or fails to converge for problems having largely inactive or violate constraints (Choi and Youn 2001; Youn *et al.* 2003). Equation (5) can replace the probabilistic constraint in (1) with the performance measure, referred to as PMA.

2.2 First-order reliability analysis in PMA

Reliability analysis in PMA can be formulated as the inverse of reliability analysis in RIA. The first-order probabilistic performance measure $G_{p,\mathrm{FORM}}$ is obtained from a nonlinear optimization problem (Tu and Choi 1999; Choi and Youn 2001) in U-space, defined as

$$\begin{aligned} &\text{minimize} \quad G(\mathbf{U}) \\ &\text{subject to} \quad \|\mathbf{U}\| = \beta_t \end{aligned} \qquad (6)$$

where the optimum point on the target reliability surface is identified as the Most Probable Point (MPP) $\mathbf{u}^*_{\beta=\beta_t}$ with a prescribed reliability $\beta_t = \|\mathbf{u}^*_{\beta=\beta_t}\|$, which is denoted as MPP in the paper. Unlike RIA, only the direction vector $\mathbf{u}^*_{\beta=\beta_t}/\|\mathbf{u}^*_{\beta=\beta_t}\|$ needs to be determined by exploring the spherical equality constraint $\|\mathbf{U}\| = \beta_t$. General optimization algorithms can be employed to solve the optimization problem in (6). However, the HMV method is shown to be well suited for PMA due to its robustness and efficiency (Choi and Youn 2001; Youn *et al.* 2003).

2.3 Reliability analysis tools for PMA

As described in Sect. 1, the HMV method, which selectively combines the AMV and CMV methods, is adopted to solve the inverse problem of PMA. This section introduces the HMV method, by combining the AMV and CMV methods.

2.3.1 Advanced Mean-Value (AMV) method

Formulation of the first-order AMV method begins with the Mean Value (MV) method, defined as

$$\mathbf{u}^*_{\mathrm{MV}} = \beta_t \mathbf{n}(\mathbf{0})$$

$$\text{where} \quad \mathbf{n}(\mathbf{0}) = -\frac{\nabla_{\mathrm{X}} G(\boldsymbol{\mu})}{\|\nabla_{\mathrm{X}} G(\boldsymbol{\mu})\|} = -\frac{\nabla_U G(\mathbf{0})}{\|\nabla_U G(\mathbf{0})\|} \qquad (7)$$

That is, to minimize the performance function $G(\mathbf{U})$ (the cost function in (6)), the normalized steepest descent direction $\mathbf{n}(\boldsymbol{\mu})$ is defined at the mean-value point. The AMV method iteratively updates the direction vector of the steepest descent direction at the probable point $\mathbf{u}^{(k)}_{\mathrm{AMV}}$ initially obtained using the MV method. The AMV method can be formulated as

$$\mathbf{u}^{(0)}_{\mathrm{AMV}} = \mathbf{0}\,,\ \mathbf{u}^{1}_{\mathrm{AMV}} = \mathbf{u}^*_{\mathrm{MV}},\ \mathbf{u}^{k+1}_{\mathrm{AMV}} = \beta_t \mathbf{n}\left(\mathbf{u}^{(k)}_{\mathrm{AMV}}\right) \qquad (8)$$

where

$$\mathbf{n}\left(\mathbf{u}^{(k)}_{\mathrm{AMV}}\right) = -\frac{\nabla_U G\left(\mathbf{u}^{(k)}_{\mathrm{AMV}}\right)}{\left\|\nabla_U G\left(\mathbf{u}^{(k)}_{\mathrm{AMV}}\right)\right\|} \qquad (9)$$

As has been shown (Choi and Youn 2001; Youn *et al.* 2003), this method exhibits instability and inefficiency in solving a concave function since it only updates the direction using the current MPP.

2.3.2 Conjugate Mean-Value (CMV) method

When applied to a concave function, the AMV method tends to be slow in the rate of convergence and/or divergence due to the lack of updated information during iterative reliability analyses. This kind of difficulty can be overcome by using both the current and the previous MPP information to develop the proposed Conjugate Mean Value (CMV) method (Choi and Youn 2001). The new search direction is obtained by combining $\mathbf{n}(\mathbf{u}_{\mathrm{CMV}}^{(k-2)})$, $\mathbf{n}(\mathbf{u}_{\mathrm{CMV}}^{(k-1)})$, and $\mathbf{n}(\mathbf{u}_{\mathrm{CMV}}^{(k)})$ with an equal weight, such that it is directed towards the diagonal of the three consecutive steepest descent directions. That is,

$$\mathbf{u}_{\mathrm{CMV}}^{(0)} = \mathbf{0}\,,\ \mathbf{u}_{\mathrm{CMV}}^{1} = \mathbf{u}_{\mathrm{AMV}}^{(1)}\,,\ \mathbf{u}_{\mathrm{CMV}}^{(2)} = \mathbf{u}_{\mathrm{AMV}}^{(2)}$$

$$\mathbf{u}_{\mathrm{CMV}}^{(k+1)} = \beta_t \frac{\mathbf{n}\left(\mathbf{u}_{\mathrm{CMV}}^{(k)}\right) + \mathbf{n}\left(\mathbf{u}_{\mathrm{CMV}}^{(k-1)}\right) + \mathbf{n}\left(\mathbf{u}_{\mathrm{CMV}}^{(k-2)}\right)}{\left\|\mathbf{n}\left(\mathbf{u}_{\mathrm{CMV}}^{(k)}\right) + \mathbf{n}\left(\mathbf{u}_{\mathrm{CMV}}^{(k-1)}\right) + \mathbf{n}\left(\mathbf{u}_{\mathrm{CMV}}^{(k-2)}\right)\right\|}$$

