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A Tradeoff Cut Approach to Multiple Objective Optimization

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There is a need to develop user-oriented math programming techniques for resolution of decision problems in which several objectives must be considered. One approach, the Geoffrion-Dyer-Feinberg algorithm, allows interaction between the computer and the decision maker during the solution process. The interactive approach is adopted in this paper. However, our approach focuses on reducing the feasible region of the decision space rather than improving the stored image of the overall preference function. In so doing, the problem is reduced to a series of pairwise tradeoffs between the objectives. This obviates the need for any type of choice among vectors on the part of the decision maker and stays reasonably within his capability to supply necessary information for problem solution.

WHEN PLANNING the allocation of resources, designing or controlling a system, or, in general, making a decision, one is often confronted with a need to simultaneously consider several objectives. These problems, found both in the public and private sector, have become more and more complex due to the active participation of special-interest and consumer groups, legislative bodies, and the public media. This widening of the participating community creates the incentive to represent individually and treat those factors relevant to various interest groups. As a consequence, decision makers are frequently beset with having to find solutions to problems which have become characterized by multiple, noncommensurate, and often conflicting objectives.

Mathematical techniques for the resolution of these problems have recently received growing attention. Viable approaches to these complex decision problems include those of Belenson and Kapur (1973), Benayoun et al. (1971), Geoffrion (1970), Haimes et al. (1975), Oppenheimer (1978), and Zionts and Wallenius (1976).

Geoffrion et al. (1972) presents a formalization of the local preference technique proposed in a former paper (1970). This approach, the Geof-

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frion-Dyer-Feinberg (GDF) algorithm, is an interactive search procedure which questions the decision maker about his tradeoff preferences at each step in order to advance to improved feasible solutions. In the performance of this search, explicit knowledge of the overall preference function is not essential. Instead, by questioning the decision maker, local information regarding his preferences is received and this, in turn, is sufficient to determine a direction in which to proceed to preferred solutions. The step size is found by once again interacting with the decision maker to find the point on this direction ray which is most preferred.

This same interactive approach is adopted with the technique developed in this paper. However, unlike what is seen in the literature and the GDF algorithm in particular, this technique focuses its attention on reducing the decision space. Through the use of "tradeoff cuts," the convex envelope containing the best-compromise solution is refined. By repeatedly locating an improved feasible solution and then modifying the problem by means of these tradeoff cuts, one converges to what is defined later as the best-compromise solution (Musselman [1978]).

Motivation for this tradeoff cut approach arose from some practical difficulties encountered with the GDF algorithm. These included step-size resolution difficulties and the need for a sensitivity analysis of the decision maker's inputs supplied during the solution process. The step-size resolution problem involved two areas of concern: first, the likelihood of transferring misleading information to the decision maker and, secondly, the difficulty involved with assessing one's preference among vectors.

With the GDF algorithm, a linear approximation to the decision maker's overall preference function is always maximized over the original decision space to determine the next feasible direction in which to proceed. Yet, as will be shown, regions of this decision space can be removed from further consideration in view of the decision maker's tradeoffs. Consequently, the solution to the direction-finding subproblem might possibly be a point in a region that is of no further interest. In the step-size subproblem which follows, this may lead to the needless and unpropitious consideration of points located in an unfavorable region.

Furthermore, with each step-size subproblem, the decision maker is presented with a set of alternatives. These alternatives consist of vectors whose components are the various objective functions evaluated along the feasible direction line segment. From this set, the decision maker is asked to select his most preferred vector. Understandably, as the number of objective functions increase, this selection process becomes more and more difficult to perform due to the decision maker's limited processing capacity, fallible memory, and selective perception. Circumventing this vector decision would obviously enhance the solution process.

