



Balancing Commonality and Performance within the Concurrent Design of Multiple Products in a Product Family

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Abstract: Product family design involves the concurrent design of multiple products based on a common product platform to satisfy a variety of markets. The success of the resulting product family relies heavily on properly balancing the commonality of the product platform with the individual product performance within the product family. To help resolve this tradeoff, we present the Product Variety Tradeoff Evaluation Method for assessing alternative product platform concepts with varying levels of commonality. Two examples are presented to demonstrate the proposed method at both the detailed and the early stages of design. The redesign of a planetary gear transmission for a family of four cordless drills demonstrates the use of the method in the detailed stages of design, while the design of a family of three General Aviation Aircraft demonstrates the use of the method in the preliminary stages of design. The emphasis in this paper is on the effectiveness of the proposed method in evaluating this tradeoff and not on the results of the examples, *per se*.

Key Words: product platform, product family design, product variety, commonality.

1. Introduction to Product Family and Product Platform Design

In an effort to improve customization for today's highly competitive global marketplace, many companies are utilizing product families to increase product variety and shorten product lead-times while reducing costs. A product family is a group of related products, which share common features, components and subsystems, yet satisfy a variety of market niches. Product family design involves the concurrent design of multiple products to maximize commonality across a set of products without compromising their individual performance. Resolving this tradeoff between commonality and performance requires the skillful coordination of design, manufacturing, marketing, and management to realize an innovative product platform that can yield an assortment of distinct products. A successful product platform carefully balances the performance and commonality of the individual products in the family, yielding a suitable compromise between these two

conflicting objectives. In reality, this tradeoff is further complicated by a variety of factors such as aesthetics, cost, quality, reliability, manufacturability, and serviceability (Prasad, 1996); however, our focus here is primarily on the technical performance of the individual products within a family to establish a foundation for resolving this tradeoff.

As noted by Robertson and Ulrich (1998), the architecture of the product platform dictates the nature of the tradeoff between individual product performance and commonality. Consequently, we propose a method to examine this tradeoff within families of products derived from product platforms with varying levels of commonality. The Product Variety Tradeoff Evaluation Method (PVTEM) described in the next section provides a novel approach to evaluate this tradeoff and provides decision support for integrated product development teams to realize effective product platforms. After describing the PVTEM in the next section, two examples are presented to demonstrate its application in both the detailed and the early stages of design. In Section 3, the redesign of a transmission for a family of four cordless drills is discussed to demonstrate the use of the PVTEM for detailed design. In

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Section 4, the design of a family of three aircraft is presented to demonstrate the use of the PVTEM in the preliminary stages of design. Closing remarks are offered in Section 5.

2. The Product Variety Tradeoff Evaluation Method

Since product family designers often generate a range of options for product platforms, the Product Variety Tradeoff Evaluation Method (PVTEM) introduced by Simpson (1998) was developed to help designers evaluate these options to help resolve the tradeoff between platform commonality and individual product performance within a product family. The steps of the PVTEM are listed in Table 1 and are followed verbatim in the two example problems given in Sections 3 and 4 to demonstrate their application. By following these steps, product platform designers can determine the appropriate level of commonality for a product platform.

The PVTEM helps designers resolve the tradeoff between product platform commonality and the individual performance of the products within the corresponding family of products derived from the common product platform. This is achieved through the use of two indices—the non-commonality index (NCI) and the performance deviation index (PDI)—and a product variety tradeoff chart as noted in Step 6. The two indices, NCI and PDI, are described in detail in the next section, and the product variety tradeoff chart is presented in Section 2.2.

Table 1. Steps of the product variety tradeoff evaluation method

1. Describe design alternatives and provide meaningful acronyms.
2. Identify performance parameters and design variables for inclusion in the evaluation indices (PDI and NCI). Identification should be directed toward determining those parameters and design variables relevant to product performance and commonality, respectively. Consider only those design variables that may change among variants in the product family.
3. Determine relevant ranges for the design variables (in the NCI) and target values for the performance parameters (in the PDI).
4. Assign weights to the design variables in the NCI and to the performance parameters in the PDI. Weights should be guided by the relative difficulty of varying the design variable in the final design (for the NCI) and by the relative importance of each performance parameter to the end user (for the PDI).
5. Determine overall PDI's and NCI's for each product family design alternative (follow the procedure outlined in Table 2 and Table 3).
6. Compare product family design alternatives using a Product Variety Tradeoff Chart (as shown in Figure 1).

2.1. Non-Commonality and Performance Deviation Indices

In the PVTEM, product variety tradeoff studies are performed using two measures: the NCI and the PDI. While a variety of indices have been proposed for assessing commonality within product families (see, e.g., Martin and Ishii, 1996, 1997; Siddique and Rosen, 1998; Kota *et al.*, 2000), *NCI has been developed to assess parametric variation within a product family*. NCI is a normalized measure of the variability of design variable settings across members of the product family; it is a weighted-sum of average relative variations in design variables within a family of products. A smaller NCI indicates less variation among design variable settings across the family, and, hence, more commonality within the family. Steps for computing the non-commonality index are given in Table 2. When comparing NCI's between different platform designs, it is assumed that similar sets of design variables characterize each design (i.e., the same configuration is used and only the values of the design parameters are changing).

While the NCI assesses parametric variation within a product family, the performance deviation index (PDI) provides a measure of how well products within the family meet their individual performance targets. PDI is a weighted sum of deviations from performance targets across a family of products; the steps for calculating

Table 2. Steps for computing non-commonality index (NCI)

1. List relevant design variables (x_j) (e.g., radius, thickness, etc.) for each of i products in the family.
2. Compute the mean value (μ_j) for each design variable across the family.

$$\mu_j = \frac{\sum_{i=1}^{\# \text{products}} x_{ij}}{i} \quad (1)$$

3. Compute the absolute value of the difference between μ_j and x_j for each product i .

$$\text{diff}_{ij} = |\mu_j - x_{ij}| \quad (2)$$

4. Normalize the differences by the design variable range (i.e., [upper bound – lower bound]). The design variable range should reflect the feasible range of the design variable across the family of products.

$$\text{Ndiff}_{ij} = \frac{|\mu_j - x_{ij}|}{\text{Range}} \quad (3)$$

5. Compute the average across products of the normalized differences for each design variable in turn. This value is DI_j (the dissimilarity index for design variable j).

$$DI_j = \frac{\sum_{i=1}^{\# \text{products}} \text{Ndiff}_{ij}}{\# \text{products}} \quad (4)$$

6. Weight the DI_j for each design variable. The weights should reflect the relative difficulty of varying that parameter.
7. Sum the weighted DI 's into an NCI.

$$\text{NCI} = \sum w_j DI_j, i = 1, \dots, \# \text{products}; j = 1, \dots, \# \text{variables} \quad (5)$$

Table 3. Steps for computing performance deviation index (PDI)

1. Specify the performance parameters of interest.
2. Specify goals or target values for each performance parameter (j) for each member of the product family (i).
3. Calculate normalized deviation (d_{ij}) of the value of each performance parameter from its target value for each product in the family.

e.g.,
$$\frac{|\text{Target Mass}_i - \text{Actual Mass}_i|}{\text{Target Mass}_i}$$

4. Choose weights (w_j) for each performance parameter. Weights should be based upon the importance of that performance parameter to the end user.
5. Sum the weighted, normalized performance parameter deviations for each product. Call each product's sum Z_i .

e.g.,
$$Z_i = w_1 d_{i1}^+ + w_2 d_{i2}^- \quad (6)$$

6. Assign a weight to each product in the family. The weight should reflect the product's importance to the family as a whole. (For example, the weight may reflect the relative demand for the product.)
7. Sum the weighted Z_i 's into a PDI.