$$\text{for } k \geq 2 \tag{10}$$

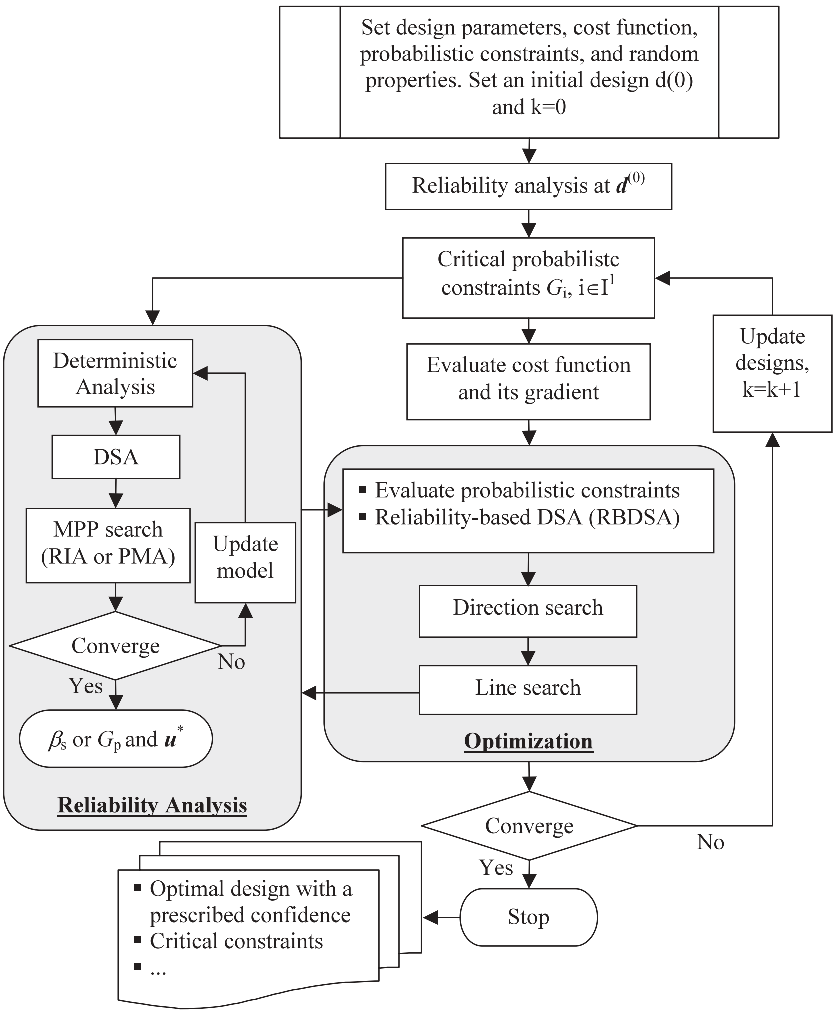
where

$$\mathbf{n}\left(\mathbf{u}_{\mathrm{CMV}}^{(k)}\right) = -\frac{\nabla_U G\left(\mathbf{u}_{\mathrm{CMV}}^{(k)}\right)}{\left\|\nabla_U G\left(\mathbf{u}_{\mathrm{CMV}}^{(k)}\right)\right\|} \tag{11}$$

Consequently, the conjugate steepest descent direction significantly improves the rate of convergence as well as the stability compared to the AMV method for the concave performance function. However, the CMV method is shown to be inefficient for the convex function, even though the method converges for the concave performance function.

2.3.3
Hybrid Mean-Value (HMV) method

To develop an appropriate MPP search method, the type of the performance function must first be identified. The function type can be determined by employing the steep-



[1] I : Index set of ε-active and violate constraints

Fig. 1 Flow chart for RBDO numerical process est descent direction at three consecutive iterations, as follows:

$$\zeta^{(k+1)} = \left(\mathbf{n}^{(k+1)} - \mathbf{n}^{(k)}\right) \cdot \left(\mathbf{n}^{(k)} - \mathbf{n}^{(k-1)}\right)$$

$$\mathrm{sign}\left(\zeta^{(k+1)}\right) > 0 : \textit{Convex type at } \mathbf{u}_{\mathrm{HMV}}^{(k+1)} \textit{ w.r.t. design } \mathbf{d}$$

$$\leq 0 : \textit{Concave type at } \mathbf{u}_{\mathrm{HMV}}^{(k+1)} \textit{ w.r.t. design } \mathbf{d} \quad (12)$$

where $\zeta^{(k+1)}$ is the criterion for the performance function type at the $(k+1)^{th}$ step and $\mathbf{n}^{(k)}$ is the steepest descent direction for a performance function at MPP $\mathbf{u}_{\mathrm{HMV}}^{(k)}$ at the k^{th} iteration. Once the performance function type is identified, one of two numerical algorithms, either AMV or CMV, is adaptively selected for the MPP search. This MPP search method is called the Hybrid Mean Value (HMV) method (Choi and Youn 2001). The HMV method needs to be further studied for non-convex (or non-concave) response, such as a response with an inflection point in the n-dimensional space, as it works very well with an inflection point in the 2-dimensional space (Choi and Youn 2001; Youn *et al.* 2003).