1. PROBLEM DEFINITION

The multiple objective optimization problem of specific interest is stated as follows:

$$\text{maximize} \quad U(f_1(\underset{\sim}{x}), f_2(\underset{\sim}{x}), \cdots, f_r(\underset{\sim}{x})) \tag{1}$$

$$\text{subject to} \quad \underset{\sim}{x} \in X = \{\underset{\sim}{x} \mid g_j(\underset{\sim}{x}) \le 0, j = 1, 2, \cdots, m\} \tag{2}$$

where $\underset{\sim}{x}$ is an n-dimensional vector of decision variables, $f_i(\underset{\sim}{x})$, $i = 1, 2, \cdots, r$, are r distinct objective functions of the decision vector $\underset{\sim}{x}$, X forms the constrained set of feasible decisions, and U is the decision maker's overall preference function defined on the range of $\underset{\sim}{f}$.

The set X and each f_i are assumed known. The overall preference function U, however, is not identified explicitly; rather, it is established locally at various iteration points by questioning the decision maker. The feasible point at which U is maximized is termed the best-compromise solution.

It is assumed that the functions U, f_i, and g_j are continuously differentiable. The functions U and f_i are concave, while the functions g_j are convex. Furthermore, U is increasing in each f_i; and X is a nonempty, compact, convex set.

As mentioned, the technique developed in this paper is an interactive procedure in that the decision maker is periodically asked to supply information during the solution process. It concentrates on reducing the feasible region in decision space through the use of tradeoff cuts. In supplying tradeoffs between the objectives, the decision maker indirectly establishes a cutting plane in *objective* space which removes from further consideration large areas of the *decision* space. Although the decision maker is not aware of the extent to which he has reduced his decision space, he is assured that the best-compromise solution has not been carelessly eliminated. The cutting plane established from these tradeoffs focuses on the most relevant portion of the decision space, but the accomplishment of this is based more on eliminating the inferior portion of the space than in identifying the superior portion. Basically, the cut acts as a restriction on where *not* to proceed.

At each iteration of the algorithm, the decision maker eliminates an alternative which is represented by some point $\underset{\sim}{f}' = \underset{\sim}{f}(\underset{\sim}{x}')$ in objective space. U being concave and differentiable,

$$U(\underset{\sim}{f}) \ge \mathrm{U}(\underset{\sim}{f}') \Rightarrow \textstyle\sum_{i=1}^{r} \lambda_i (f_i - f_i') \ge 0,$$

where $\lambda_i = \delta U / \delta f_i | \underset{\sim}{f} = \underset{\sim}{f}'$. Thus, in searching for improved values of U, one need only concentrate in that half-space for which

$$\textstyle\sum_{i=1}^{r} \lambda_i (f_i - f_i') \ge 0, \tag{3}$$

or equivalently,

$$\sum_{i=1}^{r} w_i(f_i - f_i') \geq 0, \tag{4}$$

where $w_i = \lambda_i/\lambda_1$ are the marginal rates of substitution between pairs of objectives. The left-hand side of the above inequality is termed a tradeoff cut, and the corresponding set of feasible decision points which satisfies this inequality is necessarily convex.

The optimization method works as follows. When the decision maker furnishes tradeoffs at a particular point, a tradeoff cut is formed in objective space. This nonnegative half-space translates into a convex set in decision space. The intersection of this set with the current feasible region in decision space defines a convex set which must contain the best-compromise solution.

The question which now arises is how does one judiciously select the next point at which to question the decision maker about his tradeoff preferences. This problem is the same as trying to advantageously locate the next point in the restricted region of the decision space. The image of the best-compromise solution is obviously on the boundary of the feasible region in objective space. However, a decision point whose image is on the boundary is not necessarily found on the border of the feasible set in decision space. Thus, to concentrate on the extreme points or even to focus on the constraint boundaries of the decision space would be a mistake. One must consider the entire restricted region when searching for the best-compromise solution. A point near the "center" of this constrained set would be beneficial, for it could eliminate an ample portion of this set on the next cut. A method analogous to Polak's (1971) modified method of centers technique is used to locate this next point in decision space. The procedure is based on determining a usable direction $\underline{h}$ on the kth iteration of the algorithm by means of the following linear programming problem:

$$\text{minimize } M \tag{5}$$

subject to

$$-M + g_j(\underline{x}') + \nabla_{\underline{x}} g_j(\underline{x}')^t \cdot \underline{h} \leq 0 \qquad j = 1, 2, \cdots, m_k \tag{6}$$

$$-1 \leq h_i \leq 1, \qquad i = 1, 2, \cdots, n, \tag{7}$$

where $\alpha = 1/\lambda_1$, $m_k = m + k$, and $g_j(\underline{x}) \leq 0, j = 1, 2, \cdots, m_k$ represents the initial m constraints plus the k tradeoff cuts.