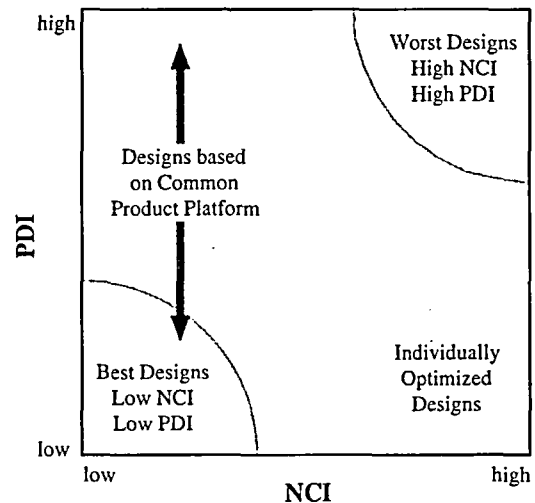
$$PDI = \sum_i w_i Z_i \quad \text{where } i = \# \text{ of products in family} \quad (7)$$

PDI are given in Table 3. An assumption behind the PDI is that the products in the family seek to satisfy goals for performance and that the constraints can be met for each product. These goals for product performance characteristics reflect customer requirements for specific market segments to which derivative products are targeted. Thus, the PDI is an indirect measure of the effectiveness of the product platform in facilitating customization of individual derivative products. When comparing PDI values between different platform designs, it is assumed that similar performance characteristics and targets characterize each design.

Admittedly, NCI and PDI are *ad hoc* as are most evaluation measures and indices. Their definitions are somewhat arbitrary, and they do not permit absolute measures of commonality or performance; rather, the measures are defined relative to the specific targets and ranges. However, NCI and PDI are useful for focusing attention on the tradeoff between performance and commonality within a product family, providing a basis for creating a product variety tradeoff chart as discussed in the next section.

2.2. The Product Variety Tradeoff Chart

In conjunction with NCI and PDI, the product variety tradeoff chart shown in Figure 1 provides a visual aid for designers for comparing product platform designs. As shown in the figure, low PDI and NCI values are desirable; hence, the most preferred designs fall in the lower left of the chart. The least preferred designs appear in the upper right of the chart.

**Figure 1. Product variety tradeoff chart.**

As shown in the figure, individually optimized designs tend to meet performance targets quite well but often lack commonality; hence, they fall in the lower right portion of the chart since they tend to have low PDI but high NCI. Product families aim to have higher commonality (lower NCI) but at the expense of losing performance. Thus, designs based on common product platforms tend to cluster along the left-hand side of the chart, and the best product platform has a low NCI combined with the lowest possible PDI.

To demonstrate the use of the product variety tradeoff chart in assessing alternative product platform concepts and the computation of NCI and PDI for a family of products, two examples are offered in the following sections. In the next section, the redesign of a transmission for a family of four cordless drills is presented to demonstrate the use of the PVTEM during detailed design. This is followed in Section 4 with the design of a family of three General Aviation aircraft based on different seating configurations to demonstrate the applicability of the proposed method for the preliminary stages of design.

3. Transmission Redesign for a Family of Cordless Drills

Our first example is the redesign of transmissions for a family of four cordless drills based on 9.6 Volt (V), 12.0 V, 14.4 V and 18.0 V variants. The objective in this example is to impose product platform commonality on the transmissions of all four drills with minimal compromise in performance. The performance requirements for the planetary gear transmissions are to meet pre-existing targets for output speeds as closely as possible, with minimum mass, and a minimum number of components. These performance requirements can be easily understood from a basic physical description of

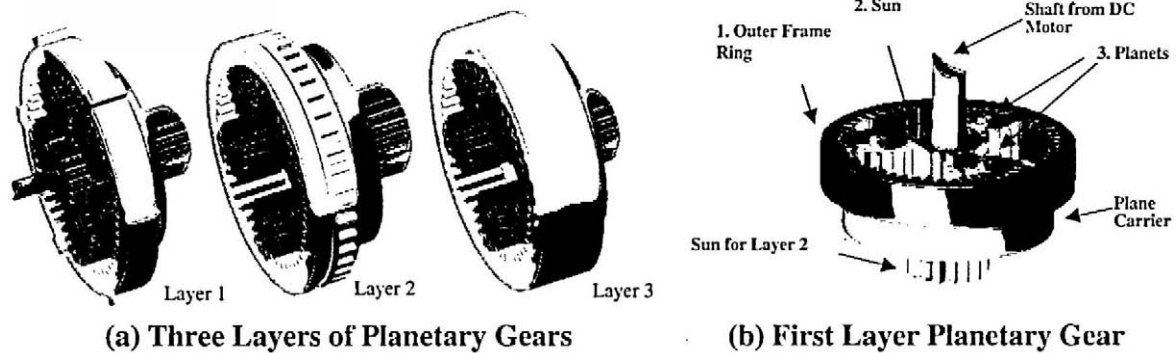


Figure 2. Planetary gear transmission assembly; (a) Three layers of planetary gears; (b) First layer planetary gear.

the transmissions, and the derivation of the mathematical model for the performance of each drill can be found in (Conner *et al.*, 1999). The function of the transmission in the drills is to transmit power from the motor to the chuck and its accessory. Each transmission consists of three layers of planetary gears, as shown in an exploded view in Figure 2a; a single layer of planetary gears is shown in Figure 2b. The transmission must transmit power in such a way that the output speed to the chuck (ω_{out}) is a fraction of the input speed from the motor (ω_{in}). This fraction (ω_{out}/ω_{in}) is called the gear ratio.

An important functional requirement for the customer is a specific set of output speeds (generally, 1400 and 400 rpm for high and low gears, respectively). The transmissions reside near the chuck in the drill casing. Due to their position, horizontally extended from the handle of the drill, low transmission mass is a desirable performance characteristic to lower the moment exerted on the user's wrist. The third performance requirement is to minimize the number of components. This requirement stems from the desire to limit the manufacturing complexity of the transmission assembly. The number of components is a function of the number of planets, which can vary from three to five.