2.4 Numerical procedure in RBDO model

Based on the aforementioned findings, complete numerical procedures of the RBDO model have been presented in connection with the probabilistic constraint. The RBDO numerical procedure is illustrated in detail in Fig. 1.

The reliability analysis in the left shaded box of Fig. 1 that calculates the probabilistic constraint as well as its sensitivity is called a sub-optimization loop. As explained, PMA entails the HMV method for an effective reliability analysis. This procedure is continually employed during an RBDO iteration, and is analogous to the evaluation of a deterministic constraint in deterministic optimizations. The primary optimization procedure shown in the center shaded box of Fig. 1 carries out the design optimization subject to specified probabilistic constraints. In the main optimization loop, a set of active probabilistic constraints must be identified prior to the sub-optimization loop, so that a computation burden can be relieved during an entire RBDO process. The main difference between RBDO and deterministic design optimization can be found in the constraint evaluation. The deterministic constraint and its sensitivity are evaluated at the design point during a deterministic design optimization process. In RBDO, the probabilistic constraint and its sensitivity are computed at the MPP on the failure surface in RIA (Choi and Youn 2001; Youn *et al.* 2003), or on the prescribed reliability sphere in PMA. In comparison with the evaluation of the deterministic constraint in conventional deterministic design optimization, computation of the probabilistic constraint is much more expensive, since every probabilistic constraint evaluation requires several deterministic analyses.

3 Model description for crashworthiness of side impact

3.1 Model description for crashworthiness of vehicle side impact

A large-scale application of the proposed RBDO methodology in the design of vehicle side impact is illustrated in Fig. 2. The system model includes a full-vehicle FE structural model, an FE side impact dummy model, and an FE deformable side impact barrier model. The system model consists of 85 941 shell elements and 96 122 nodes. In the FE simulation of the side impact event, the barrier has an initial velocity of 49.89 kph (31 mph) and impacts the vehicle structure. The CPU time for one nonlinear FE simulation using the RADIOSS software is approximately 20 hours on an SGI Origin 2000. The design objective is to enhance side impact crash performance while minimizing the vehicle weight.

3.2 Safety regulation of vehicle side impact

For side impact protection, the vehicle design must meet internal and regulated side impact requirements specified by the vehicle market. Two primary side impact protection guidelines are the National Highway Traffic Safety Administration (NHTSA) side impact procedure for the Federal Motor Vehicle Safety Standard (FMVSS) and Canadian Motor Vehicle Safety Standard (CMVSS), and the European Enhanced Vehicle-Safety Committee (EEVC) side impact procedure (for European Vehicles). In this study, the EEVC side impact test configuration was used. The dummy's responses are the main metric

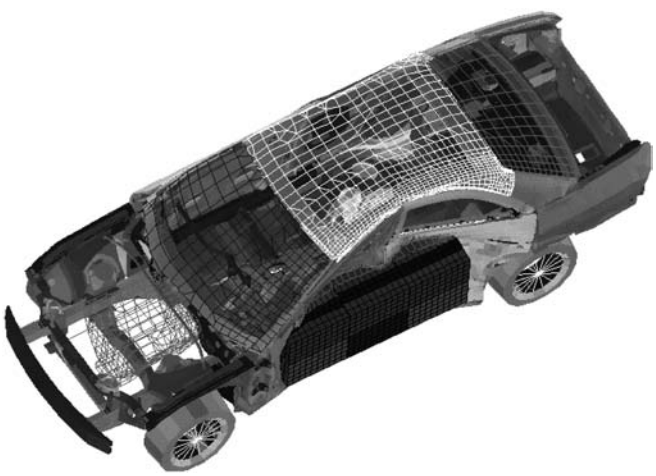


Fig. 2 Vehicle side impact model

Table 1 Regulations and requirements for vehicle side impact

Performance		EEVC regulation	Top safety rating criteria	Initial design
Abdomen Load (kN)		2.5	≤ 1.0	0.663
Rib Deflection	Upper	42	≤ 22	28.5
	Middle			29.0
	Lower			34.0
VC (m/s)	Upper	1.0	≤ 0.32	0.22
	Middle			0.21
	Lower			0.31
Pubic symphysis force (kN)		6	≤ 3.0	4.067
HIC		1000	≤ 650	229.4

in side impact studies, which includes Head Injury Criterion (HIC), abdomen load, pubic symphysis force, VC's (viscous criteria), and rib deflections (upper, middle, and lower). The dummy's responses must meet EEVC criteria, as described in Table 1. Other concerns in side impact design are the velocity of the B-pillar at the middle point and the velocity of the front door at the B-pillar.

In this research, the optimization task is to maintain or improve the dummy safety performance as measured by the side impact top safety-rating criteria shown in Table 1, while reducing the weight. The safety-rating score consists of four measurements of the dummy: HIC, abdomen load, rib deflection or VC, and pubic symphysis force. Top safety-rating for each of these items is 4 points with a sliding scale. A maximum of 16 points may be obtained for the vehicle side impact.