Determining the best feasible direction $\underline{h}$ solely on the basis of the "feasibility" constraints (6) does not avoid the necessity for aligning $\underline{h}$ in a direction which improves the overall preference function U. This requirement is implicitly guaranteed through the inclusion of the current tradeoff cut (i.e., the last constraint in (6)).

To locate the next point at which the decision maker assesses his tradeoffs, one moves in the usable direction $\underset{\sim}{h}$ until he has minimized the maximum value of all the constraint expressions g_j, for $j = 1, 2, \cdots, m_k$. This is a one-dimensional search which can be performed without the need for decision maker interaction. The image of this new point in objective space is the next tradeoff point (i.e., the point in objective space at which the decision maker assesses his tradeoffs).

The purpose of this "center" point is to establish the base step-size to be used in the step length determination scheme. In this scheme, two constraints, ρ_1 and ρ_2, are used to adjust the step-size to guarantee locating a decision point which is known to be preferred to the former one. This adjustment procedure will be explained once the algorithm has been introduced.

2. THE ALGORITHM

Let $m_0 = m$, the initial number of constraints associated with problems (1)–(2). Define

$$C_0 \equiv \{\underset{\sim}{x} \mid g_j(\underset{\sim}{x}) \leq 0, j = 1, 2, \cdots, m_0\}.$$

On each iteration a new constraint is added, reflecting the tradeoff cut which was made in objective space. Therefore, after the kth iteration, let

$$C_k \equiv \{\underset{\sim}{x} \mid g_j(x) \leq 0, j = 1, 2, \cdots, m_k\},$$

where $m_k = m_0 + k$ and $g_j(\underset{\sim}{x}) \leq 0, j = 1, 2, \cdots, m_k$, represent the initial m constraints plus the k tradeoff cuts. The set C_k is assumed to have an interior and contain at least one point $\underset{\sim}{x}^c$ for which $g_j(\underset{\sim}{x}^c) < 0$, for all $j = 1, 2, \cdots, m_k$. This together with the fact that the functions $g_j(\underset{\sim}{x})$ are convex and continuously differentiable ensures that the constraint qualification (Simmons [1975]) holds at any feasible point $\underset{\sim}{x}$ in C_k. Let d be defined at any point $\underset{\sim}{y}$ in C_k as $d(\underset{\sim}{y}) \equiv \text{Max}\{g_j(\underset{\sim}{y}), j = 1, 2, \cdots, m_k\}$. Select ρ_1 and ρ_2 to be greater than one. As before, let $\alpha \equiv 1/(\delta U/\delta f_1)$ be evaluated at the current decision point.

Step 0. Select $\underset{\sim}{x}^0 \in C_0$ and set $k = 0$.
Step 1. Determine tradeoffs at $\underset{\sim}{f}(\underset{\sim}{x}^0)$.
Step 2. Set $\underset{\sim}{x} = \underset{\sim}{x}^k$ and $t = 1$.
Step 3. Solve (5)–(7) to obtain $(M(\underset{\sim}{x}), \underset{\sim}{h})$.
Step 4. If $M(\underset{\sim}{x}) \geq 0$, Stop! $\underset{\sim}{x}$ is the best-compromise solution. Otherwise, continue.
Step 5. Compute (e.g., via the golden section search) λ^*, the smallest positive scaler such that $d(\underset{\sim}{x} + \lambda^* \underset{\sim}{h}) = \text{Min}\{d(\underset{\sim}{x} + \lambda \underset{\sim}{h}) \mid \lambda \geq 0\}$.
Step 6. Set $\underset{\sim}{z} = \lambda^* \underset{\sim}{h}$.