Heavier drills (e.g., the 18 V version) input more power and torque into the transmission assembly. In order to transfer this increased power without exceeding allowable bending and contact stresses on the teeth of each component gear, the number of planets and/or the mass of the transmission (a function of face-widths and diameters) generally increase. Thus, with an outer diameter constrained by the inner diameter of the casing, the transmissions for more powerful drills tend to be heavier and include more planets than those for less powerful drills.

3.1. Transmission Design Problem Formulation and Solution

The strategy for embodying each transmission layer is to satisfy the three performance goals (i.e., gear ratio,

mass, and number of components) as closely as possible without violating the physical constraints on the design. Toward this end, the compromise Decision Support Problem (DSP) shown in Figure 3 is formulated to design a single layer of a planetary transmission. The compromise DSP is a multiobjective mathematical construct which is a hybrid formulation based on mathematical programming and goal programming (Mistree *et al.*, 1993). The compromise DSP is used to determine the values of the design variables that satisfy a set of constraints and achieve a set of potentially conflicting goals as closely as possible. In the compromise DSP, goals are formulated as equalities with appropriate targets to evaluate the tradeoffs between multiple goals effectively. Notice that each system goal is modeled using two deviation variables (d_i^-, d_i^+), representing under-achievement or over-achievement of each goal with respect to the target value. To handle tradeoffs, the objective is to minimize the deviation function, Z , which is a function of all of the deviation variables. A comprehensive discussion of deviation variables, deviation functions, system constraints, goals, bounds, and the solution algorithm can be found in (e.g., Mistree *et al.*, 1993).

As shown in Figure 3, for each layer of each transmission, there are five independent system variables: diametral pitch (P_d), number of teeth on the outer ring (N_1), number of teeth on the sun (N_2), number of planets (N_p), and face-width (F). Gear material is not varied in this example. The transmissions are designed to meet pre-existing targets for output speeds as closely as possible with minimum mass and a minimum number of components without exceeding allowable bending or contact stresses in the sun and planet gears over an infinite lifetime (i.e., 10^7 cycles). Also, the maximum outer diameter of the transmission is constrained to its value in pre-existing drills to avoid interference with drill casings. Since the outer ring is typically composed of a lightweight, synthetic material and experiences low stress levels, it is not included in stress calculations, and only the sun and the number of planets are included in mass calculations.

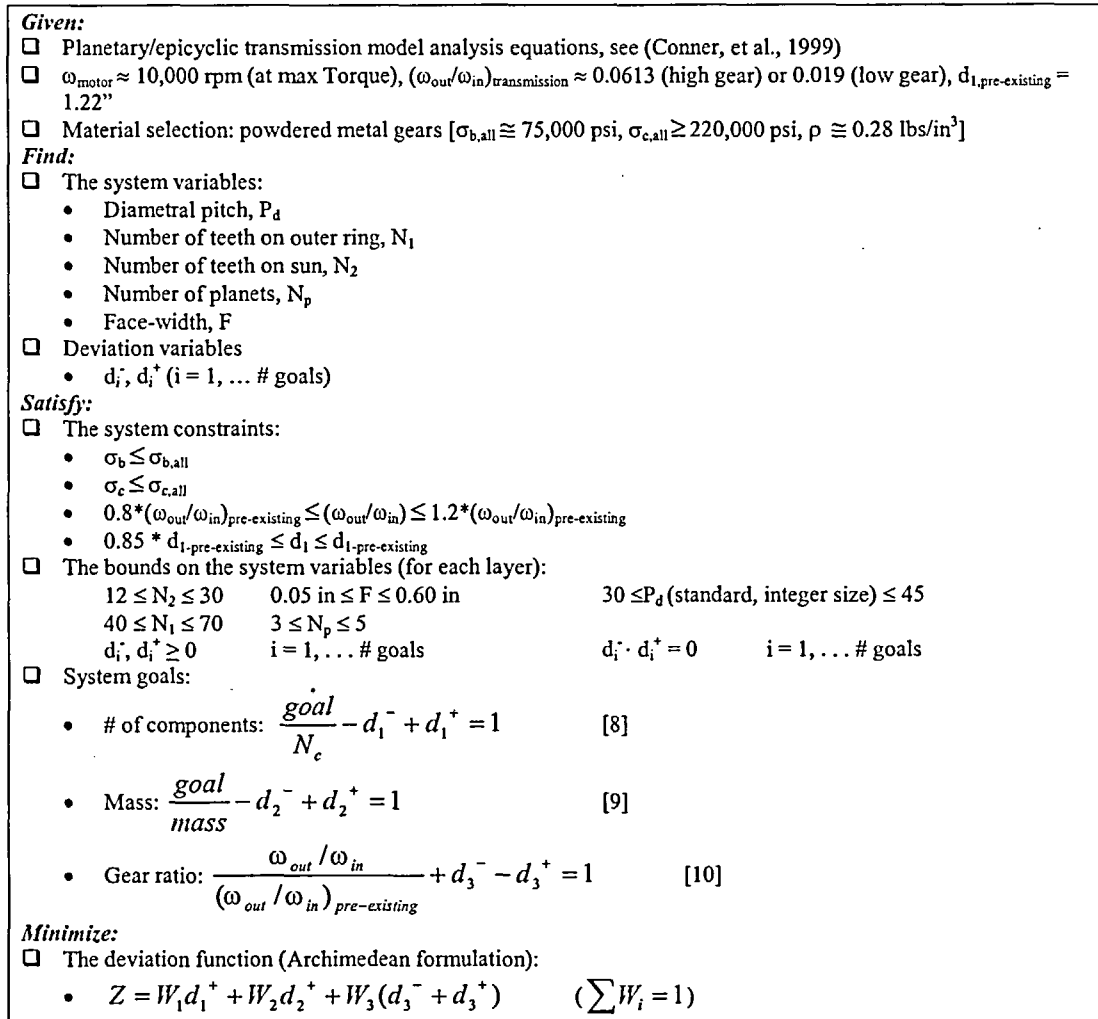


Figure 3. Compromise DSP for planetary gear transmission.

Three families of transmissions are specified in Table 4. The first family is a group of individually designed benchmark transmissions. Each transmission in the benchmark group is designed to meet its variant-specific performance targets without any regard to commonality among the products. The second and third families are based on product platforms, where the product platform is the set of common design variables around which each family of products is derived. For each platform-based family, the set of common design variables includes all five design variables for layers 1 and 2 and four of the five design variables for layer 3. The fifth design variable in layer 3 is identified as a scale factor that can be varied to meet the individual performance requirements of each member of the family while preserving commonality among the remaining design variables that compose the product platform. The reasoning for selecting these two possible scale factors is explained in Conner *et al.* (1999); the use of scale factors in product platform design is elaborated in Simpson *et al.* (2001).

