LESSON TITLE

Prepared By

Faculty Name

GAUSSIAN ELIMINATION AND GAUSS JORDAN ELIMINATION

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COURSE OBJECTIVES

After completing this lesson, students must be able to:

- Determine the size of a matrix and write an augmented or
- coefficient matrix from a system of linear equations.
- Use matrices and Gaussian elimination with back-substitution to solve a system of linear equations.
- Use matrices and Gauss-Jordan elimination to solve a system of linear equations.



Matrices index in your subscript

Definition of a Matrix

If m and n are positive integers, an $m \times n$ (read "m by n") matrix is a rectangular array

	Column 1	Column 2	Column 3	 Column <i>n</i>
Row	$ \begin{array}{c c} 1 & a_{11} \\ 2 & a_{21} \\ 3 & a_{31} \\ \vdots \\ m & a_{m1} \end{array} $	a_{12}	a_{13}	 a_{1n}
Row	a_{21}	a_{22}	a_{23}	 a_{1n} a_{2n} a_{3n} \vdots a_{mn}
Row	a_{31}	a_{32}	a_{33}	 a_{3n}
•	•	•	•	
:		:	:	:
Row	$m \mid a_{m1}$	a_{m2}	a_{m3}	 a_{mn}

in which each **entry**, a_{ij} , of the matrix is a number. An $m \times n$ matrix has m rows and *n* columns. Matrices are usually denoted by capital letters.



Representation of Matrices

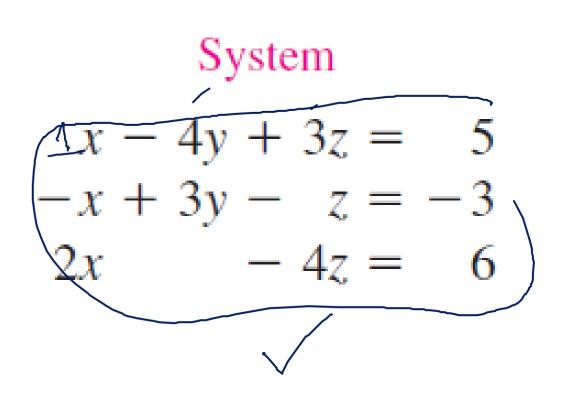
- **1.** An uppercase letter such as A, B, or C
- 2. A representative element enclosed in brackets, such as $[a_{ij}]$, $[b_{ij}]$, or $[c_{ij}]$
- 3. A rectangular array of numbers

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\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}
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SIZE OF MATRICES

A matrix with m rows and n columns is said to be of size $m \times n$.

REPRESENTATION OF SYSTEMS OF LINEAR EQUATIONS



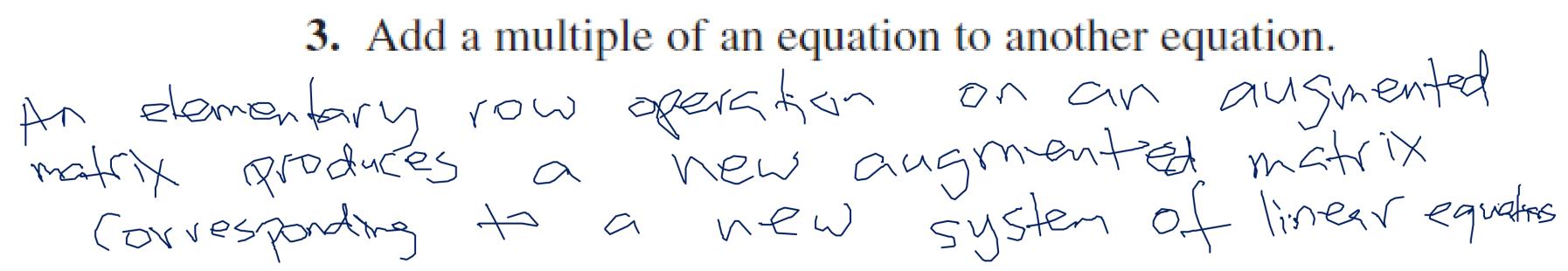
Augmented Matrix

Coefficient Matrix

$$\begin{bmatrix} 1 & -4 & 3 \\ -1 & 3 & -1 \\ 2 & 0 & -4 \end{bmatrix}$$

ELEMENTARY ROW OPERATIONS

- 1. Interchange two equations.
- 2. Multiply an equation by a nonzero constant.