Step 7. If $\alpha\nabla_{x}U(\underset{\sim}{x} + t\underset{\sim}{z})^{t}\cdot(-t\underset{\sim}{z})$ (i.e., $\sum_{i=1}^{r} w_i\nabla_{x}f_i(\underset{\sim}{x} + t\underset{\sim}{z})^{t}\cdot(-t\underset{\sim}{z})$)
< 0, go to Step 12
$= 0$, go to Step 14
> 0, continue.
Step 8. Set $t = t/\rho_1$.
Step 9. If $\alpha\nabla_{x}U(\underset{\sim}{x} + t\underset{\sim}{z})^{t}\cdot(-t\underset{\sim}{z}) > 0$, return to Step 8. Otherwise, go to Step 14.
Step 10. If $\underset{\sim}{x} + t\underset{\sim}{z} \in C_k$, continue. Otherwise, go to Step 13.
Step 11. If $\alpha\nabla_{x}U(\underset{\sim}{x} + t\underset{\sim}{z})^{t}\cdot(-t\underset{\sim}{z})$
< 0, continue
$= 0$, go to Step 14
> 0, go to Step 13.
Step 12. Set $t = \rho_2 t$ and return to Step 10.
Step 13. Set $t = t/\rho_2$.
Step 14. Set $\underset{\sim}{x}^{k+1} = \underset{\sim}{x} + t\underset{\sim}{z}$, $k = k + 1$, and return to Step 2.

This algorithm consists of four parts. The first part, Steps 0–3, determines the termination index M and the usable direction $\underset{\sim}{h}$ to be used in locating the "center" of the current feasible region. The next part is the algorithm's termination rule which is found at Step 4. The third part, Step 5, involves locating the center point of the current feasible region. Finally, Steps 6–14 comprise the algorithm's step length determination scheme.

In executing the algorithm, a sequence is constructed which is either finite (in which case the last point generated is the best-compromise solution) or else it is infinite and has the property that any limit point of this sequence is a best-compromise solution.

Interaction with the decision maker may occur at Steps 1, 7, 9, and 11. Although these interactions appear at a number of steps in the algorithm, one is precluded from interacting at all of these locations due to the algorithm's logic. Note also that each interaction requests the same type of information, namely local tradeoffs between pairs of objectives. This information is then used again in Step 3 to define the next LP problem.

It is with the step length determination scheme, in Steps 6–14, that the constants ρ_1 and ρ_2 are used. ρ_1 is seen in Step 8, at which point too large a step size has been taken. The result is that the point at which U reaches its maximum (along $\underset{\sim}{x} + t\underset{\sim}{z}$) has been passed. Since no statement can be made concerning the degree of improvement—if, indeed, there has been any—a decrease in the step size is necessary. In Steps 8–9, the step size is decreased by a factor of ρ_1 until the point at which U reaches its maximum has been passed again (this time in the opposite direction). The algorithm then sets the image of this new point as the next tradeoff point and continues.

High values for ρ_1 would possibly result in reaching a point of guaranteed improvement in fewer iterations and, in turn, require fewer interactions with the decision maker. On the other hand, small values for ρ_1 would allow for a "better" cut to be taken. That is, the cut would intersect the path $\underset{\sim}{x} + t\underset{\sim}{z}$ at a point closer to where U reaches its maximum. In taking cuts closer to this maximizing point, one is apt to reduce the feasible decision space in a more efficient manner. Current practice is to set $\rho_1 = 2$.

The other constant ρ_2 appears in Step 12. Upon entering Steps 10–13 of the algorithm, the maximum of U along the path $\underset{\sim}{x} + t\underset{\sim}{z}$ has not been passed yet. Consequently the step size is increased by a factor of ρ_2. Similar to the situation with ρ_1, the considerations one must address in determining a value for ρ_2 result in conflict. A large ρ_2 is beneficial because the next step would most likely be infeasible and, consequently, would not require any interaction with the decision maker. Yet, a small value for ρ_2 would increase the possibility of obtaining a better cut. In practice, setting $\rho_2 = 2$ has yielded favorable results.