Given these sets of specifications for the three product families, we can now employ the Product Variety Tradeoff Evaluation Method to help resolve the tradeoff between commonality and performance within the product family.

3.2 Evaluation of Design Alternatives Using PVTEM

To demonstrate the PVTEM, the steps outlined previously in Table 1 are used to evaluate the three product families.

Step 1 *Describe design alternatives and provide meaningful acronyms.* The three families that are considered in this example are listed in Table 5. The first is a group of individually designed benchmark transmissions. Meanwhile, the second and third families are based on common product platforms with different scaling factors.

Table 4. Specifications for benchmark transmissions and product families

Drill	Design Variables															System Performance			
	Layer 1					Layer 2					Layer 3					Mass	N _c	Gear Ratio	
	N _p	N ₁	P _d	N ₂	F	N _p	N ₁	P _d	N ₂	F	N _p	N ₁	P _d	N ₂	F			High	Low
Benchmark Transmissions																			
9.6V	3	48	45	12	0.05	3	54	45	24	0.122	5	44	39	20	0.251	0.063	20	0.0625	0.019
12V	3	48	44	12	0.052	3	50	45	22	0.178	5	44	38	20	0.305	0.079	20	0.0625	0.019
14.4V	3	48	41	12	0.054	5	54	45	24	0.111	5	50	45	22	0.372	0.087	22	0.0611	0.019
18V	4	48	40	12	0.054	5	50	45	22	0.177	5	54	44	24	0.432	0.122	23	0.0615	0.019
Product Family N _p (Platform scaled around Number of Planets, N _p)																			
9.6V	4	48	40	12	0.054	5	50	45	22	0.176	4	44	38	20	0.44	0.104	22	0.0625	0.019
12V	4	48	40	12	0.054	5	50	45	22	0.176	4	44	38	20	0.44	0.104	22	0.0625	0.019
14.4V	4	48	40	12	0.054	5	50	45	22	0.176	5	44	38	20	0.44	0.114	23	0.0625	0.019
18V	4	48	40	12	0.054	5	50	45	22	0.176	5	44	38	20	0.44	0.114	23	0.0625	0.019
Product Family F (Platform scaled around Facewidth, F)																			
9.6V	4	48	40	12	0.054	5	50	45	22	0.176	5	54	44	24	0.21	0.063	20	0.0625	0.019
12V	4	48	40	12	0.054	5	50	45	22	0.176	5	54	44	24	0.27	0.079	20	0.0625	0.019
14.4V	4	48	40	12	0.054	5	50	45	22	0.176	5	54	44	24	0.32	0.087	22	0.0611	0.019
18V	4	48	40	12	0.054	5	50	45	22	0.176	5	54	44	24	0.44	0.122	23	0.0615	0.019

Table 5. Platform design alternatives for drill transmissions

Acronym	Description
Benchmark	The benchmark family of individually designed drills.
Platform- N_p	A redesigned family of drills based on a common platform and with the number of planets in the third layer as the scale factor.
Platform- F	A redesigned family of drills based on a common platform and with the face-width of the third layer as the scale factor.

Step 2 Identify performance parameters and design variables for inclusion in the evaluation indices (PDI and NCI). There are five design variables of interest within each family:

1. Number of planets per layer (N_{p1} , N_{p2} , and N_{p3}),
2. Face-width per layer (F_1 , F_2 , F_3),
3. Number of teeth on the outer frame ring per layer (N_{1-1} , N_{1-2} , N_{1-3}),
4. Number of teeth on the sun per layer (N_{2-1} , N_{2-2} , N_{2-3}), and
5. Diametral pitch per layer (P_{d1} , P_{d2} , P_{d3}).

The performance parameters for inclusion in the performance deviation index are: (1) mass, (2) number of components, (3) high gear ratio, and (4) low gear ratio as discussed in Section 3.1.

Step 3 Determine relevant ranges (for the design variables in the NCI) and target values (for the performance parameters in the PDI).

Step 4 Assign weights to the design variables in the NCI and to the performance parameters in the PDI.

Table 6. Design variables with weights and ranges

Design Variable	Range	Weight (Divide by 3 to find weight per layer)
N_p	3	0.1
F	0.4 (in)	0.1
N_1	12	0.25
N_2	13	0.25
P_d	7	0.3

Table 7. Transmission performance parameters with weights and targets

Performance Parameter	Target	Weight
Mass	0.03lbs	0.05
N_c	18	0.2
High gear ratio	0.0613	0.375
Low gear ratio	0.019	0.375

The ranges, targets, and importance weightings for each design variable and performance parameter are presented in Table 6 and Table 7, respectively. The weights for the design variables shown in Table 6 are chosen based on the perceived difficulty of varying the design variable.

Our "rule of thumb" for performance parameters is to assign weights based on the importance of the performance parameter to the final product. Assigning higher weights to the output speeds implies that output speeds are much more important to the overall performance of the final product than the mass of the transmission or the number of components in the transmission. Although compactness and low mass are important performance parameters for drills, mass is given a low weight in the PDI because variations in mass are small

compared to the total mass of the drill. Precise output speeds are the most important functional characteristics of the drills.

Targets for the high and low gear ratios are averages of the pre-existing drill parameters. The target for the number of components is 18 and is based upon a minimum of three planets per layer. The total number of components per layer is the number of planets plus 3 (for the sun, outer frame ring, and planet carrier). The mass target is chosen to be slightly less than the mass of the sun and planet gears currently being produced. Ranges for the design variables are estimated by the difference between the highest and lowest values of the variables among the design alternatives.

Step 5 Determine overall PDIs and NCIs for each product family design alternative. The PDI and NCI are calculated for each of the three alternatives described in Step 1. The PDI and NCI values for each alternative are given in Table 8 based on Equations (1)–(7).

There are two interesting features of the values in Table 8. First, the NCI values of the product platform-based families are 96% lower than the benchmark group of individually designed products. On the other hand, PDI has increased by 27%–38% from the benchmark group to the product platform-based families. This increase in PDI is due to higher mass and increased numbers of components in both of the platform-based families.

Table 8. NCIs and PDIs for transmission design alternatives

Platform Alternative	NCI	PDI
Benchmark	0.609	0.137
Platform- N_p	0.022	0.189
Platform- F	0.023	0.174

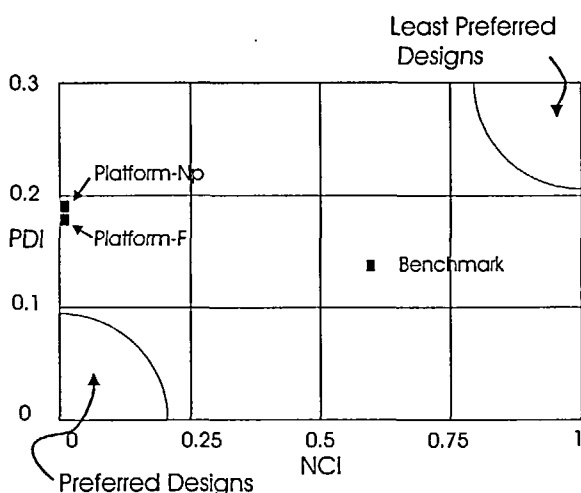


Figure 4. Products variety tradeoff chart for gear transmission families.