3.3 Response surface methodology for vehicle side impact

Due to the expense involved in crash analysis, the response surface method (RSM) is employed to demonstrate the proposed RBDO methodology for vehicle crashworthiness. For this purpose, optimal Latin Hypercube Sampling (LHS) (Myers and Montgomery 1995) is employed to generate a global surrogate model, coupled with a quadratic backward-Stepwise Regression (SR) method (Madsen *et al.* 1986). The optimal LHS design is directed based on a non-negative entropy criterion to minimize the bias part of the mean square error by distributing the sample points uniformly over the entire design region, where the non-negative entropy criterion is $H(X) = E[-\ln f_X(x)]$. The lower the entropy, the more precise the data will be on the approximate response surface. Moreover, to construct a reasonably accurate response surface with less computational effort, the optimal LHS demands $3N$ sampling points, with N as the total number of system variables including both design and random parameters. The quadratic backward-SR begins with a model that includes all quadratic candidate regressors. By deleting trivial regressors one at a time, the quadratic backward-SR develops a stepwise final regression model, which only contains sets of regressors that have large effects on the response. The quadratic backward-SR method with the optimal LHS generates global response surfaces for an objective function and constraints, and can be summarized as follows (Gu *et al.* 2001):

$$\mathrm{Cost(weight)} = 1.98 + 4.90x_1 + 6.67x_2 + 6.98x_3 + 4.01x_4 + 1.78x_5 + 2.73x_7$$

$$\mathrm{Load}_{\mathrm{Abdomen}} = 1.16 - 0.3717x_2x_4 - 0.00931x_2x_{10} - 0.484x_3x_9 + 0.01343x_6x_{10}$$

$$\mathrm{Deflection}_{\mathrm{rib\_u}} = 28.98 + 3.818x_3 - 4.2x_1x_2 + 0.0207x_5x_{10} + 6.63x_6x_9 - 7.7x_7x_8 + 0.32x_9x_{10}$$

$$\mathrm{Deflection}_{\mathrm{rib\_m}} = 33.86 + 2.95x_3 + 0.1792x_{10} - 5.057x_1x_2 - 11.0x_2x_8 - 0.0215x_5x_{10} - 9.98x_7x_8 + 22.0x_8x_9$$

$$\mathrm{Deflection}_{\mathrm{rib\_1}} = 46.36 - 9.9x_2 - 12.9x_1x_8 + 0.1107x_3x_{10}$$

$$\mathrm{VC}_{\mathrm{upper}} = 0.261 - 0.0159x_1x_2 - 0.188x_1x_8 - 0.019x_2x_7 + 0.0144x_3x_5 + 0.0008757x_5x_{10} + 0.08045x_6x_9 + 0.00139x_8x_{11} + 0.00001575x_{10}x_{11}$$

$$\mathrm{VC}_{\mathrm{middle}} = 0.214 + 0.00817x_5 - 0.131x_1x_8 - 0.0704x_1x_9 + 0.03099x_2x_6 - 0.018x_2x_7 + 0.0208x_3x_8 + 0.121x_3x_9 - 0.00364x_5x_6 + 0.0007715x_5x_{10} - 0.0005354x_6x_{10} + 0.00121x_8x_{11}$$

$$\mathrm{VC}_{\mathrm{lower}} = 0.74 - 0.061x_2 - 0.163x_3x_8 + 0.001232x_3x_{10} - 0.166x_7x_9 + 0.227x_2^2$$

$$\mathrm{Force}_{\mathrm{public}} = 4.72 - 0.5x_4 - 0.19x_2x_3 - 0.0122x_4x_{10} + 0.009325x_6x_{10} + 0.000191x_{11}^2$$

$$\text{Velocity}_{\text{B-pillar}} = 10.58 - 0.674x_1x_2 - 1.95x_2x_8 + 0.02054x_3x_{10} - 0.0198x_4x_{10} + 0.028x_6x_{10}$$

$$\text{Velocity}_{\text{door}} = 16.45 - 0.489x_3x_7 - 0.843x_5x_6 + 0.0432x_9x_{10} - 0.0556x_9x_{11} - 0.000786x_{11}^2 \quad (13)$$

4 Reliability-based crashworthiness design optimization

4.1 RBDO model of crashworthiness for vehicle side impact

In engineering design, the deterministic design optimization model has been widely used to systematically reduce costs and to improve model performance. However, the existence of uncertainties in either the manufacturing process or engineering simulations requires a reliability-based design. The RBDO vehicle model for crashworthiness is formulated as follows:

$$\begin{aligned}
&\text{minimize} \quad \text{Weight}(\mathbf{d})\\
&\text{subject to} \quad P(\text{abdomen load} \leq 1.0\,\text{kN}) \geq P_s\\
&P(\text{upper/middle/lower VC} \leq 0.32\,\text{mls}) \geq P_s\\
&P(\text{upper/middle/lower rib deflection} \leq 32\,\text{mm}) \geq P_s\\
&P(\text{public symphysis force, } F \leq 4.0\,\text{kN}) \geq P_s\\
&P(\text{velocity of B-pillar at middle point} \leq 9.9\,\text{mm/ms}) \geq P_s\\
&P(\text{velocity of front door at B-pillar} \leq 15.7\,\text{mm/ms}) \geq P_s\\
&\mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U, \ \ \mathbf{d} \in R^9 \text{ and } \mathbf{X} \in R^{11} \qquad (14)
\end{aligned}$$

There are nine design parameters used in the design optimization of vehicle side impact. The design parameters are the thickness ($d_1 - d_7$) and material properties of critical parts (d_8, d_9), as shown in Table 2. All thickness design variables (unit, mm) are continuous; however, two material design variables (unit, GPa) are discrete, and can be either mild or high strength steel. The two non-design variable random parameters are barrier height and hitting position, which can vary from -30 mm to 30 mm according to the physical test. In the side impact CAE model, it is assumed that all random variables are normally distributed around their nominal value. In probabilistic constraints, performance requirements could be relaxed from performance values for the top-safety rating listed in Table 1, since the top safety rating could be unachievable in reality.