3. TRADEOFF SENSITIVITY

The importance of tradeoffs to the solution process and the difficulty a decision maker has in making confident assessments of these values underscore the need for a sensitivity analysis. With the tradeoff cut approach, the sensitivity of the decision maker's uncertainty can be judged with regard to the feasibility of the final solution. That is, limits on the range of tradeoffs at each step can be determined such that the final solution remains feasible. These limits are based on the fact that the best-compromise solution can only be eliminated by producing a tradeoff cut for which this final solution is no longer feasible. As long as the decision maker's tradeoffs are such that the corresponding tradeoff cut does not cause this point to become infeasible, this final compromise is not in jeopardy.

If, in the process of running the algorithm, a vector $\underset{\sim}{f}^c$ is found for which the decision maker is satisfied, the process is usually terminated prior to locating the best-compromise solution. In this situation, $\underset{\sim}{f}^c$ is assumed to be the final compromise.

Letting $w_i = (\delta U/\delta f_i)/(\delta U/\delta f_1)$, the tradeoff cut at $\underset{\sim}{f}'$ is: $\sum_{i=1}^{r} w_i(f_i - f_i') \geq 0$, where f_i' is the value of the ith objective at the current tradeoff point $\underset{\sim}{f}'$. Since the final compromise must satisfy this cut, $\sum_{i=1}^{r} w_i(f_i^c - f_i') \geq 0$, where f_i^c is the value of the ith objective at the final compromise. For any given tradeoff ratio w_j (which is necessarily nonnegative), $w_j(f_j^c - f_j') + \sum_{i=1, i\neq j}^{r} w_i(f_i^c - f_i') \geq 0$. This establishes the following restrictions on w_j:

$$
\begin{aligned}
w_j &\geq -\textstyle\sum_{i=1, i\neq j}^{r} w_i(f_i^c - f_i')/(f_j^c - f_j'), && \text{if} \quad (f_j^c - f_j') \geq 0,\\
0 \leq w_j &\leq -\textstyle\sum_{i=1, i\neq j}^{r} w_i(f_i^c - f_i')/(f_j^c - f_j'), && \text{if} \quad (f_j^c - f_j') < 0,\\
w_j &\leq \infty, && \text{if} \quad (f_j^c - f_j') = 0.
\end{aligned}
$$

Reiterating, these limits on w_j indicate what range is possible for a given tradeoff ratio such that the final compromise does not become infeasible. These limits are obtained for each tradeoff ratio and at every tradeoff assessment point of the algorithm.

Practically speaking, care must be taken to understand what these limits mean. One should not assume that by using one of these extreme tradeoff values, the final solution will necessarily be the same. Indeed, even one change at a given tradeoff point would most likely cause a different sequence of points to be followed. Unfortunately, since the decision maker's tradeoff preferences are unknown at locations other than those found in the original sequence, this alternate sequence is unable to be generated.

Yet, the importance of these limits is recognized in the fact that they represent the degree of criticality associated with each tradeoff. If, e.g., a given tradeoff is near the maximum allowable, one would realize that a small degree of tradeoff error would jeopardize the solution. So, if a decision maker establishes a range within which a particular tradeoff is known to exist, this range can be judged against these limits to indicate to what extent his tradeoff uncertainty undermines the final result.

4. APPLICATION

This tradeoff cut approach to multiple objective optimization has been applied to the solution of a water resources problem. The problem is one of optimal planning for storm-drainage systems in urban areas and is described in detail by Dendrou et al. (1978). That problem formulation essentially consisted of a hierarchically structured linear program with a simulation model acting as a constraint. A tractable multiple-objective approximation of that formulation is presented here for the purpose of demonstrating the applicability of this type of an approach to urban drainage planning and design.

The problem is to examine, in terms of its storm drainage needs, a particular subbasin within a watershed. The subbasin is assumed to be hydrologically independent of other subbasins, having its own drainage network, on-site detention storage facility, treatment plant, and tributary to a receiving water body. The various factors involved in this problem as well as how they conceptually interlink are illustrated in Figure 1.