Step 6 Compare product family design alternatives using the product variety tradeoff chart. Figure 4 is a product variety tradeoff chart illustrating the tradeoff between performance and commonality between the transmission family design alternatives. Recall that the members of the benchmark group are designed individually without regard to commonality. The members of the platform-based families exhibit complete design variable commonality, with the exception of the scale factor (N_p or F) which is varied to meet individual performance requirements. As shown in Figure 4, the benchmark group of products has a relatively low PDI value but high NCI value (i.e., the group meets its performance targets well but does not exhibit a high degree of commonality). The platform-based families have lower NCI's than the benchmark with a small corresponding increase in PDI.

According to the product variety tradeoff chart in Figure 4, a designer should strongly consider the platform-based families. The platform-based families offer a 96% increase in commonality over the benchmark group with only a 27%–38% increase in deviation from performance targets. Since the platform-based families offer nearly complete commonality, the manufacturer *should* be able to design and manufacture this family more efficiently than the benchmark group. Thus, the final price to the consumer will be less, and the lead-time for new product development will decrease as well, allowing more variants to enter the market. If these and other benefits of commonality to the producer are valuable enough to compensate the consumer for a slightly higher PDI (i.e., slightly worse performance relative to target), the producer should market one of the platform-based design alternatives. On the other hand, if the performance decline (as well as other drawbacks like manufacturing investments) outweighs the benefits of commonality or if consumers simply find the declining performance relative to target unacceptable, the producer should continue marketing the pre-existing (benchmark) group of products. A cross-functional team composed of design, manufacturing, and marketing personnel should carefully consider the relevant market forces and cost drivers before making a decision, using the PVTEM to provide decision support for resolving this tradeoff.

4. Designing a Family of General Aviation Aircraft

Our second example is the preliminary design of a family of General Aviation Aircraft (GAA) based on three different seating configurations. The term General Aviation encompasses all flights except military operations and commercial carriers. Its potential buyers form a diverse group that include weekend and recreational

pilots, training pilots and instructors, traveling business executives and even small commercial operators. Satisfying a group with such diverse needs and economic potential poses a constant challenge for the General Aviation industry because it is impossible to satisfy all of the market needs with a single aircraft. Hence, we seek to design a family of three aircraft to accommodate 2, 4, or 6 people that can easily be adapted to satisfy distinct groups of customer demands. For this example, the configuration for the GAA is a fixed-wing, single-engine, single-pilot, propeller-driven aircraft. The family of aircraft is to be scaled around the 2, 4, and 6 seat configurations, and the challenge is to determine the best values of key parameters for the fuselage, wing, and engine to satisfy a variety of performance and economic requirements. The problem formulation is described in the next section; Section 4.2 follows with a detailed account of the product variety tradeoff within the GAA family.

4.1. General Aviation Aircraft Problem Formulation

The general configuration of each aircraft has been fixed at three propeller blades, high wing position, and retractable landing gear based on previous work (Simpson *et al.*, 1996, 1999a). The design variables and corresponding ranges of interest in this study are:

1. Cruise speed, CSPD [Mach 0.24, Mach 0.48];
2. Aspect ratio, AR [7, 11];
3. Propeller diameter, DPRP [5.0 ft, 5.96 ft];
4. Wing loading, WL [19.0 lb/ft², 25.0 lb/ft²];
5. Engine activity factor, AF [85, 110]; and
6. Seat width, SW [14.0 in, 20.0 in].

The baseline configuration and remaining parameters are derived from a Beechcraft Bonanza B36TC, a four-to-six seat, single-engine business and utility aircraft that is one of the most popular GAA sold in recent years.

For aircraft sizing and performance estimation, we utilize the GASP (General Aviation Synthesis Program (NASA, 1978)). Input variables for GASP are general descriptors of aircraft type, size, and missing requirements. The numerical output from GASP includes many aircraft design characteristics such as aircraft empty weight, fuel weight, direct operating cost, and maximum flight range. To reduce the computational expense of performing these calculations, statistical approximations (i.e., response surface models) are employed to provide simplified, yet accurate, approximations of each performance parameter as a function of the input design variables (Simpson *et al.*, 1996).

There are a total of nine responses that are of interest for each aircraft: takeoff noise, direct operating cost, ride roughness, empty weight and fuel weight, purchase price, and maximum cruise speed, flight range and

Table 9. Constraint and goal targets for the two, four, and six seat GAA

	Acronym	2 Seater	4 Seater	6 Seater
<i>Constraints</i>				
Maximum take-off noise [db]	NOISE	75	75	75
Maximum direct operating cost [\$ /h]	DOC	80	80	80
Maximum ride roughness coefficient	ROUGH	2.0	2.0	2.0
Maximum aircraft empty weight [lbs]	WEMP	2200	2200	2200
Maximum aircraft fuel weight [lbs]	WFUEL	450	475	500
Minimum flight range [nm]	RANGE	2000	2000	2000
<i>Goals</i>				
Aircraft fuel weight [lbs]	WFUEL	450	400	350
Aircraft empty weight [lbs]	WEMP	1900	1950	2000
Direct operating cost [\$ /h]	DOC	60	60	60
Purchase price [\$ (in 1970)]	PURCH	41,000	42,000	43,000
Maximum lift/drag ratio	LDMAX	17	17	17
Maximum cruise speed [kts]	VCRMX	200	200	200
Maximum flight range [nm]	RANGE	2500	2500	2500

lift/drag ratio. We seek to find values of the design variables that:

- lower direct operating cost, purchase price, empty weight and fuel weight to their targets;
- raise maximum cruise speed, flight range, and lift/drag ratio to their targets; and
- satisfy constraints on the maximum takeoff noise, ride roughness, direct operating cost, empty weight, and fuel weight, and minimum flight range.

The constraint values and target values for the goals employed in this example are listed in Table 9. As such, these constraints and targets define the performance requirements for each aircraft.