4.2 Designs of crashworthiness for vehicle side impact

As summarized in Table 3, the total safety-rating score is compared between initial and deterministic optimum designs (Gu *et al.* 2001), which were obtained using sequential quadratic programming (SQP) (Arora 1989) based on the SR surrogate model. For the deterministic optimum design, as shown in Table 3, vehicle performance improves in terms of human safety, since a total safety rating score is increased from 12.342 to 12.924, while weight is minimized. The deterministic optimum design is listed in Table 4. However, as previously mentioned, the deterministic optimum design could be unreliable due to system uncertainties.

Figure 3 displays the reliability assessment of pubic force for side impact at the deterministic optimum design. Pubic force (= 3.99) at the deterministic optimum design could lead to system failure due to system uncertainties, since the deterministic optimum design is pushed to the failure boundary (= 4.00) of pubic force as shown in (14). Therefore, it is necessary to carry out RBDO with a given target reliability, as shown in (14).

Table 2 Properties of design and random parameters of vehicle side impact model

Random Variable	Std Dev.	Distr. type	d_1	$\mathbf{d}^L$	$\mathbf{d}$	$\mathbf{d}^U$
1(B-pillar inner)	0.030	Normal	1	0.500	1.000	1.500
2(B-pillar reinforce)	0.030	Normal	2	0.500	1.000	1.500
3(Floor side inner)	0.030	Normal	3	0.500	1.000	1.500
4(Cross member)	0.030	Normal	4	0.500	1.000	1.500
5(Door beam)	0.030	Normal	5	0.500	1.000	1.500
6(Door belt line)	0.030	Normal	6	0.500	1.000	1.500
7(Roof rail)	0.030	Normal	1	0.500	1.000	1.500
8(Mat. B-pillar inner)	0.006	Normal	8	0.192	0.300	0.345
9(Mat. floor side inner)	0.006	Normal	9	0.192	0.300	0.345
10(Barrier height)	10.0	Normal	10^{th} and 11^{th} random variables are not regarded as design variables			
11(Barrier hitting)	10.0	Normal				

Attempts were made to perform RBDO for crashworthiness by estimating the probabilistic constraints using the MCS method. However, in Yang *et al.* (2000) and Gu *et al.* (2001), neither reference led to a successful RBDO result, since although the MCS method is capable of evaluating reliability, it is not effective at providing guidelines for an improved design. Therefore, PMA with the HMV method, PMA with the AMV method, and RIA with the HLRF method in RBDO are used in this paper to perform RBDO for vehicle crashworthiness. In order to satisfy diverse demands on a reliable engineering system, RBDO for vehicle crashworthiness is solved

Table 3 Safety-rating scores for initial design and deterministic optimum design

Performance	Top safety-rating score	Initial design		Deterministic optimum design	
		Result	Score	Result	Score
Weight		29.05		24.12	
$\text{Load}_{\text{abdomen}}$	≤ 1.0	0.64	4.000	0.51	4.000
$\text{Deflection}_{\text{upper}}$		31.36		30.39	
$\text{Deflection}_{\text{middle}}$	≤ 22	31.54		28.88	
$\text{Deflection}_{\text{lower}}$		32.59	2.075	31.66	2.244
VC_{upper}		0.21		0.24	
VC_{middle}	≤ 0.32	0.20		0.21	
VC_{lower}		0.26		0.30	
$\text{Force}_{\text{pubic}}$	≤ 3.0	4.30	2.267	3.99	2.680
HIC	≤ 650	229.4	4.000	209.8	4.000
$\text{Velocity}_{\text{B-Pillar}}$	N.A.	9.22	N.A.	9.51	N.A.
$\text{Velocity}_{\text{door}}$		15.12		15.70	
Total score	16.0	12.342		12.924	

Table 4 Design points for crashworthiness of vehicle side impact

Design parameter	Initial design	Deterministic optimum design	RBDO 90(%)		RBDO (99.87%)
			MCS	PMA using HMV	
1. B-pillar inner	1.000	0.500	0.776	0.500	0.511
2. B-pillar reinforce	1.000	1.360	1.200	1.309	1.417
3. Floor side inner	1.000	0.500	0.896	0.500	0.500
4. Cross member	1.000	1.202	1.467	1.240	1.352
5. Door beam	1.000	0.500	0.550	0.610	0.658c
6. Door belt line	1.000	1.120	1.440	1.500	1.473
7. Roof rail	1.000	0.500	0.550	0.500	0.500
8. Mat. floor side inner	MS	HSS	HSS	HSS	HSS
9. Mat. floor side	MS	MS	HSS	MS	MS

Table 5 90% RBDO cost and history for crashworthiness of vehicle side impact (PMA with HMV)

Iter.	Cost	d_1,X_1	d_2,X_2	d_3,X_3	d_4,X_4	d_5,X_5	d_6,X_6	d_7,X_7	d_8,X_8	d_9,X_9	X_{10}	X_{11}
0	29.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.300	0.300	0.000	0.000
1	23.51	0.500	1.277	0.500	1.174	0.561	1.500	0.500	0.345	0.198	0.000	0.000
2	24.08	0.500	1.308	0.500	1.237	0.610	1.500	0.500	0.345	0.192	0.000	0.000
3	24.05	0.500	1.308	0.500	1.237	0.609	1.500	0.500	0.345	0.192	0.000	0.000
4	24.10	0.500	1.310	0.500	1.243	0.614	1.500	0.500	0.345	0.192	0.000	0.000
5	24.07	0.500	1.309	0.500	1.240	0.610	1.500	0.500	0.345	0.192	0.000	0.000
Opt.	24.07	0.500	1.309	0.500	1.240	0.610	1.500	0.500	0.345	0.192	0.000	0.000