Runoff due to rainfall and snowfall is channeled to a treatment facility before being discharged into the receiving body of water. That which is not able to be immediately treated is rerouted to a temporary storage facility to await treatment. Once the storage facility has filled, any further runoff either overflows into the receiving body of water or it backs up.

The subbasin's storm drainage system is assumed to be characterized by three decision variables: x_1 = local detention storage capacity (basin·

inches), x_2 = maximum treatment rate (basin·inches/hour), and x_3 = maximum allowable overflow rate (basin·inches/hour). Restricting the overflow rate makes it possible to incorporate into the analysis damages caused by local flooding. Once the detention storage facility is filled and the overflow rate is at its maximum, the drainage system's conveying capacity is significantly reduced. This results in local flooding, the consequence of which is structural damage and economic disruption to the area.

The hydrologic performance of the subbasin was examined by using an urban hydrologic simulation program termed LANDSTORM (Dendrou

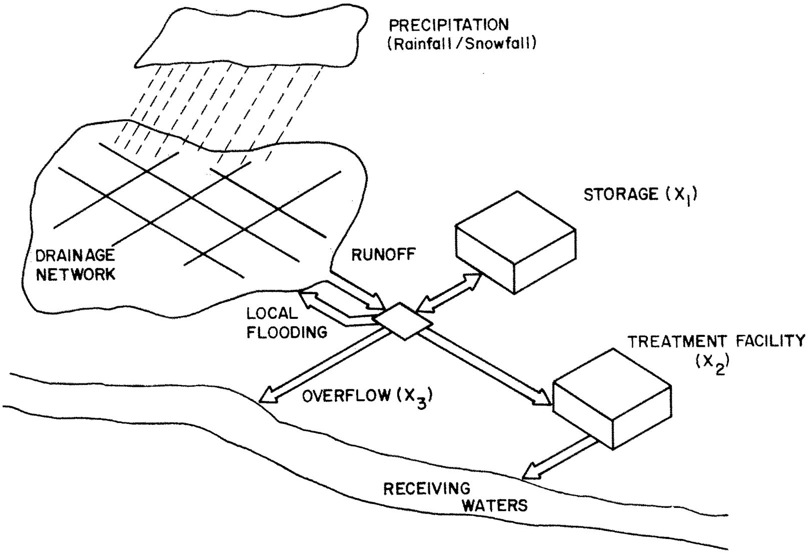

Figure 1. Conceptualization of a subbasin's storm drainage system.

et al. [1978]). The program oupled a landuse forecasting model with an urban hydrologic simulation model called STORM (Roesner et al. [1974]). Based on such inputs as surface characteristics, growth requirements and zoning policies, the simulation program generated an urban growth scenario for the subbasin on a lot-by-lot basis. Following this, it simulated the performance of the given storm drainage system (i.e., (x_1, x_2, x_3)) over a specified period of time and under weather conditions representative of the area. Twenty-three years of recorded precipitation data from the area was used as input to the simulation. The model was then simulated over a 20-year period to obtain an estimate of the hydrologic performance of the subbasin's drainage system.

Output from the simulation consisted of the drainage system's yearly statistical performance. Relevant data included:

1. Average number of floods per year (y_1)
2. Average flood volume per year (y_2)
3. Average number of pounds per year of suspended solids (y_{31})
4. Average number of pounds per year of settleable solids (y_{32})
5. Average number of pounds per year of biochemical oxygen demand $-BOD(y_{33})$
6. Average number of pounds per year of total nitrogen$-N(y_{34})$
7. Average number of pounds per year of orthophosphate$-PO_4(y_{35})$.