Given the constraints and goal targets listed in Table 9, the corresponding compromise DSP formulation for the GAA platform is shown in Figure 5. The compromise DSP is used to determine the values of the six design variables that constitute the common aircraft platform from which the three aircraft are derived by scaling the fuselage to accommodate 2, 4, or 6 people. The common product platform is the “best” setting of the design variables that satisfies all six constraints for each aircraft; lowers fuel weight, empty weight, direct operating cost, and purchase price to their targets (i.e., minimizes d_i^+ , $i = 1, 2, 3, 4$) for each aircraft; and

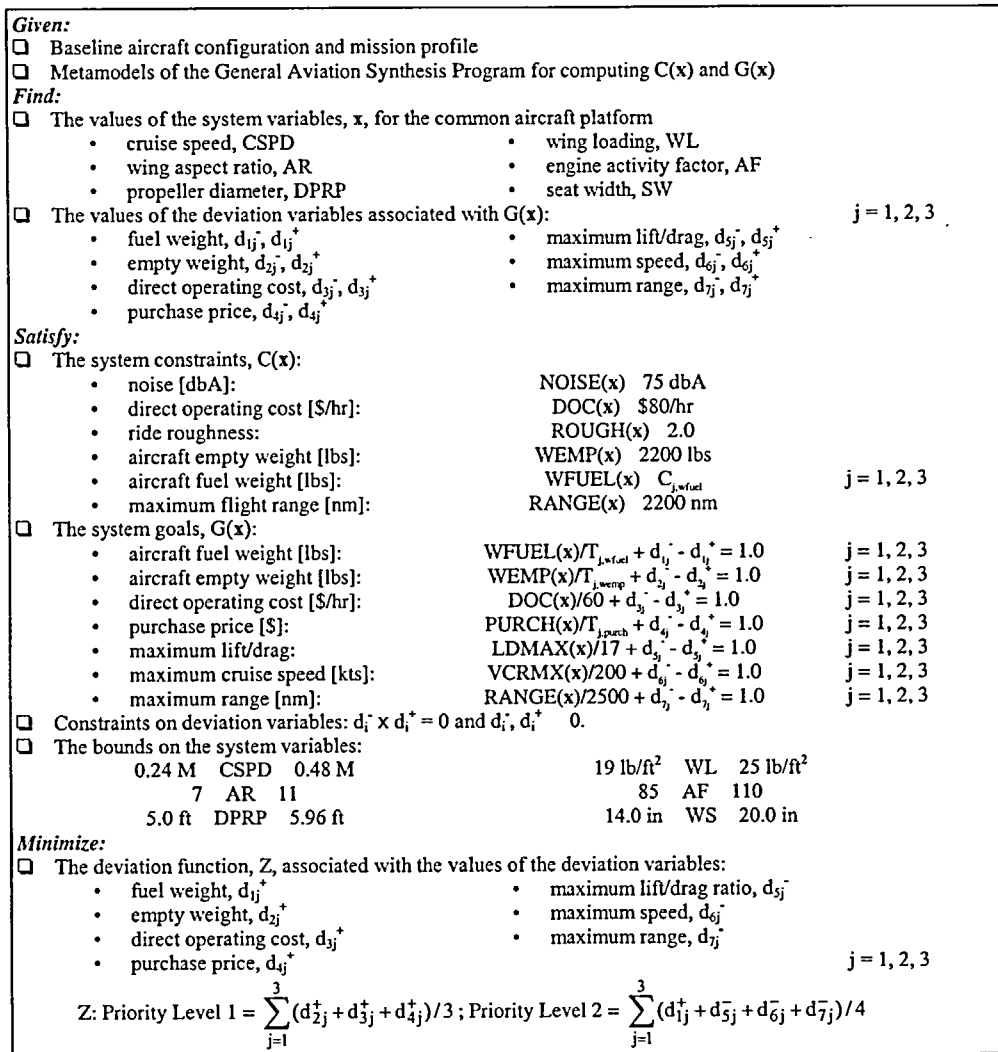


Figure 5. Compromise DSP for GAA platform.

increases maximum lift/drag, maximum cruise speed, and maximum range to their targets (i.e., minimize $d_i^-, i = 5, 6, 7$) for each aircraft. The deviation function, Z , used in this example utilizes a Preemptive formulation, consisting of two priority levels. The goals associated with the first priority level ($d_{2j}^+, d_{3j}^+, d_{4j}^+$) are economic related while the goals associated with the second priority level ($d_{1j}^+, d_{5j}^-, d_{6j}^-, d_{7j}^-$) are related to the technical performance of each aircraft. Thus, we place emphasis on first satisfying the economic targets before trying to satisfy the technical targets for each aircraft.

The compromise DSP given in Figure 5 is solved once to find the settings of the common platform variables which are good for all three aircraft. The compromise DSP is also modified and solved three times to find the best settings for each individual aircraft without any regard to commonality. The results of solving these compromise DSPs are listed in Table 10. The aircraft family is based on the common platform that has the same settings for all six design variables for all three

aircraft. The group of benchmark aircraft is designed individually by solving three compromise DSPs, one for each aircraft with no regard for commonality. The corresponding performance of each aircraft and deviation function value for each level based on goal target satisfaction for the given design variable settings are listed in the bottom half of Table 10.

As can be seen in the table, there is little variation between the design variable settings for the benchmark aircraft even though they have been individually designed. For instance, the benchmark aircraft share nearly common settings for the cruise speed and propeller diameter. The aspect ratio for the family of aircraft is contained within the range of the benchmark designs but is on the low end. The propeller diameter for the family of aircraft is slightly higher than that of the benchmark aircraft. The wing loading for the family of aircraft is about 2 lb/ft² lower than that of the benchmark designs, whose value only varies by about 0.7 lb/ft² between the three aircraft. The engine activity

Table 10. Individual aircraft in platform-based family and benchmark aircraft

	Aircraft family based on common aircraft platform			Group of individually designed benchmark aircraft		
	2 Seater	4 Seater	6 Seater	2 Seater	4 Seater	6 Seater
<i>Design Variable Value</i>						
CSPD [Mach]		0.242		0.240	0.243	0.240
AR		8.09		10.00	8.50	7.96
DPRP [ft]		5.20		5.00	5.06	5.00
WL [lb/ft ²]		22.63		24.49	24.27	24.91
AF		89.40		91.37	109.09	85.00
SW [in]		18.72		18.18	15.42	14.39
<i>Performance Responses</i>						
WFUEL [lbs]	447.25	411.34	385.69	450.04	474.07	494.81
WEMP [lbs]	1889.73	1924.56	1949.77	1891.95	1865.35	1843.77
DOC [\$ /h]	61.60	62.85	63.38	60.34	59.99	60.21
PURCH [\$]	41959.4	42502.8	42989.1	42049.5	41778.0	41106.4
LDMAX	15.91	15.63	15.55	17.11	16.23	15.85
VCRMV [kts]	192.24	189.51	189.07	193.46	196.63	194.29
RANGE [nm]	2446	2393	2377	1997	2133	2058
<i>Deviation Function Value</i>						
PLEV1 (economic goals)	0.0167	0.0198	0.0188	0.0104	0.000	0.0011
PLEV2 (performance goals)	0.0311	0.0510	0.0728	0.0585	0.0986	0.1717

factors for the benchmark aircraft vary from 85 to 109; the aircraft platform has a value of 89.4 which falls within this range. Finally, the seat widths for the benchmark designs are lower than for the aircraft platform, with the largest benchmark aircraft converging almost to the lower bound of 14 in.