Table 6 90% RBDO probabilistic constraints history for crashworthiness of vehicle side impact (PMA with HMV)

Iter.	G_{p1}^{PMA}	G_{p2}^{PMA}	G_{p3}^{PMA}	G_{p4}^{PMA}	G_{p5}^{PMA}	G_{p6}^{PMA}	G_{p7}^{PMA}	G_{p8}^{PMA}	G_{p9}^{PMA}	G_{p10}^{PMA}	No. of analysis
0	0.299	−2.070	2.496	0.120	0.098	0.113	0.045	−0.071	0.206	0.357	29
1	0.336	−0.313	2.781	0.951	0.075	0.094	0.025	−0.034	0.234	−0.066	32
2	0.384	0.003	3.042	1.090	0.076	0.094	0.025	0.000	0.282	0.003	35
3	0.381	−0.013	3.027	1.081	0.076	0.094	0.025	−0.001	0.280	−0.003	35
4	0.385	0.005	3.040	1.083	0.076	0.094	0.025	0.001	0.283	0.005	35
5	0.384	0.000	3.039	1.088	0.076	0.094	0.025	0.000	0.282	0.000	36
Opt.	0.384	0.000	3.039	1.088	0.076	0.094	0.025	0.000	0.282	0.000	202

Table 7 90% RBDO cost and design history for crashworthiness of vehicle side impact (PMA with AMV)

Iter.	Cost	d_1,X_1	d_2,X_2	d_3,X_3	d_4,X_4	d_5,X_5	d_6,X_6	d_7,X_7	d_8,X_8	d_9,X_9	X_{10}	X_{11}
0	29.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.300	0.300	0.000	0.000
1	23.50	0.500	1.278	0.504	1.173	0.555	1.500	0.500	0.345	0.192	0.000	0.000
2	24.02	0.500	1.308	0.500	1.237	0.608	1.500	0.500	0.345	0.192	0.000	0.000
3	24.05	0.500	1.308	0.500	1.237	0.609	1.500	0.500	0.345	0.192	0.000	0.000
4	24.07	0.500	1.309	0.500	1.239	0.610	1.500	0.500	0.345	0.192	0.000	0.000
Opt.	24.07	0.500	1.309	0.500	1.239	0.610	1.500	0.500	0.345	0.192	0.000	0.000

Table 8 90% RBDO probabilistic constraints history for crashworthiness of vehicle side impact (PMA with AMV)

Iter.	G_{p1}^{PMA}	G_{p2}^{PMA}	G_{p3}^{PMA}	G_{p4}^{PMA}	G_{p5}^{PMA}	G_{p6}^{PMA}	G_{p7}^{PMA}	G_{p8}^{PMA}	G_{p9}^{PMA}	G_{p10}^{PMA}	No. of analysis
0	0.299	−2.070	2.496	0.120	0.098	0.113	0.045	−0.071	0.206	0.357	82
1	0.334	−0.313	2.825	2.227	0.076	0.094	0.025	−0.033	0.233	0.130	83
2	0.384	0.003	3.042	1.090	0.080	0.090	0.025	0.000	0.282	0.002	84
3	0.381	−0.013	3.027	1.081	0.076	0.094	0.025	−0.001	0.280	−0.003	80
4	0.384	0.000	3.039	1.088	0.076	0.094	0.025	0.000	0.282	0.000	80
Opt.	0.384	0.000	3.039	1.088	0.076	0.094	0.025	0.000	0.282	0.000	409

Table 9 90% RBDO cost and design history for crashworthiness of vehicle side impact (RIA with HLRF)

Iter.	Cost	d_1, X_1	d_2, X_2	d_3,X_3	d_4,X_4	d_5,X_5	d_6,X_6	d_7,X_7	d_8,X_8	d_9,X_9	X_{10}	X_{11}
0	29.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.300	0.300	0.000	0.000
1	24.00	0.500	1.374	0.500	1.164	0.500	0.920	0.500	0.345	0.192	0.000	0.000
2	25.03	0.500	1.325	0.500	1.303	0.947	1.255	0.500	0.345	0.257	0.000	0.000
3	24.09	0.500	1.312	0.500	1.202	0.681	1.497	0.500	0.345	0.331	0.000	0.000
4	24.22	0.500	1.309	0.500	1.239	0.696	1.500	0.500	0.345	0.327	0.000	0.000
5	24.15	0.500	1.309	0.500	1.239	0.653	1.500	0.500	0.345	0.267	0.000	0.000
6	24.15	0.500	1.309	0.500	1.239	0.654	1.500	0.500	0.345	0.266	0.000	0.000
7	24.17	0.500	1.309	0.500	1.239	0.657	1.500	0.500	0.345	0.264	0.000	0.000
8	24.17	0.500	1.309	0.500	1.239	0.667	1.500	0.500	0.345	0.258	0.000	0.000
Opt.	24.17	0.500	1.309	0.500	1.239	0.667	1.500	0.500	0.345	0.258	0.000	0.000

for two target reliabilities, $P_s = 90\%$ and 99.87% (3-σ design). For $P_s = 90\%$, the optimization histories using different approaches and methods are shown in Table 5 through 10. For $P_s = 99.87\%$, the optimization results using PMA with the HMV method are shown in Tables 11 and 12. RBDO procedures using different approaches and

