NAE-3SAT: formula evaluates true with not-all-equal at each clause

<u>NAE-3SAT</u> $x_i \in \{0,1\}$

$$f \equiv (x_1 \vee x_2 \vee \neg x_3) \wedge (x_3 \vee \neg x_1 \vee \neg x_6) \wedge (\neg x_2 \vee x_4 \vee x_5) \wedge (\neg x_4 \vee \neg x_5 \vee x_6)$$

$$w_i \equiv (-1)^{x_i} \qquad \mathrm{NAE}(w_1, w_2, w_3) = \frac{3}{4} - \frac{1}{4}(w_1 w_2 + w_1 w_3 + w_2 w_3)$$

$$x_1, \ldots, x_6 \to w_1, \ldots, w_6 \qquad \neg x_1, \ldots, \neg x_6 \to w_6, \ldots, w_{12}$$

$$\begin{aligned} \min_{w_i} & (w_1 w_2 + w_1 w_9 + w_2 w_9 + w_3 w_7 + w_3 w_{12} + w_7 w_{12}) \\ + & \quad (w_4 w_8 + w_5 w_8 + w_4 w_5 + w_6 w_{10} + w_6 w_{11} + w_{10} w_{11}) \\ + & \quad 10 \sum_{i=1}^{6} w_i w_{i+6} \end{aligned}$$

<u>compute the ground state of the Hamiltonian below</u>

$$\begin{aligned} H = & X_1 X_2 + X_1 X_4 + X_2 X_4 + Y_1 Z_3 + Y_1 Z_4 + Z_3 Z_4 \\ + & \quad Z_1 Y_2 + Z_1 X_3 + Y_2 X_3 + Y_3 Y_4 + Z_2 Y_3 + Z_2 Y_4 \\ + & \quad 10\,(X_1 Z_3 + Z_1 X_2 + Y_1 X_4 + Y_2 Y_4 + Z_2 X_3 + Y_3 Z_4) \end{aligned}$$

The ground state is close to

$$\begin{aligned} \rho(\mathbf{w}) = & \frac{1}{2}\left(I + \frac{(-1)^{w_1}}{\sqrt{3}} X + \frac{(-1)^{w_3}}{\sqrt{3}} Y + \frac{(-1)^{w_8}}{\sqrt{3}} Z\right) \\ & \otimes \frac{1}{2}\left(I + \frac{(-1)^{w_2}}{\sqrt{3}} X + \frac{(-1)^{w_4}}{\sqrt{3}} Y + \frac{(-1)^{w_{11}}}{\sqrt{3}} Z\right) \\ & \otimes \frac{1}{2}\left(I + \frac{(-1)^{w_5}}{\sqrt{3}} X + \frac{(-1)^{w_6}}{\sqrt{3}} Y + \frac{(-1)^{w_7}}{\sqrt{3}} Z\right) \\ & \otimes \frac{1}{2}\left(I + \frac{(-1)^{w_9}}{\sqrt{3}} X + \frac{(-1)^{w_{10}}}{\sqrt{3}} Y + \frac{(-1)^{w_{12}}}{\sqrt{3}} Z\right) \end{aligned}$$

<u>Steps to find satisfying assignment:</u>

1. Compute ground state
2. Compute reduced-density-matrix of every qubit pair and compare its distance to the product states of (3,1)-QRAC