All of the aircraft in Table 10 are feasible. As for performance, the responses listed in Table 10 can be directly compared against the targets listed in Table 9 to determine how well each goal has been achieved. An easier approach is to examine the deviation function values which indicate goal target satisfaction. As seen in Table 10, the first level deviation function value for the four seat benchmark design is zero, indicating that the aircraft met all of its targets. The 2 and 6 seat benchmark designs also both fare well at achieving their targets, having PLEV1 values of 0.01 and 0.001, respectively. The platform-based aircraft on the other hand, have PLEV1 values that are much higher, and the four seat design does not achieve all of its targets as did its benchmark counterpart. In an effort to improve the performance of the individual aircraft within the product family, a product variety tradeoff study is conducted in the next section.

4.2. Evaluation of GAA Platform Alternatives Using PVTEM

To evaluate a variety of GAA platform alternatives with varying levels of commonality to improve the performance of the family of aircraft, the PVTEM is used to construct a product variety tradeoff chart for families of GAA based on alternative platforms. As we

did in the first example, we proceed through the steps listed in Table 1 to demonstrate the PVTEM.

Step 1 *Describe design alternatives and provide meaningful acronyms.* The GAA platform design alternatives are listed in Table 11. In total, 33 product families are considered based on product platforms with varying levels of commonality.

Step 2 *Identify performance parameters and design variables for inclusion in the evaluation indices (PDI and NCI).*

Step 3 *Determine relevant ranges (for the design variables in the NCI) and target values (for the performance parameters in the PDI).* The performance parameters and targets are listed in the bottom half of Table 9; in order for a design to be considered, however, the constraints listed in the top half of Table 9 must be satisfied. There are six design variables of interest, the ranges of which are listed at the beginning of Section 4.1.

Step 4 *Assign weights to the design variables in the NCI and to the performance parameters in the PDI.* The important weightings for the design variables used in this example are listed in Table 12 and are based on rank ordering the design variables with regard to the relative ease and cost with which they can be allowed to vary. The more difficult or costly it is to allow a particular variable to change, the more important it is to have that variable stay the same across derivative products. Cruise speed is the least costly variable to allow to vary between designs because it is easy to vary the cruise speed throughout the mission without having to make any modifications to the aircraft; therefore, it is

given a rank of 1. Meanwhile, seat width (SW) is the most expensive to allow to vary because it is costly not to have the same fuselage width for all three aircraft since fuselage width is a direct function of seat width. Consequently, seat width is ranked the highest (6th). The remaining weights are derived from a pair-wise comparison of the design variables for the purposes of demonstration.

Step 5 Determine overall PDIs and NCIs for each product family design alternative. In total, 33 different product families are considered as discussed in Step 1.

Table 11. GAA platform design alternatives

Acronym	Description
Benchmark	The benchmark group of aircraft based on each aircraft being individually optimized.
Platform6	The family of aircraft based on having all 6 design variables common across all three aircraft.
Platform5(x_i) $i = 1, \dots, 6$	One of 6 families of aircraft based on making five variables common while allowing variable x_i to vary between aircraft, e.g., Platform 5 (AF) is the family of aircraft based on the common aircraft platform composed of all variables except AF which is allowed to vary freely for each aircraft.
Platform4(x_i, x_j) $i = 1, \dots, 6$ $j = 1, \dots, 6, j \neq i$	One of 15 families of aircraft based on making four variables common while allowing variables x_i and x_j to vary between aircraft, e.g., Platform 4 (AF, AR) is the family of aircraft based on the common aircraft platform composed of all variables except AF and AR which are allowed to vary freely for each aircraft.
Platform3(x_i, x_j, x_k) $i = 1, \dots, 5$ $j = 1, \dots, 5$ $k = 1, \dots, 5$ $k \neq j \neq i$	One of 10 families of aircraft based on making three variables common while allowing variables x_i , x_j , and x_k to vary between aircraft, e.g., Platform 3 (AF, AR, SW) is the family of aircraft based on the common aircraft platform composed of all variables except AF, AR, and SW which vary freely for each aircraft. <i>To reduce the number of possible combinations by half, cruise speed (CSPD) is taken as a common platform variable for all Platform3 designs.</i>

Table 12. Relative importance of the design variables

Design Variable	Rank Order (lower is better)	Relative Importance
Engine activity factor, AF	3	0.1429
Aspect ratio, AR	5	0.2381
Cruise speed, CSPD	1	0.0476
Propeller diameter, DPRP	2	0.0952
Wing loading, WL	4	0.1905
Seat width, SW	6	0.2857

To obtain these results, $(6 \times 3) + (15 \times 3) + (10 \times 3) = 93$ additional compromise DSPs are solved, one compromise DSP for each aircraft for each platform to find the best settings of the non-platform variables as described in Table 11. The resulting NCI and PDI values for each family are listed in Table 13. The complete results can be found in Simpson (1998) since they are too lengthy to include here.

Despite the apparently high levels of commonality seen in Table 10, the NCI value for the benchmark group of aircraft (0.1795) is relatively high when compared to the other platform alternatives shown in Table 13; however, this lack of commonality between

Table 13. NCI and PDI values for GAA example product variety tradeoff study

	NCI	PDI
Benchmark – Each aircraft is optimized; all variables can vary	0.1795	0.0038
Platform 6 – Each aircraft is designed to have same settings for all 6 design variables	0.0000	0.0184
Platform 5 (x_i) – Allow 1 variable to vary between aircraft		
AF	0.0178	0.0181
AR	0.0040	0.0181
CSPD	0.0000	0.0182
DPRP	0.0026	0.0179
WL	0.0171	0.0155
SW	0.0381	0.0117
Platform 4 (x_i, x_j) Allow 2 variables to vary between aircraft		
AF AR	0.0147	0.0181
AF CSPD	0.0178	0.0181
AF DPRP	0.0155	0.0175
AF WL	0.0234	0.0150
AF SW	0.0509	0.0113
AR CSPD	0.0041	0.0181
AR DPRP	0.0172	0.0175
AR WL	0.0230	0.0147
AR SW	0.0559	0.0096
AR DPRP	0.0027	0.0179
CSPD WL	0.0233	0.0152
CSPD SW	0.0382	0.0117
DPRP WL	0.0249	0.0154
DPRP SW	0.0528	0.0106
WL SW	0.0702	0.0086
Platform 3 (x_i, x_j, x_k) Allow 3 variables to vary between aircraft from		
AF AR DPRP	0.0110	0.0181
AF AR WL	0.0370	0.0146
AF AR SW	0.0848	0.0100
AF DPRP WL	0.0495	0.0154
AF DPRP SW	0.0672	0.0147
AF WL SW	0.1068	0.0081
AR DPRP WL	0.0405	0.0151
AR DPRP SW	0.0572	0.0107
AR WL SW	0.0803	0.0068
DPRP WL SW	0.0701	0.0075