Table 10 90% RBDO probabilistic constraints history for crashworthiness of vehicle side impact (RIA with HLRF)

Iter.	G_{p1}^{PMA}	G_{p2}^{PMA}	G_{p3}^{PMA}	G_{p4}^{PMA}	G_{p5}^{PMA}	G_{p6}^{PMA}	G_{p7}^{PMA}	G_{p8}^{PMA}	G_{p9}^{PMA}	G_{p10}^{PMA}	No. of analysis
0	5.423	−1.792	1.549	1.835	6.014	11.72	3.300	−2.145	0.703	2.275	90
1	11.21	0.984	1.990	4.129	6.351	18.82	4.063	−1.410	3.546	−2.425	96
2	7.261	0.193	1.670	1.549	4.381	12.30	4.913	0.126	1.875	1.607	185
3	4.520	0.041	1.207	0.256	4.297	13.54	5.694	−1.014	0.946	−0.009	50
4	4.671	0.002	1.219	0.309	4.327	13.67	5.632	−0.008	0.995	0.020	56
5	4.521	0.000	1.478	1.206	4.869	16.04	4.943	0.003	0.996	−0.087	95
6	4.519	0.000	1.480	1.213	4.870	16.05	4.938	0.003	0.996	−0.084	190
7	4.503	0.000	1.511	1.316	4.889	16.15	4.866	0.000	0.995	−0.029	190
8	4.500	−0.001	0.152	1.340	4.888	16.15	4.849	0.002	0.995	−0.001	190
Opt.	4.500	−0.001	70.152	1.340	4.888	16.15	4.849	0.002	0.995	−0.001	1142

Table 11 99.87% RBDO cost and design history for crashworthiness of vehicle side impact

Iter.	Cost	d_1,X_1	d_2,X_2	d_3,X_3	d_4,X_4	d_5,X_5	d_6,X_6	d_7,X_7	d_8,X_8	d_9,X_9	X_{10}	X_{11}
0	29.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.300	0.300	0.000	0.000
1	29.72	0.952	1.304	0.867	1.047	0.937	1.300	0.903	0.340	0.300	0.000	0.000
2	25.22	0.500	1.415	0.500	1.251	0.825	1.428	0.500	0.345	0.192	0.000	0.000
3	25.41	0.566	1.393	0.500	1.336	0.651	1.495	0.500	0.345	0.192	0.000	0.000
4	25.39	0.511	1.417	0.500	1.352	0.658	1.473	0.500	0.345	0.192	0.000	0.000
Opt.	25.39	0.511	1.417	0.500	1.352	0.658	1.473	0.500	0.345	0.192	0.000	0.000

Table 12 99.87% RBDO probabilistic constraints history for crashworthiness of vehicle side impact

Iter.	G_{p1}	G_{p2}	G_{p3}	G_{p4}	G_{p5}	G_{p6}	G_{p7}	G_{p8}	G_{p9}	G_{p10}	No. of analysis
0	0.214	−4.065	−0.274	0.147	0.076	0.097	0.021	−0.124	0.300	0.121	47
1	0.296	−0.335	2.662	1.151	0.083	0.104	0.050	−0.035	0.000	0.195	100
2	0.346	−0.067	0.936	1.121	0.054	0.088	0.014	−0.029	−0.034	0.175	46
3	0.348	−0.002	1.141	1.473	0.063	0.091	0.014	0.000	−0.001	0.002	97
4	0.383	0.001	0.921	1.230	0.059	0.088	0.013	0.000	0.000	0.000	50
Opt.	0.383	0.001	0.921	1.230	0.059	0.088	0.013	0.000	0.000	0.000	340

methods are shown to be consistent, since they produces very close optimum designs. In terms of a total number of analyses, PMA with the HMV method is much more efficient than the others. In this example, analysis indicates FE analysis plus design sensitivity analysis. For a better result interpretation, an e-active set criterion to determine critical probabilistic constraints is set to be relatively large in this example.

The RBDO results using the MCS method (Yang *et al.* 2000; Gu *et al.* 2001) are compared to RBDO results using PMA with the HMV method, as shown in Table 13. Note that all RBDO results have higher total scores than those in Table 3. The RBDO results of PMA with 90% target reliability reduces vehicle weight more, whereas this vehicle design turns out to be less safe than others. With higher target reliability (99.87%), the RBDO result of PMA gives safer vehicle design, which requires more vehicle weight instead. It is found that the RBDO result obtained using MCS method achieves the largest safety-rating score, but it generates an infeasible design, since the MCS method misleads the RBDO process. In other words, the MCS method is inadequate for RBDO due to its inaccuracy as well as inefficiency, unlike a reliability analysis or output probability analysis.