Gaussian Elimination with Back-Substitution

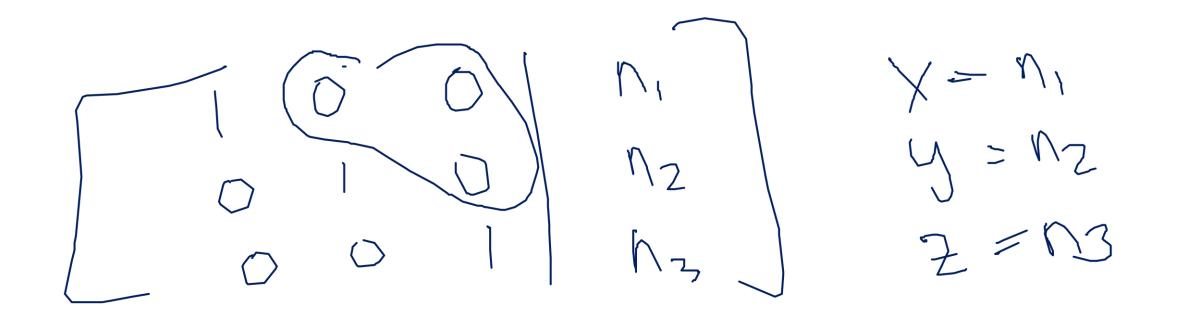
- 1. Write the augmented matrix of the system of linear equations.
- 2. Use elementary row operations to rewrite the matrix in row-echelon form.
- 3. Write the system of linear equations corresponding to the matrix in row-echelon form, and use back-substitution to find the solution.



 $\frac{2}{3} \frac{3}{0} \frac{3}{-4} \frac{7}{17} \frac{7$



1 -2 3 9 0 1 3 2 (ow-echolon for x-24) x - 2y + 3 = 5 y + 3z = 2









Row-Echelon Form and Reduced Row-Echelon Form

A matrix in row-echelon form has the following properties.

- 1. Any rows consisting entirely of zeros occur at the bottom of the matrix.
- 2. For each row that does not consist entirely of zeros, the first nonzero entry is 1 (called a leading 1).
- 3. For two successive (nonzero) rows, the leading 1 in the higher row is farther to the left than the leading 1 in the lower row.

A matrix in row-echelon form is in reduced row-echelon form when every column that has a leading 1 has zeros in every position above and below its leading 1.



Determine whether each matrix is in row-echelon form. If it is, determine whether the matrix is in reduced row-echelon form.