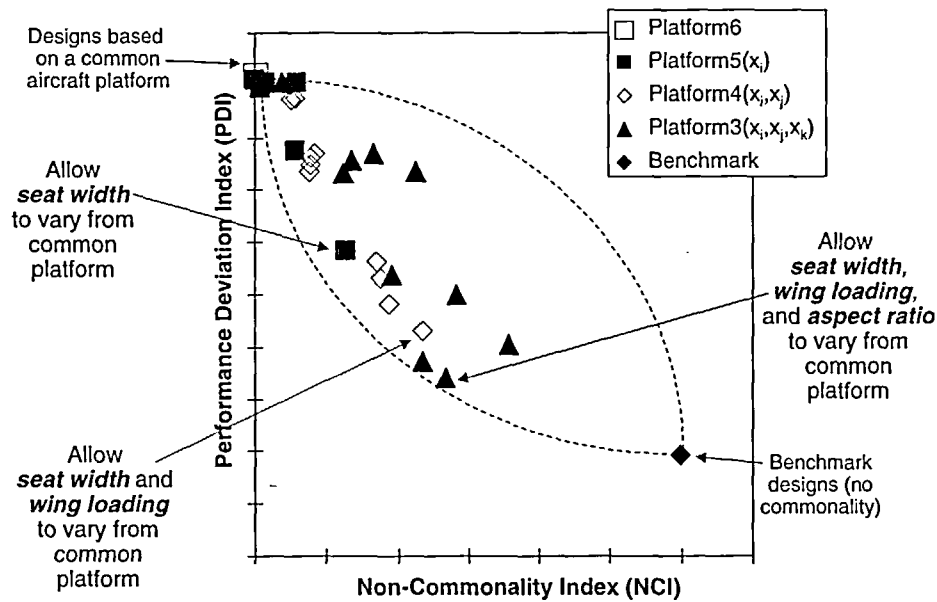


Figure 6. Product variety tradeoff chart for GAA example.

aircraft yields a remarkably low PDI value (0.0038). Meanwhile, the family of aircraft based on having all six design variables common has a PDI value nearly six times that of the benchmark group of aircraft, but its NCI value is zero because the family takes full advantage of commonality. The remaining alternatives fall between these two extremes as expected, demonstrating the tradeoff between commonality and performance. The gray shaded rows in Table 13 indicate the best increase in PDI that can be achieved by allowing 1, 2, or 3 variables to vary at a given time from the common product platform. So, if only one variable is allowed to vary between aircraft, then allowing seat width (SW) to vary yields the best improvement in performance. If two can vary, then wing loading (WL) and seat width (SW) should be allowed to vary. Finally, if three can vary, then varying aspect ratio (AR), wing loading (WL), and seat width (SW) yields the best improvement.

Step 6 Compare product family design alternatives using the product variety tradeoff chart. A plot of NCI versus PDI for the data in Table 13 is shown in Figure 6. This product variety tradeoff chart provides a visual display of the tradeoff between commonality and performance within the aircraft families derived from product platforms with varying levels of commonality. Notice that the designs based on the aircraft platform with all 6 variables common (Platform6) yields the top left point in Figure 6. Meanwhile, the group of individually designed benchmark aircraft provides the bottom right point, and the different families of aircraft, based on varying levels of platform commonality, lie between these two extremes. The families progress from left to right as the level of commonality of the product

platform is relaxed and from top to bottom, depending upon the impact of the non-platform variables. For a given level of commonality, the best product platform is the one that yields the family of products with the lowest PDI value since it offers the best compromise between commonality and performance. These platforms, noted in the figure, are the same as those highlighted in gray in Table 13.

Of all the platforms considered, the family with the lowest NCI and PDI values is Platform3 (AR, WL, SW) which allows the aspect ratio (AR), the wing loading (WL), and the seat width (SW) to vary between the three aircraft in the family while making propeller diameter, engine activity factor, and cruise speed common for all three aircraft. As in the previous example, the final decision as to the appropriate level of commonality for the three aircraft should be made by a multidisciplinary team consisting of design, manufacturing, and marketing personnel who have considered all of the market forces and cost drivers that impact this decision.

5. Concluding Remarks

The Product Variety Tradeoff Evaluation Method and its accompanying commonality and performance indices provide a means to balance commonality and performance within a product family. The NCI and PDI values for a variety of platform alternatives can be plotted in a product variety tradeoff chart to gain a visual representation of this tradeoff, enabling informed decisions to be made regarding the appropriate level of commonality within a product family. However, as mentioned in both examples it is important that a multidisciplinary, cross-functional team consisting of

design, manufacturing, and marketing personnel consider all of the market forces and cost factors that impact this decision to realize a successful product platform.

One of the weaknesses of the PVTEM brings our attention to the need for further work in metrics for product family design. The product variety tradeoff chart enables designers to compare the commonality of alternative product platform concepts and the corresponding performance of the individual products within the family derived from each platform. In this regard, the commonality index serves as a proxy for the efficiency of a product platform; however, relying on a commonality index requires several assumptions. The major assumption is that increased commonality implies greater ease and reduced cost of assembly and manufacture which may not always be the case. NCI and PDI metrics should be tailored to each problem to accommodate relevant factors such as aesthetics, cost, quality, reliability, manufacturability, and serviceability as needed. Furthermore, we must assume that the increased level of commonality does not affect a customer's perception of the product and adversely impact market sales, as was the case with Chrysler's K-car in the 1980s. Another assumption is that the underlying product platform architecture around which the product family is derived is good for the entire family. The GAA and transmission examples demonstrate how to use the PVTEM to explore commonality and performance tradeoffs in parametric variety during the preliminary and detailed stages of design. Configuration-based variety (e.g., modularity) is equally important, and NCI and PDI would need to be modified to assess module-based variety in addition to parametric variety as demonstrated in this paper. Cross-functional product design teams need to consider architectural and parametric variations when trying to determine the best product platform for a family of products.

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Nomenclature

d_i^-, d_i^+ = Deviation variables in compromise DSP

DI = Average of normalized differences

DSP = Decision Support Problem

GAA = General Aviation Aircraft

μ = Mean value

NCI = Non-Commonality Index—used to assess parametric variation within a product family

PDI = Performance Deviation Index—measures how well products within a product family meet their individual performance targets

Product family = a group of related products, which share common features, components and subsystems, and satisfy a variety of market niches.

Product platform = the set of common features, components, and subassemblies shared by products within a product family

PVTEM = Product Variety Tradeoff Evaluation Method

Scale factors = parameters used to scale (i.e., "stretch" or "shrink") the product platform in one or more dimensions to realize a family of products

Z = Deviation function

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