As shown in Table 14, the constraint status is examined for the RBDO results obtained using the MCS method and PMA. For the RBDO result of the MCS method, the safety reliability of the 2^{nd}, 4^{th}, and 10^{th} constraints must be less than the target reliability, which is 90%, since inequality constraints in (14) are not sat-

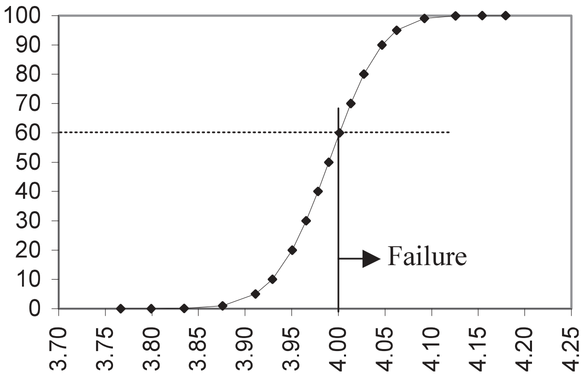


Fig. 3 CDF plot of pubic force for vehicle side impact

isfied. In other words, the RBDO result of the MCS method in Yang *et al.* (2000) and Gu *et al.* (2001) does not yield a feasible design for probabilistic constraints. However, RBDO results of PMA satisfy the probabilistic constraints with both 90% and 99.87% reliabilities, as shown in Table 14. For the RBDO results of PMA, the 4^{th} , 8^{th}, and 10^{th} constraints become active at the 90% reliable optimum design and the 4^{th}, 8^{th}, 9^{th}, and 10^{th} constraints become active at the 99.87% reliable optimum design. Consequently, the RBDO of PMA achieves a very safe vehicle design with target reliability, while minimizing vehicle weight.

The four designs: design 1 at the initial stage, design 2 at the deterministic design optimization, design 3 at PMA-RBDO with 90 % target reliability, and design 4 at PMA-RBDO with 99.87 % target reliability are compared for the vehicle safety-rating in Fig. 4. As shown in Fig. 4, the CDFs of safety-rating score for four different designs demonstrate distributive levels of human safety from vehicle side impact. The initial design has the largest deviation of safety-rating score for a vehicle side impact,

Table 13 Safety-rating scores for reliability-based optimum designs

Performance	Top safety-rating score	90% Reliability				99.87%	
		RBDO using MCS		RBDO using PMA			
		Result	Score	Result	Score	Result	Score
Weight		25.60		24.07		25.39	
$\text{Load}_{\text{abdomen}}$	≤ 1.0	0.456	4.000	0.510	4.000	0.401	4.000
$\text{Deflection}_{\text{upper}}$		27.30		29.92		29.59	
$\text{Deflection}_{\text{middle}}$	≤ 22	29.20		26.79		26.03	
$\text{Deflection}_{\text{lower}}$		29.90	2.563	31.17	2.333	30.05	2.536
VC_{upper}		0.222		0.233		0.231	
$\text{VC}_{\text{middle}}$	≤ 0.32	0.196		0.220		0.217	
VC_{lower}		0.260		0.286		0.287	
$\text{Force}_{\text{pubic}}$	≤ 3.0	3.800	2.933	3.975	2.700	3.909	2.788
HIC	≤ 650	229.4	4.000	211.3	4.000	211.7	4.000
$\text{Velocity}_{\text{B-Pillar}}$	N.A.	9.280	N.A.	9.258	N.A.	9.142	N.A.
$\text{Velocity}_{\text{door}}$		15.31		15.56		15.51	
Total score	16.0	13.496		13.033		13.324	

Table 14 Probabilistic constraint values for reliability-based design optimization

No.	Performance	Constraint requirements	90% Reliability Using MCS	Using PMA	99.87%
1	$\text{Load}_{\text{abdomen}}$	≤ 1.00	0.50	0.616	0.617
2	$\text{Deflection}_{\text{upper}}$		34.31	30.91	30.77
3	$\text{Deflection}_{\text{middle}}$	≤ 32.00	31.43	28.69	31.08
4	$\text{Deflection}_{\text{lower}}$		33.67	32.00	32.00
5	VC_{upper}		0.250	0.244	0.261
6	$\text{VC}_{\text{middle}}$	≤ 0.32	0.230	0.226	0.232
7	VC_{lower}		0.290	0.295	0.307
8	$\text{Force}_{\text{pubic}}$	≤ 4.00	3.890	4.000	4.000
9	$\text{Velocity}_{\text{B-Pillar}}$	≤ 10.00	9.600	9.618	9.900
10	$\text{Velocity}_{\text{door}}$	≤ 15.70	17.20	15.70	15.70

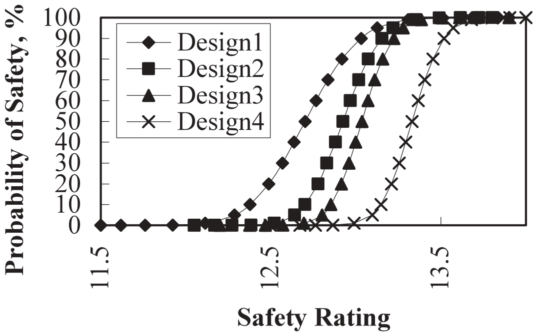


Fig. 4 CDF of safety-rating score of vehicle side impact

while the deterministic optimum design has smaller deviation and the reliability-based optimum designs have the smallest deviations. Accordingly, design 4 yields the highest safety-rating score over all probability levels and achieves the highest target reliability at the same time. In conclusion, the reliability-based optimum design not only improves the crashworthiness of a vehicle side impact, but also obtains a reliable and robust design by reducing randomness in the safety-rating score.

5 Conclusion

RBDO using PMA with the HMV method was successfully applied to the crashworthiness of a vehicle side impact. It was been found that the deterministic optimum design is pushed to the limits of the constraint boundaries, resulting in an unreliable design due to system uncertainties. Therefore, the RBDO process was required to obtain reliable designs. The RBDO method using MCS yielded unsuccessful results, since the reliability requirement was not satisfied. On the other hand, RBDO using PMA enhanced the crashworthiness performance of vehicle side impact, while minimizing the weight, increasing the safety-rating score, and achieving a high target reliability.

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