a.
$$\begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

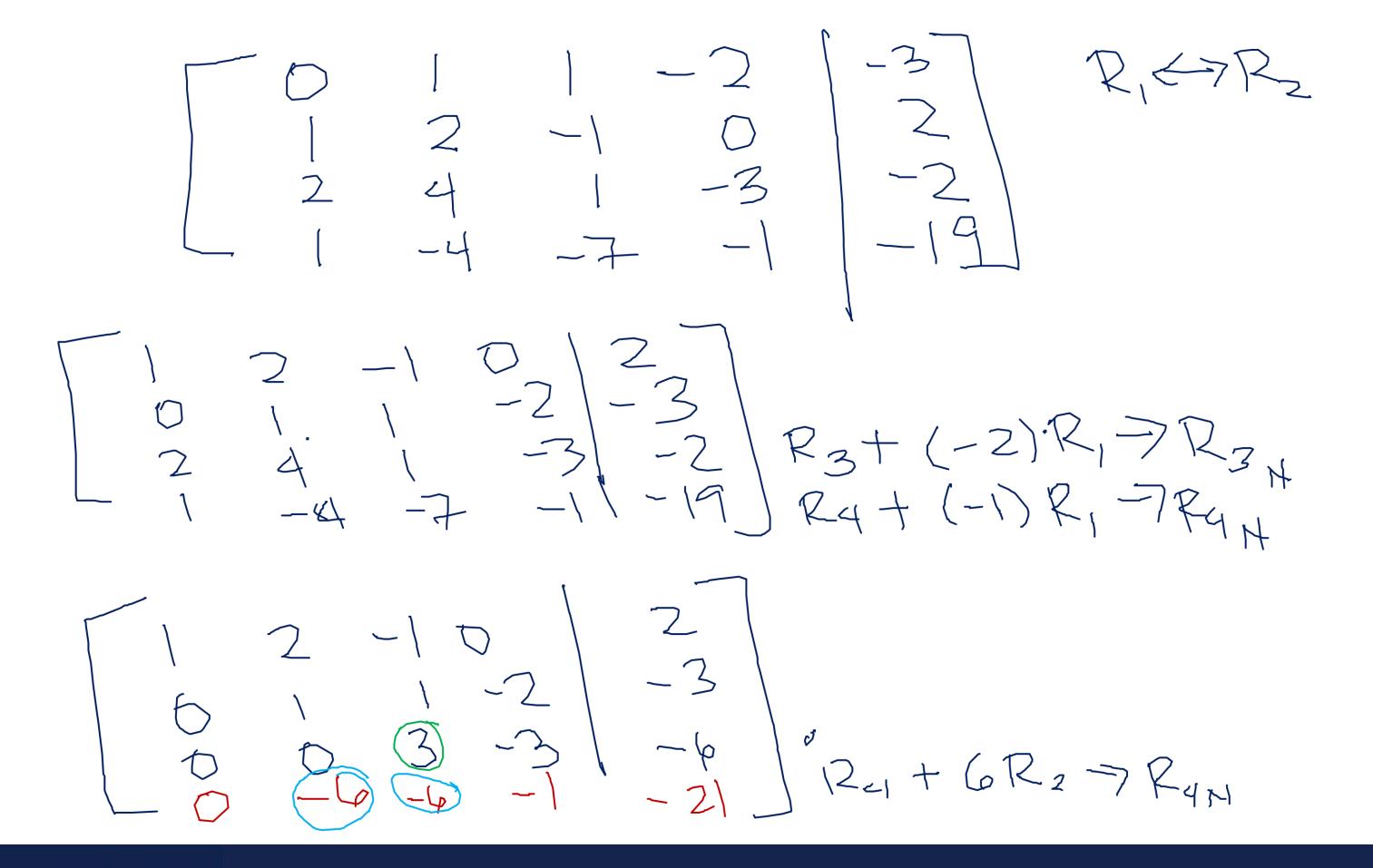
b.
$$\begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -4 \end{bmatrix} \times$$

c.
$$\begin{bmatrix} 1 & -5 & 2 & -1 & 3 \\ 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

e.
$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 1 & -3 \end{bmatrix} \times \qquad \qquad \text{f.} \begin{bmatrix} 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix}
0 & 1 & 0 & 5 \\
0 & 0 & 1 & 3 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

91, 92, 93, 94 X1, X2, X3, X4 X, Y, Z





$$\begin{bmatrix} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 0 & 0 & 3 & -3 & -4 \\ 0 & 0 & -4 & -4 & -1/-21 \end{bmatrix} R_{4} + (6R_{2} - 7R_{4}H)$$

$$\begin{bmatrix} 1 & 2 & -1 & 0 & | & 2 \\ 0 & 0 & -1/2 & | & -2/2 \\ 0 & 0 & 0 & -1/2 & | & -3/2 \\ 0 & 0 & 0 & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0 & 0 & | & -1/2 & | & -3/2 \\ 0 & 0$$





References:

The main references of this course are the following:

a. Anton, H., Rorres, C. (2010). Elementary Linear Algebra: Applications Version. United Kingdom: Wiley.

b. Anton. (2013). Elementary Linear Algebra: Applications Version, Tenth Edition Wiley E-Text Reg Card. (n.p.): John Wiley & Sons, Incorporated.

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e. Hill, R. O. (2014). Elementary Linear Algebra. United Kingdom: Elsevier Science.