Inasmuch as the basic submodel of this problem is a simulation program, it became necessary to approximate the behavior of the simulation output parameters by response surfaces. Over the region of interest,

TABLE I

ANALYTICAL MODEL OF THE SUBBASIN'S STORM DRAINAGE PROBLEM

Function	Name	Algebraic Expression
Minimize objectives		
f_1	Drainage network cost	$106{,}780.37x_2 + 106{,}780.37x_3 + 61{,}704.67$
f_2	Storage facility cost	$3{,}000x_1$
f_3	Treatment facility cost	$(305{,}700/(.06 * 2289)^{.65})\ 2289x_2$
f_4	Expected flood damage cost	$250\ (2289)\exp(-39.75x_2 + 9.9x_3 + 2.74)$
f_5	Expected economic loss due to flooding	$25(1.39/(x_1x_2) + 4940x_3 - 80)$
Subject to constraints		
g_1	Average no. of floods/year	$0.00139/(x_1x_2) + 4.94x_3 - 0.08 \le 1$
g_2	Probability of exceeding a flood depth of 0.01 basin-inches	$0.0000306/(x_1x_2) + 0.1082x_3 - 0.00986 \le 0.10$
g_{31}	Average no. of pounds/year of suspended solids	$12.307/(x_1x_2) + 49{,}408.24x_3 - 4{,}051.02 \le 50{,}000$
g_{32}	Average no. of pounds/year of settleable solids	$2.098/(x_1x_2) + 8.046.33x_3 - 696.71 \le 16{,}000$
g_{33}	Average no. of pounds/year of BOD	$2.138/(x_1x_2) + 7{,}883.39x_3 - 705.04 \le 10{,}000$
g_{34}	Average no. of pounds/year of N	$0.417/(x_1x_2) + 1{,}721.26x_3 - 136.54 \le 2{,}000$
g_{35}	Average no. of pounds/year of PO_4	$0.164/(x_1x_2) + 631.13x_3 - 54.48 \le 550$

$0.01 \le x_1 \le 0.45$, $0.01 \le x_2 \le 0.10$, and $0.01 \le x_3 \le 0.10$. A response surface was established for each of the seven output parameters listed earlier. Since the tradeoff cut algorithm assumes convexity, it was necessary to selectively choose regression models which not only satisfied this assumption but also allowed for a respectable fit. Numerous models were attempted for each of the seven parameters. In each case, the model with the best "quality" fit (in terms of the coefficient of multiple determination R^2) was selected. The multiple objective formulation of the subbasin's storm drainage problem is summarized in Table I.

When this formulation was presented to a knowledgable decision maker in the interactive mode described in this paper, the decision maker (a graduate student) was generally able to arrive at a final compromise solution within relatively few (5 to 10) iterations. On occasion, however, he did apparently display inconsistent tradeoff preferences depending on the starting point used (Musselman and Talavage [1979]). The inconsistency shown by the decision maker in starting from different locations in decision space exposes the fact that he does not always behave according to a unique overall preference function. Reduction of some of this erratic behavior is usually accomplished by performing tradeoff consistency checks at the various iteration points (Dyer [1973], Geoffrion et al. [1972], Keeney and Raiffa [1976], and Oppenheimer [1978]).

Our experience has shown that decision makers tend to be cognizant of only a few objectives at any one time. Those objectives which exhibit unusual or critical values are inclined to receive the most attention. Consequently, the decision maker might be more conscious of certain objectives at one location than at another. This phenomenon is reflected in the tradeoffs the decision maker assesses between these objectives and, thus, causes his overall preference function to deviate according to the starting point.

Often, by starting the interactive solution processes from different starting points, certain decision variables repeatedly seek the same level, while others seem to disguise their true intentions. Although the human element may be the cause of such behavior, it should not be viewed as a destructive influence. By incorporating these human inconsistencies into the decision process, one is able to recognize those decision variables having rather robust solution levels. Further analysis of the problem can then be carried out with these variables fixed at their appropriate levels. In fact, having a condensed version of the original problem might even enhance the chances of better judgements on behalf of the decision maker.

5. CONCLUSION

The interactive, multiple-objective programming approach described in this paper makes use of tradeoff cuts to reformulate the original problem into a series of tradeoff questions between pairs of objectives. This approach obviates the need for the step-size vector decision and allows for a sensitivity analysis of the decision maker's inputs supplied during the solution process. Prudent use of the decision maker is made by staying reasonably within his capability to supply the necessary information to resolve this type of complex decision problem.

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